Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/19-1.1.2.2-c-x-^m-a+b-x^2-^p

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Contents

1 Introduction 3

2 detailed summary tables of results 19

3 Listing of integrals 279

4 Appendix 4385

Chapter 1

Introduction

Local contents

1.1 Listing of CAS systems tested 4
1.2 Results 5
1.3 Time and leaf size Performance 9
1.4 list of integrals that has no closed form antiderivative 11
1.5 List of integrals solved by CAS but has no known antiderivative 12
1.6 list of integrals solved by CAS but failed verification 13
1.7 Timing 13
1.8 Verification 14
1.9 Important notes about some of the results 14
1.10 Design of the test system 17

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1071]. This is test number [19].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1071)	0.00 (0)
Mathematica	100.00 (1071)	0.00 (0)
Sympy	95.52 (1023)	4.48 (48)
Maple	71.62 (767)	28.38 (304)
Fricas	68.63 (735)	31.37 (336)
Mupad	64.89 (695)	35.11 (376)
Maxima	59.01 (632)	40.99 (439)
Giac	57.52 (616)	42.48 (455)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

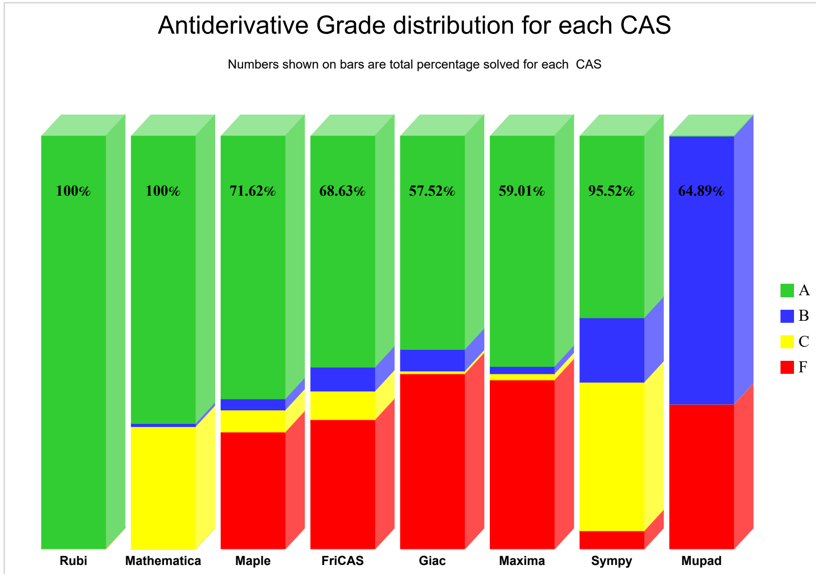
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

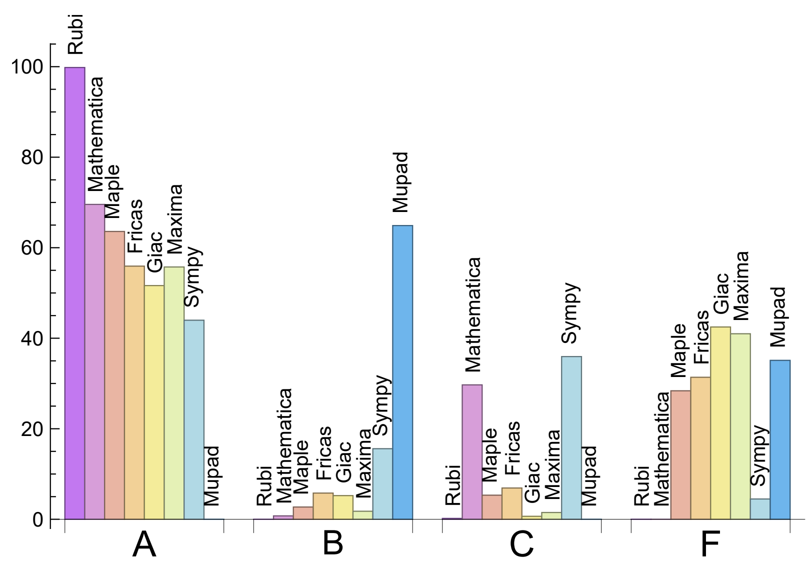
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.81	0.00	0.19	0.00
Mathematica	69.56	0.75	29.69	0.00
Maple	63.59	2.71	5.32	28.38
Fricas	55.93	5.79	6.91	31.37
Maxima	55.74	1.77	1.49	40.99
Giac	51.63	5.23	0.65	42.48
Sympy	43.98	15.59	35.95	4.48
Mupad	N/A	64.89	0.00	35.11

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	304	100.00 %	0.00 %	0.00 %
Fricas	336	93.15 %	6.85 %	0.00 %
Giac	455	100.00 %	0.00 %	0.00 %
Maxima	439	98.86 %	0.00 %	1.14 %
Sympy	48	0.00 %	56.25 %	43.75 %
Mupad	376	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

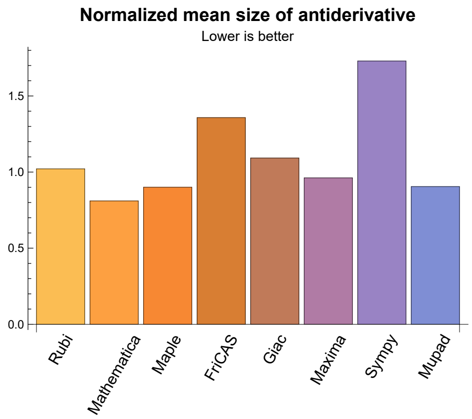
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

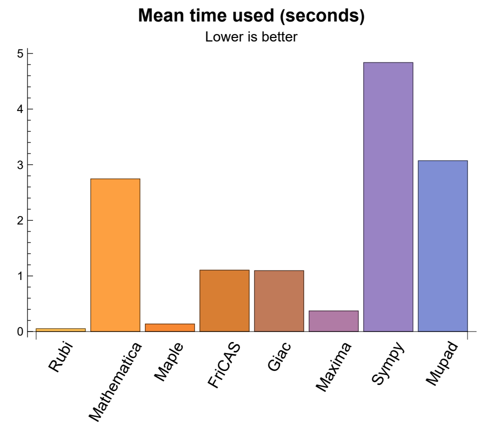
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.05	112.03	1.02	78.00	1.00
Mathematica	2.75	61.14	0.81	54.00	0.81
Maple	0.14	68.61	0.90	49.00	0.84
Maxima	0.37	69.75	0.96	55.00	0.88
Fricas	1.10	104.68	1.36	65.00	1.03
Sympy	4.84	129.57	1.73	46.00	0.94
Giac	1.10	74.16	1.09	56.50	0.88
Mupad	3.07	60.01	0.90	41.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

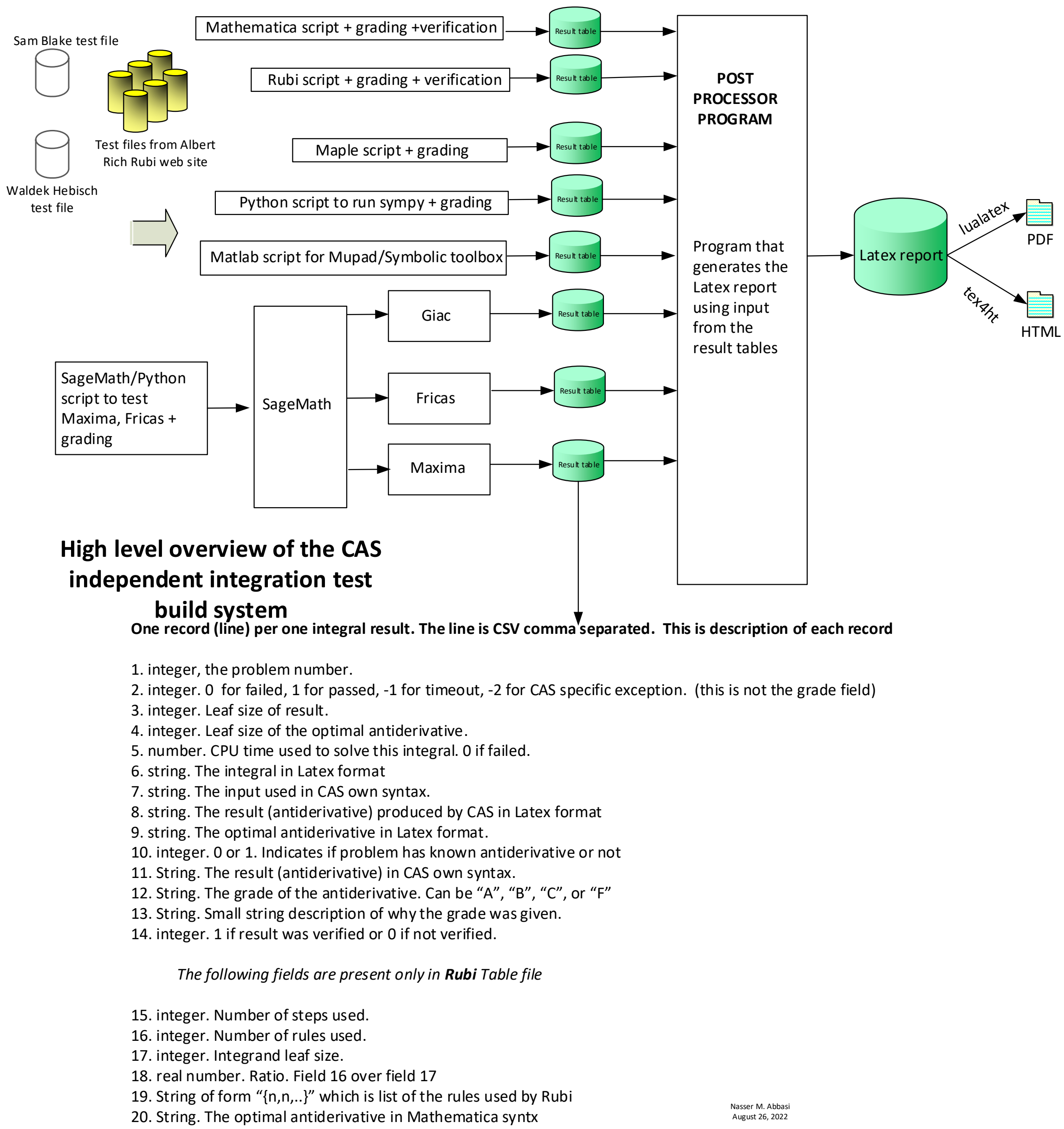
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS 20
2.2 Detailed conclusion table per each integral for all CAS systems 32
2.3 Detailed conclusion table specific for Rubi results 247

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi 21
2.1.2 Mathematica 22
2.1.3 Maple 23
2.1.4 Maxima 25
2.1.5 FriCAS 26
2.1.6 Sympy 27
2.1.7 Giac 29
2.1.8 Mupad 30

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071 }

B grade: { }

C grade: { 662, 664 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 560, 561, 562, 563, 564, 565, 567, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 694, 695, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 738, 739, 740, 741, 742, 743, 744, 745, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 784, 785, 786, 884, 885, 886, 887, 925, 926, 927, 928, 929, 930, 931, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 955, 956, 957, 958, 959, 970, 971, 972, 973, 974, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071 }

B grade: { 38, 65, 90, 101, 102, 196, 197, 559 }

C grade: { 338, 475, 477, 479, 481, 548, 566, 568, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 661, 662, 663, 664, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 696, 697, 698, 699, 700, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 746, 747, 748, 749, 750, 762, 763, 764, 765, 766, 767, 779, 780, 781, 782, 783, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 932, 933, 934, 935, 936, 949, 950, 951, 952, 953, 954, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 975, 976, 977, 978, 979, 991, 992, 993, 994, 995, 996, 997, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1061, 1063 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 341, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 608, 610, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 642, 644, 646, 648, 650, 661, 663, 665, 666, 667, 668, 677, 678, 679, 680, 689, 690, 691, 692, 701, 702, 703, 704, 705, 714, 715, 716, 717, 726, 727, 728, 729, 742, 743, 744, 745, 759, 760, 761, 775, 776, 777, 778, 928, 929, 930, 931, 940, 941, 942, 943, 946, 947, 948, 957, 958, 959, 972, 973, 974, 982, 983, 984, 987, 988, 989, 990, 1039, 1040, 1041, 1042, 1061, 1063, 1065 }

B grade: { 38, 65, 90, 101, 102, 196, 197, 198, 337, 339, 340, 382, 402, 422, 433, 510, 521, 523, 607, 609, 611, 637, 639, 640, 641, 643, 645, 647, 649 }

C grade: { 352, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918 }

F grade: { 344, 345, 346, 347, 348, 349, 350, 351, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 662, 664, 669, 670, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 685, 686, 687, 688, 693, 694, 695, 696, 697, 698, 699, 700, 706, 707, 708, 709, 710, 711, 712, 713, 718, 719, 720, 721, 722, 723, 724, 725, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 944, 945, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 975, 976, 977, 978, 979, 980, 981, 985, 986, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 518, 520, 522, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 569, 572, 574, 576, 577, 578, 580, 581, 582, 584, 585, 586, 588, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 694, 695, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 742, 745, 775, 778, 1039, 1040, 1041, 1042, 1061, 1063, 1065 }

B grade: { 38, 65, 90, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 312, 337, 517, 519, 521, 523 }

C grade: { 475, 477, 479, 480, 481, 482, 484, 566, 568, 570, 571, 573, 575, 579, 583, 587 }

F grade: { 257, 258, 261, 262, 263, 344, 345, 346, 347, 348, 349, 350, 351, 352, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 696, 697, 698, 699, 700, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 738, 739, 740, 741, 743, 744, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 525, 527, 529, 530, 531, 532, 533, 534, 535, 536, 539, 540, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 569, 572, 574, 578, 579, 581, 582, 583, 584, 585, 586, 587, 588, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 689, 690, 691, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 720, 726, 727, 728, 729, 730, 732, 742, 743, 744, 745, 759, 760, 761, 775, 776, 777, 778, 928, 929, 930, 931, 940, 941, 942, 943, 946, 947, 948, 955, 956, 957, 958, 959, 972, 973, 974, 982, 983, 984, 987, 988, 989, 990, 1039, 1040, 1041, 1042, 1061, 1063, 1065 }

B grade: { 33, 38, 58, 65, 90, 91, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 221, 248, 250, 252, 263, 312, 335, 337, 338, 339, 340, 341, 372, 383, 390, 403, 415, 416, 434, 435, 510, 511, 521, 524, 526, 528, 537, 538, 548, 576, 577, 580, 692, 701, 718, 719, 731, 944, 945, 985, 986 }

C grade: { 475, 477, 479, 480, 481, 482, 484, 566, 568, 570, 571, 573, 575, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 650 }

F grade: { 344, 345, 346, 347, 348, 349, 350, 351, 352, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 696, 697, 698, 699, 700, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 949, 950, 951, 952, 953, 954, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 975, 976, 977, 978, 979, 980, 981, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 136, 138, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 231, 232, 233, 234, 235, 236, 237, 238, 242, 243, 244, 245, 247, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 302, 312, 313, 314, 315, 316, 317, 318, 319, 320, 338, 353, 354, 357, 358, 359, 360, 361, 362, 363, 364, 366, 369, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 417, 418, 419, 420, 421, 422, 423, 427, 428, 429, 430, 431, 432, 433, 441, 442, 443, 444, 446, 447, 448, 449, 451, 452, 454, 457, 459, 463, 465, 468, 470, 474, 476, 479, 481, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 554, 555, 556, 557, 558, 559, 565, 566, 567, 568, 569, 570, 576, 580, 584, 608, 610, 612, 638, 640, 642, 644, 646, 648, 672, 673, 674, 675, 676, 679, 684, 685, 686, 687, 688, 689, 696, 697, 698, 699, 700, 705, 709, 710, 711, 712, 713, 717, 721, 722, 723, 724, 725, 728, 729, 733, 734, 735, 736, 737, 775, 776, 946, 947, 972, 973, 987, 988, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038 }

B grade: { 17, 33, 38, 57, 58, 65, 89, 90, 91, 101, 102, 131, 133, 135, 137, 139, 141, 156, 158, 174, 186, 196, 197, 198, 199, 201, 202, 203, 204, 228, 230, 239, 240, 241, 246, 248, 257, 258, 261, 262, 263, 299, 301, 306, 307, 308, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 355, 356, 365, 367, 368, 370, 371, 372, 383, 384, 385, 386, 387, 388, 389, 390, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 434, 435, 436, 437, 438, 439, 440, 445, 450, 456, 467, 478, 502, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 649, 665, 666, 667, 668, 677, 678, 680, 690, 691, 692, 701, 702, 703, 704, 714, 715, 716, 726, 727, 742, 928, 929, 974, 989, 1039, 1040, 1041, 1042 }

C grade: { 344, 345, 346, 347, 348, 349, 350, 351, 352, 453, 455, 458, 460, 461, 462, 464, 466, 469, 471, 472, 473, 475, 477, 480, 482, 483, 484, 549, 550, 551, 552, 553, 560, 561, 562, 563, 564, 571, 572, 573, 574, 575, 577, 578, 579, 581, 582, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 609, 611, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 641, 643, 645, 647, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 669, 670, 671, 681, 682, 683, 693, 694, 695, 706, 707, 708, 718, 719, 720, 730, 731, 732, 739, 740, 741, 746, 747, 748, 749, 751, 752, 753, 755, 756, 757, 758, 763, 764, 765, 768, 769, 770, 773, 774, 779, 780, 781, 782, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 944, 945, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 960, 961, 962, 963, 964, 965, 966, 967, 968, 970, 971, 975, 976, 977, 978, 980, 981, 982, 983, 984, 985, 986, 992, 993, 994, 995, 996, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1052, 1053, 1054, 1055, 1058, 1059, 1067, 1068, 1069, 1070, 1071 }

F grade: { 303, 304, 305, 309, 310, 311, 424, 425, 426, 738, 743, 744, 745, 750, 754, 759, 760, 761, 762, 766, 767, 771, 772, 777, 778, 783, 930, 931, 942, 943, 948, 959, 969, 979, 990, 991, 997, 1050, 1051, 1056, 1057, 1060, 1061, 1062, 1063, 1064, 1065, 1066 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 441, 442, 443, 444, 445, 446, 447, 448, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 480, 482, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 539, 540, 541, 542, 543, 544, 545, 546, 547, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 569, 571, 573, 578, 582, 584, 586, 588, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 694, 695, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 1040, 1041, 1042 }

B grade: { 38, 65, 90, 101, 102, 196, 197, 312, 337, 339, 340, 341, 342, 365, 366, 367, 368, 382, 383, 384, 385, 386, 402, 403, 404, 405, 406, 407, 408, 432, 433, 434, 435, 436, 437, 438, 439, 440, 450, 461, 472, 537, 538, 548, 549, 550, 552, 576, 577, 579, 580, 581, 583, 585, 587, 1039 }

C grade: { 474, 476, 478, 484, 565, 567, 575 }

F grade: { 344, 345, 346, 347, 348, 349, 350, 351, 352, 475, 477, 479, 481, 483, 566, 568, 570, 572, 574, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 696, 697, 698, 699, 700, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 399, 400, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 428, 429, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 465, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 479, 480, 481, 482, 483, 484, 485, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 504, 505, 507, 509, 510, 511, 512, 513, 514, 515, 516, 518, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 661, 663, 665, 666, 667, 668, 669, 670, 671, 674, 675, 677, 678, 679, 680, 681, 682, 683, 686, 687, 689, 690, 691, 692, 693, 694, 695, 698, 699, 701, 702, 703, 704, 705, 706, 707, 708, 711, 712, 714, 715, 716, 717, 718, 719, 720, 723, 724, 726, 727, 728, 729, 730, 731, 732, 735, 736, 789, 790, 795, 796, 801, 802, 807, 808, 811, 812, 813, 814, 818, 819, 825, 826, 832, 833, 839, 840, 846, 847, 853, 854, 857, 858, 859, 860, 861, 862, 866, 867, 873, 874, 880, 881, 887, 888, 894, 895, 901, 902, 908, 909, 915, 916, 928, 929, 930, 931, 940, 941, 942, 943, 946, 947, 948, 957, 958, 959, 972, 973, 974, 982, 983, 984, 987, 988, 989, 990, 1013, 1014, 1021, 1022, 1028, 1029, 1035, 1036, 1039, 1040, 1041, 1042, 1048, 1049, 1061, 1063, 1065 }

C grade: { }

F grade: { 344, 345, 346, 347, 348, 349, 350, 351, 352, 361, 362, 378, 379, 382, 397, 398, 401, 402, 425, 426, 427, 430, 431, 432, 433, 464, 466, 475, 477, 486, 497, 506, 508, 517, 519, 555, 566, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 662, 664, 672, 673, 676, 684, 685, 688, 696, 697, 700, 709, 710, 713, 721, 722, 725, 733, 734, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 791, 792, 793, 794, 797, 798, 799, 800, 803, 804, 805, 806, 809, 810, 815, 816, 817, 820, 821, 822, 823, 824, 827, 828, 829, 830, 831, 834, 835, 836, 837, 838, 841, 842, 843, 844, 845, 848, 849, 850, 851, 852, 855, 856, 863, 864, 865, 868, 869, 870, 871, 872, 875, 876, 877, 878, 879, 882, 883, 884, 885, 886, 889, 890, 891, 892, 893, 896, 897, 898, 899, 900, 903, 904, 905, 906, 907, 910, 911, 912, 913, 914, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 944, 945, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 975, 976, 977, 978, 979, 980, 981, 985, 986, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1015, 1016, 1017, 1018, 1019, 1020, 1023, 1024, 1025, 1026, 1027, 1030, 1031, 1032, 1033, 1034, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.002	0.058	0.329	0.941	0.005	1.449	0.029

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.001	0.020	0.275	0.993	0.005	1.649	0.021

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.001	0.028	0.272	1.313	0.005	1.349	0.020

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	14	13	12	13	13
N.S.	1	1.00	1.00	0.88	0.82	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.001	0.021	0.272	1.117	0.006	1.626	0.021

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.001	0.000	0.003	0.278	1.224	0.005	1.717	0.016

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	14	11
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85
time (sec)	N/A	0.002	0.001	0.012	0.309	1.829	0.019	2.094	0.021

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	13	5	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.00
time (sec)	N/A	0.003	0.001	0.006	0.287	1.369	0.016	1.734	0.023

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	11
N.S.	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	0.85
time (sec)	N/A	0.004	0.002	0.008	0.292	1.674	0.029	1.334	4.925

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.003	0.002	0.016	0.288	0.955	0.034	1.581	0.026

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.004	0.002	0.008	0.285	2.823	0.036	0.964	0.024

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.003	0.002	0.009	0.276	1.431	0.041	0.972	0.027

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.003	0.002	0.008	0.282	1.186	0.042	2.319	0.026

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.013	0.001	0.038	0.284	1.627	0.007	1.927	0.037

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.007	0.001	0.029	0.288	0.758	0.007	1.870	0.031

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.012	0.001	0.023	0.274	1.053	0.007	1.071	0.031

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.007	0.001	0.030	0.276	1.453	0.006	1.213	0.034

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	24	24	14	24
N.S.	1	1.00	1.00	0.94	0.88	1.50	1.50	0.88	1.50
time (sec)	N/A	0.002	0.002	0.020	0.265	1.268	0.007	1.089	0.031

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.005	0.001	0.008	0.275	1.463	0.006	0.930	0.029

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	21	20	24	21
N.S.	1	1.00	1.00	0.96	1.04	0.91	0.87	1.04	0.91
time (sec)	N/A	0.009	0.001	0.017	0.282	1.471	0.023	1.111	0.033

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.006	0.000	0.012	0.281	1.041	0.021	1.141	0.032

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	24	27	24	32	23
N.S.	1	1.00	1.00	0.89	0.89	1.00	0.89	1.19	0.85
time (sec)	N/A	0.009	0.001	0.014	0.305	1.038	0.038	1.374	4.937

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	24
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.04
time (sec)	N/A	0.007	0.001	0.013	0.286	1.045	0.043	1.447	0.026

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	26	28	24	34	24
N.S.	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00
time (sec)	N/A	0.009	0.001	0.014	0.291	0.904	0.059	1.345	0.043

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	25
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89
time (sec)	N/A	0.007	0.001	0.013	0.278	1.055	0.061	1.162	0.034

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26
N.S.	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37
time (sec)	N/A	0.002	0.001	0.013	0.283	1.217	0.066	0.703	0.034

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.007	0.001	0.013	0.292	0.889	0.071	1.395	0.036

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.010	0.001	0.014	0.286	1.245	0.076	2.698	0.035

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.007	0.001	0.014	0.277	1.025	0.080	1.081	0.035

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.019	0.002	0.034	0.294	1.083	0.009	1.589	0.042

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.018	0.002	0.031	0.305	1.194	0.008	1.442	0.040

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.017	0.002	0.029	0.270	1.323	0.008	0.994	0.044

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	36	35	35	37	35	35
N.S.	1	1.00	1.26	1.06	1.03	1.03	1.09	1.03	1.03
time (sec)	N/A	0.022	0.002	0.022	0.279	1.121	0.009	0.668	0.043

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	35	37	14	35
N.S.	1	1.00	1.00	0.94	0.88	2.19	2.31	0.88	2.19
time (sec)	N/A	0.002	0.001	0.020	0.284	0.886	0.009	0.630	0.057

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	36	33	37	36	33
N.S.	1	1.00	1.00	0.87	0.92	0.85	0.95	0.92	0.85
time (sec)	N/A	0.013	0.003	0.013	0.290	0.695	0.026	0.559	0.036

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	36	38	37	46	34
N.S.	1	1.00	1.00	0.88	0.90	0.95	0.92	1.15	0.85
time (sec)	N/A	0.015	0.005	0.022	0.290	0.692	0.043	0.468	0.036

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	37	39	37	46	37
N.S.	1	1.00	1.00	0.88	0.92	0.98	0.92	1.15	0.92
time (sec)	N/A	0.014	0.003	0.015	0.293	0.947	0.067	0.511	4.902

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	39	39	37	47	36
N.S.	1	1.00	1.00	0.87	1.00	1.00	0.95	1.21	0.92
time (sec)	N/A	0.013	0.003	0.014	0.285	0.684	0.092	0.500	0.045

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	35	35	37	35	37
N.S.	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95
time (sec)	N/A	0.002	0.005	0.013	0.331	0.785	0.104	0.576	0.027

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	37	37	39	37	37
N.S.	1	1.00	1.08	0.90	0.92	0.92	0.98	0.92	0.92
time (sec)	N/A	0.013	0.003	0.015	0.285	1.055	0.115	0.552	0.057

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.014	0.003	0.016	0.286	1.201	0.126	0.541	0.047

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.014	0.006	0.016	0.264	0.942	0.137	0.556	0.031

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.010	0.002	0.036	0.286	0.738	0.008	0.561	0.041

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.010	0.002	0.030	0.282	0.825	0.008	0.522	0.040

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.009	0.002	0.031	0.313	0.757	0.008	0.541	0.041

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.008	0.001	0.009	0.274	0.803	0.007	0.466	0.038

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	29	32	32
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	0.94
time (sec)	N/A	0.009	0.003	0.013	0.267	1.585	0.025	0.658	0.041

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	36	36	34	36
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97
time (sec)	N/A	0.009	0.003	0.013	0.288	0.920	0.047	0.550	4.802

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	33	37	34	33	34
N.S.	1	1.00	1.00	0.97	0.97	1.09	1.00	0.97	1.00
time (sec)	N/A	0.010	0.004	0.014	0.294	0.791	0.071	0.629	0.029

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	37	37	39	37	35
N.S.	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.90
time (sec)	N/A	0.010	0.003	0.015	0.299	1.102	0.094	0.641	0.027

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.011	0.003	0.014	0.279	0.941	0.105	0.555	0.032

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.010	0.005	0.015	0.271	1.161	0.117	0.590	0.029

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83
time (sec)	N/A	0.033	0.002	0.035	0.327	0.750	0.010	0.565	0.026

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83
time (sec)	N/A	0.032	0.002	0.033	0.281	0.850	0.011	0.545	0.023

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.030	0.002	0.035	0.342	0.677	0.011	0.491	0.024

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	58	57	57	65	57	57
N.S.	1	1.00	0.96	0.81	0.79	0.79	0.90	0.79	0.79
time (sec)	N/A	0.063	0.002	0.035	0.274	0.992	0.012	0.493	0.024

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	57	56	56	63	56	56
N.S.	1	1.00	1.25	1.08	1.06	1.06	1.19	1.06	1.06
time (sec)	N/A	0.045	0.002	0.031	0.272	0.737	0.012	0.509	0.024

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	66	57	56	56	63	56	56
N.S.	1	1.00	1.94	1.68	1.65	1.65	1.85	1.65	1.65
time (sec)	N/A	0.025	0.002	0.031	0.276	0.937	0.012	0.533	0.025

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	57	65	14	57
N.S.	1	1.00	1.00	0.94	0.88	3.56	4.06	0.88	3.56
time (sec)	N/A	0.002	0.002	0.022	0.299	0.876	0.012	0.495	0.023

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	58	55	65	58	55
N.S.	1	1.00	1.00	0.86	0.89	0.85	1.00	0.89	0.85
time (sec)	N/A	0.023	0.003	0.016	0.293	1.205	0.036	0.463	0.028

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	58	61	63	68	56
N.S.	1	1.00	1.00	0.89	0.91	0.95	0.98	1.06	0.88
time (sec)	N/A	0.024	0.004	0.017	0.288	1.528	0.052	0.448	0.030

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	59	61	63	70	59
N.S.	1	1.00	1.00	0.89	0.92	0.95	0.98	1.09	0.92
time (sec)	N/A	0.025	0.005	0.017	0.284	1.652	0.078	0.458	0.031

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	61	61	65	72	59
N.S.	1	1.00	1.00	0.89	0.95	0.95	1.02	1.12	0.92
time (sec)	N/A	0.024	0.004	0.026	0.276	1.030	0.110	0.466	0.040

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	61	61	63	70	59
N.S.	1	1.00	1.00	0.89	0.95	0.95	0.98	1.09	0.92
time (sec)	N/A	0.023	0.004	0.018	0.272	0.750	0.146	0.540	0.042

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	61	61	61	69	58
N.S.	1	1.00	1.00	0.86	0.94	0.94	0.94	1.06	0.89
time (sec)	N/A	0.023	0.003	0.017	0.307	1.007	0.184	0.479	4.772

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	69	58	57	57	61	57	59
N.S.	1	1.00	3.63	3.05	3.00	3.00	3.21	3.00	3.11
time (sec)	N/A	0.002	0.003	0.027	0.276	2.063	0.197	0.514	4.751

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	67	58	59	59	63	59	58
N.S.	1	1.00	1.68	1.45	1.48	1.48	1.58	1.48	1.45
time (sec)	N/A	0.012	0.005	0.029	0.274	1.457	0.213	0.483	4.736

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	58	59	59	63	59	58
N.S.	1	1.00	1.08	0.94	0.95	0.95	1.02	0.95	0.94
time (sec)	N/A	0.020	0.003	0.031	0.279	1.227	0.229	0.507	0.041

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.022	0.003	0.033	0.283	0.711	0.246	0.493	0.040

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.021	0.003	0.034	0.284	0.878	0.261	0.485	0.041

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.017	0.002	0.059	0.284	0.978	0.010	0.507	0.024

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83
time (sec)	N/A	0.016	0.002	0.057	0.279	0.797	0.010	0.681	0.024

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.015	0.002	0.059	0.276	0.712	0.010	0.808	0.025

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	56	56
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.015	0.002	0.055	0.285	0.963	0.010	0.994	0.024

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	55	54	54	61	54	54
N.S.	1	1.00	1.00	0.89	0.87	0.87	0.98	0.87	0.87
time (sec)	N/A	0.013	0.001	0.017	0.291	1.025	0.009	1.596	0.022

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	55	59	58	55	55
N.S.	1	1.00	1.00	0.92	0.90	0.97	0.95	0.90	0.90
time (sec)	N/A	0.015	0.003	0.025	0.270	1.060	0.033	1.342	0.026

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	55	59	60	55	57
N.S.	1	1.00	1.00	0.92	0.92	0.98	1.00	0.92	0.95
time (sec)	N/A	0.014	0.003	0.026	0.283	1.382	0.055	0.971	0.025

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	58	59	63	58	58
N.S.	1	1.00	1.00	0.89	0.92	0.94	1.00	0.92	0.92
time (sec)	N/A	0.015	0.003	0.026	0.272	1.103	0.082	1.573	0.047

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	58	59	61	58	59
N.S.	1	1.00	1.00	0.92	0.95	0.97	1.00	0.95	0.97
time (sec)	N/A	0.015	0.003	0.026	0.305	1.772	0.113	1.121	4.789

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	57	59	60	57	57
N.S.	1	1.00	1.00	0.92	0.95	0.98	1.00	0.95	0.95
time (sec)	N/A	0.015	0.004	0.029	0.307	1.012	0.149	1.032	0.039

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	59	59	63	59	58
N.S.	1	1.00	1.00	0.89	0.91	0.91	0.97	0.91	0.89
time (sec)	N/A	0.016	0.003	0.028	0.308	1.101	0.181	1.660	0.038

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	59	59	63	59	58
N.S.	1	1.00	1.00	0.87	0.88	0.88	0.94	0.88	0.87
time (sec)	N/A	0.016	0.003	0.030	0.297	1.296	0.194	1.320	0.038

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.016	0.003	0.031	0.288	1.494	0.208	0.811	4.747

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.017	0.003	0.030	0.278	1.150	0.228	1.394	0.040

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.016	0.004	0.032	0.329	0.992	0.239	1.037	0.040

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	108	91	90	90	105	90	90
N.S.	1	1.00	0.84	0.71	0.70	0.70	0.81	0.70	0.70
time (sec)	N/A	0.149	0.003	0.071	0.332	1.080	0.016	1.062	0.103

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	91	90	90	107	90	90
N.S.	1	1.00	0.98	0.83	0.82	0.82	0.97	0.82	0.82
time (sec)	N/A	0.117	0.002	0.063	0.298	1.278	0.017	1.234	4.566

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	106	91	90	90	104	90	90
N.S.	1	1.00	1.16	1.00	0.99	0.99	1.14	0.99	0.99
time (sec)	N/A	0.096	0.002	0.062	0.281	1.069	0.016	1.341	4.593

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	106	91	90	90	105	90	90
N.S.	1	1.00	1.47	1.26	1.25	1.25	1.46	1.25	1.25
time (sec)	N/A	0.076	0.002	0.059	0.279	1.396	0.017	0.825	0.090

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	103	90	89	89	102	89	89
N.S.	1	1.00	1.94	1.70	1.68	1.68	1.92	1.68	1.68
time (sec)	N/A	0.057	0.002	0.056	0.275	1.022	0.017	0.917	0.092

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	106	91	90	90	105	90	90
N.S.	1	1.00	3.12	2.68	2.65	2.65	3.09	2.65	2.65
time (sec)	N/A	0.031	0.003	0.044	0.274	1.261	0.017	0.727	0.092

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	90	99	14	14
N.S.	1	1.00	1.00	0.94	0.88	5.62	6.19	0.88	0.88
time (sec)	N/A	0.002	0.002	0.046	0.281	1.049	0.017	1.349	4.612

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	88	102	91	88
N.S.	1	1.00	1.00	0.89	0.91	0.88	1.02	0.91	0.88
time (sec)	N/A	0.038	0.004	0.114	0.295	1.668	0.048	0.867	4.618

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	91	94	100	101	89
N.S.	1	1.00	1.00	0.91	0.92	0.95	1.01	1.02	0.90
time (sec)	N/A	0.041	0.004	0.034	0.279	1.157	0.065	0.820	0.059

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	92	94	104	103	92
N.S.	1	1.00	1.00	0.89	0.91	0.93	1.03	1.02	0.91
time (sec)	N/A	0.040	0.006	0.035	0.298	1.153	0.091	0.801	0.055

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	89	91	94	97	102	91
N.S.	1	1.00	1.00	0.95	0.97	1.00	1.03	1.09	0.97
time (sec)	N/A	0.039	0.005	0.036	0.281	1.098	0.126	0.997	0.053

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	94	94	100	105	92
N.S.	1	1.00	1.00	0.93	0.97	0.97	1.03	1.08	0.95
time (sec)	N/A	0.037	0.004	0.033	0.315	1.231	0.164	1.064	0.052

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	90	94	94	99	105	91
N.S.	1	1.00	1.00	0.95	0.99	0.99	1.04	1.11	0.96
time (sec)	N/A	0.037	0.004	0.037	0.300	1.432	0.208	0.876	5.110

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	94	94	99	105	92
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.98	1.04	0.91
time (sec)	N/A	0.036	0.004	0.046	0.303	1.284	0.261	0.936	0.061

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	94	94	99	103	94
N.S.	1	1.00	1.00	0.91	0.95	0.95	1.00	1.04	0.95
time (sec)	N/A	0.034	0.004	0.035	0.318	1.873	0.311	1.510	5.148

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	94	94	97	102	91
N.S.	1	1.00	1.00	0.89	0.94	0.94	0.97	1.02	0.91
time (sec)	N/A	0.035	0.004	0.033	0.302	1.054	0.368	1.591	5.089

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	100	91	90	90	97	90	92
N.S.	1	1.00	5.26	4.79	4.74	4.74	5.11	4.74	4.84
time (sec)	N/A	0.002	0.003	0.035	0.270	1.565	0.387	1.286	0.076

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	91	92	92	99	92	92
N.S.	1	1.00	2.65	2.28	2.30	2.30	2.48	2.30	2.30
time (sec)	N/A	0.012	0.003	0.036	0.277	1.168	0.412	0.921	0.077

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	104	91	92	92	99	92	91
N.S.	1	1.00	1.68	1.47	1.48	1.48	1.60	1.48	1.47
time (sec)	N/A	0.019	0.003	0.037	0.294	1.062	0.438	0.786	0.079

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	91	92	92	99	92	92
N.S.	1	1.00	1.26	1.08	1.10	1.10	1.18	1.10	1.10
time (sec)	N/A	0.029	0.003	0.043	0.284	1.007	0.457	0.739	4.967

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	92	92	99	92	92
N.S.	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.039	0.003	0.042	0.411	0.628	0.486	0.619	0.080

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	92	92	99	92	92
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.92	0.85	0.85
time (sec)	N/A	0.037	0.004	0.043	0.293	0.481	0.511	0.744	4.893

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	92	92	99	92	92
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.92	0.85	0.85
time (sec)	N/A	0.035	0.003	0.046	0.295	0.824	0.531	0.694	4.862

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	92	92	99	92	91
N.S.	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.86
time (sec)	N/A	0.035	0.003	0.047	0.309	0.565	0.561	0.702	0.084

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	90
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83
time (sec)	N/A	0.030	0.002	0.073	0.303	0.623	0.013	0.806	4.945

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	90
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83
time (sec)	N/A	0.027	0.002	0.059	0.294	0.950	0.013	0.714	0.097

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	90
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83
time (sec)	N/A	0.027	0.002	0.061	0.307	0.644	0.013	1.372	0.096

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	105	90	90
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.85
time (sec)	N/A	0.026	0.002	0.069	0.310	0.985	0.013	1.841	4.970

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	88	87	87	102	87	87
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.01	0.86	0.86
time (sec)	N/A	0.023	0.001	0.021	0.287	0.846	0.012	1.229	0.053

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	88	92	99	88	88
N.S.	1	1.00	1.00	0.89	0.88	0.92	0.99	0.88	0.88
time (sec)	N/A	0.025	0.009	0.105	0.278	0.660	0.045	1.012	0.057

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	89	92	100	89	91
N.S.	1	1.00	1.00	0.91	0.91	0.94	1.02	0.91	0.93
time (sec)	N/A	0.026	0.007	0.031	0.290	1.007	0.068	1.184	0.053

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	92	102	91	91
N.S.	1	1.00	1.00	0.89	0.91	0.92	1.02	0.91	0.91
time (sec)	N/A	0.025	0.010	0.029	0.295	0.757	0.094	0.903	4.959

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	91	92	102	91	91
N.S.	1	1.00	1.00	0.87	0.89	0.90	1.00	0.89	0.89
time (sec)	N/A	0.025	0.005	0.031	0.281	0.821	0.120	0.953	4.798

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	91	92	100	91	91
N.S.	1	1.00	1.00	0.87	0.89	0.90	0.98	0.89	0.89
time (sec)	N/A	0.025	0.011	0.034	0.271	1.156	0.165	1.135	0.047

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	92	99	91	91
N.S.	1	1.00	1.00	0.89	0.91	0.92	0.99	0.91	0.91
time (sec)	N/A	0.025	0.006	0.032	0.295	1.440	0.206	1.041	4.578

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	91	92	97	91	92
N.S.	1	1.00	1.00	0.91	0.93	0.94	0.99	0.93	0.94
time (sec)	N/A	0.025	0.008	0.033	0.306	0.926	0.241	0.972	0.074

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	88	90	92	95	90	90
N.S.	1	1.00	1.00	0.89	0.91	0.93	0.96	0.91	0.91
time (sec)	N/A	0.027	0.004	0.034	0.325	1.054	0.299	0.874	4.525

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	91	92	92	99	92	91
N.S.	1	1.00	1.00	0.88	0.88	0.88	0.95	0.88	0.88
time (sec)	N/A	0.027	0.008	0.033	0.285	1.029	0.346	1.111	0.074

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	92	92	99	92	92
N.S.	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.026	0.008	0.036	0.382	0.740	0.365	1.585	0.076

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	68	67	68	69	67
N.S.	1	1.00	1.00	0.86	0.86	0.85	0.86	0.87	0.85
time (sec)	N/A	0.039	0.005	0.049	0.292	0.869	0.060	1.186	0.064

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	71	72	170	119	77	65
N.S.	1	1.00	1.00	0.88	0.89	2.10	1.47	0.95	0.80
time (sec)	N/A	0.026	0.023	0.058	0.490	0.977	0.073	1.428	0.056

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	57	56	56	58	56
N.S.	1	1.00	1.00	0.86	0.86	0.85	0.85	0.88	0.85
time (sec)	N/A	0.030	0.005	0.093	0.290	1.520	0.056	1.098	0.083

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	60	148	107	65	54
N.S.	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.79
time (sec)	N/A	0.019	0.019	0.085	0.489	0.786	0.068	0.806	0.052

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	46	45	44	47	45
N.S.	1	1.00	1.00	0.87	0.87	0.85	0.83	0.89	0.85
time (sec)	N/A	0.024	0.005	0.033	0.264	1.566	0.052	1.225	4.727

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	50	126	95	55	43
N.S.	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.78
time (sec)	N/A	0.017	0.019	0.036	0.492	1.169	0.064	0.831	0.070

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	33	32	35	33
N.S.	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82
time (sec)	N/A	0.019	0.005	0.039	0.268	0.983	0.049	0.747	4.646

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	37	99	80	40	32
N.S.	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.76
time (sec)	N/A	0.015	0.015	0.035	0.566	1.103	0.059	0.602	0.069

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
N.S.	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.013	0.004	0.022	0.344	1.278	0.043	0.556	0.037

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	23
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74
time (sec)	N/A	0.008	0.007	0.030	0.551	1.069	0.050	0.664	0.034

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.002	0.002	0.019	0.279	1.448	0.031	0.638	4.632

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.003	0.003	0.029	0.481	1.044	0.043	0.540	4.695

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	24	18
N.S.	1	1.00	1.00	0.95	1.05	0.82	0.68	1.09	0.82
time (sec)	N/A	0.008	0.004	0.030	0.395	0.843	0.076	0.489	0.081

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	26
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76
time (sec)	N/A	0.008	0.009	0.049	0.488	0.712	0.063	0.499	4.620

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	43	31
N.S.	1	1.00	1.00	0.91	0.94	0.94	0.89	1.23	0.89
time (sec)	N/A	0.018	0.005	0.036	0.279	0.954	0.105	0.488	0.074

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	40	106	87	40	37
N.S.	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86
time (sec)	N/A	0.012	0.015	0.054	0.583	0.917	0.083	0.462	4.672

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	47	45	42	57	46
N.S.	1	1.00	1.00	0.90	0.96	0.92	0.86	1.16	0.94
time (sec)	N/A	0.020	0.005	0.037	0.281	1.293	0.125	0.445	0.078

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	52	132	100	52	48
N.S.	1	1.00	1.00	0.90	0.90	2.28	1.72	0.90	0.83
time (sec)	N/A	0.017	0.016	0.048	0.528	1.179	0.103	0.453	0.063

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	58	58	56	70	58
N.S.	1	1.00	1.00	0.89	0.92	0.92	0.89	1.11	0.92
time (sec)	N/A	0.024	0.005	0.040	0.280	1.164	0.148	0.408	4.645

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	61	62	154	112	62	59
N.S.	1	1.00	1.00	0.88	0.90	2.23	1.62	0.90	0.86
time (sec)	N/A	0.022	0.019	0.042	0.528	1.595	0.122	0.494	0.065

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	66	69	69	68	81	68
N.S.	1	1.00	1.00	0.88	0.92	0.92	0.91	1.08	0.91
time (sec)	N/A	0.029	0.005	0.045	0.352	1.231	0.164	0.468	4.667

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	83	88	88	104	88	103	90
N.S.	1	1.00	0.88	0.94	0.94	1.11	0.94	1.10	0.96
time (sec)	N/A	0.054	0.025	0.043	0.384	1.320	0.124	0.534	0.085

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	87	93	234	151	95	88
N.S.	1	1.00	0.89	0.83	0.89	2.23	1.44	0.90	0.84
time (sec)	N/A	0.029	0.047	0.047	0.488	1.176	0.142	0.489	0.071

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	78	77	93	80	92	79
N.S.	1	1.00	0.87	0.94	0.93	1.12	0.96	1.11	0.95
time (sec)	N/A	0.046	0.015	0.041	0.272	1.464	0.119	1.004	4.485

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	76	82	212	134	84	77
N.S.	1	1.00	0.89	0.83	0.89	2.30	1.46	0.91	0.84
time (sec)	N/A	0.024	0.041	0.056	0.493	0.922	0.135	1.463	4.555

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	66	65	81	66	80	68
N.S.	1	1.00	0.86	0.94	0.93	1.16	0.94	1.14	0.97
time (sec)	N/A	0.038	0.016	0.043	0.286	0.637	0.110	0.729	0.069

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	65	71	190	124	73	66
N.S.	1	1.00	0.90	0.82	0.90	2.41	1.57	0.92	0.84
time (sec)	N/A	0.020	0.033	0.046	0.502	1.294	0.127	0.854	4.586

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	55	54	70	53	67	57
N.S.	1	1.00	0.86	0.96	0.95	1.23	0.93	1.18	1.00
time (sec)	N/A	0.029	0.013	0.059	0.269	0.654	0.101	0.792	0.077

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	54	59	164	107	61	56
N.S.	1	1.00	0.91	0.82	0.89	2.48	1.62	0.92	0.85
time (sec)	N/A	0.018	0.032	0.054	0.631	0.624	0.119	0.758	0.089

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	44	43	56	39	49	45
N.S.	1	1.00	0.86	1.00	0.98	1.27	0.89	1.11	1.02
time (sec)	N/A	0.022	0.011	0.036	0.299	0.775	0.091	0.721	0.046

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	42	45	136	83	42	43
N.S.	1	1.00	0.93	0.76	0.82	2.47	1.51	0.76	0.78
time (sec)	N/A	0.012	0.023	0.046	0.491	0.997	0.104	1.302	4.591

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	48	29
N.S.	1	1.00	0.82	0.91	0.97	1.06	0.88	1.45	0.88
time (sec)	N/A	0.017	0.008	0.031	0.279	1.090	0.070	1.199	0.046

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73
time (sec)	N/A	0.008	0.014	0.044	0.541	1.570	0.081	0.943	4.756

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.002	0.001	0.021	0.292	1.789	0.054	0.722	0.031

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.007	0.017	0.044	0.504	1.622	0.084	1.057	4.740

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	42	37	47	34	47	34
N.S.	1	1.00	0.87	1.11	0.97	1.24	0.89	1.24	0.89
time (sec)	N/A	0.019	0.014	0.041	0.290	1.113	0.122	1.025	4.701

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	45	49	136	92	47	44
N.S.	1	1.00	0.95	0.79	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.013	0.026	0.059	0.494	1.351	0.119	0.908	0.073

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	55	52	73	51	51	51
N.S.	1	1.00	0.84	1.12	1.06	1.49	1.04	1.04	1.04
time (sec)	N/A	0.025	0.025	0.045	0.296	1.499	0.156	0.978	0.078

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	55	64	172	114	59	58
N.S.	1	1.00	0.99	0.81	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.017	0.028	0.051	0.476	1.237	0.148	1.139	4.729

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	65	70	90	68	86	67
N.S.	1	1.00	0.86	0.98	1.06	1.36	1.03	1.30	1.02
time (sec)	N/A	0.033	0.035	0.045	0.273	1.383	0.190	1.170	4.804

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	67	75	198	126	70	70
N.S.	1	1.00	0.99	0.83	0.93	2.44	1.56	0.86	0.86
time (sec)	N/A	0.022	0.030	0.045	0.498	1.416	0.173	1.100	4.850

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	77	79	99	78	99	78
N.S.	1	1.00	0.85	0.96	0.99	1.24	0.98	1.24	0.98
time (sec)	N/A	0.036	0.035	0.048	0.273	1.212	0.206	1.175	0.119

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	77	86	220	138	81	80
N.S.	1	1.00	0.97	0.82	0.91	2.34	1.47	0.86	0.85
time (sec)	N/A	0.030	0.034	0.047	0.490	1.368	0.196	2.374	4.560

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	88	92	112	94	110	89
N.S.	1	1.00	0.85	0.95	0.99	1.20	1.01	1.18	0.96
time (sec)	N/A	0.044	0.056	0.054	0.279	1.192	0.238	1.787	4.731

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	97	105	111	137	119	114	111
N.S.	1	1.00	0.85	0.92	0.97	1.20	1.04	1.00	0.97
time (sec)	N/A	0.070	0.023	0.054	0.283	1.505	0.208	1.314	4.725

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	85	96	99	125	104	102	100
N.S.	1	1.00	0.85	0.96	0.99	1.25	1.04	1.02	1.00
time (sec)	N/A	0.057	0.022	0.048	0.316	1.432	0.201	1.239	0.077

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	75	84	89	115	92	92	90
N.S.	1	1.00	0.86	0.97	1.02	1.32	1.06	1.06	1.03
time (sec)	N/A	0.050	0.017	0.044	0.284	1.568	0.187	1.707	4.493

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	73	77	103	78	80	78
N.S.	1	1.00	0.85	0.99	1.04	1.39	1.05	1.08	1.05
time (sec)	N/A	0.040	0.017	0.062	0.306	1.317	0.176	1.397	0.083

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	62	66	91	68	62	68
N.S.	1	1.00	0.74	0.95	1.02	1.40	1.05	0.95	1.05
time (sec)	N/A	0.032	0.043	0.038	0.304	1.871	0.165	1.112	4.749

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	46	55	69	53	42	52
N.S.	1	1.00	0.80	0.94	1.12	1.41	1.08	0.86	1.06
time (sec)	N/A	0.026	0.012	0.034	0.276	1.463	0.131	1.477	0.059

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.002	0.005	0.033	0.267	1.126	0.108	2.385	0.032

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	26	27	14	28
N.S.	1	1.00	1.00	0.94	0.88	1.62	1.69	0.88	1.75
time (sec)	N/A	0.002	0.002	0.021	0.266	1.324	0.093	1.111	4.623

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	59	60	90	56	59	56
N.S.	1	1.00	0.80	1.09	1.11	1.67	1.04	1.09	1.04
time (sec)	N/A	0.025	0.023	0.043	0.282	0.965	0.178	1.330	4.675

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	72	77	119	80	82	75
N.S.	1	1.00	0.88	1.07	1.15	1.78	1.19	1.22	1.12
time (sec)	N/A	0.036	0.039	0.050	0.283	1.019	0.226	2.066	0.080

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	83	92	134	90	80	88
N.S.	1	1.00	0.86	0.97	1.07	1.56	1.05	0.93	1.02
time (sec)	N/A	0.041	0.034	0.052	0.294	0.997	0.245	1.922	4.672

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	94	103	145	104	110	101
N.S.	1	1.00	0.89	0.99	1.08	1.53	1.09	1.16	1.06
time (sec)	N/A	0.047	0.046	0.055	0.285	1.382	0.269	1.416	4.692

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	96	105	114	156	116	119	111
N.S.	1	1.00	0.86	0.94	1.02	1.39	1.04	1.06	0.99
time (sec)	N/A	0.053	0.041	0.066	0.310	0.998	0.300	1.625	4.855

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	99	85	105	278	162	96	99
N.S.	1	1.00	0.89	0.77	0.95	2.50	1.46	0.86	0.89
time (sec)	N/A	0.032	0.042	0.054	0.489	1.084	0.214	1.337	0.076

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	74	93	256	144	84	87
N.S.	1	1.00	0.90	0.76	0.95	2.61	1.47	0.86	0.89
time (sec)	N/A	0.028	0.034	0.048	0.497	1.201	0.209	1.933	0.070

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	63	82	230	133	73	77
N.S.	1	1.00	0.91	0.74	0.96	2.71	1.56	0.86	0.91
time (sec)	N/A	0.025	0.032	0.045	0.520	1.433	0.192	1.381	4.725

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	51	68	202	107	54	64
N.S.	1	1.00	0.89	0.69	0.92	2.73	1.45	0.73	0.86
time (sec)	N/A	0.017	0.031	0.041	0.521	1.133	0.176	1.609	4.746

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	47	59	188	110	45	56
N.S.	1	1.00	0.86	0.73	0.92	2.94	1.72	0.70	0.88
time (sec)	N/A	0.013	0.027	0.045	0.490	0.925	0.141	1.258	4.771

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	49	62	190	110	50	55
N.S.	1	1.00	0.89	0.75	0.95	2.92	1.69	0.77	0.85
time (sec)	N/A	0.012	0.020	0.045	0.479	0.940	0.130	1.348	4.738

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	57	58	188	105	45	55
N.S.	1	1.00	0.89	0.92	0.94	3.03	1.69	0.73	0.89
time (sec)	N/A	0.011	0.025	0.053	0.498	0.790	0.134	1.353	4.656

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	54	71	202	116	57	66
N.S.	1	1.00	0.89	0.71	0.93	2.66	1.53	0.75	0.87
time (sec)	N/A	0.018	0.029	0.053	0.502	0.929	0.181	1.356	4.673

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	64	86	238	138	71	80
N.S.	1	1.00	0.91	0.74	0.99	2.74	1.59	0.82	0.92
time (sec)	N/A	0.024	0.029	0.059	0.624	0.847	0.210	0.925	4.672

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	75	97	264	150	80	92
N.S.	1	1.00	0.90	0.75	0.97	2.64	1.50	0.80	0.92
time (sec)	N/A	0.029	0.036	0.062	0.498	1.290	0.234	1.785	5.021

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	86	108	286	162	93	102
N.S.	1	1.00	0.89	0.76	0.96	2.53	1.43	0.82	0.90
time (sec)	N/A	0.036	0.038	0.067	0.510	1.262	0.258	1.241	4.985

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	169	203	242	346	260	168	242
N.S.	1	1.00	0.78	0.94	1.12	1.60	1.20	0.78	1.12
time (sec)	N/A	0.176	0.032	0.121	0.297	1.285	0.957	1.429	5.295

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	158	192	231	335	245	157	230
N.S.	1	1.00	0.77	0.94	1.13	1.63	1.20	0.77	1.12
time (sec)	N/A	0.149	0.020	0.141	0.415	1.306	0.928	1.209	0.401

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	145	181	220	322	233	139	220
N.S.	1	1.00	0.77	0.96	1.17	1.71	1.24	0.74	1.17
time (sec)	N/A	0.128	0.023	0.105	0.402	0.962	0.878	1.163	0.425

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	116	166	209	300	219	119	207
N.S.	1	1.00	0.65	0.93	1.17	1.68	1.22	0.66	1.16
time (sec)	N/A	0.114	0.024	0.097	0.299	1.471	0.751	1.725	5.362

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	101	150	190	190	202	99	192
N.S.	1	1.00	5.32	7.89	10.00	10.00	10.63	5.21	10.11
time (sec)	N/A	0.002	0.014	0.089	0.306	0.984	0.678	0.961	0.115

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	90	133	179	179	190	88	181
N.S.	1	1.00	2.31	3.41	4.59	4.59	4.87	2.26	4.64
time (sec)	N/A	0.012	0.011	0.090	0.290	1.131	0.618	1.017	4.886

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	79	116	168	168	178	77	170
N.S.	1	1.00	1.36	2.00	2.90	2.90	3.07	1.33	2.93
time (sec)	N/A	0.018	0.013	0.087	0.331	0.871	0.567	1.732	0.105

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	68	99	157	157	167	66	159
N.S.	1	1.00	0.88	1.29	2.04	2.04	2.17	0.86	2.06
time (sec)	N/A	0.026	0.013	0.088	0.307	0.955	0.525	1.787	0.097

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	57	82	146	146	155	55	148
N.S.	1	1.00	0.63	0.90	1.60	1.60	1.70	0.60	1.63
time (sec)	N/A	0.049	0.013	0.088	0.304	1.228	0.487	1.327	4.819

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	65	135	135	143	44	136
N.S.	1	1.00	0.64	0.90	1.88	1.88	1.99	0.61	1.89
time (sec)	N/A	0.039	0.009	0.096	0.285	0.841	0.458	1.257	0.099

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	48	124	124	131	33	125
N.S.	1	1.00	0.66	0.91	2.34	2.34	2.47	0.62	2.36
time (sec)	N/A	0.028	0.010	0.087	0.296	1.198	0.432	1.144	4.828

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	113	113	119	22	114
N.S.	1	1.00	0.71	0.91	3.32	3.32	3.50	0.65	3.35
time (sec)	N/A	0.018	0.006	0.086	0.302	1.533	0.411	1.252	0.108

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	103	110	14	14
N.S.	1	1.00	1.00	0.94	0.88	6.44	6.88	0.88	0.88
time (sec)	N/A	0.002	0.002	0.078	0.283	1.120	0.395	1.389	0.126

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	120	178	214	398	223	136	210
N.S.	1	1.00	0.72	1.07	1.29	2.40	1.34	0.82	1.27
time (sec)	N/A	0.088	0.069	0.116	0.305	1.284	0.581	1.140	5.464

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	136	191	231	427	245	159	229
N.S.	1	1.00	0.74	1.04	1.26	2.32	1.33	0.86	1.24
time (sec)	N/A	0.129	0.084	0.129	0.322	1.419	0.667	1.558	0.522

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	151	202	246	442	260	174	243
N.S.	1	1.00	0.70	0.93	1.13	2.04	1.20	0.80	1.12
time (sec)	N/A	0.151	0.065	0.131	0.332	1.252	0.701	1.366	5.775

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	162	213	257	453	270	187	255
N.S.	1	1.00	0.72	0.94	1.14	2.00	1.19	0.83	1.13
time (sec)	N/A	0.160	0.081	0.137	0.350	1.433	0.719	1.592	1.088

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	166	151	248	718	314	162	241
N.S.	1	1.00	0.72	0.65	1.07	3.11	1.36	0.70	1.04
time (sec)	N/A	0.114	0.059	0.124	0.592	1.213	0.916	1.439	4.898

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	155	140	236	692	299	150	231
N.S.	1	1.00	0.71	0.64	1.08	3.17	1.37	0.69	1.06
time (sec)	N/A	0.096	0.053	0.101	0.497	1.496	0.882	1.143	0.402

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	144	128	222	664	274	131	218
N.S.	1	1.00	0.70	0.62	1.07	3.21	1.32	0.63	1.05
time (sec)	N/A	0.084	0.049	0.098	0.527	1.493	0.826	2.230	0.444

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	134	124	213	650	277	122	210
N.S.	1	1.00	0.68	0.63	1.08	3.30	1.41	0.62	1.07
time (sec)	N/A	0.081	0.051	0.115	0.551	1.282	0.683	1.226	4.964

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	138	124	219	654	289	128	207
N.S.	1	1.00	0.70	0.63	1.11	3.30	1.46	0.65	1.05
time (sec)	N/A	0.083	0.048	0.101	0.689	1.852	0.639	1.189	4.757

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	138	122	221	654	291	128	205
N.S.	1	1.00	0.69	0.61	1.11	3.29	1.46	0.64	1.03
time (sec)	N/A	0.082	0.040	0.114	0.502	1.124	0.608	1.476	4.753

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	138	122	221	654	291	128	205
N.S.	1	1.00	0.69	0.61	1.10	3.27	1.46	0.64	1.02
time (sec)	N/A	0.079	0.049	0.101	0.518	1.541	0.568	1.239	0.146

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	138	122	221	654	291	128	205
N.S.	1	1.00	0.69	0.61	1.10	3.25	1.45	0.64	1.02
time (sec)	N/A	0.078	0.047	0.100	0.529	0.939	0.539	1.001	4.770

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	138	122	221	654	291	128	204
N.S.	1	1.00	0.68	0.60	1.09	3.24	1.44	0.63	1.01
time (sec)	N/A	0.077	0.041	0.101	0.532	1.049	0.511	1.251	4.736

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	138	122	221	654	291	128	204
N.S.	1	1.00	0.68	0.60	1.09	3.22	1.43	0.63	1.00
time (sec)	N/A	0.074	0.044	0.107	0.541	1.075	0.495	1.403	4.794

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	138	122	221	654	291	128	204
N.S.	1	1.00	0.68	0.60	1.08	3.21	1.43	0.63	1.00
time (sec)	N/A	0.071	0.045	0.102	0.545	1.241	0.477	0.958	4.738

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	138	124	219	654	286	128	206
N.S.	1	1.00	0.67	0.60	1.07	3.19	1.40	0.62	1.00
time (sec)	N/A	0.070	0.040	0.100	0.507	1.357	0.467	0.833	0.167

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	131	204	212	650	272	122	209
N.S.	1	1.00	0.72	1.13	1.17	3.59	1.50	0.67	1.15
time (sec)	N/A	0.066	0.064	0.089	0.523	0.682	0.476	1.388	4.743

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	147	131	225	664	282	134	220
N.S.	1	1.00	0.70	0.63	1.08	3.18	1.35	0.64	1.05
time (sec)	N/A	0.087	0.066	0.105	0.538	1.089	0.624	1.600	5.089

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	157	141	240	700	304	148	234
N.S.	1	1.00	0.71	0.64	1.09	3.18	1.38	0.67	1.06
time (sec)	N/A	0.095	0.057	0.131	0.540	0.974	0.653	1.312	5.106

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	169	153	251	726	316	159	246
N.S.	1	1.00	0.73	0.66	1.08	3.12	1.36	0.68	1.06
time (sec)	N/A	0.106	0.059	0.121	0.543	0.771	0.694	1.204	5.892

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	25	23	22	26	23
N.S.	1	1.00	1.00	0.89	0.89	0.82	0.79	0.93	0.82
time (sec)	N/A	0.016	0.005	0.024	0.291	0.714	0.048	1.243	0.049

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	42	80	49	29	23
N.S.	1	1.00	1.00	0.87	1.35	2.58	1.58	0.94	0.74
time (sec)	N/A	0.008	0.007	0.031	0.496	0.902	0.054	1.501	4.567

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	15	12	16	15
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.75	1.00	0.94
time (sec)	N/A	0.002	0.003	0.021	0.280	0.927	0.037	0.934	0.033

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	31	68	46	18	16
N.S.	1	1.00	1.00	0.67	1.29	2.83	1.92	0.75	0.67
time (sec)	N/A	0.004	0.003	0.036	0.520	0.905	0.047	1.164	0.177

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	25	20	15	26	21
N.S.	1	1.00	1.00	0.96	1.09	0.87	0.65	1.13	0.91
time (sec)	N/A	0.009	0.005	0.029	0.295	1.383	0.082	1.103	4.537

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	44	82	58	31	25
N.S.	1	1.00	1.00	0.88	1.33	2.48	1.76	0.94	0.76
time (sec)	N/A	0.008	0.008	0.030	0.571	1.198	0.068	1.291	4.610

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	35	33	31	43	31
N.S.	1	1.00	1.00	0.91	1.00	0.94	0.89	1.23	0.89
time (sec)	N/A	0.017	0.006	0.034	0.306	1.503	0.112	1.190	0.073

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	29	32	35	42	29	53	32
N.S.	1	1.00	0.83	0.91	1.00	1.20	0.83	1.51	0.91
time (sec)	N/A	0.018	0.010	0.031	0.284	1.327	0.077	1.244	0.044

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	37	52	127	71	39	34
N.S.	1	1.00	1.02	0.80	1.13	2.76	1.54	0.85	0.74
time (sec)	N/A	0.008	0.021	0.037	0.490	0.938	0.085	2.521	4.684

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	16	16	15	16	15
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.88	0.94	0.88
time (sec)	N/A	0.002	0.002	0.021	0.303	0.664	0.057	4.863	0.021

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	37	52	126	71	39	34
N.S.	1	1.00	1.02	0.80	1.13	2.74	1.54	0.85	0.74
time (sec)	N/A	0.008	0.012	0.038	0.510	0.911	0.087	2.193	4.514

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	35	44	41	53	34	51	36
N.S.	1	1.00	0.88	1.10	1.02	1.32	0.85	1.28	0.90
time (sec)	N/A	0.019	0.014	0.039	0.284	0.555	0.128	1.213	0.055

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	45	65	140	83	50	45
N.S.	1	1.00	0.97	0.78	1.12	2.41	1.43	0.86	0.78
time (sec)	N/A	0.012	0.025	0.040	0.478	0.698	0.125	4.509	4.628

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	56	57	80	49	56	55
N.S.	1	1.00	0.85	1.08	1.10	1.54	0.94	1.08	1.06
time (sec)	N/A	0.027	0.023	0.045	0.283	0.610	0.162	1.213	4.591

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	33	38	38	36	26	37
N.S.	1	1.00	1.25	1.65	1.90	1.90	1.80	1.30	1.85
time (sec)	N/A	0.003	0.007	0.034	0.273	0.905	0.109	1.772	0.038

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	50	76	188	105	53	54
N.S.	1	1.00	0.84	0.75	1.13	2.81	1.57	0.79	0.81
time (sec)	N/A	0.013	0.022	0.043	0.531	0.929	0.133	1.164	4.616

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	16	26	26	16	26
N.S.	1	1.00	1.00	0.94	0.94	1.53	1.53	0.94	1.53
time (sec)	N/A	0.002	0.002	0.023	0.296	0.933	0.095	1.211	0.026

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	59	73	188	99	49	55
N.S.	1	1.00	0.88	0.92	1.14	2.94	1.55	0.77	0.86
time (sec)	N/A	0.012	0.028	0.039	0.612	1.357	0.138	1.879	4.600

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	62	62	92	56	63	57
N.S.	1	1.00	0.79	1.09	1.09	1.61	0.98	1.11	1.00
time (sec)	N/A	0.027	0.026	0.049	0.282	1.436	0.185	1.697	0.065

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	54	86	202	107	61	66
N.S.	1	1.00	0.88	0.69	1.10	2.59	1.37	0.78	0.85
time (sec)	N/A	0.018	0.031	0.046	0.496	1.224	0.188	2.497	4.605

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	75	79	121	78	84	76
N.S.	1	1.00	0.87	1.09	1.14	1.75	1.13	1.22	1.10
time (sec)	N/A	0.037	0.039	0.049	0.384	1.447	0.232	1.719	4.610

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	25	33	60	60	60	39	59
N.S.	1	1.00	0.69	0.92	1.67	1.67	1.67	1.08	1.64
time (sec)	N/A	0.022	0.008	0.042	0.334	1.107	0.188	2.015	4.595

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	81	70	124	324	160	77	96
N.S.	1	1.00	0.74	0.64	1.14	2.97	1.47	0.71	0.88
time (sec)	N/A	0.026	0.035	0.065	0.511	1.319	0.223	1.396	4.759

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	16	48	49	16	48
N.S.	1	1.00	1.00	0.94	0.94	2.82	2.88	0.94	2.82
time (sec)	N/A	0.002	0.002	0.035	0.269	0.923	0.177	2.564	4.690

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	79	103	117	320	146	71	99
N.S.	1	1.00	0.79	1.03	1.17	3.20	1.46	0.71	0.99
time (sec)	N/A	0.023	0.029	0.054	0.476	1.082	0.234	1.559	4.795

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	98	106	180	104	85	101
N.S.	1	1.00	0.74	1.08	1.16	1.98	1.14	0.93	1.11
time (sec)	N/A	0.046	0.024	0.058	0.301	1.694	0.293	2.137	5.238

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	92	76	130	334	155	83	110
N.S.	1	1.00	0.78	0.64	1.10	2.83	1.31	0.70	0.93
time (sec)	N/A	0.033	0.039	0.061	0.505	1.199	0.308	1.627	5.175

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	83	111	123	209	126	106	120
N.S.	1	1.00	0.78	1.05	1.16	1.97	1.19	1.00	1.13
time (sec)	N/A	0.063	0.048	0.065	0.275	0.910	0.353	1.871	5.351

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	12	18	14
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.80	1.20	0.93
time (sec)	N/A	0.007	0.004	0.061	0.266	0.831	0.043	1.480	5.108

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	17	15	12	18	16
N.S.	1	1.00	1.00	0.89	0.94	0.83	0.67	1.00	0.89
time (sec)	N/A	0.007	0.004	0.033	0.274	0.711	0.046	1.879	0.056

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	24	28	22	32	22
N.S.	1	1.00	1.00	0.88	0.92	1.08	0.85	1.23	0.85
time (sec)	N/A	0.011	0.004	0.043	0.279	1.814	0.081	1.667	0.059

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	24	28	22	32	22
N.S.	1	1.00	1.00	0.85	0.89	1.04	0.81	1.19	0.81
time (sec)	N/A	0.013	0.004	0.039	0.279	1.250	0.082	1.136	0.059

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	20	0	82	83	23	23
N.S.	1	1.00	0.93	0.67	0.00	2.73	2.77	0.77	0.77
time (sec)	N/A	0.017	0.009	0.048	0.000	0.833	0.064	1.438	0.167

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	44	33	0	102	87	33	29
N.S.	1	1.00	1.19	0.89	0.00	2.76	2.35	0.89	0.78
time (sec)	N/A	0.021	0.012	0.061	0.000	0.979	0.094	1.172	0.293

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	34	28	40	22	36	24
N.S.	1	1.00	0.89	1.21	1.00	1.43	0.79	1.29	0.86
time (sec)	N/A	0.013	0.010	0.068	0.268	0.793	0.078	1.379	0.035

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	33	28	40	22	36	26
N.S.	1	1.00	0.87	1.10	0.93	1.33	0.73	1.20	0.87
time (sec)	N/A	0.015	0.010	0.054	0.279	1.190	0.082	1.327	0.036

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	106	60	37	38
N.S.	1	1.00	1.06	1.00	0.00	3.12	1.76	1.09	1.12
time (sec)	N/A	0.018	0.011	0.056	0.000	1.447	0.107	1.431	5.138

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	105	66	36	38
N.S.	1	1.00	1.06	1.00	0.00	3.09	1.94	1.06	1.12
time (sec)	N/A	0.008	0.007	0.033	0.000	1.459	0.100	1.470	4.638

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	0	182	104	58	46
N.S.	1	1.00	1.00	0.78	0.00	3.64	2.08	1.16	0.92
time (sec)	N/A	0.034	0.015	0.058	0.000	1.390	0.144	1.616	4.876

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.003	0.012	0.077	0.271	1.633	0.454	1.497	4.505

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	13	18	19	13	15
N.S.	1	1.00	0.90	0.67	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.003	0.010	0.046	0.283	1.189	0.265	1.448	0.026

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.003	0.011	0.053	0.304	1.283	0.143	1.069	0.025

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	16	19	13	15
N.S.	1	1.00	1.00	0.67	0.62	0.76	0.90	0.62	0.71
time (sec)	N/A	0.003	0.010	0.025	0.264	1.917	0.569	1.324	0.025

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	20	14	13	14	17	13	14
N.S.	1	1.00	1.05	0.74	0.68	0.74	0.89	0.68	0.74
time (sec)	N/A	0.003	0.010	0.023	0.321	1.469	0.062	1.566	0.026

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	14	17	13	15
N.S.	1	1.00	1.00	0.74	0.68	0.74	0.89	0.68	0.79
time (sec)	N/A	0.003	0.013	0.028	0.307	1.078	0.138	1.082	0.026

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	14	13	15	17	13	15
N.S.	1	1.00	0.89	0.74	0.68	0.79	0.89	0.68	0.79
time (sec)	N/A	0.003	0.012	0.037	0.275	1.335	0.174	1.610	0.027

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	14	13	13	19	13	15
N.S.	1	1.00	0.89	0.74	0.68	0.68	1.00	0.68	0.79
time (sec)	N/A	0.003	0.013	0.028	0.271	1.577	0.285	1.013	0.027

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.006	0.015	0.066	0.294	1.588	0.696	0.958	0.045

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	26
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.72
time (sec)	N/A	0.006	0.015	0.056	0.304	1.572	0.457	1.213	0.037

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	26
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.72
time (sec)	N/A	0.006	0.014	0.058	0.268	1.579	0.273	1.139	0.038

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	27	34	24	26
N.S.	1	1.00	0.83	0.69	0.67	0.75	0.94	0.67	0.72
time (sec)	N/A	0.006	0.014	0.055	0.317	0.968	0.753	1.440	0.037

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	24	26	32	24	26
N.S.	1	1.00	0.88	0.74	0.71	0.76	0.94	0.71	0.76
time (sec)	N/A	0.006	0.014	0.033	0.405	1.516	0.137	1.055	0.035

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	24	26	32	24	26
N.S.	1	1.00	0.88	0.74	0.71	0.76	0.94	0.71	0.76
time (sec)	N/A	0.006	0.017	0.049	0.281	1.036	0.225	1.295	0.038

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	24	26	32	24	26
N.S.	1	1.00	0.88	0.74	0.71	0.76	0.94	0.71	0.76
time (sec)	N/A	0.006	0.017	0.041	0.276	1.537	0.250	2.112	0.037

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	25	26	32	25	26
N.S.	1	1.00	0.88	0.74	0.74	0.76	0.94	0.74	0.76
time (sec)	N/A	0.006	0.019	0.046	0.285	1.143	0.325	1.197	0.032

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.009	0.017	0.063	0.274	1.164	1.038	1.045	0.044

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.009	0.016	0.061	0.280	1.274	0.704	2.054	0.044

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.009	0.016	0.062	0.272	1.458	0.460	1.148	0.042

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	38	49	35	35
N.S.	1	1.00	0.80	0.71	0.69	0.75	0.96	0.69	0.69
time (sec)	N/A	0.009	0.016	0.058	0.336	1.049	0.977	1.037	0.042

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	36	35	37	48	35	35
N.S.	1	1.00	0.84	0.73	0.71	0.76	0.98	0.71	0.71
time (sec)	N/A	0.009	0.016	0.037	0.287	1.158	0.250	1.935	0.044

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	35	37	46	35	35
N.S.	1	1.00	0.87	0.77	0.74	0.79	0.98	0.74	0.74
time (sec)	N/A	0.008	0.019	0.044	0.272	1.136	0.363	1.114	0.044

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	36	35	37	48	35	35
N.S.	1	1.00	0.84	0.73	0.71	0.76	0.98	0.71	0.71
time (sec)	N/A	0.008	0.020	0.046	0.269	1.286	0.417	1.201	0.045

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	36	37	46	36	37
N.S.	1	1.00	0.87	0.77	0.77	0.79	0.98	0.77	0.79
time (sec)	N/A	0.009	0.021	0.050	0.310	1.099	0.529	1.356	0.042

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	128	125	194	170	136	196	67
N.S.	1	1.00	0.60	0.58	0.90	0.79	0.63	0.91	0.31
time (sec)	N/A	0.147	0.156	0.065	0.527	2.178	15.162	1.186	4.486

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	119	116	186	165	124	178	54
N.S.	1	1.00	0.58	0.57	0.91	0.81	0.61	0.87	0.26
time (sec)	N/A	0.111	0.125	0.051	0.488	1.447	4.502	1.336	0.087

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	118	115	185	124	110	178	55
N.S.	1	1.00	0.58	0.57	0.92	0.61	0.54	0.88	0.27
time (sec)	N/A	0.108	0.128	0.049	0.559	1.017	1.727	1.377	0.094

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	91	106	172	126	104	182	38
N.S.	1	1.00	0.47	0.55	0.90	0.66	0.54	0.95	0.20
time (sec)	N/A	0.092	0.109	0.028	0.526	0.979	0.973	1.103	0.071

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	92	106	172	126	104	182	37
N.S.	1	1.00	0.48	0.55	0.90	0.66	0.54	0.95	0.19
time (sec)	N/A	0.089	0.102	0.029	0.496	1.240	1.583	1.566	0.084

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	117	115	186	142	114	190	54
N.S.	1	1.00	0.58	0.57	0.92	0.70	0.56	0.94	0.27
time (sec)	N/A	0.104	0.143	0.049	0.480	2.231	3.468	0.823	4.501

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	119	116	187	167	128	178	53
N.S.	1	1.00	0.58	0.57	0.92	0.82	0.63	0.87	0.26
time (sec)	N/A	0.109	0.142	0.051	0.497	1.144	8.353	0.756	0.090

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	127	125	198	193	139	200	66
N.S.	1	1.00	0.59	0.58	0.92	0.90	0.65	0.93	0.31
time (sec)	N/A	0.117	0.160	0.056	0.490	2.204	28.944	0.925	4.481

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	138	136	206	192	338	196	80
N.S.	1	1.00	0.60	0.59	0.90	0.83	1.47	0.85	0.35
time (sec)	N/A	0.117	0.266	0.089	0.525	1.575	75.550	1.005	4.515

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	128	124	195	185	393	199	64
N.S.	1	1.00	0.59	0.57	0.89	0.85	1.80	0.91	0.29
time (sec)	N/A	0.101	0.254	0.053	0.519	1.272	49.482	1.091	0.081

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	127	127	195	187	323	199	64
N.S.	1	1.00	0.58	0.58	0.89	0.86	1.48	0.91	0.29
time (sec)	N/A	0.099	0.242	0.049	0.624	1.241	27.074	0.941	4.651

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	128	127	194	182	400	199	64
N.S.	1	1.00	0.59	0.58	0.89	0.83	1.83	0.91	0.29
time (sec)	N/A	0.102	0.231	0.048	0.545	0.870	16.589	0.868	0.086

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	128	124	194	179	316	199	64
N.S.	1	1.00	0.59	0.57	0.89	0.82	1.45	0.91	0.29
time (sec)	N/A	0.101	0.237	0.049	0.539	1.356	23.730	1.727	0.090

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	138	136	208	208	512	210	77
N.S.	1	1.00	0.60	0.59	0.90	0.90	2.23	0.91	0.33
time (sec)	N/A	0.114	0.279	0.084	0.683	1.102	49.025	1.691	0.083

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	138	136	209	228	425	196	77
N.S.	1	1.00	0.60	0.59	0.91	0.99	1.85	0.85	0.33
time (sec)	N/A	0.114	0.282	0.084	0.516	1.464	108.926	1.167	4.683

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	149	147	221	251	0	220	87
N.S.	1	1.00	0.61	0.60	0.91	1.03	0.00	0.91	0.36
time (sec)	N/A	0.132	0.306	0.087	0.571	1.724	0.000	1.047	4.690

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	138	139	218	254	0	209	87
N.S.	1	1.00	0.58	0.58	0.91	1.06	0.00	0.87	0.36
time (sec)	N/A	0.117	0.337	0.064	0.499	1.665	0.000	1.496	4.688

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	136	138	222	260	0	212	85
N.S.	1	1.00	0.56	0.57	0.92	1.07	0.00	0.88	0.35
time (sec)	N/A	0.118	0.336	0.064	0.529	1.971	0.000	1.240	0.083

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	137	138	221	257	666	211	85
N.S.	1	1.00	0.57	0.57	0.91	1.06	2.75	0.87	0.35
time (sec)	N/A	0.114	0.322	0.065	0.486	1.338	155.125	0.731	4.669

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	138	150	217	250	887	209	86
N.S.	1	1.00	0.58	0.63	0.91	1.05	3.71	0.87	0.36
time (sec)	N/A	0.116	0.226	0.065	0.492	1.850	99.921	1.209	0.083

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	138	147	217	241	627	209	86
N.S.	1	1.00	0.58	0.62	0.91	1.01	2.62	0.87	0.36
time (sec)	N/A	0.113	0.217	0.055	0.491	1.598	140.111	0.739	4.667

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	149	145	230	263	0	220	99
N.S.	1	1.00	0.59	0.58	0.92	1.05	0.00	0.88	0.39
time (sec)	N/A	0.130	0.373	0.107	0.502	1.333	0.000	0.708	0.102

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	149	145	231	283	0	208	99
N.S.	1	1.00	0.59	0.58	0.92	1.13	0.00	0.83	0.39
time (sec)	N/A	0.129	0.359	0.113	0.489	1.705	0.000	0.823	4.657

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	160	156	243	306	0	232	109
N.S.	1	1.00	0.61	0.59	0.92	1.16	0.00	0.88	0.41
time (sec)	N/A	0.143	0.373	0.131	0.498	1.369	0.000	0.753	4.648

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	48	58	86	117	92	194	33
N.S.	1	1.00	0.83	1.00	1.48	2.02	1.59	3.34	0.57
time (sec)	N/A	0.023	0.037	0.031	0.498	1.545	1.002	0.955	0.083

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	59	67	84	117	105	84	47
N.S.	1	1.00	0.55	0.62	0.78	1.08	0.97	0.78	0.44
time (sec)	N/A	0.042	0.085	0.891	0.509	1.724	0.566	0.735	0.079

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	55	62	79	110	99	79	42
N.S.	1	1.00	0.54	0.61	0.78	1.09	0.98	0.78	0.42
time (sec)	N/A	0.040	0.053	0.784	0.509	1.894	0.313	0.776	0.044

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	54	62	79	110	97	79	42
N.S.	1	1.00	0.55	0.63	0.80	1.11	0.98	0.80	0.42
time (sec)	N/A	0.040	0.052	0.795	0.497	2.192	0.194	1.130	0.041

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	41	56	74	107	90	74	37
N.S.	1	1.00	0.45	0.61	0.80	1.16	0.98	0.80	0.40
time (sec)	N/A	0.038	0.044	0.584	0.493	1.428	0.152	0.756	0.037

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	39	56	74	107	90	74	37
N.S.	1	1.00	0.42	0.61	0.80	1.16	0.98	0.80	0.40
time (sec)	N/A	0.036	0.044	0.331	0.494	1.730	0.188	0.957	0.027

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	53	62	79	120	97	79	42
N.S.	1	1.00	0.54	0.63	0.80	1.21	0.98	0.80	0.42
time (sec)	N/A	0.038	0.059	0.444	0.524	1.264	0.343	0.819	0.037

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	56	62	79	129	99	79	42
N.S.	1	1.00	0.55	0.61	0.78	1.28	0.98	0.78	0.42
time (sec)	N/A	0.037	0.067	0.628	0.525	0.946	0.484	0.610	0.039

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	62	67	86	136	105	86	48
N.S.	1	1.00	0.57	0.62	0.80	1.26	0.97	0.80	0.44
time (sec)	N/A	0.040	0.073	0.776	0.489	1.006	0.882	0.672	0.040

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	72	74	91	148	277	91	55
N.S.	1	1.00	0.59	0.61	0.75	1.21	2.27	0.75	0.45
time (sec)	N/A	0.042	0.133	0.457	0.533	1.499	1.181	0.562	4.617

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	65	69	86	141	264	86	51
N.S.	1	1.00	0.58	0.61	0.76	1.25	2.34	0.76	0.45
time (sec)	N/A	0.043	0.129	0.768	0.511	1.323	0.873	0.989	4.623

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	63	69	86	140	257	86	51
N.S.	1	1.00	0.56	0.61	0.76	1.24	2.27	0.76	0.45
time (sec)	N/A	0.050	0.127	0.622	0.503	1.397	0.634	0.806	4.677

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	64	69	86	140	257	86	50
N.S.	1	1.00	0.57	0.61	0.76	1.24	2.27	0.76	0.44
time (sec)	N/A	0.042	0.112	0.439	0.487	1.268	0.514	0.735	0.035

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	65	69	86	141	264	86	50
N.S.	1	1.00	0.58	0.61	0.76	1.25	2.34	0.76	0.44
time (sec)	N/A	0.041	0.106	0.613	0.554	1.054	0.605	3.204	4.663

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	72	74	92	148	366	92	55
N.S.	1	1.00	0.59	0.61	0.75	1.21	3.00	0.75	0.45
time (sec)	N/A	0.044	0.166	0.439	0.504	1.218	0.840	2.038	4.677

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	72	74	92	158	366	91	55
N.S.	1	1.00	0.59	0.61	0.75	1.30	3.00	0.75	0.45
time (sec)	N/A	0.045	0.152	0.466	0.624	1.055	1.392	1.019	0.077

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	77	79	97	163	384	98	59
N.S.	1	1.00	0.59	0.60	0.74	1.24	2.93	0.75	0.45
time (sec)	N/A	0.047	0.157	0.456	0.562	1.617	2.635	0.976	0.070

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	72	77	99	173	481	94	62
N.S.	1	1.00	0.56	0.60	0.77	1.34	3.73	0.73	0.48
time (sec)	N/A	0.050	0.194	0.464	0.505	1.450	3.014	1.509	4.733

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	72	77	99	175	481	94	62
N.S.	1	1.00	0.56	0.60	0.77	1.36	3.73	0.73	0.48
time (sec)	N/A	0.051	0.171	0.447	0.500	1.294	2.451	0.701	0.066

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	70	77	97	171	481	92	62
N.S.	1	1.00	0.54	0.60	0.75	1.33	3.73	0.71	0.48
time (sec)	N/A	0.051	0.158	0.470	0.503	1.090	1.664	0.944	4.687

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	72	81	99	175	481	94	61
N.S.	1	1.00	0.56	0.63	0.77	1.36	3.73	0.73	0.47
time (sec)	N/A	0.053	0.102	0.437	0.509	0.952	1.234	0.929	0.037

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	72	81	99	173	481	94	61
N.S.	1	1.00	0.56	0.63	0.77	1.34	3.73	0.73	0.47
time (sec)	N/A	0.057	0.104	0.465	0.501	1.206	1.548	2.991	4.750

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	77	82	102	178	653	99	65
N.S.	1	1.00	0.56	0.59	0.74	1.29	4.73	0.72	0.47
time (sec)	N/A	0.055	0.202	0.624	0.509	1.200	2.196	2.065	4.718

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	77	82	102	188	653	99	65
N.S.	1	1.00	0.56	0.59	0.74	1.36	4.73	0.72	0.47
time (sec)	N/A	0.052	0.211	0.630	0.515	0.914	3.604	1.313	4.721

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	82	87	107	193	678	106	69
N.S.	1	1.00	0.56	0.59	0.73	1.31	4.61	0.72	0.47
time (sec)	N/A	0.054	0.187	0.454	0.506	0.843	6.645	0.904	0.075

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	24	23	23	26	24	11
N.S.	1	1.00	1.00	1.60	1.53	1.53	1.73	1.60	0.73
time (sec)	N/A	0.006	0.020	0.186	0.515	1.063	0.097	1.107	0.032

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	100	79	69	68	108	94	68	57
N.S.	1	1.37	1.08	0.95	0.93	1.48	1.29	0.93	0.78
time (sec)	N/A	0.178	0.107	2.544	0.524	1.161	0.367	0.795	4.767

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	431	97	367	1999	540	389
N.S.	1	1.00	0.91	4.44	1.00	3.78	20.61	5.57	4.01
time (sec)	N/A	0.030	0.101	0.030	0.302	1.207	0.674	0.907	5.113

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	72	290	79	251	1221	365	272
N.S.	1	1.00	0.91	3.67	1.00	3.18	15.46	4.62	3.44
time (sec)	N/A	0.022	0.057	0.021	0.296	1.125	0.505	1.234	4.988

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	72	61	157	683	224	167
N.S.	1	1.00	0.92	1.18	1.00	2.57	11.20	3.67	2.74
time (sec)	N/A	0.015	0.042	0.029	0.267	1.202	0.355	1.645	4.793

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	51	43	85	306	117	93
N.S.	1	1.00	0.93	1.19	1.00	1.98	7.12	2.72	2.16
time (sec)	N/A	0.010	0.033	0.026	0.287	1.249	0.239	1.204	4.751

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	25	33	94	43	34
N.S.	1	1.00	1.00	1.20	1.00	1.32	3.76	1.72	1.36
time (sec)	N/A	0.005	0.021	0.014	0.299	1.777	0.126	0.895	4.801

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	88	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	2.26	0.00	-0.03
time (sec)	N/A	0.006	0.022	0.020	0.000	0.000	0.539	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	374	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	9.59	0.00	-0.03
time (sec)	N/A	0.006	0.023	0.024	0.000	0.000	2.873	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	1556	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	39.90	0.00	-0.03
time (sec)	N/A	0.005	0.023	0.036	0.000	0.000	10.048	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	0	0	92	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	2.09	0.00	-0.02
time (sec)	N/A	0.008	0.027	0.019	0.000	0.000	1.420	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	95	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	2.16	0.00	-0.02
time (sec)	N/A	0.007	0.005	0.018	0.000	0.000	0.547	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	39	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.03	0.00	-0.03
time (sec)	N/A	0.006	0.025	0.019	0.000	0.000	3.239	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	0	0	102	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	2.17	0.00	-0.02
time (sec)	N/A	0.009	0.028	0.019	0.000	0.000	4.962	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	92	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.04	0.00	-0.02
time (sec)	N/A	0.008	0.029	0.019	0.000	0.000	7.694	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	92	0	0	343	0	-1
N.S.	1	1.00	1.06	2.56	0.00	0.00	9.53	0.00	-0.03
time (sec)	N/A	0.006	0.025	0.058	0.000	0.000	3.012	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	82	73	57	110	57	55
N.S.	1	1.00	0.76	1.02	0.91	0.71	1.38	0.71	0.69
time (sec)	N/A	0.031	0.027	0.051	0.266	0.559	0.252	1.363	4.587

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	50	58	53	46	87	43	44
N.S.	1	1.00	0.85	0.98	0.90	0.78	1.47	0.73	0.75
time (sec)	N/A	0.024	0.024	0.047	0.412	0.725	0.148	1.544	4.659

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	33	34	63	29	33
N.S.	1	1.00	1.00	0.89	0.87	0.89	1.66	0.76	0.87
time (sec)	N/A	0.015	0.022	0.033	0.359	0.885	0.088	1.099	4.650

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78
time (sec)	N/A	0.002	0.002	0.039	0.295	0.846	0.056	1.158	4.627

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	39	27	77	56	33	29
N.S.	1	1.00	1.00	1.05	0.73	2.08	1.51	0.89	0.78
time (sec)	N/A	0.017	0.025	0.033	0.330	1.019	0.697	1.882	4.658

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	63	51	106	42	46	35
N.S.	1	1.00	1.00	1.34	1.09	2.26	0.89	0.98	0.74
time (sec)	N/A	0.019	0.056	0.040	0.317	1.104	0.927	1.062	4.795

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	87	73	131	92	72	54
N.S.	1	1.00	0.87	1.23	1.03	1.85	1.30	1.01	0.76
time (sec)	N/A	0.027	0.089	0.043	0.292	1.241	2.192	2.365	4.910

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	73	111	93	157	117	92	74
N.S.	1	1.00	0.77	1.17	0.98	1.65	1.23	0.97	0.78
time (sec)	N/A	0.038	0.105	0.060	0.286	1.058	5.266	1.087	4.942

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	74	82	69	146	117	64	-1
N.S.	1	1.00	0.79	0.87	0.73	1.55	1.24	0.68	-0.01
time (sec)	N/A	0.022	0.065	0.039	0.277	1.019	5.099	1.498	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	58	49	119	92	50	-1
N.S.	1	1.00	0.87	0.83	0.70	1.70	1.31	0.71	-0.01
time (sec)	N/A	0.015	0.042	0.037	0.291	1.221	2.018	1.613	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	48	36	28	94	41	37	35
N.S.	1	1.00	1.04	0.78	0.61	2.04	0.89	0.80	0.76
time (sec)	N/A	0.007	0.034	0.027	0.295	1.249	0.973	1.082	4.673

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	60	28	88	56	57	56
N.S.	1	1.00	1.07	1.43	0.67	2.10	1.33	1.36	1.33
time (sec)	N/A	0.008	0.040	0.032	0.282	1.217	0.738	1.530	4.816

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	42	59	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	2.00	2.81	0.81
time (sec)	N/A	0.003	0.040	0.037	0.277	1.240	0.368	1.352	4.567

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	37	36	38	68	112	37
N.S.	1	1.00	0.95	0.84	0.82	0.86	1.55	2.55	0.84
time (sec)	N/A	0.007	0.050	0.042	0.283	0.717	0.470	1.048	4.767

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	53	61	56	49	359	138	73
N.S.	1	1.00	0.78	0.90	0.82	0.72	5.28	2.03	1.07
time (sec)	N/A	0.012	0.057	0.052	0.280	0.767	0.654	1.384	4.707

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	64	85	76	60	575	166	93
N.S.	1	1.00	0.70	0.92	0.83	0.65	6.25	1.80	1.01
time (sec)	N/A	0.019	0.064	0.069	0.267	0.780	0.866	1.091	4.946

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	82	73	68	133	57	64
N.S.	1	1.00	0.62	1.02	0.91	0.85	1.66	0.71	0.80
time (sec)	N/A	0.032	0.033	0.048	0.298	1.026	0.368	1.135	4.604

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	58	53	57	109	43	53
N.S.	1	1.00	0.66	0.98	0.90	0.97	1.85	0.73	0.90
time (sec)	N/A	0.025	0.027	0.043	0.275	0.857	0.269	0.924	4.659

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	34	33	45	85	29	42
N.S.	1	1.00	0.74	0.89	0.87	1.18	2.24	0.76	1.11
time (sec)	N/A	0.017	0.025	0.038	0.339	0.909	0.180	0.654	4.637

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	61	14	14
N.S.	1	1.00	1.00	0.83	0.78	1.78	3.39	0.78	0.78
time (sec)	N/A	0.003	0.003	0.030	0.287	0.971	0.116	1.324	4.567

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	53	40	100	78	48	42
N.S.	1	1.00	0.93	0.98	0.74	1.85	1.44	0.89	0.78
time (sec)	N/A	0.024	0.034	0.033	0.330	1.098	1.093	1.031	4.696

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	77	63	119	88	63	47
N.S.	1	1.00	0.90	1.22	1.00	1.89	1.40	1.00	0.75
time (sec)	N/A	0.025	0.075	0.051	0.333	1.665	1.287	1.489	4.816

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	101	90	136	71	70	52
N.S.	1	1.00	0.87	1.49	1.32	2.00	1.04	1.03	0.76
time (sec)	N/A	0.026	0.095	0.044	0.370	1.731	1.754	0.870	4.905

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	73	125	110	157	119	92	72
N.S.	1	1.00	0.79	1.36	1.20	1.71	1.29	1.00	0.78
time (sec)	N/A	0.037	0.097	0.055	0.324	1.324	3.870	0.875	4.941

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	84	149	130	179	148	109	89
N.S.	1	1.00	0.72	1.28	1.12	1.54	1.28	0.94	0.77
time (sec)	N/A	0.046	0.128	0.069	0.299	1.664	11.621	0.698	5.212

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	85	98	87	168	148	76	-1
N.S.	1	1.00	0.74	0.85	0.76	1.46	1.29	0.66	-0.01
time (sec)	N/A	0.030	0.069	0.040	0.290	1.254	11.513	1.543	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	74	74	67	145	119	63	-1
N.S.	1	1.00	0.81	0.81	0.74	1.59	1.31	0.69	-0.01
time (sec)	N/A	0.021	0.067	0.036	0.274	1.787	3.638	1.020	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	52	43	124	70	49	37
N.S.	1	1.00	0.92	0.80	0.66	1.91	1.08	0.75	0.57
time (sec)	N/A	0.011	0.056	0.032	0.330	1.276	1.550	1.930	4.608

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	76	43	112	88	73	40
N.S.	1	1.00	0.94	1.21	0.68	1.78	1.40	1.16	0.63
time (sec)	N/A	0.012	0.082	0.038	0.278	1.292	1.280	1.354	5.153

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	100	66	112	78	114	-1
N.S.	1	1.00	0.93	1.64	1.08	1.84	1.28	1.87	-0.02
time (sec)	N/A	0.013	0.075	0.039	0.327	0.927	1.135	0.973	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	35	68	86	17
N.S.	1	1.00	1.00	0.86	0.81	1.67	3.24	4.10	0.81
time (sec)	N/A	0.003	0.056	0.043	0.329	0.849	0.452	0.711	5.295

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	37	36	49	94	166	71
N.S.	1	1.00	0.70	0.84	0.82	1.11	2.14	3.77	1.61
time (sec)	N/A	0.007	0.071	0.055	0.335	1.114	0.575	1.299	5.655

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	61	56	60	420	192	91
N.S.	1	1.00	0.62	0.90	0.82	0.88	6.18	2.82	1.34
time (sec)	N/A	0.013	0.081	0.075	0.291	0.856	0.792	0.700	5.710

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	85	76	71	648	220	111
N.S.	1	1.00	0.58	0.92	0.83	0.77	7.04	2.39	1.21
time (sec)	N/A	0.020	0.086	0.103	0.299	1.374	1.051	1.014	5.781

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	82	73	79	158	57	75
N.S.	1	1.00	0.62	1.02	0.91	0.99	1.98	0.71	0.94
time (sec)	N/A	0.031	0.032	0.053	0.294	1.311	0.631	1.166	4.863

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	58	53	68	133	43	64
N.S.	1	1.00	0.66	0.98	0.90	1.15	2.25	0.73	1.08
time (sec)	N/A	0.025	0.027	0.054	0.360	1.498	0.473	1.315	4.752

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	34	33	56	109	29	53
N.S.	1	1.00	0.74	0.89	0.87	1.47	2.87	0.76	1.39
time (sec)	N/A	0.016	0.026	0.039	0.297	1.609	0.349	0.977	4.744

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	43	85	14	14
N.S.	1	1.00	1.00	0.83	0.78	2.39	4.72	0.78	0.78
time (sec)	N/A	0.002	0.003	0.037	0.275	1.187	0.223	1.479	4.595

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	67	54	126	105	62	59
N.S.	1	1.00	0.86	0.93	0.75	1.75	1.46	0.86	0.82
time (sec)	N/A	0.031	0.038	0.039	0.291	1.219	2.259	1.035	4.682

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	91	76	142	112	82	66
N.S.	1	1.00	0.85	1.14	0.95	1.78	1.40	1.02	0.82
time (sec)	N/A	0.032	0.080	0.057	0.280	1.404	2.104	0.851	4.948

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	115	104	145	117	88	71
N.S.	1	1.00	0.81	1.34	1.21	1.69	1.36	1.02	0.83
time (sec)	N/A	0.034	0.091	0.063	0.273	1.408	2.389	0.664	5.007

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	70	139	127	158	99	87	72
N.S.	1	1.00	0.79	1.56	1.43	1.78	1.11	0.98	0.81
time (sec)	N/A	0.035	0.116	0.056	0.304	1.268	2.837	0.630	5.201

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	84	163	147	179	150	109	89
N.S.	1	1.00	0.74	1.44	1.30	1.58	1.33	0.96	0.79
time (sec)	N/A	0.046	0.111	0.073	0.291	1.448	7.630	0.708	5.430

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	95	187	167	201	175	126	106
N.S.	1	1.00	0.69	1.36	1.22	1.47	1.28	0.92	0.77
time (sec)	N/A	0.058	0.148	0.097	0.327	1.316	30.761	0.685	5.678

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	96	114	105	190	175	91	-1
N.S.	1	1.00	0.71	0.84	0.77	1.40	1.29	0.67	-0.01
time (sec)	N/A	0.039	0.108	0.046	0.279	1.306	30.411	0.827	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	85	90	85	167	150	77	-1
N.S.	1	1.00	0.76	0.80	0.76	1.49	1.34	0.69	-0.01
time (sec)	N/A	0.029	0.086	0.042	0.284	1.149	7.373	0.810	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	68	58	146	97	63	37
N.S.	1	1.00	0.85	0.81	0.69	1.74	1.15	0.75	0.44
time (sec)	N/A	0.016	0.078	0.035	0.288	0.927	2.632	0.946	4.460

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	92	59	140	117	87	40
N.S.	1	1.00	0.88	1.11	0.71	1.69	1.41	1.05	0.48
time (sec)	N/A	0.018	0.089	0.043	0.427	1.375	2.335	0.846	5.050

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	71	116	84	141	112	132	-1
N.S.	1	1.00	0.83	1.35	0.98	1.64	1.30	1.53	-0.01
time (sec)	N/A	0.019	0.107	0.052	0.384	1.196	2.110	0.920	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	140	104	140	105	168	-1
N.S.	1	1.00	0.83	1.71	1.27	1.71	1.28	2.05	-0.01
time (sec)	N/A	0.019	0.104	0.050	0.312	1.060	2.344	0.978	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	46	95	113	71
N.S.	1	1.00	1.00	0.86	0.81	2.19	4.52	5.38	3.38
time (sec)	N/A	0.003	0.072	0.067	0.282	0.974	0.615	1.380	5.073

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	37	36	60	121	220	91
N.S.	1	1.00	0.70	0.84	0.82	1.36	2.75	5.00	2.07
time (sec)	N/A	0.008	0.086	0.079	0.272	1.118	0.803	1.289	5.369

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	61	56	71	481	246	111
N.S.	1	1.00	0.62	0.90	0.82	1.04	7.07	3.62	1.63
time (sec)	N/A	0.014	0.096	0.116	0.301	0.883	1.081	0.779	5.618

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	85	76	82	721	274	131
N.S.	1	1.00	0.58	0.92	0.83	0.89	7.84	2.98	1.42
time (sec)	N/A	0.023	0.107	0.214	0.356	1.171	1.425	0.729	5.981

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	109	96	93	1012	300	151
N.S.	1	1.00	0.55	0.94	0.83	0.80	8.72	2.59	1.30
time (sec)	N/A	0.028	0.119	0.421	0.293	1.723	1.834	0.757	6.365

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	75	133	116	104	1346	328	171
N.S.	1	1.00	0.54	0.95	0.83	0.74	9.61	2.34	1.22
time (sec)	N/A	0.039	0.141	0.993	0.291	1.150	2.359	1.538	6.869

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	94	178	153	145	301	113	141
N.S.	1	1.00	0.58	1.11	0.95	0.90	1.87	0.70	0.88
time (sec)	N/A	0.070	0.059	0.458	0.293	1.112	3.214	1.180	4.837

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	83	154	133	134	277	99	130
N.S.	1	1.00	0.59	1.10	0.95	0.96	1.98	0.71	0.93
time (sec)	N/A	0.059	0.048	0.225	0.285	1.080	2.698	1.002	4.836

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	72	130	113	123	253	85	119
N.S.	1	1.00	0.59	1.07	0.93	1.01	2.07	0.70	0.98
time (sec)	N/A	0.051	0.039	0.127	0.289	0.787	2.181	1.786	4.809

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	61	106	93	112	230	71	108
N.S.	1	1.00	0.60	1.05	0.92	1.11	2.28	0.70	1.07
time (sec)	N/A	0.042	0.035	0.079	0.308	0.955	1.786	1.018	4.741

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	82	73	101	204	57	97
N.S.	1	1.00	0.62	1.02	0.91	1.26	2.55	0.71	1.21
time (sec)	N/A	0.034	0.032	0.064	0.342	0.863	1.483	3.831	4.735

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	58	53	90	180	43	86
N.S.	1	1.00	0.66	0.98	0.90	1.53	3.05	0.73	1.46
time (sec)	N/A	0.025	0.029	0.057	0.274	0.777	1.213	1.629	4.600

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	34	33	78	156	29	29
N.S.	1	1.00	0.74	0.89	0.87	2.05	4.11	0.76	0.76
time (sec)	N/A	0.016	0.029	0.049	0.334	0.602	0.958	1.521	4.739

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	65	133	14	14
N.S.	1	1.00	1.00	0.83	0.78	3.61	7.39	0.78	0.78
time (sec)	N/A	0.003	0.004	0.042	0.296	0.757	0.739	1.389	4.813

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	84	95	82	170	160	90	87
N.S.	1	1.00	0.78	0.88	0.76	1.57	1.48	0.83	0.81
time (sec)	N/A	0.049	0.050	0.047	0.309	1.098	10.723	1.017	5.309

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	90	119	106	188	167	116	95
N.S.	1	1.00	0.76	1.01	0.90	1.59	1.42	0.98	0.81
time (sec)	N/A	0.051	0.087	0.088	0.319	1.257	10.308	1.050	5.445

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	92	143	136	192	175	124	132
N.S.	1	1.00	0.73	1.13	1.08	1.52	1.39	0.98	1.05
time (sec)	N/A	0.054	0.112	0.082	0.298	1.094	9.775	1.119	5.651

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	92	167	156	190	175	124	149
N.S.	1	1.00	0.73	1.33	1.24	1.51	1.39	0.98	1.18
time (sec)	N/A	0.052	0.107	0.081	0.275	1.058	9.342	0.949	5.992

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	92	191	178	189	173	122	105
N.S.	1	1.00	0.72	1.49	1.39	1.48	1.35	0.95	0.82
time (sec)	N/A	0.052	0.138	0.088	0.276	1.395	9.732	0.870	6.197

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	92	215	201	202	153	121	106
N.S.	1	1.00	0.70	1.64	1.53	1.54	1.17	0.92	0.81
time (sec)	N/A	0.057	0.157	0.117	0.295	1.454	10.671	1.192	6.621

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	106	239	221	223	204	143	123
N.S.	1	1.00	0.68	1.54	1.43	1.44	1.32	0.92	0.79
time (sec)	N/A	0.070	0.164	0.214	0.282	1.912	47.160	2.558	6.678

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	117	263	241	245	0	160	140
N.S.	1	1.00	0.65	1.47	1.35	1.37	0.00	0.89	0.78
time (sec)	N/A	0.090	0.187	0.415	0.302	1.138	0.000	1.121	7.225

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	129	170	161	255	0	133	-1
N.S.	1	1.00	0.64	0.84	0.80	1.26	0.00	0.66	-0.00
time (sec)	N/A	0.078	0.149	0.069	0.301	1.130	0.000	1.040	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	118	146	141	234	0	119	-1
N.S.	1	1.00	0.66	0.82	0.79	1.31	0.00	0.67	-0.01
time (sec)	N/A	0.060	0.139	0.059	0.287	1.301	0.000	0.888	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	107	122	121	211	204	105	-1
N.S.	1	1.00	0.69	0.79	0.79	1.37	1.32	0.68	-0.01
time (sec)	N/A	0.049	0.130	0.056	0.327	1.370	46.707	0.916	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	93	100	88	190	151	91	37
N.S.	1	1.00	0.76	0.82	0.72	1.56	1.24	0.75	0.30
time (sec)	N/A	0.028	0.116	0.041	0.270	1.340	10.321	1.280	4.620

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	124	91	184	173	115	40
N.S.	1	1.00	0.77	1.01	0.74	1.50	1.41	0.93	0.33
time (sec)	N/A	0.030	0.122	0.062	0.278	0.909	9.832	0.896	6.031

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	148	120	189	175	160	-1
N.S.	1	1.00	0.74	1.16	0.94	1.48	1.37	1.25	-0.01
time (sec)	N/A	0.031	0.138	0.060	0.324	0.825	9.388	0.620	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	95	172	140	191	175	200	-1
N.S.	1	1.00	0.74	1.33	1.09	1.48	1.36	1.55	-0.01
time (sec)	N/A	0.034	0.154	0.071	0.336	0.992	9.735	0.896	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	93	196	160	187	167	240	-1
N.S.	1	1.00	0.74	1.56	1.27	1.48	1.33	1.90	-0.01
time (sec)	N/A	0.034	0.159	0.074	0.319	0.891	10.208	0.722	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	90	220	180	184	160	276	-1
N.S.	1	1.00	0.73	1.77	1.45	1.48	1.29	2.23	-0.01
time (sec)	N/A	0.035	0.164	0.092	0.298	1.094	10.938	0.859	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	68	150	167	111
N.S.	1	1.00	1.00	0.86	0.81	3.24	7.14	7.95	5.29
time (sec)	N/A	0.004	0.110	0.145	0.337	1.085	1.329	0.782	6.277

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	37	36	82	175	328	131
N.S.	1	1.00	0.70	0.84	0.82	1.86	3.98	7.45	2.98
time (sec)	N/A	0.009	0.125	0.301	0.338	1.535	1.652	0.714	6.849

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	61	56	93	604	354	151
N.S.	1	1.00	0.62	0.90	0.82	1.37	8.88	5.21	2.22
time (sec)	N/A	0.015	0.141	0.622	0.280	1.656	2.089	0.671	7.416

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	85	76	104	867	382	171
N.S.	1	1.00	0.58	0.92	0.83	1.13	9.42	4.15	1.86
time (sec)	N/A	0.022	0.152	1.543	0.302	1.452	2.606	0.806	8.173

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	109	96	115	1182	408	191
N.S.	1	1.00	0.55	0.94	0.83	0.99	10.19	3.52	1.65
time (sec)	N/A	0.029	0.177	4.020	0.291	1.523	3.235	0.693	8.840

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	75	133	116	126	1540	436	211
N.S.	1	1.00	0.54	0.95	0.83	0.90	11.00	3.11	1.51
time (sec)	N/A	0.041	0.180	10.290	0.320	2.034	3.918	0.668	9.629

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	86	157	136	137	1950	462	231
N.S.	1	1.00	0.52	0.96	0.83	0.84	11.89	2.82	1.41
time (sec)	N/A	0.052	0.229	26.683	0.310	1.606	4.816	0.605	10.465

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	41	40	28	61	34	25
N.S.	1	1.00	0.59	0.89	0.87	0.61	1.33	0.74	0.54
time (sec)	N/A	0.015	0.016	0.060	0.492	1.183	0.332	0.614	4.547

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	46	45	42	75	43	30
N.S.	1	1.00	0.78	0.73	0.71	0.67	1.19	0.68	0.48
time (sec)	N/A	0.011	0.037	0.107	0.482	1.094	4.533	0.612	0.028

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	27	26	23	44	23	20
N.S.	1	1.00	0.71	0.87	0.84	0.74	1.42	0.74	0.65
time (sec)	N/A	0.011	0.013	0.039	0.496	1.109	0.145	0.646	0.021

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	32	31	37	54	36	23
N.S.	1	1.00	0.98	0.71	0.69	0.82	1.20	0.80	0.51
time (sec)	N/A	0.007	0.029	0.077	0.614	1.114	1.639	0.622	0.034

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	27	11	16
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.80	0.73	1.07
time (sec)	N/A	0.002	0.001	0.038	0.287	1.202	0.062	0.622	0.019

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	20	19	29	22	29	16
N.S.	1	1.00	1.37	0.74	0.70	1.07	0.81	1.07	0.59
time (sec)	N/A	0.002	0.021	0.068	0.506	1.247	0.067	0.543	0.032

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	19	44	39	38	22
N.S.	1	1.00	1.00	0.83	0.63	1.47	1.30	1.27	0.73
time (sec)	N/A	0.011	0.018	0.092	0.508	1.182	0.592	0.567	0.028

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	35	34	21	35	19	40	19
N.S.	1	1.00	1.40	1.36	0.84	1.40	0.76	1.60	0.76
time (sec)	N/A	0.004	0.027	0.081	0.489	1.502	0.078	0.508	0.028

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	41	35	57	24	43	25
N.S.	1	1.00	1.00	1.05	0.90	1.46	0.62	1.10	0.64
time (sec)	N/A	0.012	0.030	0.092	0.522	1.631	0.801	0.513	0.030

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	20	34	42	27
N.S.	1	1.00	1.00	0.83	0.78	1.11	1.89	2.33	1.50
time (sec)	N/A	0.002	0.030	0.046	0.613	1.219	0.451	0.584	0.033

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	55	49	64	63	55	45
N.S.	1	1.00	0.81	0.96	0.86	1.12	1.11	0.96	0.79
time (sec)	N/A	0.015	0.045	0.092	0.494	1.448	1.865	0.600	0.031

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	41	40	28	61	52	28
N.S.	1	1.00	0.70	0.89	0.87	0.61	1.33	1.13	0.61
time (sec)	N/A	0.014	0.015	0.070	0.498	1.027	0.334	0.567	4.532

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	46	45	45	165	33	32
N.S.	1	1.00	0.83	0.73	0.71	0.71	2.62	0.52	0.51
time (sec)	N/A	0.010	0.106	0.156	0.506	1.311	4.552	0.525	4.563

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	27	26	23	44	32	23
N.S.	1	1.00	0.71	0.87	0.84	0.74	1.42	1.03	0.74
time (sec)	N/A	0.011	0.013	0.053	0.553	0.897	0.150	0.730	0.022

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	32	31	40	122	26	27
N.S.	1	1.00	1.04	0.71	0.69	0.89	2.71	0.58	0.60
time (sec)	N/A	0.007	0.080	0.130	0.501	0.855	1.727	0.640	0.035

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	18	27	11	18
N.S.	1	1.00	1.00	0.80	0.73	1.20	1.80	0.73	1.20
time (sec)	N/A	0.002	0.001	0.043	0.316	0.717	0.070	0.565	0.020

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	41	20	19	32	22	19	18
N.S.	1	1.00	1.52	0.74	0.70	1.19	0.81	0.70	0.67
time (sec)	N/A	0.003	0.041	0.108	0.595	1.357	0.069	0.551	0.024

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	28	75	40	32
N.S.	1	1.00	1.00	0.83	1.17	0.93	2.50	1.33	1.07
time (sec)	N/A	0.012	0.019	0.103	0.571	1.014	0.668	0.568	4.552

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	39	34	21	36	20	39	21
N.S.	1	1.00	1.56	1.36	0.84	1.44	0.80	1.56	0.84
time (sec)	N/A	0.004	0.040	0.132	0.522	1.043	0.084	0.501	0.027

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	41	51	38	97	45	35
N.S.	1	1.00	1.00	1.05	1.31	0.97	2.49	1.15	0.90
time (sec)	N/A	0.012	0.034	0.117	0.504	1.091	0.862	0.584	4.655

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	21	76	73	31
N.S.	1	1.00	1.00	0.83	0.78	1.17	4.22	4.06	1.72
time (sec)	N/A	0.002	0.024	0.049	0.589	1.016	0.481	0.523	0.033

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	55	65	45	139	57	49
N.S.	1	1.00	0.81	0.96	1.14	0.79	2.44	1.00	0.86
time (sec)	N/A	0.015	0.036	0.102	0.501	1.567	1.951	0.575	0.030

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	41	40	28	61	34	28
N.S.	1	1.00	0.59	0.89	0.87	0.61	1.33	0.74	0.61
time (sec)	N/A	0.013	0.016	0.076	0.482	1.625	0.333	0.580	5.314

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	61	57	42	165	44	-1
N.S.	1	1.00	0.68	0.85	0.79	0.58	2.29	0.61	-0.01
time (sec)	N/A	0.013	0.035	0.109	0.497	1.129	4.595	0.538	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	27	26	23	44	23	23
N.S.	1	1.00	0.71	0.87	0.84	0.74	1.42	0.74	0.74
time (sec)	N/A	0.011	0.013	0.059	0.496	1.267	0.148	0.559	5.511

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	47	43	37	122	37	-1
N.S.	1	1.00	0.81	0.87	0.80	0.69	2.26	0.69	-0.02
time (sec)	N/A	0.009	0.031	0.095	0.490	0.890	1.638	0.517	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	27	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.80	0.73	0.73
time (sec)	N/A	0.002	0.001	0.055	0.276	1.086	0.068	0.502	0.060

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	31	29	22	30	29
N.S.	1	1.00	1.03	0.97	0.86	0.81	0.61	0.83	0.81
time (sec)	N/A	0.004	0.022	0.085	0.491	1.308	0.068	0.690	5.032

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	19	28	80	24	24
N.S.	1	1.00	1.00	0.83	0.63	0.93	2.67	0.80	0.80
time (sec)	N/A	0.011	0.016	0.135	0.559	1.252	0.671	0.748	5.338

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	48	33	35	19	44	39
N.S.	1	1.00	1.03	1.41	0.97	1.03	0.56	1.29	1.15
time (sec)	N/A	0.005	0.026	0.097	0.537	1.244	0.082	0.802	5.548

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	41	35	38	97	29	29
N.S.	1	1.00	1.00	1.05	0.90	0.97	2.49	0.74	0.74
time (sec)	N/A	0.011	0.026	0.131	0.515	0.987	0.829	0.803	5.330

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	20	76	42	31
N.S.	1	1.00	1.00	0.83	0.78	1.11	4.22	2.33	1.72
time (sec)	N/A	0.002	0.025	0.056	0.547	0.888	0.489	0.607	5.098

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	55	49	45	139	41	43
N.S.	1	1.00	0.81	0.96	0.86	0.79	2.44	0.72	0.75
time (sec)	N/A	0.015	0.037	0.129	0.612	0.794	1.898	0.685	5.190

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	41	40	28	68	34	27
N.S.	1	1.00	0.59	0.89	0.87	0.61	1.48	0.74	0.59
time (sec)	N/A	0.014	0.017	0.048	0.490	1.248	0.344	0.657	5.127

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	53	55	45	72	83	0	-1
N.S.	1	1.00	0.74	0.76	0.62	1.00	1.15	0.00	-0.01
time (sec)	N/A	0.013	0.035	0.133	0.514	1.227	4.567	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	27	26	23	49	23	22
N.S.	1	1.00	0.71	0.87	0.84	0.74	1.58	0.74	0.71
time (sec)	N/A	0.010	0.013	0.035	0.506	1.120	0.154	0.568	5.145

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	41	31	67	61	0	-1
N.S.	1	1.00	0.89	0.76	0.57	1.24	1.13	0.00	-0.02
time (sec)	N/A	0.009	0.029	0.122	0.486	2.012	1.652	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	18	31	11	11
N.S.	1	1.00	1.00	0.80	0.73	1.20	2.07	0.73	0.73
time (sec)	N/A	0.002	0.001	0.037	0.278	1.100	0.075	0.532	0.070

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	29	19	59	34	0	28
N.S.	1	1.00	1.14	0.81	0.53	1.64	0.94	0.00	0.78
time (sec)	N/A	0.004	0.021	0.122	0.497	0.948	0.161	0.000	5.027

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	52	44	24	24
N.S.	1	1.00	1.00	0.83	1.17	1.73	1.47	0.80	0.80
time (sec)	N/A	0.011	0.015	0.114	0.493	0.653	0.610	0.534	4.732

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	39	43	21	64	32	0	41
N.S.	1	1.00	1.15	1.26	0.62	1.88	0.94	0.00	1.21
time (sec)	N/A	0.005	0.030	0.118	0.518	0.877	0.175	0.000	4.775

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	41	51	65	27	29	29
N.S.	1	1.00	1.00	1.05	1.31	1.67	0.69	0.74	0.74
time (sec)	N/A	0.011	0.023	0.140	0.580	0.758	0.822	0.530	4.792

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	37	0	31
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.06	0.00	1.72
time (sec)	N/A	0.002	0.032	0.039	0.521	0.969	0.477	0.000	4.740

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	55	65	72	68	43	43
N.S.	1	1.00	0.81	0.96	1.14	1.26	1.19	0.75	0.75
time (sec)	N/A	0.014	0.036	0.114	0.503	0.662	2.354	0.469	4.695

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	39	58	53	35	68	46	36
N.S.	1	1.00	0.70	1.04	0.95	0.62	1.21	0.82	0.64
time (sec)	N/A	0.023	0.023	0.035	0.281	0.720	0.253	0.476	4.674

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	63	51	124	95	54	-1
N.S.	1	1.00	0.86	0.86	0.70	1.70	1.30	0.74	-0.01
time (sec)	N/A	0.014	0.057	0.034	0.289	0.893	2.491	0.463	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	34	33	23	44	30	24
N.S.	1	1.00	0.75	0.94	0.92	0.64	1.22	0.83	0.67
time (sec)	N/A	0.015	0.018	0.035	0.277	1.186	0.206	0.515	4.729

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	39	31	93	42	40	56
N.S.	1	1.00	1.04	0.80	0.63	1.90	0.86	0.82	1.14
time (sec)	N/A	0.008	0.037	0.032	0.269	1.059	1.080	0.482	4.818

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87
time (sec)	N/A	0.002	0.002	0.026	0.280	1.194	0.169	0.449	4.584

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	0.80
time (sec)	N/A	0.004	0.005	0.021	0.272	0.969	0.433	0.506	0.123

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	17	60	19	22	19
N.S.	1	1.00	1.00	1.16	0.68	2.40	0.76	0.88	0.76
time (sec)	N/A	0.011	0.019	0.025	0.270	1.128	0.447	0.524	4.873

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	19	30	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	1.00	1.58	0.89
time (sec)	N/A	0.003	0.028	0.031	0.270	0.815	0.308	0.515	4.600

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	48	36	105	42	51	38
N.S.	1	1.00	1.00	0.96	0.72	2.10	0.84	1.02	0.76
time (sec)	N/A	0.018	0.052	0.038	0.284	0.908	1.149	0.537	4.753

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	37	36	27	46	55	25
N.S.	1	1.00	0.70	0.84	0.82	0.61	1.05	1.25	0.57
time (sec)	N/A	0.007	0.040	0.034	0.307	0.919	0.402	0.648	4.622

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	72	56	135	97	75	57
N.S.	1	1.00	0.84	0.97	0.76	1.82	1.31	1.01	0.77
time (sec)	N/A	0.027	0.080	0.044	0.275	1.212	2.561	0.657	4.732

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	57	53	46	68	52	41
N.S.	1	1.00	0.69	1.04	0.96	0.84	1.24	0.95	0.75
time (sec)	N/A	0.023	0.027	0.049	0.288	1.179	0.299	0.812	4.721

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	61	49	159	71	51	-1
N.S.	1	1.00	0.88	0.90	0.72	2.34	1.04	0.75	-0.01
time (sec)	N/A	0.014	0.072	0.050	0.332	1.119	1.694	0.657	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	33	32	34	41	32	22
N.S.	1	1.00	0.75	1.03	1.00	1.06	1.28	1.00	0.69
time (sec)	N/A	0.015	0.021	0.043	0.273	1.136	0.250	0.607	4.698

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	29	130	37	39	36
N.S.	1	1.00	1.07	0.86	0.67	3.02	0.86	0.91	0.84
time (sec)	N/A	0.009	0.051	0.034	0.281	1.465	0.766	0.524	0.091

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	24	24	14	14
N.S.	1	1.00	1.00	0.94	0.88	1.50	1.50	0.88	0.88
time (sec)	N/A	0.003	0.002	0.028	0.276	0.879	0.240	0.530	0.040

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	14
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88
time (sec)	N/A	0.002	0.029	0.033	0.320	0.659	0.294	0.499	0.029

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	43	31	126	184	39	33
N.S.	1	1.00	1.00	1.05	0.76	3.07	4.49	0.95	0.80
time (sec)	N/A	0.019	0.034	0.037	0.331	0.677	0.788	0.502	4.776

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	35	34	35	46	50	35
N.S.	1	1.00	0.74	0.92	0.89	0.92	1.21	1.32	0.92
time (sec)	N/A	0.006	0.042	0.046	0.275	0.645	0.405	0.498	4.635

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	67	51	171	73	72	53
N.S.	1	1.00	0.87	0.97	0.74	2.48	1.06	1.04	0.77
time (sec)	N/A	0.026	0.062	0.064	0.275	0.792	1.735	0.467	4.937

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	42	59	54	50	233	106	38
N.S.	1	1.00	0.64	0.89	0.82	0.76	3.53	1.61	0.58
time (sec)	N/A	0.011	0.058	0.056	0.348	0.821	0.579	0.495	5.125

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	72	83	89	227	367	65	-1
N.S.	1	1.00	0.79	0.91	0.98	2.49	4.03	0.71	-0.01
time (sec)	N/A	0.022	0.111	0.095	0.285	0.735	2.734	0.480	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	39	57	52	58	138	44	38
N.S.	1	1.00	0.72	1.06	0.96	1.07	2.56	0.81	0.70
time (sec)	N/A	0.023	0.028	0.054	0.315	0.643	0.357	0.491	5.201

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	58	59	65	199	303	51	-1
N.S.	1	1.00	0.91	0.92	1.02	3.11	4.73	0.80	-0.02
time (sec)	N/A	0.015	0.097	0.039	0.298	0.816	1.444	0.461	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	34	33	47	92	24	24
N.S.	1	1.00	0.78	0.94	0.92	1.31	2.56	0.67	0.67
time (sec)	N/A	0.015	0.022	0.036	0.300	0.691	0.347	0.481	5.174

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	54	34	37	44	17	17
N.S.	1	1.00	1.00	2.57	1.62	1.76	2.10	0.81	0.81
time (sec)	N/A	0.003	0.045	0.032	0.273	0.694	0.369	0.493	5.128

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	35	46	14	14
N.S.	1	1.00	1.00	0.83	0.78	1.94	2.56	0.78	0.78
time (sec)	N/A	0.002	0.003	0.033	0.299	0.898	0.329	0.631	4.944

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	32	31	47	95	27	28
N.S.	1	1.00	0.74	0.82	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.004	0.043	0.034	0.286	1.414	0.407	1.102	4.885

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	62	45	197	740	50	47
N.S.	1	1.00	0.92	1.05	0.76	3.34	12.54	0.85	0.80
time (sec)	N/A	0.025	0.057	0.049	0.284	1.762	1.377	1.332	5.199

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	56	50	59	165	64	42
N.S.	1	1.00	0.70	0.93	0.83	0.98	2.75	1.07	0.70
time (sec)	N/A	0.009	0.061	0.063	0.357	1.274	0.605	1.581	5.181

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	71	86	66	241	864	73	73
N.S.	1	1.00	0.81	0.98	0.75	2.74	9.82	0.83	0.83
time (sec)	N/A	0.036	0.082	0.093	0.347	0.956	2.789	0.957	5.253

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	53	80	72	72	354	121	78
N.S.	1	1.00	0.62	0.93	0.84	0.84	4.12	1.41	0.91
time (sec)	N/A	0.016	0.066	0.077	0.278	1.271	0.847	0.982	5.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	94	127	285	359	3181	91	-1
N.S.	1	1.00	0.72	0.97	2.18	2.74	24.28	0.69	-0.01
time (sec)	N/A	0.037	0.166	0.127	0.338	1.479	24.895	0.910	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	61	105	92	102	454	72	80
N.S.	1	1.00	0.65	1.12	0.98	1.09	4.83	0.77	0.85
time (sec)	N/A	0.039	0.033	0.102	0.303	0.907	0.884	1.057	4.945

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	80	103	255	331	2980	78	-1
N.S.	1	1.00	0.75	0.97	2.41	3.12	28.11	0.74	-0.01
time (sec)	N/A	0.030	0.145	0.054	0.295	1.160	13.187	0.808	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	82	73	91	364	55	63
N.S.	1	1.00	0.67	1.09	0.97	1.21	4.85	0.73	0.84
time (sec)	N/A	0.030	0.031	0.052	0.290	1.475	0.887	0.812	4.880

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	144	103	59	95	17	68
N.S.	1	1.00	1.00	6.86	4.90	2.81	4.52	0.81	3.24
time (sec)	N/A	0.003	0.078	0.039	0.310	1.455	0.730	0.841	4.764

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	58	53	80	272	41	41
N.S.	1	1.00	0.66	0.98	0.90	1.36	4.61	0.69	0.69
time (sec)	N/A	0.024	0.029	0.048	0.362	1.400	0.871	0.555	4.805

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	120	85	71	199	29	68
N.S.	1	1.00	0.70	2.73	1.93	1.61	4.52	0.66	1.55
time (sec)	N/A	0.008	0.093	0.042	0.306	1.227	0.777	0.539	4.876

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	34	33	69	180	24	24
N.S.	1	1.00	0.74	0.89	0.87	1.82	4.74	0.63	0.63
time (sec)	N/A	0.015	0.028	0.047	0.270	1.013	0.864	0.624	4.824

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	96	70	82	517	43	70
N.S.	1	1.00	0.62	1.41	1.03	1.21	7.60	0.63	1.03
time (sec)	N/A	0.013	0.078	0.043	0.295	0.929	0.957	0.543	4.761

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	57	90	14	14
N.S.	1	1.00	1.00	0.83	0.78	3.17	5.00	0.78	0.78
time (sec)	N/A	0.003	0.003	0.044	0.317	1.003	0.850	0.526	4.597

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	74	61	91	1265	55	61
N.S.	1	1.00	0.66	0.96	0.79	1.18	16.43	0.71	0.79
time (sec)	N/A	0.011	0.068	0.043	0.274	1.025	1.028	0.584	4.602

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	76	100	73	329	5250	81	75
N.S.	1	1.00	0.80	1.05	0.77	3.46	55.26	0.85	0.79
time (sec)	N/A	0.040	0.070	0.075	0.310	1.854	3.897	0.545	4.853

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	98	82	103	400	90	76
N.S.	1	1.00	0.64	0.98	0.82	1.03	4.00	0.90	0.76
time (sec)	N/A	0.018	0.091	0.099	0.295	1.450	1.396	0.621	4.715

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	93	124	96	373	5540	104	113
N.S.	1	1.00	0.74	0.98	0.76	2.96	43.97	0.83	0.90
time (sec)	N/A	0.052	0.093	0.134	0.335	1.359	7.461	0.640	4.959

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	75	122	108	116	668	147	97
N.S.	1	1.00	0.57	0.92	0.82	0.88	5.06	1.11	0.73
time (sec)	N/A	0.029	0.099	0.138	0.307	1.416	1.974	0.629	4.803

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	41	40	23	44	34	21
N.S.	1	1.00	0.59	0.89	0.87	0.50	0.96	0.74	0.46
time (sec)	N/A	0.014	0.015	0.050	0.504	1.051	0.234	0.569	0.025

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	34	33	37	39	36	25
N.S.	1	1.00	0.98	0.76	0.73	0.82	0.87	0.80	0.56
time (sec)	N/A	0.007	0.036	0.081	0.587	1.365	0.157	0.769	0.031

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	27	26	18	27	23	15
N.S.	1	1.00	0.71	0.87	0.84	0.58	0.87	0.74	0.48
time (sec)	N/A	0.010	0.012	0.033	0.539	1.195	0.101	1.511	0.018

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	20	19	29	22	29	17
N.S.	1	1.00	1.37	0.74	0.70	1.07	0.81	1.07	0.63
time (sec)	N/A	0.004	0.027	0.071	0.496	0.889	0.069	1.547	0.033

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	10	11	9
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.60
time (sec)	N/A	0.002	0.001	0.029	0.298	1.027	0.048	1.386	0.015

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	7	6	16	7	29	6
N.S.	1	1.00	2.00	0.70	0.60	1.60	0.70	2.90	0.60
time (sec)	N/A	0.001	0.012	0.057	0.496	0.899	0.046	1.279	0.027

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	9	35	8	29	12
N.S.	1	1.00	1.00	0.75	0.45	1.75	0.40	1.45	0.60
time (sec)	N/A	0.007	0.015	0.069	0.498	1.086	0.442	1.179	0.037

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	18	15	23	12
N.S.	1	1.00	1.00	0.83	0.78	1.00	0.83	1.28	0.67
time (sec)	N/A	0.002	0.021	0.034	0.484	0.935	0.360	0.804	0.022

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	24	57	44	43	25
N.S.	1	1.00	1.00	0.77	0.62	1.46	1.13	1.10	0.64
time (sec)	N/A	0.011	0.031	0.079	0.504	1.038	1.055	1.934	0.028

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	30	29	28	32	42	19
N.S.	1	1.00	0.68	0.81	0.78	0.76	0.86	1.14	0.51
time (sec)	N/A	0.005	0.031	0.047	0.633	1.385	0.641	1.167	0.021

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	44	38	64	63	55	33
N.S.	1	1.00	0.81	0.77	0.67	1.12	1.11	0.96	0.58
time (sec)	N/A	0.016	0.039	0.089	0.499	1.436	2.483	1.226	0.029

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	41	40	23	46	43	23
N.S.	1	1.00	0.59	0.89	0.87	0.50	1.00	0.93	0.50
time (sec)	N/A	0.014	0.015	0.058	0.496	1.191	0.235	1.299	0.041

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	34	33	40	39	26	27
N.S.	1	1.00	1.04	0.76	0.73	0.89	0.87	0.58	0.60
time (sec)	N/A	0.007	0.067	0.128	0.536	1.509	0.157	0.992	0.030

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	27	26	18	29	23	18
N.S.	1	1.00	0.71	0.87	0.84	0.58	0.94	0.74	0.58
time (sec)	N/A	0.010	0.012	0.050	0.501	0.947	0.101	1.221	0.017

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	40	20	19	32	22	19	19
N.S.	1	1.00	1.48	0.74	0.70	1.19	0.81	0.70	0.70
time (sec)	N/A	0.004	0.043	0.114	0.518	1.110	0.075	0.847	0.022

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	0.73
time (sec)	N/A	0.002	0.001	0.036	0.276	1.053	0.051	1.297	4.556

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	24	7	6	19	7	19	6
N.S.	1	1.00	2.40	0.70	0.60	1.90	0.70	1.90	0.60
time (sec)	N/A	0.001	0.018	0.108	0.488	1.306	0.050	0.976	0.008

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	25	18	26	31	20
N.S.	1	1.00	1.00	0.75	1.25	0.90	1.30	1.55	1.00
time (sec)	N/A	0.007	0.014	0.079	0.498	1.214	0.463	1.201	0.116

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	36	33	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.00	1.83	0.78
time (sec)	N/A	0.002	0.025	0.043	0.500	1.269	0.375	1.206	0.022

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	40	38	99	45	35
N.S.	1	1.00	1.00	0.77	1.03	0.97	2.54	1.15	0.90
time (sec)	N/A	0.012	0.030	0.095	0.525	1.873	1.095	1.067	0.026

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	30	29	21	80	73	22
N.S.	1	1.00	0.68	0.81	0.78	0.57	2.16	1.97	0.59
time (sec)	N/A	0.005	0.036	0.061	0.605	1.033	0.639	1.075	0.021

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	44	54	45	136	57	49
N.S.	1	1.00	0.81	0.77	0.95	0.79	2.39	1.00	0.86
time (sec)	N/A	0.018	0.033	0.092	0.495	1.428	2.540	0.967	4.505

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	41	40	23	44	34	22
N.S.	1	1.00	0.59	0.89	0.87	0.50	0.96	0.74	0.48
time (sec)	N/A	0.014	0.014	0.055	0.490	1.402	0.239	1.213	4.740

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	49	45	37	39	37	-1
N.S.	1	1.00	0.81	0.91	0.83	0.69	0.72	0.69	-0.02
time (sec)	N/A	0.009	0.036	0.096	0.508	1.424	0.158	0.806	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	27	26	18	27	23	18
N.S.	1	1.00	0.71	0.87	0.84	0.58	0.87	0.74	0.58
time (sec)	N/A	0.010	0.012	0.051	0.487	1.107	0.105	0.916	4.861

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	31	29	22	30	29
N.S.	1	1.00	1.03	0.97	0.86	0.81	0.61	0.83	0.81
time (sec)	N/A	0.005	0.029	0.094	0.498	1.356	0.070	0.798	0.101

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.73
time (sec)	N/A	0.002	0.001	0.047	0.278	1.333	0.047	0.903	0.145

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	22	18	16	7	30	16
N.S.	1	1.00	2.26	1.16	0.95	0.84	0.37	1.58	0.84
time (sec)	N/A	0.003	0.003	0.078	0.484	1.739	0.049	0.789	4.746

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	9	18	26	14	20
N.S.	1	1.00	1.00	0.75	0.45	0.90	1.30	0.70	1.00
time (sec)	N/A	0.007	0.015	0.121	0.501	1.546	0.465	0.780	0.122

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	18	37	23	14
N.S.	1	1.00	1.00	0.83	0.78	1.00	2.06	1.28	0.78
time (sec)	N/A	0.002	0.022	0.051	0.498	1.411	0.378	0.704	4.747

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	24	38	99	29	29
N.S.	1	1.00	1.00	0.77	0.62	0.97	2.54	0.74	0.74
time (sec)	N/A	0.012	0.026	0.132	0.534	2.351	1.083	0.908	4.881

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	30	29	28	68	42	31
N.S.	1	1.00	0.68	0.81	0.78	0.76	1.84	1.14	0.84
time (sec)	N/A	0.005	0.028	0.056	0.494	1.593	0.633	0.886	4.874

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	44	38	45	136	41	57
N.S.	1	1.00	0.81	0.77	0.67	0.79	2.39	0.72	1.00
time (sec)	N/A	0.017	0.035	0.127	0.495	1.731	2.494	1.096	5.319

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	41	40	23	49	34	23
N.S.	1	1.00	0.59	0.89	0.87	0.50	1.07	0.74	0.50
time (sec)	N/A	0.015	0.014	0.037	0.489	0.998	0.241	1.113	5.295

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	43	33	67	53	0	-1
N.S.	1	1.00	0.91	0.80	0.61	1.24	0.98	0.00	-0.02
time (sec)	N/A	0.009	0.036	0.122	0.484	0.759	0.248	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	27	26	18	31	23	18
N.S.	1	1.00	0.71	0.87	0.84	0.58	1.00	0.74	0.58
time (sec)	N/A	0.010	0.012	0.033	0.505	1.453	0.110	0.802	5.067

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	29	19	59	36	0	31
N.S.	1	1.00	1.14	0.81	0.53	1.64	1.00	0.00	0.86
time (sec)	N/A	0.005	0.025	0.110	0.486	0.902	0.161	0.000	0.124

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	14	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.93	0.73	0.73
time (sec)	N/A	0.002	0.001	0.028	0.282	0.922	0.058	1.055	4.986

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	6	47	17	0	15
N.S.	1	1.00	1.00	0.84	0.32	2.47	0.89	0.00	0.79
time (sec)	N/A	0.002	0.003	0.095	0.513	1.188	0.140	0.000	0.107

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	25	43	8	14	14
N.S.	1	1.00	1.00	0.75	1.25	2.15	0.40	0.70	0.70
time (sec)	N/A	0.007	0.013	0.098	0.511	0.958	0.451	0.598	5.272

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	15	0	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.83	0.00	0.78
time (sec)	N/A	0.002	0.024	0.032	0.500	1.314	0.384	0.000	5.038

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	40	65	46	29	29
N.S.	1	1.00	1.00	0.77	1.03	1.67	1.18	0.74	0.74
time (sec)	N/A	0.011	0.023	0.108	0.515	1.851	1.089	0.526	5.179

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	30	29	21	36	0	31
N.S.	1	1.00	0.68	0.81	0.78	0.57	0.97	0.00	0.84
time (sec)	N/A	0.005	0.035	0.039	0.493	0.829	0.627	0.000	5.082

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	44	54	72	65	43	60
N.S.	1	1.00	0.81	0.77	0.95	1.26	1.14	0.75	1.05
time (sec)	N/A	0.016	0.028	0.115	0.504	1.525	2.530	0.500	5.070

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	28	21	11	65	14	35	11
N.S.	1	1.00	1.65	1.24	0.65	3.82	0.82	2.06	0.65
time (sec)	N/A	0.002	0.019	0.039	0.305	1.711	0.404	0.780	0.036

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	21	11	58	37	41	15
N.S.	1	1.00	1.94	1.24	0.65	3.41	2.18	2.41	0.88
time (sec)	N/A	0.002	0.034	0.035	0.489	1.544	0.437	0.698	0.036

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	24	57	37	36	20
N.S.	1	1.00	1.00	0.84	0.96	2.28	1.48	1.44	0.80
time (sec)	N/A	0.004	0.005	0.033	0.280	1.351	0.443	0.769	0.082

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	12	68	17	42	25
N.S.	1	1.00	1.00	0.81	0.46	2.62	0.65	1.62	0.96
time (sec)	N/A	0.003	0.005	0.033	0.282	1.148	0.414	0.677	0.092

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	21	13	59	17	36	20
N.S.	1	1.00	1.47	1.11	0.68	3.11	0.89	1.89	1.05
time (sec)	N/A	0.004	0.019	0.046	0.314	0.849	0.423	0.609	5.119

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	21	13	62	46	43	25
N.S.	1	1.00	1.74	1.11	0.68	3.26	2.42	2.26	1.32
time (sec)	N/A	0.002	0.023	0.039	0.490	1.622	0.445	1.189	0.101

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	26	74	46	41	22
N.S.	1	1.00	1.00	0.85	0.96	2.74	1.70	1.52	0.81
time (sec)	N/A	0.005	0.005	0.038	0.288	1.458	0.445	1.349	0.129

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	14	74	20	47	27
N.S.	1	1.00	1.00	0.82	0.50	2.64	0.71	1.68	0.96
time (sec)	N/A	0.004	0.005	0.036	0.286	1.232	0.418	2.021	5.095

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	0.80
time (sec)	N/A	0.004	0.003	0.000	0.336	1.094	0.470	0.998	0.002

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	13	72	46	43	25
N.S.	1	1.00	1.00	0.81	0.50	2.77	1.77	1.65	0.96
time (sec)	N/A	0.003	0.004	0.033	0.570	1.047	0.464	0.907	0.114

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	26	63	46	41	22
N.S.	1	1.00	1.00	0.85	0.96	2.33	1.70	1.52	0.81
time (sec)	N/A	0.004	0.005	0.030	0.303	1.207	0.477	0.697	0.125

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	14	74	20	47	27
N.S.	1	1.00	1.00	0.82	0.50	2.64	0.71	1.68	0.96
time (sec)	N/A	0.004	0.005	0.031	0.307	1.501	0.446	0.762	5.125

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	6	23	19	28	14
N.S.	1	1.00	1.00	0.94	0.38	1.44	1.19	1.75	0.88
time (sec)	N/A	0.002	0.004	0.041	0.526	1.210	0.439	0.648	4.738

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	103	152	0	76	46	0	-1
N.S.	1	1.00	0.56	0.83	0.00	0.41	0.25	0.00	-0.01
time (sec)	N/A	0.092	10.048	0.115	0.000	0.215	17.360	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	85	221	0	71	46	0	-1
N.S.	1	1.00	0.28	0.73	0.00	0.24	0.15	0.00	-0.00
time (sec)	N/A	0.167	10.054	0.069	0.000	0.279	5.192	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	85	138	0	57	46	0	-1
N.S.	1	1.00	0.56	0.90	0.00	0.37	0.30	0.00	-0.01
time (sec)	N/A	0.062	9.983	0.060	0.000	0.327	1.387	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	56	205	0	48	46	0	-1
N.S.	1	1.00	0.21	0.76	0.00	0.18	0.17	0.00	-0.00
time (sec)	N/A	0.135	8.108	0.052	0.000	0.259	0.503	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	54	119	0	42	46	0	-1
N.S.	1	1.00	0.43	0.94	0.00	0.33	0.37	0.00	-0.01
time (sec)	N/A	0.053	7.191	0.050	0.000	0.266	0.459	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	54	194	0	49	49	0	-1
N.S.	1	1.00	0.21	0.74	0.00	0.19	0.19	0.00	-0.00
time (sec)	N/A	0.136	10.015	0.066	0.000	0.315	0.635	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	56	120	0	44	49	0	-1
N.S.	1	1.00	0.44	0.95	0.00	0.35	0.39	0.00	-0.01
time (sec)	N/A	0.051	10.016	0.071	0.000	0.252	1.470	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	56	219	0	63	53	0	-1
N.S.	1	1.00	0.18	0.72	0.00	0.21	0.17	0.00	-0.00
time (sec)	N/A	0.158	10.015	0.074	0.000	0.272	5.360	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	102	163	0	90	46	0	-1
N.S.	1	1.00	0.48	0.77	0.00	0.42	0.22	0.00	-0.00
time (sec)	N/A	0.090	10.054	0.053	0.000	0.362	27.906	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	89	232	0	85	46	0	-1
N.S.	1	1.00	0.27	0.71	0.00	0.26	0.14	0.00	-0.00
time (sec)	N/A	0.177	10.052	0.057	0.000	0.267	9.323	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	89	150	0	69	46	0	-1
N.S.	1	1.00	0.49	0.83	0.00	0.38	0.25	0.00	-0.01
time (sec)	N/A	0.075	10.047	0.043	0.000	0.316	2.797	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	57	218	0	63	46	0	-1
N.S.	1	1.00	0.19	0.73	0.00	0.21	0.15	0.00	-0.00
time (sec)	N/A	0.156	10.015	0.044	0.000	0.334	1.389	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	55	134	0	55	46	0	-1
N.S.	1	1.00	0.36	0.88	0.00	0.36	0.30	0.00	-0.01
time (sec)	N/A	0.061	9.698	0.041	0.000	0.170	1.006	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	55	208	0	60	49	0	-1
N.S.	1	1.00	0.19	0.70	0.00	0.20	0.17	0.00	-0.00
time (sec)	N/A	0.156	10.020	0.056	0.000	0.355	1.031	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	57	125	0	53	49	0	-1
N.S.	1	1.00	0.38	0.82	0.00	0.35	0.32	0.00	-0.01
time (sec)	N/A	0.062	10.013	0.048	0.000	0.371	2.163	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	57	216	0	60	53	0	-1
N.S.	1	1.00	0.19	0.73	0.00	0.20	0.18	0.00	-0.00
time (sec)	N/A	0.154	10.013	0.058	0.000	0.339	5.436	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	57	135	0	53	53	0	-1
N.S.	1	1.00	0.38	0.89	0.00	0.35	0.35	0.00	-0.01
time (sec)	N/A	0.064	10.021	0.077	0.000	0.317	19.521	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	57	234	0	78	53	0	-1
N.S.	1	1.00	0.17	0.71	0.00	0.24	0.16	0.00	-0.00
time (sec)	N/A	0.181	10.028	0.086	0.000	0.415	51.685	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	74	237	0	56	53	0	-1
N.S.	1	1.00	0.58	1.85	0.00	0.44	0.41	0.00	-0.01
time (sec)	N/A	0.046	9.952	0.143	0.000	0.264	5.115	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	74	133	0	45	53	0	-1
N.S.	1	1.00	0.63	1.14	0.00	0.38	0.45	0.00	-0.01
time (sec)	N/A	0.055	8.372	0.097	0.000	0.248	1.391	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	51	229	0	39	53	0	-1
N.S.	1	1.00	0.52	2.31	0.00	0.39	0.54	0.00	-0.01
time (sec)	N/A	0.030	7.334	0.078	0.000	0.407	0.484	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	51	124	0	40	53	0	-1
N.S.	1	1.00	0.54	1.32	0.00	0.43	0.56	0.00	-0.01
time (sec)	N/A	0.036	6.814	0.081	0.000	0.252	0.438	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	51	225	0	46	56	0	-1
N.S.	1	1.00	0.52	2.30	0.00	0.47	0.57	0.00	-0.01
time (sec)	N/A	0.030	10.014	0.099	0.000	0.184	0.601	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	51	129	0	46	49	0	-1
N.S.	1	1.00	0.53	1.34	0.00	0.48	0.51	0.00	-0.01
time (sec)	N/A	0.037	10.013	0.086	0.000	0.303	1.448	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	87	141	0	62	44	0	-1
N.S.	1	1.00	0.56	0.90	0.00	0.40	0.28	0.00	-0.01
time (sec)	N/A	0.063	10.031	0.053	0.000	0.296	11.470	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	69	210	0	54	44	0	-1
N.S.	1	1.00	0.25	0.77	0.00	0.20	0.16	0.00	-0.00
time (sec)	N/A	0.136	10.034	0.052	0.000	0.225	3.497	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	69	125	0	41	44	0	-1
N.S.	1	1.00	0.54	0.98	0.00	0.32	0.35	0.00	-0.01
time (sec)	N/A	0.051	10.029	0.049	0.000	0.493	0.897	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	56	132	0	27	44	0	-1
N.S.	1	1.00	0.24	0.56	0.00	0.11	0.19	0.00	-0.00
time (sec)	N/A	0.117	10.014	0.040	0.000	0.264	0.430	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	54	104	0	22	44	0	-1
N.S.	1	1.00	0.56	1.07	0.00	0.23	0.45	0.00	-0.01
time (sec)	N/A	0.041	10.025	0.039	0.000	0.307	0.512	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	54	196	0	51	48	0	-1
N.S.	1	1.00	0.20	0.73	0.00	0.19	0.18	0.00	-0.00
time (sec)	N/A	0.137	10.015	0.048	0.000	0.446	0.780	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	56	123	0	45	48	0	-1
N.S.	1	1.00	0.43	0.95	0.00	0.35	0.37	0.00	-0.01
time (sec)	N/A	0.049	10.015	0.052	0.000	0.264	1.993	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	56	219	0	65	51	0	-1
N.S.	1	1.00	0.18	0.72	0.00	0.21	0.17	0.00	-0.00
time (sec)	N/A	0.161	10.016	0.055	0.000	0.331	7.507	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	74	128	0	86	44	0	-1
N.S.	1	1.00	0.48	0.84	0.00	0.56	0.29	0.00	-0.01
time (sec)	N/A	0.060	10.026	0.091	0.000	0.297	14.036	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	60	197	0	76	44	0	-1
N.S.	1	1.00	0.23	0.74	0.00	0.29	0.17	0.00	-0.00
time (sec)	N/A	0.133	10.036	0.066	0.000	0.662	4.080	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	59	115	0	61	44	0	-1
N.S.	1	1.00	0.47	0.92	0.00	0.49	0.35	0.00	-0.01
time (sec)	N/A	0.052	10.021	0.045	0.000	0.250	1.112	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	59	197	0	65	44	0	-1
N.S.	1	1.00	0.22	0.74	0.00	0.24	0.17	0.00	-0.00
time (sec)	N/A	0.139	10.021	0.048	0.000	0.298	0.599	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	59	114	0	58	44	0	-1
N.S.	1	1.00	0.47	0.90	0.00	0.46	0.35	0.00	-0.01
time (sec)	N/A	0.052	10.020	0.051	0.000	0.187	0.844	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	57	197	0	83	48	0	-1
N.S.	1	1.00	0.19	0.67	0.00	0.28	0.16	0.00	-0.00
time (sec)	N/A	0.162	10.013	0.082	0.000	0.320	1.614	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	59	124	0	79	48	0	-1
N.S.	1	1.00	0.38	0.81	0.00	0.51	0.31	0.00	-0.01
time (sec)	N/A	0.064	10.016	0.078	0.000	0.172	4.469	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	59	219	0	101	51	0	-1
N.S.	1	1.00	0.18	0.66	0.00	0.31	0.15	0.00	-0.00
time (sec)	N/A	0.178	10.020	0.079	0.000	0.226	15.713	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	80	219	0	108	44	0	-1
N.S.	1	1.00	0.52	1.41	0.00	0.70	0.28	0.00	-0.01
time (sec)	N/A	0.062	10.054	0.076	0.000	0.222	13.839	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	74	385	0	118	44	0	-1
N.S.	1	1.00	0.24	1.27	0.00	0.39	0.14	0.00	-0.00
time (sec)	N/A	0.159	10.036	0.075	0.000	0.313	4.063	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	79	218	0	98	44	0	-1
N.S.	1	1.00	0.51	1.40	0.00	0.63	0.28	0.00	-0.01
time (sec)	N/A	0.065	10.049	0.056	0.000	0.235	2.104	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	59	382	0	103	44	0	-1
N.S.	1	1.00	0.20	1.26	0.00	0.34	0.15	0.00	-0.00
time (sec)	N/A	0.159	10.017	0.055	0.000	0.277	1.489	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	79	216	0	97	44	0	-1
N.S.	1	1.00	0.50	1.38	0.00	0.62	0.28	0.00	-0.01
time (sec)	N/A	0.061	10.027	0.081	0.000	0.232	2.389	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	57	384	0	119	48	0	-1
N.S.	1	1.00	0.17	1.15	0.00	0.36	0.14	0.00	-0.00
time (sec)	N/A	0.189	10.016	0.108	0.000	0.188	4.455	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	59	227	0	115	48	0	-1
N.S.	1	1.00	0.32	1.23	0.00	0.62	0.26	0.00	-0.01
time (sec)	N/A	0.077	10.025	0.115	0.000	0.345	8.635	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	59	410	0	137	51	0	-1
N.S.	1	1.00	0.16	1.13	0.00	0.38	0.14	0.00	-0.00
time (sec)	N/A	0.205	10.031	0.100	0.000	0.254	27.709	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	61	235	0	50	51	0	-1
N.S.	1	1.00	0.57	2.20	0.00	0.47	0.48	0.00	-0.01
time (sec)	N/A	0.029	10.035	0.092	0.000	0.473	3.476	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	61	131	0	42	51	0	-1
N.S.	1	1.00	0.69	1.49	0.00	0.48	0.58	0.00	-0.01
time (sec)	N/A	0.035	10.019	0.087	0.000	0.265	0.891	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	165	0	20	51	0	-1
N.S.	1	1.00	0.79	2.46	0.00	0.30	0.76	0.00	-0.01
time (sec)	N/A	0.021	10.021	0.062	0.000	0.136	0.422	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	117	0	21	51	0	-1
N.S.	1	1.00	0.89	1.86	0.00	0.33	0.81	0.00	-0.02
time (sec)	N/A	0.026	10.030	0.056	0.000	0.321	0.502	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	51	228	0	48	54	0	-1
N.S.	1	1.00	0.48	2.13	0.00	0.45	0.50	0.00	-0.01
time (sec)	N/A	0.031	10.022	0.076	0.000	0.334	0.764	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	53	132	0	47	54	0	-1
N.S.	1	1.00	0.54	1.35	0.00	0.48	0.55	0.00	-0.01
time (sec)	N/A	0.036	10.016	0.080	0.000	0.262	1.986	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	59	230	0	74	51	0	-1
N.S.	1	1.00	0.54	2.09	0.00	0.67	0.46	0.00	-0.01
time (sec)	N/A	0.032	10.032	0.086	0.000	0.170	4.032	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	59	126	0	63	51	0	-1
N.S.	1	1.00	0.63	1.34	0.00	0.67	0.54	0.00	-0.01
time (sec)	N/A	0.037	10.017	0.086	0.000	0.245	1.018	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	58	227	0	63	51	0	-1
N.S.	1	1.00	0.57	2.25	0.00	0.62	0.50	0.00	-0.01
time (sec)	N/A	0.029	8.566	0.066	0.000	0.255	0.592	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	59	122	0	61	51	0	-1
N.S.	1	1.00	0.61	1.27	0.00	0.64	0.53	0.00	-0.01
time (sec)	N/A	0.034	9.836	0.072	0.000	0.252	0.829	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	58	228	0	76	54	0	-1
N.S.	1	1.00	0.41	1.63	0.00	0.54	0.39	0.00	-0.01
time (sec)	N/A	0.040	10.022	0.080	0.000	0.438	1.607	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	58	133	0	80	54	0	-1
N.S.	1	1.00	0.44	1.01	0.00	0.61	0.41	0.00	-0.01
time (sec)	N/A	0.046	10.024	0.074	0.000	0.363	4.685	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	66	0	0	36	0	-1
N.S.	1	1.00	1.14	3.14	0.00	0.00	1.71	0.00	-0.05
time (sec)	N/A	0.007	10.022	0.115	0.000	0.000	0.386	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	23	73	0	13	32	0	-1
N.S.	1	1.00	0.34	1.09	0.00	0.19	0.48	0.00	-0.01
time (sec)	N/A	0.028	10.022	0.083	0.000	0.174	0.362	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	64	66	0	0	0	54	0	-1
N.S.	1	1.28	1.32	0.00	0.00	0.00	1.08	0.00	-0.02
time (sec)	N/A	0.014	0.190	0.011	0.000	0.000	1.454	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	54	0	-1
N.S.	1	1.26	1.30	0.00	0.00	0.00	1.08	0.00	-0.02
time (sec)	N/A	0.014	0.137	0.011	0.000	0.000	0.552	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	53	0	-1
N.S.	1	1.26	1.30	0.00	0.00	0.00	1.06	0.00	-0.02
time (sec)	N/A	0.013	0.176	0.013	0.000	0.000	0.476	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	66	68	0	0	0	53	0	-1
N.S.	1	1.38	1.42	0.00	0.00	0.00	1.10	0.00	-0.02
time (sec)	N/A	0.014	0.203	0.011	0.000	0.000	0.675	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	66	68	0	0	0	53	0	-1
N.S.	1	1.32	1.36	0.00	0.00	0.00	1.06	0.00	-0.02
time (sec)	N/A	0.014	0.291	0.012	0.000	0.000	1.509	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	54	0	-1
N.S.	1	1.26	1.30	0.00	0.00	0.00	1.08	0.00	-0.02
time (sec)	N/A	0.015	0.170	0.013	0.000	0.000	1.286	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	48	0	-1
N.S.	1	1.26	1.30	0.00	0.00	0.00	0.96	0.00	-0.02
time (sec)	N/A	0.015	0.177	0.013	0.000	0.000	0.872	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	53	0	-1
N.S.	1	1.26	1.30	0.00	0.00	0.00	1.06	0.00	-0.02
time (sec)	N/A	0.013	0.004	0.000	0.000	0.000	0.472	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	57	57	0	0	0	41	0	-1
N.S.	1	1.24	1.24	0.00	0.00	0.00	0.89	0.00	-0.02
time (sec)	N/A	0.013	0.166	0.013	0.000	0.000	1.460	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	66	65	0	0	0	53	0	-1
N.S.	1	1.29	1.27	0.00	0.00	0.00	1.04	0.00	-0.02
time (sec)	N/A	0.015	0.168	0.011	0.000	0.000	4.137	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	97	16	16	16	202	0	24
N.S.	1	1.00	5.71	0.94	0.94	0.94	11.88	0.00	1.41
time (sec)	N/A	0.008	0.338	0.032	0.323	2.580	4.737	0.000	5.193

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	127	97	0	16	18	105	0	-1
N.S.	1	7.47	5.71	0.00	0.94	1.06	6.18	0.00	-0.06
time (sec)	N/A	0.047	0.033	0.017	0.330	1.257	2.655	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	131	14	13	16	97	0	13
N.S.	1	1.00	8.73	0.93	0.87	1.07	6.47	0.00	0.87
time (sec)	N/A	0.008	0.411	0.031	0.331	1.631	23.523	0.000	5.431

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	123	131	0	13	26	94	0	-1
N.S.	1	8.20	8.73	0.00	0.87	1.73	6.27	0.00	-0.07
time (sec)	N/A	0.047	0.035	0.015	0.331	1.537	2.880	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	57	1795	57	55
N.S.	1	1.00	0.62	0.59	0.80	0.71	22.44	0.71	0.69
time (sec)	N/A	0.034	0.032	0.058	0.295	1.027	1.349	1.305	5.254

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	700	43	44
N.S.	1	1.00	0.66	0.61	0.80	0.78	11.86	0.73	0.75
time (sec)	N/A	0.026	0.026	0.043	0.277	1.330	0.897	0.817	4.803

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	25	30	34	223	29	33
N.S.	1	1.00	1.00	0.66	0.79	0.89	5.87	0.76	0.87
time (sec)	N/A	0.017	0.023	0.038	0.325	1.948	0.590	1.133	4.720

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	42	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	0.78
time (sec)	N/A	0.002	0.002	0.034	0.270	1.350	0.066	1.113	4.657

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	126	0	97	102	46	98	115
N.S.	1	1.00	1.25	0.00	0.96	1.01	0.46	0.97	1.14
time (sec)	N/A	0.055	0.099	0.012	0.508	1.891	0.531	1.629	4.735

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	135	0	103	155	42	115	125
N.S.	1	1.00	1.26	0.00	0.96	1.45	0.39	1.07	1.17
time (sec)	N/A	0.050	0.153	0.006	0.513	1.425	0.584	1.394	4.880

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	158	0	155	199	42	140	217
N.S.	1	1.00	1.17	0.00	1.15	1.47	0.31	1.04	1.61
time (sec)	N/A	0.066	0.162	0.006	0.516	1.560	1.061	1.562	5.109

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	94	0	0	0	29	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.199	5.859	0.007	0.000	0.000	0.435	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	62	0	0	0	29	0	-1
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.118	5.565	0.005	0.000	0.000	0.402	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	46	0	0	0	26	0	37
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.10	0.00	0.14
time (sec)	N/A	0.094	5.541	0.010	0.000	0.000	0.384	0.000	4.592

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	49	0	0	0	29	0	40
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.11	0.00	0.15
time (sec)	N/A	0.095	5.792	0.005	0.000	0.000	0.416	0.000	4.777

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	51	0	0	0	34	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.119	10.010	0.005	0.000	0.000	0.461	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	57	1795	57	55
N.S.	1	1.00	0.62	0.59	0.80	0.71	22.44	0.71	0.69
time (sec)	N/A	0.033	0.034	0.049	0.328	0.966	1.450	1.718	4.629

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	700	43	44
N.S.	1	1.00	0.66	0.61	0.80	0.78	11.86	0.73	0.75
time (sec)	N/A	0.026	0.030	0.043	0.281	0.812	1.009	1.741	4.663

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	25	30	35	66	29	33
N.S.	1	1.00	1.03	0.66	0.79	0.92	1.74	0.76	0.87
time (sec)	N/A	0.017	0.027	0.040	0.283	1.060	0.150	2.482	4.715

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	42	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	0.78
time (sec)	N/A	0.003	0.003	0.033	0.342	1.065	0.092	1.918	4.583

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	126	0	97	122	46	98	125
N.S.	1	1.00	1.25	0.00	0.96	1.21	0.46	0.97	1.24
time (sec)	N/A	0.048	0.082	0.011	0.495	0.983	0.559	3.116	4.666

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	136	0	103	290	42	116	136
N.S.	1	1.00	1.31	0.00	0.99	2.79	0.40	1.12	1.31
time (sec)	N/A	0.048	0.144	0.006	0.502	0.976	0.615	1.939	4.889

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	158	0	155	380	42	141	212
N.S.	1	1.00	1.17	0.00	1.15	2.81	0.31	1.04	1.57
time (sec)	N/A	0.066	0.190	0.004	0.510	0.968	1.102	2.904	5.161

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	94	0	0	0	29	0	-1
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.333	6.870	0.006	0.000	0.000	0.480	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	62	0	0	0	29	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.254	6.388	0.006	0.000	0.000	0.440	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	46	0	0	0	26	0	37
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.220	6.266	0.008	0.000	0.000	0.408	0.000	5.167

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	538	49	0	0	0	29	0	40
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.216	6.769	0.003	0.000	0.000	0.432	0.000	5.467

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	51	0	0	0	34	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.06	0.00	-0.00
time (sec)	N/A	0.264	10.016	0.006	0.000	0.000	0.473	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	68	136	57	64
N.S.	1	1.00	0.62	0.59	0.80	0.85	1.70	0.71	0.80
time (sec)	N/A	0.034	0.037	0.049	0.286	0.846	0.497	1.157	5.188

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	57	112	43	53
N.S.	1	1.00	0.66	0.61	0.80	0.97	1.90	0.73	0.90
time (sec)	N/A	0.025	0.030	0.046	0.306	0.898	0.375	1.737	5.100

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	45	88	29	42
N.S.	1	1.00	0.74	0.66	0.79	1.18	2.32	0.76	1.11
time (sec)	N/A	0.016	0.026	0.043	0.277	1.098	0.278	2.355	5.054

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	65	14	14
N.S.	1	1.00	1.00	0.83	0.78	1.78	3.61	0.78	0.78
time (sec)	N/A	0.002	0.003	0.038	0.301	0.951	0.190	1.920	4.990

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	135	0	109	111	49	110	133
N.S.	1	1.00	1.15	0.00	0.93	0.95	0.42	0.94	1.14
time (sec)	N/A	0.055	0.096	0.013	0.495	0.801	0.818	2.428	5.001

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	140	0	116	129	46	131	141
N.S.	1	1.00	1.21	0.00	1.00	1.11	0.40	1.13	1.22
time (sec)	N/A	0.058	0.139	0.004	0.515	0.743	0.891	1.438	5.416

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	148	0	152	174	42	139	191
N.S.	1	1.00	1.12	0.00	1.15	1.32	0.32	1.05	1.45
time (sec)	N/A	0.059	0.179	0.005	0.531	0.988	0.955	1.745	5.491

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	79	0	0	0	29	0	-1
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.164	7.709	0.006	0.000	0.000	0.595	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	67	0	0	0	29	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.142	7.271	0.007	0.000	0.000	0.538	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	47	0	0	0	26	0	37
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.09	0.00	0.13
time (sec)	N/A	0.117	6.783	0.009	0.000	0.000	0.522	0.000	5.164

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	50	0	0	0	29	0	40
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.14
time (sec)	N/A	0.112	7.329	0.004	0.000	0.000	0.535	0.000	5.613

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	52	0	0	0	34	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.121	10.020	0.004	0.000	0.000	0.534	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	24	56	9	25
N.S.	1	1.00	1.00	0.77	0.69	1.85	4.31	0.69	1.92
time (sec)	N/A	0.002	0.003	0.096	0.295	1.030	0.361	1.603	5.131

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	46	1690	61	48
N.S.	1	1.00	0.62	0.59	0.80	0.58	21.12	0.76	0.60
time (sec)	N/A	0.034	0.034	0.045	0.283	1.405	1.311	1.120	5.316

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	35	631	47	36
N.S.	1	1.00	0.66	0.61	0.80	0.59	10.69	0.80	0.61
time (sec)	N/A	0.026	0.027	0.039	0.333	1.001	0.863	1.018	5.207

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	24	178	30	24
N.S.	1	1.00	0.74	0.66	0.79	0.63	4.68	0.79	0.63
time (sec)	N/A	0.018	0.023	0.036	0.270	1.034	0.553	1.058	5.006

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	24	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.78
time (sec)	N/A	0.003	0.002	0.033	0.312	1.205	0.176	1.351	4.671

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	103	0	86	235	41	87	106
N.S.	1	1.00	1.20	0.00	1.00	2.73	0.48	1.01	1.23
time (sec)	N/A	0.038	0.063	0.010	0.505	0.726	0.456	1.607	4.828

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	136	0	118	344	41	119	138
N.S.	1	1.00	1.24	0.00	1.07	3.13	0.37	1.08	1.25
time (sec)	N/A	0.050	0.118	0.005	0.560	0.795	0.655	1.626	5.011

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	143	0	158	326	41	142	201
N.S.	1	1.00	1.04	0.00	1.14	2.36	0.30	1.03	1.46
time (sec)	N/A	0.064	0.138	0.005	0.558	1.174	1.467	1.528	5.063

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	79	0	0	0	27	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.270	5.467	0.007	0.000	0.000	0.407	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	62	0	0	0	27	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.223	5.087	0.007	0.000	0.000	0.380	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	46	0	0	0	24	0	37
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.178	5.075	0.016	0.000	0.000	0.372	0.000	4.634

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	49	0	0	0	27	0	40
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.215	5.328	0.005	0.000	0.000	0.411	0.000	4.830

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	51	0	0	0	32	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.06	0.00	-0.00
time (sec)	N/A	0.260	10.021	0.005	0.000	0.000	0.458	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	46	1690	61	48
N.S.	1	1.00	0.62	0.59	0.80	0.58	21.12	0.76	0.60
time (sec)	N/A	0.034	0.033	0.044	0.279	1.140	1.313	0.960	4.749

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	35	631	47	36
N.S.	1	1.00	0.66	0.61	0.80	0.59	10.69	0.80	0.61
time (sec)	N/A	0.025	0.026	0.040	0.291	1.257	0.868	1.152	4.766

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	25	30	23	178	30	24
N.S.	1	1.00	0.71	0.66	0.79	0.61	4.68	0.79	0.63
time (sec)	N/A	0.017	0.021	0.037	0.291	1.153	0.557	0.794	4.786

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	24	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.78
time (sec)	N/A	0.003	0.002	0.035	0.271	0.661	0.194	0.782	4.695

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	101	0	86	123	41	87	102
N.S.	1	1.00	1.17	0.00	1.00	1.43	0.48	1.01	1.19
time (sec)	N/A	0.038	0.062	0.012	0.534	0.770	0.475	1.329	4.843

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	135	0	118	182	41	118	130
N.S.	1	1.00	1.26	0.00	1.10	1.70	0.38	1.10	1.21
time (sec)	N/A	0.048	0.097	0.005	0.524	0.662	0.675	1.187	5.052

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	143	0	158	174	41	142	193
N.S.	1	1.00	1.04	0.00	1.14	1.26	0.30	1.03	1.40
time (sec)	N/A	0.066	0.138	0.004	0.511	0.930	1.526	1.151	5.144

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	79	0	0	0	27	0	-1
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.121	5.510	0.006	0.000	0.000	0.424	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	62	0	0	0	27	0	-1
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.096	5.095	0.007	0.000	0.000	0.393	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	46	0	0	0	24	0	37
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.10	0.00	0.15
time (sec)	N/A	0.077	5.013	0.014	0.000	0.000	0.379	0.000	5.366

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	49	0	0	0	27	0	40
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.15
time (sec)	N/A	0.090	5.551	0.005	0.000	0.000	0.437	0.000	5.458

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	51	0	0	0	32	0	-1
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.118	10.021	0.005	0.000	0.000	0.489	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	58	1584	70	55
N.S.	1	1.00	0.62	0.59	0.80	0.72	19.80	0.88	0.69
time (sec)	N/A	0.034	0.031	0.065	0.286	0.997	1.378	0.611	5.440

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	36	47	46	561	52	41
N.S.	1	1.00	0.64	0.61	0.80	0.78	9.51	0.88	0.69
time (sec)	N/A	0.029	0.029	0.056	0.291	0.977	0.878	0.518	5.361

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	30	35	46	34	24
N.S.	1	1.00	0.71	0.63	0.79	0.92	1.21	0.89	0.63
time (sec)	N/A	0.017	0.024	0.051	0.287	1.880	0.283	0.722	5.591

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	24	26	14	14
N.S.	1	1.00	1.00	0.83	0.78	1.33	1.44	0.78	0.78
time (sec)	N/A	0.003	0.002	0.062	0.300	1.130	0.272	0.710	5.393

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	121	0	100	327	41	101	123
N.S.	1	1.00	1.16	0.00	0.96	3.14	0.39	0.97	1.18
time (sec)	N/A	0.049	0.110	0.012	0.489	0.710	0.532	1.660	5.590

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	135	0	136	453	41	134	178
N.S.	1	1.00	1.10	0.00	1.11	3.68	0.33	1.09	1.45
time (sec)	N/A	0.062	0.157	0.003	0.560	0.782	0.870	1.248	5.645

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	154	0	176	437	41	154	224
N.S.	1	1.00	0.97	0.00	1.11	2.75	0.26	0.97	1.41
time (sec)	N/A	0.076	0.207	0.005	0.501	0.769	2.426	1.502	5.672

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	65	0	0	0	27	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.264	6.544	0.007	0.000	0.000	0.420	0.000	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	55	0	0	0	27	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.227	5.988	0.012	0.000	0.000	0.417	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	58	0	0	0	24	0	37
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.04	0.00	0.07
time (sec)	N/A	0.217	5.694	0.015	0.000	0.000	0.398	0.000	5.416

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	52	0	0	0	27	0	40
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.262	6.448	0.005	0.000	0.000	0.493	0.000	5.575

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	599	599	54	0	0	0	32	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.309	10.019	0.006	0.000	0.000	0.544	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	233	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	1.786	0.006	0.000	0.000	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	204	0	0	0	46	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.114	1.151	0.007	0.000	0.000	21.162	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	173	0	0	0	46	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.094	0.805	0.005	0.000	0.000	0.774	0.000	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	182	0	0	0	49	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.096	0.594	0.007	0.000	0.000	1.596	0.000	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	35	25	78	0	-1
N.S.	1	1.00	0.93	0.75	1.25	0.89	2.79	0.00	-0.04
time (sec)	N/A	0.005	0.742	0.041	0.295	1.562	39.417	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	31	0	46	0	0	-1
N.S.	1	1.00	0.81	0.54	0.00	0.81	0.00	0.00	-0.02
time (sec)	N/A	0.011	1.730	0.044	0.000	0.861	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	47	42	0	57	0	0	-1
N.S.	1	1.00	0.55	0.49	0.00	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.020	2.462	0.042	0.000	0.982	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	58	53	64	68	0	0	-1
N.S.	1	1.00	0.51	0.47	0.57	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.027	4.255	0.046	0.293	0.973	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	103	0	0	0	46	0	-1
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.680	10.052	0.005	0.000	0.000	92.491	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	85	0	0	0	46	0	-1
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.510	10.034	0.005	0.000	0.000	3.734	0.000	0.000

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	54	0	0	0	46	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.472	10.022	0.006	0.000	0.000	0.622	0.000	0.000

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	56	0	0	0	32	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.468	10.023	0.006	0.000	0.000	7.612	0.000	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	56	0	0	0	0	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.507	10.011	0.007	0.000	0.000	0.000	0.000	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	-0.02
time (sec)	N/A	0.013	10.021	0.006	0.000	0.000	0.949	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	-0.02
time (sec)	N/A	0.013	10.013	0.007	0.000	0.000	0.577	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	49	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.013	10.015	0.006	0.000	0.000	1.024	0.000	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	259	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	2.405	0.006	0.000	0.000	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	230	0	0	0	46	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.24	0.00	-0.01
time (sec)	N/A	0.121	1.374	0.006	0.000	0.000	45.092	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	204	0	0	0	46	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.108	0.951	0.007	0.000	0.000	3.999	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	203	0	0	0	49	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.105	0.744	0.010	0.000	0.000	5.032	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	200	0	0	0	53	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.103	0.776	0.010	0.000	0.000	39.345	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	43	0	0	-1
N.S.	1	1.00	0.93	0.75	0.00	1.54	0.00	0.00	-0.04
time (sec)	N/A	0.004	1.968	0.045	0.000	1.378	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	36	31	0	57	0	0	-1
N.S.	1	1.00	0.63	0.54	0.00	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	2.635	0.049	0.000	0.945	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	47	42	0	68	0	0	-1
N.S.	1	1.00	0.55	0.49	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.018	3.816	0.050	0.000	1.023	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	102	0	0	0	0	0	-1
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.631	10.059	0.008	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	89	0	0	0	46	0	-1
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.543	10.042	0.005	0.000	0.000	9.022	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	55	0	0	0	46	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.496	10.027	0.007	0.000	0.000	2.746	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	57	0	0	0	32	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.495	10.014	0.007	0.000	0.000	9.726	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	57	0	0	0	0	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	10.022	0.009	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	57	0	0	0	0	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.570	10.029	0.008	0.000	0.000	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	46	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.78	0.00	-0.02
time (sec)	N/A	0.014	10.014	0.006	0.000	0.000	5.085	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	46	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.78	0.00	-0.02
time (sec)	N/A	0.013	10.013	0.007	0.000	0.000	2.721	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	49	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.014	10.013	0.008	0.000	0.000	2.758	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	233	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	5.515	0.010	0.000	0.000	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	207	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	2.226	0.008	0.000	0.000	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	174	0	0	0	44	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.098	1.242	0.008	0.000	0.000	18.727	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	146	0	0	0	44	0	-1
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.080	0.814	0.013	0.000	0.000	0.681	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	35	25	36	0	-1
N.S.	1	1.00	0.93	0.75	1.25	0.89	1.29	0.00	-0.04
time (sec)	N/A	0.004	0.812	0.036	0.300	2.216	2.131	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	34	29	0	35	78	0	-1
N.S.	1	1.00	0.60	0.51	0.00	0.61	1.37	0.00	-0.02
time (sec)	N/A	0.010	1.298	0.043	0.000	1.121	81.742	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	47	42	0	46	0	0	-1
N.S.	1	1.00	0.55	0.49	0.00	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.017	3.284	0.042	0.000	0.927	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	58	53	64	57	0	0	-1
N.S.	1	1.00	0.51	0.47	0.57	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.027	4.952	0.045	0.287	0.655	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	87	0	0	0	44	0	-1
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.561	10.028	0.010	0.000	0.000	99.016	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	66	0	0	0	44	0	-1
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.473	10.021	0.004	0.000	0.000	3.213	0.000	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	54	0	0	0	31	0	-1
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.446	10.014	0.013	0.000	0.000	0.980	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	56	0	0	0	48	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.477	10.026	0.009	0.000	0.000	18.025	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	56	0	0	0	0	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	10.019	0.010	0.000	0.000	0.000	0.000	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.014	10.012	0.013	0.000	0.000	0.839	0.000	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	-0.02
time (sec)	N/A	0.014	10.016	0.017	0.000	0.000	0.703	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	48	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.014	10.013	0.009	0.000	0.000	1.729	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	93	0	0	0	29	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.24	0.00	-0.01
time (sec)	N/A	0.035	7.035	0.005	0.000	0.000	0.484	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	62	0	0	0	29	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.024	6.984	0.006	0.000	0.000	0.463	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	46	0	0	0	26	0	37
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.35	0.00	0.49
time (sec)	N/A	0.012	6.963	0.010	0.000	0.000	0.445	0.000	4.657

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	0	0	0	29	0	40
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.40	0.00	0.56
time (sec)	N/A	0.014	7.239	0.004	0.000	0.000	0.438	0.000	4.805

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	51	0	0	0	34	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.022	10.019	0.004	0.000	0.000	0.548	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	51	0	0	0	34	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.033	10.011	0.004	0.000	0.000	0.610	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	0	0	0	31	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.033	7.015	0.007	0.000	0.000	0.509	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	64	0	0	0	31	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.025	6.944	0.008	0.000	0.000	0.452	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0	38
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	0.49
time (sec)	N/A	0.013	6.894	0.009	0.000	0.000	0.443	0.000	4.802

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	50	0	0	0	31	0	41
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.41	0.00	0.54
time (sec)	N/A	0.016	7.013	0.005	0.000	0.000	0.460	0.000	4.980

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	52	0	0	0	36	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.024	10.013	0.004	0.000	0.000	0.527	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	52	0	0	0	36	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.035	10.014	0.004	0.000	0.000	0.624	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	93	0	0	0	29	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.20	0.00	-0.01
time (sec)	N/A	0.036	8.401	0.007	0.000	0.000	0.612	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	62	0	0	0	29	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.24	0.00	-0.01
time (sec)	N/A	0.026	8.088	0.006	0.000	0.000	0.552	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	46	0	0	0	26	0	37
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.28	0.00	0.40
time (sec)	N/A	0.016	7.707	0.008	0.000	0.000	0.573	0.000	4.777

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	49	0	0	0	29	0	40
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.33	0.00	0.45
time (sec)	N/A	0.018	8.176	0.004	0.000	0.000	0.548	0.000	5.029

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	51	0	0	0	34	0	-1
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.027	10.021	0.004	0.000	0.000	0.612	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	51	0	0	0	34	0	-1
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.038	10.012	0.007	0.000	0.000	0.676	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	0	0	0	31	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.032	8.446	0.005	0.000	0.000	0.670	0.000	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	64	0	0	0	31	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.023	8.090	0.006	0.000	0.000	0.551	0.000	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0	38
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	0.49
time (sec)	N/A	0.013	7.802	0.009	0.000	0.000	0.478	0.000	4.849

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	50	0	0	0	31	0	41
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.41	0.00	0.54
time (sec)	N/A	0.015	8.260	0.003	0.000	0.000	0.586	0.000	5.170

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	52	0	0	0	36	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.023	10.014	0.005	0.000	0.000	0.620	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	52	0	0	0	36	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.034	10.010	0.005	0.000	0.000	0.665	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	47	0	0	0	26	0	37
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.28	0.00	0.40
time (sec)	N/A	0.018	8.533	0.010	0.000	0.000	0.620	0.000	4.859

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	48	0	0	0	27	0	38
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.28	0.00	0.40
time (sec)	N/A	0.019	8.598	0.010	0.000	0.000	0.639	0.000	4.838

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	47	0	0	0	26	0	37
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.23	0.00	0.33
time (sec)	N/A	0.023	8.907	0.010	0.000	0.000	0.799	0.000	4.823

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	48	0	0	0	27	0	38
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.28	0.00	0.40
time (sec)	N/A	0.019	8.775	0.009	0.000	0.000	0.792	0.000	4.816

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	90	0	0	0	27	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.18	0.00	-0.01
time (sec)	N/A	0.040	6.880	0.006	0.000	0.000	0.476	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	79	0	0	0	27	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.027	6.659	0.006	0.000	0.000	0.458	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	62	0	0	0	27	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.018	6.522	0.008	0.000	0.000	0.460	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	46	0	0	0	24	0	37
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.34	0.00	0.52
time (sec)	N/A	0.010	6.461	0.014	0.000	0.000	0.431	0.000	4.845

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	49	0	0	0	27	0	40
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.29	0.00	0.43
time (sec)	N/A	0.018	6.684	0.004	0.000	0.000	0.469	0.000	5.088

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	51	0	0	0	32	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.027	10.015	0.005	0.000	0.000	0.560	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	51	0	0	0	32	0	-1
N.S.	1	1.00	0.34	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.037	10.015	0.005	0.000	0.000	0.662	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	89	0	0	0	29	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.035	6.814	0.006	0.000	0.000	0.570	0.000	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	79	0	0	0	29	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.023	6.683	0.005	0.000	0.000	0.481	0.000	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.015	6.537	0.006	0.000	0.000	0.443	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0	38
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.45	0.00	0.66
time (sec)	N/A	0.008	6.526	0.015	0.000	0.000	0.408	0.000	4.872

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	50	0	0	0	29	0	41
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.37	0.00	0.52
time (sec)	N/A	0.015	6.688	0.006	0.000	0.000	0.436	0.000	5.085

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	52	0	0	0	34	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.023	10.018	0.005	0.000	0.000	0.509	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	52	0	0	0	34	0	-1
N.S.	1	1.00	0.40	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.033	10.020	0.004	0.000	0.000	0.604	0.000	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	90	0	0	0	27	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.033	7.129	0.006	0.000	0.000	0.480	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	78	0	0	0	27	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.022	7.036	0.005	0.000	0.000	0.448	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	62	0	0	0	27	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.015	6.717	0.007	0.000	0.000	0.425	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	0	0	0	24	0	37
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.43	0.00	0.66
time (sec)	N/A	0.008	6.517	0.013	0.000	0.000	0.408	0.000	4.883

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	49	0	0	0	27	0	40
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.36	0.00	0.53
time (sec)	N/A	0.014	7.067	0.006	0.000	0.000	0.475	0.000	5.070

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	51	0	0	0	32	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.022	10.012	0.005	0.000	0.000	0.592	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	51	0	0	0	32	0	-1
N.S.	1	1.00	0.40	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.032	10.023	0.005	0.000	0.000	0.701	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	91	0	0	0	29	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.032	7.126	0.005	0.000	0.000	0.598	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	77	0	0	0	29	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.025	7.013	0.006	0.000	0.000	0.473	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.016	6.536	0.005	0.000	0.000	0.455	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0	38
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.45	0.00	0.66
time (sec)	N/A	0.008	6.380	0.018	0.000	0.000	0.405	0.000	4.909

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	50	0	0	0	29	0	41
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.37	0.00	0.53
time (sec)	N/A	0.016	6.945	0.005	0.000	0.000	0.520	0.000	5.100

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	52	0	0	0	34	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.025	10.019	0.004	0.000	0.000	0.563	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	52	0	0	0	34	0	-1
N.S.	1	1.00	0.40	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.035	10.024	0.005	0.000	0.000	0.678	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	78	0	0	0	27	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.034	7.659	0.006	0.000	0.000	0.526	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	65	0	0	0	27	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.024	7.533	0.005	0.000	0.000	0.502	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	53	0	0	0	27	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.015	7.168	0.011	0.000	0.000	0.509	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	24	0	37
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.43	0.00	0.66
time (sec)	N/A	0.008	6.939	0.015	0.000	0.000	0.469	0.000	4.921

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	52	0	0	0	27	0	40
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.36	0.00	0.53
time (sec)	N/A	0.015	7.407	0.005	0.000	0.000	0.574	0.000	5.156

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	54	0	0	0	32	0	-1
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.023	10.012	0.004	0.000	0.000	0.664	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	54	0	0	0	32	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.032	10.022	0.005	0.000	0.000	0.777	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	78	0	0	0	29	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.032	7.652	0.007	0.000	0.000	0.490	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	0	0	0	29	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.024	7.646	0.007	0.000	0.000	0.513	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	29	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.015	7.111	0.011	0.000	0.000	0.532	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	26	0	38
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.34	0.00	0.49
time (sec)	N/A	0.014	7.031	0.019	0.000	0.000	0.464	0.000	4.913

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	53	0	0	0	29	0	41
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.29	0.00	0.41
time (sec)	N/A	0.022	7.747	0.006	0.000	0.000	0.545	0.000	5.097

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	55	0	0	0	34	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.034	10.027	0.005	0.000	0.000	0.667	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	55	0	0	0	34	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.041	10.022	0.005	0.000	0.000	0.837	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	55	0	0	0	24	0	37
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.31	0.00	0.47
time (sec)	N/A	0.012	7.945	0.014	0.000	0.000	0.522	0.000	4.879

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	24	0	37
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.31	0.00	0.47
time (sec)	N/A	0.013	8.757	0.014	0.000	0.000	0.772	0.000	4.882

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	0	0	0	24	0	37
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.25	0.00	0.38
time (sec)	N/A	0.018	8.845	0.016	0.000	0.000	0.996	0.000	4.894

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	26	0	38
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.32	0.00	0.47
time (sec)	N/A	0.015	7.913	0.015	0.000	0.000	0.550	0.000	4.870

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	74	0	0	0	26	0	38
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.26	0.00	0.38
time (sec)	N/A	0.020	8.704	0.016	0.000	0.000	0.746	0.000	4.864

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	0	0	0	26	0	38
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.26	0.00	0.38
time (sec)	N/A	0.021	8.878	0.018	0.000	0.000	1.009	0.000	4.812

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	54	20	0	0	27	0	-1
N.S.	1	1.00	0.55	0.20	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.022	6.148	0.084	0.000	0.000	0.475	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	49	20	0	0	27	0	-1
N.S.	1	1.00	0.60	0.25	0.00	0.00	0.33	0.00	-0.01
time (sec)	N/A	0.016	5.863	0.056	0.000	0.000	0.400	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	41	20	0	0	27	0	-1
N.S.	1	1.00	0.65	0.32	0.00	0.00	0.43	0.00	-0.02
time (sec)	N/A	0.010	5.768	0.054	0.000	0.000	0.383	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	24	18	0	0	26	0	16
N.S.	1	1.00	0.56	0.42	0.00	0.00	0.60	0.00	0.37
time (sec)	N/A	0.005	5.742	0.090	0.000	0.000	0.366	0.000	0.094

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	27	20	0	0	29	0	36
N.S.	1	1.00	0.43	0.32	0.00	0.00	0.46	0.00	0.57
time (sec)	N/A	0.010	5.860	0.069	0.000	0.000	0.389	0.000	5.024

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	29	20	0	0	32	0	-1
N.S.	1	1.00	0.35	0.24	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.016	10.022	0.113	0.000	0.000	0.537	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	29	20	0	0	32	0	-1
N.S.	1	1.00	0.29	0.20	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.021	10.019	0.063	0.000	0.000	0.607	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	54	20	0	0	29	0	-1
N.S.	1	1.00	0.65	0.24	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.016	6.099	0.098	0.000	0.000	0.478	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	20	0	0	29	0	-1
N.S.	1	1.00	0.75	0.31	0.00	0.00	0.45	0.00	-0.02
time (sec)	N/A	0.011	5.864	0.085	0.000	0.000	0.429	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	20	0	0	29	0	-1
N.S.	1	1.00	0.87	0.43	0.00	0.00	0.62	0.00	-0.02
time (sec)	N/A	0.007	5.753	0.073	0.000	0.000	0.443	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	18	0	0	27	0	16
N.S.	1	1.00	0.86	0.64	0.00	0.00	0.96	0.00	0.57
time (sec)	N/A	0.002	5.774	0.050	0.000	0.000	0.362	0.000	0.091

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	27	20	0	0	31	0	36
N.S.	1	1.00	0.57	0.43	0.00	0.00	0.66	0.00	0.77
time (sec)	N/A	0.006	5.871	0.078	0.000	0.000	0.400	0.000	5.047

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	29	20	0	0	34	0	-1
N.S.	1	1.00	0.43	0.30	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.011	10.015	0.078	0.000	0.000	0.511	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	29	20	0	0	34	0	-1
N.S.	1	1.00	0.34	0.24	0.00	0.00	0.40	0.00	-0.01
time (sec)	N/A	0.016	10.008	0.079	0.000	0.000	0.575	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	54	20	0	0	27	0	-1
N.S.	1	1.00	0.65	0.24	0.00	0.00	0.33	0.00	-0.01
time (sec)	N/A	0.018	6.153	0.087	0.000	0.000	0.432	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	20	0	0	27	0	-1
N.S.	1	1.00	0.75	0.31	0.00	0.00	0.42	0.00	-0.02
time (sec)	N/A	0.012	5.921	0.060	0.000	0.000	0.408	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	20	0	0	27	0	-1
N.S.	1	1.00	0.87	0.43	0.00	0.00	0.57	0.00	-0.02
time (sec)	N/A	0.006	5.558	0.057	0.000	0.000	0.391	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	18	0	0	26	0	16
N.S.	1	1.00	0.89	0.67	0.00	0.00	0.96	0.00	0.59
time (sec)	N/A	0.002	5.392	0.041	0.000	0.000	0.386	0.000	0.082

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	27	20	0	0	29	0	36
N.S.	1	1.00	0.55	0.41	0.00	0.00	0.59	0.00	0.73
time (sec)	N/A	0.007	5.872	0.057	0.000	0.000	0.453	0.000	4.989

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	29	20	0	0	32	0	-1
N.S.	1	1.00	0.43	0.30	0.00	0.00	0.48	0.00	-0.01
time (sec)	N/A	0.011	10.021	0.064	0.000	0.000	0.505	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	29	20	0	0	32	0	-1
N.S.	1	1.00	0.34	0.24	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.016	10.007	0.069	0.000	0.000	0.599	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	59	20	0	0	29	0	-1
N.S.	1	1.00	0.71	0.24	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.017	6.264	0.101	0.000	0.000	0.462	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	20	0	0	29	0	-1
N.S.	1	1.00	0.83	0.31	0.00	0.00	0.45	0.00	-0.02
time (sec)	N/A	0.012	6.172	0.076	0.000	0.000	0.399	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	20	0	0	29	0	-1
N.S.	1	1.00	1.00	0.43	0.00	0.00	0.62	0.00	-0.02
time (sec)	N/A	0.007	5.761	0.077	0.000	0.000	0.386	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	0	0	27	0	16
N.S.	1	1.00	1.00	0.67	0.00	0.00	1.00	0.00	0.59
time (sec)	N/A	0.002	5.554	0.052	0.000	0.000	0.349	0.000	4.762

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	27	20	0	0	31	0	36
N.S.	1	1.00	0.55	0.41	0.00	0.00	0.63	0.00	0.73
time (sec)	N/A	0.007	5.987	0.080	0.000	0.000	0.467	0.000	5.028

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	29	20	0	0	34	0	-1
N.S.	1	1.00	0.43	0.30	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.012	10.014	0.084	0.000	0.000	0.503	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	29	20	0	0	34	0	-1
N.S.	1	1.00	0.34	0.24	0.00	0.00	0.40	0.00	-0.01
time (sec)	N/A	0.017	10.009	0.088	0.000	0.000	0.580	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	68	42	0	0	29	0	-1
N.S.	1	1.00	0.26	0.16	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.099	5.877	0.113	0.000	0.000	0.490	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	63	42	0	0	29	0	-1
N.S.	1	1.00	0.26	0.18	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.079	5.680	0.102	0.000	0.000	0.416	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	57	42	0	0	29	0	-1
N.S.	1	1.00	0.26	0.19	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.068	5.589	0.095	0.000	0.000	0.397	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	43	40	0	0	27	0	34
N.S.	1	1.00	0.22	0.20	0.00	0.00	0.14	0.00	0.17
time (sec)	N/A	0.058	5.567	0.059	0.000	0.000	0.356	0.000	4.743

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	46	42	0	0	31	0	36
N.S.	1	1.00	0.21	0.19	0.00	0.00	0.14	0.00	0.16
time (sec)	N/A	0.070	5.659	0.096	0.000	0.000	0.401	0.000	4.886

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	48	42	0	0	34	0	-1
N.S.	1	1.00	0.20	0.17	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.081	10.017	0.147	0.000	0.000	0.495	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	48	42	0	0	34	0	-1
N.S.	1	1.00	0.18	0.16	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.094	10.017	0.101	0.000	0.000	0.518	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	68	23	0	0	34	0	-1
N.S.	1	1.00	0.26	0.09	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.099	5.903	0.068	0.000	0.000	0.453	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	63	23	0	0	34	0	-1
N.S.	1	1.00	0.26	0.10	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.084	5.683	0.108	0.000	0.000	0.481	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	58	23	0	0	34	0	-1
N.S.	1	1.00	0.26	0.10	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.068	5.603	0.061	0.000	0.000	0.402	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	43	21	0	0	32	0	34
N.S.	1	1.00	0.21	0.10	0.00	0.00	0.16	0.00	0.17
time (sec)	N/A	0.058	5.559	0.046	0.000	0.000	0.396	0.000	4.879

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	46	23	0	0	36	0	36
N.S.	1	1.00	0.21	0.10	0.00	0.00	0.16	0.00	0.16
time (sec)	N/A	0.072	5.700	0.060	0.000	0.000	0.401	0.000	5.038

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	48	23	0	0	39	0	-1
N.S.	1	1.00	0.20	0.09	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.088	10.020	0.056	0.000	0.000	0.462	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	48	23	0	0	39	0	-1
N.S.	1	1.00	0.18	0.09	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.101	10.020	0.064	0.000	0.000	0.530	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	68	42	0	0	31	0	-1
N.S.	1	1.00	0.49	0.30	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.043	6.080	0.104	0.000	0.000	0.483	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	63	42	0	0	31	0	-1
N.S.	1	1.00	0.52	0.35	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.037	5.926	0.097	0.000	0.000	0.404	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	57	42	0	0	31	0	-1
N.S.	1	1.00	0.56	0.41	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.028	5.577	0.094	0.000	0.000	0.382	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	43	40	0	0	29	0	34
N.S.	1	1.00	0.52	0.49	0.00	0.00	0.35	0.00	0.41
time (sec)	N/A	0.021	5.569	0.061	0.000	0.000	0.381	0.000	4.871

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	46	42	0	0	29	0	23
N.S.	1	1.00	0.44	0.40	0.00	0.00	0.28	0.00	0.22
time (sec)	N/A	0.027	5.863	0.097	0.000	0.000	0.469	0.000	5.068

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	48	42	0	0	32	0	-1
N.S.	1	1.00	0.39	0.34	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.035	10.010	0.099	0.000	0.000	0.531	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	48	42	0	0	32	0	-1
N.S.	1	1.00	0.34	0.30	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.045	10.015	0.112	0.000	0.000	0.581	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	68	23	0	0	36	0	-1
N.S.	1	1.00	0.49	0.17	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.044	6.169	0.089	0.000	0.000	0.420	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	63	23	0	0	36	0	-1
N.S.	1	1.00	0.52	0.19	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.036	6.030	0.065	0.000	0.000	0.407	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	58	23	0	0	36	0	-1
N.S.	1	1.00	0.56	0.22	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.029	5.601	0.068	0.000	0.000	0.401	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	43	21	0	0	34	0	34
N.S.	1	1.00	0.51	0.25	0.00	0.00	0.40	0.00	0.40
time (sec)	N/A	0.020	5.434	0.042	0.000	0.000	0.360	0.000	4.612

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	46	23	0	0	34	0	36
N.S.	1	1.00	0.44	0.22	0.00	0.00	0.32	0.00	0.34
time (sec)	N/A	0.029	5.890	0.075	0.000	0.000	0.436	0.000	4.731

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	48	23	0	0	37	0	-1
N.S.	1	1.00	0.39	0.19	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.036	10.020	0.079	0.000	0.000	0.528	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	48	23	0	0	37	0	-1
N.S.	1	1.00	0.34	0.16	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.045	10.010	0.079	0.000	0.000	0.591	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	102	0	0	0	46	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.082	10.047	0.007	0.000	0.000	16.981	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	85	0	0	0	46	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.063	10.034	0.006	0.000	0.000	1.557	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	0	0	0	46	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.52	0.00	-0.01
time (sec)	N/A	0.054	10.023	0.007	0.000	0.000	0.699	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	56	0	0	0	32	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.058	10.027	0.007	0.000	0.000	2.171	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	56	0	0	0	36	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.066	10.015	0.007	0.000	0.000	19.405	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	56	0	0	0	36	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.079	10.015	0.008	0.000	0.000	177.696	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	109	0	0	0	46	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.066	0.323	0.006	0.000	0.000	6.181	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	0	0	0	46	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.40	0.00	-0.01
time (sec)	N/A	0.052	0.188	0.006	0.000	0.000	0.946	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	92	0	0	0	49	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.46	0.00	-0.01
time (sec)	N/A	0.048	0.183	0.006	0.000	0.000	1.301	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	25	78	0	37
N.S.	1	1.00	0.93	0.75	0.00	0.89	2.79	0.00	1.32
time (sec)	N/A	0.005	0.115	0.049	0.000	1.072	5.527	0.000	4.916

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	31	0	46	124	0	51
N.S.	1	1.00	0.81	0.54	0.00	0.81	2.18	0.00	0.89
time (sec)	N/A	0.012	0.133	0.054	0.000	1.040	55.147	0.000	4.976

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	47	42	0	57	0	0	65
N.S.	1	1.00	0.55	0.49	0.00	0.67	0.00	0.00	0.76
time (sec)	N/A	0.017	0.148	0.060	0.000	1.356	0.000	0.000	5.002

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	58	53	0	68	0	0	79
N.S.	1	1.00	0.51	0.47	0.00	0.60	0.00	0.00	0.70
time (sec)	N/A	0.028	1.137	0.063	0.000	1.101	0.000	0.000	5.022

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	88	0	0	0	48	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.068	10.046	0.006	0.000	0.000	1.490	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	55	0	0	0	39	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.058	10.019	0.006	0.000	0.000	0.722	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	57	0	0	0	36	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.062	10.015	0.005	0.000	0.000	2.177	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	57	0	0	0	39	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.072	10.019	0.008	0.000	0.000	19.122	0.000	0.000

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	57	0	0	0	39	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.081	10.018	0.007	0.000	0.000	177.499	0.000	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	177	0	0	0	48	0	-1
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.264	0.589	0.006	0.000	0.000	6.204	0.000	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	162	0	0	0	48	0	-1
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.205	0.361	0.007	0.000	0.000	0.966	0.000	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	161	0	0	0	51	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.202	0.315	0.007	0.000	0.000	1.257	0.000	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	0	35	178	0	38
N.S.	1	1.00	0.93	0.76	0.00	1.21	6.14	0.00	1.31
time (sec)	N/A	0.004	0.129	0.054	0.000	1.714	5.821	0.000	4.847

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	32	0	46	462	0	51
N.S.	1	1.00	0.81	0.54	0.00	0.78	7.83	0.00	0.86
time (sec)	N/A	0.011	0.141	0.058	0.000	1.322	55.501	0.000	4.892

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	48	43	0	58	0	0	65
N.S.	1	1.00	0.55	0.49	0.00	0.66	0.00	0.00	0.74
time (sec)	N/A	0.019	0.146	0.059	0.000	1.630	0.000	0.000	4.911

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	59	54	0	69	0	0	79
N.S.	1	1.00	0.50	0.46	0.00	0.59	0.00	0.00	0.68
time (sec)	N/A	0.031	1.149	0.063	0.000	1.745	0.000	0.000	4.930

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	314	44	0	-1
N.S.	1	1.00	0.83	0.00	0.00	2.68	0.38	0.00	-0.01
time (sec)	N/A	0.047	0.273	0.007	0.000	1.613	1.294	0.000	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	0	0	241	44	0	-1
N.S.	1	1.00	0.78	0.00	0.00	2.90	0.53	0.00	-0.01
time (sec)	N/A	0.034	0.169	0.018	0.000	1.243	0.748	0.000	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	25	36	0	25
N.S.	1	1.00	0.93	0.75	0.00	0.89	1.29	0.00	0.89
time (sec)	N/A	0.004	0.134	0.051	0.000	1.200	2.124	0.000	4.946

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	36	31	0	35	80	0	40
N.S.	1	1.00	0.63	0.54	0.00	0.61	1.40	0.00	0.70
time (sec)	N/A	0.010	0.156	0.049	0.000	1.103	25.482	0.000	4.997

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	47	42	0	46	0	0	54
N.S.	1	1.00	0.55	0.49	0.00	0.54	0.00	0.00	0.64
time (sec)	N/A	0.018	0.187	0.052	0.000	1.302	0.000	0.000	5.005

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	87	0	0	0	44	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.047	10.024	0.012	0.000	0.000	37.363	0.000	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	69	0	0	0	44	0	-1
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.034	10.019	0.008	0.000	0.000	3.988	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	56	0	0	0	44	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.53	0.00	-0.01
time (sec)	N/A	0.022	10.013	0.012	0.000	0.000	0.525	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	54	0	0	0	31	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.025	10.027	0.010	0.000	0.000	1.084	0.000	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	56	0	0	0	34	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.035	10.023	0.011	0.000	0.000	8.110	0.000	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	56	0	0	0	34	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.048	10.023	0.012	0.000	0.000	73.090	0.000	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	161	0	0	340	46	0	-1
N.S.	1	1.00	0.52	0.00	0.00	1.10	0.15	0.00	-0.00
time (sec)	N/A	0.197	0.499	0.008	0.000	1.413	1.278	0.000	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	125	0	0	267	46	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.98	0.17	0.00	-0.00
time (sec)	N/A	0.169	0.307	0.019	0.000	1.217	0.814	0.000	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	0	26	88	0	26
N.S.	1	1.00	0.93	0.76	0.00	0.90	3.03	0.00	0.90
time (sec)	N/A	0.004	0.140	0.047	0.000	1.289	2.190	0.000	5.124

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	32	0	36	343	0	41
N.S.	1	1.00	0.63	0.54	0.00	0.61	5.81	0.00	0.69
time (sec)	N/A	0.011	0.174	0.046	0.000	1.442	25.966	0.000	5.145

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	48	43	0	47	0	0	55
N.S.	1	1.00	0.55	0.49	0.00	0.53	0.00	0.00	0.62
time (sec)	N/A	0.018	0.188	0.118	0.000	1.325	0.000	0.000	5.251

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	71	0	0	0	46	0	-1
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.038	10.024	0.010	0.000	0.000	3.985	0.000	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	57	0	0	0	46	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.027	10.020	0.012	0.000	0.000	0.562	0.000	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	0	0	0	32	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.47	0.00	-0.01
time (sec)	N/A	0.017	10.013	0.010	0.000	0.000	1.151	0.000	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	57	0	0	0	39	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.028	10.017	0.011	0.000	0.000	8.086	0.000	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	57	0	0	0	36	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.039	10.022	0.010	0.000	0.000	72.841	0.000	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	66	0	0	0	44	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.053	10.023	0.006	0.000	0.000	1.086	0.000	0.000

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	0	0	0	31	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.47	0.00	-0.02
time (sec)	N/A	0.047	10.023	0.016	0.000	0.000	0.987	0.000	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	56	0	0	0	48	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.49	0.00	-0.01
time (sec)	N/A	0.054	10.013	0.010	0.000	0.000	4.831	0.000	0.000

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	56	0	0	0	34	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.065	10.017	0.010	0.000	0.000	52.368	0.000	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	56	0	0	0	0	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	10.018	0.009	0.000	0.000	0.000	0.000	0.000

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	0	44	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.049	0.315	0.008	0.000	0.000	4.561	0.000	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	67	0	0	0	44	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.52	0.00	-0.01
time (sec)	N/A	0.041	0.213	0.015	0.000	0.000	0.752	0.000	0.000

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	0	25	36	0	22
N.S.	1	1.00	0.92	0.81	0.00	0.96	1.38	0.00	0.85
time (sec)	N/A	0.005	0.168	0.046	0.000	1.686	1.689	0.000	4.903

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	34	29	0	35	78	0	40
N.S.	1	1.00	0.62	0.53	0.00	0.64	1.42	0.00	0.73
time (sec)	N/A	0.010	0.219	0.119	0.000	1.303	17.072	0.000	4.982

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	47	42	0	46	483	0	54
N.S.	1	1.00	0.57	0.51	0.00	0.55	5.82	0.00	0.65
time (sec)	N/A	0.017	0.273	0.049	0.000	1.380	141.652	0.000	5.172

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	0	0	0	46	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.058	10.022	0.006	0.000	0.000	1.186	0.000	0.000

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	0	0	0	32	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.47	0.00	-0.01
time (sec)	N/A	0.051	10.015	0.020	0.000	0.000	0.955	0.000	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	57	0	0	0	39	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.056	10.026	0.009	0.000	0.000	4.712	0.000	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	57	0	0	0	36	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.071	10.019	0.011	0.000	0.000	52.065	0.000	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	57	0	0	0	0	0	-1
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	10.016	0.011	0.000	0.000	0.000	0.000	0.000

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	164	0	0	0	46	0	-1
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.209	0.507	0.009	0.000	0.000	4.308	0.000	0.000

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	127	0	0	0	46	0	-1
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.177	0.328	0.014	0.000	0.000	0.699	0.000	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	22	0	26	90	0	23
N.S.	1	1.00	0.93	0.81	0.00	0.96	3.33	0.00	0.85
time (sec)	N/A	0.005	0.173	0.051	0.000	0.976	1.535	0.000	5.076

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	30	0	34	352	0	41
N.S.	1	1.00	0.61	0.53	0.00	0.60	6.18	0.00	0.72
time (sec)	N/A	0.011	0.230	0.050	0.000	0.747	15.653	0.000	5.096

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	48	43	0	47	1263	0	55
N.S.	1	1.00	0.56	0.50	0.00	0.55	14.69	0.00	0.64
time (sec)	N/A	0.019	0.303	0.122	0.000	0.619	133.641	0.000	5.153

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	131	0	0	389	44	0	-1
N.S.	1	1.00	0.90	0.00	0.00	2.66	0.30	0.00	-0.01
time (sec)	N/A	0.061	0.398	0.012	0.000	0.781	14.417	0.000	0.000

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	91	0	0	319	44	0	-1
N.S.	1	1.00	0.85	0.00	0.00	2.98	0.41	0.00	-0.01
time (sec)	N/A	0.045	0.296	0.015	0.000	1.198	2.309	0.000	0.000

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	0	31	34	0	29
N.S.	1	1.00	0.92	0.81	0.00	1.19	1.31	0.00	1.12
time (sec)	N/A	0.005	0.189	0.047	0.000	0.860	1.524	0.000	4.968

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	34	29	0	48	78	0	57
N.S.	1	1.00	0.62	0.53	0.00	0.87	1.42	0.00	1.04
time (sec)	N/A	0.011	0.229	0.059	0.000	1.143	8.529	0.000	5.097

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	47	42	0	61	384	0	70
N.S.	1	1.00	0.57	0.51	0.00	0.73	4.63	0.00	0.84
time (sec)	N/A	0.019	0.387	0.062	0.000	1.036	80.888	0.000	5.154

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	58	53	0	72	0	0	85
N.S.	1	1.00	0.53	0.49	0.00	0.66	0.00	0.00	0.78
time (sec)	N/A	0.028	0.474	0.066	0.000	1.456	0.000	0.000	5.197

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	87	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	10.032	0.014	0.000	0.000	0.000	0.000	0.000

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	74	0	0	0	44	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.036	10.026	0.010	0.000	0.000	42.376	0.000	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	60	0	0	0	44	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.49	0.00	-0.01
time (sec)	N/A	0.025	10.030	0.016	0.000	0.000	4.212	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	0	0	0	44	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.70	0.00	-0.02
time (sec)	N/A	0.017	10.019	0.011	0.000	0.000	1.038	0.000	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	57	0	0	0	48	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.52	0.00	-0.01
time (sec)	N/A	0.025	10.019	0.012	0.000	0.000	3.202	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	59	0	0	0	34	0	-1
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.035	10.027	0.013	0.000	0.000	27.776	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	59	0	0	0	0	0	-1
N.S.	1	1.00	0.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	10.017	0.013	0.000	0.000	0.000	0.000	0.000

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.013	10.013	0.009	0.000	0.000	6.601	0.000	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.013	10.024	0.012	0.000	0.000	1.850	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.012	10.014	0.014	0.000	0.000	0.577	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.013	10.017	0.019	0.000	0.000	0.626	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	44	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.79	0.00	-0.02
time (sec)	N/A	0.013	10.013	0.018	0.000	0.000	1.012	0.000	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	48	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.013	10.023	0.009	0.000	0.000	1.997	0.000	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.015	10.032	0.015	0.000	0.000	14.295	0.000	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.013	10.025	0.016	0.000	0.000	7.760	0.000	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.013	10.018	0.018	0.000	0.000	4.137	0.000	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.014	10.015	0.023	0.000	0.000	4.477	0.000	0.000

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	44	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.75	0.00	-0.02
time (sec)	N/A	0.013	10.022	0.020	0.000	0.000	8.530	0.000	0.000

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	48	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.81	0.00	-0.02
time (sec)	N/A	0.013	10.016	0.015	0.000	0.000	15.558	0.000	0.000

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	105	0	0	0	29	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.291	8.790	0.007	0.000	0.000	0.553	0.000	0.000

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	93	0	0	0	29	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.212	8.816	0.005	0.000	0.000	0.508	0.000	0.000

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	62	0	0	0	29	0	-1
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.183	8.565	0.005	0.000	0.000	0.449	0.000	0.000

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	46	0	0	0	26	0	37
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.10	0.00	0.14
time (sec)	N/A	0.150	8.518	0.010	0.000	0.000	0.429	0.000	4.884

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	49	0	0	0	29	0	40
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.11	0.00	0.15
time (sec)	N/A	0.148	8.630	0.004	0.000	0.000	0.478	0.000	5.079

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	51	0	0	0	34	0	-1
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.179	10.024	0.005	0.000	0.000	0.536	0.000	0.000

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	51	0	0	0	34	0	-1
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.206	10.011	0.005	0.000	0.000	0.607	0.000	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	51	0	0	0	34	0	-1
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.235	10.016	0.005	0.000	0.000	0.735	0.000	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	90	0	0	0	27	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.512	8.751	0.007	0.000	0.000	0.525	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	79	0	0	0	27	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.442	8.626	0.006	0.000	0.000	0.464	0.000	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	62	0	0	0	27	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.366	8.411	0.006	0.000	0.000	0.461	0.000	0.000

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	46	0	0	0	24	0	37
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.04	0.00	0.06
time (sec)	N/A	0.321	8.387	0.016	0.000	0.000	0.408	0.000	4.879

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	586	49	0	0	0	27	0	40
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.372	8.418	0.005	0.000	0.000	0.465	0.000	5.087

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	633	51	0	0	0	32	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.414	10.020	0.005	0.000	0.000	0.525	0.000	0.000

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	51	0	0	0	32	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.474	10.011	0.006	0.000	0.000	0.600	0.000	0.000

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	89	0	0	0	27	0	-1
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.203	9.495	0.008	0.000	0.000	0.457	0.000	0.000

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	79	0	0	0	27	0	-1
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.184	9.186	0.006	0.000	0.000	0.490	0.000	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	62	0	0	0	27	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.150	8.470	0.007	0.000	0.000	0.439	0.000	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	46	0	0	0	24	0	37
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.15
time (sec)	N/A	0.137	8.203	0.016	0.000	0.000	0.416	0.000	4.906

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	49	0	0	0	27	0	40
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.15
time (sec)	N/A	0.152	9.079	0.004	0.000	0.000	0.518	0.000	5.100

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	51	0	0	0	32	0	-1
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.173	10.015	0.004	0.000	0.000	0.599	0.000	0.000

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	51	0	0	0	32	0	-1
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.208	10.011	0.005	0.000	0.000	0.707	0.000	0.000

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	654	79	0	0	0	27	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.506	9.374	0.006	0.000	0.000	0.493	0.000	0.000

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	65	0	0	0	27	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.434	9.231	0.007	0.000	0.000	0.489	0.000	0.000

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	583	583	58	0	0	0	27	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.390	8.675	0.010	0.000	0.000	0.489	0.000	0.000

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	49	0	0	0	24	0	37
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.04	0.00	0.07
time (sec)	N/A	0.284	8.526	0.014	0.000	0.000	0.471	0.000	4.943

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	52	0	0	0	27	0	40
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.04	0.00	0.07
time (sec)	N/A	0.412	9.177	0.005	0.000	0.000	0.595	0.000	5.121

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	54	0	0	0	32	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.468	10.011	0.005	0.000	0.000	0.714	0.000	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	54	0	0	0	32	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.531	10.010	0.005	0.000	0.000	0.841	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	132	106	148	1923	260	183
N.S.	1	1.00	0.95	1.32	1.06	1.48	19.23	2.60	1.83
time (sec)	N/A	0.045	0.073	0.057	0.290	1.298	3.001	1.164	4.966

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	80	73	98	920	132	117
N.S.	1	1.00	0.89	1.11	1.01	1.36	12.78	1.83	1.62
time (sec)	N/A	0.030	0.050	0.052	0.287	1.466	1.365	1.221	4.902

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	42	47	58	333	51	68
N.S.	1	1.00	0.83	0.88	0.98	1.21	6.94	1.06	1.42
time (sec)	N/A	0.020	0.040	0.049	0.281	0.932	0.505	1.046	4.887

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	21	25	87	21	21
N.S.	1	1.00	0.96	0.96	0.91	1.09	3.78	0.91	0.91
time (sec)	N/A	0.003	0.002	0.045	0.274	0.898	0.200	1.028	4.892

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	0	0	39	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.95	0.00	-0.02
time (sec)	N/A	0.016	0.032	0.014	0.000	0.000	0.907	0.000	0.000

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	42	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.00	0.00	-0.02
time (sec)	N/A	0.016	0.030	0.019	0.000	0.000	2.032	0.000	0.000

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0	-1
N.S.	1	1.22	1.22	0.00	0.00	0.00	0.65	0.00	-0.02
time (sec)	N/A	0.009	0.042	0.027	0.000	0.000	5.655	0.000	0.000

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0	-1
N.S.	1	1.22	1.22	0.00	0.00	0.00	0.65	0.00	-0.02
time (sec)	N/A	0.009	0.039	0.021	0.000	0.000	3.198	0.000	0.000

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0	-1
N.S.	1	1.22	1.22	0.00	0.00	0.00	0.65	0.00	-0.02
time (sec)	N/A	0.010	0.031	0.018	0.000	0.000	1.809	0.000	0.000

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	44	0	0	0	22	0	41
N.S.	1	1.26	1.26	0.00	0.00	0.00	0.63	0.00	1.17
time (sec)	N/A	0.007	0.025	0.012	0.000	0.000	1.064	0.000	5.410

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	47	47	0	0	0	26	0	58
N.S.	1	1.24	1.24	0.00	0.00	0.00	0.68	0.00	1.53
time (sec)	N/A	0.010	0.036	0.019	0.000	0.000	1.700	0.000	5.029

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	-1
N.S.	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.096	0.013	0.000	0.000	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	-1
N.S.	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.127	0.012	0.000	0.000	0.000	0.000	0.000

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	37	0	-1
N.S.	1	1.21	1.21	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.009	0.109	0.013	0.000	0.000	171.886	0.000	0.000

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	37	0	-1
N.S.	1	1.21	1.21	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.009	0.088	0.012	0.000	0.000	25.417	0.000	0.000

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	37	0	-1
N.S.	1	1.22	1.22	0.00	0.00	0.00	0.92	0.00	-0.02
time (sec)	N/A	0.009	0.122	0.012	0.000	0.000	16.237	0.000	0.000

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	41	0	-1
N.S.	1	1.22	1.22	0.00	0.00	0.00	1.02	0.00	-0.02
time (sec)	N/A	0.010	0.149	0.011	0.000	0.000	86.072	0.000	0.000

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	-1
N.S.	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.150	0.013	0.000	0.000	0.000	0.000	0.000

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	-1
N.S.	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.121	0.012	0.000	0.000	0.000	0.000	0.000

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	63	0	0	0	51	0	-1
N.S.	1	1.15	1.19	0.00	0.00	0.00	0.96	0.00	-0.02
time (sec)	N/A	0.012	0.024	0.036	0.000	0.000	9.965	0.000	0.000

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	-0.02
time (sec)	N/A	0.014	0.008	0.037	0.000	0.000	10.004	0.000	0.000

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	-1
N.S.	1	1.32	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.033	0.037	0.000	0.000	0.000	0.000	0.000

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	62	81	84	106	0	0	154
N.S.	1	1.00	0.59	0.77	0.80	1.01	0.00	0.00	1.47
time (sec)	N/A	0.041	0.029	0.067	0.301	0.895	0.000	0.000	5.089

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	-1
N.S.	1	1.32	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.033	0.035	0.000	0.000	0.000	0.000	0.000

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	62	45	59	67	0	0	96
N.S.	1	1.00	0.93	0.67	0.88	1.00	0.00	0.00	1.43
time (sec)	N/A	0.013	0.029	0.072	0.293	0.881	0.000	0.000	5.020

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	-1
N.S.	1	1.32	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.033	0.038	0.000	0.000	0.000	0.000	0.000

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	37	34	0	0	52
N.S.	1	1.00	0.97	0.97	1.23	1.13	0.00	0.00	1.73
time (sec)	N/A	0.004	0.026	0.071	0.317	1.062	0.000	0.000	5.053

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	-1
N.S.	1	1.32	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.032	0.034	0.000	0.000	0.000	0.000	0.000

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	56	56	0	0	0	37	0	-1
N.S.	1	1.30	1.30	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.012	0.023	0.050	0.000	0.000	133.854	0.000	0.000

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	69	65	0	0	0	24	0	-1
N.S.	1	1.33	1.25	0.00	0.00	0.00	0.46	0.00	-0.02
time (sec)	N/A	0.014	0.029	0.031	0.000	0.000	6.954	0.000	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	64	61	0	0	0	41	0	-1
N.S.	1	1.31	1.24	0.00	0.00	0.00	0.84	0.00	-0.02
time (sec)	N/A	0.015	0.031	0.036	0.000	0.000	111.866	0.000	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	69	65	0	0	0	48	0	-1
N.S.	1	1.33	1.25	0.00	0.00	0.00	0.92	0.00	-0.02
time (sec)	N/A	0.015	0.031	0.036	0.000	0.000	125.929	0.000	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	64	61	0	0	0	41	0	-1
N.S.	1	1.31	1.24	0.00	0.00	0.00	0.84	0.00	-0.02
time (sec)	N/A	0.016	0.031	0.036	0.000	0.000	168.187	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [662] had the largest ratio of [43]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	11	0.091
2	A	2	1	1.00	11	0.091
3	A	2	1	1.00	11	0.091
4	A	2	1	1.00	9	0.111
5	A	1	0	1.00	7	0.000
6	A	2	1	1.00	11	0.091
7	A	2	1	1.00	11	0.091
8	A	2	1	1.00	11	0.091
9	A	2	1	1.00	11	0.091
10	A	2	1	1.00	11	0.091
11	A	2	1	1.00	11	0.091
12	A	2	1	1.00	11	0.091
13	A	3	2	1.00	13	0.154
14	A	2	1	1.00	13	0.077
15	A	3	2	1.00	13	0.154
16	A	2	1	1.00	13	0.077
17	A	1	1	1.00	11	0.091
18	A	2	1	1.00	9	0.111
19	A	3	2	1.00	13	0.154
20	A	2	1	1.00	13	0.077
21	A	3	2	1.00	13	0.154
22	A	2	1	1.00	13	0.077
23	A	3	2	1.00	13	0.154
24	A	2	1	1.00	13	0.077
25	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	1	1.00	13	0.077
27	A	3	2	1.00	13	0.154
28	A	2	1	1.00	13	0.077
29	A	3	2	1.00	13	0.154
30	A	3	2	1.00	13	0.154
31	A	3	2	1.00	13	0.154
32	A	3	2	1.00	13	0.154
33	A	1	1	1.00	11	0.091
34	A	3	2	1.00	13	0.154
35	A	3	2	1.00	13	0.154
36	A	3	2	1.00	13	0.154
37	A	3	2	1.00	13	0.154
38	A	1	1	1.00	13	0.077
39	A	3	3	1.00	13	0.231
40	A	3	2	1.00	13	0.154
41	A	3	2	1.00	13	0.154
42	A	2	1	1.00	13	0.077
43	A	2	1	1.00	13	0.077
44	A	2	1	1.00	13	0.077
45	A	2	1	1.00	9	0.111
46	A	2	1	1.00	13	0.077
47	A	2	1	1.00	13	0.077
48	A	2	1	1.00	13	0.077
49	A	2	1	1.00	13	0.077
50	A	2	1	1.00	13	0.077
51	A	2	1	1.00	13	0.077
52	A	3	2	1.00	13	0.154
53	A	3	2	1.00	13	0.154
54	A	3	2	1.00	13	0.154
55	A	3	2	1.00	13	0.154
56	A	3	2	1.00	13	0.154
57	A	3	2	1.00	13	0.154
58	A	1	1	1.00	11	0.091
59	A	3	2	1.00	13	0.154
60	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	13	0.154
62	A	3	2	1.00	13	0.154
63	A	3	2	1.00	13	0.154
64	A	3	2	1.00	13	0.154
65	A	1	1	1.00	13	0.077
66	A	3	3	1.00	13	0.231
67	A	4	3	1.00	13	0.231
68	A	3	2	1.00	13	0.154
69	A	3	2	1.00	13	0.154
70	A	2	1	1.00	13	0.077
71	A	2	1	1.00	13	0.077
72	A	2	1	1.00	13	0.077
73	A	2	1	1.00	13	0.077
74	A	2	1	1.00	9	0.111
75	A	2	1	1.00	13	0.077
76	A	2	1	1.00	13	0.077
77	A	2	1	1.00	13	0.077
78	A	2	1	1.00	13	0.077
79	A	2	1	1.00	13	0.077
80	A	2	1	1.00	13	0.077
81	A	2	1	1.00	13	0.077
82	A	2	1	1.00	13	0.077
83	A	2	1	1.00	13	0.077
84	A	2	1	1.00	13	0.077
85	A	3	2	1.00	13	0.154
86	A	3	2	1.00	13	0.154
87	A	3	2	1.00	13	0.154
88	A	3	2	1.00	13	0.154
89	A	3	2	1.00	13	0.154
90	A	3	2	1.00	13	0.154
91	A	1	1	1.00	11	0.091
92	A	3	2	1.00	13	0.154
93	A	3	2	1.00	13	0.154
94	A	3	2	1.00	13	0.154
95	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	2	1.00	13	0.154
97	A	3	2	1.00	13	0.154
98	A	3	2	1.00	13	0.154
99	A	3	2	1.00	13	0.154
100	A	3	2	1.00	13	0.154
101	A	1	1	1.00	13	0.077
102	A	3	3	1.00	13	0.231
103	A	4	3	1.00	13	0.231
104	A	5	3	1.00	13	0.231
105	A	6	3	1.00	13	0.231
106	A	3	2	1.00	13	0.154
107	A	3	2	1.00	13	0.154
108	A	3	2	1.00	13	0.154
109	A	2	1	1.00	13	0.077
110	A	2	1	1.00	13	0.077
111	A	2	1	1.00	13	0.077
112	A	2	1	1.00	13	0.077
113	A	2	1	1.00	9	0.111
114	A	2	1	1.00	13	0.077
115	A	2	1	1.00	13	0.077
116	A	2	1	1.00	13	0.077
117	A	2	1	1.00	13	0.077
118	A	2	1	1.00	13	0.077
119	A	2	1	1.00	13	0.077
120	A	2	1	1.00	13	0.077
121	A	2	1	1.00	13	0.077
122	A	2	1	1.00	13	0.077
123	A	2	1	1.00	13	0.077
124	A	3	2	1.00	13	0.154
125	A	3	2	1.00	13	0.154
126	A	3	2	1.00	13	0.154
127	A	3	2	1.00	13	0.154
128	A	3	2	1.00	13	0.154
129	A	3	2	1.00	13	0.154
130	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	2	1.00	13	0.154
132	A	3	2	1.00	13	0.154
133	A	2	2	1.00	13	0.154
134	A	1	1	1.00	11	0.091
135	A	1	1	1.00	9	0.111
136	A	4	4	1.00	13	0.308
137	A	2	2	1.00	13	0.154
138	A	3	2	1.00	13	0.154
139	A	3	2	1.00	13	0.154
140	A	3	2	1.00	13	0.154
141	A	4	2	1.00	13	0.154
142	A	3	2	1.00	13	0.154
143	A	5	2	1.00	13	0.154
144	A	3	2	1.00	13	0.154
145	A	3	2	1.00	13	0.154
146	A	4	3	1.00	13	0.231
147	A	3	2	1.00	13	0.154
148	A	4	3	1.00	13	0.231
149	A	3	2	1.00	13	0.154
150	A	4	3	1.00	13	0.231
151	A	3	2	1.00	13	0.154
152	A	4	3	1.00	13	0.231
153	A	3	2	1.00	13	0.154
154	A	3	3	1.00	13	0.231
155	A	3	2	1.00	13	0.154
156	A	2	2	1.00	13	0.154
157	A	1	1	1.00	11	0.091
158	A	2	2	1.00	9	0.222
159	A	3	2	1.00	13	0.154
160	A	3	3	1.00	13	0.231
161	A	3	2	1.00	13	0.154
162	A	4	3	1.00	13	0.231
163	A	3	2	1.00	13	0.154
164	A	5	3	1.00	13	0.231
165	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	3	1.00	13	0.231
167	A	3	2	1.00	13	0.154
168	A	3	2	1.00	13	0.154
169	A	3	2	1.00	13	0.154
170	A	3	2	1.00	13	0.154
171	A	3	2	1.00	13	0.154
172	A	3	2	1.00	13	0.154
173	A	3	2	1.00	13	0.154
174	A	1	1	1.00	13	0.077
175	A	1	1	1.00	11	0.091
176	A	3	2	1.00	13	0.154
177	A	3	2	1.00	13	0.154
178	A	3	2	1.00	13	0.154
179	A	3	2	1.00	13	0.154
180	A	3	2	1.00	13	0.154
181	A	5	3	1.00	13	0.231
182	A	5	3	1.00	13	0.231
183	A	5	3	1.00	13	0.231
184	A	4	3	1.00	13	0.231
185	A	3	2	1.00	13	0.154
186	A	3	3	1.00	13	0.231
187	A	3	2	1.00	9	0.222
188	A	4	3	1.00	13	0.231
189	A	5	3	1.00	13	0.231
190	A	6	3	1.00	13	0.231
191	A	7	3	1.00	13	0.231
192	A	3	2	1.00	13	0.154
193	A	3	2	1.00	13	0.154
194	A	3	2	1.00	13	0.154
195	A	3	2	1.00	13	0.154
196	A	1	1	1.00	13	0.077
197	A	3	3	1.00	13	0.231
198	A	4	3	1.00	13	0.231
199	A	5	3	1.00	13	0.231
200	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	3	2	1.00	13	0.154
202	A	3	2	1.00	13	0.154
203	A	3	2	1.00	13	0.154
204	A	1	1	1.00	11	0.091
205	A	3	2	1.00	13	0.154
206	A	3	2	1.00	13	0.154
207	A	3	2	1.00	13	0.154
208	A	3	2	1.00	13	0.154
209	A	12	3	1.00	13	0.231
210	A	12	3	1.00	13	0.231
211	A	11	3	1.00	13	0.231
212	A	10	2	1.00	13	0.154
213	A	10	3	1.00	13	0.231
214	A	10	3	1.00	13	0.231
215	A	10	3	1.00	13	0.231
216	A	10	3	1.00	13	0.231
217	A	10	3	1.00	13	0.231
218	A	10	3	1.00	13	0.231
219	A	10	3	1.00	13	0.231
220	A	10	3	1.00	13	0.231
221	A	10	2	1.00	9	0.222
222	A	11	3	1.00	13	0.231
223	A	12	3	1.00	13	0.231
224	A	13	3	1.00	13	0.231
225	A	3	2	1.00	14	0.143
226	A	2	2	1.00	14	0.143
227	A	1	1	1.00	12	0.083
228	A	1	1	1.00	10	0.100
229	A	4	4	1.00	14	0.286
230	A	2	2	1.00	14	0.143
231	A	3	2	1.00	14	0.143
232	A	3	2	1.00	14	0.143
233	A	2	2	1.00	14	0.143
234	A	1	1	1.00	12	0.083
235	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	2	1.00	14	0.143
237	A	3	3	1.00	14	0.214
238	A	3	2	1.00	14	0.143
239	A	1	1	1.00	14	0.071
240	A	3	3	1.00	14	0.214
241	A	1	1	1.00	12	0.083
242	A	3	2	1.00	10	0.200
243	A	3	2	1.00	14	0.143
244	A	4	3	1.00	14	0.214
245	A	3	2	1.00	14	0.143
246	A	3	2	1.00	14	0.143
247	A	5	3	1.00	14	0.214
248	A	1	1	1.00	12	0.083
249	A	5	2	1.00	10	0.200
250	A	3	2	1.00	14	0.143
251	A	6	3	1.00	14	0.214
252	A	3	2	1.00	14	0.143
253	A	4	4	1.00	13	0.308
254	A	4	4	1.00	13	0.308
255	A	3	2	1.00	13	0.154
256	A	3	2	1.00	13	0.154
257	A	1	1	1.00	10	0.100
258	A	1	1	1.00	18	0.056
259	A	3	2	1.00	13	0.154
260	A	3	2	1.00	13	0.154
261	A	1	1	1.00	14	0.071
262	A	1	1	1.00	15	0.067
263	A	1	1	1.00	20	0.050
264	A	2	1	1.00	13	0.077
265	A	2	1	1.00	13	0.077
266	A	2	1	1.00	13	0.077
267	A	2	1	1.00	13	0.077
268	A	2	1	1.00	13	0.077
269	A	2	1	1.00	13	0.077
270	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	2	1	1.00	13	0.077
272	A	2	1	1.00	15	0.067
273	A	2	1	1.00	15	0.067
274	A	2	1	1.00	15	0.067
275	A	2	1	1.00	15	0.067
276	A	2	1	1.00	15	0.067
277	A	2	1	1.00	15	0.067
278	A	2	1	1.00	15	0.067
279	A	2	1	1.00	15	0.067
280	A	2	1	1.00	15	0.067
281	A	2	1	1.00	15	0.067
282	A	2	1	1.00	15	0.067
283	A	2	1	1.00	15	0.067
284	A	2	1	1.00	15	0.067
285	A	2	1	1.00	15	0.067
286	A	2	1	1.00	15	0.067
287	A	2	1	1.00	15	0.067
288	A	12	8	1.00	15	0.533
289	A	11	8	1.00	15	0.533
290	A	11	8	1.00	15	0.533
291	A	10	7	1.00	15	0.467
292	A	10	7	1.00	15	0.467
293	A	11	8	1.00	15	0.533
294	A	11	8	1.00	15	0.533
295	A	12	8	1.00	15	0.533
296	A	12	9	1.00	15	0.600
297	A	11	8	1.00	15	0.533
298	A	11	8	1.00	15	0.533
299	A	11	8	1.00	15	0.533
300	A	11	8	1.00	15	0.533
301	A	12	9	1.00	15	0.600
302	A	12	9	1.00	15	0.600
303	A	13	9	1.00	15	0.600
304	A	12	8	1.00	15	0.533
305	A	12	9	1.00	15	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	12	9	1.00	15	0.600
307	A	12	8	1.00	15	0.533
308	A	12	8	1.00	15	0.533
309	A	13	9	1.00	15	0.600
310	A	13	9	1.00	15	0.600
311	A	14	9	1.00	15	0.600
312	A	4	4	1.00	16	0.250
313	A	12	8	1.00	13	0.615
314	A	11	8	1.00	13	0.615
315	A	11	8	1.00	13	0.615
316	A	10	7	1.00	13	0.538
317	A	10	7	1.00	13	0.538
318	A	11	8	1.00	13	0.615
319	A	11	8	1.00	13	0.615
320	A	12	8	1.00	13	0.615
321	A	12	9	1.00	13	0.692
322	A	11	8	1.00	13	0.615
323	A	11	8	1.00	13	0.615
324	A	11	8	1.00	13	0.615
325	A	11	8	1.00	13	0.615
326	A	12	9	1.00	13	0.692
327	A	12	9	1.00	13	0.692
328	A	13	9	1.00	13	0.692
329	A	12	8	1.00	13	0.615
330	A	12	9	1.00	13	0.692
331	A	12	9	1.00	13	0.692
332	A	12	8	1.00	13	0.615
333	A	12	8	1.00	13	0.615
334	A	13	9	1.00	13	0.692
335	A	13	9	1.00	13	0.692
336	A	14	9	1.00	13	0.692
337	A	4	4	1.00	15	0.267
338	A	11	7	1.37	13	0.538
339	A	2	1	1.00	13	0.077
340	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	2	1	1.00	13	0.077
342	A	2	1	1.00	13	0.077
343	A	2	1	1.00	11	0.091
344	A	1	1	1.00	13	0.077
345	A	1	1	1.00	13	0.077
346	A	1	1	1.00	13	0.077
347	A	1	1	1.00	17	0.059
348	A	1	1	1.00	15	0.067
349	A	1	1	1.00	17	0.059
350	A	1	1	1.00	17	0.059
351	A	1	1	1.00	17	0.059
352	A	1	1	1.00	16	0.062
353	A	3	2	1.00	15	0.133
354	A	3	2	1.00	15	0.133
355	A	3	2	1.00	15	0.133
356	A	1	1	1.00	13	0.077
357	A	4	4	1.00	15	0.267
358	A	4	4	1.00	15	0.267
359	A	5	5	1.00	15	0.333
360	A	6	5	1.00	15	0.333
361	A	5	4	1.00	15	0.267
362	A	4	4	1.00	15	0.267
363	A	3	3	1.00	11	0.273
364	A	3	3	1.00	15	0.200
365	A	1	1	1.00	15	0.067
366	A	2	2	1.00	15	0.133
367	A	3	2	1.00	15	0.133
368	A	4	2	1.00	15	0.133
369	A	3	2	1.00	15	0.133
370	A	3	2	1.00	15	0.133
371	A	3	2	1.00	15	0.133
372	A	1	1	1.00	13	0.077
373	A	5	4	1.00	15	0.267
374	A	5	5	1.00	15	0.333
375	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	6	5	1.00	15	0.333
377	A	7	5	1.00	15	0.333
378	A	6	4	1.00	15	0.267
379	A	5	4	1.00	15	0.267
380	A	4	3	1.00	11	0.273
381	A	4	4	1.00	15	0.267
382	A	4	3	1.00	15	0.200
383	A	1	1	1.00	15	0.067
384	A	2	2	1.00	15	0.133
385	A	3	2	1.00	15	0.133
386	A	4	2	1.00	15	0.133
387	A	3	2	1.00	15	0.133
388	A	3	2	1.00	15	0.133
389	A	3	2	1.00	15	0.133
390	A	1	1	1.00	13	0.077
391	A	6	4	1.00	15	0.267
392	A	6	5	1.00	15	0.333
393	A	6	5	1.00	15	0.333
394	A	6	4	1.00	15	0.267
395	A	7	5	1.00	15	0.333
396	A	8	5	1.00	15	0.333
397	A	7	4	1.00	15	0.267
398	A	6	4	1.00	15	0.267
399	A	5	3	1.00	11	0.273
400	A	5	4	1.00	15	0.267
401	A	5	4	1.00	15	0.267
402	A	5	3	1.00	15	0.200
403	A	1	1	1.00	15	0.067
404	A	2	2	1.00	15	0.133
405	A	3	2	1.00	15	0.133
406	A	4	2	1.00	15	0.133
407	A	5	2	1.00	15	0.133
408	A	6	2	1.00	15	0.133
409	A	3	2	1.00	15	0.133
410	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	3	2	1.00	15	0.133
412	A	3	2	1.00	15	0.133
413	A	3	2	1.00	15	0.133
414	A	3	2	1.00	15	0.133
415	A	3	2	1.00	15	0.133
416	A	1	1	1.00	13	0.077
417	A	8	4	1.00	15	0.267
418	A	8	5	1.00	15	0.333
419	A	8	5	1.00	15	0.333
420	A	8	5	1.00	15	0.333
421	A	8	5	1.00	15	0.333
422	A	8	4	1.00	15	0.267
423	A	9	5	1.00	15	0.333
424	A	10	5	1.00	15	0.333
425	A	10	4	1.00	15	0.267
426	A	9	4	1.00	15	0.267
427	A	8	4	1.00	15	0.267
428	A	7	3	1.00	11	0.273
429	A	7	4	1.00	15	0.267
430	A	7	4	1.00	15	0.267
431	A	7	4	1.00	15	0.267
432	A	7	4	1.00	15	0.267
433	A	7	3	1.00	15	0.200
434	A	1	1	1.00	15	0.067
435	A	2	2	1.00	15	0.133
436	A	3	2	1.00	15	0.133
437	A	4	2	1.00	15	0.133
438	A	5	2	1.00	15	0.133
439	A	6	2	1.00	15	0.133
440	A	7	2	1.00	15	0.133
441	A	3	2	1.00	15	0.133
442	A	4	3	1.00	15	0.200
443	A	3	2	1.00	15	0.133
444	A	3	3	1.00	15	0.200
445	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	2	2	1.00	11	0.182
447	A	4	4	1.00	15	0.267
448	A	2	2	1.00	15	0.133
449	A	4	4	1.00	15	0.267
450	A	1	1	1.00	15	0.067
451	A	5	5	1.00	15	0.333
452	A	3	2	1.00	15	0.133
453	A	4	3	1.00	15	0.200
454	A	3	2	1.00	15	0.133
455	A	3	3	1.00	15	0.200
456	A	1	1	1.00	13	0.077
457	A	2	2	1.00	11	0.182
458	A	4	4	1.00	15	0.267
459	A	2	2	1.00	15	0.133
460	A	4	4	1.00	15	0.267
461	A	1	1	1.00	15	0.067
462	A	5	5	1.00	15	0.333
463	A	3	2	1.00	15	0.133
464	A	5	4	1.00	15	0.267
465	A	3	2	1.00	15	0.133
466	A	4	4	1.00	15	0.267
467	A	1	1	1.00	13	0.077
468	A	3	3	1.00	11	0.273
469	A	4	4	1.00	15	0.267
470	A	3	3	1.00	15	0.200
471	A	4	4	1.00	15	0.267
472	A	1	1	1.00	15	0.067
473	A	5	5	1.00	15	0.333
474	A	3	2	1.00	15	0.133
475	A	5	4	1.00	15	0.267
476	A	3	2	1.00	15	0.133
477	A	4	4	1.00	15	0.267
478	A	1	1	1.00	13	0.077
479	A	3	3	1.00	11	0.273
480	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	3	3	1.00	15	0.200
482	A	4	4	1.00	15	0.267
483	A	1	1	1.00	15	0.067
484	A	5	5	1.00	15	0.333
485	A	3	2	1.00	15	0.133
486	A	4	3	1.00	15	0.200
487	A	3	2	1.00	15	0.133
488	A	3	3	1.00	15	0.200
489	A	1	1	1.00	13	0.077
490	A	2	2	1.00	11	0.182
491	A	3	3	1.00	15	0.200
492	A	1	1	1.00	15	0.067
493	A	4	4	1.00	15	0.267
494	A	2	2	1.00	15	0.133
495	A	5	4	1.00	15	0.267
496	A	3	2	1.00	15	0.133
497	A	4	4	1.00	15	0.267
498	A	3	2	1.00	15	0.133
499	A	3	3	1.00	15	0.200
500	A	1	1	1.00	13	0.077
501	A	1	1	1.00	11	0.091
502	A	4	4	1.00	15	0.267
503	A	2	2	1.00	15	0.133
504	A	5	5	1.00	15	0.333
505	A	3	2	1.00	15	0.133
506	A	5	4	1.00	15	0.267
507	A	3	2	1.00	15	0.133
508	A	4	3	1.00	15	0.200
509	A	3	2	1.00	15	0.133
510	A	1	1	1.00	15	0.067
511	A	1	1	1.00	13	0.077
512	A	2	2	1.00	11	0.182
513	A	5	4	1.00	15	0.267
514	A	3	3	1.00	15	0.200
515	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	4	3	1.00	15	0.200
517	A	7	4	1.00	15	0.267
518	A	3	2	1.00	15	0.133
519	A	6	3	1.00	15	0.200
520	A	3	2	1.00	15	0.133
521	A	1	1	1.00	15	0.067
522	A	3	2	1.00	15	0.133
523	A	2	2	1.00	15	0.133
524	A	3	2	1.00	15	0.133
525	A	3	2	1.00	15	0.133
526	A	1	1	1.00	13	0.077
527	A	4	2	1.00	11	0.182
528	A	7	4	1.00	15	0.267
529	A	5	3	1.00	15	0.200
530	A	8	5	1.00	15	0.333
531	A	6	3	1.00	15	0.200
532	A	3	2	1.00	15	0.133
533	A	3	2	1.00	15	0.133
534	A	3	2	1.00	15	0.133
535	A	2	2	1.00	15	0.133
536	A	1	1	1.00	13	0.077
537	A	1	1	1.00	11	0.091
538	A	3	3	1.00	15	0.200
539	A	1	1	1.00	15	0.067
540	A	4	4	1.00	15	0.267
541	A	2	2	1.00	15	0.133
542	A	5	4	1.00	15	0.267
543	A	3	2	1.00	15	0.133
544	A	3	2	1.00	15	0.133
545	A	3	2	1.00	15	0.133
546	A	2	2	1.00	15	0.133
547	A	1	1	1.00	13	0.077
548	A	1	1	1.00	11	0.091
549	A	3	3	1.00	15	0.200
550	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	4	4	1.00	15	0.267
552	A	2	2	1.00	15	0.133
553	A	5	4	1.00	15	0.267
554	A	3	2	1.00	15	0.133
555	A	4	3	1.00	15	0.200
556	A	3	2	1.00	15	0.133
557	A	3	3	1.00	15	0.200
558	A	1	1	1.00	13	0.077
559	A	2	2	1.00	11	0.182
560	A	3	3	1.00	15	0.200
561	A	1	1	1.00	15	0.067
562	A	4	4	1.00	15	0.267
563	A	2	2	1.00	15	0.133
564	A	5	4	1.00	15	0.267
565	A	3	2	1.00	15	0.133
566	A	4	3	1.00	15	0.200
567	A	3	2	1.00	15	0.133
568	A	3	3	1.00	15	0.200
569	A	1	1	1.00	13	0.077
570	A	2	2	1.00	11	0.182
571	A	3	3	1.00	15	0.200
572	A	1	1	1.00	15	0.067
573	A	4	4	1.00	15	0.267
574	A	2	2	1.00	15	0.133
575	A	5	4	1.00	15	0.267
576	A	1	1	1.00	11	0.091
577	A	1	1	1.00	12	0.083
578	A	2	2	1.00	11	0.182
579	A	2	2	1.00	12	0.167
580	A	1	1	1.00	11	0.091
581	A	1	1	1.00	12	0.083
582	A	2	2	1.00	13	0.154
583	A	2	2	1.00	14	0.143
584	A	2	2	1.00	11	0.182
585	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	2	2	1.00	13	0.154
587	A	2	2	1.00	14	0.143
588	A	2	2	1.00	13	0.154
589	A	5	4	1.00	19	0.210
590	A	6	6	1.00	19	0.316
591	A	4	4	1.00	19	0.210
592	A	5	5	1.00	19	0.263
593	A	3	3	1.00	19	0.158
594	A	5	5	1.00	19	0.263
595	A	3	3	1.00	19	0.158
596	A	6	6	1.00	19	0.316
597	A	6	4	1.00	19	0.210
598	A	7	6	1.00	19	0.316
599	A	5	4	1.00	19	0.210
600	A	6	5	1.00	19	0.263
601	A	4	3	1.00	19	0.158
602	A	6	6	1.00	19	0.316
603	A	4	4	1.00	19	0.210
604	A	6	5	1.00	19	0.263
605	A	4	3	1.00	19	0.158
606	A	7	6	1.00	19	0.316
607	A	6	6	1.00	22	0.273
608	A	5	5	1.00	22	0.227
609	A	5	5	1.00	22	0.227
610	A	4	4	1.00	22	0.182
611	A	5	5	1.00	22	0.227
612	A	4	4	1.00	22	0.182
613	A	4	3	1.00	19	0.158
614	A	5	5	1.00	19	0.263
615	A	3	3	1.00	19	0.158
616	A	4	4	1.00	19	0.210
617	A	2	2	1.00	19	0.105
618	A	5	5	1.00	19	0.263
619	A	3	3	1.00	19	0.158
620	A	6	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	4	4	1.00	19	0.210
622	A	5	5	1.00	19	0.263
623	A	3	3	1.00	19	0.158
624	A	5	5	1.00	19	0.263
625	A	3	3	1.00	19	0.158
626	A	6	6	1.00	19	0.316
627	A	4	4	1.00	19	0.210
628	A	7	6	1.00	19	0.316
629	A	4	3	1.00	19	0.158
630	A	6	6	1.00	19	0.316
631	A	4	4	1.00	19	0.210
632	A	6	5	1.00	19	0.263
633	A	4	3	1.00	19	0.158
634	A	7	6	1.00	19	0.316
635	A	5	4	1.00	19	0.210
636	A	8	6	1.00	19	0.316
637	A	5	5	1.00	22	0.227
638	A	4	4	1.00	22	0.182
639	A	4	4	1.00	22	0.182
640	A	3	3	1.00	22	0.136
641	A	5	5	1.00	22	0.227
642	A	4	4	1.00	22	0.182
643	A	5	5	1.00	22	0.227
644	A	4	4	1.00	22	0.182
645	A	5	5	1.00	22	0.227
646	A	4	4	1.00	22	0.182
647	A	6	6	1.00	22	0.273
648	A	5	5	1.00	22	0.227
649	A	2	2	1.00	20	0.100
650	A	2	2	1.00	17	0.118
651	A	2	2	1.28	15	0.133
652	A	2	2	1.26	15	0.133
653	A	2	2	1.26	15	0.133
654	A	2	2	1.38	15	0.133
655	A	2	2	1.32	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	2	2	1.26	17	0.118
657	A	2	2	1.26	17	0.118
658	A	2	2	1.26	15	0.133
659	A	2	2	1.24	17	0.118
660	A	2	2	1.29	17	0.118
661	A	1	1	1.00	31	0.032
662	C	5	2	7.47	43	0.047
663	A	1	1	1.00	29	0.034
664	C	5	2	8.20	38	0.053
665	A	3	2	1.00	15	0.133
666	A	3	2	1.00	15	0.133
667	A	3	2	1.00	15	0.133
668	A	1	1	1.00	13	0.077
669	A	6	6	1.00	15	0.400
670	A	6	6	1.00	15	0.400
671	A	7	7	1.00	15	0.467
672	A	5	4	1.00	15	0.267
673	A	4	4	1.00	15	0.267
674	A	3	3	1.00	11	0.273
675	A	3	3	1.00	15	0.200
676	A	4	4	1.00	15	0.267
677	A	3	2	1.00	15	0.133
678	A	3	2	1.00	15	0.133
679	A	3	2	1.00	15	0.133
680	A	1	1	1.00	13	0.077
681	A	6	6	1.00	15	0.400
682	A	6	6	1.00	15	0.400
683	A	7	7	1.00	15	0.467
684	A	7	6	1.00	15	0.400
685	A	6	6	1.00	15	0.400
686	A	5	5	1.00	11	0.454
687	A	5	5	1.00	15	0.333
688	A	6	6	1.00	15	0.400
689	A	3	2	1.00	15	0.133
690	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	3	2	1.00	15	0.133
692	A	1	1	1.00	13	0.077
693	A	7	6	1.00	15	0.400
694	A	7	7	1.00	15	0.467
695	A	7	6	1.00	15	0.400
696	A	6	4	1.00	15	0.267
697	A	5	4	1.00	15	0.267
698	A	4	3	1.00	11	0.273
699	A	4	4	1.00	15	0.267
700	A	4	3	1.00	15	0.200
701	A	1	1	1.00	11	0.091
702	A	3	2	1.00	15	0.133
703	A	3	2	1.00	15	0.133
704	A	3	2	1.00	15	0.133
705	A	1	1	1.00	13	0.077
706	A	5	5	1.00	15	0.333
707	A	6	6	1.00	15	0.400
708	A	7	6	1.00	15	0.400
709	A	6	5	1.00	15	0.333
710	A	5	5	1.00	15	0.333
711	A	4	4	1.00	11	0.364
712	A	5	5	1.00	15	0.333
713	A	6	5	1.00	15	0.333
714	A	3	2	1.00	15	0.133
715	A	3	2	1.00	15	0.133
716	A	3	2	1.00	15	0.133
717	A	1	1	1.00	13	0.077
718	A	5	5	1.00	15	0.333
719	A	6	6	1.00	15	0.400
720	A	7	6	1.00	15	0.400
721	A	4	3	1.00	15	0.200
722	A	3	3	1.00	15	0.200
723	A	2	2	1.00	11	0.182
724	A	3	3	1.00	15	0.200
725	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	3	2	1.00	15	0.133
727	A	3	2	1.00	15	0.133
728	A	3	2	1.00	15	0.133
729	A	1	1	1.00	13	0.077
730	A	6	6	1.00	15	0.400
731	A	7	7	1.00	15	0.467
732	A	8	7	1.00	15	0.467
733	A	6	6	1.00	15	0.400
734	A	5	5	1.00	15	0.333
735	A	5	5	1.00	11	0.454
736	A	6	6	1.00	15	0.400
737	A	7	6	1.00	15	0.400
738	A	6	5	1.00	19	0.263
739	A	5	5	1.00	19	0.263
740	A	4	4	1.00	19	0.210
741	A	4	4	1.00	19	0.210
742	A	1	1	1.00	19	0.053
743	A	2	2	1.00	19	0.105
744	A	3	2	1.00	19	0.105
745	A	4	2	1.00	19	0.105
746	A	6	5	1.00	19	0.263
747	A	5	5	1.00	19	0.263
748	A	4	4	1.00	19	0.210
749	A	4	4	1.00	19	0.210
750	A	5	5	1.00	19	0.263
751	A	2	2	1.00	19	0.105
752	A	2	2	1.00	19	0.105
753	A	2	2	1.00	19	0.105
754	A	7	5	1.00	19	0.263
755	A	6	5	1.00	19	0.263
756	A	5	4	1.00	19	0.210
757	A	5	5	1.00	19	0.263
758	A	5	4	1.00	19	0.210
759	A	1	1	1.00	19	0.053
760	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	3	2	1.00	19	0.105
762	A	7	5	1.00	19	0.263
763	A	6	5	1.00	19	0.263
764	A	5	4	1.00	19	0.210
765	A	5	5	1.00	19	0.263
766	A	5	4	1.00	19	0.210
767	A	6	5	1.00	19	0.263
768	A	2	2	1.00	19	0.105
769	A	2	2	1.00	19	0.105
770	A	2	2	1.00	19	0.105
771	A	6	4	1.00	19	0.210
772	A	5	4	1.00	19	0.210
773	A	4	4	1.00	19	0.210
774	A	3	3	1.00	19	0.158
775	A	1	1	1.00	19	0.053
776	A	2	2	1.00	19	0.105
777	A	3	2	1.00	19	0.105
778	A	4	2	1.00	19	0.105
779	A	5	4	1.00	19	0.210
780	A	4	4	1.00	19	0.210
781	A	3	3	1.00	19	0.158
782	A	4	4	1.00	19	0.210
783	A	5	4	1.00	19	0.210
784	A	2	2	1.00	19	0.105
785	A	2	2	1.00	19	0.105
786	A	2	2	1.00	19	0.105
787	A	5	4	1.00	15	0.267
788	A	4	4	1.00	15	0.267
789	A	3	3	1.00	11	0.273
790	A	3	3	1.00	15	0.200
791	A	4	4	1.00	15	0.267
792	A	5	4	1.00	15	0.267
793	A	5	4	1.00	16	0.250
794	A	4	4	1.00	16	0.250
795	A	3	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	3	3	1.00	16	0.188
797	A	4	4	1.00	16	0.250
798	A	5	4	1.00	16	0.250
799	A	6	5	1.00	15	0.333
800	A	5	5	1.00	15	0.333
801	A	4	4	1.00	11	0.364
802	A	4	4	1.00	15	0.267
803	A	5	5	1.00	15	0.333
804	A	6	5	1.00	15	0.333
805	A	5	4	1.00	16	0.250
806	A	4	4	1.00	16	0.250
807	A	3	3	1.00	12	0.250
808	A	3	3	1.00	16	0.188
809	A	4	4	1.00	16	0.250
810	A	5	4	1.00	16	0.250
811	A	4	3	1.00	11	0.273
812	A	4	3	1.00	12	0.250
813	A	5	4	1.00	11	0.364
814	A	4	3	1.00	12	0.250
815	A	6	4	1.00	15	0.267
816	A	5	4	1.00	15	0.267
817	A	4	4	1.00	15	0.267
818	A	3	3	1.00	11	0.273
819	A	4	4	1.00	15	0.267
820	A	5	4	1.00	15	0.267
821	A	6	4	1.00	15	0.267
822	A	5	3	1.00	16	0.188
823	A	4	3	1.00	16	0.188
824	A	3	3	1.00	16	0.188
825	A	2	2	1.00	12	0.167
826	A	3	3	1.00	16	0.188
827	A	4	3	1.00	16	0.188
828	A	5	3	1.00	16	0.188
829	A	5	3	1.00	15	0.200
830	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	3	3	1.00	15	0.200
832	A	2	2	1.00	11	0.182
833	A	3	3	1.00	15	0.200
834	A	4	3	1.00	15	0.200
835	A	5	3	1.00	15	0.200
836	A	5	3	1.00	16	0.188
837	A	4	3	1.00	16	0.188
838	A	3	3	1.00	16	0.188
839	A	2	2	1.00	12	0.167
840	A	3	3	1.00	16	0.188
841	A	4	3	1.00	16	0.188
842	A	5	3	1.00	16	0.188
843	A	5	3	1.00	15	0.200
844	A	4	3	1.00	15	0.200
845	A	3	3	1.00	15	0.200
846	A	2	2	1.00	11	0.182
847	A	3	3	1.00	15	0.200
848	A	4	3	1.00	15	0.200
849	A	5	3	1.00	15	0.200
850	A	5	4	1.00	16	0.250
851	A	4	4	1.00	16	0.250
852	A	3	3	1.00	16	0.188
853	A	3	3	1.00	12	0.250
854	A	4	4	1.00	16	0.250
855	A	5	4	1.00	16	0.250
856	A	6	4	1.00	16	0.250
857	A	3	3	1.00	11	0.273
858	A	3	3	1.00	11	0.273
859	A	4	3	1.00	11	0.273
860	A	3	3	1.00	12	0.250
861	A	4	3	1.00	12	0.250
862	A	4	3	1.00	12	0.250
863	A	5	3	1.00	15	0.200
864	A	4	3	1.00	15	0.200
865	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	2	2	1.00	11	0.182
867	A	3	3	1.00	15	0.200
868	A	4	3	1.00	15	0.200
869	A	5	3	1.00	15	0.200
870	A	4	2	1.00	15	0.133
871	A	3	2	1.00	15	0.133
872	A	2	2	1.00	15	0.133
873	A	1	1	1.00	11	0.091
874	A	2	2	1.00	15	0.133
875	A	3	2	1.00	15	0.133
876	A	4	2	1.00	15	0.133
877	A	4	2	1.00	15	0.133
878	A	3	2	1.00	15	0.133
879	A	2	2	1.00	15	0.133
880	A	1	1	1.00	11	0.091
881	A	2	2	1.00	15	0.133
882	A	3	2	1.00	15	0.133
883	A	4	2	1.00	15	0.133
884	A	4	2	1.00	15	0.133
885	A	3	2	1.00	15	0.133
886	A	2	2	1.00	15	0.133
887	A	1	1	1.00	11	0.091
888	A	2	2	1.00	15	0.133
889	A	3	2	1.00	15	0.133
890	A	4	2	1.00	15	0.133
891	A	7	5	1.00	15	0.333
892	A	6	5	1.00	15	0.333
893	A	5	5	1.00	15	0.333
894	A	4	4	1.00	11	0.364
895	A	5	5	1.00	15	0.333
896	A	6	5	1.00	15	0.333
897	A	7	5	1.00	15	0.333
898	A	7	5	1.00	15	0.333
899	A	6	5	1.00	15	0.333
900	A	5	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	4	4	1.00	11	0.364
902	A	5	5	1.00	15	0.333
903	A	6	5	1.00	15	0.333
904	A	7	5	1.00	15	0.333
905	A	5	3	1.00	15	0.200
906	A	4	3	1.00	15	0.200
907	A	3	3	1.00	15	0.200
908	A	2	2	1.00	11	0.182
909	A	3	3	1.00	15	0.200
910	A	4	3	1.00	15	0.200
911	A	5	3	1.00	15	0.200
912	A	5	3	1.00	15	0.200
913	A	4	3	1.00	15	0.200
914	A	3	3	1.00	15	0.200
915	A	2	2	1.00	11	0.182
916	A	3	3	1.00	15	0.200
917	A	4	3	1.00	15	0.200
918	A	5	3	1.00	15	0.200
919	A	8	7	1.00	19	0.368
920	A	7	7	1.00	19	0.368
921	A	6	6	1.00	19	0.316
922	A	6	6	1.00	19	0.316
923	A	7	7	1.00	19	0.368
924	A	8	7	1.00	19	0.368
925	A	7	7	1.00	19	0.368
926	A	6	6	1.00	19	0.316
927	A	6	6	1.00	19	0.316
928	A	1	1	1.00	19	0.053
929	A	2	2	1.00	19	0.105
930	A	3	2	1.00	19	0.105
931	A	4	2	1.00	19	0.105
932	A	7	7	1.00	20	0.350
933	A	6	6	1.00	20	0.300
934	A	6	6	1.00	20	0.300
935	A	7	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	8	7	1.00	20	0.350
937	A	13	10	1.00	20	0.500
938	A	12	9	1.00	20	0.450
939	A	12	9	1.00	20	0.450
940	A	1	1	1.00	20	0.050
941	A	2	2	1.00	20	0.100
942	A	3	2	1.00	20	0.100
943	A	4	2	1.00	20	0.100
944	A	6	6	1.00	19	0.316
945	A	5	5	1.00	19	0.263
946	A	1	1	1.00	19	0.053
947	A	2	2	1.00	19	0.105
948	A	3	2	1.00	19	0.105
949	A	6	5	1.00	19	0.263
950	A	5	5	1.00	19	0.263
951	A	4	4	1.00	19	0.210
952	A	4	4	1.00	19	0.210
953	A	5	5	1.00	19	0.263
954	A	6	5	1.00	19	0.263
955	A	12	9	1.00	20	0.450
956	A	11	8	1.00	20	0.400
957	A	1	1	1.00	20	0.050
958	A	2	2	1.00	20	0.100
959	A	3	2	1.00	20	0.100
960	A	5	5	1.00	20	0.250
961	A	4	4	1.00	20	0.200
962	A	3	3	1.00	20	0.150
963	A	4	4	1.00	20	0.200
964	A	5	4	1.00	20	0.200
965	A	6	6	1.00	19	0.316
966	A	5	5	1.00	19	0.263
967	A	6	6	1.00	19	0.316
968	A	7	6	1.00	19	0.316
969	A	8	6	1.00	19	0.316
970	A	6	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	5	5	1.00	19	0.263
972	A	1	1	1.00	19	0.053
973	A	2	2	1.00	19	0.105
974	A	3	2	1.00	19	0.105
975	A	6	6	1.00	20	0.300
976	A	5	5	1.00	20	0.250
977	A	6	6	1.00	20	0.300
978	A	7	6	1.00	20	0.300
979	A	8	6	1.00	20	0.300
980	A	12	9	1.00	20	0.450
981	A	11	8	1.00	20	0.400
982	A	1	1	1.00	20	0.050
983	A	2	2	1.00	20	0.100
984	A	3	2	1.00	20	0.100
985	A	7	7	1.00	19	0.368
986	A	6	6	1.00	19	0.316
987	A	1	1	1.00	19	0.053
988	A	2	2	1.00	19	0.105
989	A	3	2	1.00	19	0.105
990	A	4	2	1.00	19	0.105
991	A	6	4	1.00	19	0.210
992	A	5	4	1.00	19	0.210
993	A	4	4	1.00	19	0.210
994	A	3	3	1.00	19	0.158
995	A	4	4	1.00	19	0.210
996	A	5	4	1.00	19	0.210
997	A	6	4	1.00	19	0.210
998	A	2	2	1.00	19	0.105
999	A	2	2	1.00	19	0.105
1000	A	2	2	1.00	19	0.105
1001	A	2	2	1.00	19	0.105
1002	A	2	2	1.00	19	0.105
1003	A	2	2	1.00	19	0.105
1004	A	2	2	1.00	19	0.105
1005	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1006	A	2	2	1.00	19	0.105
1007	A	2	2	1.00	19	0.105
1008	A	2	2	1.00	19	0.105
1009	A	2	2	1.00	19	0.105
1010	A	7	5	1.00	15	0.333
1011	A	6	5	1.00	15	0.333
1012	A	5	5	1.00	15	0.333
1013	A	4	4	1.00	11	0.364
1014	A	4	4	1.00	15	0.267
1015	A	5	5	1.00	15	0.333
1016	A	6	5	1.00	15	0.333
1017	A	7	5	1.00	15	0.333
1018	A	9	7	1.00	15	0.467
1019	A	8	7	1.00	15	0.467
1020	A	7	7	1.00	15	0.467
1021	A	6	6	1.00	11	0.546
1022	A	7	7	1.00	15	0.467
1023	A	8	7	1.00	15	0.467
1024	A	9	7	1.00	15	0.467
1025	A	6	4	1.00	15	0.267
1026	A	5	4	1.00	15	0.267
1027	A	4	4	1.00	15	0.267
1028	A	3	3	1.00	11	0.273
1029	A	4	4	1.00	15	0.267
1030	A	5	4	1.00	15	0.267
1031	A	6	4	1.00	15	0.267
1032	A	9	8	1.00	15	0.533
1033	A	8	8	1.00	15	0.533
1034	A	7	7	1.00	15	0.467
1035	A	5	5	1.00	11	0.454
1036	A	8	8	1.00	15	0.533
1037	A	9	8	1.00	15	0.533
1038	A	10	8	1.00	15	0.533
1039	A	3	2	1.00	13	0.154
1040	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1041	A	3	2	1.00	13	0.154
1042	A	1	1	1.00	11	0.091
1043	A	2	2	1.00	13	0.154
1044	A	2	2	1.00	13	0.154
1045	A	2	2	1.22	13	0.154
1046	A	2	2	1.22	13	0.154
1047	A	2	2	1.22	13	0.154
1048	A	2	2	1.26	9	0.222
1049	A	2	2	1.24	13	0.154
1050	A	2	2	1.21	15	0.133
1051	A	2	2	1.21	15	0.133
1052	A	2	2	1.21	15	0.133
1053	A	2	2	1.21	15	0.133
1054	A	2	2	1.22	15	0.133
1055	A	2	2	1.22	15	0.133
1056	A	2	2	1.21	15	0.133
1057	A	2	2	1.21	15	0.133
1058	A	2	2	1.15	13	0.154
1059	A	2	2	1.00	15	0.133
1060	A	2	2	1.32	17	0.118
1061	A	3	2	1.00	17	0.118
1062	A	2	2	1.32	17	0.118
1063	A	2	2	1.00	17	0.118
1064	A	2	2	1.32	17	0.118
1065	A	1	1	1.00	17	0.059
1066	A	2	2	1.32	17	0.118
1067	A	2	2	1.30	17	0.118
1068	A	2	2	1.33	15	0.133
1069	A	2	2	1.31	17	0.118
1070	A	2	2	1.33	17	0.118
1071	A	2	2	1.31	17	0.118

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^4(a+bx^2)\,dx$ 280
3.2 $\int x^3(a+bx^2)\,dx$ 283
3.3 $\int x^2(a+bx^2)\,dx$ 286
3.4 $\int x(a+bx^2)\,dx$ 289
3.5 $\int (a+bx^2)\,dx$ 292
3.6 $\int \frac{a+bx^2}{x}\,dx$ 295
3.7 $\int \frac{a+bx^2}{x^2}\,dx$ 298
3.8 $\int \frac{a+bx^2}{x^3}\,dx$ 301
3.9 $\int \frac{a+bx^2}{x^4}\,dx$ 304
3.10 $\int \frac{a+bx^2}{x^5}\,dx$ 307
3.11 $\int \frac{a+bx^2}{x^6}\,dx$ 310
3.12 $\int \frac{a+bx^2}{x^7}\,dx$ 313
3.13 $\int x^5(a+bx^2)^2\,dx$ 316
3.14 $\int x^4(a+bx^2)^2\,dx$ 319
3.15 $\int x^3(a+bx^2)^2\,dx$ 322
3.16 $\int x^2(a+bx^2)^2\,dx$ 325
3.17 $\int x(a+bx^2)^2\,dx$ 328
3.18 $\int (a+bx^2)^2\,dx$ 331
3.19 $\int \frac{(a+bx^2)^2}{x}\,dx$ 334
3.20 $\int \frac{(a+bx^2)^2}{x^2}\,dx$ 337
3.21 $\int \frac{(a+bx^2)^2}{x^3}\,dx$ 340
3.22 $\int \frac{(a+bx^2)^2}{x^4}\,dx$ 343
3.23 $\int \frac{(a+bx^2)^2}{x^5}\,dx$ 346
3.24 $\int \frac{(a+bx^2)^2}{x^6}\,dx$ 349

3.25 $\int \frac{(a+bx^2)^2}{x^7}\,dx$ . . . 352
3.26 $\int \frac{(a+bx^2)^2}{x^8}\,dx$ . . . 355
3.27 $\int \frac{(a+bx^2)^2}{x^9}\,dx$ . . . 358
3.28 $\int \frac{(a+bx^2)^2}{x^{10}}\,dx$ . . . 361
3.29 $\int x^9(a+bx^2)^3\,dx$ . . . 364
3.30 $\int x^7(a+bx^2)^3\,dx$ . . . 367
3.31 $\int x^5(a+bx^2)^3\,dx$ . . . 370
3.32 $\int x^3(a+bx^2)^3\,dx$ . . . 373
3.33 $\int x(a+bx^2)^3\,dx$ . . . 376
3.34 $\int \frac{(a+bx^2)^3}{x}\,dx$ . . . 379
3.35 $\int \frac{(a+bx^2)^3}{x^3}\,dx$ . . . 382
3.36 $\int \frac{(a+bx^2)^3}{x^5}\,dx$ . . . 385
3.37 $\int \frac{(a+bx^2)^3}{x^7}\,dx$ . . . 388
3.38 $\int \frac{(a+bx^2)^3}{x^9}\,dx$ . . . 391
3.39 $\int \frac{(a+bx^2)^3}{x^{11}}\,dx$ . . . 394
3.40 $\int \frac{(a+bx^2)^3}{x^{13}}\,dx$ . . . 397
3.41 $\int \frac{(a+bx^2)^3}{x^{15}}\,dx$ . . . 400
3.42 $\int x^6(a+bx^2)^3\,dx$ . . . 403
3.43 $\int x^4(a+bx^2)^3\,dx$ . . . 406
3.44 $\int x^2(a+bx^2)^3\,dx$ . . . 409
3.45 $\int (a+bx^2)^3\,dx$ . . . 412
3.46 $\int \frac{(a+bx^2)^3}{x^2}\,dx$ . . . 415
3.47 $\int \frac{(a+bx^2)^3}{x^4}\,dx$ . . . 418
3.48 $\int \frac{(a+bx^2)^3}{x^6}\,dx$ . . . 421
3.49 $\int \frac{(a+bx^2)^3}{x^8}\,dx$ . . . 424
3.50 $\int \frac{(a+bx^2)^3}{x^{10}}\,dx$ . . . 427
3.51 $\int \frac{(a+bx^2)^3}{x^{12}}\,dx$ . . . 430
3.52 $\int x^{13}(a+bx^2)^5\,dx$ . . . 433
3.53 $\int x^{11}(a+bx^2)^5\,dx$ . . . 436
3.54 $\int x^9(a+bx^2)^5\,dx$ . . . 439
3.55 $\int x^7(a+bx^2)^5\,dx$ . . . 442
3.56 $\int x^5(a+bx^2)^5\,dx$ . . . 445
3.57 $\int x^3(a+bx^2)^5\,dx$ . . . 448
3.58 $\int x(a+bx^2)^5\,dx$ . . . 451
3.59 $\int \frac{(a+bx^2)^5}{x}\,dx$ . . . 454

3.60 $\int \frac{(a+bx^2)^5}{x^3}\,dx$ . . . 457
3.61 $\int \frac{(a+bx^2)^5}{x^5}\,dx$ . . . 460
3.62 $\int \frac{(a+bx^2)^5}{x^7}\,dx$ . . . 463
3.63 $\int \frac{(a+bx^2)^5}{x^9}\,dx$ . . . 466
3.64 $\int \frac{(a+bx^2)^5}{x^{11}}\,dx$ . . . 469
3.65 $\int \frac{(a+bx^2)^5}{x^{13}}\,dx$ . . . 472
3.66 $\int \frac{(a+bx^2)^5}{x^{15}}\,dx$ . . . 475
3.67 $\int \frac{(a+bx^2)^5}{x^{17}}\,dx$ . . . 478
3.68 $\int \frac{(a+bx^2)^5}{x^{19}}\,dx$ . . . 482
3.69 $\int \frac{(a+bx^2)^5}{x^{21}}\,dx$ . . . 485
3.70 $\int x^8(a+bx^2)^5\,dx$ . . . 488
3.71 $\int x^6(a+bx^2)^5\,dx$ . . . 491
3.72 $\int x^4(a+bx^2)^5\,dx$ . . . 494
3.73 $\int x^2(a+bx^2)^5\,dx$ . . . 497
3.74 $\int (a+bx^2)^5\,dx$ . . . 500
3.75 $\int \frac{(a+bx^2)^5}{x^2}\,dx$ . . . 503
3.76 $\int \frac{(a+bx^2)^5}{x^4}\,dx$ . . . 506
3.77 $\int \frac{(a+bx^2)^5}{x^6}\,dx$ . . . 509
3.78 $\int \frac{(a+bx^2)^5}{x^8}\,dx$ . . . 512
3.79 $\int \frac{(a+bx^2)^5}{x^{10}}\,dx$ . . . 515
3.80 $\int \frac{(a+bx^2)^5}{x^{12}}\,dx$ . . . 518
3.81 $\int \frac{(a+bx^2)^5}{x^{14}}\,dx$ . . . 521
3.82 $\int \frac{(a+bx^2)^5}{x^{16}}\,dx$ . . . 524
3.83 $\int \frac{(a+bx^2)^5}{x^{18}}\,dx$ . . . 527
3.84 $\int \frac{(a+bx^2)^5}{x^{20}}\,dx$ . . . 530
3.85 $\int x^{13}(a+bx^2)^8\,dx$ . . . 533
3.86 $\int x^{11}(a+bx^2)^8\,dx$ . . . 536
3.87 $\int x^9(a+bx^2)^8\,dx$ . . . 539
3.88 $\int x^7(a+bx^2)^8\,dx$ . . . 542
3.89 $\int x^5(a+bx^2)^8\,dx$ . . . 545
3.90 $\int x^3(a+bx^2)^8\,dx$ . . . 548
3.91 $\int x(a+bx^2)^8\,dx$ . . . 551
3.92 $\int \frac{(a+bx^2)^8}{x}\,dx$ . . . 554
3.93 $\int \frac{(a+bx^2)^8}{x^3}\,dx$ . . . 557
3.94 $\int \frac{(a+bx^2)^8}{x^5}\,dx$ . . . 560

3.95 $\int \frac{(a+bx^2)^8}{x^7}\,dx$ 563
3.96 $\int \frac{(a+bx^2)^8}{x^9}\,dx$ 566
3.97 $\int \frac{(a+bx^2)^8}{x^{11}}\,dx$ 569
3.98 $\int \frac{(a+bx^2)^8}{x^{13}}\,dx$ 572
3.99 $\int \frac{(a+bx^2)^8}{x^{15}}\,dx$ 575
3.100 $\int \frac{(a+bx^2)^8}{x^{17}}\,dx$ 578
3.101 $\int \frac{(a+bx^2)^8}{x^{19}}\,dx$ 581
3.102 $\int \frac{(a+bx^2)^8}{x^{21}}\,dx$ 584
3.103 $\int \frac{(a+bx^2)^8}{x^{23}}\,dx$ 588
3.104 $\int \frac{(a+bx^2)^8}{x^{25}}\,dx$ 592
3.105 $\int \frac{(a+bx^2)^8}{x^{27}}\,dx$ 596
3.106 $\int \frac{(a+bx^2)^8}{x^{29}}\,dx$ 600
3.107 $\int \frac{(a+bx^2)^8}{x^{31}}\,dx$ 603
3.108 $\int \frac{(a+bx^2)^8}{x^{33}}\,dx$ 606
3.109 $\int x^8(a+bx^2)^8\,dx$ 609
3.110 $\int x^6(a+bx^2)^8\,dx$ 612
3.111 $\int x^4(a+bx^2)^8\,dx$ 615
3.112 $\int x^2(a+bx^2)^8\,dx$ 618
3.113 $\int (a+bx^2)^8\,dx$ 621
3.114 $\int \frac{(a+bx^2)^8}{x^2}\,dx$ 624
3.115 $\int \frac{(a+bx^2)^8}{x^4}\,dx$ 627
3.116 $\int \frac{(a+bx^2)^8}{x^6}\,dx$ 630
3.117 $\int \frac{(a+bx^2)^8}{x^8}\,dx$ 633
3.118 $\int \frac{(a+bx^2)^8}{x^{10}}\,dx$ 636
3.119 $\int \frac{(a+bx^2)^8}{x^{12}}\,dx$ 639
3.120 $\int \frac{(a+bx^2)^8}{x^{14}}\,dx$ 642
3.121 $\int \frac{(a+bx^2)^8}{x^{16}}\,dx$ 645
3.122 $\int \frac{(a+bx^2)^8}{x^{18}}\,dx$ 648
3.123 $\int \frac{(a+bx^2)^8}{x^{20}}\,dx$ 651
3.124 $\int \frac{x^{11}}{a+bx^2}\,dx$ 654
3.125 $\int \frac{x^{10}}{a+bx^2}\,dx$ 657
3.126 $\int \frac{x^9}{a+bx^2}\,dx$ 660
3.127 $\int \frac{x^8}{a+bx^2}\,dx$ 663
3.128 $\int \frac{x^7}{a+bx^2}\,dx$ 666

3.129 $\int \frac{x^6}{a+bx^2}\,dx$ 669
3.130 $\int \frac{x^5}{a+bx^2}\,dx$ 673
3.131 $\int \frac{x^4}{a+bx^2}\,dx$ 676
3.132 $\int \frac{x^3}{a+bx^2}\,dx$ 680
3.133 $\int \frac{x^2}{a+bx^2}\,dx$ 683
3.134 $\int \frac{x}{a+bx^2}\,dx$ 687
3.135 $\int \frac{1}{a+bx^2}\,dx$ 690
3.136 $\int \frac{1}{x(a+bx^2)}\,dx$ 693
3.137 $\int \frac{1}{x^2(a+bx^2)}\,dx$ 696
3.138 $\int \frac{1}{x^3(a+bx^2)}\,dx$ 700
3.139 $\int \frac{1}{x^4(a+bx^2)}\,dx$ 703
3.140 $\int \frac{1}{x^5(a+bx^2)}\,dx$ 707
3.141 $\int \frac{1}{x^6(a+bx^2)}\,dx$ 710
3.142 $\int \frac{1}{x^7(a+bx^2)}\,dx$ 714
3.143 $\int \frac{1}{x^8(a+bx^2)}\,dx$ 717
3.144 $\int \frac{1}{x^9(a+bx^2)}\,dx$ 721
3.145 $\int \frac{x^{13}}{(a+bx^2)^2}\,dx$ 724
3.146 $\int \frac{x^{12}}{(a+bx^2)^2}\,dx$ 727
3.147 $\int \frac{x^{11}}{(a+bx^2)^2}\,dx$ 731
3.148 $\int \frac{x^{10}}{(a+bx^2)^2}\,dx$ 734
3.149 $\int \frac{x^9}{(a+bx^2)^2}\,dx$ 738
3.150 $\int \frac{x^8}{(a+bx^2)^2}\,dx$ 741
3.151 $\int \frac{x^7}{(a+bx^2)^2}\,dx$ 745
3.152 $\int \frac{x^6}{(a+bx^2)^2}\,dx$ 748
3.153 $\int \frac{x^5}{(a+bx^2)^2}\,dx$ 752
3.154 $\int \frac{x^4}{(a+bx^2)^2}\,dx$ 755
3.155 $\int \frac{x^3}{(a+bx^2)^2}\,dx$ 759
3.156 $\int \frac{x^2}{(a+bx^2)^2}\,dx$ 762
3.157 $\int \frac{x}{(a+bx^2)^2}\,dx$ 765
3.158 $\int \frac{1}{(a+bx^2)^2}\,dx$ 768
3.159 $\int \frac{1}{x(a+bx^2)^2}\,dx$ 771
3.160 $\int \frac{1}{x^2(a+bx^2)^2}\,dx$ 774
3.161 $\int \frac{1}{x^3(a+bx^2)^2}\,dx$ 778
3.162 $\int \frac{1}{x^4(a+bx^2)^2}\,dx$ 781
3.163 $\int \frac{1}{x^5(a+bx^2)^2}\,dx$ 785
3.164 $\int \frac{1}{x^6(a+bx^2)^2}\,dx$ 788

3.165 $\int \frac{1}{x^7(a+bx^2)^2}\,dx$ . . . 792
3.166 $\int \frac{1}{x^8(a+bx^2)^2}\,dx$ . . . 795
3.167 $\int \frac{1}{x^9(a+bx^2)^2}\,dx$ . . . 799
3.168 $\int \frac{x^{15}}{(a+bx^2)^3}\,dx$ . . . 802
3.169 $\int \frac{x^{13}}{(a+bx^2)^3}\,dx$ . . . 806
3.170 $\int \frac{x^{11}}{(a+bx^2)^3}\,dx$ . . . 810
3.171 $\int \frac{x^9}{(a+bx^2)^3}\,dx$ . . . 813
3.172 $\int \frac{x^7}{(a+bx^2)^3}\,dx$ . . . 816
3.173 $\int \frac{x^5}{(a+bx^2)^3}\,dx$ . . . 819
3.174 $\int \frac{x^3}{(a+bx^2)^3}\,dx$ . . . 822
3.175 $\int \frac{x}{(a+bx^2)^3}\,dx$ . . . 825
3.176 $\int \frac{1}{x(a+bx^2)^3}\,dx$ . . . 828
3.177 $\int \frac{1}{x^3(a+bx^2)^3}\,dx$ . . . 831
3.178 $\int \frac{1}{x^5(a+bx^2)^3}\,dx$ . . . 834
3.179 $\int \frac{1}{x^7(a+bx^2)^3}\,dx$ . . . 837
3.180 $\int \frac{1}{x^9(a+bx^2)^3}\,dx$ . . . 840
3.181 $\int \frac{x^{12}}{(a+bx^2)^3}\,dx$ . . . 843
3.182 $\int \frac{x^{10}}{(a+bx^2)^3}\,dx$ . . . 847
3.183 $\int \frac{x^8}{(a+bx^2)^3}\,dx$ . . . 851
3.184 $\int \frac{x^6}{(a+bx^2)^3}\,dx$ . . . 855
3.185 $\int \frac{x^4}{(a+bx^2)^3}\,dx$ . . . 859
3.186 $\int \frac{x^2}{(a+bx^2)^3}\,dx$ . . . 863
3.187 $\int \frac{1}{(a+bx^2)^3}\,dx$ . . . 867
3.188 $\int \frac{1}{x^2(a+bx^2)^3}\,dx$ . . . 871
3.189 $\int \frac{1}{x^4(a+bx^2)^3}\,dx$ . . . 875
3.190 $\int \frac{1}{x^6(a+bx^2)^3}\,dx$ . . . 879
3.191 $\int \frac{1}{x^8(a+bx^2)^3}\,dx$ . . . 883
3.192 $\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx$ . . . 887
3.193 $\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx$ . . . 891
3.194 $\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx$ . . . 895
3.195 $\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx$ . . . 899
3.196 $\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx$ . . . 903
3.197 $\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx$ . . . 906
3.198 $\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx$ . . . 910

3.199 $\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx$ . . . 914
3.200 $\int \frac{x^{9}}{(a+bx^2)^{10}}\,dx$ . . . 918
3.201 $\int \frac{x^{7}}{(a+bx^2)^{10}}\,dx$ . . . 921
3.202 $\int \frac{x^{5}}{(a+bx^2)^{10}}\,dx$ . . . 924
3.203 $\int \frac{x^{3}}{(a+bx^2)^{10}}\,dx$ . . . 927
3.204 $\int \frac{x}{(a+bx^2)^{10}}\,dx$ . . . 930
3.205 $\int \frac{1}{x(a+bx^2)^{10}}\,dx$ . . . 933
3.206 $\int \frac{1}{x^3(a+bx^2)^{10}}\,dx$ . . . 937
3.207 $\int \frac{1}{x^5(a+bx^2)^{10}}\,dx$ . . . 941
3.208 $\int \frac{1}{x^7(a+bx^2)^{10}}\,dx$ . . . 945
3.209 $\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx$ . . . 949
3.210 $\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx$ . . . 954
3.211 $\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx$ . . . 959
3.212 $\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx$ . . . 964
3.213 $\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx$ . . . 969
3.214 $\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx$ . . . 974
3.215 $\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx$ . . . 979
3.216 $\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx$ . . . 984
3.217 $\int \frac{x^{8}}{(a+bx^2)^{10}}\,dx$ . . . 989
3.218 $\int \frac{x^{6}}{(a+bx^2)^{10}}\,dx$ . . . 994
3.219 $\int \frac{x^{4}}{(a+bx^2)^{10}}\,dx$ . . . 999
3.220 $\int \frac{x^{2}}{(a+bx^2)^{10}}\,dx$ . . . 1004
3.221 $\int \frac{1}{(a+bx^2)^{10}}\,dx$ . . . 1009
3.222 $\int \frac{1}{x^2(a+bx^2)^{10}}\,dx$ . . . 1016
3.223 $\int \frac{1}{x^4(a+bx^2)^{10}}\,dx$ . . . 1021
3.224 $\int \frac{1}{x^6(a+bx^2)^{10}}\,dx$ . . . 1026
3.225 $\int \frac{x^3}{a-bx^2}\,dx$ . . . 1032
3.226 $\int \frac{x^2}{a-bx^2}\,dx$ . . . 1035
3.227 $\int \frac{x}{a-bx^2}\,dx$ . . . 1039
3.228 $\int \frac{1}{a-bx^2}\,dx$ . . . 1042
3.229 $\int \frac{1}{x(a-bx^2)}\,dx$ . . . 1045
3.230 $\int \frac{1}{x^2(a-bx^2)}\,dx$ . . . 1048
3.231 $\int \frac{1}{x^3(a-bx^2)}\,dx$ . . . 1052
3.232 $\int \frac{x^3}{(a-bx^2)^2}\,dx$ . . . 1055
3.233 $\int \frac{x^2}{(a-bx^2)^2}\,dx$ . . . 1058

3.234 $\int \frac{x}{(a-bx^2)^2}\,dx$ 1061
3.235 $\int \frac{1}{(a-bx^2)^2}\,dx$ 1064
3.236 $\int \frac{1}{x(a-bx^2)^2}\,dx$ 1067
3.237 $\int \frac{1}{x^2(a-bx^2)^2}\,dx$ 1070
3.238 $\int \frac{1}{x^3(a-bx^2)^2}\,dx$ 1074
3.239 $\int \frac{x^3}{(a-bx^2)^3}\,dx$ 1077
3.240 $\int \frac{x^2}{(a-bx^2)^3}\,dx$ 1080
3.241 $\int \frac{x}{(a-bx^2)^3}\,dx$ 1084
3.242 $\int \frac{1}{(a-bx^2)^3}\,dx$ 1087
3.243 $\int \frac{1}{x(a-bx^2)^3}\,dx$ 1091
3.244 $\int \frac{1}{x^2(a-bx^2)^3}\,dx$ 1094
3.245 $\int \frac{1}{x^3(a-bx^2)^3}\,dx$ 1098
3.246 $\int \frac{x^3}{(a-bx^2)^5}\,dx$ 1101
3.247 $\int \frac{x^2}{(a-bx^2)^5}\,dx$ 1104
3.248 $\int \frac{x}{(a-bx^2)^5}\,dx$ 1108
3.249 $\int \frac{1}{(a-bx^2)^5}\,dx$ 1111
3.250 $\int \frac{1}{x(a-bx^2)^5}\,dx$ 1115
3.251 $\int \frac{1}{x^2(a-bx^2)^5}\,dx$ 1118
3.252 $\int \frac{1}{x^3(a-bx^2)^5}\,dx$ 1122
3.253 $\int \frac{1}{x(1+bx^2)}\,dx$ 1125
3.254 $\int \frac{1}{x(-1+bx^2)}\,dx$ 1128
3.255 $\int \frac{1}{x^3(1+bx^2)}\,dx$ 1131
3.256 $\int \frac{1}{x^3(-1+bx^2)}\,dx$ 1134
3.257 $\int \frac{1}{-1+a+ax^2}\,dx$ 1137
3.258 $\int \frac{1}{-c-d+(c-d)x^2}\,dx$ 1140
3.259 $\int \frac{1}{x(1+bx^2)^2}\,dx$ 1143
3.260 $\int \frac{1}{x(-1+bx^2)^2}\,dx$ 1146
3.261 $\int \frac{1}{a+(b-ac)x^2}\,dx$ 1149
3.262 $\int \frac{1}{a-(b-ac)x^2}\,dx$ 1152
3.263 $\int \frac{1}{c(a-d)-(b-c)x^2}\,dx$ 1155
3.264 $\int x^{7/2}(a+bx^2)\,dx$ 1158
3.265 $\int x^{5/2}(a+bx^2)\,dx$ 1161
3.266 $\int x^{3/2}(a+bx^2)\,dx$ 1164
3.267 $\int \sqrt{x}\,(a+bx^2)\,dx$ 1167
3.268 $\int \frac{a+bx^2}{\sqrt{x}}\,dx$ 1170
3.269 $\int \frac{a+bx^2}{x^{3/2}}\,dx$ 1173
3.270 $\int \frac{a+bx^2}{x^{5/2}}\,dx$ 1176

3.271 $\int \frac{a+bx^2}{x^{7/2}}\,dx$ . . . 1179
3.272 $\int x^{7/2}(a+bx^2)^2\,dx$ . . . 1182
3.273 $\int x^{5/2}(a+bx^2)^2\,dx$ . . . 1185
3.274 $\int x^{3/2}(a+bx^2)^2\,dx$ . . . 1188
3.275 $\int \sqrt{x}\,(a+bx^2)^2\,dx$ . . . 1191
3.276 $\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx$ . . . 1194
3.277 $\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx$ . . . 1197
3.278 $\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx$ . . . 1200
3.279 $\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx$ . . . 1203
3.280 $\int x^{7/2}(a+bx^2)^3\,dx$ . . . 1206
3.281 $\int x^{5/2}(a+bx^2)^3\,dx$ . . . 1209
3.282 $\int x^{3/2}(a+bx^2)^3\,dx$ . . . 1212
3.283 $\int \sqrt{x}\,(a+bx^2)^3\,dx$ . . . 1215
3.284 $\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx$ . . . 1218
3.285 $\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx$ . . . 1221
3.286 $\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx$ . . . 1224
3.287 $\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx$ . . . 1227
3.288 $\int \frac{x^{7/2}}{a+bx^2}\,dx$ . . . 1230
3.289 $\int \frac{x^{5/2}}{a+bx^2}\,dx$ . . . 1236
3.290 $\int \frac{x^{3/2}}{a+bx^2}\,dx$ . . . 1242
3.291 $\int \frac{\sqrt{x}}{a+bx^2}\,dx$ . . . 1247
3.292 $\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx$ . . . 1252
3.293 $\int \frac{1}{x^{3/2}(a+bx^2)}\,dx$ . . . 1257
3.294 $\int \frac{1}{x^{5/2}(a+bx^2)}\,dx$ . . . 1263
3.295 $\int \frac{1}{x^{7/2}(a+bx^2)}\,dx$ . . . 1268
3.296 $\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx$ . . . 1274
3.297 $\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx$ . . . 1280
3.298 $\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx$ . . . 1286
3.299 $\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx$ . . . 1292
3.300 $\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx$ . . . 1298
3.301 $\int \frac{1}{x^{3/2}(a+bx^2)^2}\,dx$ . . . 1304
3.302 $\int \frac{1}{x^{5/2}(a+bx^2)^2}\,dx$ . . . 1310
3.303 $\int \frac{1}{x^{7/2}(a+bx^2)^2}\,dx$ . . . 1316
3.304 $\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx$ . . . 1322

3.305 $\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx$ 1327
3.306 $\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx$ 1332
3.307 $\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx$ 1338
3.308 $\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx$ 1344
3.309 $\int \frac{1}{x^{3/2}(a+bx^2)^3}\,dx$ 1350
3.310 $\int \frac{1}{x^{5/2}(a+bx^2)^3}\,dx$ 1356
3.311 $\int \frac{1}{x^{7/2}(a+bx^2)^3}\,dx$ 1362
3.312 $\int \frac{\sqrt{x}}{a-bx^2}\,dx$ 1368
3.313 $\int \frac{x^{7/2}}{1+x^2}\,dx$ 1372
3.314 $\int \frac{x^{5/2}}{1+x^2}\,dx$ 1377
3.315 $\int \frac{x^{3/2}}{1+x^2}\,dx$ 1382
3.316 $\int \frac{\sqrt{x}}{1+x^2}\,dx$ 1387
3.317 $\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$ 1392
3.318 $\int \frac{1}{x^{3/2}(1+x^2)}\,dx$ 1397
3.319 $\int \frac{1}{x^{5/2}(1+x^2)}\,dx$ 1402
3.320 $\int \frac{1}{x^{7/2}(1+x^2)}\,dx$ 1407
3.321 $\int \frac{x^{7/2}}{(1+x^2)^2}\,dx$ 1412
3.322 $\int \frac{x^{5/2}}{(1+x^2)^2}\,dx$ 1417
3.323 $\int \frac{x^{3/2}}{(1+x^2)^2}\,dx$ 1422
3.324 $\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx$ 1427
3.325 $\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx$ 1432
3.326 $\int \frac{1}{x^{3/2}(1+x^2)^2}\,dx$ 1437
3.327 $\int \frac{1}{x^{5/2}(1+x^2)^2}\,dx$ 1442
3.328 $\int \frac{1}{x^{7/2}(1+x^2)^2}\,dx$ 1447
3.329 $\int \frac{x^{7/2}}{(1+x^2)^3}\,dx$ 1453
3.330 $\int \frac{x^{5/2}}{(1+x^2)^3}\,dx$ 1459
3.331 $\int \frac{x^{3/2}}{(1+x^2)^3}\,dx$ 1465
3.332 $\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx$ 1471
3.333 $\int \frac{1}{\sqrt{x}\,(1+x^2)^3}\,dx$ 1477
3.334 $\int \frac{1}{x^{3/2}(1+x^2)^3}\,dx$ 1483
3.335 $\int \frac{1}{x^{5/2}(1+x^2)^3}\,dx$ 1489
3.336 $\int \frac{1}{x^{7/2}(1+x^2)^3}\,dx$ 1495

3.337 $\int \frac{\sqrt{x}}{1-x^2}\,dx$ . . . 1501
3.338 $\int \frac{x^{2/3}}{1+x^2}\,dx$ . . . 1504
3.339 $\int x^m(a+bx^2)^5\,dx$ . . . 1508
3.340 $\int x^m(a+bx^2)^4\,dx$ . . . 1513
3.341 $\int x^m(a+bx^2)^3\,dx$ . . . 1517
3.342 $\int x^m(a+bx^2)^2\,dx$ . . . 1521
3.343 $\int x^m(a+bx^2)\,dx$ . . . 1524
3.344 $\int \frac{x^m}{a+bx^2}\,dx$ . . . 1527
3.345 $\int \frac{x^m}{(a+bx^2)^2}\,dx$ . . . 1530
3.346 $\int \frac{x^m}{(a+bx^2)^3}\,dx$ . . . 1533
3.347 $\int \frac{(cx)^{1+m}}{a+bx^2}\,dx$ . . . 1537
3.348 $\int \frac{(cx)^m}{a+bx^2}\,dx$ . . . 1540
3.349 $\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx$ . . . 1543
3.350 $\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx$ . . . 1546
3.351 $\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx$ . . . 1549
3.352 $\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2}\,dx$ . . . 1552
3.353 $\int x^7\sqrt{a+bx^2}\,dx$ . . . 1555
3.354 $\int x^5\sqrt{a+bx^2}\,dx$ . . . 1558
3.355 $\int x^3\sqrt{a+bx^2}\,dx$ . . . 1561
3.356 $\int x\sqrt{a+bx^2}\,dx$ . . . 1564
3.357 $\int \frac{\sqrt{a+bx^2}}{x}\,dx$ . . . 1567
3.358 $\int \frac{\sqrt{a+bx^2}}{x^3}\,dx$ . . . 1571
3.359 $\int \frac{\sqrt{a+bx^2}}{x^5}\,dx$ . . . 1575
3.360 $\int \frac{\sqrt{a+bx^2}}{x^7}\,dx$ . . . 1579
3.361 $\int x^4\sqrt{a+bx^2}\,dx$ . . . 1583
3.362 $\int x^2\sqrt{a+bx^2}\,dx$ . . . 1587
3.363 $\int \sqrt{a+bx^2}\,dx$ . . . 1591
3.364 $\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$ . . . 1595
3.365 $\int \frac{\sqrt{a+bx^2}}{x^4}\,dx$ . . . 1599
3.366 $\int \frac{\sqrt{a+bx^2}}{x^6}\,dx$ . . . 1602
3.367 $\int \frac{\sqrt{a+bx^2}}{x^8}\,dx$ . . . 1605
3.368 $\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx$ . . . 1609
3.369 $\int x^7(a+bx^2)^{3/2}\,dx$ . . . 1613
3.370 $\int x^5(a+bx^2)^{3/2}\,dx$ . . . 1616
3.371 $\int x^3(a+bx^2)^{3/2}\,dx$ . . . 1619

3.372 $\int x(a+bx^2)^{3/2}\,dx$ . . . 1622
3.373 $\int \frac{(a+bx^2)^{3/2}}{x}\,dx$ . . . 1625
3.374 $\int \frac{(a+bx^2)^{3/2}}{x^3}\,dx$ . . . 1629
3.375 $\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx$ . . . 1633
3.376 $\int \frac{(a+bx^2)^{3/2}}{x^7}\,dx$ . . . 1637
3.377 $\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx$ . . . 1641
3.378 $\int x^4(a+bx^2)^{3/2}\,dx$ . . . 1646
3.379 $\int x^2(a+bx^2)^{3/2}\,dx$ . . . 1650
3.380 $\int (a+bx^2)^{3/2}\,dx$ . . . 1654
3.381 $\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx$ . . . 1658
3.382 $\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx$ . . . 1662
3.383 $\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx$ . . . 1666
3.384 $\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx$ . . . 1669
3.385 $\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx$ . . . 1672
3.386 $\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx$ . . . 1676
3.387 $\int x^7(a+bx^2)^{5/2}\,dx$ . . . 1680
3.388 $\int x^5(a+bx^2)^{5/2}\,dx$ . . . 1684
3.389 $\int x^3(a+bx^2)^{5/2}\,dx$ . . . 1687
3.390 $\int x(a+bx^2)^{5/2}\,dx$ . . . 1690
3.391 $\int \frac{(a+bx^2)^{5/2}}{x}\,dx$ . . . 1693
3.392 $\int \frac{(a+bx^2)^{5/2}}{x^3}\,dx$ . . . 1697
3.393 $\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx$ . . . 1701
3.394 $\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx$ . . . 1705
3.395 $\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx$ . . . 1709
3.396 $\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx$ . . . 1714
3.397 $\int x^4(a+bx^2)^{5/2}\,dx$ . . . 1720
3.398 $\int x^2(a+bx^2)^{5/2}\,dx$ . . . 1725
3.399 $\int (a+bx^2)^{5/2}\,dx$ . . . 1729
3.400 $\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx$ . . . 1733
3.401 $\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx$ . . . 1737
3.402 $\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx$ . . . 1741
3.403 $\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx$ . . . 1746
3.404 $\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx$ . . . 1749

3.405 $\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx$ 1752
3.406 $\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx$ 1756
3.407 $\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx$ 1760
3.408 $\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx$ 1765
3.409 $\int x^{15}(a+bx^2)^{9/2}\,dx$ 1770
3.410 $\int x^{13}(a+bx^2)^{9/2}\,dx$ 1775
3.411 $\int x^{11}(a+bx^2)^{9/2}\,dx$ 1780
3.412 $\int x^{9}(a+bx^2)^{9/2}\,dx$ 1784
3.413 $\int x^{7}(a+bx^2)^{9/2}\,dx$ 1788
3.414 $\int x^{5}(a+bx^2)^{9/2}\,dx$ 1792
3.415 $\int x^{3}(a+bx^2)^{9/2}\,dx$ 1795
3.416 $\int x(a+bx^2)^{9/2}\,dx$ 1798
3.417 $\int \frac{(a+bx^2)^{9/2}}{x}\,dx$ 1801
3.418 $\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx$ 1805
3.419 $\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx$ 1809
3.420 $\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx$ 1814
3.421 $\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx$ 1819
3.422 $\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx$ 1824
3.423 $\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx$ 1829
3.424 $\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx$ 1836
3.425 $\int x^{6}(a+bx^2)^{9/2}\,dx$ 1843
3.426 $\int x^{4}(a+bx^2)^{9/2}\,dx$ 1849
3.427 $\int x^{2}(a+bx^2)^{9/2}\,dx$ 1855
3.428 $\int (a+bx^2)^{9/2}\,dx$ 1861
3.429 $\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx$ 1865
3.430 $\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx$ 1871
3.431 $\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx$ 1877
3.432 $\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx$ 1883
3.433 $\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx$ 1889
3.434 $\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx$ 1894
3.435 $\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx$ 1897
3.436 $\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx$ 1901
3.437 $\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx$ 1905
3.438 $\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx$ 1910

3.439 $\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx$ 1915
3.440 $\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx$ 1921
3.441 $\int x^5\sqrt{9+4x^2}\,dx$ 1927
3.442 $\int x^4\sqrt{9+4x^2}\,dx$ 1930
3.443 $\int x^3\sqrt{9+4x^2}\,dx$ 1934
3.444 $\int x^2\sqrt{9+4x^2}\,dx$ 1937
3.445 $\int x\sqrt{9+4x^2}\,dx$ 1940
3.446 $\int \sqrt{9+4x^2}\,dx$ 1943
3.447 $\int \frac{\sqrt{9+4x^2}}{x}\,dx$ 1946
3.448 $\int \frac{\sqrt{9+4x^2}}{x^2}\,dx$ 1950
3.449 $\int \frac{\sqrt{9+4x^2}}{x^3}\,dx$ 1953
3.450 $\int \frac{\sqrt{9+4x^2}}{x^4}\,dx$ 1957
3.451 $\int \frac{\sqrt{9+4x^2}}{x^5}\,dx$ 1960
3.452 $\int x^5\sqrt{9-4x^2}\,dx$ 1964
3.453 $\int x^4\sqrt{9-4x^2}\,dx$ 1967
3.454 $\int x^3\sqrt{9-4x^2}\,dx$ 1971
3.455 $\int x^2\sqrt{9-4x^2}\,dx$ 1974
3.456 $\int x\sqrt{9-4x^2}\,dx$ 1978
3.457 $\int \sqrt{9-4x^2}\,dx$ 1981
3.458 $\int \frac{\sqrt{9-4x^2}}{x}\,dx$ 1984
3.459 $\int \frac{\sqrt{9-4x^2}}{x^2}\,dx$ 1988
3.460 $\int \frac{\sqrt{9-4x^2}}{x^3}\,dx$ 1991
3.461 $\int \frac{\sqrt{9-4x^2}}{x^4}\,dx$ 1995
3.462 $\int \frac{\sqrt{9-4x^2}}{x^5}\,dx$ 1998
3.463 $\int x^5\sqrt{-9+4x^2}\,dx$ 2002
3.464 $\int x^4\sqrt{-9+4x^2}\,dx$ 2005
3.465 $\int x^3\sqrt{-9+4x^2}\,dx$ 2009
3.466 $\int x^2\sqrt{-9+4x^2}\,dx$ 2012
3.467 $\int x\sqrt{-9+4x^2}\,dx$ 2016
3.468 $\int \sqrt{-9+4x^2}\,dx$ 2019
3.469 $\int \frac{\sqrt{-9+4x^2}}{x}\,dx$ 2022
3.470 $\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx$ 2026
3.471 $\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx$ 2029
3.472 $\int \frac{\sqrt{-9+4x^2}}{x^4}\,dx$ 2033
3.473 $\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx$ 2036

3.474 $\int x^5\sqrt{-9-4x^2}\,dx$ 2040
3.475 $\int x^4\sqrt{-9-4x^2}\,dx$ 2043
3.476 $\int x^3\sqrt{-9-4x^2}\,dx$ 2047
3.477 $\int x^2\sqrt{-9-4x^2}\,dx$ 2050
3.478 $\int x\sqrt{-9-4x^2}\,dx$ 2054
3.479 $\int \sqrt{-9-4x^2}\,dx$ 2057
3.480 $\int \frac{\sqrt{-9-4x^2}}{x}\,dx$ 2060
3.481 $\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx$ 2064
3.482 $\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx$ 2068
3.483 $\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx$ 2072
3.484 $\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx$ 2075
3.485 $\int \frac{x^5}{\sqrt{a+bx^2}}\,dx$ 2079
3.486 $\int \frac{x^4}{\sqrt{a+bx^2}}\,dx$ 2082
3.487 $\int \frac{x^3}{\sqrt{a+bx^2}}\,dx$ 2086
3.488 $\int \frac{x^2}{\sqrt{a+bx^2}}\,dx$ 2089
3.489 $\int \frac{x}{\sqrt{a+bx^2}}\,dx$ 2093
3.490 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$ 2096
3.491 $\int \frac{1}{x\sqrt{a+bx^2}}\,dx$ 2099
3.492 $\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx$ 2103
3.493 $\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx$ 2106
3.494 $\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx$ 2110
3.495 $\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx$ 2113
3.496 $\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx$ 2117
3.497 $\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx$ 2120
3.498 $\int \frac{x^3}{(a+bx^2)^{3/2}}\,dx$ 2124
3.499 $\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx$ 2127
3.500 $\int \frac{x}{(a+bx^2)^{3/2}}\,dx$ 2131
3.501 $\int \frac{1}{(a+bx^2)^{3/2}}\,dx$ 2134
3.502 $\int \frac{1}{x(a+bx^2)^{3/2}}\,dx$ 2137
3.503 $\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx$ 2141
3.504 $\int \frac{1}{x^3(a+bx^2)^{3/2}}\,dx$ 2144
3.505 $\int \frac{1}{x^4(a+bx^2)^{3/2}}\,dx$ 2148
3.506 $\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx$ 2151

3.507 $\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx$ . . . 2155
3.508 $\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx$ . . . 2158
3.509 $\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx$ . . . 2162
3.510 $\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx$ . . . 2165
3.511 $\int \frac{x}{(a+bx^2)^{5/2}}\,dx$ . . . 2168
3.512 $\int \frac{1}{(a+bx^2)^{5/2}}\,dx$ . . . 2171
3.513 $\int \frac{1}{x(a+bx^2)^{5/2}}\,dx$ . . . 2174
3.514 $\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx$ . . . 2178
3.515 $\int \frac{1}{x^3(a+bx^2)^{5/2}}\,dx$ . . . 2182
3.516 $\int \frac{1}{x^4(a+bx^2)^{5/2}}\,dx$ . . . 2187
3.517 $\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx$ . . . 2191
3.518 $\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx$ . . . 2197
3.519 $\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx$ . . . 2201
3.520 $\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx$ . . . 2207
3.521 $\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx$ . . . 2211
3.522 $\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx$ . . . 2215
3.523 $\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx$ . . . 2218
3.524 $\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx$ . . . 2222
3.525 $\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx$ . . . 2225
3.526 $\int \frac{x}{(a+bx^2)^{9/2}}\,dx$ . . . 2229
3.527 $\int \frac{1}{(a+bx^2)^{9/2}}\,dx$ . . . 2232
3.528 $\int \frac{1}{x(a+bx^2)^{9/2}}\,dx$ . . . 2236
3.529 $\int \frac{1}{x^2(a+bx^2)^{9/2}}\,dx$ . . . 2242
3.530 $\int \frac{1}{x^3(a+bx^2)^{9/2}}\,dx$ . . . 2246
3.531 $\int \frac{1}{x^4(a+bx^2)^{9/2}}\,dx$ . . . 2253
3.532 $\int \frac{x^5}{\sqrt{9+4x^2}}\,dx$ . . . 2258
3.533 $\int \frac{x^4}{\sqrt{9+4x^2}}\,dx$ . . . 2261
3.534 $\int \frac{x^3}{\sqrt{9+4x^2}}\,dx$ . . . 2264
3.535 $\int \frac{x^2}{\sqrt{9+4x^2}}\,dx$ . . . 2267
3.536 $\int \frac{x}{\sqrt{9+4x^2}}\,dx$ . . . 2270
3.537 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$ . . . 2273
3.538 $\int \frac{1}{x\sqrt{9+4x^2}}\,dx$ . . . 2276

3.539 $\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx$ 2279
3.540 $\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx$ 2282
3.541 $\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx$ 2286
3.542 $\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx$ 2289
3.543 $\int \frac{x^5}{\sqrt{9-4x^2}}\,dx$ 2293
3.544 $\int \frac{x^4}{\sqrt{9-4x^2}}\,dx$ 2296
3.545 $\int \frac{x^3}{\sqrt{9-4x^2}}\,dx$ 2299
3.546 $\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$ 2302
3.547 $\int \frac{x}{\sqrt{9-4x^2}}\,dx$ 2305
3.548 $\int \frac{1}{\sqrt{9-4x^2}}\,dx$ 2308
3.549 $\int \frac{1}{x\sqrt{9-4x^2}}\,dx$ 2311
3.550 $\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx$ 2315
3.551 $\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx$ 2318
3.552 $\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx$ 2322
3.553 $\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx$ 2325
3.554 $\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx$ 2329
3.555 $\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx$ 2332
3.556 $\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx$ 2336
3.557 $\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx$ 2339
3.558 $\int \frac{x}{\sqrt{-9+4x^2}}\,dx$ 2342
3.559 $\int \frac{1}{\sqrt{-9+4x^2}}\,dx$ 2345
3.560 $\int \frac{1}{x\sqrt{-9+4x^2}}\,dx$ 2348
3.561 $\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx$ 2351
3.562 $\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx$ 2354
3.563 $\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx$ 2358
3.564 $\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx$ 2361
3.565 $\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx$ 2365
3.566 $\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx$ 2368
3.567 $\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx$ 2372
3.568 $\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx$ 2375

3.569 $\int \frac{x}{\sqrt{-9-4x^2}}\,dx$ . . . 2378
3.570 $\int \frac{1}{\sqrt{-9-4x^2}}\,dx$ . . . 2381
3.571 $\int \frac{1}{x\sqrt{-9-4x^2}}\,dx$ . . . 2384
3.572 $\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx$ . . . 2387
3.573 $\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx$ . . . 2390
3.574 $\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx$ . . . 2394
3.575 $\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx$ . . . 2397
3.576 $\int \frac{1}{\sqrt{9+bx^2}}\,dx$ . . . 2401
3.577 $\int \frac{1}{\sqrt{9-bx^2}}\,dx$ . . . 2404
3.578 $\int \frac{1}{\sqrt{-9+bx^2}}\,dx$ . . . 2407
3.579 $\int \frac{1}{\sqrt{-9-bx^2}}\,dx$ . . . 2410
3.580 $\int \frac{1}{\sqrt{\pi+bx^2}}\,dx$ . . . 2413
3.581 $\int \frac{1}{\sqrt{\pi-bx^2}}\,dx$ . . . 2416
3.582 $\int \frac{1}{\sqrt{-\pi+bx^2}}\,dx$ . . . 2419
3.583 $\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx$ . . . 2422
3.584 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$ . . . 2425
3.585 $\int \frac{1}{\sqrt{a-bx^2}}\,dx$ . . . 2428
3.586 $\int \frac{1}{\sqrt{-a+bx^2}}\,dx$ . . . 2431
3.587 $\int \frac{1}{\sqrt{-a-bx^2}}\,dx$ . . . 2434
3.588 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$ . . . 2437
3.589 $\int (cx)^{7/2}\sqrt{a+bx^2}\,dx$ . . . 2440
3.590 $\int (cx)^{5/2}\sqrt{a+bx^2}\,dx$ . . . 2445
3.591 $\int (cx)^{3/2}\sqrt{a+bx^2}\,dx$ . . . 2451
3.592 $\int \sqrt{cx}\,\sqrt{a+bx^2}\,dx$ . . . 2455
3.593 $\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx$ . . . 2460
3.594 $\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx$ . . . 2464
3.595 $\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx$ . . . 2469
3.596 $\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx$ . . . 2473
3.597 $\int (cx)^{7/2}\left(a+bx^2\right)^{3/2}\,dx$ . . . 2479
3.598 $\int (cx)^{5/2}\left(a+bx^2\right)^{3/2}\,dx$ . . . 2484
3.599 $\int (cx)^{3/2}\left(a+bx^2\right)^{3/2}\,dx$ . . . 2490

3.600 $\int \sqrt{cx}\,(a+bx^2)^{3/2}\,dx$ . . . 2495
3.601 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx$ . . . 2501
3.602 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx$ . . . 2505
3.603 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx$ . . . 2511
3.604 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx$ . . . 2515
3.605 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx$ . . . 2521
3.606 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx$ . . . 2525
3.607 $\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx$ . . . 2531
3.608 $\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx$ . . . 2537
3.609 $\int \sqrt{cx}\,\sqrt{3a-2ax^2}\,dx$ . . . 2542
3.610 $\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx$ . . . 2547
3.611 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx$ . . . 2551
3.612 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx$ . . . 2556
3.613 $\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx$ . . . 2560
3.614 $\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx$ . . . 2564
3.615 $\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx$ . . . 2569
3.616 $\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx$ . . . 2573
3.617 $\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx$ . . . 2577
3.618 $\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx$ . . . 2581
3.619 $\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx$ . . . 2586
3.620 $\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx$ . . . 2590
3.621 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx$ . . . 2596
3.622 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx$ . . . 2600
3.623 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx$ . . . 2605
3.624 $\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx$ . . . 2609
3.625 $\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx$ . . . 2614
3.626 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/2}}\,dx$ . . . 2618
3.627 $\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}}\,dx$ . . . 2625
3.628 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}\,dx$ . . . 2629

3.629 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx$ 2635

3.630 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx$ 2639

3.631 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx$ 2644

3.632 $\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx$ 2648

3.633 $\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx$ 2653

3.634 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}\,dx$ 2657

3.635 $\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/2}}\,dx$ 2663

3.636 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}}\,dx$ 2668

3.637 $\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx$ 2674

3.638 $\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx$ 2679

3.639 $\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx$ 2683

3.640 $\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx$ 2687

3.641 $\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx$ 2691

3.642 $\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx$ 2696

3.643 $\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx$ 2700

3.644 $\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx$ 2705

3.645 $\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx$ 2709

3.646 $\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx$ 2714

3.647 $\int \frac{1}{(cx)^{3/2}(3a-2ax^2)^{3/2}}\,dx$ 2718

3.648 $\int \frac{1}{(cx)^{5/2}(3a-2ax^2)^{3/2}}\,dx$ 2723

3.649 $\int \frac{1}{\sqrt{x}\,\sqrt{1-a^2x^2}}\,dx$ 2728

3.650 $\int \frac{1}{\sqrt{x}\,\sqrt{1+ax^2}}\,dx$ 2731

3.651 $\int x^m(a+bx^2)^{3/2}\,dx$ 2734

3.652 $\int x^m\sqrt{a+bx^2}\,dx$ 2737

3.653 $\int \frac{x^m}{\sqrt{a+bx^2}}\,dx$ 2740

3.654 $\int \frac{x^m}{(a+bx^2)^{3/2}}\,dx$ 2743

3.655 $\int \frac{x^m}{(a+bx^2)^{5/2}}\,dx$ 2746

3.656 $\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx$ 2749

3.657 $\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx$ 2752

3.658 $\int \frac{x^m}{\sqrt{a+bx^2}}\,dx$ 2755

3.659 $\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx$ 2758
3.660 $\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx$ 2761
3.661 $\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx$ 2764
3.662 $\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)dx$ 2767
3.663 $\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx$ 2771
3.664 $\int \left(-\frac{bx^{1+m}}{(a+bx^2)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)dx$ 2774
3.665 $\int x^7\sqrt[3]{a+bx^2}\,dx$ 2778
3.666 $\int x^5\sqrt[3]{a+bx^2}\,dx$ 2782
3.667 $\int x^3\sqrt[3]{a+bx^2}\,dx$ 2786
3.668 $\int x\sqrt[3]{a+bx^2}\,dx$ 2789
3.669 $\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx$ 2792
3.670 $\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx$ 2796
3.671 $\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx$ 2800
3.672 $\int x^4\sqrt[3]{a+bx^2}\,dx$ 2805
3.673 $\int x^2\sqrt[3]{a+bx^2}\,dx$ 2810
3.674 $\int \sqrt[3]{a+bx^2}\,dx$ 2814
3.675 $\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx$ 2818
3.676 $\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx$ 2822
3.677 $\int x^7(a+bx^2)^{2/3}\,dx$ 2826
3.678 $\int x^5(a+bx^2)^{2/3}\,dx$ 2830
3.679 $\int x^3(a+bx^2)^{2/3}\,dx$ 2834
3.680 $\int x(a+bx^2)^{2/3}\,dx$ 2837
3.681 $\int \frac{(a+bx^2)^{2/3}}{x}\,dx$ 2840
3.682 $\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx$ 2844
3.683 $\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx$ 2849
3.684 $\int x^4(a+bx^2)^{2/3}\,dx$ 2854
3.685 $\int x^2(a+bx^2)^{2/3}\,dx$ 2859
3.686 $\int (a+bx^2)^{2/3}\,dx$ 2864
3.687 $\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx$ 2869
3.688 $\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx$ 2874
3.689 $\int x^7(a+bx^2)^{4/3}\,dx$ 2879
3.690 $\int x^5(a+bx^2)^{4/3}\,dx$ 2882
3.691 $\int x^3(a+bx^2)^{4/3}\,dx$ 2885

3.692 $\int x(a+bx^2)^{4/3}\,dx$ 2888
3.693 $\int \frac{(a+bx^2)^{4/3}}{x}\,dx$ 2891
3.694 $\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx$ 2895
3.695 $\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx$ 2900
3.696 $\int x^4(a+bx^2)^{4/3}\,dx$ 2905
3.697 $\int x^2(a+bx^2)^{4/3}\,dx$ 2910
3.698 $\int (a+bx^2)^{4/3}\,dx$ 2915
3.699 $\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx$ 2919
3.700 $\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx$ 2923
3.701 $\int x(-1+x^2)^{7/3}\,dx$ 2927
3.702 $\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx$ 2930
3.703 $\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx$ 2934
3.704 $\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx$ 2938
3.705 $\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx$ 2941
3.706 $\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx$ 2944
3.707 $\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx$ 2948
3.708 $\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx$ 2953
3.709 $\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx$ 2958
3.710 $\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx$ 2963
3.711 $\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$ 2968
3.712 $\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx$ 2972
3.713 $\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx$ 2977
3.714 $\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx$ 2982
3.715 $\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx$ 2986
3.716 $\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx$ 2990
3.717 $\int \frac{x}{(a+bx^2)^{2/3}}\,dx$ 2993
3.718 $\int \frac{1}{x(a+bx^2)^{2/3}}\,dx$ 2996
3.719 $\int \frac{1}{x^3(a+bx^2)^{2/3}}\,dx$ 3000
3.720 $\int \frac{1}{x^5(a+bx^2)^{2/3}}\,dx$ 3005
3.721 $\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx$ 3010
3.722 $\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx$ 3014
3.723 $\int \frac{1}{(a+bx^2)^{2/3}}\,dx$ 3018

3.724 $\int \frac{1}{x^2(a+bx^2)^{2/3}}\,dx$ 3022

3.725 $\int \frac{1}{x^4(a+bx^2)^{2/3}}\,dx$ 3026

3.726 $\int \frac{x^7}{(a+bx^2)^{4/3}}\,dx$ 3030

3.727 $\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx$ 3034

3.728 $\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx$ 3038

3.729 $\int \frac{x}{(a+bx^2)^{4/3}}\,dx$ 3041

3.730 $\int \frac{1}{x(a+bx^2)^{4/3}}\,dx$ 3044

3.731 $\int \frac{1}{x^3(a+bx^2)^{4/3}}\,dx$ 3049

3.732 $\int \frac{1}{x^5(a+bx^2)^{4/3}}\,dx$ 3054

3.733 $\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx$ 3059

3.734 $\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx$ 3064

3.735 $\int \frac{1}{(a+bx^2)^{4/3}}\,dx$ 3069

3.736 $\int \frac{1}{x^2(a+bx^2)^{4/3}}\,dx$ 3074

3.737 $\int \frac{1}{x^4(a+bx^2)^{4/3}}\,dx$ 3079

3.738 $\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$ 3084

3.739 $\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$ 3089

3.740 $\int \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,dx$ 3094

3.741 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx$ 3099

3.742 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx$ 3104

3.743 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx$ 3107

3.744 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx$ 3110

3.745 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx$ 3113

3.746 $\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx$ 3116

3.747 $\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx$ 3121

3.748 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx$ 3126

3.749 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx$ 3131

3.750 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx$ 3136

3.751 $\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx$ 3141

3.752 $\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx$ 3144

3.753 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx$ 3147

3.754 $\int (cx)^{13/3}\,(a+bx^2)^{4/3}\,dx$ 3150

3.755 $\int (cx)^{7/3}\,(a+bx^2)^{4/3}\,dx$ 3155

3.756 $\int \sqrt[3]{cx}\,(a+bx^2)^{4/3}\,dx$ . . . 3160
3.757 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx$ . . . 3165
3.758 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx$ . . . 3170
3.759 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx$ . . . 3175
3.760 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx$ . . . 3178
3.761 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx$ . . . 3181
3.762 $\int (cx)^{10/3}\,(a+bx^2)^{4/3}\,dx$ . . . 3184
3.763 $\int (cx)^{4/3}\,(a+bx^2)^{4/3}\,dx$ . . . 3189
3.764 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx$ . . . 3194
3.765 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx$ . . . 3199
3.766 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx$ . . . 3204
3.767 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx$ . . . 3209
3.768 $\int (cx)^{2/3}\,(a+bx^2)^{4/3}\,dx$ . . . 3214
3.769 $\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx$ . . . 3217
3.770 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx$ . . . 3220
3.771 $\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx$ . . . 3223
3.772 $\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx$ . . . 3228
3.773 $\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx$ . . . 3233
3.774 $\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx$ . . . 3238
3.775 $\int \frac{1}{(cx)^{5/3}(a+bx^2)^{2/3}}\,dx$ . . . 3242
3.776 $\int \frac{1}{(cx)^{11/3}(a+bx^2)^{2/3}}\,dx$ . . . 3245
3.777 $\int \frac{1}{(cx)^{17/3}(a+bx^2)^{2/3}}\,dx$ . . . 3248
3.778 $\int \frac{1}{(cx)^{23/3}(a+bx^2)^{2/3}}\,dx$ . . . 3251
3.779 $\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx$ . . . 3254
3.780 $\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx$ . . . 3259
3.781 $\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx$ . . . 3264
3.782 $\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx$ . . . 3269
3.783 $\int \frac{1}{(cx)^{14/3}(a+bx^2)^{2/3}}\,dx$ . . . 3274
3.784 $\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}}\,dx$ . . . 3279
3.785 $\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx$ . . . 3282
3.786 $\int \frac{1}{(cx)^{4/3}(a+bx^2)^{2/3}}\,dx$ . . . 3285

3.787 $\int x^4 \sqrt[4]{a+bx^2}\, dx$ . . . 3288
3.788 $\int x^2 \sqrt[4]{a+bx^2}\, dx$ . . . 3292
3.789 $\int \sqrt[4]{a+bx^2}\, dx$ . . . 3296
3.790 $\int \frac{\sqrt[4]{a+bx^2}}{x^2}\, dx$ . . . 3299
3.791 $\int \frac{\sqrt[4]{a+bx^2}}{x^4}\, dx$ . . . 3302
3.792 $\int \frac{\sqrt[4]{a+bx^2}}{x^6}\, dx$ . . . 3306
3.793 $\int x^4 \sqrt[4]{a-bx^2}\, dx$ . . . 3310
3.794 $\int x^2 \sqrt[4]{a-bx^2}\, dx$ . . . 3314
3.795 $\int \sqrt[4]{a-bx^2}\, dx$ . . . 3318
3.796 $\int \frac{\sqrt[4]{a-bx^2}}{x^2}\, dx$ . . . 3321
3.797 $\int \frac{\sqrt[4]{a-bx^2}}{x^4}\, dx$ . . . 3324
3.798 $\int \frac{\sqrt[4]{a-bx^2}}{x^6}\, dx$ . . . 3328
3.799 $\int x^4 (a+bx^2)^{3/4}\, dx$ . . . 3332
3.800 $\int x^2 (a+bx^2)^{3/4}\, dx$ . . . 3336
3.801 $\int (a+bx^2)^{3/4}\, dx$ . . . 3340
3.802 $\int \frac{(a+bx^2)^{3/4}}{x^2}\, dx$ . . . 3344
3.803 $\int \frac{(a+bx^2)^{3/4}}{x^4}\, dx$ . . . 3348
3.804 $\int \frac{(a+bx^2)^{3/4}}{x^6}\, dx$ . . . 3352
3.805 $\int x^4 (a-bx^2)^{3/4}\, dx$ . . . 3356
3.806 $\int x^2 (a-bx^2)^{3/4}\, dx$ . . . 3360
3.807 $\int (a-bx^2)^{3/4}\, dx$ . . . 3364
3.808 $\int \frac{(a-bx^2)^{3/4}}{x^2}\, dx$ . . . 3367
3.809 $\int \frac{(a-bx^2)^{3/4}}{x^4}\, dx$ . . . 3370
3.810 $\int \frac{(a-bx^2)^{3/4}}{x^6}\, dx$ . . . 3374
3.811 $\int (a+bx^2)^{5/4}\, dx$ . . . 3378
3.812 $\int (a-bx^2)^{5/4}\, dx$ . . . 3382
3.813 $\int (a+bx^2)^{7/4}\, dx$ . . . 3386
3.814 $\int (a-bx^2)^{7/4}\, dx$ . . . 3390
3.815 $\int \frac{x^6}{\sqrt[4]{a+bx^2}}\, dx$ . . . 3394
3.816 $\int \frac{x^4}{\sqrt[4]{a+bx^2}}\, dx$ . . . 3398
3.817 $\int \frac{x^2}{\sqrt[4]{a+bx^2}}\, dx$ . . . 3402
3.818 $\int \frac{1}{\sqrt[4]{a+bx^2}}\, dx$ . . . 3406
3.819 $\int \frac{1}{x^2 \sqrt[4]{a+bx^2}}\, dx$ . . . 3409
3.820 $\int \frac{1}{x^4 \sqrt[4]{a+bx^2}}\, dx$ . . . 3413

3.821 $\int \frac{1}{x^6 \sqrt[4]{a+bx^2}}\,dx$ 3417
3.822 $\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx$ 3421
3.823 $\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx$ 3425
3.824 $\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx$ 3429
3.825 $\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$ 3432
3.826 $\int \frac{1}{x^2 \sqrt[4]{a-bx^2}}\,dx$ 3435
3.827 $\int \frac{1}{x^4 \sqrt[4]{a-bx^2}}\,dx$ 3438
3.828 $\int \frac{1}{x^6 \sqrt[4]{a-bx^2}}\,dx$ 3442
3.829 $\int \frac{x^6}{(a+bx^2)^{3/4}}\,dx$ 3446
3.830 $\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx$ 3450
3.831 $\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx$ 3454
3.832 $\int \frac{1}{(a+bx^2)^{3/4}}\,dx$ 3457
3.833 $\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx$ 3460
3.834 $\int \frac{1}{x^4(a+bx^2)^{3/4}}\,dx$ 3463
3.835 $\int \frac{1}{x^6(a+bx^2)^{3/4}}\,dx$ 3467
3.836 $\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx$ 3471
3.837 $\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx$ 3475
3.838 $\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx$ 3479
3.839 $\int \frac{1}{(a-bx^2)^{3/4}}\,dx$ 3482
3.840 $\int \frac{1}{x^2(a-bx^2)^{3/4}}\,dx$ 3485
3.841 $\int \frac{1}{x^4(a-bx^2)^{3/4}}\,dx$ 3488
3.842 $\int \frac{1}{x^6(a-bx^2)^{3/4}}\,dx$ 3492
3.843 $\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx$ 3496
3.844 $\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx$ 3500
3.845 $\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx$ 3504
3.846 $\int \frac{1}{(a+bx^2)^{5/4}}\,dx$ 3507
3.847 $\int \frac{1}{x^2(a+bx^2)^{5/4}}\,dx$ 3510
3.848 $\int \frac{1}{x^4(a+bx^2)^{5/4}}\,dx$ 3513
3.849 $\int \frac{1}{x^6(a+bx^2)^{5/4}}\,dx$ 3516
3.850 $\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx$ 3520
3.851 $\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx$ 3524
3.852 $\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx$ 3528

3.853 $\int \frac{1}{(a-bx^2)^{5/4}}\,dx$ 3531
3.854 $\int \frac{1}{x^2(a-bx^2)^{5/4}}\,dx$ 3534
3.855 $\int \frac{1}{x^4(a-bx^2)^{5/4}}\,dx$ 3538
3.856 $\int \frac{1}{x^6(a-bx^2)^{5/4}}\,dx$ 3542
3.857 $\int \frac{1}{(a+bx^2)^{7/4}}\,dx$ 3546
3.858 $\int \frac{1}{(a+bx^2)^{9/4}}\,dx$ 3549
3.859 $\int \frac{1}{(a+bx^2)^{11/4}}\,dx$ 3552
3.860 $\int \frac{1}{(a-bx^2)^{7/4}}\,dx$ 3556
3.861 $\int \frac{1}{(a-bx^2)^{9/4}}\,dx$ 3559
3.862 $\int \frac{1}{(a-bx^2)^{11/4}}\,dx$ 3563
3.863 $\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx$ 3567
3.864 $\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx$ 3571
3.865 $\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx$ 3574
3.866 $\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$ 3577
3.867 $\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx$ 3580
3.868 $\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx$ 3583
3.869 $\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx$ 3586
3.870 $\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx$ 3590
3.871 $\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx$ 3593
3.872 $\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx$ 3596
3.873 $\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx$ 3599
3.874 $\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx$ 3602
3.875 $\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx$ 3605
3.876 $\int \frac{1}{x^6\sqrt[4]{2-3x^2}}\,dx$ 3608
3.877 $\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx$ 3611
3.878 $\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx$ 3614
3.879 $\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx$ 3617
3.880 $\int \frac{1}{(2+3x^2)^{3/4}}\,dx$ 3620
3.881 $\int \frac{1}{x^2(2+3x^2)^{3/4}}\,dx$ 3623
3.882 $\int \frac{1}{x^4(2+3x^2)^{3/4}}\,dx$ 3626
3.883 $\int \frac{1}{x^6(2+3x^2)^{3/4}}\,dx$ 3629
3.884 $\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx$ 3632

3.885 $\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx$ . . . 3635

3.886 $\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx$ . . . 3638

3.887 $\int \frac{1}{(2-3x^2)^{3/4}}\,dx$ . . . 3641

3.888 $\int \frac{1}{x^2(2-3x^2)^{3/4}}\,dx$ . . . 3644

3.889 $\int \frac{1}{x^4(2-3x^2)^{3/4}}\,dx$ . . . 3647

3.890 $\int \frac{1}{x^6(2-3x^2)^{3/4}}\,dx$ . . . 3650

3.891 $\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx$ . . . 3653

3.892 $\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx$ . . . 3658

3.893 $\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx$ . . . 3663

3.894 $\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$ . . . 3667

3.895 $\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx$ . . . 3671

3.896 $\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx$ . . . 3675

3.897 $\int \frac{1}{x^6\sqrt[4]{-2+3x^2}}\,dx$ . . . 3680

3.898 $\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx$ . . . 3685

3.899 $\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx$ . . . 3690

3.900 $\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$ . . . 3695

3.901 $\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$ . . . 3699

3.902 $\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx$ . . . 3703

3.903 $\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx$ . . . 3707

3.904 $\int \frac{1}{x^6\sqrt[4]{-2-3x^2}}\,dx$ . . . 3712

3.905 $\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx$ . . . 3717

3.906 $\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx$ . . . 3721

3.907 $\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx$ . . . 3725

3.908 $\int \frac{1}{(-2+3x^2)^{3/4}}\,dx$ . . . 3729

3.909 $\int \frac{1}{x^2(-2+3x^2)^{3/4}}\,dx$ . . . 3732

3.910 $\int \frac{1}{x^4(-2+3x^2)^{3/4}}\,dx$ . . . 3736

3.911 $\int \frac{1}{x^6(-2+3x^2)^{3/4}}\,dx$ . . . 3740

3.912 $\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx$ . . . 3744

3.913 $\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx$ . . . 3748

3.914 $\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx$ . . . 3752

3.915 $\int \frac{1}{(-2-3x^2)^{3/4}}\,dx$ . . . 3756

3.916 $\int \frac{1}{x^2(-2-3x^2)^{3/4}}\,dx$ . . . 3759

3.917 $\int \frac{1}{x^4(-2-3x^2)^{3/4}}\,dx$ . . . 3763
3.918 $\int \frac{1}{x^6(-2-3x^2)^{3/4}}\,dx$ . . . 3767
3.919 $\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx$ . . . 3771
3.920 $\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx$ . . . 3776
3.921 $\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx$ . . . 3781
3.922 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx$ . . . 3786
3.923 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx$ . . . 3791
3.924 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx$ . . . 3796
3.925 $\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx$ . . . 3801
3.926 $\int \sqrt{cx}\,\sqrt[4]{a+bx^2}\,dx$ . . . 3806
3.927 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx$ . . . 3810
3.928 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx$ . . . 3814
3.929 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx$ . . . 3817
3.930 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx$ . . . 3820
3.931 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx$ . . . 3823
3.932 $\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx$ . . . 3826
3.933 $\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx$ . . . 3831
3.934 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx$ . . . 3836
3.935 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx$ . . . 3841
3.936 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx$ . . . 3846
3.937 $\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx$ . . . 3851
3.938 $\int \sqrt{cx}\,\sqrt[4]{a-bx^2}\,dx$ . . . 3856
3.939 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx$ . . . 3861
3.940 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx$ . . . 3866
3.941 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx$ . . . 3869
3.942 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx$ . . . 3872
3.943 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx$ . . . 3875
3.944 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx$ . . . 3878
3.945 $\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx$ . . . 3882
3.946 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx$ . . . 3886

3.947 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx$ 3889
3.948 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx$ 3892
3.949 $\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx$ 3895
3.950 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx$ 3899
3.951 $\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx$ 3903
3.952 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx$ 3907
3.953 $\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx$ 3911
3.954 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx$ 3915
3.955 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx$ 3919
3.956 $\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a-bx^2}}\,dx$ 3924
3.957 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx$ 3929
3.958 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx$ 3932
3.959 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx$ 3935
3.960 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx$ 3938
3.961 $\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx$ 3942
3.962 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx$ 3946
3.963 $\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx$ 3949
3.964 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx$ 3953
3.965 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx$ 3957
3.966 $\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx$ 3962
3.967 $\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/4}}\,dx$ 3966
3.968 $\int \frac{1}{(cx)^{9/2}(a+bx^2)^{3/4}}\,dx$ 3971
3.969 $\int \frac{1}{(cx)^{13/2}(a+bx^2)^{3/4}}\,dx$ 3976
3.970 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx$ 3981
3.971 $\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx$ 3985
3.972 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/4}}\,dx$ 3989
3.973 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/4}}\,dx$ 3992
3.974 $\int \frac{1}{(cx)^{11/2}(a+bx^2)^{3/4}}\,dx$ 3995
3.975 $\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx$ 3999

3.976 $\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx$ . . . 4004
3.977 $\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx$ . . . 4008
3.978 $\int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}}\,dx$ . . . 4013
3.979 $\int \frac{1}{(cx)^{13/2}(a-bx^2)^{3/4}}\,dx$ . . . 4018
3.980 $\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx$ . . . 4023
3.981 $\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx$ . . . 4028
3.982 $\int \frac{1}{(cx)^{3/2}(a-bx^2)^{3/4}}\,dx$ . . . 4033
3.983 $\int \frac{1}{(cx)^{7/2}(a-bx^2)^{3/4}}\,dx$ . . . 4036
3.984 $\int \frac{1}{(cx)^{11/2}(a-bx^2)^{3/4}}\,dx$ . . . 4039
3.985 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}}\,dx$ . . . 4043
3.986 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx$ . . . 4048
3.987 $\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx$ . . . 4052
3.988 $\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/4}}\,dx$ . . . 4055
3.989 $\int \frac{1}{(cx)^{9/2}(a+bx^2)^{5/4}}\,dx$ . . . 4058
3.990 $\int \frac{1}{(cx)^{13/2}(a+bx^2)^{5/4}}\,dx$ . . . 4062
3.991 $\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx$ . . . 4065
3.992 $\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx$ . . . 4069
3.993 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx$ . . . 4073
3.994 $\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx$ . . . 4077
3.995 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}}\,dx$ . . . 4080
3.996 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/4}}\,dx$ . . . 4084
3.997 $\int \frac{1}{(cx)^{11/2}(a+bx^2)^{5/4}}\,dx$ . . . 4088
3.998 $\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx$ . . . 4092
3.999 $\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx$ . . . 4095
3.1000 $\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx$ . . . 4098
3.1001 $\int \frac{1}{\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}\,dx$ . . . 4101
3.1002 $\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx$ . . . 4104
3.1003 $\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx$ . . . 4107
3.1004 $\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx$ . . . 4110
3.1005 $\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx$ . . . 4113

3.1006 $\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx$ 4116

3.1007 $\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx$ 4119

3.1008 $\int \frac{1}{(cx)^{3/4}(a+bx^2)^{7/4}}\,dx$ 4122

3.1009 $\int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}}\,dx$ 4125

3.1010 $\int x^6\sqrt[6]{a+bx^2}\,dx$ 4128

3.1011 $\int x^4\sqrt[6]{a+bx^2}\,dx$ 4133

3.1012 $\int x^2\sqrt[6]{a+bx^2}\,dx$ 4138

3.1013 $\int \sqrt[6]{a+bx^2}\,dx$ 4143

3.1014 $\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx$ 4148

3.1015 $\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx$ 4153

3.1016 $\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx$ 4158

3.1017 $\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx$ 4163

3.1018 $\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx$ 4168

3.1019 $\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx$ 4174

3.1020 $\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx$ 4180

3.1021 $\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx$ 4186

3.1022 $\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx$ 4191

3.1023 $\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx$ 4196

3.1024 $\int \frac{1}{x^6\sqrt[6]{a+bx^2}}\,dx$ 4202

3.1025 $\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx$ 4208

3.1026 $\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx$ 4213

3.1027 $\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx$ 4218

3.1028 $\int \frac{1}{(a+bx^2)^{5/6}}\,dx$ 4223

3.1029 $\int \frac{1}{x^2(a+bx^2)^{5/6}}\,dx$ 4227

3.1030 $\int \frac{1}{x^4(a+bx^2)^{5/6}}\,dx$ 4232

3.1031 $\int \frac{1}{x^6(a+bx^2)^{5/6}}\,dx$ 4237

3.1032 $\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx$ 4242

3.1033 $\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx$ 4248

3.1034 $\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx$ 4254

3.1035 $\int \frac{1}{(a+bx^2)^{7/6}}\,dx$ 4260

3.1036 $\int \frac{1}{x^2(a+bx^2)^{7/6}}\,dx$ 4265

3.1037 $\int \frac{1}{x^4(a+bx^2)^{7/6}}\,dx$ 4271

3.1038 $\int \frac{1}{x^6(a+bx^2)^{7/6}}\,dx$ 4277

3.1039 $\int x^7(a+bx^2)^p\,dx$ 4283
3.1040 $\int x^5(a+bx^2)^p\,dx$ 4287
3.1041 $\int x^3(a+bx^2)^p\,dx$ 4291
3.1042 $\int x(a+bx^2)^p\,dx$ 4295
3.1043 $\int \frac{(a+bx^2)^p}{x}\,dx$ 4298
3.1044 $\int \frac{(a+bx^2)^p}{x^3}\,dx$ 4301
3.1045 $\int x^6(a+bx^2)^p\,dx$ 4304
3.1046 $\int x^4(a+bx^2)^p\,dx$ 4307
3.1047 $\int x^2(a+bx^2)^p\,dx$ 4310
3.1048 $\int (a+bx^2)^p\,dx$ 4313
3.1049 $\int \frac{(a+bx^2)^p}{x^2}\,dx$ 4316
3.1050 $\int x^{7/2}(a+bx^2)^p\,dx$ 4319
3.1051 $\int x^{5/2}(a+bx^2)^p\,dx$ 4322
3.1052 $\int x^{3/2}(a+bx^2)^p\,dx$ 4325
3.1053 $\int \sqrt{x}\,(a+bx^2)^p\,dx$ 4328
3.1054 $\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx$ 4331
3.1055 $\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx$ 4334
3.1056 $\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx$ 4337
3.1057 $\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx$ 4340
3.1058 $\int x^m(a+bx^2)^p\,dx$ 4343
3.1059 $\int (cx)^m\,(a+bx^2)^p\,dx$ 4346
3.1060 $\int x^{-8-2p}(a+bx^2)^p\,dx$ 4349
3.1061 $\int x^{-7-2p}(a+bx^2)^p\,dx$ 4352
3.1062 $\int x^{-6-2p}(a+bx^2)^p\,dx$ 4355
3.1063 $\int x^{-5-2p}(a+bx^2)^p\,dx$ 4358
3.1064 $\int x^{-4-2p}(a+bx^2)^p\,dx$ 4361
3.1065 $\int x^{-3-2p}(a+bx^2)^p\,dx$ 4364
3.1066 $\int x^{-2-2p}(a+bx^2)^p\,dx$ 4367
3.1067 $\int x^{-1-2p}(a+bx^2)^p\,dx$ 4370
3.1068 $\int x^{-2p}(a+bx^2)^p\,dx$ 4373
3.1069 $\int x^{1-2p}(a+bx^2)^p\,dx$ 4376
3.1070 $\int x^{2-2p}(a+bx^2)^p\,dx$ 4379
3.1071 $\int x^{3-2p}(a+bx^2)^p\,dx$ 4382

3.1 $\int x^4(a + bx^2)\ dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

```
[Out] 1/5*a*x^5+1/7*b*x^7
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{14\}$

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2),x]
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^4(a + bx^2)\ dx &= \int (ax^4 + bx^6)\ dx \\ &= \frac{ax^5}{5} + \frac{bx^7}{7} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2),x]
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

Maple [A]
time = 0.06, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{1}{5}a\,x^5+\frac{1}{7}b\,x^7$	14
default	$\frac{1}{5}a\,x^5+\frac{1}{7}b\,x^7$	14
norman	$\frac{1}{5}a\,x^5+\frac{1}{7}b\,x^7$	14
risch	$\frac{1}{5}a\,x^5+\frac{1}{7}b\,x^7$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/7*b*x^7
```

Maxima [A]
time = 0.33, size = 13, normalized size = 0.76

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Fricas [A]
time = 0.94, size = 13, normalized size = 0.76

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^5}{5}+\frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a),x)
```

```
[Out] a*x**5/5 + b*x**7/7
```

Giac [**A**]
time = 1.45, size = 13, normalized size = 0.76

$$\frac{1}{7}\,bx^7 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Mupad [**B**]
time = 0.03, size = 13, normalized size = 0.76

$$\frac{b\,x^7}{7} + \frac{a\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2),x)
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

3.2 $\int x^3(a+bx^2)\,dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{14\}$

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2),x]
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)\,dx&=\int\left(ax^3+bx^5\right)\,dx\\&=\frac{ax^4}{4}+\frac{bx^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2),x]
[Out] (a*x^4)/4 + (b*x^6)/6
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
default	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
norman	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
risch	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^{6}+\frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Fricas [A]
time = 0.99, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^{6}+\frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^{4}}{4}+\frac{bx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

Giac [A]
time = 1.65, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^6}{6}+\frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2),x)
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

3.3 $\int x^2(a+bx^2)\,dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{14\}$

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2),x]
[Out] (a*x^3)/3 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)\,dx &= \int \left(ax^2+bx^4\right)\,dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2),x]
[Out] (a*x^3)/3 + (b*x^5)/5
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
default	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/3*a*x^3+1/5*b*x^5
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="maxima")
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A]
time = 1.31, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="fricas")
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

Giac [A]
time = 1.35, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^5}{5} + \frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

3.4 $\int x(a+bx^2)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2),x]
[Out] (a*x^2)/2 + (b*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\,dx &= \int \left(ax+bx^3\right)\,dx\\ &= \frac{ax^2}{2}+\frac{bx^4}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2),x]
[Out] (a*x^2)/2 + (b*x^4)/4
```

Maple [A]
time = 0.02, size = 15, normalized size = 0.88

method	result	size
gosper	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4$	14
norman	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4$	14
default	$\frac{(b\,x^2+a)^2}{4b}$	15
risch	$\frac{b\,x^4}{4}+\frac{a\,x^2}{2}+\frac{a^2}{4b}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)^2/b
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.82

$$\frac{(bx^2+a)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^2/b
```

Fricas [A]
time = 1.12, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a),x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

Giac [A]
time = 1.63, size = 13, normalized size = 0.76

$$\frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^4}{4} + \frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2),x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

3.5 $\int (a + bx^2)\, dx$

Optimal. Leaf size=12

$$ax + \frac{bx^3}{3}$$

[Out] a*x+1/3*b*x^3

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[a + b*x^2,x]

[Out] a*x + (b*x^3)/3

Rubi steps

$$\int (a + bx^2)\, dx = ax + \frac{bx^3}{3}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*x^2,x]

[Out] a*x + (b*x^3)/3

Maple [A]
time = 0.00, size = 11, normalized size = 0.92

method	result	size
gosper	$ax + \frac{1}{3}b\,x^3$	11
default	$ax + \frac{1}{3}b\,x^3$	11

norman	$ax+\frac{1}{3}b\,x^3$	11
risch	$ax+\frac{1}{3}b\,x^3$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^2+a,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/3*b*x^3
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + a*x
```

Fricas [A]
time = 1.22, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$ax+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**2+a,x)
```

```
[Out] a*x + b*x**3/3
```

Giac [A]
time = 1.72, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + a*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^3}{3}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^2,x)
```

```
[Out] a*x + (b*x^3)/3
```

3.6 $\int \frac{a+bx^2}{x}\,dx$

Optimal. Leaf size=13

$$\frac{bx^2}{2} + a\log(x)$$

[Out] 1/2*b*x^2+a*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$a\log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/x,x]

[Out] (b*x^2)/2 + a*Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x}\,dx &= \int \left(\frac{a}{x} + bx\right)dx \\ &= \frac{bx^2}{2} + a\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{bx^2}{2} + a\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/x,x]

[Out] (b*x^2)/2 + a*Log[x]

Maple [A]
time = 0.01, size = 12, normalized size = 0.92

method	result	size
default	$\frac{b\,x^2}{2}+a\ln\left(x\right)$	12
norman	$\frac{b\,x^2}{2}+a\ln\left(x\right)$	12
risch	$\frac{b\,x^2}{2}+a\ln\left(x\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*x^2+a*ln(x)
```

Maxima [A]
time = 0.31, size = 14, normalized size = 1.08

$$\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

Fricas [A]
time = 1.83, size = 11, normalized size = 0.85

$$\frac{1}{2}\,bx^2+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.77

$$a\log\left(x\right)+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x,x)
```

```
[Out] a*log(x) + b*x**2/2
```

Giac [A]
time = 2.09, size = 14, normalized size = 1.08

$$\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.85

$$\frac{b\,x^2}{2}+a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x,x)
```

```
[Out] (b*x^2)/2 + a*log(x)
```

3.7 $\int \frac{a+bx^2}{x^2} dx$

Optimal. Leaf size=10

$$-\frac{a}{x}+bx$$

[Out] -a/x+b*x

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$bx-\frac{a}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/x^2,x]

[Out] -(a/x) + b*x

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^2}\,dx &= \int \left(b+\frac{a}{x^2}\right)\,dx\\ &= -\frac{a}{x}+bx\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{a}{x}+bx$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/x^2,x]

[Out] -(a/x) + b*x

Maple [A]
time = 0.01, size = 11, normalized size = 1.10

method	result	size
default	$-\frac{a}{x}+bx$	11
risch	$-\frac{a}{x}+bx$	11
gosper	$-\frac{-b\,x^{2}+a}{x}$	14
norman	$\frac{b\,x^{2}-a}{x}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x+b*x
```

Maxima [A]
time = 0.29, size = 10, normalized size = 1.00

$$bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] b*x - a/x
```

Fricas [A]
time = 1.37, size = 13, normalized size = 1.30

$$\frac{bx^{2}-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] (b*x^2 - a)/x
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.50

$$-\frac{a}{x}+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2,x)
```

```
[Out] -a/x + b*x
```

Giac [A]
time = 1.73, size = 10, normalized size = 1.00

$$bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] b*x - a/x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$b\,x - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^2,x)
```

```
[Out] b*x - a/x
```

3.8 $\int \frac{a+bx^2}{x^3} \, dx$

Optimal. Leaf size=13

$$-\frac{a}{2x^2} + b\log(x)$$

```
[Out] -1/2*a/x^2+b*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$b\log(x) - \frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^3,x]
[Out] -1/2*a/x^2 + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^3} \, dx &= \int \left(\frac{a}{x^3} + \frac{b}{x}\right) dx \\ &= -\frac{a}{2x^2} + b\log(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{a}{2x^2} + b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^3,x]
[Out] -1/2*a/x^2 + b*Log[x]
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.92

method	result	size
default	$-\frac{a}{2x^2}+b\ln(x)$	12
norman	$-\frac{a}{2x^2}+b\ln(x)$	12
risch	$-\frac{a}{2x^2}+b\ln(x)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a/x^2+b*ln(x)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 1.08

$$\frac{1}{2}\,b\log\left(x^2\right)-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="maxima")
[Out] 1/2*b*log(x^2) - 1/2*a/x^2
```

Fricas [A]
time = 1.67, size = 17, normalized size = 1.31

$$\frac{2\,bx^2\log\left(x\right)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b*x^2*log(x) - a)/x^2
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.77

$$-\frac{a}{2x^2}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3,x)
[Out] -a/(2*x**2) + b*log(x)
```

Giac [A]
time = 1.33, size = 20, normalized size = 1.54

$$\frac{1}{2}\,b\log\left(x^2\right)-\frac{bx^2+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

Mupad [B]
time = 4.93, size = 11, normalized size = 0.85

$$b\,\ln\left(x\right)-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^3,x)
```

```
[Out] b*log(x) - a/(2*x^2)
```

3.9 $\int \frac{a+bx^2}{x^4} \, dx$

Optimal. Leaf size=15

$$-\frac{a}{3x^3}-\frac{b}{x}$$

```
[Out] -1/3*a/x^3-b/x
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{14\}$

$$-\frac{a}{3x^3}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^4} \, dx &= \int \left(\frac{a}{x^4}+\frac{b}{x^2}\right) \, dx \\ &= -\frac{a}{3x^3}-\frac{b}{x} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/x
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.93

method	result	size
gosper	$-\frac{3b\,x^2+a}{3x^3}$	14
default	$-\frac{a}{3x^3}-\frac{b}{x}$	14
norman	$\frac{-b\,x^2-\frac{a}{3}}{x^3}$	15
risch	$\frac{-b\,x^2-\frac{a}{3}}{x^3}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3-b/x
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.87

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

Fricas [A]
time = 0.95, size = 13, normalized size = 0.87

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.93

$$\frac{-a-3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4,x)
```

```
[Out] (-a - 3*b*x**2)/(3*x**3)
```

Giac [A]
time = 1.58, size = 13, normalized size = 0.87

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.87

$$-\frac{3\,b\,x^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^4,x)
```

```
[Out] -(a + 3*b*x^2)/(3*x^3)
```

3.10 $\int \frac{a+bx^2}{x^5} dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

```
[Out] -1/4*a/x^4-1/2*b/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^5}\,dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^3}\right) dx\\ &= -\frac{a}{4x^4}-\frac{b}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(2*x^2)
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
gosper	$-\frac{2b\,x^2+a}{4x^4}$	14
default	$-\frac{a}{4x^4}-\frac{b}{2x^2}$	14
norman	$\frac{-\frac{b\,x^2}{2}-\frac{a}{4}}{x^4}$	15
risch	$\frac{-\frac{b\,x^2}{2}-\frac{a}{4}}{x^4}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a/x^4-1/2*b/x^2
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.76

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

Fricas [A]
time = 2.82, size = 13, normalized size = 0.76

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="fricas")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.82

$$\frac{-a-2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5,x)
```

```
[Out] (-a - 2*b*x**2)/(4*x**4)
```

Giac [A]
time = 0.96, size = 13, normalized size = 0.76

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$-\frac{2\,b\,x^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^5,x)
```

```
[Out] -(a + 2*b*x^2)/(4*x^4)
```

3.11 $\int \frac{a+bx^2}{x^6} dx$

Optimal. Leaf size=17

$$-\frac{a}{5x^5} - \frac{b}{3x^3}$$

```
[Out] -1/5*a/x^5-1/3*b/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{5x^5} - \frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^6}\,dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^4}\right)dx\\ &= -\frac{a}{5x^5}-\frac{b}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{5x^5} - \frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/(3*x^3)
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{a}{5x^5}-\frac{b}{3x^3}$	14
norman	$\frac{-\frac{b\,x^2}{3}-\frac{a}{5}}{x^5}$	15
risch	$\frac{-\frac{b\,x^2}{3}-\frac{a}{5}}{x^5}$	15
gosper	$-\frac{5b\,x^2+3a}{15x^5}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*a/x^5-1/3*b/x^3
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.88

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="maxima")
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Fricas [A]
time = 1.43, size = 15, normalized size = 0.88

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="fricas")
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.88

$$\frac{-3a-5bx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**6,x)
```

```
[Out] (-3*a - 5*b*x**2)/(15*x**5)
```

Giac [A]
time = 0.97, size = 15, normalized size = 0.88

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="giac")
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$-\frac{5\,b\,x^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^6,x)
[Out] -(3*a + 5*b*x^2)/(15*x^5)
```

3.12 $\int \frac{a+bx^2}{x^7} dx$

Optimal. Leaf size=17

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

[Out] -1/6*a/x^6-1/4*b/x^4

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/x^7,x]

[Out] -1/6*a/x^6 - b/(4*x^4)

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^7}\,dx &= \int \left(\frac{a}{x^7}+\frac{b}{x^5}\right) dx\\ &= -\frac{a}{6x^6}-\frac{b}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/x^7,x]

[Out] -1/6*a/x^6 - b/(4*x^4)

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{a}{6x^6}-\frac{b}{4x^4}$	14
norman	$\frac{-\frac{b\,x^2}{4}-\frac{a}{6}}{x^6}$	15
risch	$\frac{-\frac{b\,x^2}{4}-\frac{a}{6}}{x^6}$	15
gosper	$-\frac{3b\,x^2+2a}{12x^6}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a/x^6-1/4*b/x^4
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.88

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="maxima")
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

Fricas [A]
time = 1.19, size = 15, normalized size = 0.88

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="fricas")
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.88

$$\frac{-2a-3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**7,x)
```

```
[Out] (-2*a - 3*b*x**2)/(12*x**6)
```

Giac [A]
time = 2.32, size = 15, normalized size = 0.88

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$-\frac{3\,b\,x^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^7,x)
```

```
[Out] -(2*a + 3*b*x^2)/(12*x^6)
```

3.13 $\int x^5(a+bx^2)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

```
[Out] 1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^2\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^2\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(a^2x^2+2abx^3+b^2x^4\right)\,dx,x,x^2\right)\\&=\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{6}a^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}b^2x^{10}$	25
default	$\frac{1}{6}a^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}b^2x^{10}$	25
norman	$\frac{1}{6}a^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}b^2x^{10}$	25
risch	$\frac{1}{6}a^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}b^2x^{10}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10}+\frac{1}{4}\,abx^8+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Fricas [A]
time = 1.63, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10}+\frac{1}{4}\,abx^8+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**6/6 + a*b*x**8/4 + b**2*x**10/10
```

Giac [A]
time = 1.93, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10} + \frac{1}{4}\,abx^8 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^6}{6} + \frac{a\,b\,x^8}{4} + \frac{b^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^6)/6 + (b^2*x^10)/10 + (a*b*x^8)/4
```

3.14 $\int x^4(a+bx^2)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2,x]
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\,dx&=\int\left(a^2x^4+2abx^6+b^2x^8\right)\,dx\\&=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2,x]
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Maple [A]
time = 0.03, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
default	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
norman	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
risch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Fricas [A]
time = 0.76, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

Giac [A]
time = 1.87, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

3.15 $\int x^3(a+bx^2)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3(a+bx^2)^2 \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2 \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x + 2abx^2 + b^2x^3\right) \, dx, x, x^2\right) \\
&= \frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
default	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Fricas [A]
time = 1.05, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

Giac [A]
time = 1.07, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

3.16 $\int x^2(a+bx^2)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2,x]
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\,dx &= \int \left(a^2x^2+2abx^4+b^2x^6\right)\,dx\\ &= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2,x]
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Maple [A]
time = 0.03, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
default	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Fricas [A]
time = 1.45, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Giac [A]
time = 1.21, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

3.17 $\int x(a+bx^2)^2\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^2)^3}{6b}$$

[Out] 1/6*(b*x^2+a)^3/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\frac{(a+bx^2)^3}{6b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^2,x]

[Out] (a + b*x^2)^3/(6*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^2\,dx = \frac{\left(a+bx^2\right)^3}{6b}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^3}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^2,x]

[Out] (a + b*x^2)^3/(6*b)

Maple [A]
time = 0.02, size = 15, normalized size = 0.94

method	result	size
default	$\frac{(b\,x^2+a)^3}{6b}$	15
gosper	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
norman	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
risch	$\frac{b^2x^6}{6}+\frac{ab\,x^4}{2}+\frac{a^2x^2}{2}+\frac{a^3}{6b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x^2+a)^3/b
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.88

$$\frac{(bx^2+a)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^3/b
```

Fricas [A]
time = 1.27, size = 24, normalized size = 1.50

$$\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.
time = 0.01, size = 24, normalized size = 1.50

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

Giac [A]
time = 1.09, size = 14, normalized size = 0.88

$$\frac{(bx^2 + a)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*(b*x^2 + a)^3/b
```

Mupad [B]
time = 0.03, size = 24, normalized size = 1.50

$$\frac{a^2\,x^2}{2} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

3.18 $\int (a+bx^2)^2 \, dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {200}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 \, dx &= \int \left(a^2+2abx^2+b^2x^4\right) \, dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A]
time = 0.01, size = 22, normalized size = 0.88

method	result	size
gosper	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
default	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
norman	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
risch	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A]
time = 1.46, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Giac [A]
time = 0.93, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2,x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

3.19 $\int \frac{(a+bx^2)^2}{x} \, dx$

Optimal. Leaf size=23

$$abx^2 + \frac{b^2x^4}{4} + a^2\log(x)$$

[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$a^2\log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^2/x,x]

[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(2ab + \frac{a^2}{x} + b^2x\right) \, dx, x, x^2\right) \\
&= abx^2 + \frac{b^2x^4}{4} + a^2\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$abx^2 + \frac{b^2x^4}{4} + a^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x,x]
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.96

method	result	size
default	$ab\,x^2 + \frac{b^2x^4}{4} + a^2\ln(x)$	22
norman	$ab\,x^2 + \frac{b^2x^4}{4} + a^2\ln(x)$	22
risch	$\frac{b^2x^4}{4} + ab\,x^2 + a^2 + a^2\ln(x)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 1.04

$$\frac{1}{4}\,b^2x^4 + abx^2 + \frac{1}{2}\,a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="maxima")
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Fricas [A]
time = 1.47, size = 21, normalized size = 0.91

$$\frac{1}{4}\,b^2x^4 + abx^2 + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="fricas")
[Out] 1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A]
time = 0.02, size = 20, normalized size = 0.87

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

Giac [A]
time = 1.11, size = 24, normalized size = 1.04

$$\frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.91

$$a^2\ln(x) + \frac{b^2x^4}{4} + a\,b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x,x)
```

```
[Out] a^2*log(x) + (b^2*x^4)/4 + a*b*x^2
```

3.20 $\int \frac{(a+bx^2)^2}{x^2} \, dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^2}{x^2} \, dx &= \int \left(2ab + \frac{a^2}{x^2} + b^2x^2\right) dx \\ &= -\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Maple [A]
time = 0.01, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
risch	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
norman	$\frac{\frac{1}{3}b^2x^4+2ab\,x^2-a^2}{x}$	26
gosper	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Fricas [A]
time = 1.04, size = 25, normalized size = 1.04

$$\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="fricas")
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A]
time = 0.02, size = 19, normalized size = 0.79

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

Giac [A]
time = 1.14, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Mupad [B]
time = 0.03, size = 22, normalized size = 0.92

$$\frac{b^2\,x^3}{3} - \frac{a^2}{x} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^2,x)
```

```
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

3.21 $\int \frac{(a+bx^2)^2}{x^3} \, dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)$$

[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^2/x^3,x]

[Out] -1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right) dx, x, x^2\right)\\
&= -\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Maple [A]
time = 0.01, size = 24, normalized size = 0.89

method	result	size
default	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln\left(x\right)$	24
risch	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln\left(x\right)$	24
norman	$\frac{-\frac{a^2}{2}+\frac{b^2x^4}{2}}{x^2}+2ab\ln\left(x\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Maxima [A]
time = 0.30, size = 24, normalized size = 0.89

$$\frac{1}{2}\,b^2x^2+ab\log\left(x^2\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

Fricas [A]
time = 1.04, size = 27, normalized size = 1.00

$$\frac{b^2x^4+4\,abx^2\log\left(x\right)-a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

Sympy [A]
time = 0.04, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

Giac [A]
time = 1.37, size = 32, normalized size = 1.19

$$\frac{1}{2}b^2x^2 + ab\log(x^2) - \frac{2abx^2 + a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

Mupad [B]
time = 4.94, size = 23, normalized size = 0.85

$$\frac{b^2x^2}{2} - \frac{a^2}{2x^2} + 2ab\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^3,x)
```

```
[Out] (b^2*x^2)/2 - a^2/(2*x^2) + 2*a*b*log(x)
```

3.22 $\int \frac{(a+bx^2)^2}{x^4}\,dx$

Optimal. Leaf size=23

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^4}\,dx &= \int \left(b^2+\frac{a^2}{x^4}+\frac{2ab}{x^2}\right)dx\\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Maple [A]
time = 0.01, size = 22, normalized size = 0.96

method	result	size
default	$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$	22
risch	$b^2x + \frac{-2ab\,x^2 - \frac{1}{3}a^2}{x^3}$	24
gosper	$-\frac{-3b^2x^4 + 6ab\,x^2 + a^2}{3x^3}$	25
norman	$\frac{b^2x^4 - 2ab\,x^2 - \frac{1}{3}a^2}{x^3}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

Maxima [A]
time = 0.29, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Fricas [A]
time = 1.05, size = 26, normalized size = 1.13

$$\frac{3\,b^2x^4 - 6\,abx^2 - a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.96

$$b^2x + \frac{-a^2 - 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4,x)
```

```
[Out] b**2*x + (-a**2 - 6*a*b*x**2)/(3*x**3)
```

Giac [A]
time = 1.45, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Mupad [B]
time = 0.03, size = 24, normalized size = 1.04

$$b^2\,x - \frac{\frac{a^2}{3} + 2\,b\,a\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^4,x)
```

```
[Out] b^2*x - (a^2/3 + 2*a*b*x^2)/x^3
```

3.23 $\int \frac{(a+bx^2)^2}{x^5} dx$

Optimal. Leaf size=24

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

```
[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^5} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^3} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{x^3} + \frac{2ab}{x^2} + \frac{b^2}{x}\right) dx, x, x^2\right) \\
&= -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^5,x]
[Out] -1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]
```

Maple [A]
time = 0.01, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2\ln(x)$	23
norman	$\frac{-\frac{1}{4}a^2 - ab\,x^2}{x^4} + b^2\ln(x)$	25
risch	$\frac{-\frac{1}{4}a^2 - ab\,x^2}{x^4} + b^2\ln(x)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 1.08

$$\frac{1}{2}\,b^2\log\left(x^2\right) - \frac{4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="maxima")
[Out] 1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

Fricas [A]
time = 0.90, size = 28, normalized size = 1.17

$$\frac{4\,b^2x^4\log\left(x\right) - 4\,abx^2 - a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="fricas")
[Out] 1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

Sympy [A]
time = 0.06, size = 24, normalized size = 1.00

$$b^2 \log(x) + \frac{-a^2 - 4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5,x)
```

```
[Out] b**2*log(x) + (-a**2 - 4*a*b*x**2)/(4*x**4)
```

Giac [A]
time = 1.35, size = 34, normalized size = 1.42

$$\frac{1}{2} b^2 \log\left(x^2\right) - \frac{3\,b^2x^4 + 4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

Mupad [B]
time = 0.04, size = 24, normalized size = 1.00

$$b^2 \ln(x) - \frac{\frac{a^2}{4} + b\,a\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^5,x)
```

```
[Out] b^2*log(x) - (a^2/4 + a*b*x^2)/x^4
```

3.24 $\int \frac{(a+bx^2)^2}{x^6} dx$

Optimal. Leaf size=28

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^6} dx &= \int \left(\frac{a^2}{x^6} + \frac{2ab}{x^4} + \frac{b^2}{x^2}\right) dx \\ &= -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.89

method	result	size
default	$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$	25
norman	$\frac{-b^2x^4-\frac{2}{3}ab\,x^2-\frac{1}{5}a^2}{x^5}$	26
risch	$\frac{-b^2x^4-\frac{2}{3}ab\,x^2-\frac{1}{5}a^2}{x^5}$	26
gosper	$-\frac{15b^2x^4+10ab\,x^2+3a^2}{15x^5}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="maxima")
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Fricas [A]
time = 1.06, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="fricas")
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Sympy [A]
time = 0.06, size = 27, normalized size = 0.96

$$\frac{-3a^2-10abx^2-15b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6,x)
```

```
[Out] (-3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

Giac [A]
time = 1.16, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.89

$$-\frac{\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}+b^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^6,x)
```

```
[Out] -(a^2/5 + b^2*x^4 + (2*a*b*x^2)/3)/x^5
```

3.25 $\int \frac{(a+bx^2)^2}{x^7} dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^3}{6ax^6}$$

```
[Out] -1/6*(b*x^2+a)^3/a/x^6
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{(a+bx^2)^3}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^7,x]
```

```
[Out] -1/6*(a + b*x^2)^3/(a*x^6)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^7} dx = -\frac{(a+bx^2)^3}{6ax^6}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.58

$$-\frac{a^2}{6x^6} - \frac{ab}{2x^4} - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Maple [A]
time = 0.01, size = 25, normalized size = 1.32

method	result	size
gosper	$-\frac{3b^2x^4+3ab\,x^2+a^2}{6x^6}$	25
default	$-\frac{ab}{2x^4}-\frac{a^2}{6x^6}-\frac{b^2}{2x^2}$	25
norman	$\frac{-\frac{1}{2}b^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}a^2}{x^6}$	26
risch	$\frac{-\frac{1}{2}b^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}a^2}{x^6}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7,x,method=_RETURNVERBOSE)
[Out] -1/2*a*b/x^4-1/6*a^2/x^6-1/2*b^2/x^2
```

Maxima [A]
time = 0.28, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="maxima")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Fricas [A]
time = 1.22, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="fricas")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Sympy [A]
time = 0.07, size = 26, normalized size = 1.37

$$\frac{-a^2-3abx^2-3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7,x)
```

```
[Out] (-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

Giac [A]
time = 0.70, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Mupad [B]
time = 0.03, size = 26, normalized size = 1.37

$$-\frac{\frac{a^2}{6}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^7,x)
```

```
[Out] -(a^2/6 + (b^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

3.26 $\int \frac{(a+bx^2)^2}{x^8} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^8} dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right) dx\\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.83

method	result	size
default	$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$	25
norman	$\frac{-\frac{1}{3}b^2x^4-\frac{2}{5}ab\,x^2-\frac{1}{7}a^2}{x^7}$	26
risch	$\frac{-\frac{1}{3}b^2x^4-\frac{2}{5}ab\,x^2-\frac{1}{7}a^2}{x^7}$	26
gosper	$-\frac{35b^2x^4+42ab\,x^2+15a^2}{105x^7}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^8,x,method=_RETURNVERBOSE)
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="maxima")
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Fricas [A]
time = 0.89, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="fricas")
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Sympy [A]
time = 0.07, size = 27, normalized size = 0.90

$$\frac{-15a^2-42abx^2-35b^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**8,x)
```

```
[Out] (-15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)
```

Giac [A]
time = 1.40, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{7}+\frac{2\,a\,b\,x^2}{5}+\frac{b^2\,x^4}{3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^8,x)
```

```
[Out] -(a^2/7 + (b^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

3.27 $\int \frac{(a+bx^2)^2}{x^9} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

```
[Out] -1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^9,x]
```

```
[Out] -1/8*a^2/x^8 - (a*b)/(3*x^6) - b^2/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^5}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)\,dx, x, x^2\right)\\ &= -\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^9,x]
[Out] -1/8*a^2/x^8 - (a*b)/(3*x^6) - b^2/(4*x^4)
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.83

method	result	size
default	$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$	25
norman	$\frac{-\frac{1}{4}b^2x^4-\frac{1}{3}ab\,x^2-\frac{1}{8}a^2}{x^8}$	26
risch	$\frac{-\frac{1}{4}b^2x^4-\frac{1}{3}ab\,x^2-\frac{1}{8}a^2}{x^8}$	26
gosper	$-\frac{6b^2x^4+8ab\,x^2+3a^2}{24x^8}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^9,x,method=_RETURNVERBOSE)
[Out] -1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="maxima")
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Fricas [A]
time = 1.25, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Sympy [A]
time = 0.08, size = 27, normalized size = 0.90

$$\frac{-3a^2 - 8abx^2 - 6b^2x^4}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**9,x)
```

```
[Out] (-3*a**2 - 8*a*b*x**2 - 6*b**2*x**4)/(24*x**8)
```

Giac [A]
time = 2.70, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^4 + 8\,abx^2 + 3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{8} + \frac{a\,b\,x^2}{3} + \frac{b^2\,x^4}{4}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^9,x)
```

```
[Out] -(a^2/8 + (b^2*x^4)/4 + (a*b*x^2)/3)/x^8
```

3.28 $\int \frac{(a+bx^2)^2}{x^{10}}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

```
[Out] -1/9*a^2/x^9-2/7*a*b/x^7-1/5*b^2/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^10,x]
```

```
[Out] -1/9*a^2/x^9 - (2*a*b)/(7*x^7) - b^2/(5*x^5)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^8}+\frac{b^2}{x^6}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^10,x]
```

```
[Out] -1/9*a^2/x^9 - (2*a*b)/(7*x^7) - b^2/(5*x^5)
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.83

method	result	size
default	$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$	25
norman	$\frac{-\frac{1}{5}b^2x^4-\frac{2}{7}ab\,x^2-\frac{1}{9}a^2}{x^9}$	26
risch	$\frac{-\frac{1}{5}b^2x^4-\frac{2}{7}ab\,x^2-\frac{1}{9}a^2}{x^9}$	26
gosper	$-\frac{63b^2x^4+90ab\,x^2+35a^2}{315x^9}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^10,x,method=_RETURNVERBOSE)
[Out] -1/9*a^2/x^9-2/7*a*b/x^7-1/5*b^2/x^5
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.87

$$-\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="maxima")
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

Fricas [A]
time = 1.03, size = 26, normalized size = 0.87

$$-\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="fricas")
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

Sympy [A]
time = 0.08, size = 27, normalized size = 0.90

$$\frac{-35a^2-90abx^2-63b^2x^4}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**10,x)
```

```
[Out] (-35*a**2 - 90*a*b*x**2 - 63*b**2*x**4)/(315*x**9)
```

Giac [A]
time = 1.08, size = 26, normalized size = 0.87

$$-\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{9}+\frac{2\,a\,b\,x^2}{7}+\frac{b^2\,x^4}{5}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^10,x)
```

```
[Out] -(a^2/9 + (b^2*x^4)/5 + (2*a*b*x^2)/7)/x^9
```

3.29 $\int x^9(a+bx^2)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

```
[Out] 1/10*a^3*x^10+1/4*a^2*b*x^12+3/14*a*b^2*x^14+1/16*b^3*x^16
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^9\left(a+bx^2\right)^3\,dx &= \frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^3\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)\,dx,x,x^2\right)\\
&= \frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

Maple [A]
time = 0.03, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{10}a^3x^{10}+\frac{1}{4}a^2b\,x^{12}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{16}b^3x^{16}$	36
default	$\frac{1}{10}a^3x^{10}+\frac{1}{4}a^2b\,x^{12}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{16}b^3x^{16}$	36
norman	$\frac{1}{10}a^3x^{10}+\frac{1}{4}a^2b\,x^{12}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{16}b^3x^{16}$	36
risch	$\frac{1}{10}a^3x^{10}+\frac{1}{4}a^2b\,x^{12}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{16}b^3x^{16}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*a^3*x^10+1/4*a^2*b*x^12+3/14*a*b^2*x^14+1/16*b^3*x^16
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.81

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Fricas [A]
time = 1.08, size = 35, normalized size = 0.81

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Sympy [A]
time = 0.01, size = 37, normalized size = 0.86

$$\frac{a^3x^{10}}{10} + \frac{a^2bx^{12}}{4} + \frac{3ab^2x^{14}}{14} + \frac{b^3x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**3,x)
[Out] a**3*x**10/10 + a**2*b*x**12/4 + 3*a*b**2*x**14/14 + b**3*x**16/16
```

Giac [A]
time = 1.59, size = 35, normalized size = 0.81

$$\frac{1}{16}\,b^3x^{16} + \frac{3}{14}\,ab^2x^{14} + \frac{1}{4}\,a^2bx^{12} + \frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^{10}}{10} + \frac{a^2\,b\,x^{12}}{4} + \frac{3\,a\,b^2\,x^{14}}{14} + \frac{b^3\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^3,x)
[Out] (a^3*x^10)/10 + (b^3*x^16)/16 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14
```

3.30 $\int x^7 (a+bx^2)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

```
[Out] 1/8*a^3*x^8+3/10*a^2*b*x^10+1/4*a*b^2*x^12+1/14*b^3*x^14
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^3 \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^3 \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right) \, dx, x, x^2\right)\\
&= \frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^3,x]
[Out] (a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

Maple [A]
time = 0.03, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{8}a^3x^8+\frac{3}{10}a^2b\,x^{10}+\frac{1}{4}a\,b^2x^{12}+\frac{1}{14}b^3x^{14}$	36
default	$\frac{1}{8}a^3x^8+\frac{3}{10}a^2b\,x^{10}+\frac{1}{4}a\,b^2x^{12}+\frac{1}{14}b^3x^{14}$	36
norman	$\frac{1}{8}a^3x^8+\frac{3}{10}a^2b\,x^{10}+\frac{1}{4}a\,b^2x^{12}+\frac{1}{14}b^3x^{14}$	36
risch	$\frac{1}{8}a^3x^8+\frac{3}{10}a^2b\,x^{10}+\frac{1}{4}a\,b^2x^{12}+\frac{1}{14}b^3x^{14}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/8*a^3*x^8+3/10*a^2*b*x^10+1/4*a*b^2*x^12+1/14*b^3*x^14
```

Maxima [A]
time = 0.30, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Fricas [A]
time = 1.19, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Sympy [A]
time = 0.01, size = 37, normalized size = 0.86

$$\frac{a^3x^8}{8}+\frac{3a^2bx^{10}}{10}+\frac{ab^2x^{12}}{4}+\frac{b^3x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**8/8 + 3*a**2*b*x**10/10 + a*b**2*x**12/4 + b**3*x**14/14
```

Giac [A]
time = 1.44, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^8}{8}+\frac{3\,a^2\,b\,x^{10}}{10}+\frac{a\,b^2\,x^{12}}{4}+\frac{b^3\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^8)/8 + (b^3*x^14)/14 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4
```

3.31 $\int x^5(a+bx^2)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

```
[Out] 1/6*a^3*x^6+3/8*a^2*b*x^8+3/10*a*b^2*x^10+1/12*b^3*x^12
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 45}

$$\frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^3\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)\,dx,x,x^2\right)\\&=\frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

Maple [A]
time = 0.03, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{6}a^3x^6+\frac{3}{8}a^2b\,x^8+\frac{3}{10}a\,b^2x^{10}+\frac{1}{12}b^3x^{12}$	36
default	$\frac{1}{6}a^3x^6+\frac{3}{8}a^2b\,x^8+\frac{3}{10}a\,b^2x^{10}+\frac{1}{12}b^3x^{12}$	36
norman	$\frac{1}{6}a^3x^6+\frac{3}{8}a^2b\,x^8+\frac{3}{10}a\,b^2x^{10}+\frac{1}{12}b^3x^{12}$	36
risch	$\frac{1}{6}a^3x^6+\frac{3}{8}a^2b\,x^8+\frac{3}{10}a\,b^2x^{10}+\frac{1}{12}b^3x^{12}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^3*x^6+3/8*a^2*b*x^8+3/10*a*b^2*x^10+1/12*b^3*x^12
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.81

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Fricas [A]
time = 1.32, size = 35, normalized size = 0.81

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Sympy [A]
time = 0.01, size = 39, normalized size = 0.91

$$\frac{a^3x^6}{6} + \frac{3a^2bx^8}{8} + \frac{3ab^2x^{10}}{10} + \frac{b^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**3,x)
[Out] a**3*x**6/6 + 3*a**2*b*x**8/8 + 3*a*b**2*x**10/10 + b**3*x**12/12
```

Giac [A]
time = 0.99, size = 35, normalized size = 0.81

$$\frac{1}{12}\,b^3x^{12} + \frac{3}{10}\,ab^2x^{10} + \frac{3}{8}\,a^2bx^8 + \frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^6}{6} + \frac{3\,a^2\,b\,x^8}{8} + \frac{3\,a\,b^2\,x^{10}}{10} + \frac{b^3\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^3,x)
[Out] (a^3*x^6)/6 + (b^3*x^12)/12 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10
```

3.32 $\int x^3(a+bx^2)^3\,dx$

Optimal. Leaf size=34

$$-\frac{a(a+bx^2)^4}{8b^2}+\frac{(a+bx^2)^5}{10b^2}$$

```
[Out] -1/8*a*(b*x^2+a)^4/b^2+1/10*(b*x^2+a)^5/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\frac{(a+bx^2)^5}{10b^2}-\frac{a(a+bx^2)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^3,x]
```

```
[Out] -1/8*(a*(a + b*x^2)^4)/b^2 + (a + b*x^2)^5/(10*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^3\,dx&=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,x^2\right)\\&=-\frac{a(a+bx^2)^4}{8b^2}+\frac{(a+bx^2)^5}{10b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.26

$$\frac{a^3x^4}{4}+\frac{1}{2}a^2bx^6+\frac{3}{8}ab^2x^8+\frac{b^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^4)/4 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8 + (b^3*x^10)/10
```

Maple [A]
time = 0.02, size = 36, normalized size = 1.06

method	result	size
gosper	$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$	36
default	$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$	36
norman	$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$	36
risch	$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^3*x^10+3/8*a*b^2*x^8+1/2*a^2*b*x^6+1/4*a^3*x^4
```

Maxima [A]
time = 0.28, size = 35, normalized size = 1.03

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{2}\,a^2bx^6+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Fricas [A]
time = 1.12, size = 35, normalized size = 1.03

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{2}\,a^2bx^6+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Sympy [A]
time = 0.01, size = 37, normalized size = 1.09

$$\frac{a^3x^4}{4} + \frac{a^2bx^6}{2} + \frac{3ab^2x^8}{8} + \frac{b^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**4/4 + a**2*b*x**6/2 + 3*a*b**2*x**8/8 + b**3*x**10/10
```

Giac [A]
time = 0.67, size = 35, normalized size = 1.03

$$\frac{1}{10}\,b^3x^{10} + \frac{3}{8}\,ab^2x^8 + \frac{1}{2}\,a^2bx^6 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Mupad [B]
time = 0.04, size = 35, normalized size = 1.03

$$\frac{a^3\,x^4}{4} + \frac{a^2\,b\,x^6}{2} + \frac{3\,a\,b^2\,x^8}{8} + \frac{b^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^10)/10 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8
```

3.33 $\int x(a+bx^2)^3\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^2)^4}{8b}$$

[Out] 1/8*(b*x^2+a)^4/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\frac{(a+bx^2)^4}{8b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^3,x]

[Out] (a + b*x^2)^4/(8*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^3\,dx = \frac{(a+bx^2)^4}{8b}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^4}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^3,x]

[Out] (a + b*x^2)^4/(8*b)

Maple [A]
time = 0.02, size = 15, normalized size = 0.94

method	result	size
default	$\frac{\left(b\,x^{2}+a\right)^{4}}{8b}$	15
gosper	$\frac{1}{8}b^{3}x^{8}+\frac{1}{2}a\,b^{2}x^{6}+\frac{3}{4}a^{2}b\,x^{4}+\frac{1}{2}a^{3}x^{2}$	36
norman	$\frac{1}{8}b^{3}x^{8}+\frac{1}{2}a\,b^{2}x^{6}+\frac{3}{4}a^{2}b\,x^{4}+\frac{1}{2}a^{3}x^{2}$	36
risch	$\frac{b^{3}x^{8}}{8}+\frac{a\,b^{2}x^{6}}{2}+\frac{3a^{2}b\,x^{4}}{4}+\frac{a^{3}x^{2}}{2}+\frac{a^{4}}{8b}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(b*x^2+a)^4/b
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.88

$$\frac{\left(bx^{2}+a\right)^{4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^4/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.
time = 0.89, size = 35, normalized size = 2.19

$$\frac{1}{8}\,b^{3}x^{8}+\frac{1}{2}\,ab^{2}x^{6}+\frac{3}{4}\,a^{2}bx^{4}+\frac{1}{2}\,a^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.
time = 0.01, size = 37, normalized size = 2.31

$$\frac{a^{3}x^{2}}{2}+\frac{3a^{2}bx^{4}}{4}+\frac{ab^{2}x^{6}}{2}+\frac{b^{3}x^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**2/2 + 3*a**2*b*x**4/4 + a*b**2*x**6/2 + b**3*x**8/8
```

Giac [A]
time = 0.63, size = 14, normalized size = 0.88

$$\frac{(bx^2 + a)^4}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2 + a)^4/b
```

Mupad [B]
time = 0.06, size = 35, normalized size = 2.19

$$\frac{a^3\,x^2}{2} + \frac{3\,a^2\,b\,x^4}{4} + \frac{a\,b^2\,x^6}{2} + \frac{b^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^2)/2 + (b^3*x^8)/8 + (3*a^2*b*x^4)/4 + (a*b^2*x^6)/2
```

3.34 $\int \frac{(a+bx^2)^3}{x} \, dx$

Optimal. Leaf size=39

$$\frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6} + a^3\log(x)$$

```
[Out] 3/2*a^2*b*x^2+3/4*a*b^2*x^4+1/6*b^3*x^6+a^3*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$a^3\log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(3a^2b + \frac{a^3}{x} + 3ab^2x + b^3x^2\right) \, dx, x, x^2\right) \\
&= \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6} + a^3\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 39, normalized size = 1.00

$$\frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6} + a^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x,x]
[Out] (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

Maple [A]
time = 0.01, size = 34, normalized size = 0.87

method	result	size
default	$\frac{3a^2b\,x^2}{2} + \frac{3a\,b^2x^4}{4} + \frac{b^3x^6}{6} + a^3\ln(x)$	34
norman	$\frac{3a^2b\,x^2}{2} + \frac{3a\,b^2x^4}{4} + \frac{b^3x^6}{6} + a^3\ln(x)$	34
risch	$\frac{3a^2b\,x^2}{2} + \frac{3a\,b^2x^4}{4} + \frac{b^3x^6}{6} + a^3\ln(x)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
[Out] 3/2*a^2*b*x^2+3/4*a*b^2*x^4+1/6*b^3*x^6+a^3*ln(x)
```

Maxima [A]
time = 0.29, size = 36, normalized size = 0.92

$$\frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + \frac{1}{2}\,a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="maxima")
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

Fricas [A]
time = 0.69, size = 33, normalized size = 0.85

$$\frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="fricas")
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

Sympy [A]
time = 0.03, size = 37, normalized size = 0.95

$$a^{3}\log\left(x\right)+\frac{3a^{2}bx^{2}}{2}+\frac{3ab^{2}x^{4}}{4}+\frac{b^{3}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/4 + b**3*x**6/6
```

Giac [A]
time = 0.56, size = 36, normalized size = 0.92

$$\frac{1}{6}b^{3}x^{6}+\frac{3}{4}ab^{2}x^{4}+\frac{3}{2}a^{2}bx^{2}+\frac{1}{2}a^{3}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

Mupad [B]
time = 0.04, size = 33, normalized size = 0.85

$$a^{3}\,\ln\left(x\right)+\frac{b^{3}\,x^{6}}{6}+\frac{3\,a^{2}\,b\,x^{2}}{2}+\frac{3\,a\,b^{2}\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x,x)
```

```
[Out] a^3*log(x) + (b^3*x^6)/6 + (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4
```

3.35 $\int \frac{(a+bx^2)^3}{x^3} dx$

Optimal. Leaf size=40

$$-\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)\,dx,x,x^2\right)\\
&= -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.00

$$-\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^3,x]
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.88

method	result	size
default	$-\frac{a^3}{2x^2}+\frac{3a\,b^2x^2}{2}+\frac{b^3x^4}{4}+3a^2b\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{2}a^3+\frac{1}{4}b^3x^6+\frac{3}{2}a\,b^2x^4}{x^2}+3a^2b\ln\left(x\right)$	37
risch	$\frac{b^3x^4}{4}+\frac{3a\,b^2x^2}{2}+\frac{9a^2b}{4}-\frac{a^3}{2x^2}+3a^2b\ln\left(x\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Maxima [A]
time = 0.29, size = 36, normalized size = 0.90

$$\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+\frac{3}{2}\,a^2b\log\left(x^2\right)-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="maxima")
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*a^3/x^2
```

Fricas [A]
time = 0.69, size = 38, normalized size = 0.95

$$\frac{b^3x^6+6\,ab^2x^4+12\,a^2bx^2\log\left(x\right)-2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="fricas")
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [A]
time = 0.04, size = 37, normalized size = 0.92

$$-\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

Giac [A]
time = 0.47, size = 46, normalized size = 1.15

$$\frac{1}{4}\,b^3x^4 + \frac{3}{2}\,ab^2x^2 + \frac{3}{2}\,a^2b\log\left(x^2\right) - \frac{3\,a^2bx^2 + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/
x^2
```

Mupad [B]
time = 0.04, size = 34, normalized size = 0.85

$$\frac{b^3\,x^4}{4} - \frac{a^3}{2\,x^2} + \frac{3\,a\,b^2\,x^2}{2} + 3\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^3,x)
```

```
[Out] (b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

3.36 $\int \frac{(a+bx^2)^3}{x^5} dx$

Optimal. Leaf size=40

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{2x^2}+\frac{b^3x^2}{2}+3ab^2\log(x)$$

```
[Out] -1/4*a^3/x^4-3/2*a^2*b/x^2+1/2*b^3*x^2+3*a*b^2*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{2x^2}+3ab^2\log(x)+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^5,x]
```

```
[Out] -1/4*a^3/x^4 - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right)\,dx,x,x^2\right)\\
&= -\frac{a^3}{4x^4}-\frac{3a^2b}{2x^2}+\frac{b^3x^2}{2}+3ab^2\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.00

$$-\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + \frac{b^3x^2}{2} + 3ab^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^5,x]
[Out] -1/4*a^3/x^4 - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.88

method	result	size
default	$-\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + \frac{b^3x^2}{2} + 3a\,b^2\ln(x)$	35
norman	$\frac{-\frac{1}{4}a^3 + \frac{1}{2}b^3x^6 - \frac{3}{2}a^2b\,x^2}{x^4} + 3a\,b^2\ln(x)$	37
risch	$\frac{b^3x^2}{2} + \frac{-\frac{3}{2}a^2b\,x^2 - \frac{1}{4}a^3}{x^4} + 3a\,b^2\ln(x)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a^3/x^4-3/2*a^2*b/x^2+1/2*b^3*x^2+3*a*b^2*ln(x)
```

Maxima [A]
time = 0.29, size = 37, normalized size = 0.92

$$\frac{1}{2}\,b^3x^2 + \frac{3}{2}\,ab^2\log\left(x^2\right) - \frac{6\,a^2bx^2 + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="maxima")
[Out] 1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(6*a^2*b*x^2 + a^3)/x^4
```

Fricas [A]
time = 0.95, size = 39, normalized size = 0.98

$$\frac{2\,b^3x^6 + 12\,ab^2x^4\log\left(x\right) - 6\,a^2bx^2 - a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="fricas")
[Out] 1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

Sympy [A]
time = 0.07, size = 37, normalized size = 0.92

$$3ab^2\log(x) + \frac{b^3x^2}{2} + \frac{-a^3 - 6a^2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**5,x)
```

```
[Out] 3*a*b**2*log(x) + b**3*x**2/2 + (-a**3 - 6*a**2*b*x**2)/(4*x**4)
```

Giac [A]
time = 0.51, size = 46, normalized size = 1.15

$$\frac{1}{2}\,b^3x^2 + \frac{3}{2}\,ab^2\log\left(x^2\right) - \frac{9\,ab^2x^4 + 6\,a^2bx^2 + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(9*a*b^2*x^4 + 6*a^2*b*x^2 + a^3)/x^
4
```

Mupad [B]
time = 4.90, size = 37, normalized size = 0.92

$$\frac{b^3\,x^2}{2} - \frac{\frac{a^3}{4} + \frac{3\,b\,a^2\,x^2}{2}}{x^4} + 3\,a\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^5,x)
```

```
[Out] (b^3*x^2)/2 - (a^3/4 + (3*a^2*b*x^2)/2)/x^4 + 3*a*b^2*log(x)
```

3.37 $\int \frac{(a+bx^2)^3}{x^7} \, dx$

Optimal. Leaf size=39

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

```
[Out] -1/6*a^3/x^6-3/4*a^2*b/x^4-3/2*a*b^2/x^2+b^3*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^4}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)dx,x,x^2\right)\\
&= -\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 39, normalized size = 1.00

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^7,x]
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

Maple [A]
time = 0.01, size = 34, normalized size = 0.87

method	result	size
default	$-\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3a\,b^2}{2x^2}+b^3\ln\left(x\right)$	34
norman	$\frac{-\frac{1}{6}a^3-\frac{3}{2}a\,b^2x^4-\frac{3}{4}a^2b\,x^2}{x^6}+b^3\ln\left(x\right)$	36
risch	$\frac{-\frac{1}{6}a^3-\frac{3}{2}a\,b^2x^4-\frac{3}{4}a^2b\,x^2}{x^6}+b^3\ln\left(x\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a^3/x^6-3/4*a^2*b/x^4-3/2*a*b^2/x^2+b^3*ln(x)
```

Maxima [A]
time = 0.29, size = 39, normalized size = 1.00

$$\frac{1}{2}b^3\log\left(x^2\right)-\frac{18\,ab^2x^4+9\,a^2bx^2+2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="maxima")
[Out] 1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x^6
```

Fricas [A]
time = 0.68, size = 39, normalized size = 1.00

$$\frac{12\,b^3x^6\log\left(x\right)-18\,ab^2x^4-9\,a^2bx^2-2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="fricas")
[Out] 1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

Sympy [A]
time = 0.09, size = 37, normalized size = 0.95

$$b^3 \log(x) + \frac{-2a^3 - 9a^2bx^2 - 18ab^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**7,x)
```

```
[Out] b**3*log(x) + (-2*a**3 - 9*a**2*b*x**2 - 18*a*b**2*x**4)/(12*x**6)
```

Giac [A]
time = 0.50, size = 47, normalized size = 1.21

$$\frac{1}{2}\, b^3 \log\left(x^2\right) - \frac{11\, b^3x^6 + 18\, ab^2x^4 + 9\, a^2bx^2 + 2\, a^3}{12\, x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(x^2) - 1/12*(11*b^3*x^6 + 18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x
^6
```

Mupad [B]
time = 0.05, size = 36, normalized size = 0.92

$$b^3\, \ln(x) - \frac{\frac{a^3}{6} + \frac{3\, a^2\, b\, x^2}{4} + \frac{3\, a\, b^2\, x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^7,x)
```

```
[Out] b^3*log(x) - (a^3/6 + (3*a^2*b*x^2)/4 + (3*a*b^2*x^4)/2)/x^6
```

3.38 $\int \frac{(a+bx^2)^3}{x^9} dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^4}{8ax^8}$$

[Out] -1/8*(b*x^2+a)^4/a/x^8

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{(a+bx^2)^4}{8ax^8}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^3/x^9,x]

[Out] -1/8*(a + b*x^2)^4/(a*x^8)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^3}{x^9} dx = -\frac{(a+bx^2)^4}{8ax^8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.
time = 0.00, size = 43, normalized size = 2.26

$$-\frac{a^3}{8x^8} - \frac{a^2b}{2x^6} - \frac{3ab^2}{4x^4} - \frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^3/x^9,x]

[Out] -1/8*a^3/x^8 - (a^2*b)/(2*x^6) - (3*a*b^2)/(4*x^4) - b^3/(2*x^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 0.01, size = 36, normalized size = 1.89

method	result	size
gosper	$-\frac{4b^{3}x^{6}+6a\,b^{2}x^{4}+4a^{2}b\,x^{2}+a^{3}}{8x^{8}}$	36
default	$-\frac{3a\,b^{2}}{4x^{4}}-\frac{a^{2}b}{2x^{6}}-\frac{b^{3}}{2x^{2}}-\frac{a^{3}}{8x^{8}}$	36
norman	$\frac{-\frac{1}{2}b^{3}x^{6}-\frac{3}{4}a\,b^{2}x^{4}-\frac{1}{2}a^{2}b\,x^{2}-\frac{1}{8}a^{3}}{x^{8}}$	37
risch	$\frac{-\frac{1}{2}b^{3}x^{6}-\frac{3}{4}a\,b^{2}x^{4}-\frac{1}{2}a^{2}b\,x^{2}-\frac{1}{8}a^{3}}{x^{8}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*a*b^2/x^4-1/2*a^2*b/x^6-1/2*b^3/x^2-1/8*a^3/x^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 0.33, size = 35, normalized size = 1.84

$$-\frac{4\,b^{3}x^{6}+6\,ab^{2}x^{4}+4\,a^{2}bx^{2}+a^{3}}{8\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 0.78, size = 35, normalized size = 1.84

$$-\frac{4\,b^{3}x^{6}+6\,ab^{2}x^{4}+4\,a^{2}bx^{2}+a^{3}}{8\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.
time = 0.10, size = 37, normalized size = 1.95

$$\frac{-a^{3}-4a^{2}bx^{2}-6ab^{2}x^{4}-4b^{3}x^{6}}{8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**9,x)
```

```
[Out] (-a**3 - 4*a**2*b*x**2 - 6*a*b**2*x**4 - 4*b**3*x**6)/(8*x**8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 0.58, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^6 + 6\,ab^2x^4 + 4\,a^2bx^2 + a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Mupad [B]
time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{a^3}{8} + \frac{a^2\,b\,x^2}{2} + \frac{3\,a\,b^2\,x^4}{4} + \frac{b^3\,x^6}{2}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^9,x)
```

```
[Out] -(a^3/8 + (b^3*x^6)/2 + (a^2*b*x^2)/2 + (3*a*b^2*x^4)/4)/x^8
```

3.39 $\int \frac{(a+bx^2)^3}{x^{11}} \, dx$

Optimal. Leaf size=40

$$-\frac{(a+bx^2)^4}{10ax^{10}}+\frac{b(a+bx^2)^4}{40a^2x^8}$$

```
[Out] -1/10*(b*x^2+a)^4/a/x^10+1/40*b*(b*x^2+a)^4/a^2/x^8
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 47, 37}

$$\frac{b(a+bx^2)^4}{40a^2x^8}-\frac{(a+bx^2)^4}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^11,x]
```

```
[Out] -1/10*(a + b*x^2)^4/(a*x^10) + (b*(a + b*x^2)^4)/(40*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^6}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^4}{10ax^{10}} - \frac{b\text{Subst}\left(\int \frac{(a+bx)^3}{x^5}\,dx, x, x^2\right)}{10a}$$
$$= -\frac{(a+bx^2)^4}{10ax^{10}} + \frac{b(a+bx^2)^4}{40a^2x^8}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.08

$$-\frac{a^3}{10x^{10}} - \frac{3a^2b}{8x^8} - \frac{ab^2}{2x^6} - \frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^11,x]
[Out] -1/10*a^3/x^10 - (3*a^2*b)/(8*x^8) - (a*b^2)/(2*x^6) - b^3/(4*x^4)
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.90

method	result	size
default	$-\frac{b^3}{4x^4} - \frac{a\,b^2}{2x^6} - \frac{a^3}{10x^{10}} - \frac{3a^2b}{8x^8}$	36
norman	$\frac{-\frac{1}{4}b^3x^6 - \frac{1}{2}a\,b^2x^4 - \frac{3}{8}a^2b\,x^2 - \frac{1}{10}a^3}{x^{10}}$	37
risch	$\frac{-\frac{1}{4}b^3x^6 - \frac{1}{2}a\,b^2x^4 - \frac{3}{8}a^2b\,x^2 - \frac{1}{10}a^3}{x^{10}}$	37
gosper	$-\frac{10b^3x^6 + 20a\,b^2x^4 + 15a^2b\,x^2 + 4a^3}{40x^{10}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^11,x,method=_RETURNVERBOSE)
[Out] -1/4*b^3/x^4-1/2*a*b^2/x^6-1/10*a^3/x^10-3/8*a^2*b/x^8
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^6 + 20\,ab^2x^4 + 15\,a^2bx^2 + 4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="maxima")
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Fricas [A]
time = 1.05, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="fricas")
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Sympy [A]
time = 0.12, size = 39, normalized size = 0.98

$$\frac{-4a^3-15a^2bx^2-20ab^2x^4-10b^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**11,x)
[Out] (-4*a**3 - 15*a**2*b*x**2 - 20*a*b**2*x**4 - 10*b**3*x**6)/(40*x**10)
```

Giac [A]
time = 0.55, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="giac")
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Mupad [B]
time = 0.06, size = 37, normalized size = 0.92

$$-\frac{\frac{a^3}{10}+\frac{3\,a^2\,b\,x^2}{8}+\frac{a\,b^2\,x^4}{2}+\frac{b^3\,x^6}{4}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^11,x)
[Out] -(a^3/10 + (b^3*x^6)/4 + (3*a^2*b*x^2)/8 + (a*b^2*x^4)/2)/x^10
```

3.40 $\int \frac{(a+bx^2)^3}{x^{13}} \, dx$

Optimal. Leaf size=43

$$-\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

```
[Out] -1/12*a^3/x^12-3/10*a^2*b/x^10-3/8*a*b^2/x^8-1/6*b^3/x^6
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^13,x]
```

```
[Out] -1/12*a^3/x^12 - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^{13}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^7} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right) dx, x, x^2\right) \\
&= -\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^13,x]
[Out] -1/12*a^3/x^12 - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.84

method	result	size
default	$-\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3a\,b^2}{8x^8}-\frac{b^3}{6x^6}$	36
norman	$\frac{-\frac{1}{6}b^3x^6-\frac{3}{8}a\,b^2x^4-\frac{3}{10}a^2b\,x^2-\frac{1}{12}a^3}{x^{12}}$	37
risch	$\frac{-\frac{1}{6}b^3x^6-\frac{3}{8}a\,b^2x^4-\frac{3}{10}a^2b\,x^2-\frac{1}{12}a^3}{x^{12}}$	37
gosper	$-\frac{20b^3x^6+45a\,b^2x^4+36a^2b\,x^2+10a^3}{120x^{12}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^13,x,method=_RETURNVERBOSE)
[Out] -1/12*a^3/x^12-3/10*a^2*b/x^10-3/8*a*b^2/x^8-1/6*b^3/x^6
```

Maxima [A]
time = 0.29, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="maxima")
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Fricas [A]
time = 1.20, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Sympy [A]
time = 0.13, size = 39, normalized size = 0.91

$$\frac{-10a^3 - 36a^2bx^2 - 45ab^2x^4 - 20b^3x^6}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**13,x)
[Out] (-10*a**3 - 36*a**2*b*x**2 - 45*a*b**2*x**4 - 20*b**3*x**6)/(120*x**12)
```

Giac [A]
time = 0.54, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^6 + 45\,ab^2x^4 + 36\,a^2bx^2 + 10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="giac")
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Mupad [B]
time = 0.05, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{12} + \frac{3\,a^2\,b\,x^2}{10} + \frac{3\,a\,b^2\,x^4}{8} + \frac{b^3\,x^6}{6}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^13,x)
[Out] -(a^3/12 + (b^3*x^6)/6 + (3*a^2*b*x^2)/10 + (3*a*b^2*x^4)/8)/x^12
```

3.41 $\int \frac{(a+bx^2)^3}{x^{15}} dx$

Optimal. Leaf size=43

$$-\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

```
[Out] -1/14*a^3/x^14-1/4*a^2*b/x^12-3/10*a*b^2/x^10-1/8*b^3/x^8
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^15,x]
```

```
[Out] -1/14*a^3/x^14 - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^{15}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^8}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right)dx, x, x^2\right)\\
&= -\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.00

$$-\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^15,x]
[Out] -1/14*a^3/x^14 - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.84

method	result	size
default	$-\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3a\,b^2}{10x^{10}}-\frac{b^3}{8x^8}$	36
norman	$\frac{-\frac{1}{14}a^3-\frac{1}{4}a^2b\,x^2-\frac{3}{10}a\,b^2x^4-\frac{1}{8}b^3x^6}{x^{14}}$	37
risch	$\frac{-\frac{1}{14}a^3-\frac{1}{4}a^2b\,x^2-\frac{3}{10}a\,b^2x^4-\frac{1}{8}b^3x^6}{x^{14}}$	37
gosper	$-\frac{35b^3x^6+84a\,b^2x^4+70a^2b\,x^2+20a^3}{280x^{14}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^15,x,method=_RETURNVERBOSE)
[Out] -1/14*a^3/x^14-1/4*a^2*b/x^12-3/10*a*b^2/x^10-1/8*b^3/x^8
```

Maxima [A]
time = 0.26, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="maxima")
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Fricas [A]
time = 0.94, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="fricas")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Sympy [A]
time = 0.14, size = 39, normalized size = 0.91

$$\frac{-20a^3 - 70a^2bx^2 - 84ab^2x^4 - 35b^3x^6}{280x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**15,x)
[Out] (-20*a**3 - 70*a**2*b*x**2 - 84*a*b**2*x**4 - 35*b**3*x**6)/(280*x**14)
```

Giac [A]
time = 0.56, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^6 + 84\,ab^2x^4 + 70\,a^2bx^2 + 20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="giac")
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Mupad [B]
time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{14} + \frac{a^2\,b\,x^2}{4} + \frac{3\,a\,b^2\,x^4}{10} + \frac{b^3\,x^6}{8}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^15,x)
[Out] -(a^3/14 + (b^3*x^6)/8 + (a^2*b*x^2)/4 + (3*a*b^2*x^4)/10)/x^14
```

3.42 $\int x^6(a+bx^2)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

```
[Out] 1/7*a^3*x^7+1/3*a^2*b*x^9+3/11*a*b^2*x^11+1/13*b^3*x^13
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a+bx^2\right)^3\,dx&=\int\left(a^3x^6+3a^2bx^8+3ab^2x^{10}+b^3x^{12}\right)\,dx\\&=\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{7}a^3x^7+\frac{1}{3}a^2b\,x^9+\frac{3}{11}a\,b^2x^{11}+\frac{1}{13}b^3x^{13}$	36
default	$\frac{1}{7}a^3x^7+\frac{1}{3}a^2b\,x^9+\frac{3}{11}a\,b^2x^{11}+\frac{1}{13}b^3x^{13}$	36
norman	$\frac{1}{7}a^3x^7+\frac{1}{3}a^2b\,x^9+\frac{3}{11}a\,b^2x^{11}+\frac{1}{13}b^3x^{13}$	36
risch	$\frac{1}{7}a^3x^7+\frac{1}{3}a^2b\,x^9+\frac{3}{11}a\,b^2x^{11}+\frac{1}{13}b^3x^{13}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/7*a^3*x^7+1/3*a^2*b*x^9+3/11*a*b^2*x^11+1/13*b^3*x^13
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.81

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Fricas [A]
time = 0.74, size = 35, normalized size = 0.81

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Sympy [A]
time = 0.01, size = 37, normalized size = 0.86

$$\frac{a^3x^7}{7}+\frac{a^2bx^9}{3}+\frac{3ab^2x^{11}}{11}+\frac{b^3x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**7/7 + a**2*b*x**9/3 + 3*a*b**2*x**11/11 + b**3*x**13/13
```

Giac [A]
time = 0.56, size = 35, normalized size = 0.81

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^7}{7}+\frac{a^2\,b\,x^9}{3}+\frac{3\,a\,b^2\,x^{11}}{11}+\frac{b^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^7)/7 + (b^3*x^13)/13 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11
```

3.43 $\int x^4 (a+bx^2)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

```
[Out] 1/5*a^3*x^5+3/7*a^2*b*x^7+1/3*a*b^2*x^9+1/11*b^3*x^11
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^3\,dx &= \int \left(a^3x^4+3a^2bx^6+3ab^2x^8+b^3x^{10}\right)\,dx\\ &= \frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

Maple [A]
time = 0.03, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{5}a^3x^5+\frac{3}{7}a^2b\,x^7+\frac{1}{3}a\,b^2x^9+\frac{1}{11}b^3x^{11}$	36
default	$\frac{1}{5}a^3x^5+\frac{3}{7}a^2b\,x^7+\frac{1}{3}a\,b^2x^9+\frac{1}{11}b^3x^{11}$	36
norman	$\frac{1}{5}a^3x^5+\frac{3}{7}a^2b\,x^7+\frac{1}{3}a\,b^2x^9+\frac{1}{11}b^3x^{11}$	36
risch	$\frac{1}{5}a^3x^5+\frac{3}{7}a^2b\,x^7+\frac{1}{3}a\,b^2x^9+\frac{1}{11}b^3x^{11}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/5*a^3*x^5+3/7*a^2*b*x^7+1/3*a*b^2*x^9+1/11*b^3*x^11
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.81

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Fricas [A]
time = 0.83, size = 35, normalized size = 0.81

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Sympy [A]
time = 0.01, size = 37, normalized size = 0.86

$$\frac{a^3x^5}{5}+\frac{3a^2bx^7}{7}+\frac{ab^2x^9}{3}+\frac{b^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**5/5 + 3*a**2*b*x**7/7 + a*b**2*x**9/3 + b**3*x**11/11
```

Giac [A]
time = 0.52, size = 35, normalized size = 0.81

$$\frac{1}{11}\,b^3x^{11} + \frac{1}{3}\,ab^2x^9 + \frac{3}{7}\,a^2bx^7 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^5}{5} + \frac{3\,a^2\,b\,x^7}{7} + \frac{a\,b^2\,x^9}{3} + \frac{b^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^5)/5 + (b^3*x^11)/11 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3
```

3.44 $\int x^2(a+bx^2)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

```
[Out] 1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*b^2*x^7+1/9*b^3*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {276}

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^3,x]
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^3\,dx&=\int\left(a^3x^2+3a^2bx^4+3ab^2x^6+b^3x^8\right)\,dx\\&=\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^3,x]
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

Maple [A]
time = 0.03, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}a\,b^2x^7+\frac{1}{9}b^3x^9$	36
default	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}a\,b^2x^7+\frac{1}{9}b^3x^9$	36
norman	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}a\,b^2x^7+\frac{1}{9}b^3x^9$	36
risch	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}a\,b^2x^7+\frac{1}{9}b^3x^9$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*b^2*x^7+1/9*b^3*x^9
```

Maxima [A]
time = 0.31, size = 35, normalized size = 0.81

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Fricas [A]
time = 0.76, size = 35, normalized size = 0.81

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Sympy [A]
time = 0.01, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3}+\frac{3a^2bx^5}{5}+\frac{3ab^2x^7}{7}+\frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*a*b**2*x**7/7 + b**3*x**9/9
```

Giac [A]
time = 0.54, size = 35, normalized size = 0.81

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^3}{3}+\frac{3\,a^2\,b\,x^5}{5}+\frac{3\,a\,b^2\,x^7}{7}+\frac{b^3\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^9)/9 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7
```

3.45 $\int (a+bx^2)^3 \, dx$

Optimal. Leaf size=35

$$a^3x+a^2bx^3+\frac{3}{5}ab^2x^5+\frac{b^3x^7}{7}$$

```
[Out] a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {200}

$$a^3x+a^2bx^3+\frac{3}{5}ab^2x^5+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^3 \, dx &= \int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right) \, dx\\ &= a^3x+a^2bx^3+\frac{3}{5}ab^2x^5+\frac{b^3x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$a^3x+a^2bx^3+\frac{3}{5}ab^2x^5+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Maple [A]
time = 0.01, size = 32, normalized size = 0.91

method	result	size
gosper	$a^{3}x+a^{2}b\,x^{3}+\frac{3}{5}a\,b^{2}x^{5}+\frac{1}{7}b^{3}x^{7}$	32
default	$a^{3}x+a^{2}b\,x^{3}+\frac{3}{5}a\,b^{2}x^{5}+\frac{1}{7}b^{3}x^{7}$	32
norman	$a^{3}x+a^{2}b\,x^{3}+\frac{3}{5}a\,b^{2}x^{5}+\frac{1}{7}b^{3}x^{7}$	32
risch	$a^{3}x+a^{2}b\,x^{3}+\frac{3}{5}a\,b^{2}x^{5}+\frac{1}{7}b^{3}x^{7}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.89

$$\frac{1}{7}\,b^{3}x^{7}+\frac{3}{5}\,ab^{2}x^{5}+a^{2}bx^{3}+a^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Fricas [A]
time = 0.80, size = 31, normalized size = 0.89

$$\frac{1}{7}\,b^{3}x^{7}+\frac{3}{5}\,ab^{2}x^{5}+a^{2}bx^{3}+a^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Sympy [A]
time = 0.01, size = 32, normalized size = 0.91

$$a^{3}x+a^{2}bx^{3}+\frac{3ab^{2}x^{5}}{5}+\frac{b^{3}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*b*x**3 + 3*a*b**2*x**5/5 + b**3*x**7/7
```

Giac [A]
time = 0.47, size = 31, normalized size = 0.89

$$\frac{1}{7}\,b^3x^7+\frac{3}{5}\,ab^2x^5+a^2bx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.89

$$a^3\,x+a^2\,b\,x^3+\frac{3\,a\,b^2\,x^5}{5}+\frac{b^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3,x)
```

```
[Out] a^3*x + (b^3*x^7)/7 + a^2*b*x^3 + (3*a*b^2*x^5)/5
```

3.46 $\int \frac{(a+bx^2)^3}{x^2} \, dx$

Optimal. Leaf size=34

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

```
[Out] -a^3/x+3*a^2*b*x+a*b^2*x^3+1/5*b^3*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^2} \, dx &= \int \left(3a^2b+\frac{a^3}{x^2}+3ab^2x^2+b^3x^4\right) \, dx\\ &= -\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.00

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

Maple [A]
time = 0.01, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{a^3}{x}+3a^2bx+a\,b^2x^3+\frac{b^3x^5}{5}$	33
risch	$-\frac{a^3}{x}+3a^2bx+a\,b^2x^3+\frac{b^3x^5}{5}$	33
norman	$\frac{\frac{1}{5}b^3x^6+a\,b^2x^4+3a^2b\,x^2-a^3}{x}$	36
gosper	$-\frac{-b^3x^6-5a\,b^2x^4-15a^2b\,x^2+5a^3}{5x}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
[Out] -a^3/x+3*a^2*b*x+a*b^2*x^3+1/5*b^3*x^5
```

Maxima [A]
time = 0.27, size = 32, normalized size = 0.94

$$\frac{1}{5}\,b^3x^5+ab^2x^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="maxima")
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

Fricas [A]
time = 1.58, size = 36, normalized size = 1.06

$$\frac{b^3x^6+5\,ab^2x^4+15\,a^2bx^2-5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="fricas")
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

Sympy [A]
time = 0.02, size = 29, normalized size = 0.85

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*x + a*b**2*x**3 + b**3*x**5/5
```

Giac [A]
time = 0.66, size = 32, normalized size = 0.94

$$\frac{1}{5}\,b^3x^5 + ab^2x^3 + 3\,a^2bx - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

Mupad [B]
time = 0.04, size = 32, normalized size = 0.94

$$\frac{b^3\,x^5}{5} - \frac{a^3}{x} + a\,b^2\,x^3 + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^2,x)
```

```
[Out] (b^3*x^5)/5 - a^3/x + a*b^2*x^3 + 3*a^2*b*x
```

3.47 $\int \frac{(a+bx^2)^3}{x^4} dx$

Optimal. Leaf size=37

$$-\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

```
[Out] -1/3*a^3/x^3-3*a^2*b/x+3*a*b^2*x+1/3*b^3*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^4} dx &= \int \left(3ab^2 + \frac{a^3}{x^4} + \frac{3a^2b}{x^2} + b^3x^2\right) dx \\ &= -\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.00

$$-\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

Maple [A]
time = 0.01, size = 34, normalized size = 0.92

method	result	size
default	$-\frac{a^{3}}{3x^{3}}-\frac{3a^{2}b}{x}+3a\,b^{2}x+\frac{b^{3}x^{3}}{3}$	34
gosper	$-\frac{-b^{3}x^{6}-9a\,b^{2}x^{4}+9a^{2}b\,x^{2}+a^{3}}{3x^{3}}$	36
risch	$\frac{b^{3}x^{3}}{3}+3a\,b^{2}x+\frac{-3a^{2}b\,x^{2}-\frac{1}{3}a^{3}}{x^{3}}$	36
norman	$\frac{\frac{1}{3}b^{3}x^{6}+3a\,b^{2}x^{4}-3a^{2}b\,x^{2}-\frac{1}{3}a^{3}}{x^{3}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^3/x^3-3*a^2*b/x+3*a*b^2*x+1/3*b^3*x^3
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.92

$$\frac{1}{3}\,b^3x^3+3\,ab^2x-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Fricas [A]
time = 0.92, size = 36, normalized size = 0.97

$$\frac{b^3x^6+9\,ab^2x^4-9\,a^2bx^2-a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

Sympy [A]
time = 0.05, size = 36, normalized size = 0.97

$$3ab^2x+\frac{b^3x^3}{3}+\frac{-a^3-9a^2bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**4,x)
```

```
[Out] 3*a*b**2*x + b**3*x**3/3 + (-a**3 - 9*a**2*b*x**2)/(3*x**3)
```

Giac [A]
time = 0.55, size = 34, normalized size = 0.92

$$\frac{1}{3}\,b^3x^3 + 3\,ab^2x - \frac{9\,a^2bx^2 + a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Mupad [B]
time = 4.80, size = 36, normalized size = 0.97

$$\frac{b^3\,x^3}{3} - \frac{\frac{a^3}{3} + 3\,b\,a^2\,x^2}{x^3} + 3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^4,x)
```

```
[Out] (b^3*x^3)/3 - (a^3/3 + 3*a^2*b*x^2)/x^3 + 3*a*b^2*x
```

3.48 $\int \frac{(a+bx^2)^3}{x^6} \, dx$

Optimal. Leaf size=34

$$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x$$

```
[Out] -1/5*a^3/x^5-a^2*b/x^3-3*a*b^2/x+b^3*x
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^6,x]
```

```
[Out] -1/5*a^3/x^5 - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^3}{x^6} \, dx &= \int \left(b^3+\frac{a^3}{x^6}+\frac{3a^2b}{x^4}+\frac{3ab^2}{x^2}\right) \, dx \\ &= -\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x \end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.00

$$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^6,x]
```

```
[Out] -1/5*a^3/x^5 - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

Maple [A]
time = 0.01, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3a\,b^2}{x}+b^3x$	33
risch	$b^3x+\frac{-3a\,b^2x^4-a^2b\,x^2-\frac{1}{5}a^3}{x^5}$	35
gosper	$-\frac{-5b^3x^6+15a\,b^2x^4+5a^2b\,x^2+a^3}{5x^5}$	36
norman	$\frac{b^3x^6-3a\,b^2x^4-a^2b\,x^2-\frac{1}{5}a^3}{x^5}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*a^3/x^5-a^2*b/x^3-3*a*b^2/x+b^3*x
```

Maxima [A]
time = 0.29, size = 33, normalized size = 0.97

$$b^3x-\frac{15\,ab^2x^4+5\,a^2bx^2+a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="maxima")
[Out] b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

Fricas [A]
time = 0.79, size = 37, normalized size = 1.09

$$\frac{5\,b^3x^6-15\,ab^2x^4-5\,a^2bx^2-a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="fricas")
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

Sympy [A]
time = 0.07, size = 34, normalized size = 1.00

$$b^3x+\frac{-a^3-5a^2bx^2-15ab^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**6,x)
```

```
[Out] b**3*x + (-a**3 - 5*a**2*b*x**2 - 15*a*b**2*x**4)/(5*x**5)
```

Giac [A]
time = 0.63, size = 33, normalized size = 0.97

$$b^3x - \frac{15\,ab^2x^4 + 5\,a^2bx^2 + a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="giac")
```

```
[Out] b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

Mupad [B]
time = 0.03, size = 34, normalized size = 1.00

$$b^3\,x - \frac{\frac{a^3}{5} + a^2\,b\,x^2 + 3\,a\,b^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^6,x)
```

```
[Out] b^3*x - (a^3/5 + a^2*b*x^2 + 3*a*b^2*x^4)/x^5
```

3.49 $\int \frac{(a+bx^2)^3}{x^8} dx$

Optimal. Leaf size=39

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

```
[Out] -1/7*a^3/x^7-3/5*a^2*b/x^5-a*b^2/x^3-b^3/x
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^8}\,dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^4}+\frac{b^3}{x^2}\right) dx\\ &= -\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 39, normalized size = 1.00

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.92

method	result	size
default	$-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{a\,b^2}{x^3}-\frac{b^3}{x}$	36
norman	$\frac{-b^3x^6-a\,b^2x^4-\frac{3}{5}a^2b\,x^2-\frac{1}{7}a^3}{x^7}$	37
risch	$\frac{-b^3x^6-a\,b^2x^4-\frac{3}{5}a^2b\,x^2-\frac{1}{7}a^3}{x^7}$	37
gosper	$-\frac{35b^3x^6+35a\,b^2x^4+21a^2b\,x^2+5a^3}{35x^7}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^3/x^7-3/5*a^2*b/x^5-a*b^2/x^3-b^3/x
```

Maxima [A]
time = 0.30, size = 37, normalized size = 0.95

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="maxima")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Fricas [A]
time = 1.10, size = 37, normalized size = 0.95

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Sympy [A]
time = 0.09, size = 39, normalized size = 1.00

$$\frac{-5a^3-21a^2bx^2-35ab^2x^4-35b^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**8,x)
```

```
[Out] (-5*a**3 - 21*a**2*b*x**2 - 35*a*b**2*x**4 - 35*b**3*x**6)/(35*x**7)
```

Giac [A]
time = 0.64, size = 37, normalized size = 0.95

$$-\frac{35\,b^3x^6 + 35\,ab^2x^4 + 21\,a^2bx^2 + 5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.90

$$-\frac{\frac{a^3}{7} + \frac{3\,a^2\,b\,x^2}{5} + a\,b^2\,x^4 + b^3\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^8,x)
```

```
[Out] -(a^3/7 + b^3*x^6 + (3*a^2*b*x^2)/5 + a*b^2*x^4)/x^7
```

3.50 $\int \frac{(a+bx^2)^3}{x^{10}} \, dx$

Optimal. Leaf size=43

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

```
[Out] -1/9*a^3/x^9-3/7*a^2*b/x^7-3/5*a*b^2/x^5-1/3*b^3/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^10,x]
```

```
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^3}{x^{10}} \, dx &= \int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^8}+\frac{3ab^2}{x^6}+\frac{b^3}{x^4}\right) dx \\ &= -\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^10,x]
```

```
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

Maple [A]
time = 0.01, size = 36, normalized size = 0.84

method	result	size
default	$-\frac{a^{3}}{9x^{9}}-\frac{3a^{2}b}{7x^{7}}-\frac{3a\,b^{2}}{5x^{5}}-\frac{b^{3}}{3x^{3}}$	36
norman	$\frac{-\frac{1}{3}b^{3}x^{6}-\frac{3}{5}a\,b^{2}x^{4}-\frac{3}{7}a^{2}b\,x^{2}-\frac{1}{9}a^{3}}{x^{9}}$	37
risch	$\frac{-\frac{1}{3}b^{3}x^{6}-\frac{3}{5}a\,b^{2}x^{4}-\frac{3}{7}a^{2}b\,x^{2}-\frac{1}{9}a^{3}}{x^{9}}$	37
gosper	$-\frac{105b^{3}x^{6}+189a\,b^{2}x^{4}+135a^{2}b\,x^{2}+35a^{3}}{315x^{9}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^10,x,method=_RETURNVERBOSE)
[Out] -1/9*a^3/x^9-3/7*a^2*b/x^7-3/5*a*b^2/x^5-1/3*b^3/x^3
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.86

$$-\frac{105\,b^{3}x^{6}+189\,ab^{2}x^{4}+135\,a^{2}bx^{2}+35\,a^{3}}{315\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="maxima")
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Fricas [A]
time = 0.94, size = 37, normalized size = 0.86

$$-\frac{105\,b^{3}x^{6}+189\,ab^{2}x^{4}+135\,a^{2}bx^{2}+35\,a^{3}}{315\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="fricas")
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Sympy [A]
time = 0.10, size = 39, normalized size = 0.91

$$\frac{-35a^{3}-135a^{2}bx^{2}-189ab^{2}x^{4}-105b^{3}x^{6}}{315x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**10,x)
```

```
[Out] (-35*a**3 - 135*a**2*b*x**2 - 189*a*b**2*x**4 - 105*b**3*x**6)/(315*x**9)
```

Giac [A]
time = 0.55, size = 37, normalized size = 0.86

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Mupad [B]
time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{9}+\frac{3\,a^2\,b\,x^2}{7}+\frac{3\,a\,b^2\,x^4}{5}+\frac{b^3\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^10,x)
```

```
[Out] -(a^3/9 + (b^3*x^6)/3 + (3*a^2*b*x^2)/7 + (3*a*b^2*x^4)/5)/x^9
```

3.51 $\int \frac{(a+bx^2)^3}{x^{12}} \, dx$

Optimal. Leaf size=43

$$-\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

```
[Out] -1/11*a^3/x^11-1/3*a^2*b/x^9-3/7*a*b^2/x^7-1/5*b^3/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^12,x]
```

```
[Out] -1/11*a^3/x^11 - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^3}{x^{12}} \, dx &= \int \left(\frac{a^3}{x^{12}}+\frac{3a^2b}{x^{10}}+\frac{3ab^2}{x^8}+\frac{b^3}{x^6}\right) \, dx \\ &= -\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.00

$$-\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^12,x]
```

```
[Out] -1/11*a^3/x^11 - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.84

method	result	size
default	$-\frac{a^3}{11x^{11}} - \frac{a^2b}{3x^9} - \frac{3a\,b^2}{7x^7} - \frac{b^3}{5x^5}$	36
norman	$\frac{-\frac{1}{5}b^3x^6 - \frac{3}{7}a\,b^2x^4 - \frac{1}{3}a^2b\,x^2 - \frac{1}{11}a^3}{x^{11}}$	37
risch	$\frac{-\frac{1}{5}b^3x^6 - \frac{3}{7}a\,b^2x^4 - \frac{1}{3}a^2b\,x^2 - \frac{1}{11}a^3}{x^{11}}$	37
gosper	$-\frac{231b^3x^6+495a\,b^2x^4+385a^2b\,x^2+105a^3}{1155x^{11}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^12,x,method=_RETURNVERBOSE)
[Out] -1/11*a^3/x^11-1/3*a^2*b/x^9-3/7*a*b^2/x^7-1/5*b^3/x^5
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.86

$$-\frac{231\,b^3x^6 + 495\,ab^2x^4 + 385\,a^2bx^2 + 105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="maxima")
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Fricas [A]
time = 1.16, size = 37, normalized size = 0.86

$$-\frac{231\,b^3x^6 + 495\,ab^2x^4 + 385\,a^2bx^2 + 105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="fricas")
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Sympy [A]
time = 0.12, size = 39, normalized size = 0.91

$$\frac{-105a^3 - 385a^2bx^2 - 495ab^2x^4 - 231b^3x^6}{1155x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**12,x)
```

```
[Out] (-105*a**3 - 385*a**2*b*x**2 - 495*a*b**2*x**4 - 231*b**3*x**6)/(1155*x**11
)
```

Giac [A]
time = 0.59, size = 37, normalized size = 0.86

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="giac")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Mupad [B]
time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{11}+\frac{a^2\,b\,x^2}{3}+\frac{3\,a\,b^2\,x^4}{7}+\frac{b^3\,x^6}{5}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^12,x)
```

```
[Out] -(a^3/11 + (b^3*x^6)/5 + (a^2*b*x^2)/3 + (3*a*b^2*x^4)/7)/x^11
```

3.52 $\int x^{13}(a+bx^2)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

```
[Out] 1/14*a^5*x^14+5/16*a^4*b*x^16+5/9*a^3*b^2*x^18+1/2*a^2*b^3*x^20+5/22*a*b^4*
x^22+1/24*b^5*x^24
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 45}

$$\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2 +
 (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{13}\left(a+bx^2\right)^5\,dx &= \frac{1}{2}\text{Subst}\left(\int x^6(a+bx)^5\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right)\,dx,x,x^2\right)\\
&= \frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2 +
 (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{14}a^5x^{14}+\frac{5}{16}a^4b\,x^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{24}b^5x^{24}$	58
default	$\frac{1}{14}a^5x^{14}+\frac{5}{16}a^4b\,x^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{24}b^5x^{24}$	58
norman	$\frac{1}{14}a^5x^{14}+\frac{5}{16}a^4b\,x^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{24}b^5x^{24}$	58
risch	$\frac{1}{14}a^5x^{14}+\frac{5}{16}a^4b\,x^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{24}b^5x^{24}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*a^5*x^14+5/16*a^4*b*x^16+5/9*a^3*b^2*x^18+1/2*a^2*b^3*x^20+5/22*a*b^4*
x^22+1/24*b^5*x^24
```

Maxima [A]
time = 0.33, size = 57, normalized size = 0.83

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

Fricas [A]
time = 0.75, size = 57, normalized size = 0.83

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

Sympy [A]
time = 0.01, size = 65, normalized size = 0.94

$$\frac{a^5x^{14}}{14}+\frac{5a^4bx^{16}}{16}+\frac{5a^3b^2x^{18}}{9}+\frac{a^2b^3x^{20}}{2}+\frac{5ab^4x^{22}}{22}+\frac{b^5x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**14/14 + 5*a**4*b*x**16/16 + 5*a**3*b**2*x**18/9 + a**2*b**3*x**20/2
 + 5*a*b**4*x**22/22 + b**5*x**24/24
```

Giac [A]
time = 0.57, size = 57, normalized size = 0.83

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

Mupad [B]
time = 0.03, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{14}}{14}+\frac{5\,a^4\,b\,x^{16}}{16}+\frac{5\,a^3\,b^2\,x^{18}}{9}+\frac{a^2\,b^3\,x^{20}}{2}+\frac{5\,a\,b^4\,x^{22}}{22}+\frac{b^5\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^14)/14 + (b^5*x^24)/24 + (5*a^4*b*x^16)/16 + (5*a*b^4*x^22)/22 + (5*
a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2
```

3.53 $\int x^{11}(a+bx^2)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

```
[Out] 1/12*a^5*x^12+5/14*a^4*b*x^14+5/8*a^3*b^2*x^16+5/9*a^2*b^3*x^18+1/4*a*b^4*x
^20+1/22*b^5*x^22
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^2\right)^5\,dx&=\frac{1}{2}\text{Subst}\left(\int x^5(a+bx)^5\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)\,dx,x,x^2\right)\\&=\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{12}a^5x^{12}+\frac{5}{14}a^4b\,x^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}a\,b^4x^{20}+\frac{1}{22}b^5x^{22}$	58
default	$\frac{1}{12}a^5x^{12}+\frac{5}{14}a^4b\,x^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}a\,b^4x^{20}+\frac{1}{22}b^5x^{22}$	58
norman	$\frac{1}{12}a^5x^{12}+\frac{5}{14}a^4b\,x^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}a\,b^4x^{20}+\frac{1}{22}b^5x^{22}$	58
risch	$\frac{1}{12}a^5x^{12}+\frac{5}{14}a^4b\,x^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}a\,b^4x^{20}+\frac{1}{22}b^5x^{22}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a^5*x^12+5/14*a^4*b*x^14+5/8*a^3*b^2*x^16+5/9*a^2*b^3*x^18+1/4*a*b^4*x
^20+1/22*b^5*x^22
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.83

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

Fricas [A]
time = 0.85, size = 57, normalized size = 0.83

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

Sympy [A]
time = 0.01, size = 65, normalized size = 0.94

$$\frac{a^5x^{12}}{12}+\frac{5a^4bx^{14}}{14}+\frac{5a^3b^2x^{16}}{8}+\frac{5a^2b^3x^{18}}{9}+\frac{ab^4x^{20}}{4}+\frac{b^5x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**12/12 + 5*a**4*b*x**14/14 + 5*a**3*b**2*x**16/8 + 5*a**2*b**3*x**18
/9 + a*b**4*x**20/4 + b**5*x**22/22
```

Giac [A]
time = 0.55, size = 57, normalized size = 0.83

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{12}}{12}+\frac{5\,a^4\,b\,x^{14}}{14}+\frac{5\,a^3\,b^2\,x^{16}}{8}+\frac{5\,a^2\,b^3\,x^{18}}{9}+\frac{a\,b^4\,x^{20}}{4}+\frac{b^5\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^12)/12 + (b^5*x^22)/22 + (5*a^4*b*x^14)/14 + (a*b^4*x^20)/4 + (5*a^3
*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
```

3.54 $\int x^9(a+bx^2)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

```
[Out] 1/10*a^5*x^10+5/12*a^4*b*x^12+5/7*a^3*b^2*x^14+5/8*a^2*b^3*x^16+5/18*a*b^4*
x^18+1/20*b^5*x^20
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^5\,dx&=\frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^5\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)\,dx,x,x^2\right)\\&=\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{10}a^5x^{10}+\frac{5}{12}a^4b\,x^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{20}b^5x^{20}$	58
default	$\frac{1}{10}a^5x^{10}+\frac{5}{12}a^4b\,x^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{20}b^5x^{20}$	58
norman	$\frac{1}{10}a^5x^{10}+\frac{5}{12}a^4b\,x^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{20}b^5x^{20}$	58
risch	$\frac{1}{10}a^5x^{10}+\frac{5}{12}a^4b\,x^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{20}b^5x^{20}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*a^5*x^10+5/12*a^4*b*x^12+5/7*a^3*b^2*x^14+5/8*a^2*b^3*x^16+5/18*a*b^4*
x^18+1/20*b^5*x^20
```

Maxima [A]
time = 0.34, size = 57, normalized size = 0.83

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

Fricas [A]
time = 0.68, size = 57, normalized size = 0.83

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

Sympy [A]
time = 0.01, size = 66, normalized size = 0.96

$$\frac{a^5x^{10}}{10}+\frac{5a^4bx^{12}}{12}+\frac{5a^3b^2x^{14}}{7}+\frac{5a^2b^3x^{16}}{8}+\frac{5ab^4x^{18}}{18}+\frac{b^5x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**10/10 + 5*a**4*b*x**12/12 + 5*a**3*b**2*x**14/7 + 5*a**2*b**3*x**16
/8 + 5*a*b**4*x**18/18 + b**5*x**20/20
```

Giac [A]
time = 0.49, size = 57, normalized size = 0.83

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{10}}{10}+\frac{5\,a^4\,b\,x^{12}}{12}+\frac{5\,a^3\,b^2\,x^{14}}{7}+\frac{5\,a^2\,b^3\,x^{16}}{8}+\frac{5\,a\,b^4\,x^{18}}{18}+\frac{b^5\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^10)/10 + (b^5*x^20)/20 + (5*a^4*b*x^12)/12 + (5*a*b^4*x^18)/18 + (5*
a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
```

3.55 $\int x^7(a+bx^2)^5\,dx$

Optimal. Leaf size=72

$$-\frac{a^3(a+bx^2)^6}{12b^4}+\frac{3a^2(a+bx^2)^7}{14b^4}-\frac{3a(a+bx^2)^8}{16b^4}+\frac{(a+bx^2)^9}{18b^4}$$

```
[Out] -1/12*a^3*(b*x^2+a)^6/b^4+3/14*a^2*(b*x^2+a)^7/b^4-3/16*a*(b*x^2+a)^8/b^4+1
/18*(b*x^2+a)^9/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 45}

$$-\frac{a^3(a+bx^2)^6}{12b^4}+\frac{3a^2(a+bx^2)^7}{14b^4}+\frac{(a+bx^2)^9}{18b^4}-\frac{3a(a+bx^2)^8}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^5,x]
```

```
[Out] -1/12*(a^3*(a + b*x^2)^6)/b^4 + (3*a^2*(a + b*x^2)^7)/(14*b^4) - (3*a*(a +
b*x^2)^8)/(16*b^4) + (a + b*x^2)^9/(18*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^5\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^5\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right)\,dx,x,x^2\right)\\ &= -\frac{a^3(a+bx^2)^6}{12b^4}+\frac{3a^2(a+bx^2)^7}{14b^4}-\frac{3a(a+bx^2)^8}{16b^4}+\frac{(a+bx^2)^9}{18b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 0.96

$$\frac{a^5x^8}{8}+\frac{1}{2}a^4bx^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}ab^4x^{16}+\frac{b^5x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^8)/8 + (a^4*b*x^10)/2 + (5*a^3*b^2*x^12)/6 + (5*a^2*b^3*x^14)/7 + (5
*a*b^4*x^16)/16 + (b^5*x^18)/18
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.81

method	result	size
gosper	$\frac{1}{8}a^5x^8+\frac{1}{2}a^4b\,x^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{18}b^5x^{18}$	58
default	$\frac{1}{8}a^5x^8+\frac{1}{2}a^4b\,x^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{18}b^5x^{18}$	58
norman	$\frac{1}{8}a^5x^8+\frac{1}{2}a^4b\,x^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{18}b^5x^{18}$	58
risch	$\frac{1}{8}a^5x^8+\frac{1}{2}a^4b\,x^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{18}b^5x^{18}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a^5*x^8+1/2*a^4*b*x^10+5/6*a^3*b^2*x^12+5/7*a^2*b^3*x^14+5/16*a*b^4*x^1
6+1/18*b^5*x^18
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.79

$$\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

Fricas [A]
time = 0.99, size = 57, normalized size = 0.79

$$\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

Sympy [A]
time = 0.01, size = 65, normalized size = 0.90

$$\frac{a^5x^8}{8}+\frac{a^4bx^{10}}{2}+\frac{5a^3b^2x^{12}}{6}+\frac{5a^2b^3x^{14}}{7}+\frac{5ab^4x^{16}}{16}+\frac{b^5x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**8/8 + a**4*b*x**10/2 + 5*a**3*b**2*x**12/6 + 5*a**2*b**3*x**14/7 +
5*a*b**4*x**16/16 + b**5*x**18/18
```

Giac [A]
time = 0.49, size = 57, normalized size = 0.79

$$\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.79

$$\frac{a^5\,x^8}{8}+\frac{a^4\,b\,x^{10}}{2}+\frac{5\,a^3\,b^2\,x^{12}}{6}+\frac{5\,a^2\,b^3\,x^{14}}{7}+\frac{5\,a\,b^4\,x^{16}}{16}+\frac{b^5\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^8)/8 + (b^5*x^18)/18 + (a^4*b*x^10)/2 + (5*a*b^4*x^16)/16 + (5*a^3*b
^2*x^12)/6 + (5*a^2*b^3*x^14)/7
```

3.56 $\int x^5(a+bx^2)^5\,dx$

Optimal. Leaf size=53

$$\frac{a^2(a+bx^2)^6}{12b^3}-\frac{a(a+bx^2)^7}{7b^3}+\frac{(a+bx^2)^8}{16b^3}$$

```
[Out] 1/12*a^2*(b*x^2+a)^6/b^3-1/7*a*(b*x^2+a)^7/b^3+1/16*(b*x^2+a)^8/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\frac{a^2(a+bx^2)^6}{12b^3}+\frac{(a+bx^2)^8}{16b^3}-\frac{a(a+bx^2)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^5,x]
```

```
[Out] (a^2*(a + b*x^2)^6)/(12*b^3) - (a*(a + b*x^2)^7)/(7*b^3) + (a + b*x^2)^8/(1
6*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^5\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^5\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right)dx,x,x^2\right)\\ &= \frac{a^2(a+bx^2)^6}{12b^3}-\frac{a(a+bx^2)^7}{7b^3}+\frac{(a+bx^2)^8}{16b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.25

$$\frac{a^5x^6}{6}+\frac{5}{8}a^4bx^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^8)/8 + a^3*b^2*x^10 + (5*a^2*b^3*x^12)/6 + (5*a*b^
4*x^14)/14 + (b^5*x^16)/16
```

Maple [A]
time = 0.03, size = 57, normalized size = 1.08

method	result	size
gosper	$\frac{1}{6}a^5x^6+\frac{5}{8}a^4b\,x^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}a\,b^4x^{14}+\frac{1}{16}b^5x^{16}$	57
default	$\frac{1}{6}a^5x^6+\frac{5}{8}a^4b\,x^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}a\,b^4x^{14}+\frac{1}{16}b^5x^{16}$	57
norman	$\frac{1}{6}a^5x^6+\frac{5}{8}a^4b\,x^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}a\,b^4x^{14}+\frac{1}{16}b^5x^{16}$	57
risch	$\frac{1}{6}a^5x^6+\frac{5}{8}a^4b\,x^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}a\,b^4x^{14}+\frac{1}{16}b^5x^{16}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^5*x^6+5/8*a^4*b*x^8+a^3*b^2*x^10+5/6*a^2*b^3*x^12+5/14*a*b^4*x^14+1/1
6*b^5*x^16
```

Maxima [A]
time = 0.27, size = 56, normalized size = 1.06

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

Fricas [A]
time = 0.74, size = 56, normalized size = 1.06

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

Sympy [A]
time = 0.01, size = 63, normalized size = 1.19

$$\frac{a^5x^6}{6}+\frac{5a^4bx^8}{8}+a^3b^2x^{10}+\frac{5a^2b^3x^{12}}{6}+\frac{5ab^4x^{14}}{14}+\frac{b^5x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**8/8 + a**3*b**2*x**10 + 5*a**2*b**3*x**12/6 + 5*a
*b**4*x**14/14 + b**5*x**16/16
```

Giac [A]
time = 0.51, size = 56, normalized size = 1.06

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

Mupad [B]
time = 0.02, size = 56, normalized size = 1.06

$$\frac{a^5\,x^6}{6}+\frac{5\,a^4\,b\,x^8}{8}+a^3\,b^2\,x^{10}+\frac{5\,a^2\,b^3\,x^{12}}{6}+\frac{5\,a\,b^4\,x^{14}}{14}+\frac{b^5\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^6)/6 + (b^5*x^16)/16 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^14)/14 + a^3*b^2
*x^10 + (5*a^2*b^3*x^12)/6
```

3.57 $\int x^3(a+bx^2)^5\,dx$

Optimal. Leaf size=34

$$-\frac{a(a+bx^2)^6}{12b^2}+\frac{(a+bx^2)^7}{14b^2}$$

```
[Out] -1/12*a*(b*x^2+a)^6/b^2+1/14*(b*x^2+a)^7/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{(a+bx^2)^7}{14b^2}-\frac{a(a+bx^2)^6}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^5,x]
```

```
[Out] -1/12*(a*(a + b*x^2)^6)/b^2 + (a + b*x^2)^7/(14*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^5\,dx&=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^5\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right)\,dx,x,x^2\right)\\&=-\frac{a(a+bx^2)^6}{12b^2}+\frac{(a+bx^2)^7}{14b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.94

$$\frac{a^5x^4}{4}+\frac{5}{6}a^4bx^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}ab^4x^{12}+\frac{b^5x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^4)/4 + (5*a^4*b*x^6)/6 + (5*a^3*b^2*x^8)/4 + a^2*b^3*x^10 + (5*a*b^4
*x^12)/12 + (b^5*x^14)/14
```

Maple [A]
time = 0.03, size = 57, normalized size = 1.68

method	result	size
gosper	$\frac{1}{4}a^5x^4+\frac{5}{6}a^4b\,x^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}a\,b^4x^{12}+\frac{1}{14}b^5x^{14}$	57
default	$\frac{1}{4}a^5x^4+\frac{5}{6}a^4b\,x^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}a\,b^4x^{12}+\frac{1}{14}b^5x^{14}$	57
norman	$\frac{1}{4}a^5x^4+\frac{5}{6}a^4b\,x^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}a\,b^4x^{12}+\frac{1}{14}b^5x^{14}$	57
risch	$\frac{1}{4}a^5x^4+\frac{5}{6}a^4b\,x^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}a\,b^4x^{12}+\frac{1}{14}b^5x^{14}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^5*x^4+5/6*a^4*b*x^6+5/4*a^3*b^2*x^8+a^2*b^3*x^10+5/12*a*b^4*x^12+1/14
*b^5*x^14
```

Maxima [A]
time = 0.28, size = 56, normalized size = 1.65

$$\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

Fricas [A]
time = 0.94, size = 56, normalized size = 1.65

$$\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(27) = 54$.
time = 0.01, size = 63, normalized size = 1.85

$$\frac{a^5x^4}{4} + \frac{5a^4bx^6}{6} + \frac{5a^3b^2x^8}{4} + a^2b^3x^{10} + \frac{5ab^4x^{12}}{12} + \frac{b^5x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**4/4 + 5*a**4*b*x**6/6 + 5*a**3*b**2*x**8/4 + a**2*b**3*x**10 + 5*a*
b**4*x**12/12 + b**5*x**14/14
```

Giac [**A**]
time = 0.53, size = 56, normalized size = 1.65

$$\frac{1}{14}\,b^5x^{14} + \frac{5}{12}\,ab^4x^{12} + a^2b^3x^{10} + \frac{5}{4}\,a^3b^2x^8 + \frac{5}{6}\,a^4bx^6 + \frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

Mupad [**B**]
time = 0.02, size = 56, normalized size = 1.65

$$\frac{a^5\,x^4}{4} + \frac{5\,a^4\,b\,x^6}{6} + \frac{5\,a^3\,b^2\,x^8}{4} + a^2\,b^3\,x^{10} + \frac{5\,a\,b^4\,x^{12}}{12} + \frac{b^5\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^4)/4 + (b^5*x^14)/14 + (5*a^4*b*x^6)/6 + (5*a*b^4*x^12)/12 + (5*a^3*
b^2*x^8)/4 + a^2*b^3*x^10
```

3.58 $\int x(a+bx^2)^5\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^2)^6}{12b}$$

```
[Out] 1/12*(b*x^2+a)^6/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{267\}$

$$\frac{(a+bx^2)^6}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^5,x]
```

```
[Out] (a + b*x^2)^6/(12*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x(a+bx^2)^5\,dx=\frac{(a+bx^2)^6}{12b}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^6}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^5,x]
```

```
[Out] (a + b*x^2)^6/(12*b)
```

Maple [A]
time = 0.02, size = 15, normalized size = 0.94

method	result	size
default	$\frac{(b\,x^2+a)^6}{12b}$	15
gosper	$\frac{1}{12}b^5x^{12}+\frac{1}{2}a\,b^4x^{10}+\frac{5}{4}a^2b^3x^8+\frac{5}{3}a^3b^2x^6+\frac{5}{4}a^4b\,x^4+\frac{1}{2}a^5x^2$	58
norman	$\frac{1}{12}b^5x^{12}+\frac{1}{2}a\,b^4x^{10}+\frac{5}{4}a^2b^3x^8+\frac{5}{3}a^3b^2x^6+\frac{5}{4}a^4b\,x^4+\frac{1}{2}a^5x^2$	58
risch	$\frac{b^5x^{12}}{12}+\frac{a\,b^4x^{10}}{2}+\frac{5a^2b^3x^8}{4}+\frac{5a^3b^2x^6}{3}+\frac{5a^4b\,x^4}{4}+\frac{a^5x^2}{2}+\frac{a^6}{12b}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(b*x^2+a)^6/b
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.88

$$\frac{(bx^2+a)^6}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^6/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(14) = 28$.
time = 0.88, size = 57, normalized size = 3.56

$$\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a^
4*b*x^4 + 1/2*a^5*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(10) = 20$.
time = 0.01, size = 65, normalized size = 4.06

$$\frac{a^5x^2}{2}+\frac{5a^4bx^4}{4}+\frac{5a^3b^2x^6}{3}+\frac{5a^2b^3x^8}{4}+\frac{ab^4x^{10}}{2}+\frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**2/2 + 5*a**4*b*x**4/4 + 5*a**3*b**2*x**6/3 + 5*a**2*b**3*x**8/4 + a
*b**4*x**10/2 + b**5*x**12/12
```

Giac [A]
time = 0.49, size = 14, normalized size = 0.88

$$\frac{(bx^2 + a)^6}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*(b*x^2 + a)^6/b
```

Mupad [B]
time = 0.02, size = 57, normalized size = 3.56

$$\frac{a^5\,x^2}{2} + \frac{5\,a^4\,b\,x^4}{4} + \frac{5\,a^3\,b^2\,x^6}{3} + \frac{5\,a^2\,b^3\,x^8}{4} + \frac{a\,b^4\,x^{10}}{2} + \frac{b^5\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^2)/2 + (b^5*x^12)/12 + (5*a^4*b*x^4)/4 + (a*b^4*x^10)/2 + (5*a^3*b^2
*x^6)/3 + (5*a^2*b^3*x^8)/4
```

3.59 $\int \frac{(a+bx^2)^5}{x}\,dx$

Optimal. Leaf size=65

$$\frac{5}{2}a^4bx^2+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^2b^3x^6+\frac{5}{8}ab^4x^8+\frac{b^5x^{10}}{10}+a^5\log(x)$$

```
[Out] 5/2*a^4*b*x^2+5/2*a^3*b^2*x^4+5/3*a^2*b^3*x^6+5/8*a*b^4*x^8+1/10*b^5*x^10+a
^5*ln(x)
```

Rubi [A]

time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 45}

$$a^5\log(x)+\frac{5}{2}a^4bx^2+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^2b^3x^6+\frac{5}{8}ab^4x^8+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x,x]
```

```
[Out] (5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 +
 (b^5*x^10)/10 + a^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)\,dx,x,x^2\right)\\&=\frac{5}{2}a^4bx^2+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^2b^3x^6+\frac{5}{8}ab^4x^8+\frac{b^5x^{10}}{10}+a^5\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 1.00

$$\frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10} + a^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x,x]
```

```
[Out] (5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 +
 (b^5*x^10)/10 + a^5*Log[x]
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.86

method	result	size
default	$\frac{5a^4b\,x^2}{2}+\frac{5a^3b^2x^4}{2}+\frac{5a^2b^3x^6}{3}+\frac{5a\,b^4x^8}{8}+\frac{b^5x^{10}}{10}+a^5\ln\left(x\right)$	56
norman	$\frac{5a^4b\,x^2}{2}+\frac{5a^3b^2x^4}{2}+\frac{5a^2b^3x^6}{3}+\frac{5a\,b^4x^8}{8}+\frac{b^5x^{10}}{10}+a^5\ln\left(x\right)$	56
risch	$\frac{5a^4b\,x^2}{2}+\frac{5a^3b^2x^4}{2}+\frac{5a^2b^3x^6}{3}+\frac{5a\,b^4x^8}{8}+\frac{b^5x^{10}}{10}+a^5\ln\left(x\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*a^4*b*x^2+5/2*a^3*b^2*x^4+5/3*a^2*b^3*x^6+5/8*a*b^4*x^8+1/10*b^5*x^10+a
^5*ln(x)
```

Maxima [A]
time = 0.29, size = 58, normalized size = 0.89

$$\frac{1}{10}\,b^5x^{10} + \frac{5}{8}\,ab^4x^8 + \frac{5}{3}\,a^2b^3x^6 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{2}\,a^4bx^2 + \frac{1}{2}\,a^5\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + 1/2*a^5*log(x^2)
```

Fricas [A]
time = 1.20, size = 55, normalized size = 0.85

$$\frac{1}{10}\,b^5x^{10} + \frac{5}{8}\,ab^4x^8 + \frac{5}{3}\,a^2b^3x^6 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{2}\,a^4bx^2 + a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

Sympy [A]
time = 0.04, size = 65, normalized size = 1.00

$$a^{5}\log\left(x\right)+\frac{5a^{4}bx^{2}}{2}+\frac{5a^{3}b^{2}x^{4}}{2}+\frac{5a^{2}b^{3}x^{6}}{3}+\frac{5ab^{4}x^{8}}{8}+\frac{b^{5}x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**3*b**2*x**4/2 + 5*a**2*b**3*x**6/3 + 5
*a*b**4*x**8/8 + b**5*x**10/10
```

Giac [A]
time = 0.46, size = 58, normalized size = 0.89

$$\frac{1}{10}\,b^{5}x^{10}+\frac{5}{8}\,ab^{4}x^{8}+\frac{5}{3}\,a^{2}b^{3}x^{6}+\frac{5}{2}\,a^{3}b^{2}x^{4}+\frac{5}{2}\,a^{4}bx^{2}+\frac{1}{2}\,a^{5}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + 1/2*a^5*log(x^2)
```

Mupad [B]
time = 0.03, size = 55, normalized size = 0.85

$$a^{5}\,\ln\left(x\right)+\frac{b^{5}\,x^{10}}{10}+\frac{5\,a^{4}\,b\,x^{2}}{2}+\frac{5\,a\,b^{4}\,x^{8}}{8}+\frac{5\,a^{3}\,b^{2}\,x^{4}}{2}+\frac{5\,a^{2}\,b^{3}\,x^{6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x,x)
```

```
[Out] a^5*log(x) + (b^5*x^10)/10 + (5*a^4*b*x^2)/2 + (5*a*b^4*x^8)/8 + (5*a^3*b^2
*x^4)/2 + (5*a^2*b^3*x^6)/3
```

3.60 $\int \frac{(a+bx^2)^5}{x^3}\,dx$

Optimal. Leaf size=64

$$-\frac{a^5}{2x^2}+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}+5a^4b\log(x)$$

```
[Out] -1/2*a^5/x^2+5*a^3*b^2*x^2+5/2*a^2*b^3*x^4+5/6*a*b^4*x^6+1/8*b^5*x^8+5*a^4*
b*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^5}{2x^2}+5a^4b\log(x)+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x
^8)/8 + 5*a^4*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right)dx,x,x^2\right)\\
&= -\frac{a^5}{2x^2}+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}+5a^4b\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{2x^2}+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}+5a^4b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x
^8)/8 + 5*a^4*b*Log[x]
```

Maple [A]
time = 0.02, size = 57, normalized size = 0.89

method	result	size
default	$-\frac{a^5}{2x^2}+5a^3b^2x^2+\frac{5a^2b^3x^4}{2}+\frac{5a\,b^4x^6}{6}+\frac{b^5x^8}{8}+5a^4b\ln(x)$	57
risch	$-\frac{a^5}{2x^2}+5a^3b^2x^2+\frac{5a^2b^3x^4}{2}+\frac{5a\,b^4x^6}{6}+\frac{b^5x^8}{8}+5a^4b\ln(x)$	57
norman	$\frac{-\frac{1}{2}a^5+\frac{1}{8}b^5x^{10}+\frac{5}{6}a\,b^4x^8+\frac{5}{2}a^2b^3x^6+5a^3b^2x^4}{x^2}+5a^4b\ln(x)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^5/x^2+5*a^3*b^2*x^2+5/2*a^2*b^3*x^4+5/6*a*b^4*x^6+1/8*b^5*x^8+5*a^4*
b*ln(x)
```

Maxima [A]
time = 0.29, size = 58, normalized size = 0.91

$$\frac{1}{8}\,b^5x^8+\frac{5}{6}\,ab^4x^6+\frac{5}{2}\,a^2b^3x^4+5\,a^3b^2x^2+\frac{5}{2}\,a^4b\log\left(x^2\right)-\frac{a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*l
og(x^2) - 1/2*a^5/x^2
```

Fricas [A]
time = 1.53, size = 61, normalized size = 0.95

$$\frac{3\,b^5x^{10}+20\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4+120\,a^4bx^2\log\left(x\right)-12\,a^5}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a^
4*b*x^2*log(x) - 12*a^5)/x^2
```

Sympy [A]
time = 0.05, size = 63, normalized size = 0.98

$$-\frac{a^5}{2x^2} + 5a^4b\log{(x)} + 5a^3b^2x^2 + \frac{5a^2b^3x^4}{2} + \frac{5ab^4x^6}{6} + \frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**3,x)
```

```
[Out] -a**5/(2*x**2) + 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 +
5*a*b**4*x**6/6 + b**5*x**8/8
```

Giac [A]
time = 0.45, size = 68, normalized size = 1.06

$$\frac{1}{8}\,b^5x^8 + \frac{5}{6}\,ab^4x^6 + \frac{5}{2}\,a^2b^3x^4 + 5\,a^3b^2x^2 + \frac{5}{2}\,a^4b\log\left(x^2\right) - \frac{5\,a^4bx^2 + a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*l
og(x^2) - 1/2*(5*a^4*b*x^2 + a^5)/x^2
```

Mupad [B]
time = 0.03, size = 56, normalized size = 0.88

$$\frac{b^5\,x^8}{8} - \frac{a^5}{2\,x^2} + \frac{5\,a\,b^4\,x^6}{6} + 5\,a^4\,b\,\ln\left(x\right) + 5\,a^3\,b^2\,x^2 + \frac{5\,a^2\,b^3\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^3,x)
```

```
[Out] (b^5*x^8)/8 - a^5/(2*x^2) + (5*a*b^4*x^6)/6 + 5*a^4*b*log(x) + 5*a^3*b^2*x^
2 + (5*a^2*b^3*x^4)/2
```

3.61 $\int \frac{(a+bx^2)^5}{x^5} dx$

Optimal. Leaf size=64

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}+10a^3b^2\log(x)$$

```
[Out] -1/4*a^5/x^4-5/2*a^4*b/x^2+5*a^2*b^3*x^2+5/4*a*b^4*x^4+1/6*b^5*x^6+10*a^3*b
^2*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+10a^3b^2\log(x)+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x
^6)/6 + 10*a^3*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right)dx, x, x^2\right)\\
&= -\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}+10a^3b^2\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}+10a^3b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x
^6)/6 + 10*a^3*b^2*Log[x]
```

Maple [A]
time = 0.02, size = 57, normalized size = 0.89

method	result	size
default	$-\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+5a^2b^3x^2+\frac{5a\,b^4x^4}{4}+\frac{b^5x^6}{6}+10a^3b^2\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{4}a^5+\frac{1}{6}b^5x^{10}+\frac{5}{4}a\,b^4x^8+5a^2b^3x^6-\frac{5}{2}a^4b\,x^2}{x^4}+10a^3b^2\ln\left(x\right)$	59
risch	$\frac{b^5x^6}{6}+\frac{5a\,b^4x^4}{4}+5a^2b^3x^2+\frac{-\frac{5}{2}a^4b\,x^2-\frac{1}{4}a^5}{x^4}+10a^3b^2\ln\left(x\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^5/x^4-5/2*a^4*b/x^2+5*a^2*b^3*x^2+5/4*a*b^4*x^4+1/6*b^5*x^6+10*a^3*b
^2*ln(x)
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.92

$$\frac{1}{6}\,b^5x^6+\frac{5}{4}\,ab^4x^4+5\,a^2b^3x^2+5\,a^3b^2\log\left(x^2\right)-\frac{10\,a^4bx^2+a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(10*
a^4*b*x^2 + a^5)/x^4
```

Fricas [A]
time = 1.65, size = 61, normalized size = 0.95

$$\frac{2\,b^5x^{10}+15\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4\log\left(x\right)-30\,a^4bx^2-3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x) -
 30*a^4*b*x^2 - 3*a^5)/x^4
```

Sympy [A]
time = 0.08, size = 63, normalized size = 0.98

$$10a^3b^2\log{(x)} + 5a^2b^3x^2 + \frac{5ab^4x^4}{4} + \frac{b^5x^6}{6} + \frac{-a^5 - 10a^4bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**5,x)
```

```
[Out] 10*a**3*b**2*log(x) + 5*a**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6 + (-
a**5 - 10*a**4*b*x**2)/(4*x**4)
```

Giac [A]
time = 0.46, size = 70, normalized size = 1.09

$$\frac{1}{6}\,b^5x^6 + \frac{5}{4}\,ab^4x^4 + 5\,a^2b^3x^2 + 5\,a^3b^2\log\left(x^2\right) - \frac{30\,a^3b^2x^4 + 10\,a^4bx^2 + a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(30*
a^3*b^2*x^4 + 10*a^4*b*x^2 + a^5)/x^4
```

Mupad [B]
time = 0.03, size = 59, normalized size = 0.92

$$\frac{b^5\,x^6}{6} - \frac{\frac{a^5}{4} + \frac{5\,b\,a^4\,x^2}{2}}{x^4} + \frac{5\,a\,b^4\,x^4}{4} + 5\,a^2\,b^3\,x^2 + 10\,a^3\,b^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^5,x)
```

```
[Out] (b^5*x^6)/6 - (a^5/4 + (5*a^4*b*x^2)/2)/x^4 + (5*a*b^4*x^4)/4 + 5*a^2*b^3*x
^2 + 10*a^3*b^2*log(x)
```

3.62 $\int \frac{(a+bx^2)^5}{x^7} dx$

Optimal. Leaf size=64

$$-\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}+10a^2b^3\log(x)$$

```
[Out] -1/6*a^5/x^6-5/4*a^4*b/x^4-5*a^3*b^2/x^2+5/2*a*b^4*x^2+1/4*b^5*x^4+10*a^2*b
^3*ln(x)
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$-\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^7,x]
```

```
[Out] -1/6*a^5/x^6 - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5
*x^4)/4 + 10*a^2*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^4}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)dx,x,x^2\right)\\
&= -\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}+10a^2b^3\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}+10a^2b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^7,x]
```

```
[Out] -1/6*a^5/x^6 - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5
*x^4)/4 + 10*a^2*b^3*Log[x]
```

Maple [A]
time = 0.03, size = 57, normalized size = 0.89

method	result	size
default	$-\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5a\,b^4x^2}{2}+\frac{b^5x^4}{4}+10a^2b^3\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{6}a^5+\frac{1}{4}b^5x^{10}+\frac{5}{2}a\,b^4x^8-5a^3b^2x^4-\frac{5}{4}a^4b\,x^2}{x^6}+10a^2b^3\ln\left(x\right)$	59
risch	$\frac{b^5x^4}{4}+\frac{5a\,b^4x^2}{2}+\frac{25a^2b^3}{4}+\frac{-5a^3b^2x^4-\frac{5}{4}a^4b\,x^2-\frac{1}{6}a^5}{x^6}+10a^2b^3\ln\left(x\right)$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^5/x^6-5/4*a^4*b/x^4-5*a^3*b^2/x^2+5/2*a*b^4*x^2+1/4*b^5*x^4+10*a^2*b
^3*ln(x)
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.95

$$\frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+5\,a^2b^3\log\left(x^2\right)-\frac{60\,a^3b^2x^4+15\,a^4bx^2+2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(60*a^3*b^2*x^4 + 1
5*a^4*b*x^2 + 2*a^5)/x^6
```

Fricas [A]
time = 1.03, size = 61, normalized size = 0.95

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log\left(x\right)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4 -
 15*a^4*b*x^2 - 2*a^5)/x^6
```

Sympy [A]
time = 0.11, size = 65, normalized size = 1.02

$$10a^2b^3\log\left(x\right)+\frac{5ab^4x^2}{2}+\frac{b^5x^4}{4}+\frac{-2a^5-15a^4bx^2-60a^3b^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**7,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4 + (-2*a**5 - 15*a**4*b*
x**2 - 60*a**3*b**2*x**4)/(12*x**6)
```

Giac [A]
time = 0.47, size = 72, normalized size = 1.12

$$\frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+5\,a^2b^3\log\left(x^2\right)-\frac{110\,a^2b^3x^6+60\,a^3b^2x^4+15\,a^4bx^2+2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(110*a^2*b^3*x^6 +
60*a^3*b^2*x^4 + 15*a^4*b*x^2 + 2*a^5)/x^6
```

Mupad [B]
time = 0.04, size = 59, normalized size = 0.92

$$\frac{b^5\,x^4}{4}-\frac{\frac{a^5}{6}+\frac{5\,a^4\,b\,x^2}{4}+5\,a^3\,b^2\,x^4}{x^6}+\frac{5\,a\,b^4\,x^2}{2}+10\,a^2\,b^3\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^7,x)
```

```
[Out] (b^5*x^4)/4 - (a^5/6 + (5*a^4*b*x^2)/4 + 5*a^3*b^2*x^4)/x^6 + (5*a*b^4*x^2)
/2 + 10*a^2*b^3*log(x)
```

3.63 $\int \frac{(a+bx^2)^5}{x^9}\,dx$

Optimal. Leaf size=64

$$-\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+\frac{b^5x^2}{2}+5ab^4\log(x)$$

```
[Out] -1/8*a^5/x^8-5/6*a^4*b/x^6-5/2*a^3*b^2/x^4-5*a^2*b^3/x^2+1/2*b^5*x^2+5*a*b^
4*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^9,x]
```

```
[Out] -1/8*a^5/x^8 - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
(b^5*x^2)/2 + 5*a*b^4*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^5}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx,x,x^2\right)\\
&= -\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+\frac{b^5x^2}{2}+5ab^4\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+\frac{b^5x^2}{2}+5ab^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^9,x]
```

```
[Out] -1/8*a^5/x^8 - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
(b^5*x^2)/2 + 5*a*b^4*Log[x]
```

Maple [A]
time = 0.02, size = 57, normalized size = 0.89

method	result	size
default	$-\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+\frac{b^5x^2}{2}+5a\,b^4\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{8}a^5+\frac{1}{2}b^5x^{10}-5a^2b^3x^6-\frac{5}{2}a^3b^2x^4-\frac{5}{6}a^4b\,x^2}{x^8}+5a\,b^4\ln\left(x\right)$	59
risch	$\frac{b^5x^2}{2}+\frac{-5a^2b^3x^6-\frac{5}{2}a^3b^2x^4-\frac{5}{6}a^4b\,x^2-\frac{1}{8}a^5}{x^8}+5a\,b^4\ln\left(x\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a^5/x^8-5/6*a^4*b/x^6-5/2*a^3*b^2/x^4-5*a^2*b^3/x^2+1/2*b^5*x^2+5*a*b^
4*ln(x)
```

Maxima [A]
time = 0.27, size = 61, normalized size = 0.95

$$\frac{1}{2}\,b^5x^2+\frac{5}{2}\,ab^4\log\left(x^2\right)-\frac{120\,a^2b^3x^6+60\,a^3b^2x^4+20\,a^4bx^2+3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(120*a^2*b^3*x^6 + 60*a^3*b^2*x^4 +
 20*a^4*b*x^2 + 3*a^5)/x^8
```

Fricas [A]
time = 0.75, size = 61, normalized size = 0.95

$$\frac{12\,b^5x^{10}+120\,ab^4x^8\log\left(x\right)-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^4
 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

Sympy [A]
time = 0.15, size = 63, normalized size = 0.98

$$5ab^4\log(x) + \frac{b^5x^2}{2} + \frac{-3a^5 - 20a^4bx^2 - 60a^3b^2x^4 - 120a^2b^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**9,x)
```

```
[Out] 5*a*b**4*log(x) + b**5*x**2/2 + (-3*a**5 - 20*a**4*b*x**2 - 60*a**3*b**2*x*
*4 - 120*a**2*b**3*x**6)/(24*x**8)
```

Giac [A]
time = 0.54, size = 70, normalized size = 1.09

$$\frac{1}{2}b^5x^2 + \frac{5}{2}ab^4\log\left(x^2\right) - \frac{125\,ab^4x^8 + 120\,a^2b^3x^6 + 60\,a^3b^2x^4 + 20\,a^4bx^2 + 3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(125*a*b^4*x^8 + 120*a^2*b^3*x^6 +
60*a^3*b^2*x^4 + 20*a^4*b*x^2 + 3*a^5)/x^8
```

Mupad [B]
time = 0.04, size = 59, normalized size = 0.92

$$\frac{b^5\,x^2}{2} - \frac{\frac{a^5}{8} + \frac{5\,a^4\,b\,x^2}{6} + \frac{5\,a^3\,b^2\,x^4}{2} + 5\,a^2\,b^3\,x^6}{x^8} + 5\,a\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^9,x)
```

```
[Out] (b^5*x^2)/2 - (a^5/8 + (5*a^4*b*x^2)/6 + (5*a^3*b^2*x^4)/2 + 5*a^2*b^3*x^6)
/x^8 + 5*a*b^4*log(x)
```

3.64 $\int \frac{(a+bx^2)^5}{x^{11}} dx$

Optimal. Leaf size=65

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

```
[Out] -1/10*a^5/x^10-5/8*a^4*b/x^8-5/3*a^3*b^2/x^6-5/2*a^2*b^3/x^4-5/2*a*b^4/x^2+
b^5*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x
^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^{11}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx,x,x^2\right)\\
&= -\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x
^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.86

method	result	size
default	$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5a\,b^4}{2x^2}+b^5\ln\left(x\right)$	56
norman	$\frac{-\frac{1}{10}a^5-\frac{5}{2}a\,b^4x^8-\frac{5}{2}a^2b^3x^6-\frac{5}{3}a^3b^2x^4-\frac{5}{8}a^4b\,x^2}{x^{10}}+b^5\ln\left(x\right)$	58
risch	$\frac{-\frac{1}{10}a^5-\frac{5}{2}a\,b^4x^8-\frac{5}{2}a^2b^3x^6-\frac{5}{3}a^3b^2x^4-\frac{5}{8}a^4b\,x^2}{x^{10}}+b^5\ln\left(x\right)$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^5/x^10-5/8*a^4*b/x^8-5/3*a^3*b^2/x^6-5/2*a^2*b^3/x^4-5/2*a*b^4/x^2+
b^5*ln(x)
```

Maxima [A]
time = 0.31, size = 61, normalized size = 0.94

$$\frac{1}{2}\,b^5\log\left(x^2\right)-\frac{300\,ab^4x^8+300\,a^2b^3x^6+200\,a^3b^2x^4+75\,a^4bx^2+12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*log(x^2) - 1/120*(300*a*b^4*x^8 + 300*a^2*b^3*x^6 + 200*a^3*b^2*x^4
 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

Fricas [A]
time = 1.01, size = 61, normalized size = 0.94

$$\frac{120\,b^5x^{10}\log\left(x\right)-300\,ab^4x^8-300\,a^2b^3x^6-200\,a^3b^2x^4-75\,a^4bx^2-12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2*
x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

Sympy [A]
time = 0.18, size = 61, normalized size = 0.94

$$b^5\log(x)+\frac{-12a^5-75a^4bx^2-200a^3b^2x^4-300a^2b^3x^6-300ab^4x^8}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**11,x)
```

```
[Out] b**5*log(x) + (-12*a**5 - 75*a**4*b*x**2 - 200*a**3*b**2*x**4 - 300*a**2*b*
*3*x**6 - 300*a*b**4*x**8)/(120*x**10)
```

Giac [A]
time = 0.48, size = 69, normalized size = 1.06

$$\frac{1}{2}\,b^5\log\left(x^2\right)-\frac{137\,b^5x^{10}+300\,ab^4x^8+300\,a^2b^3x^6+200\,a^3b^2x^4+75\,a^4bx^2+12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(x^2) - 1/120*(137*b^5*x^10 + 300*a*b^4*x^8 + 300*a^2*b^3*x^6 +
200*a^3*b^2*x^4 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

Mupad [B]
time = 4.77, size = 58, normalized size = 0.89

$$b^5\,\ln\left(x\right)-\frac{\frac{a^5}{10}+\frac{5\,a^4\,b\,x^2}{8}+\frac{5\,a^3\,b^2\,x^4}{3}+\frac{5\,a^2\,b^3\,x^6}{2}+\frac{5\,a\,b^4\,x^8}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^11,x)
```

```
[Out] b^5*log(x) - (a^5/10 + (5*a^4*b*x^2)/8 + (5*a*b^4*x^8)/2 + (5*a^3*b^2*x^4)/
3 + (5*a^2*b^3*x^6)/2)/x^10
```

3.65 $\int \frac{(a+bx^2)^5}{x^{13}} \, dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

```
[Out] -1/12*(b*x^2+a)^6/a/x^12
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^13,x]
```

```
[Out] -1/12*(a + b*x^2)^6/(a*x^12)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^5}{x^{13}} \, dx = -\frac{(a+bx^2)^6}{12ax^{12}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(19) = 38.
time = 0.00, size = 69, normalized size = 3.63

$$-\frac{a^5}{12x^{12}}-\frac{a^4b}{2x^{10}}-\frac{5a^3b^2}{4x^8}-\frac{5a^2b^3}{3x^6}-\frac{5ab^4}{4x^4}-\frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (a^4*b)/(2*x^10) - (5*a^3*b^2)/(4*x^8) - (5*a^2*b^3)/(3*x^
6) - (5*a*b^4)/(4*x^4) - b^5/(2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.
time = 0.03, size = 58, normalized size = 3.05

method	result	size
gosper	$-\frac{6b^5x^{10}+15a\,b^4x^8+20a^2b^3x^6+15a^3b^2x^4+6a^4b\,x^2+a^5}{12x^{12}}$	58
default	$-\frac{a^4b}{2x^{10}}-\frac{5a\,b^4}{4x^4}-\frac{5a^2b^3}{3x^6}-\frac{b^5}{2x^2}-\frac{5a^3b^2}{4x^8}-\frac{a^5}{12x^{12}}$	58
norman	$\frac{-\frac{1}{2}b^5x^{10}-\frac{5}{4}a\,b^4x^8-\frac{5}{3}a^2b^3x^6-\frac{5}{4}a^3b^2x^4-\frac{1}{2}a^4b\,x^2-\frac{1}{12}a^5}{x^{12}}$	59
risch	$\frac{-\frac{1}{2}b^5x^{10}-\frac{5}{4}a\,b^4x^8-\frac{5}{3}a^2b^3x^6-\frac{5}{4}a^3b^2x^4-\frac{1}{2}a^4b\,x^2-\frac{1}{12}a^5}{x^{12}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^4*b/x^10-5/4*a*b^4/x^4-5/3*a^2*b^3/x^6-1/2*b^5/x^2-5/4*a^3*b^2/x^8-1
/12*a^5/x^12
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.
time = 0.28, size = 57, normalized size = 3.00

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.
time = 2.06, size = 57, normalized size = 3.00

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.

time = 0.20, size = 61, normalized size = 3.21

$$\frac{-a^5 - 6a^4bx^2 - 15a^3b^2x^4 - 20a^2b^3x^6 - 15ab^4x^8 - 6b^5x^{10}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**13,x)
```

```
[Out] (-a**5 - 6*a**4*b*x**2 - 15*a**3*b**2*x**4 - 20*a**2*b**3*x**6 - 15*a*b**4*
x**8 - 6*b**5*x**10)/(12*x**12)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(17) = 34.
time = 0.51, size = 57, normalized size = 3.00

$$-\frac{6\,b^5x^{10} + 15\,ab^4x^8 + 20\,a^2b^3x^6 + 15\,a^3b^2x^4 + 6\,a^4bx^2 + a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

Mupad [B]
time = 4.75, size = 59, normalized size = 3.11

$$-\frac{\frac{a^5}{12} + \frac{a^4\,b\,x^2}{2} + \frac{5\,a^3\,b^2\,x^4}{4} + \frac{5\,a^2\,b^3\,x^6}{3} + \frac{5\,a\,b^4\,x^8}{4} + \frac{b^5\,x^{10}}{2}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^13,x)
```

```
[Out] -(a^5/12 + (b^5*x^10)/2 + (a^4*b*x^2)/2 + (5*a*b^4*x^8)/4 + (5*a^3*b^2*x^4)
/4 + (5*a^2*b^3*x^6)/3)/x^12
```

3.66 $\int \frac{(a+bx^2)^5}{x^{15}} dx$

Optimal. Leaf size=40

$$-\frac{(a+bx^2)^6}{14ax^{14}}+\frac{b(a+bx^2)^6}{84a^2x^{12}}$$

```
[Out] -1/14*(b*x^2+a)^6/a/x^14+1/84*b*(b*x^2+a)^6/a^2/x^12
```

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 47, 37}

$$\frac{b(a+bx^2)^6}{84a^2x^{12}}-\frac{(a+bx^2)^6}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^15,x]
```

```
[Out] -1/14*(a + b*x^2)^6/(a*x^14) + (b*(a + b*x^2)^6)/(84*a^2*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^6}{14ax^{14}} - \frac{b\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^2\right)}{14a}$$
$$= -\frac{(a+bx^2)^6}{14ax^{14}} + \frac{b(a+bx^2)^6}{84a^2x^{12}}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.68

$$-\frac{a^5}{14x^{14}} - \frac{5a^4b}{12x^{12}} - \frac{a^3b^2}{x^{10}} - \frac{5a^2b^3}{4x^8} - \frac{5ab^4}{6x^6} - \frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^15,x]
```

```
[Out] -1/14*a^5/x^14 - (5*a^4*b)/(12*x^12) - (a^3*b^2)/x^10 - (5*a^2*b^3)/(4*x^8)
 - (5*a*b^4)/(6*x^6) - b^5/(4*x^4)
```

Maple [A]
time = 0.03, size = 58, normalized size = 1.45

method	result	size
default	$-\frac{a^3b^2}{x^{10}} - \frac{b^5}{4x^4} - \frac{5a\,b^4}{6x^6} - \frac{a^5}{14x^{14}} - \frac{5a^2b^3}{4x^8} - \frac{5a^4b}{12x^{12}}$	58
norman	$\frac{-\frac{1}{14}a^5-\frac{5}{12}a^4b\,x^2-a^3b^2x^4-\frac{5}{4}a^2b^3x^6-\frac{5}{6}a\,b^4x^8-\frac{1}{4}b^5x^{10}}{x^{14}}$	59
risch	$\frac{-\frac{1}{14}a^5-\frac{5}{12}a^4b\,x^2-a^3b^2x^4-\frac{5}{4}a^2b^3x^6-\frac{5}{6}a\,b^4x^8-\frac{1}{4}b^5x^{10}}{x^{14}}$	59
gosper	$-\frac{21b^5x^{10}+70a\,b^4x^8+105a^2b^3x^6+84a^3b^2x^4+35a^4b\,x^2+6a^5}{84x^{14}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*b^2/x^10-1/4*b^5/x^4-5/6*a*b^4/x^6-1/14*a^5/x^14-5/4*a^2*b^3/x^8-5/12*
a^4*b/x^12
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{10} + 70\,ab^4x^8 + 105\,a^2b^3x^6 + 84\,a^3b^2x^4 + 35\,a^4bx^2 + 6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="maxima")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

Fricas [A]
time = 1.46, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

Sympy [A]
time = 0.21, size = 63, normalized size = 1.58

$$\frac{-6a^5-35a^4bx^2-84a^3b^2x^4-105a^2b^3x^6-70ab^4x^8-21b^5x^{10}}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**15,x)
```

```
[Out] (-6*a**5 - 35*a**4*b*x**2 - 84*a**3*b**2*x**4 - 105*a**2*b**3*x**6 - 70*a*b
**4*x**8 - 21*b**5*x**10)/(84*x**14)
```

Giac [A]
time = 0.48, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

Mupad [B]
time = 4.74, size = 58, normalized size = 1.45

$$-\frac{\frac{a^5}{14}+\frac{5\,a^4\,b\,x^2}{12}+a^3\,b^2\,x^4+\frac{5\,a^2\,b^3\,x^6}{4}+\frac{5\,a\,b^4\,x^8}{6}+\frac{b^5\,x^{10}}{4}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^15,x)
```

```
[Out] -(a^5/14 + (b^5*x^10)/4 + (5*a^4*b*x^2)/12 + (5*a*b^4*x^8)/6 + a^3*b^2*x^4
+ (5*a^2*b^3*x^6)/4)/x^14
```

3.67 $\int \frac{(a+bx^2)^5}{x^{17}}\,dx$

Optimal. Leaf size=62

$$-\frac{(a+bx^2)^6}{16ax^{16}}+\frac{b(a+bx^2)^6}{56a^2x^{14}}-\frac{b^2(a+bx^2)^6}{336a^3x^{12}}$$

```
[Out] -1/16*(b*x^2+a)^6/a/x^16+1/56*b*(b*x^2+a)^6/a^2/x^14-1/336*b^2*(b*x^2+a)^6/
a^3/x^12
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 47, 37}

$$-\frac{b^2(a+bx^2)^6}{336a^3x^{12}}+\frac{b(a+bx^2)^6}{56a^2x^{14}}-\frac{(a+bx^2)^6}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^17,x]
```

```
[Out] -1/16*(a + b*x^2)^6/(a*x^16) + (b*(a + b*x^2)^6)/(56*a^2*x^14) - (b^2*(a +
b*x^2)^6)/(336*a^3*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{17}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^9}\,dx, x, x^2\right)\\ &= -\frac{(a+bx^2)^6}{16ax^{16}} - \frac{b\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^2\right)}{8a}\\ &= -\frac{(a+bx^2)^6}{16ax^{16}} + \frac{b(a+bx^2)^6}{56a^2x^{14}} + \frac{b^2\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^2\right)}{56a^2}\\ &= -\frac{(a+bx^2)^6}{16ax^{16}} + \frac{b(a+bx^2)^6}{56a^2x^{14}} - \frac{b^2(a+bx^2)^6}{336a^3x^{12}}\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 67, normalized size = 1.08

$$-\frac{a^5}{16x^{16}} - \frac{5a^4b}{14x^{14}} - \frac{5a^3b^2}{6x^{12}} - \frac{a^2b^3}{x^{10}} - \frac{5ab^4}{8x^8} - \frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^17,x]
```

```
[Out] -1/16*a^5/x^16 - (5*a^4*b)/(14*x^14) - (5*a^3*b^2)/(6*x^12) - (a^2*b^3)/x^1
0 - (5*a*b^4)/(8*x^8) - b^5/(6*x^6)
```

Maple [**A**]
time = 0.03, size = 58, normalized size = 0.94

method	result	size
default	$-\frac{a^2b^3}{x^{10}} - \frac{b^5}{6x^6} - \frac{5a^4b}{14x^{14}} - \frac{5a\,b^4}{8x^8} - \frac{a^5}{16x^{16}} - \frac{5a^3b^2}{6x^{12}}$	58
norman	$\frac{-\frac{1}{16}a^5 - \frac{5}{14}a^4b\,x^2 - \frac{5}{6}a^3b^2x^4 - a^2b^3x^6 - \frac{5}{8}a\,b^4x^8 - \frac{1}{6}b^5x^{10}}{x^{16}}$	59
risch	$\frac{-\frac{1}{16}a^5 - \frac{5}{14}a^4b\,x^2 - \frac{5}{6}a^3b^2x^4 - a^2b^3x^6 - \frac{5}{8}a\,b^4x^8 - \frac{1}{6}b^5x^{10}}{x^{16}}$	59
gosper	$-\frac{56b^5x^{10} + 210a\,b^4x^8 + 336a^2b^3x^6 + 280a^3b^2x^4 + 120a^4b\,x^2 + 21a^5}{336x^{16}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*b^3/x^10-1/6*b^5/x^6-5/14*a^4*b/x^14-5/8*a*b^4/x^8-1/16*a^5/x^16-5/6*a
^3*b^2/x^12
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="maxima")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

Fricas [A]
time = 1.23, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

Sympy [A]
time = 0.23, size = 63, normalized size = 1.02

$$\frac{-21a^5-120a^4bx^2-280a^3b^2x^4-336a^2b^3x^6-210ab^4x^8-56b^5x^{10}}{336x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**17,x)
```

```
[Out] (-21*a**5 - 120*a**4*b*x**2 - 280*a**3*b**2*x**4 - 336*a**2*b**3*x**6 - 210
*a*b**4*x**8 - 56*b**5*x**10)/(336*x**16)
```

Giac [A]
time = 0.51, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

Mupad [B]
time = 0.04, size = 58, normalized size = 0.94

$$-\frac{\frac{a^5}{16}+\frac{5\,a^4\,b\,x^2}{14}+\frac{5\,a^3\,b^2\,x^4}{6}+a^2\,b^3\,x^6+\frac{5\,a\,b^4\,x^8}{8}+\frac{b^5\,x^{10}}{6}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^17,x)
[Out] -(a^5/16 + (b^5*x^10)/6 + (5*a^4*b*x^2)/14 + (5*a*b^4*x^8)/8 + (5*a^3*b^2*x
^4)/6 + a^2*b^3*x^6)/x^16
```

3.68 $\int \frac{(a+bx^2)^5}{x^{19}} \, dx$

Optimal. Leaf size=69

$$-\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

```
[Out] -1/18*a^5/x^18-5/16*a^4*b/x^16-5/7*a^3*b^2/x^14-5/6*a^2*b^3/x^12-1/2*a*b^4/
x^10-1/8*b^5/x^8
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^19,x]
```

```
[Out] -1/18*a^5/x^18 - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/(
6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^{19}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^{10}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right) dx, x, x^2\right)\\
&= -\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^19,x]
```

```
[Out] -1/18*a^5/x^18 - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/(
6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.84

method	result	size
default	$-\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{a\,b^4}{2x^{10}}-\frac{b^5}{8x^8}$	58
norman	$\frac{-\frac{1}{18}a^5-\frac{5}{16}a^4b\,x^2-\frac{5}{7}a^3b^2x^4-\frac{5}{6}a^2b^3x^6-\frac{1}{2}a\,b^4x^8-\frac{1}{8}b^5x^{10}}{x^{18}}$	59
risch	$\frac{-\frac{1}{18}a^5-\frac{5}{16}a^4b\,x^2-\frac{5}{7}a^3b^2x^4-\frac{5}{6}a^2b^3x^6-\frac{1}{2}a\,b^4x^8-\frac{1}{8}b^5x^{10}}{x^{18}}$	59
gosper	$-\frac{126b^5x^{10}+504a\,b^4x^8+840a^2b^3x^6+720a^3b^2x^4+315a^4b\,x^2+56a^5}{1008x^{18}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*a^5/x^18-5/16*a^4*b/x^16-5/7*a^3*b^2/x^14-5/6*a^2*b^3/x^12-1/2*a*b^4/
x^10-1/8*b^5/x^8
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.86

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="maxima")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

Fricas [A]
time = 0.71, size = 59, normalized size = 0.86

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

Sympy [A]
time = 0.25, size = 63, normalized size = 0.91

$$\frac{-56a^5 - 315a^4bx^2 - 720a^3b^2x^4 - 840a^2b^3x^6 - 504ab^4x^8 - 126b^5x^{10}}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**19,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x**2 - 720*a**3*b**2*x**4 - 840*a**2*b**3*x**6 - 504
*a*b**4*x**8 - 126*b**5*x**10)/(1008*x**18)
```

Giac [A]
time = 0.49, size = 59, normalized size = 0.86

$$-\frac{126\,b^5x^{10} + 504\,ab^4x^8 + 840\,a^2b^3x^6 + 720\,a^3b^2x^4 + 315\,a^4bx^2 + 56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

Mupad [B]
time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{18} + \frac{5\,a^4\,b\,x^2}{16} + \frac{5\,a^3\,b^2\,x^4}{7} + \frac{5\,a^2\,b^3\,x^6}{6} + \frac{a\,b^4\,x^8}{2} + \frac{b^5\,x^{10}}{8}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^19,x)
```

```
[Out] -(a^5/18 + (b^5*x^10)/8 + (5*a^4*b*x^2)/16 + (a*b^4*x^8)/2 + (5*a^3*b^2*x^4
)/7 + (5*a^2*b^3*x^6)/6)/x^18
```

3.69 $\int \frac{(a+bx^2)^5}{x^{21}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

```
[Out] -1/20*a^5/x^20-5/18*a^4*b/x^18-5/8*a^3*b^2/x^16-5/7*a^2*b^3/x^14-5/12*a*b^4
/x^12-1/10*b^5/x^10
```

Rubi [A]

time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^21,x]
```

```
[Out] -1/20*a^5/x^20 - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/(
7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{21}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^{11}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right)dx,x,x^2\right)\\ &= -\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^21,x]
```

```
[Out] -1/20*a^5/x^20 - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/(
7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.84

method	result	size
default	$-\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5a\,b^4}{12x^{12}}-\frac{b^5}{10x^{10}}$	58
norman	$\frac{-\frac{1}{20}a^5-\frac{5}{18}a^4b\,x^2-\frac{5}{8}a^3b^2x^4-\frac{5}{7}a^2b^3x^6-\frac{5}{12}a\,b^4x^8-\frac{1}{10}b^5x^{10}}{x^{20}}$	59
risch	$\frac{-\frac{1}{20}a^5-\frac{5}{18}a^4b\,x^2-\frac{5}{8}a^3b^2x^4-\frac{5}{7}a^2b^3x^6-\frac{5}{12}a\,b^4x^8-\frac{1}{10}b^5x^{10}}{x^{20}}$	59
gosper	$-\frac{252b^5x^{10}+1050a\,b^4x^8+1800a^2b^3x^6+1575a^3b^2x^4+700a^4b\,x^2+126a^5}{2520x^{20}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^21,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*a^5/x^20-5/18*a^4*b/x^18-5/8*a^3*b^2/x^16-5/7*a^2*b^3/x^14-5/12*a*b^4
/x^12-1/10*b^5/x^10
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.86

$$-\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="maxima")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Fricas [A]
time = 0.88, size = 59, normalized size = 0.86

$$-\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Sympy [A]
time = 0.26, size = 63, normalized size = 0.91

$$\frac{-126a^5 - 700a^4bx^2 - 1575a^3b^2x^4 - 1800a^2b^3x^6 - 1050ab^4x^8 - 252b^5x^{10}}{2520x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**21,x)
```

```
[Out] (-126*a**5 - 700*a**4*b*x**2 - 1575*a**3*b**2*x**4 - 1800*a**2*b**3*x**6 -
1050*a*b**4*x**8 - 252*b**5*x**10)/(2520*x**20)
```

Giac [A]
time = 0.48, size = 59, normalized size = 0.86

$$-\frac{252\,b^5x^{10} + 1050\,ab^4x^8 + 1800\,a^2b^3x^6 + 1575\,a^3b^2x^4 + 700\,a^4bx^2 + 126\,a^5}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="giac")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Mupad [B]
time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{20} + \frac{5\,a^4\,b\,x^2}{18} + \frac{5\,a^3\,b^2\,x^4}{8} + \frac{5\,a^2\,b^3\,x^6}{7} + \frac{5\,a\,b^4\,x^8}{12} + \frac{b^5\,x^{10}}{10}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^21,x)
```

```
[Out] -(a^5/20 + (b^5*x^10)/10 + (5*a^4*b*x^2)/18 + (5*a*b^4*x^8)/12 + (5*a^3*b^2
*x^4)/8 + (5*a^2*b^3*x^6)/7)/x^20
```

3.70 $\int x^8(a+bx^2)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

```
[Out] 1/9*a^5*x^9+5/11*a^4*b*x^11+10/13*a^3*b^2*x^13+2/3*a^2*b^3*x^15+5/17*a*b^4*
x^17+1/19*b^5*x^19
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {276}

$$\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^2\right)^5\,dx&=\int\left(a^5x^8+5a^4bx^{10}+10a^3b^2x^{12}+10a^2b^3x^{14}+5ab^4x^{16}+b^5x^{18}\right)dx\\&=\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{9}a^5x^9+\frac{5}{11}a^4b\,x^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{19}b^5x^{19}$	58
default	$\frac{1}{9}a^5x^9+\frac{5}{11}a^4b\,x^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{19}b^5x^{19}$	58
norman	$\frac{1}{9}a^5x^9+\frac{5}{11}a^4b\,x^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{19}b^5x^{19}$	58
risch	$\frac{1}{9}a^5x^9+\frac{5}{11}a^4b\,x^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{19}b^5x^{19}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^5*x^9+5/11*a^4*b*x^11+10/13*a^3*b^2*x^13+2/3*a^2*b^3*x^15+5/17*a*b^4*
x^17+1/19*b^5*x^19
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.83

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Fricas [A]
time = 0.98, size = 57, normalized size = 0.83

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Sympy [A]
time = 0.01, size = 66, normalized size = 0.96

$$\frac{a^5x^9}{9}+\frac{5a^4bx^{11}}{11}+\frac{10a^3b^2x^{13}}{13}+\frac{2a^2b^3x^{15}}{3}+\frac{5ab^4x^{17}}{17}+\frac{b^5x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**9/9 + 5*a**4*b*x**11/11 + 10*a**3*b**2*x**13/13 + 2*a**2*b**3*x**15
/3 + 5*a*b**4*x**17/17 + b**5*x**19/19
```

Giac [A]
time = 0.51, size = 57, normalized size = 0.83

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^9}{9}+\frac{5\,a^4\,b\,x^{11}}{11}+\frac{10\,a^3\,b^2\,x^{13}}{13}+\frac{2\,a^2\,b^3\,x^{15}}{3}+\frac{5\,a\,b^4\,x^{17}}{17}+\frac{b^5\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^9)/9 + (b^5*x^19)/19 + (5*a^4*b*x^11)/11 + (5*a*b^4*x^17)/17 + (10*a
^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
```

3.71 $\int x^6(a+bx^2)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

```
[Out] 1/7*a^5*x^7+5/9*a^4*b*x^9+10/11*a^3*b^2*x^11+10/13*a^2*b^3*x^13+1/3*a*b^4*x
^15+1/17*b^5*x^17
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{276\}$

$$\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/13
 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a+bx^2\right)^5\,dx &= \int \left(a^5x^6+5a^4bx^8+10a^3b^2x^{10}+10a^2b^3x^{12}+5ab^4x^{14}+b^5x^{16}\right)\,dx\\ &= \frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/13
 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{7}a^5x^7+\frac{5}{9}a^4b\,x^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}a\,b^4x^{15}+\frac{1}{17}b^5x^{17}$	58
default	$\frac{1}{7}a^5x^7+\frac{5}{9}a^4b\,x^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}a\,b^4x^{15}+\frac{1}{17}b^5x^{17}$	58
norman	$\frac{1}{7}a^5x^7+\frac{5}{9}a^4b\,x^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}a\,b^4x^{15}+\frac{1}{17}b^5x^{17}$	58
risch	$\frac{1}{7}a^5x^7+\frac{5}{9}a^4b\,x^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}a\,b^4x^{15}+\frac{1}{17}b^5x^{17}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^5*x^7+5/9*a^4*b*x^9+10/11*a^3*b^2*x^11+10/13*a^2*b^3*x^13+1/3*a*b^4*x
^15+1/17*b^5*x^17
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.83

$$\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Fricas [A]
time = 0.80, size = 57, normalized size = 0.83

$$\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Sympy [A]
time = 0.01, size = 65, normalized size = 0.94

$$\frac{a^5x^7}{7}+\frac{5a^4bx^9}{9}+\frac{10a^3b^2x^{11}}{11}+\frac{10a^2b^3x^{13}}{13}+\frac{ab^4x^{15}}{3}+\frac{b^5x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**7/7 + 5*a**4*b*x**9/9 + 10*a**3*b**2*x**11/11 + 10*a**2*b**3*x**13/
13 + a*b**4*x**15/3 + b**5*x**17/17
```

Giac [A]
time = 0.68, size = 57, normalized size = 0.83

$$\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^7}{7}+\frac{5\,a^4\,b\,x^9}{9}+\frac{10\,a^3\,b^2\,x^{11}}{11}+\frac{10\,a^2\,b^3\,x^{13}}{13}+\frac{a\,b^4\,x^{15}}{3}+\frac{b^5\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^7)/7 + (b^5*x^17)/17 + (5*a^4*b*x^9)/9 + (a*b^4*x^15)/3 + (10*a^3*b^
2*x^11)/11 + (10*a^2*b^3*x^13)/13
```

3.72 $\int x^4(a+bx^2)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

```
[Out] 1/5*a^5*x^5+5/7*a^4*b*x^7+10/9*a^3*b^2*x^9+10/11*a^2*b^3*x^11+5/13*a*b^4*x^
13+1/15*b^5*x^15
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{276\}$

$$\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 +
 (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^5\,dx&=\int\left(a^5x^4+5a^4bx^6+10a^3b^2x^8+10a^2b^3x^{10}+5ab^4x^{12}+b^5x^{14}\right)\,dx\\&=\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 +
 (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{5}a^5x^5+\frac{5}{7}a^4b\,x^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{15}b^5x^{15}$	58
default	$\frac{1}{5}a^5x^5+\frac{5}{7}a^4b\,x^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{15}b^5x^{15}$	58
norman	$\frac{1}{5}a^5x^5+\frac{5}{7}a^4b\,x^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{15}b^5x^{15}$	58
risch	$\frac{1}{5}a^5x^5+\frac{5}{7}a^4b\,x^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{15}b^5x^{15}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^5*x^5+5/7*a^4*b*x^7+10/9*a^3*b^2*x^9+10/11*a^2*b^3*x^11+5/13*a*b^4*x^
13+1/15*b^5*x^15
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.83

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Fricas [A]
time = 0.71, size = 57, normalized size = 0.83

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Sympy [A]
time = 0.01, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5}+\frac{5a^4bx^7}{7}+\frac{10a^3b^2x^9}{9}+\frac{10a^2b^3x^{11}}{11}+\frac{5ab^4x^{13}}{13}+\frac{b^5x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**7/7 + 10*a**3*b**2*x**9/9 + 10*a**2*b**3*x**11/11
 + 5*a*b**4*x**13/13 + b**5*x**15/15
```

Giac [A]
time = 0.81, size = 57, normalized size = 0.83

$$\frac{1}{15}\,b^{5}x^{15}+\frac{5}{13}\,ab^{4}x^{13}+\frac{10}{11}\,a^{2}b^{3}x^{11}+\frac{10}{9}\,a^{3}b^{2}x^{9}+\frac{5}{7}\,a^{4}bx^{7}+\frac{1}{5}\,a^{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^5}{5}+\frac{5\,a^4\,b\,x^7}{7}+\frac{10\,a^3\,b^2\,x^9}{9}+\frac{10\,a^2\,b^3\,x^{11}}{11}+\frac{5\,a\,b^4\,x^{13}}{13}+\frac{b^5\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^5)/5 + (b^5*x^15)/15 + (5*a^4*b*x^7)/7 + (5*a*b^4*x^13)/13 + (10*a^3
*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11
```

3.73 $\int x^2(a+bx^2)^5\,dx$

Optimal. Leaf size=66

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

```
[Out] 1/3*a^5*x^3+a^4*b*x^5+10/7*a^3*b^2*x^7+10/9*a^2*b^3*x^9+5/11*a*b^4*x^11+1/1
3*b^5*x^13
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{276\}$

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^
4*x^11)/11 + (b^5*x^13)/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^5\,dx&=\int\left(a^5x^2+5a^4bx^4+10a^3b^2x^6+10a^2b^3x^8+5ab^4x^{10}+b^5x^{12}\right)\,dx\\&=\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.00

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^
4*x^11)/11 + (b^5*x^13)/13
```

Maple [A]
time = 0.06, size = 57, normalized size = 0.86

method	result	size
gosper	$\frac{1}{3}a^5x^3+a^4b\,x^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}a\,b^4x^{11}+\frac{1}{13}b^5x^{13}$	57
default	$\frac{1}{3}a^5x^3+a^4b\,x^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}a\,b^4x^{11}+\frac{1}{13}b^5x^{13}$	57
norman	$\frac{1}{3}a^5x^3+a^4b\,x^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}a\,b^4x^{11}+\frac{1}{13}b^5x^{13}$	57
risch	$\frac{1}{3}a^5x^3+a^4b\,x^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}a\,b^4x^{11}+\frac{1}{13}b^5x^{13}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^5*x^3+a^4*b*x^5+10/7*a^3*b^2*x^7+10/9*a^2*b^3*x^9+5/11*a*b^4*x^11+1/1
3*b^5*x^13
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.85

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

Fricas [A]
time = 0.96, size = 56, normalized size = 0.85

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

Sympy [A]
time = 0.01, size = 63, normalized size = 0.95

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10a^3b^2x^7}{7}+\frac{10a^2b^3x^9}{9}+\frac{5ab^4x^{11}}{11}+\frac{b^5x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**3/3 + a**4*b*x**5 + 10*a**3*b**2*x**7/7 + 10*a**2*b**3*x**9/9 + 5*a
*b**4*x**11/11 + b**5*x**13/13
```

Giac [A]
time = 0.99, size = 56, normalized size = 0.85

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

Mupad [B]
time = 0.02, size = 56, normalized size = 0.85

$$\frac{a^5\,x^3}{3}+a^4\,b\,x^5+\frac{10\,a^3\,b^2\,x^7}{7}+\frac{10\,a^2\,b^3\,x^9}{9}+\frac{5\,a\,b^4\,x^{11}}{11}+\frac{b^5\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^3)/3 + (b^5*x^13)/13 + a^4*b*x^5 + (5*a*b^4*x^11)/11 + (10*a^3*b^2*x
^7)/7 + (10*a^2*b^3*x^9)/9
```

3.74 $\int (a+bx^2)^5 \, dx$

Optimal. Leaf size=62

$$a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

```
[Out] a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*x
^11
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {200}

$$a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9
)/9 + (b^5*x^11)/11
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^5 \, dx &= \int \left(a^5+5a^4bx^2+10a^3b^2x^4+10a^2b^3x^6+5ab^4x^8+b^5x^{10}\right) dx\\ &= a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 62, normalized size = 1.00

$$a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9
)/9 + (b^5*x^11)/11
```

Maple [A]
time = 0.02, size = 55, normalized size = 0.89

method	result	size
gosper	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
default	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
norman	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
risch	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*x
^11
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.87

$$\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

Fricas [A]
time = 1.03, size = 54, normalized size = 0.87

$$\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

Sympy [A]
time = 0.01, size = 61, normalized size = 0.98

$$a^5x+\frac{5a^4bx^3}{3}+2a^3b^2x^5+\frac{10a^2b^3x^7}{7}+\frac{5ab^4x^9}{9}+\frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**3/3 + 2*a**3*b**2*x**5 + 10*a**2*b**3*x**7/7 + 5*a*b**
4*x**9/9 + b**5*x**11/11
```

Giac [A]
time = 1.60, size = 54, normalized size = 0.87

$$\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

Mupad [B]
time = 0.02, size = 54, normalized size = 0.87

$$a^5\,x+\frac{5\,a^4\,b\,x^3}{3}+2\,a^3\,b^2\,x^5+\frac{10\,a^2\,b^3\,x^7}{7}+\frac{5\,a\,b^4\,x^9}{9}+\frac{b^5\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5,x)
```

```
[Out] a^5*x + (b^5*x^11)/11 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^9)/9 + 2*a^3*b^2*x^5 +
 (10*a^2*b^3*x^7)/7
```

3.75 $\int \frac{(a+bx^2)^5}{x^2} \, dx$

Optimal. Leaf size=61

$$-\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

```
[Out] -a^5/x+5*a^4*b*x+10/3*a^3*b^2*x^3+2*a^2*b^3*x^5+5/7*a*b^4*x^7+1/9*b^5*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^2,x]
```

```
[Out] -(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7
 + (b^5*x^9)/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^2} \, dx &= \int \left(5a^4b+\frac{a^5}{x^2}+10a^3b^2x^2+10a^2b^3x^4+5ab^4x^6+b^5x^8\right) dx\\ &= -\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^2,x]
```

```
[Out] -(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7
 + (b^5*x^9)/9
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.92

method	result	size
default	$-\frac{a^5}{x}+5a^4bx+\frac{10a^3b^2x^3}{3}+2a^2b^3x^5+\frac{5a\,b^4x^7}{7}+\frac{b^5x^9}{9}$	56
risch	$-\frac{a^5}{x}+5a^4bx+\frac{10a^3b^2x^3}{3}+2a^2b^3x^5+\frac{5a\,b^4x^7}{7}+\frac{b^5x^9}{9}$	56
norman	$\frac{\frac{1}{9}b^5x^{10}+\frac{5}{7}a\,b^4x^8+2a^2b^3x^6+\frac{10}{3}a^3b^2x^4+5a^4b\,x^2-a^5}{x}$	59
gosper	$-\frac{-7b^5x^{10}-45a\,b^4x^8-126a^2b^3x^6-210a^3b^2x^4-315a^4b\,x^2+63a^5}{63x}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^5/x+5*a^4*b*x+10/3*a^3*b^2*x^3+2*a^2*b^3*x^5+5/7*a*b^4*x^7+1/9*b^5*x^9
```

Maxima [A]
time = 0.27, size = 55, normalized size = 0.90

$$\frac{1}{9}\,b^5x^9+\frac{5}{7}\,ab^4x^7+2\,a^2b^3x^5+\frac{10}{3}\,a^3b^2x^3+5\,a^4bx-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

Fricas [A]
time = 1.06, size = 59, normalized size = 0.97

$$\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a
^4*b*x^2 - 63*a^5)/x
```

Sympy [A]
time = 0.03, size = 58, normalized size = 0.95

$$-\frac{a^5}{x}+5a^4bx+\frac{10a^3b^2x^3}{3}+2a^2b^3x^5+\frac{5ab^4x^7}{7}+\frac{b^5x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*x + 10*a**3*b**2*x**3/3 + 2*a**2*b**3*x**5 + 5*a*b**4*x*
*7/7 + b**5*x**9/9
```

Giac [A]
time = 1.34, size = 55, normalized size = 0.90

$$\frac{1}{9}\,b^5x^9+\frac{5}{7}\,ab^4x^7+2\,a^2b^3x^5+\frac{10}{3}\,a^3b^2x^3+5\,a^4bx-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

Mupad [B]
time = 0.03, size = 55, normalized size = 0.90

$$\frac{b^5\,x^9}{9}-\frac{a^5}{x}+\frac{5\,a\,b^4\,x^7}{7}+\frac{10\,a^3\,b^2\,x^3}{3}+2\,a^2\,b^3\,x^5+5\,a^4\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^2,x)
```

```
[Out] (b^5*x^9)/9 - a^5/x + (5*a*b^4*x^7)/7 + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5
+ 5*a^4*b*x
```

3.76 $\int \frac{(a+bx^2)^5}{x^4} \, dx$

Optimal. Leaf size=60

$$-\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10}{3}a^2b^3x^3 + ab^4x^5 + \frac{b^5x^7}{7}$$

```
[Out] -1/3*a^5/x^3-5*a^4*b/x+10*a^3*b^2*x+10/3*a^2*b^3*x^3+a*b^4*x^5+1/7*b^5*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10}{3}a^2b^3x^3 + ab^4x^5 + \frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
+ (b^5*x^7)/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^4} \, dx &= \int \left(10a^3b^2 + \frac{a^5}{x^4} + \frac{5a^4b}{x^2} + 10a^2b^3x^2 + 5ab^4x^4 + b^5x^6\right) dx \\
&= -\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10}{3}a^2b^3x^3 + ab^4x^5 + \frac{b^5x^7}{7}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10}{3}a^2b^3x^3 + ab^4x^5 + \frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^4,x]
[Out] -1/3*a^5/x^3 - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
+ (b^5*x^7)/7
```

Maple [A]
time = 0.03, size = 55, normalized size = 0.92

method	result	size
default	$-\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10a^2b^3x^3}{3}+a\,b^4x^5+\frac{b^5x^7}{7}$	55
risch	$\frac{b^5x^7}{7}+a\,b^4x^5+\frac{10a^2b^3x^3}{3}+10a^3b^2x+\frac{-5a^4b\,x^2-\frac{1}{3}a^5}{x^3}$	57
norman	$\frac{\frac{1}{7}b^5x^{10}+a\,b^4x^8+\frac{10}{3}a^2b^3x^6+10a^3b^2x^4-5a^4b\,x^2-\frac{1}{3}a^5}{x^3}$	58
gosper	$-\frac{-3b^5x^{10}-21a\,b^4x^8-70a^2b^3x^6-210a^3b^2x^4+105a^4b\,x^2+7a^5}{21x^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a^5/x^3-5*a^4*b/x+10*a^3*b^2*x+10/3*a^2*b^3*x^3+a*b^4*x^5+1/7*b^5*x^7
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.92

$$\frac{1}{7}\,b^5x^7+ab^4x^5+\frac{10}{3}\,a^2b^3x^3+10\,a^3b^2x-\frac{15\,a^4bx^2+a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="maxima")
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x
^2 + a^5)/x^3
```

Fricas [A]
time = 1.38, size = 59, normalized size = 0.98

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="fricas")
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^
4*b*x^2 - 7*a^5)/x^3
```

Sympy [A]
time = 0.06, size = 60, normalized size = 1.00

$$10a^3b^2x+\frac{10a^2b^3x^3}{3}+ab^4x^5+\frac{b^5x^7}{7}+\frac{-a^5-15a^4bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**4,x)
```

```
[Out] 10*a**3*b**2*x + 10*a**2*b**3*x**3/3 + a*b**4*x**5 + b**5*x**7/7 + (-a**5 -
 15*a**4*b*x**2)/(3*x**3)
```

Giac [A]
time = 0.97, size = 55, normalized size = 0.92

$$\frac{1}{7}\,b^5x^7 + ab^4x^5 + \frac{10}{3}\,a^2b^3x^3 + 10\,a^3b^2x - \frac{15\,a^4bx^2 + a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x
^2 + a^5)/x^3
```

Mupad [B]
time = 0.03, size = 57, normalized size = 0.95

$$\frac{b^5\,x^7}{7} - \frac{\frac{a^5}{3} + 5\,b\,a^4\,x^2}{x^3} + 10\,a^3\,b^2\,x + a\,b^4\,x^5 + \frac{10\,a^2\,b^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^4,x)
```

```
[Out] (b^5*x^7)/7 - (a^5/3 + 5*a^4*b*x^2)/x^3 + 10*a^3*b^2*x + a*b^4*x^5 + (10*a^
2*b^3*x^3)/3
```

3.77 $\int \frac{(a+bx^2)^5}{x^6} \, dx$

Optimal. Leaf size=63

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

```
[Out] -1/5*a^5/x^5-5/3*a^4*b/x^3-10*a^3*b^2/x+10*a^2*b^3*x+5/3*a*b^4*x^3+1/5*b^5*
x^5
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4
*x^3)/3 + (b^5*x^5)/5
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^6}\,dx &= \int \left(10a^2b^3+\frac{a^5}{x^6}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^2}+5ab^4x^2+b^5x^4\right)dx\\ &= -\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 63, normalized size = 1.00

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4
*x^3)/3 + (b^5*x^5)/5
```

Maple [A]
time = 0.03, size = 56, normalized size = 0.89

method	result	size
default	$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5a\,b^4x^3}{3}+\frac{b^5x^5}{5}$	56
risch	$\frac{b^5x^5}{5}+\frac{5a\,b^4x^3}{3}+10a^2b^3x+\frac{-10a^3b^2x^4-\frac{5}{3}a^4b\,x^2-\frac{1}{5}a^5}{x^5}$	58
norman	$\frac{\frac{1}{5}b^5x^{10}+\frac{5}{3}a\,b^4x^8+10a^2b^3x^6-10a^3b^2x^4-\frac{5}{3}a^4b\,x^2-\frac{1}{5}a^5}{x^5}$	59
gosper	$-\frac{-3b^5x^{10}-25a\,b^4x^8-150a^2b^3x^6+150a^3b^2x^4+25a^4b\,x^2+3a^5}{15x^5}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^5/x^5-5/3*a^4*b/x^3-10*a^3*b^2/x+10*a^2*b^3*x+5/3*a*b^4*x^3+1/5*b^5*
x^5
```

Maxima [A]
time = 0.27, size = 58, normalized size = 0.92

$$\frac{1}{5}\,b^5x^5+\frac{5}{3}\,ab^4x^3+10\,a^2b^3x-\frac{150\,a^3b^2x^4+25\,a^4bx^2+3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4
*b*x^2 + 3*a^5)/x^5
```

Fricas [A]
time = 1.10, size = 59, normalized size = 0.94

$$\frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^
4*b*x^2 - 3*a^5)/x^5
```

Sympy [A]
time = 0.08, size = 63, normalized size = 1.00

$$10a^2b^3x + \frac{5ab^4x^3}{3} + \frac{b^5x^5}{5} + \frac{-3a^5 - 25a^4bx^2 - 150a^3b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**6,x)
```

```
[Out] 10*a**2*b**3*x + 5*a*b**4*x**3/3 + b**5*x**5/5 + (-3*a**5 - 25*a**4*b*x**2
- 150*a**3*b**2*x**4)/(15*x**5)
```

Giac [A]
time = 1.57, size = 58, normalized size = 0.92

$$\frac{1}{5}\,b^5x^5 + \frac{5}{3}\,ab^4x^3 + 10\,a^2b^3x - \frac{150\,a^3b^2x^4 + 25\,a^4bx^2 + 3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4
*b*x^2 + 3*a^5)/x^5
```

Mupad [B]
time = 0.05, size = 58, normalized size = 0.92

$$\frac{b^5\,x^5}{5} - \frac{\frac{a^5}{5} + \frac{5\,a^4\,b\,x^2}{3} + 10\,a^3\,b^2\,x^4}{x^5} + 10\,a^2\,b^3\,x + \frac{5\,a\,b^4\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^6,x)
```

```
[Out] (b^5*x^5)/5 - (a^5/5 + (5*a^4*b*x^2)/3 + 10*a^3*b^2*x^4)/x^5 + 10*a^2*b^3*x
 + (5*a*b^4*x^3)/3
```

3.78 $\int \frac{(a+bx^2)^5}{x^8} \, dx$

Optimal. Leaf size=61

$$-\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}$$

```
[Out] -1/7*a^5/x^7-a^4*b/x^5-10/3*a^3*b^2/x^3-10*a^2*b^3/x+5*a*b^4*x+1/3*b^5*x^3
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^8,x]
```

```
[Out] -1/7*a^5/x^7 - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^
4*x + (b^5*x^3)/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^8} \, dx &= \int \left(5ab^4+\frac{a^5}{x^8}+\frac{5a^4b}{x^6}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^2}+b^5x^2\right) dx \\ &= -\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^8,x]
```

```
[Out] -1/7*a^5/x^7 - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^
4*x + (b^5*x^3)/3
```

Maple [A]
time = 0.03, size = 56, normalized size = 0.92

method	result	size
default	$-\frac{a^{5}}{7x^{7}}-\frac{a^{4}b}{x^{5}}-\frac{10a^{3}b^{2}}{3x^{3}}-\frac{10a^{2}b^{3}}{x}+5a\,b^{4}x+\frac{b^{5}x^{3}}{3}$	56
risch	$\frac{b^{5}x^{3}}{3}+5a\,b^{4}x+\frac{-10a^{2}b^{3}x^{6}-\frac{10}{3}a^{3}b^{2}x^{4}-a^{4}b\,x^{2}-\frac{1}{7}a^{5}}{x^{7}}$	58
norman	$\frac{\frac{1}{3}b^{5}x^{10}+5a\,b^{4}x^{8}-10a^{2}b^{3}x^{6}-\frac{10}{3}a^{3}b^{2}x^{4}-a^{4}b\,x^{2}-\frac{1}{7}a^{5}}{x^{7}}$	59
gosper	$-\frac{-7b^{5}x^{10}-105a\,b^{4}x^{8}+210a^{2}b^{3}x^{6}+70a^{3}b^{2}x^{4}+21a^{4}b\,x^{2}+3a^{5}}{21x^{7}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^5/x^7-a^4*b/x^5-10/3*a^3*b^2/x^3-10*a^2*b^3/x+5*a*b^4*x+1/3*b^5*x^3
```

Maxima [A]
time = 0.31, size = 58, normalized size = 0.95

$$\frac{1}{3}\,b^{5}x^{3}+5\,ab^{4}x-\frac{210\,a^{2}b^{3}x^{6}+70\,a^{3}b^{2}x^{4}+21\,a^{4}bx^{2}+3\,a^{5}}{21\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b
*x^2 + 3*a^5)/x^7
```

Fricas [A]
time = 1.77, size = 59, normalized size = 0.97

$$\frac{7\,b^{5}x^{10}+105\,ab^{4}x^{8}-210\,a^{2}b^{3}x^{6}-70\,a^{3}b^{2}x^{4}-21\,a^{4}bx^{2}-3\,a^{5}}{21\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^
4*b*x^2 - 3*a^5)/x^7
```

Sympy [A]
time = 0.11, size = 61, normalized size = 1.00

$$5ab^{4}x+\frac{b^{5}x^{3}}{3}+\frac{-3a^{5}-21a^{4}bx^{2}-70a^{3}b^{2}x^{4}-210a^{2}b^{3}x^{6}}{21x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**8,x)
```

```
[Out] 5*a*b**4*x + b**5*x**3/3 + (-3*a**5 - 21*a**4*b*x**2 - 70*a**3*b**2*x**4 -
210*a**2*b**3*x**6)/(21*x**7)
```

Giac [A]
time = 1.12, size = 58, normalized size = 0.95

$$\frac{1}{3}\,b^5x^3 + 5\,ab^4x - \frac{210\,a^2b^3x^6 + 70\,a^3b^2x^4 + 21\,a^4bx^2 + 3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b
*x^2 + 3*a^5)/x^7
```

Mupad [B]
time = 4.79, size = 59, normalized size = 0.97

$$-\frac{3\,a^5 + 21\,a^4\,b\,x^2 + 70\,a^3\,b^2\,x^4 + 210\,a^2\,b^3\,x^6 - 105\,a\,b^4\,x^8 - 7\,b^5\,x^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^8,x)
```

```
[Out] -(3*a^5 - 7*b^5*x^10 + 21*a^4*b*x^2 - 105*a*b^4*x^8 + 70*a^3*b^2*x^4 + 210*
a^2*b^3*x^6)/(21*x^7)
```

3.79 $\int \frac{(a+bx^2)^5}{x^{10}} \, dx$

Optimal. Leaf size=60

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

```
[Out] -1/9*a^5/x^9-5/7*a^4*b/x^7-2*a^3*b^2/x^5-10/3*a^2*b^3/x^3-5*a*b^4/x+b^5*x
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) -
 (5*a*b^4)/x + b^5*x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{10}}\,dx &= \int \left(b^5+\frac{a^5}{x^{10}}+\frac{5a^4b}{x^8}+\frac{10a^3b^2}{x^6}+\frac{10a^2b^3}{x^4}+\frac{5ab^4}{x^2}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) -
 (5*a*b^4)/x + b^5*x
```

Maple [A]
time = 0.03, size = 55, normalized size = 0.92

method	result	size
default	$-\frac{a^{5}}{9x^{9}}-\frac{5a^{4}b}{7x^{7}}-\frac{2a^{3}b^{2}}{x^{5}}-\frac{10a^{2}b^{3}}{3x^{3}}-\frac{5a\,b^{4}}{x}+b^{5}x$	55
risch	$b^{5}x+\frac{-5a\,b^{4}x^{8}-\frac{10}{3}a^{2}b^{3}x^{6}-2a^{3}b^{2}x^{4}-\frac{5}{7}a^{4}b\,x^{2}-\frac{1}{9}a^{5}}{x^{9}}$	57
norman	$\frac{b^{5}x^{10}-5a\,b^{4}x^{8}-\frac{10}{3}a^{2}b^{3}x^{6}-2a^{3}b^{2}x^{4}-\frac{5}{7}a^{4}b\,x^{2}-\frac{1}{9}a^{5}}{x^{9}}$	58
gosper	$-\frac{-63b^{5}x^{10}+315a\,b^{4}x^{8}+210a^{2}b^{3}x^{6}+126a^{3}b^{2}x^{4}+45a^{4}b\,x^{2}+7a^{5}}{63x^{9}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^5/x^9-5/7*a^4*b/x^7-2*a^3*b^2/x^5-10/3*a^2*b^3/x^3-5*a*b^4/x+b^5*x
```

Maxima [A]
time = 0.31, size = 57, normalized size = 0.95

$$b^{5}x-\frac{315\,ab^{4}x^{8}+210\,a^{2}b^{3}x^{6}+126\,a^{3}b^{2}x^{4}+45\,a^{4}bx^{2}+7\,a^{5}}{63\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*
x^2 + 7*a^5)/x^9
```

Fricas [A]
time = 1.01, size = 59, normalized size = 0.98

$$\frac{63\,b^{5}x^{10}-315\,ab^{4}x^{8}-210\,a^{2}b^{3}x^{6}-126\,a^{3}b^{2}x^{4}-45\,a^{4}bx^{2}-7\,a^{5}}{63\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*
a^4*b*x^2 - 7*a^5)/x^9
```

Sympy [A]
time = 0.15, size = 60, normalized size = 1.00

$$b^{5}x+\frac{-7a^{5}-45a^{4}bx^{2}-126a^{3}b^{2}x^{4}-210a^{2}b^{3}x^{6}-315ab^{4}x^{8}}{63x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**10,x)
```

```
[Out] b**5*x + (-7*a**5 - 45*a**4*b*x**2 - 126*a**3*b**2*x**4 - 210*a**2*b**3*x**
6 - 315*a*b**4*x**8)/(63*x**9)
```

Giac [A]
time = 1.03, size = 57, normalized size = 0.95

$$b^5x - \frac{315\,ab^4x^8 + 210\,a^2b^3x^6 + 126\,a^3b^2x^4 + 45\,a^4bx^2 + 7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="giac")
```

```
[Out] b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*
x^2 + 7*a^5)/x^9
```

Mupad [B]
time = 0.04, size = 57, normalized size = 0.95

$$b^5\,x - \frac{\frac{a^5}{9} + \frac{5\,a^4\,b\,x^2}{7} + 2\,a^3\,b^2\,x^4 + \frac{10\,a^2\,b^3\,x^6}{3} + 5\,a\,b^4\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^10,x)
```

```
[Out] b^5*x - (a^5/9 + (5*a^4*b*x^2)/7 + 5*a*b^4*x^8 + 2*a^3*b^2*x^4 + (10*a^2*b^
3*x^6)/3)/x^9
```

3.80 $\int \frac{(a+bx^2)^5}{x^{12}}\,dx$

Optimal. Leaf size=65

$$-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

```
[Out] -1/11*a^5/x^11-5/9*a^4*b/x^9-10/7*a^3*b^2/x^7-2*a^2*b^3/x^5-5/3*a*b^4/x^3-b
^5/x
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5
 - (5*a*b^4)/(3*x^3) - b^5/x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{12}}\,dx &= \int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^6}+\frac{5ab^4}{x^4}+\frac{b^5}{x^2}\right)dx\\ &= -\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5
 - (5*a*b^4)/(3*x^3) - b^5/x
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.89

method	result	size
default	$-\frac{a^{5}}{11 x^{11}}-\frac{5 a^{4} b}{9 x^{9}}-\frac{10 a^{3} b^{2}}{7 x^{7}}-\frac{2 a^{2} b^{3}}{x^{5}}-\frac{5 a\,b^{4}}{3 x^{3}}-\frac{b^{5}}{x}$	58
norman	$\frac{-b^{5} x^{10}-\frac{5}{3} a\,b^{4} x^{8}-2 a^{2} b^{3} x^{6}-\frac{10}{7} a^{3} b^{2} x^{4}-\frac{5}{9} a^{4} b\,x^{2}-\frac{1}{11} a^{5}}{x^{11}}$	59
risch	$\frac{-b^{5} x^{10}-\frac{5}{3} a\,b^{4} x^{8}-2 a^{2} b^{3} x^{6}-\frac{10}{7} a^{3} b^{2} x^{4}-\frac{5}{9} a^{4} b\,x^{2}-\frac{1}{11} a^{5}}{x^{11}}$	59
gosper	$-\frac{693 b^{5} x^{10}+1155 a\,b^{4} x^{8}+1386 a^{2} b^{3} x^{6}+990 a^{3} b^{2} x^{4}+385 a^{4} b\,x^{2}+63 a^{5}}{693 x^{11}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*a^5/x^11-5/9*a^4*b/x^9-10/7*a^3*b^2/x^7-2*a^2*b^3/x^5-5/3*a*b^4/x^3-b
^5/x
```

Maxima [A]
time = 0.31, size = 59, normalized size = 0.91

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="maxima")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

Fricas [A]
time = 1.10, size = 59, normalized size = 0.91

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

Sympy [A]
time = 0.18, size = 63, normalized size = 0.97

$$\frac{-63a^5 - 385a^4bx^2 - 990a^3b^2x^4 - 1386a^2b^3x^6 - 1155ab^4x^8 - 693b^5x^{10}}{693x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**12,x)
```

```
[Out] (-63*a**5 - 385*a**4*b*x**2 - 990*a**3*b**2*x**4 - 1386*a**2*b**3*x**6 - 11
55*a*b**4*x**8 - 693*b**5*x**10)/(693*x**11)
```

Giac [A]
time = 1.66, size = 59, normalized size = 0.91

$$-\frac{693\,b^5x^{10} + 1155\,ab^4x^8 + 1386\,a^2b^3x^6 + 990\,a^3b^2x^4 + 385\,a^4bx^2 + 63\,a^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="giac")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

Mupad [B]
time = 0.04, size = 58, normalized size = 0.89

$$-\frac{\frac{a^5}{11} + \frac{5\,a^4\,b\,x^2}{9} + \frac{10\,a^3\,b^2\,x^4}{7} + 2\,a^2\,b^3\,x^6 + \frac{5\,a\,b^4\,x^8}{3} + b^5\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^12,x)
```

```
[Out] -(a^5/11 + b^5*x^10 + (5*a^4*b*x^2)/9 + (5*a*b^4*x^8)/3 + (10*a^3*b^2*x^4)/
7 + 2*a^2*b^3*x^6)/x^11
```

3.81 $\int \frac{(a+bx^2)^5}{x^{14}}\,dx$

Optimal. Leaf size=67

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

```
[Out] -1/13*a^5/x^13-5/11*a^4*b/x^11-10/9*a^3*b^2/x^9-10/7*a^2*b^3/x^7-a*b^4/x^5-
1/3*b^5/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/
(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{14}}\,dx &= \int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^6}+\frac{b^5}{x^4}\right)dx\\ &= -\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/
(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.87

method	result	size
default	$-\frac{a^{5}}{13 x^{13}}-\frac{5 a^{4} b}{11 x^{11}}-\frac{10 a^{3} b^{2}}{9 x^{9}}-\frac{10 a^{2} b^{3}}{7 x^{7}}-\frac{a \,b^{4}}{x^{5}}-\frac{b^{5}}{3 x^{3}}$	58
norman	$\frac{-\frac{1}{3} b^{5} x^{10}-a \,b^{4} x^{8}-\frac{10}{7} a^{2} b^{3} x^{6}-\frac{10}{9} a^{3} b^{2} x^{4}-\frac{5}{11} a^{4} b \,x^{2}-\frac{1}{13} a^{5}}{x^{13}}$	59
risch	$\frac{-\frac{1}{3} b^{5} x^{10}-a \,b^{4} x^{8}-\frac{10}{7} a^{2} b^{3} x^{6}-\frac{10}{9} a^{3} b^{2} x^{4}-\frac{5}{11} a^{4} b \,x^{2}-\frac{1}{13} a^{5}}{x^{13}}$	59
gosper	$-\frac{3003 b^{5} x^{10}+9009 a \,b^{4} x^{8}+12870 a^{2} b^{3} x^{6}+10010 a^{3} b^{2} x^{4}+4095 a^{4} b \,x^{2}+693 a^{5}}{9009 x^{13}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*a^5/x^13-5/11*a^4*b/x^11-10/9*a^3*b^2/x^9-10/7*a^2*b^3/x^7-a*b^4/x^5-
1/3*b^5/x^3
```

Maxima [A]
time = 0.30, size = 59, normalized size = 0.88

$$-\frac{3003 \, b^{5} x^{10} + 9009 \, a b^{4} x^{8} + 12870 \, a^{2} b^{3} x^{6} + 10010 \, a^{3} b^{2} x^{4} + 4095 \, a^{4} b x^{2} + 693 \, a^{5}}{9009 \, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Fricas [A]
time = 1.30, size = 59, normalized size = 0.88

$$-\frac{3003 \, b^{5} x^{10} + 9009 \, a b^{4} x^{8} + 12870 \, a^{2} b^{3} x^{6} + 10010 \, a^{3} b^{2} x^{4} + 4095 \, a^{4} b x^{2} + 693 \, a^{5}}{9009 \, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Sympy [A]
time = 0.19, size = 63, normalized size = 0.94

$$\frac{-693a^5-4095a^4bx^2-10010a^3b^2x^4-12870a^2b^3x^6-9009ab^4x^8-3003b^5x^{10}}{9009x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**14,x)
```

```
[Out] (-693*a**5 - 4095*a**4*b*x**2 - 10010*a**3*b**2*x**4 - 12870*a**2*b**3*x**6
 - 9009*a*b**4*x**8 - 3003*b**5*x**10)/(9009*x**13)
```

Giac [A]
time = 1.32, size = 59, normalized size = 0.88

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Mupad [B]
time = 0.04, size = 58, normalized size = 0.87

$$-\frac{\frac{a^5}{13}+\frac{5\,a^4\,b\,x^2}{11}+\frac{10\,a^3\,b^2\,x^4}{9}+\frac{10\,a^2\,b^3\,x^6}{7}+a\,b^4\,x^8+\frac{b^5\,x^{10}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^14,x)
```

```
[Out] -(a^5/13 + (b^5*x^10)/3 + (5*a^4*b*x^2)/11 + a*b^4*x^8 + (10*a^3*b^2*x^4)/9
 + (10*a^2*b^3*x^6)/7)/x^13
```

3.82 $\int \frac{(a+bx^2)^5}{x^{16}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

```
[Out] -1/15*a^5/x^15-5/13*a^4*b/x^13-10/11*a^3*b^2/x^11-10/9*a^2*b^3/x^9-5/7*a*b^
4/x^7-1/5*b^5/x^5
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^16,x]
```

```
[Out] -1/15*a^5/x^15 - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^3
)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{16}}\,dx &= \int \left(\frac{a^5}{x^{16}}+\frac{5a^4b}{x^{14}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^8}+\frac{b^5}{x^6}\right)dx\\ &= -\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^16,x]
```

```
[Out] -1/15*a^5/x^15 - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^3
)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.84

method	result	size
default	$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5a\,b^4}{7x^7}-\frac{b^5}{5x^5}$	58
norman	$\frac{-\frac{1}{15}a^5-\frac{5}{13}a^4b\,x^2-\frac{10}{11}a^3b^2x^4-\frac{10}{9}a^2b^3x^6-\frac{5}{7}a\,b^4x^8-\frac{1}{5}b^5x^{10}}{x^{15}}$	59
risch	$\frac{-\frac{1}{15}a^5-\frac{5}{13}a^4b\,x^2-\frac{10}{11}a^3b^2x^4-\frac{10}{9}a^2b^3x^6-\frac{5}{7}a\,b^4x^8-\frac{1}{5}b^5x^{10}}{x^{15}}$	59
gosper	$-\frac{9009b^5x^{10}+32175a\,b^4x^8+50050a^2b^3x^6+40950a^3b^2x^4+17325a^4b\,x^2+3003a^5}{45045x^{15}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^5/x^15-5/13*a^4*b/x^13-10/11*a^3*b^2/x^11-10/9*a^2*b^3/x^9-5/7*a*b^
4/x^7-1/5*b^5/x^5
```

Maxima [A]
time = 0.29, size = 59, normalized size = 0.86

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Fricas [A]
time = 1.49, size = 59, normalized size = 0.86

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Sympy [A]
time = 0.21, size = 63, normalized size = 0.91

$$\frac{-3003a^5 - 17325a^4bx^2 - 40950a^3b^2x^4 - 50050a^2b^3x^6 - 32175ab^4x^8 - 9009b^5x^{10}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**16,x)
```

```
[Out] (-3003*a**5 - 17325*a**4*b*x**2 - 40950*a**3*b**2*x**4 - 50050*a**2*b**3*x*
*6 - 32175*a*b**4*x**8 - 9009*b**5*x**10)/(45045*x**15)
```

Giac [A]
time = 0.81, size = 59, normalized size = 0.86

$$-\frac{9009\,b^5x^{10} + 32175\,ab^4x^8 + 50050\,a^2b^3x^6 + 40950\,a^3b^2x^4 + 17325\,a^4bx^2 + 3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Mupad [B]
time = 4.75, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{15} + \frac{5\,a^4\,b\,x^2}{13} + \frac{10\,a^3\,b^2\,x^4}{11} + \frac{10\,a^2\,b^3\,x^6}{9} + \frac{5\,a\,b^4\,x^8}{7} + \frac{b^5\,x^{10}}{5}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^16,x)
```

```
[Out] -(a^5/15 + (b^5*x^10)/5 + (5*a^4*b*x^2)/13 + (5*a*b^4*x^8)/7 + (10*a^3*b^2*
x^4)/11 + (10*a^2*b^3*x^6)/9)/x^15
```

3.83 $\int \frac{(a+bx^2)^5}{x^{18}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

```
[Out] -1/17*a^5/x^17-1/3*a^4*b/x^15-10/13*a^3*b^2/x^13-10/11*a^2*b^3/x^11-5/9*a*b
^4/x^9-1/7*b^5/x^7
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^18,x]
```

```
[Out] -1/17*a^5/x^17 - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/(
11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{18}}\,dx &= \int \left(\frac{a^5}{x^{18}}+\frac{5a^4b}{x^{16}}+\frac{10a^3b^2}{x^{14}}+\frac{10a^2b^3}{x^{12}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^8}\right)dx\\ &= -\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^18,x]
```

```
[Out] -1/17*a^5/x^17 - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/(
11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.84

method	result	size
default	$-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5a\,b^4}{9x^9}-\frac{b^5}{7x^7}$	58
norman	$\frac{-\frac{1}{17}a^5-\frac{1}{3}a^4b\,x^2-\frac{10}{13}a^3b^2x^4-\frac{10}{11}a^2b^3x^6-\frac{5}{9}a\,b^4x^8-\frac{1}{7}b^5x^{10}}{x^{17}}$	59
risch	$\frac{-\frac{1}{17}a^5-\frac{1}{3}a^4b\,x^2-\frac{10}{13}a^3b^2x^4-\frac{10}{11}a^2b^3x^6-\frac{5}{9}a\,b^4x^8-\frac{1}{7}b^5x^{10}}{x^{17}}$	59
gosper	$-\frac{21879b^5x^{10}+85085a\,b^4x^8+139230a^2b^3x^6+117810a^3b^2x^4+51051a^4b\,x^2+9009a^5}{153153x^{17}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -1/17*a^5/x^17-1/3*a^4*b/x^15-10/13*a^3*b^2/x^13-10/11*a^2*b^3/x^11-5/9*a*b
^4/x^9-1/7*b^5/x^7
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.86

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="maxima")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Fricas [A]
time = 1.15, size = 59, normalized size = 0.86

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="fricas")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Sympy [A]
time = 0.23, size = 63, normalized size = 0.91

$$\frac{-9009a^5 - 51051a^4bx^2 - 117810a^3b^2x^4 - 139230a^2b^3x^6 - 85085ab^4x^8 - 21879b^5x^{10}}{153153x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**18,x)
```

```
[Out] (-9009*a**5 - 51051*a**4*b*x**2 - 117810*a**3*b**2*x**4 - 139230*a**2*b**3*
x**6 - 85085*a*b**4*x**8 - 21879*b**5*x**10)/(153153*x**17)
```

Giac [A]
time = 1.39, size = 59, normalized size = 0.86

$$-\frac{21879\,b^5x^{10} + 85085\,ab^4x^8 + 139230\,a^2b^3x^6 + 117810\,a^3b^2x^4 + 51051\,a^4bx^2 + 9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="giac")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Mupad [B]
time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{17} + \frac{a^4\,b\,x^2}{3} + \frac{10\,a^3\,b^2\,x^4}{13} + \frac{10\,a^2\,b^3\,x^6}{11} + \frac{5\,a\,b^4\,x^8}{9} + \frac{b^5\,x^{10}}{7}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^18,x)
```

```
[Out] -(a^5/17 + (b^5*x^10)/7 + (a^4*b*x^2)/3 + (5*a*b^4*x^8)/9 + (10*a^3*b^2*x^4
)/13 + (10*a^2*b^3*x^6)/11)/x^17
```

3.84 $\int \frac{(a+bx^2)^5}{x^{20}} \, dx$

Optimal. Leaf size=69

$$-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

```
[Out] -1/19*a^5/x^19-5/17*a^4*b/x^17-2/3*a^3*b^2/x^15-10/13*a^2*b^3/x^13-5/11*a*b
^4/x^11-1/9*b^5/x^9
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^20,x]
```

```
[Out] -1/19*a^5/x^19 - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)/
(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{20}}\,dx &= \int \left(\frac{a^5}{x^{20}}+\frac{5a^4b}{x^{18}}+\frac{10a^3b^2}{x^{16}}+\frac{10a^2b^3}{x^{14}}+\frac{5ab^4}{x^{12}}+\frac{b^5}{x^{10}}\right)dx\\ &= -\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^20,x]
```

```
[Out] -1/19*a^5/x^19 - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)/
(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.84

method	result	size
default	$-\frac{a^{5}}{19x^{19}}-\frac{5a^{4}b}{17x^{17}}-\frac{2a^{3}b^{2}}{3x^{15}}-\frac{10a^{2}b^{3}}{13x^{13}}-\frac{5a\,b^{4}}{11x^{11}}-\frac{b^{5}}{9x^{9}}$	58
norman	$\frac{-\frac{1}{19}a^{5}-\frac{5}{17}a^{4}b\,x^{2}-\frac{2}{3}a^{3}b^{2}x^{4}-\frac{10}{13}a^{2}b^{3}x^{6}-\frac{5}{11}a\,b^{4}x^{8}-\frac{1}{9}b^{5}x^{10}}{x^{19}}$	59
risch	$\frac{-\frac{1}{19}a^{5}-\frac{5}{17}a^{4}b\,x^{2}-\frac{2}{3}a^{3}b^{2}x^{4}-\frac{10}{13}a^{2}b^{3}x^{6}-\frac{5}{11}a\,b^{4}x^{8}-\frac{1}{9}b^{5}x^{10}}{x^{19}}$	59
gosper	$-\frac{46189b^{5}x^{10}+188955a\,b^{4}x^{8}+319770a^{2}b^{3}x^{6}+277134a^{3}b^{2}x^{4}+122265a^{4}b\,x^{2}+21879a^{5}}{415701x^{19}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/19*a^5/x^19-5/17*a^4*b/x^17-2/3*a^3*b^2/x^15-10/13*a^2*b^3/x^13-5/11*a*b
^4/x^11-1/9*b^5/x^9
```

Maxima [A]
time = 0.33, size = 59, normalized size = 0.86

$$-\frac{46189\,b^{5}x^{10}+188955\,ab^{4}x^{8}+319770\,a^{2}b^{3}x^{6}+277134\,a^{3}b^{2}x^{4}+122265\,a^{4}bx^{2}+21879\,a^{5}}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="maxima")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Fricas [A]
time = 0.99, size = 59, normalized size = 0.86

$$-\frac{46189\,b^{5}x^{10}+188955\,ab^{4}x^{8}+319770\,a^{2}b^{3}x^{6}+277134\,a^{3}b^{2}x^{4}+122265\,a^{4}bx^{2}+21879\,a^{5}}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="fricas")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Sympy [A]
time = 0.24, size = 63, normalized size = 0.91

$$\frac{-21879a^5 - 122265a^4bx^2 - 277134a^3b^2x^4 - 319770a^2b^3x^6 - 188955ab^4x^8 - 46189b^5x^{10}}{415701x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**20,x)
```

```
[Out] (-21879*a**5 - 122265*a**4*b*x**2 - 277134*a**3*b**2*x**4 - 319770*a**2*b**
3*x**6 - 188955*a*b**4*x**8 - 46189*b**5*x**10)/(415701*x**19)
```

Giac [A]
time = 1.04, size = 59, normalized size = 0.86

$$-\frac{46189\,b^5x^{10} + 188955\,ab^4x^8 + 319770\,a^2b^3x^6 + 277134\,a^3b^2x^4 + 122265\,a^4bx^2 + 21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="giac")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Mupad [B]
time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{19} + \frac{5\,a^4\,b\,x^2}{17} + \frac{2\,a^3\,b^2\,x^4}{3} + \frac{10\,a^2\,b^3\,x^6}{13} + \frac{5\,a\,b^4\,x^8}{11} + \frac{b^5\,x^{10}}{9}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^20,x)
```

```
[Out] -(a^5/19 + (b^5*x^10)/9 + (5*a^4*b*x^2)/17 + (5*a*b^4*x^8)/11 + (2*a^3*b^2*
x^4)/3 + (10*a^2*b^3*x^6)/13)/x^19
```

3.85 $\int x^{13}(a+bx^2)^8\,dx$

Optimal. Leaf size=129

$$\frac{a^6(a+bx^2)^9}{18b^7}-\frac{3a^5(a+bx^2)^{10}}{10b^7}+\frac{15a^4(a+bx^2)^{11}}{22b^7}-\frac{5a^3(a+bx^2)^{12}}{6b^7}+\frac{15a^2(a+bx^2)^{13}}{26b^7}-\frac{3a(a+bx^2)^{14}}{14b^7}+\frac{(a+bx}{30b}$$

```
[Out] 1/18*a^6*(b*x^2+a)^9/b^7-3/10*a^5*(b*x^2+a)^10/b^7+15/22*a^4*(b*x^2+a)^11/b
^7-5/6*a^3*(b*x^2+a)^12/b^7+15/26*a^2*(b*x^2+a)^13/b^7-3/14*a*(b*x^2+a)^14/
b^7+1/30*(b*x^2+a)^15/b^7
```

Rubi [A]

time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^6(a+bx^2)^9}{18b^7}-\frac{3a^5(a+bx^2)^{10}}{10b^7}+\frac{15a^4(a+bx^2)^{11}}{22b^7}-\frac{5a^3(a+bx^2)^{12}}{6b^7}+\frac{15a^2(a+bx^2)^{13}}{26b^7}+\frac{(a+bx^2)^{15}}{30b^7}-\frac{3a(a+bx^2)^{14}}{14b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^8,x]
```

```
[Out] (a^6*(a + b*x^2)^9)/(18*b^7) - (3*a^5*(a + b*x^2)^10)/(10*b^7) + (15*a^4*(a
 + b*x^2)^11)/(22*b^7) - (5*a^3*(a + b*x^2)^12)/(6*b^7) + (15*a^2*(a + b*x^
2)^13)/(26*b^7) - (3*a*(a + b*x^2)^14)/(14*b^7) + (a + b*x^2)^15/(30*b^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{13}(a+bx^2)^8\,dx &= \frac{1}{2}\text{Subst}\left(\int x^6(a+bx)^8\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^6(a+bx)^8}{b^6}-\frac{6a^5(a+bx)^9}{b^6}+\frac{15a^4(a+bx)^{10}}{b^6}-\frac{20a^3(a+bx)^{11}}{b^6}+15a\right.\right.\\ &= \frac{a^6(a+bx^2)^9}{18b^7}-\frac{3a^5(a+bx^2)^{10}}{10b^7}+\frac{15a^4(a+bx^2)^{11}}{22b^7}-\frac{5a^3(a+bx^2)^{12}}{6b^7}+\frac{15a^2(a+bx^2)}{26b^7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 108, normalized size = 0.84

$$\frac{a^8x^{14}}{14}+\frac{1}{2}a^7bx^{16}+\frac{14}{9}a^6b^2x^{18}+\frac{14}{5}a^5b^3x^{20}+\frac{35}{11}a^4b^4x^{22}+\frac{7}{3}a^3b^5x^{24}+\frac{14}{13}a^2b^6x^{26}+\frac{2}{7}ab^7x^{28}+\frac{b^8x^{30}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^14)/14 + (a^7*b*x^16)/2 + (14*a^6*b^2*x^18)/9 + (14*a^5*b^3*x^20)/5
+ (35*a^4*b^4*x^22)/11 + (7*a^3*b^5*x^24)/3 + (14*a^2*b^6*x^26)/13 + (2*a*b
^7*x^28)/7 + (b^8*x^30)/30
```

Maple [A]
time = 0.07, size = 91, normalized size = 0.71

method	result
gosper	$\frac{14}{9}a^6b^2x^{18}+\frac{14}{5}a^5b^3x^{20}+\frac{35}{11}a^4b^4x^{22}+\frac{7}{3}a^3b^5x^{24}+\frac{14}{13}a^2b^6x^{26}+\frac{2}{7}a\,b^7x^{28}+\frac{1}{30}b^8x^{30}+\frac{1}{14}a^8x^{14}+\frac{1}{2}a^7b$
default	$\frac{14}{9}a^6b^2x^{18}+\frac{14}{5}a^5b^3x^{20}+\frac{35}{11}a^4b^4x^{22}+\frac{7}{3}a^3b^5x^{24}+\frac{14}{13}a^2b^6x^{26}+\frac{2}{7}a\,b^7x^{28}+\frac{1}{30}b^8x^{30}+\frac{1}{14}a^8x^{14}+\frac{1}{2}a^7b$
norman	$\frac{14}{9}a^6b^2x^{18}+\frac{14}{5}a^5b^3x^{20}+\frac{35}{11}a^4b^4x^{22}+\frac{7}{3}a^3b^5x^{24}+\frac{14}{13}a^2b^6x^{26}+\frac{2}{7}a\,b^7x^{28}+\frac{1}{30}b^8x^{30}+\frac{1}{14}a^8x^{14}+\frac{1}{2}a^7b$
risch	$\frac{14}{9}a^6b^2x^{18}+\frac{14}{5}a^5b^3x^{20}+\frac{35}{11}a^4b^4x^{22}+\frac{7}{3}a^3b^5x^{24}+\frac{14}{13}a^2b^6x^{26}+\frac{2}{7}a\,b^7x^{28}+\frac{1}{30}b^8x^{30}+\frac{1}{14}a^8x^{14}+\frac{1}{2}a^7b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 14/9*a^6*b^2*x^18+14/5*a^5*b^3*x^20+35/11*a^4*b^4*x^22+7/3*a^3*b^5*x^24+14/
13*a^2*b^6*x^26+2/7*a*b^7*x^28+1/30*b^8*x^30+1/14*a^8*x^14+1/2*a^7*b*x^16
```

Maxima [A]
time = 0.33, size = 90, normalized size = 0.70

$$\frac{1}{30}\,b^8x^{30}+\frac{2}{7}\,ab^7x^{28}+\frac{14}{13}\,a^2b^6x^{26}+\frac{7}{3}\,a^3b^5x^{24}+\frac{35}{11}\,a^4b^4x^{22}+\frac{14}{5}\,a^5b^3x^{20}+\frac{14}{9}\,a^6b^2x^{18}+\frac{1}{2}\,a^7bx^{16}+\frac{1}{14}\,a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35
/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 +
 1/14*a^8*x^14
```

Fricas [A]
time = 1.08, size = 90, normalized size = 0.70

$$\frac{1}{30}\,b^8x^{30}+\frac{2}{7}\,ab^7x^{28}+\frac{14}{13}\,a^2b^6x^{26}+\frac{7}{3}\,a^3b^5x^{24}+\frac{35}{11}\,a^4b^4x^{22}+\frac{14}{5}\,a^5b^3x^{20}+\frac{14}{9}\,a^6b^2x^{18}+\frac{1}{2}\,a^7bx^{16}+\frac{1}{14}\,a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35
/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 +
 1/14*a^8*x^14
```

Sympy [A]
time = 0.02, size = 105, normalized size = 0.81

$$\frac{a^8x^{14}}{14}+\frac{a^7bx^{16}}{2}+\frac{14a^6b^2x^{18}}{9}+\frac{14a^5b^3x^{20}}{5}+\frac{35a^4b^4x^{22}}{11}+\frac{7a^3b^5x^{24}}{3}+\frac{14a^2b^6x^{26}}{13}+\frac{2ab^7x^{28}}{7}+\frac{b^8x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**14/14 + a**7*b*x**16/2 + 14*a**6*b**2*x**18/9 + 14*a**5*b**3*x**20/
5 + 35*a**4*b**4*x**22/11 + 7*a**3*b**5*x**24/3 + 14*a**2*b**6*x**26/13 + 2
*a*b**7*x**28/7 + b**8*x**30/30
```

Giac [A]
time = 1.06, size = 90, normalized size = 0.70

$$\frac{1}{30}b^8x^{30}+\frac{2}{7}ab^7x^{28}+\frac{14}{13}a^2b^6x^{26}+\frac{7}{3}a^3b^5x^{24}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{5}a^5b^3x^{20}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{2}a^7bx^{16}+\frac{1}{14}a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35
/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 +
 1/14*a^8*x^14
```

Mupad [B]
time = 0.10, size = 90, normalized size = 0.70

$$\frac{a^8\,x^{14}}{14}+\frac{a^7\,b\,x^{16}}{2}+\frac{14\,a^6\,b^2\,x^{18}}{9}+\frac{14\,a^5\,b^3\,x^{20}}{5}+\frac{35\,a^4\,b^4\,x^{22}}{11}+\frac{7\,a^3\,b^5\,x^{24}}{3}+\frac{14\,a^2\,b^6\,x^{26}}{13}+\frac{2\,a\,b^7\,x^{28}}{7}+\frac{b^8\,x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^14)/14 + (b^8*x^30)/30 + (a^7*b*x^16)/2 + (2*a*b^7*x^28)/7 + (14*a^6
*b^2*x^18)/9 + (14*a^5*b^3*x^20)/5 + (35*a^4*b^4*x^22)/11 + (7*a^3*b^5*x^24
)/3 + (14*a^2*b^6*x^26)/13
```

3.86 $\int x^{11}(a+bx^2)^8\,dx$

Optimal. Leaf size=110

$$-\frac{a^5(a+bx^2)^9}{18b^6}+\frac{a^4(a+bx^2)^{10}}{4b^6}-\frac{5a^3(a+bx^2)^{11}}{11b^6}+\frac{5a^2(a+bx^2)^{12}}{12b^6}-\frac{5a(a+bx^2)^{13}}{26b^6}+\frac{(a+bx^2)^{14}}{28b^6}$$

```
[Out] -1/18*a^5*(b*x^2+a)^9/b^6+1/4*a^4*(b*x^2+a)^10/b^6-5/11*a^3*(b*x^2+a)^11/b^
6+5/12*a^2*(b*x^2+a)^12/b^6-5/26*a*(b*x^2+a)^13/b^6+1/28*(b*x^2+a)^14/b^6
```

Rubi [A]

time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^5(a+bx^2)^9}{18b^6}+\frac{a^4(a+bx^2)^{10}}{4b^6}-\frac{5a^3(a+bx^2)^{11}}{11b^6}+\frac{5a^2(a+bx^2)^{12}}{12b^6}+\frac{(a+bx^2)^{14}}{28b^6}-\frac{5a(a+bx^2)^{13}}{26b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^8,x]
```

```
[Out] -1/18*(a^5*(a + b*x^2)^9)/b^6 + (a^4*(a + b*x^2)^10)/(4*b^6) - (5*a^3*(a +
b*x^2)^11)/(11*b^6) + (5*a^2*(a + b*x^2)^12)/(12*b^6) - (5*a*(a + b*x^2)^13
)/(26*b^6) + (a + b*x^2)^14/(28*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}(a+bx^2)^8\,dx &= \frac{1}{2}\text{Subst}\left(\int x^5(a+bx)^8\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^5(a+bx)^8}{b^5}+\frac{5a^4(a+bx)^9}{b^5}-\frac{10a^3(a+bx)^{10}}{b^5}+\frac{10a^2(a+bx)^{11}}{b^5}-5a\right.\right.\\ &= -\frac{a^5(a+bx^2)^9}{18b^6}+\frac{a^4(a+bx^2)^{10}}{4b^6}-\frac{5a^3(a+bx^2)^{11}}{11b^6}+\frac{5a^2(a+bx^2)^{12}}{12b^6}-\frac{5a(a+bx^2)^{13}}{26b^6}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 108, normalized size = 0.98

$$\frac{a^8x^{12}}{12}+\frac{4}{7}a^7bx^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{28}{9}a^5b^3x^{18}+\frac{7}{2}a^4b^4x^{20}+\frac{28}{11}a^3b^5x^{22}+\frac{7}{6}a^2b^6x^{24}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{28}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^12)/12 + (4*a^7*b*x^14)/7 + (7*a^6*b^2*x^16)/4 + (28*a^5*b^3*x^18)/9
 + (7*a^4*b^4*x^20)/2 + (28*a^3*b^5*x^22)/11 + (7*a^2*b^6*x^24)/6 + (4*a*b^
7*x^26)/13 + (b^8*x^28)/28
```

Maple [A]
time = 0.06, size = 91, normalized size = 0.83

method	result
gosper	$\frac{1}{12}a^8x^{12}+\frac{4}{7}a^7b\,x^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{28}{9}a^5b^3x^{18}+\frac{7}{2}a^4b^4x^{20}+\frac{28}{11}a^3b^5x^{22}+\frac{7}{6}a^2b^6x^{24}+\frac{4}{13}a\,b^7x^{26}+\frac{1}{28}b$
default	$\frac{1}{12}a^8x^{12}+\frac{4}{7}a^7b\,x^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{28}{9}a^5b^3x^{18}+\frac{7}{2}a^4b^4x^{20}+\frac{28}{11}a^3b^5x^{22}+\frac{7}{6}a^2b^6x^{24}+\frac{4}{13}a\,b^7x^{26}+\frac{1}{28}b$
norman	$\frac{1}{12}a^8x^{12}+\frac{4}{7}a^7b\,x^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{28}{9}a^5b^3x^{18}+\frac{7}{2}a^4b^4x^{20}+\frac{28}{11}a^3b^5x^{22}+\frac{7}{6}a^2b^6x^{24}+\frac{4}{13}a\,b^7x^{26}+\frac{1}{28}b$
risch	$\frac{1}{12}a^8x^{12}+\frac{4}{7}a^7b\,x^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{28}{9}a^5b^3x^{18}+\frac{7}{2}a^4b^4x^{20}+\frac{28}{11}a^3b^5x^{22}+\frac{7}{6}a^2b^6x^{24}+\frac{4}{13}a\,b^7x^{26}+\frac{1}{28}b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a^8*x^12+4/7*a^7*b*x^14+7/4*a^6*b^2*x^16+28/9*a^5*b^3*x^18+7/2*a^4*b^4
*x^20+28/11*a^3*b^5*x^22+7/6*a^2*b^6*x^24+4/13*a*b^7*x^26+1/28*b^8*x^28
```

Maxima [A]
time = 0.30, size = 90, normalized size = 0.82

$$\frac{1}{28}\,b^8x^{28}+\frac{4}{13}\,ab^7x^{26}+\frac{7}{6}\,a^2b^6x^{24}+\frac{28}{11}\,a^3b^5x^{22}+\frac{7}{2}\,a^4b^4x^{20}+\frac{28}{9}\,a^5b^3x^{18}+\frac{7}{4}\,a^6b^2x^{16}+\frac{4}{7}\,a^7bx^{14}+\frac{1}{12}\,a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7
/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1
/12*a^8*x^12
```

Fricas [A]
time = 1.28, size = 90, normalized size = 0.82

$$\frac{1}{28}\,b^8x^{28}+\frac{4}{13}\,ab^7x^{26}+\frac{7}{6}\,a^2b^6x^{24}+\frac{28}{11}\,a^3b^5x^{22}+\frac{7}{2}\,a^4b^4x^{20}+\frac{28}{9}\,a^5b^3x^{18}+\frac{7}{4}\,a^6b^2x^{16}+\frac{4}{7}\,a^7bx^{14}+\frac{1}{12}\,a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7
/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1
/12*a^8*x^12
```

Sympy [A]
time = 0.02, size = 107, normalized size = 0.97

$$\frac{a^8x^{12}}{12} + \frac{4a^7bx^{14}}{7} + \frac{7a^6b^2x^{16}}{4} + \frac{28a^5b^3x^{18}}{9} + \frac{7a^4b^4x^{20}}{2} + \frac{28a^3b^5x^{22}}{11} + \frac{7a^2b^6x^{24}}{6} + \frac{4ab^7x^{26}}{13} + \frac{b^8x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**12/12 + 4*a**7*b*x**14/7 + 7*a**6*b**2*x**16/4 + 28*a**5*b**3*x**18
/9 + 7*a**4*b**4*x**20/2 + 28*a**3*b**5*x**22/11 + 7*a**2*b**6*x**24/6 + 4*
a*b**7*x**26/13 + b**8*x**28/28
```

Giac [A]
time = 1.23, size = 90, normalized size = 0.82

$$\frac{1}{28}\,b^8x^{28} + \frac{4}{13}\,ab^7x^{26} + \frac{7}{6}\,a^2b^6x^{24} + \frac{28}{11}\,a^3b^5x^{22} + \frac{7}{2}\,a^4b^4x^{20} + \frac{28}{9}\,a^5b^3x^{18} + \frac{7}{4}\,a^6b^2x^{16} + \frac{4}{7}\,a^7bx^{14} + \frac{1}{12}\,a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7
/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1
/12*a^8*x^12
```

Mupad [B]
time = 4.57, size = 90, normalized size = 0.82

$$\frac{a^8\,x^{12}}{12} + \frac{4\,a^7\,b\,x^{14}}{7} + \frac{7\,a^6\,b^2\,x^{16}}{4} + \frac{28\,a^5\,b^3\,x^{18}}{9} + \frac{7\,a^4\,b^4\,x^{20}}{2} + \frac{28\,a^3\,b^5\,x^{22}}{11} + \frac{7\,a^2\,b^6\,x^{24}}{6} + \frac{4\,a\,b^7\,x^{26}}{13} + \frac{b^8\,x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^12)/12 + (b^8*x^28)/28 + (4*a^7*b*x^14)/7 + (4*a*b^7*x^26)/13 + (7*a
^6*b^2*x^16)/4 + (28*a^5*b^3*x^18)/9 + (7*a^4*b^4*x^20)/2 + (28*a^3*b^5*x^2
2)/11 + (7*a^2*b^6*x^24)/6
```

3.87 $\int x^9(a+bx^2)^8\,dx$

Optimal. Leaf size=91

$$\frac{a^4(a+bx^2)^9}{18b^5}-\frac{a^3(a+bx^2)^{10}}{5b^5}+\frac{3a^2(a+bx^2)^{11}}{11b^5}-\frac{a(a+bx^2)^{12}}{6b^5}+\frac{(a+bx^2)^{13}}{26b^5}$$

```
[Out] 1/18*a^4*(b*x^2+a)^9/b^5-1/5*a^3*(b*x^2+a)^10/b^5+3/11*a^2*(b*x^2+a)^11/b^5
-1/6*a*(b*x^2+a)^12/b^5+1/26*(b*x^2+a)^13/b^5
```

Rubi [A]
time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^4(a+bx^2)^9}{18b^5}-\frac{a^3(a+bx^2)^{10}}{5b^5}+\frac{3a^2(a+bx^2)^{11}}{11b^5}+\frac{(a+bx^2)^{13}}{26b^5}-\frac{a(a+bx^2)^{12}}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^8,x]
```

```
[Out] (a^4*(a + b*x^2)^9)/(18*b^5) - (a^3*(a + b*x^2)^10)/(5*b^5) + (3*a^2*(a + b
*x^2)^11)/(11*b^5) - (a*(a + b*x^2)^12)/(6*b^5) + (a + b*x^2)^13/(26*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^8\,dx &= \frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^8\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^4(a+bx)^8}{b^4}-\frac{4a^3(a+bx)^9}{b^4}+\frac{6a^2(a+bx)^{10}}{b^4}-\frac{4a(a+bx)^{11}}{b^4}+\frac{(a+bx}{b^4}\right.\right.\\ &= \frac{a^4(a+bx^2)^9}{18b^5}-\frac{a^3(a+bx^2)^{10}}{5b^5}+\frac{3a^2(a+bx^2)^{11}}{11b^5}-\frac{a(a+bx^2)^{12}}{6b^5}+\frac{(a+bx^2)^{13}}{26b^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 106, normalized size = 1.16

$$\frac{a^8x^{10}}{10}+\frac{2}{3}a^7bx^{12}+2a^6b^2x^{14}+\frac{7}{2}a^5b^3x^{16}+\frac{35}{9}a^4b^4x^{18}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+\frac{1}{3}ab^7x^{24}+\frac{b^8x^{26}}{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^10)/10 + (2*a^7*b*x^12)/3 + 2*a^6*b^2*x^14 + (7*a^5*b^3*x^16)/2 + (3
5*a^4*b^4*x^18)/9 + (14*a^3*b^5*x^20)/5 + (14*a^2*b^6*x^22)/11 + (a*b^7*x^2
4)/3 + (b^8*x^26)/26
```

Maple [A]
time = 0.06, size = 91, normalized size = 1.00

method	result
gosper	$\frac{35}{9}a^4b^4x^{18}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+\frac{1}{3}a\,b^7x^{24}+\frac{1}{26}b^8x^{26}+\frac{1}{10}a^8x^{10}+\frac{2}{3}a^7b\,x^{12}+2a^6b^2x^{14}+\frac{7}{2}a^5b^3x$
default	$\frac{35}{9}a^4b^4x^{18}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+\frac{1}{3}a\,b^7x^{24}+\frac{1}{26}b^8x^{26}+\frac{1}{10}a^8x^{10}+\frac{2}{3}a^7b\,x^{12}+2a^6b^2x^{14}+\frac{7}{2}a^5b^3x$
norman	$\frac{35}{9}a^4b^4x^{18}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+\frac{1}{3}a\,b^7x^{24}+\frac{1}{26}b^8x^{26}+\frac{1}{10}a^8x^{10}+\frac{2}{3}a^7b\,x^{12}+2a^6b^2x^{14}+\frac{7}{2}a^5b^3x$
risch	$\frac{35}{9}a^4b^4x^{18}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+\frac{1}{3}a\,b^7x^{24}+\frac{1}{26}b^8x^{26}+\frac{1}{10}a^8x^{10}+\frac{2}{3}a^7b\,x^{12}+2a^6b^2x^{14}+\frac{7}{2}a^5b^3x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 35/9*a^4*b^4*x^18+14/5*a^3*b^5*x^20+14/11*a^2*b^6*x^22+1/3*a*b^7*x^24+1/26*
b^8*x^26+1/10*a^8*x^10+2/3*a^7*b*x^12+2*a^6*b^2*x^14+7/2*a^5*b^3*x^16
```

Maxima [A]
time = 0.28, size = 90, normalized size = 0.99

$$\frac{1}{26}\,b^8x^{26}+\frac{1}{3}\,ab^7x^{24}+\frac{14}{11}\,a^2b^6x^{22}+\frac{14}{5}\,a^3b^5x^{20}+\frac{35}{9}\,a^4b^4x^{18}+\frac{7}{2}\,a^5b^3x^{16}+2\,a^6b^2x^{14}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{10}\,a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 + 3
5/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1/1
0*a^8*x^10
```

Fricas [A]
time = 1.07, size = 90, normalized size = 0.99

$$\frac{1}{26}\,b^8x^{26}+\frac{1}{3}\,ab^7x^{24}+\frac{14}{11}\,a^2b^6x^{22}+\frac{14}{5}\,a^3b^5x^{20}+\frac{35}{9}\,a^4b^4x^{18}+\frac{7}{2}\,a^5b^3x^{16}+2\,a^6b^2x^{14}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{10}\,a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 + 3
5/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1/1
0*a^8*x^10
```

Sympy [A]
time = 0.02, size = 104, normalized size = 1.14

$$\frac{a^8x^{10}}{10}+\frac{2a^7bx^{12}}{3}+2a^6b^2x^{14}+\frac{7a^5b^3x^{16}}{2}+\frac{35a^4b^4x^{18}}{9}+\frac{14a^3b^5x^{20}}{5}+\frac{14a^2b^6x^{22}}{11}+\frac{ab^7x^{24}}{3}+\frac{b^8x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**10/10 + 2*a**7*b*x**12/3 + 2*a**6*b**2*x**14 + 7*a**5*b**3*x**16/2
+ 35*a**4*b**4*x**18/9 + 14*a**3*b**5*x**20/5 + 14*a**2*b**6*x**22/11 + a*b
**7*x**24/3 + b**8*x**26/26
```

Giac [A]
time = 1.34, size = 90, normalized size = 0.99

$$\frac{1}{26}\,b^8x^{26}+\frac{1}{3}\,ab^7x^{24}+\frac{14}{11}\,a^2b^6x^{22}+\frac{14}{5}\,a^3b^5x^{20}+\frac{35}{9}\,a^4b^4x^{18}+\frac{7}{2}\,a^5b^3x^{16}+2\,a^6b^2x^{14}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{10}\,a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 + 3
5/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1/1
0*a^8*x^10
```

Mupad [B]
time = 4.59, size = 90, normalized size = 0.99

$$\frac{a^8\,x^{10}}{10}+\frac{2\,a^7\,b\,x^{12}}{3}+2\,a^6\,b^2\,x^{14}+\frac{7\,a^5\,b^3\,x^{16}}{2}+\frac{35\,a^4\,b^4\,x^{18}}{9}+\frac{14\,a^3\,b^5\,x^{20}}{5}+\frac{14\,a^2\,b^6\,x^{22}}{11}+\frac{a\,b^7\,x^{24}}{3}+\frac{b^8\,x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^10)/10 + (b^8*x^26)/26 + (2*a^7*b*x^12)/3 + (a*b^7*x^24)/3 + 2*a^6*b
^2*x^14 + (7*a^5*b^3*x^16)/2 + (35*a^4*b^4*x^18)/9 + (14*a^3*b^5*x^20)/5 +
(14*a^2*b^6*x^22)/11
```

3.88 $\int x^7(a+bx^2)^8 \, dx$

Optimal. Leaf size=72

$$-\frac{a^3(a+bx^2)^9}{18b^4}+\frac{3a^2(a+bx^2)^{10}}{20b^4}-\frac{3a(a+bx^2)^{11}}{22b^4}+\frac{(a+bx^2)^{12}}{24b^4}$$

```
[Out] -1/18*a^3*(b*x^2+a)^9/b^4+3/20*a^2*(b*x^2+a)^10/b^4-3/22*a*(b*x^2+a)^11/b^4
+1/24*(b*x^2+a)^12/b^4
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^3(a+bx^2)^9}{18b^4}+\frac{3a^2(a+bx^2)^{10}}{20b^4}+\frac{(a+bx^2)^{12}}{24b^4}-\frac{3a(a+bx^2)^{11}}{22b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^8,x]
```

```
[Out] -1/18*(a^3*(a + b*x^2)^9)/b^4 + (3*a^2*(a + b*x^2)^10)/(20*b^4) - (3*a*(a +
 b*x^2)^11)/(22*b^4) + (a + b*x^2)^12/(24*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7\left(a+bx^2\right)^8 \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^8 \, dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^8}{b^3}+\frac{3a^2(a+bx)^9}{b^3}-\frac{3a(a+bx)^{10}}{b^3}+\frac{(a+bx)^{11}}{b^3}\right) \, dx,x,x^2\right)\\
&= -\frac{a^3(a+bx^2)^9}{18b^4}+\frac{3a^2(a+bx^2)^{10}}{20b^4}-\frac{3a(a+bx^2)^{11}}{22b^4}+\frac{(a+bx^2)^{12}}{24b^4}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 106, normalized size = 1.47

$$\frac{a^8x^8}{8}+\frac{4}{5}a^7bx^{10}+\frac{7}{3}a^6b^2x^{12}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{28}{9}a^3b^5x^{18}+\frac{7}{5}a^2b^6x^{20}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^8)/8 + (4*a^7*b*x^10)/5 + (7*a^6*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35*
a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9 + (7*a^2*b^6*x^20)/5 + (4*a*b^7*x^22)
/11 + (b^8*x^24)/24
```

Maple [A]
time = 0.06, size = 91, normalized size = 1.26

method	result
gosper	$\frac{4}{11}a\,b^7x^{22}+\frac{1}{24}b^8x^{24}+\frac{1}{8}a^8x^8+\frac{4}{5}a^7b\,x^{10}+\frac{7}{3}a^6b^2x^{12}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{28}{9}a^3b^5x^{18}+\frac{7}{5}a^2b^6x$
default	$\frac{4}{11}a\,b^7x^{22}+\frac{1}{24}b^8x^{24}+\frac{1}{8}a^8x^8+\frac{4}{5}a^7b\,x^{10}+\frac{7}{3}a^6b^2x^{12}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{28}{9}a^3b^5x^{18}+\frac{7}{5}a^2b^6x$
norman	$\frac{4}{11}a\,b^7x^{22}+\frac{1}{24}b^8x^{24}+\frac{1}{8}a^8x^8+\frac{4}{5}a^7b\,x^{10}+\frac{7}{3}a^6b^2x^{12}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{28}{9}a^3b^5x^{18}+\frac{7}{5}a^2b^6x$
risch	$\frac{4}{11}a\,b^7x^{22}+\frac{1}{24}b^8x^{24}+\frac{1}{8}a^8x^8+\frac{4}{5}a^7b\,x^{10}+\frac{7}{3}a^6b^2x^{12}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{28}{9}a^3b^5x^{18}+\frac{7}{5}a^2b^6x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 4/11*a*b^7*x^22+1/24*b^8*x^24+1/8*a^8*x^8+4/5*a^7*b*x^10+7/3*a^6*b^2*x^12+4
*a^5*b^3*x^14+35/8*a^4*b^4*x^16+28/9*a^3*b^5*x^18+7/5*a^2*b^6*x^20
```

Maxima [A]
time = 0.28, size = 90, normalized size = 1.25

$$\frac{1}{24}\,b^8x^{24}+\frac{4}{11}\,ab^7x^{22}+\frac{7}{5}\,a^2b^6x^{20}+\frac{28}{9}\,a^3b^5x^{18}+\frac{35}{8}\,a^4b^4x^{16}+4\,a^5b^3x^{14}+\frac{7}{3}\,a^6b^2x^{12}+\frac{4}{5}\,a^7bx^{10}+\frac{1}{8}\,a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 35
/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/8*
a^8*x^8
```

Fricas [A]
time = 1.40, size = 90, normalized size = 1.25

$$\frac{1}{24}\,b^8x^{24}+\frac{4}{11}\,ab^7x^{22}+\frac{7}{5}\,a^2b^6x^{20}+\frac{28}{9}\,a^3b^5x^{18}+\frac{35}{8}\,a^4b^4x^{16}+4\,a^5b^3x^{14}+\frac{7}{3}\,a^6b^2x^{12}+\frac{4}{5}\,a^7bx^{10}+\frac{1}{8}\,a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 35
/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/8*
a^8*x^8
```

Sympy [A]
time = 0.02, size = 105, normalized size = 1.46

$$\frac{a^8x^8}{8}+\frac{4a^7bx^{10}}{5}+\frac{7a^6b^2x^{12}}{3}+4a^5b^3x^{14}+\frac{35a^4b^4x^{16}}{8}+\frac{28a^3b^5x^{18}}{9}+\frac{7a^2b^6x^{20}}{5}+\frac{4ab^7x^{22}}{11}+\frac{b^8x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**8/8 + 4*a**7*b*x**10/5 + 7*a**6*b**2*x**12/3 + 4*a**5*b**3*x**14 +
35*a**4*b**4*x**16/8 + 28*a**3*b**5*x**18/9 + 7*a**2*b**6*x**20/5 + 4*a*b**
7*x**22/11 + b**8*x**24/24
```

Giac [A]
time = 0.83, size = 90, normalized size = 1.25

$$\frac{1}{24}\,b^8x^{24}+\frac{4}{11}\,ab^7x^{22}+\frac{7}{5}\,a^2b^6x^{20}+\frac{28}{9}\,a^3b^5x^{18}+\frac{35}{8}\,a^4b^4x^{16}+4\,a^5b^3x^{14}+\frac{7}{3}\,a^6b^2x^{12}+\frac{4}{5}\,a^7bx^{10}+\frac{1}{8}\,a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 35
/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/8*
a^8*x^8
```

Mupad [B]
time = 0.09, size = 90, normalized size = 1.25

$$\frac{a^8\,x^8}{8}+\frac{4\,a^7\,b\,x^{10}}{5}+\frac{7\,a^6\,b^2\,x^{12}}{3}+4\,a^5\,b^3\,x^{14}+\frac{35\,a^4\,b^4\,x^{16}}{8}+\frac{28\,a^3\,b^5\,x^{18}}{9}+\frac{7\,a^2\,b^6\,x^{20}}{5}+\frac{4\,a\,b^7\,x^{22}}{11}+\frac{b^8\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^8)/8 + (b^8*x^24)/24 + (4*a^7*b*x^10)/5 + (4*a*b^7*x^22)/11 + (7*a^6
*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35*a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9 +
 (7*a^2*b^6*x^20)/5
```

3.89 $\int x^5(a+bx^2)^8\,dx$

Optimal. Leaf size=53

$$\frac{a^2(a+bx^2)^9}{18b^3}-\frac{a(a+bx^2)^{10}}{10b^3}+\frac{(a+bx^2)^{11}}{22b^3}$$

```
[Out] 1/18*a^2*(b*x^2+a)^9/b^3-1/10*a*(b*x^2+a)^10/b^3+1/22*(b*x^2+a)^11/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 45}

$$\frac{a^2(a+bx^2)^9}{18b^3}+\frac{(a+bx^2)^{11}}{22b^3}-\frac{a(a+bx^2)^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^8,x]
```

```
[Out] (a^2*(a + b*x^2)^9)/(18*b^3) - (a*(a + b*x^2)^10)/(10*b^3) + (a + b*x^2)^11
/(22*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx^2)^8\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^8\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^8}{b^2}-\frac{2a(a+bx)^9}{b^2}+\frac{(a+bx)^{10}}{b^2}\right)dx,x,x^2\right)\\&=\frac{a^2(a+bx^2)^9}{18b^3}-\frac{a(a+bx^2)^{10}}{10b^3}+\frac{(a+bx^2)^{11}}{22b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 103, normalized size = 1.94

$$\frac{a^8x^6}{6}+a^7bx^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}ab^7x^{20}+\frac{b^8x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^6)/6 + a^7*b*x^8 + (14*a^6*b^2*x^10)/5 + (14*a^5*b^3*x^12)/3 + 5*a^4
*b^4*x^14 + (7*a^3*b^5*x^16)/2 + (14*a^2*b^6*x^18)/9 + (2*a*b^7*x^20)/5 + (
b^8*x^22)/22
```

Maple [A]
time = 0.06, size = 90, normalized size = 1.70

method	result
gosper	$\frac{1}{6}a^8x^6+a^7b\,x^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}a\,b^7x^{20}+\frac{1}{22}b^8x^{22}$
default	$\frac{1}{6}a^8x^6+a^7b\,x^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}a\,b^7x^{20}+\frac{1}{22}b^8x^{22}$
norman	$\frac{1}{6}a^8x^6+a^7b\,x^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}a\,b^7x^{20}+\frac{1}{22}b^8x^{22}$
risch	$\frac{1}{6}a^8x^6+a^7b\,x^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}a\,b^7x^{20}+\frac{1}{22}b^8x^{22}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^8*x^6+a^7*b*x^8+14/5*a^6*b^2*x^10+14/3*a^5*b^3*x^12+5*a^4*b^4*x^14+7/
2*a^3*b^5*x^16+14/9*a^2*b^6*x^18+2/5*a*b^7*x^20+1/22*b^8*x^22
```

Maxima [A]
time = 0.27, size = 89, normalized size = 1.68

$$\frac{1}{22}\,b^8x^{22}+\frac{2}{5}\,ab^7x^{20}+\frac{14}{9}\,a^2b^6x^{18}+\frac{7}{2}\,a^3b^5x^{16}+5\,a^4b^4x^{14}+\frac{14}{3}\,a^5b^3x^{12}+\frac{14}{5}\,a^6b^2x^{10}+a^7bx^8+\frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*a
^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8*x
^6
```

Fricas [A]
time = 1.02, size = 89, normalized size = 1.68

$$\frac{1}{22}\,b^8x^{22}+\frac{2}{5}\,ab^7x^{20}+\frac{14}{9}\,a^2b^6x^{18}+\frac{7}{2}\,a^3b^5x^{16}+5\,a^4b^4x^{14}+\frac{14}{3}\,a^5b^3x^{12}+\frac{14}{5}\,a^6b^2x^{10}+a^7bx^8+\frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*a
^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8*x
^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(44) = 88$.
time = 0.02, size = 102, normalized size = 1.92

$$\frac{a^8x^6}{6}+a^7bx^8+\frac{14a^6b^2x^{10}}{5}+\frac{14a^5b^3x^{12}}{3}+5a^4b^4x^{14}+\frac{7a^3b^5x^{16}}{2}+\frac{14a^2b^6x^{18}}{9}+\frac{2ab^7x^{20}}{5}+\frac{b^8x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**6/6 + a**7*b*x**8 + 14*a**6*b**2*x**10/5 + 14*a**5*b**3*x**12/3 + 5
*a**4*b**4*x**14 + 7*a**3*b**5*x**16/2 + 14*a**2*b**6*x**18/9 + 2*a*b**7*x*
*20/5 + b**8*x**22/22
```

Giac [A]
time = 0.92, size = 89, normalized size = 1.68

$$\frac{1}{22}\,b^8x^{22}+\frac{2}{5}\,ab^7x^{20}+\frac{14}{9}\,a^2b^6x^{18}+\frac{7}{2}\,a^3b^5x^{16}+5\,a^4b^4x^{14}+\frac{14}{3}\,a^5b^3x^{12}+\frac{14}{5}\,a^6b^2x^{10}+a^7bx^8+\frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*a
^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8*x
^6
```

Mupad [B]
time = 0.09, size = 89, normalized size = 1.68

$$\frac{a^8\,x^6}{6}+a^7\,b\,x^8+\frac{14\,a^6\,b^2\,x^{10}}{5}+\frac{14\,a^5\,b^3\,x^{12}}{3}+5\,a^4\,b^4\,x^{14}+\frac{7\,a^3\,b^5\,x^{16}}{2}+\frac{14\,a^2\,b^6\,x^{18}}{9}+\frac{2\,a\,b^7\,x^{20}}{5}+\frac{b^8\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^6)/6 + (b^8*x^22)/22 + a^7*b*x^8 + (2*a*b^7*x^20)/5 + (14*a^6*b^2*x^
10)/5 + (14*a^5*b^3*x^12)/3 + 5*a^4*b^4*x^14 + (7*a^3*b^5*x^16)/2 + (14*a^2
*b^6*x^18)/9
```

3.90 $\int x^3(a+bx^2)^8\,dx$

Optimal. Leaf size=34

$$-\frac{a(a+bx^2)^9}{18b^2}+\frac{(a+bx^2)^{10}}{20b^2}$$

```
[Out] -1/18*a*(b*x^2+a)^9/b^2+1/20*(b*x^2+a)^10/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{(a+bx^2)^{10}}{20b^2}-\frac{a(a+bx^2)^9}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^8,x]
```

```
[Out] -1/18*(a*(a + b*x^2)^9)/b^2 + (a + b*x^2)^10/(20*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+bx^2\right)^8\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^8\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^8}{b}+\frac{(a+bx)^9}{b}\right)dx,x,x^2\right)\\
&= -\frac{a(a+bx^2)^9}{18b^2}+\frac{(a+bx^2)^{10}}{20b^2}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(34) = 68.
time = 0.00, size = 106, normalized size = 3.12

$$\frac{a^8x^4}{4}+\frac{4}{3}a^7bx^6+\frac{7}{2}a^6b^2x^8+\frac{28}{5}a^5b^3x^{10}+\frac{35}{6}a^4b^4x^{12}+4a^3b^5x^{14}+\frac{7}{4}a^2b^6x^{16}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^4)/4 + (4*a^7*b*x^6)/3 + (7*a^6*b^2*x^8)/2 + (28*a^5*b^3*x^10)/5 + (
35*a^4*b^4*x^12)/6 + 4*a^3*b^5*x^14 + (7*a^2*b^6*x^16)/4 + (4*a*b^7*x^18)/9
 + (b^8*x^20)/20
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(30) = 60.
time = 0.04, size = 91, normalized size = 2.68

method	result
gosper	$\frac{1}{4}a^8x^4+\frac{4}{3}a^7b\,x^6+\frac{7}{2}a^6b^2x^8+\frac{28}{5}a^5b^3x^{10}+\frac{35}{6}a^4b^4x^{12}+4a^3b^5x^{14}+\frac{7}{4}a^2b^6x^{16}+\frac{4}{9}a\,b^7x^{18}+\frac{1}{20}b^8x^{20}$
default	$\frac{1}{4}a^8x^4+\frac{4}{3}a^7b\,x^6+\frac{7}{2}a^6b^2x^8+\frac{28}{5}a^5b^3x^{10}+\frac{35}{6}a^4b^4x^{12}+4a^3b^5x^{14}+\frac{7}{4}a^2b^6x^{16}+\frac{4}{9}a\,b^7x^{18}+\frac{1}{20}b^8x^{20}$
norman	$\frac{1}{4}a^8x^4+\frac{4}{3}a^7b\,x^6+\frac{7}{2}a^6b^2x^8+\frac{28}{5}a^5b^3x^{10}+\frac{35}{6}a^4b^4x^{12}+4a^3b^5x^{14}+\frac{7}{4}a^2b^6x^{16}+\frac{4}{9}a\,b^7x^{18}+\frac{1}{20}b^8x^{20}$
risch	$\frac{1}{4}a^8x^4+\frac{4}{3}a^7b\,x^6+\frac{7}{2}a^6b^2x^8+\frac{28}{5}a^5b^3x^{10}+\frac{35}{6}a^4b^4x^{12}+4a^3b^5x^{14}+\frac{7}{4}a^2b^6x^{16}+\frac{4}{9}a\,b^7x^{18}+\frac{1}{20}b^8x^{20}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^8*x^4+4/3*a^7*b*x^6+7/2*a^6*b^2*x^8+28/5*a^5*b^3*x^10+35/6*a^4*b^4*x^
12+4*a^3*b^5*x^14+7/4*a^2*b^6*x^16+4/9*a*b^7*x^18+1/20*b^8*x^20
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(30) = 60.
time = 0.27, size = 90, normalized size = 2.65

$$\frac{1}{20}\,b^8x^{20}+\frac{4}{9}\,ab^7x^{18}+\frac{7}{4}\,a^2b^6x^{16}+4\,a^3b^5x^{14}+\frac{35}{6}\,a^4b^4x^{12}+\frac{28}{5}\,a^5b^3x^{10}+\frac{7}{2}\,a^6b^2x^8+\frac{4}{3}\,a^7bx^6+\frac{1}{4}\,a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*a
^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a^8
*x^4
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.
time = 1.26, size = 90, normalized size = 2.65

$$\frac{1}{20}\,b^8x^{20}+\frac{4}{9}\,ab^7x^{18}+\frac{7}{4}\,a^2b^6x^{16}+4\,a^3b^5x^{14}+\frac{35}{6}\,a^4b^4x^{12}+\frac{28}{5}\,a^5b^3x^{10}+\frac{7}{2}\,a^6b^2x^8+\frac{4}{3}\,a^7bx^6+\frac{1}{4}\,a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*a
^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a^8
*x^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(27) = 54$.
time = 0.02, size = 105, normalized size = 3.09

$$\frac{a^8x^4}{4}+\frac{4a^7bx^6}{3}+\frac{7a^6b^2x^8}{2}+\frac{28a^5b^3x^{10}}{5}+\frac{35a^4b^4x^{12}}{6}+4a^3b^5x^{14}+\frac{7a^2b^6x^{16}}{4}+\frac{4ab^7x^{18}}{9}+\frac{b^8x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**4/4 + 4*a**7*b*x**6/3 + 7*a**6*b**2*x**8/2 + 28*a**5*b**3*x**10/5 +
 35*a**4*b**4*x**12/6 + 4*a**3*b**5*x**14 + 7*a**2*b**6*x**16/4 + 4*a*b**7*
x**18/9 + b**8*x**20/20
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.
time = 0.73, size = 90, normalized size = 2.65

$$\frac{1}{20}\,b^8x^{20}+\frac{4}{9}\,ab^7x^{18}+\frac{7}{4}\,a^2b^6x^{16}+4\,a^3b^5x^{14}+\frac{35}{6}\,a^4b^4x^{12}+\frac{28}{5}\,a^5b^3x^{10}+\frac{7}{2}\,a^6b^2x^8+\frac{4}{3}\,a^7bx^6+\frac{1}{4}\,a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*a
^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a^8
*x^4
```

Mupad [**B**]
time = 0.09, size = 90, normalized size = 2.65

$$\frac{a^8\,x^4}{4}+\frac{4\,a^7\,b\,x^6}{3}+\frac{7\,a^6\,b^2\,x^8}{2}+\frac{28\,a^5\,b^3\,x^{10}}{5}+\frac{35\,a^4\,b^4\,x^{12}}{6}+4\,a^3\,b^5\,x^{14}+\frac{7\,a^2\,b^6\,x^{16}}{4}+\frac{4\,a\,b^7\,x^{18}}{9}+\frac{b^8\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^4)/4 + (b^8*x^20)/20 + (4*a^7*b*x^6)/3 + (4*a*b^7*x^18)/9 + (7*a^6*b
^2*x^8)/2 + (28*a^5*b^3*x^10)/5 + (35*a^4*b^4*x^12)/6 + 4*a^3*b^5*x^14 + (7
*a^2*b^6*x^16)/4
```

3.91 $\int x(a+bx^2)^8\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^2)^9}{18b}$$

[Out] 1/18*(b*x^2+a)^9/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\frac{(a+bx^2)^9}{18b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^8,x]

[Out] (a + b*x^2)^9/(18*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x(a+bx^2)^8\,dx = \frac{(a+bx^2)^9}{18b}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^9}{18b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^8,x]

[Out] (a + b*x^2)^9/(18*b)

Maple [A]
time = 0.05, size = 15, normalized size = 0.94

method	result	s
default	$\frac{(b\,x^2+a)^9}{18b}$	1
gosper	$\frac{1}{2}a^8x^2+2a^7b\,x^4+\frac{14}{3}a^6b^2x^6+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14}{3}a^3b^5x^{12}+2a^2b^6x^{14}+\frac{1}{2}a\,b^7x^{16}+\frac{1}{18}b^8x^{18}$	9
norman	$\frac{1}{2}a^8x^2+2a^7b\,x^4+\frac{14}{3}a^6b^2x^6+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14}{3}a^3b^5x^{12}+2a^2b^6x^{14}+\frac{1}{2}a\,b^7x^{16}+\frac{1}{18}b^8x^{18}$	9
risch	$\frac{b^8x^{18}}{18}+\frac{a\,b^7x^{16}}{2}+2a^2b^6x^{14}+\frac{14a^3b^5x^{12}}{3}+7a^4b^4x^{10}+7a^5b^3x^8+\frac{14a^6b^2x^6}{3}+2a^7b\,x^4+\frac{a^8x^2}{2}+\frac{a^9}{18b}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(b*x^2+a)^9/b
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.88

$$\frac{(bx^2+a)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/18*(b*x^2 + a)^9/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(14) = 28$.
time = 1.05, size = 90, normalized size = 5.62

$$\frac{1}{18}\,b^8x^{18}+\frac{1}{2}\,ab^7x^{16}+2\,a^2b^6x^{14}+\frac{14}{3}\,a^3b^5x^{12}+7\,a^4b^4x^{10}+7\,a^5b^3x^8+\frac{14}{3}\,a^6b^2x^6+2\,a^7bx^4+\frac{1}{2}\,a^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/18*b^8*x^18 + 1/2*a*b^7*x^16 + 2*a^2*b^6*x^14 + 14/3*a^3*b^5*x^12 + 7*a^4
*b^4*x^10 + 7*a^5*b^3*x^8 + 14/3*a^6*b^2*x^6 + 2*a^7*b*x^4 + 1/2*a^8*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(10) = 20$.
time = 0.02, size = 99, normalized size = 6.19

$$\frac{a^8x^2}{2}+2a^7bx^4+\frac{14a^6b^2x^6}{3}+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14a^3b^5x^{12}}{3}+2a^2b^6x^{14}+\frac{ab^7x^{16}}{2}+\frac{b^8x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**2/2 + 2*a**7*b*x**4 + 14*a**6*b**2*x**6/3 + 7*a**5*b**3*x**8 + 7*a*
*4*b**4*x**10 + 14*a**3*b**5*x**12/3 + 2*a**2*b**6*x**14 + a*b**7*x**16/2 +
 b**8*x**18/18
```

Giac [A]
time = 1.35, size = 14, normalized size = 0.88

$$\frac{(bx^2 + a)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/18*(b*x^2 + a)^9/b
```

Mupad [B]
time = 4.61, size = 14, normalized size = 0.88

$$\frac{(b\,x^2 + a)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^8,x)
```

```
[Out] (a + b*x^2)^9/(18*b)
```

3.92 $\int \frac{(a+bx^2)^8}{x} \, dx$

Optimal. Leaf size=100

$$4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}+a^8\log(x)$$

```
[Out] 4*a^7*b*x^2+7*a^6*b^2*x^4+28/3*a^5*b^3*x^6+35/4*a^4*b^4*x^8+28/5*a^3*b^5*x^
10+7/3*a^2*b^6*x^12+4/7*a*b^7*x^14+1/16*b^8*x^16+a^8*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$a^8\log(x)+4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x,x]
```

```
[Out] 4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28
*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16 +
a^8*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(8a^7b+\frac{a^8}{x}+28a^6b^2x+56a^5b^3x^2+70a^4b^4x^3+56a^3b^5x^4+28a^2b^6x^5+8ab^7\right.\right.\\ &= 4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 100, normalized size = 1.00

$$4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}+a^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x,x]
```

```
[Out] 4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28
*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16 +
a^8*Log[x]
```

Maple [A]
time = 0.11, size = 89, normalized size = 0.89

method	result
default	$4a^7b\,x^2+7a^6b^2x^4+\frac{28a^5b^3x^6}{3}+\frac{35a^4b^4x^8}{4}+\frac{28a^3b^5x^{10}}{5}+\frac{7a^2b^6x^{12}}{3}+\frac{4a\,b^7x^{14}}{7}+\frac{b^8x^{16}}{16}+a^8\ln(x)$
norman	$4a^7b\,x^2+7a^6b^2x^4+\frac{28a^5b^3x^6}{3}+\frac{35a^4b^4x^8}{4}+\frac{28a^3b^5x^{10}}{5}+\frac{7a^2b^6x^{12}}{3}+\frac{4a\,b^7x^{14}}{7}+\frac{b^8x^{16}}{16}+a^8\ln(x)$
risch	$a^8\ln(x)+\frac{176a^8}{105}+\frac{b^8x^{16}}{16}+4a^7b\,x^2+7a^6b^2x^4+\frac{28a^5b^3x^6}{3}+\frac{35a^4b^4x^8}{4}+\frac{28a^3b^5x^{10}}{5}+\frac{7a^2b^6x^{12}}{3}+\frac{4a\,b^7x^{14}}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x,x,method=_RETURNVERBOSE)
```

```
[Out] 4*a^7*b*x^2+7*a^6*b^2*x^4+28/3*a^5*b^3*x^6+35/4*a^4*b^4*x^8+28/5*a^3*b^5*x^
10+7/3*a^2*b^6*x^12+4/7*a*b^7*x^14+1/16*b^8*x^16+a^8*ln(x)
```

Maxima [A]
time = 0.29, size = 91, normalized size = 0.91

$$\frac{1}{16}\,b^8x^{16}+\frac{4}{7}\,ab^7x^{14}+\frac{7}{3}\,a^2b^6x^{12}+\frac{28}{5}\,a^3b^5x^{10}+\frac{35}{4}\,a^4b^4x^8+\frac{28}{3}\,a^5b^3x^6+7\,a^6b^2x^4+4\,a^7bx^2+\frac{1}{2}\,a^8\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*lo
g(x^2)
```

Fricas [A]
time = 1.67, size = 88, normalized size = 0.88

$$\frac{1}{16}\,b^8x^{16}+\frac{4}{7}\,ab^7x^{14}+\frac{7}{3}\,a^2b^6x^{12}+\frac{28}{5}\,a^3b^5x^{10}+\frac{35}{4}\,a^4b^4x^8+\frac{28}{3}\,a^5b^3x^6+7\,a^6b^2x^4+4\,a^7bx^2+a^8\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="fricas")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8*log(x)
```

Sympy [A]
time = 0.05, size = 102, normalized size = 1.02

$$a^{8}\log{\left(x \right)} + 4a^{7}bx^{2} + 7a^{6}b^{2}x^{4} + \frac{28a^{5}b^{3}x^{6}}{3} + \frac{35a^{4}b^{4}x^{8}}{4} + \frac{28a^{3}b^{5}x^{10}}{5} + \frac{7a^{2}b^{6}x^{12}}{3} + \frac{4ab^{7}x^{14}}{7} + \frac{b^{8}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x,x)
```

```
[Out] a**8*log(x) + 4*a**7*b*x**2 + 7*a**6*b**2*x**4 + 28*a**5*b**3*x**6/3 + 35*a
**4*b**4*x**8/4 + 28*a**3*b**5*x**10/5 + 7*a**2*b**6*x**12/3 + 4*a*b**7*x**
14/7 + b**8*x**16/16
```

Giac [A]
time = 0.87, size = 91, normalized size = 0.91

$$\frac{1}{16}\,b^{8}x^{16} + \frac{4}{7}\,ab^{7}x^{14} + \frac{7}{3}\,a^{2}b^{6}x^{12} + \frac{28}{5}\,a^{3}b^{5}x^{10} + \frac{35}{4}\,a^{4}b^{4}x^{8} + \frac{28}{3}\,a^{5}b^{3}x^{6} + 7\,a^{6}b^{2}x^{4} + 4\,a^{7}bx^{2} + \frac{1}{2}\,a^{8}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="giac")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*lo
g(x^2)
```

Mupad [B]
time = 4.62, size = 88, normalized size = 0.88

$$a^8\,\ln\left(x\right)+\frac{b^8\,x^{16}}{16}+4\,a^7\,b\,x^2+\frac{4\,a\,b^7\,x^{14}}{7}+7\,a^6\,b^2\,x^4+\frac{28\,a^5\,b^3\,x^6}{3}+\frac{35\,a^4\,b^4\,x^8}{4}+\frac{28\,a^3\,b^5\,x^{10}}{5}+\frac{7\,a^2\,b^6\,x^{12}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x,x)
```

```
[Out] a^8*log(x) + (b^8*x^16)/16 + 4*a^7*b*x^2 + (4*a*b^7*x^14)/7 + 7*a^6*b^2*x^4
 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^10)/5 + (7*a^2*b
^6*x^12)/3
```

3.93 $\int \frac{(a+bx^2)^8}{x^3} \, dx$

Optimal. Leaf size=99

$$-\frac{a^8}{2x^2}+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}+8a^7b\log(x)$$

```
[Out] -1/2*a^8/x^2+14*a^6*b^2*x^2+14*a^5*b^3*x^4+35/3*a^4*b^4*x^6+7*a^3*b^5*x^8+1
4/5*a^2*b^6*x^10+2/3*a*b^7*x^12+1/14*b^8*x^14+8*a^7*b*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$-\frac{a^8}{2x^2}+8a^7b\log(x)+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^3,x]
```

```
[Out] -1/2*a^8/x^2 + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3
*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7*b
*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(28a^6b^2+\frac{a^8}{x^2}+\frac{8a^7b}{x}+56a^5b^3x+70a^4b^4x^2+56a^3b^5x^3+28a^2b^6x^4+8ab^7x\right.\right.\\ &= -\frac{a^8}{2x^2}+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}+8\end{aligned}$$

Mathematica [A]
time = 0.00, size = 99, normalized size = 1.00

$$-\frac{a^8}{2x^2}+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}+8a^7b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^3,x]
```

```
[Out] -1/2*a^8/x^2 + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3
*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7*b
*Log[x]
```

Maple [A]
time = 0.03, size = 90, normalized size = 0.91

method	result	size
default	$-\frac{a^8}{2x^2}+14a^6b^2x^2+14a^5b^3x^4+\frac{35a^4b^4x^6}{3}+7a^3b^5x^8+\frac{14a^2b^6x^{10}}{5}+\frac{2a\,b^7x^{12}}{3}+\frac{b^8x^{14}}{14}+8a^7b\ln(x)$	90
risch	$-\frac{a^8}{2x^2}+14a^6b^2x^2+14a^5b^3x^4+\frac{35a^4b^4x^6}{3}+7a^3b^5x^8+\frac{14a^2b^6x^{10}}{5}+\frac{2a\,b^7x^{12}}{3}+\frac{b^8x^{14}}{14}+8a^7b\ln(x)$	90
norman	$\frac{-\frac{1}{2}a^8+\frac{1}{14}b^8x^{16}+\frac{2}{3}a\,b^7x^{14}+\frac{14}{5}a^2b^6x^{12}+7a^3b^5x^{10}+\frac{35}{3}a^4b^4x^8+14a^5b^3x^6+14a^6b^2x^4}{x^2}+8a^7b\ln(x)$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^8/x^2+14*a^6*b^2*x^2+14*a^5*b^3*x^4+35/3*a^4*b^4*x^6+7*a^3*b^5*x^8+1
4/5*a^2*b^6*x^10+2/3*a*b^7*x^12+1/14*b^8*x^14+8*a^7*b*ln(x)
```

Maxima [A]
time = 0.28, size = 91, normalized size = 0.92

$$\frac{1}{14}\,b^8x^{14}+\frac{2}{3}\,ab^7x^{12}+\frac{14}{5}\,a^2b^6x^{10}+7\,a^3b^5x^8+\frac{35}{3}\,a^4b^4x^6+14\,a^5b^3x^4+14\,a^6b^2x^2+4\,a^7b\log\left(x^2\right)-\frac{a^8}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*a
^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*a^8/x
^2
```

Fricas [A]
time = 1.16, size = 94, normalized size = 0.95

$$\frac{15\,b^8x^{16}+140\,ab^7x^{14}+588\,a^2b^6x^{12}+1470\,a^3b^5x^{10}+2450\,a^4b^4x^8+2940\,a^5b^3x^6+2940\,a^6b^2x^4+1680\,a^7bx^2\log(x)-105\,a^8}{210\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="fricas")
```

```
[Out] 1/210*(15*b^8*x^16 + 140*a*b^7*x^14 + 588*a^2*b^6*x^12 + 1470*a^3*b^5*x^10
+ 2450*a^4*b^4*x^8 + 2940*a^5*b^3*x^6 + 2940*a^6*b^2*x^4 + 1680*a^7*b*x^2*l
og(x) - 105*a^8)/x^2
```

Sympy [A]
time = 0.06, size = 100, normalized size = 1.01

$$-\frac{a^8}{2x^2}+8a^7b\log{(x)}+14a^6b^2x^2+14a^5b^3x^4+\frac{35a^4b^4x^6}{3}+7a^3b^5x^8+\frac{14a^2b^6x^{10}}{5}+\frac{2ab^7x^{12}}{3}+\frac{b^8x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**3,x)
```

```
[Out] -a**8/(2*x**2) + 8*a**7*b*log(x) + 14*a**6*b**2*x**2 + 14*a**5*b**3*x**4 +
35*a**4*b**4*x**6/3 + 7*a**3*b**5*x**8 + 14*a**2*b**6*x**10/5 + 2*a*b**7*x*
*12/3 + b**8*x**14/14
```

Giac [A]
time = 0.82, size = 101, normalized size = 1.02

$$\frac{1}{14}\,b^8x^{14}+\frac{2}{3}\,ab^7x^{12}+\frac{14}{5}\,a^2b^6x^{10}+7\,a^3b^5x^8+\frac{35}{3}\,a^4b^4x^6+14\,a^5b^3x^4+14\,a^6b^2x^2+4\,a^7b\log\left(x^2\right)-\frac{8\,a^7bx^2+a^8}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="giac")
```

```
[Out] 1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*a
^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*(8*a^
7*b*x^2 + a^8)/x^2
```

Mupad [B]
time = 0.06, size = 89, normalized size = 0.90

$$\frac{b^8\,x^{14}}{14}-\frac{a^8}{2\,x^2}+\frac{2\,a\,b^7\,x^{12}}{3}+8\,a^7\,b\,\ln\left(x\right)+14\,a^6\,b^2\,x^2+14\,a^5\,b^3\,x^4+\frac{35\,a^4\,b^4\,x^6}{3}+7\,a^3\,b^5\,x^8+\frac{14\,a^2\,b^6\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^3,x)
```

```
[Out] (b^8*x^14)/14 - a^8/(2*x^2) + (2*a*b^7*x^12)/3 + 8*a^7*b*log(x) + 14*a^6*b^
2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3*b^5*x^8 + (14*a^2*b^6*x
^10)/5
```

3.94 $\int \frac{(a+bx^2)^8}{x^5} \, dx$

Optimal. Leaf size=101

$$-\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}+28a^6b^2\log(x)$$

```
[Out] -1/4*a^8/x^4-4*a^7*b/x^2+28*a^5*b^3*x^2+35/2*a^4*b^4*x^4+28/3*a^3*b^5*x^6+7
/2*a^2*b^6*x^8+4/5*a*b^7*x^10+1/12*b^8*x^12+28*a^6*b^2*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^6b^2\log(x)+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^5,x]
```

```
[Out] -1/4*a^8/x^4 - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^
3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*a^
6*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^5} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^3} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(56a^5b^3+\frac{a^8}{x^3}+\frac{8a^7b}{x^2}+\frac{28a^6b^2}{x}+70a^4b^4x+56a^3b^5x^2+28a^2b^6x^3+8ab^7x^4+\right.\right.\\ &= -\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}+28a^6b^2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 101, normalized size = 1.00

$$-\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}+28a^6b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^5,x]
```

```
[Out] -1/4*a^8/x^4 - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^
3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*a^
6*b^2*Log[x]
```

Maple [A]
time = 0.04, size = 90, normalized size = 0.89

method	result	size
default	$-\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^5b^3x^2+\frac{35a^4b^4x^4}{2}+\frac{28a^3b^5x^6}{3}+\frac{7a^2b^6x^8}{2}+\frac{4a\,b^7x^{10}}{5}+\frac{b^8x^{12}}{12}+28a^6b^2\ln\left(x\right)$	90
norman	$\frac{-\frac{1}{4}a^8+\frac{1}{12}b^8x^{16}+\frac{4}{5}a\,b^7x^{14}+\frac{7}{2}a^2b^6x^{12}+\frac{28}{3}a^3b^5x^{10}+\frac{35}{2}a^4b^4x^8+28a^5b^3x^6-4a^7b\,x^2}{x^4}+28a^6b^2\ln\left(x\right)$	92
risch	$\frac{b^8x^{12}}{12}+\frac{4a\,b^7x^{10}}{5}+\frac{7a^2b^6x^8}{2}+\frac{28a^3b^5x^6}{3}+\frac{35a^4b^4x^4}{2}+28a^5b^3x^2+\frac{-4a^7b\,x^2-\frac{1}{4}a^8}{x^4}+28a^6b^2\ln\left(x\right)$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^8/x^4-4*a^7*b/x^2+28*a^5*b^3*x^2+35/2*a^4*b^4*x^4+28/3*a^3*b^5*x^6+7
/2*a^2*b^6*x^8+4/5*a*b^7*x^10+1/12*b^8*x^12+28*a^6*b^2*ln(x)
```

Maxima [A]
time = 0.30, size = 92, normalized size = 0.91

$$\frac{1}{12}\,b^8x^{12}+\frac{4}{5}\,ab^7x^{10}+\frac{7}{2}\,a^2b^6x^8+\frac{28}{3}\,a^3b^5x^6+\frac{35}{2}\,a^4b^4x^4+28\,a^5b^3x^2+14\,a^6b^2\log\left(x^2\right)-\frac{16\,a^7bx^2+a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*
a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(16*a^7*b*x^2 + a^
8)/x^4
```

Fricas [A]
time = 1.15, size = 94, normalized size = 0.93

$$\frac{5\,b^8x^{16}+48\,ab^7x^{14}+210\,a^2b^6x^{12}+560\,a^3b^5x^{10}+1050\,a^4b^4x^8+1680\,a^5b^3x^6+1680\,a^6b^2x^4\log\left(x\right)-240\,a^7bx^2-15\,a^8}{60\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="fricas")
```

```
[Out] 1/60*(5*b^8*x^16 + 48*a*b^7*x^14 + 210*a^2*b^6*x^12 + 560*a^3*b^5*x^10 + 10
50*a^4*b^4*x^8 + 1680*a^5*b^3*x^6 + 1680*a^6*b^2*x^4*log(x) - 240*a^7*b*x^2
 - 15*a^8)/x^4
```

Sympy [A]
time = 0.09, size = 104, normalized size = 1.03

$$28a^6b^2\log{(x)} + 28a^5b^3x^2 + \frac{35a^4b^4x^4}{2} + \frac{28a^3b^5x^6}{3} + \frac{7a^2b^6x^8}{2} + \frac{4ab^7x^{10}}{5} + \frac{b^8x^{12}}{12} + \frac{-a^8 - 16a^7bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**5,x)
```

```
[Out] 28*a**6*b**2*log(x) + 28*a**5*b**3*x**2 + 35*a**4*b**4*x**4/2 + 28*a**3*b**
5*x**6/3 + 7*a**2*b**6*x**8/2 + 4*a*b**7*x**10/5 + b**8*x**12/12 + (-a**8 -
 16*a**7*b*x**2)/(4*x**4)
```

Giac [A]
time = 0.80, size = 103, normalized size = 1.02

$$\frac{1}{12}\,b^8x^{12} + \frac{4}{5}\,ab^7x^{10} + \frac{7}{2}\,a^2b^6x^8 + \frac{28}{3}\,a^3b^5x^6 + \frac{35}{2}\,a^4b^4x^4 + 28\,a^5b^3x^2 + 14\,a^6b^2\log\left(x^2\right) - \frac{84\,a^6b^2x^4 + 16\,a^7bx^2 + a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="giac")
```

```
[Out] 1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*
a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(84*a^6*b^2*x^4 +
16*a^7*b*x^2 + a^8)/x^4
```

Mupad [B]
time = 0.06, size = 92, normalized size = 0.91

$$\frac{b^8\,x^{12}}{12} - \frac{\frac{a^8}{4} + 4\,b\,a^7\,x^2}{x^4} + \frac{4\,a\,b^7\,x^{10}}{5} + 28\,a^5\,b^3\,x^2 + \frac{35\,a^4\,b^4\,x^4}{2} + \frac{28\,a^3\,b^5\,x^6}{3} + \frac{7\,a^2\,b^6\,x^8}{2} + 28\,a^6\,b^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^5,x)
```

```
[Out] (b^8*x^12)/12 - (a^8/4 + 4*a^7*b*x^2)/x^4 + (4*a*b^7*x^10)/5 + 28*a^5*b^3*x
^2 + (35*a^4*b^4*x^4)/2 + (28*a^3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + 28*a^6*b
^2*log(x)
```

3.95 $\int \frac{(a+bx^2)^8}{x^7} \, dx$

Optimal. Leaf size=94

$$-\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}+56a^5b^3\log(x)$$

```
[Out] -1/6*a^8/x^6-2*a^7*b/x^4-14*a^6*b^2/x^2+35*a^4*b^4*x^2+14*a^3*b^5*x^4+14/3*
a^2*b^6*x^6+a*b^7*x^8+1/10*b^8*x^10+56*a^5*b^3*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^7,x]
```

```
[Out] -1/6*a^8/x^6 - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b
^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^4}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(70a^4b^4+\frac{a^8}{x^4}+\frac{8a^7b}{x^3}+\frac{28a^6b^2}{x^2}+\frac{56a^5b^3}{x}+56a^3b^5x+28a^2b^6x^2+8ab^7x^3+\right.\right.\\
&= -\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}+56a^5b^3\,\text{lo}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 94, normalized size = 1.00

$$-\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}+56a^5b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^7,x]
```

```
[Out] -1/6*a^8/x^6 - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b
^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[x]
```

Maple [A]
time = 0.04, size = 89, normalized size = 0.95

method	result	size
default	$-\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+35a^4b^4x^2+14a^3b^5x^4+\frac{14a^2b^6x^6}{3}+a\,b^7x^8+\frac{b^8x^{10}}{10}+56a^5b^3\ln\left(x\right)$	89
norman	$\frac{a\,b^7x^{14}-\frac{1}{6}a^8+\frac{1}{10}b^8x^{16}+\frac{14}{3}a^2b^6x^{12}+14a^3b^5x^{10}+35a^4b^4x^8-14a^6b^2x^4-2a^7b\,x^2}{x^6}+56a^5b^3\ln\left(x\right)$	91
risch	$\frac{b^8x^{10}}{10}+a\,b^7x^8+\frac{14a^2b^6x^6}{3}+14a^3b^5x^4+35a^4b^4x^2+\frac{-14a^6b^2x^4-2a^7b\,x^2-\frac{1}{6}a^8}{x^6}+56a^5b^3\ln\left(x\right)$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^8/x^6-2*a^7*b/x^4-14*a^6*b^2/x^2+35*a^4*b^4*x^2+14*a^3*b^5*x^4+14/3*
a^2*b^6*x^6+a*b^7*x^8+1/10*b^8*x^10+56*a^5*b^3*ln(x)
```

Maxima [A]
time = 0.28, size = 91, normalized size = 0.97

$$\frac{1}{10}\,b^8x^{10}+ab^7x^8+\frac{14}{3}\,a^2b^6x^6+14\,a^3b^5x^4+35\,a^4b^4x^2+28\,a^5b^3\log\left(x^2\right)-\frac{84\,a^6b^2x^4+12\,a^7bx^2+a^8}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4*
x^2 + 28*a^5*b^3*log(x^2) - 1/6*(84*a^6*b^2*x^4 + 12*a^7*b*x^2 + a^8)/x^6
```

Fricas [A]
time = 1.10, size = 94, normalized size = 1.00

$$\frac{3\,b^8x^{16}+30\,ab^7x^{14}+140\,a^2b^6x^{12}+420\,a^3b^5x^{10}+1050\,a^4b^4x^8+1680\,a^5b^3x^6\log\left(x\right)-420\,a^6b^2x^4-60\,a^7bx^2-5\,a^8}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="fricas")
```

```
[Out] 1/30*(3*b^8*x^16 + 30*a*b^7*x^14 + 140*a^2*b^6*x^12 + 420*a^3*b^5*x^10 + 10
50*a^4*b^4*x^8 + 1680*a^5*b^3*x^6*log(x) - 420*a^6*b^2*x^4 - 60*a^7*b*x^2 -
 5*a^8)/x^6
```

Sympy [A]
time = 0.13, size = 97, normalized size = 1.03

$$56a^5b^3\log{(x)} + 35a^4b^4x^2 + 14a^3b^5x^4 + \frac{14a^2b^6x^6}{3} + ab^7x^8 + \frac{b^8x^{10}}{10} + \frac{-a^8 - 12a^7bx^2 - 84a^6b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**7,x)
```

```
[Out] 56*a**5*b**3*log(x) + 35*a**4*b**4*x**2 + 14*a**3*b**5*x**4 + 14*a**2*b**6*
x**6/3 + a*b**7*x**8 + b**8*x**10/10 + (-a**8 - 12*a**7*b*x**2 - 84*a**6*b*
*2*x**4)/(6*x**6)
```

Giac [A]
time = 1.00, size = 102, normalized size = 1.09

$$\frac{1}{10}\,b^8x^{10} + ab^7x^8 + \frac{14}{3}\,a^2b^6x^6 + 14\,a^3b^5x^4 + 35\,a^4b^4x^2 + 28\,a^5b^3\log\left(x^2\right) - \frac{308\,a^5b^3x^6 + 84\,a^6b^2x^4 + 12\,a^7bx^2 + a^8}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="giac")
```

```
[Out] 1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4*
x^2 + 28*a^5*b^3*log(x^2) - 1/6*(308*a^5*b^3*x^6 + 84*a^6*b^2*x^4 + 12*a^7*
b*x^2 + a^8)/x^6
```

Mupad [B]
time = 0.05, size = 91, normalized size = 0.97

$$\frac{b^8\,x^{10}}{10} - \frac{\frac{a^8}{6} + 2\,a^7\,b\,x^2 + 14\,a^6\,b^2\,x^4}{x^6} + a\,b^7\,x^8 + 35\,a^4\,b^4\,x^2 + 14\,a^3\,b^5\,x^4 + \frac{14\,a^2\,b^6\,x^6}{3} + 56\,a^5\,b^3\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^7,x)
```

```
[Out] (b^8*x^10)/10 - (a^8/6 + 2*a^7*b*x^2 + 14*a^6*b^2*x^4)/x^6 + a*b^7*x^8 + 35
*a^4*b^4*x^2 + 14*a^3*b^5*x^4 + (14*a^2*b^6*x^6)/3 + 56*a^5*b^3*log(x)
```

3.96 $\int \frac{(a+bx^2)^8}{x^9} \, dx$

Optimal. Leaf size=97

$$-\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}+70a^4b^4\log(x)$$

```
[Out] -1/8*a^8/x^8-4/3*a^7*b/x^6-7*a^6*b^2/x^4-28*a^5*b^3/x^2+28*a^3*b^5*x^2+7*a^
2*b^6*x^4+4/3*a*b^7*x^6+1/8*b^8*x^8+70*a^4*b^4*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+70a^4b^4\log(x)+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^9,x]
```

```
[Out] -1/8*a^8/x^8 - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*
a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Lo
g[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^5}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(56a^3b^5+\frac{a^8}{x^5}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^3}+\frac{56a^5b^3}{x^2}+\frac{70a^4b^4}{x}+28a^2b^6x+8ab^7x^2+b^8\right.\right.\\
&= -\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}+70a^4b^4\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 97, normalized size = 1.00

$$-\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}+70a^4b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^9,x]
```

```
[Out] -1/8*a^8/x^8 - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*
a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Lo
g[x]
```

Maple [A]
time = 0.03, size = 90, normalized size = 0.93

method	result	size
default	$-\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+\frac{4a\,b^7x^6}{3}+\frac{b^8x^8}{8}+70a^4b^4\ln(x)$	90
norman	$\frac{-\frac{1}{8}a^8+\frac{1}{8}b^8x^{16}+\frac{4}{3}a\,b^7x^{14}+7a^2b^6x^{12}+28a^3b^5x^{10}-28a^5b^3x^6-7a^6b^2x^4-\frac{4}{3}a^7b\,x^2}{x^8}+70a^4b^4\ln(x)$	92
risch	$\frac{b^8x^8}{8}+\frac{4a\,b^7x^6}{3}+7a^2b^6x^4+28a^3b^5x^2+\frac{-28a^5b^3x^6-7a^6b^2x^4-\frac{4}{3}a^7b\,x^2-\frac{1}{8}a^8}{x^8}+70a^4b^4\ln(x)$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a^8/x^8-4/3*a^7*b/x^6-7*a^6*b^2/x^4-28*a^5*b^3/x^2+28*a^3*b^5*x^2+7*a^
2*b^6*x^4+4/3*a*b^7*x^6+1/8*b^8*x^8+70*a^4*b^4*ln(x)
```

Maxima [A]
time = 0.31, size = 94, normalized size = 0.97

$$\frac{1}{8}b^8x^8+\frac{4}{3}ab^7x^6+7\,a^2b^6x^4+28\,a^3b^5x^2+35\,a^4b^4\log\left(x^2\right)-\frac{672\,a^5b^3x^6+168\,a^6b^2x^4+32\,a^7bx^2+3\,a^8}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="maxima")
```

```
[Out] 1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*l
og(x^2) - 1/24*(672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a^7*b*x^2 + 3*a^8)/x
^8
```

Fricas [A]
time = 1.23, size = 94, normalized size = 0.97

$$\frac{3\,b^8x^{16}+32\,ab^7x^{14}+168\,a^2b^6x^{12}+672\,a^3b^5x^{10}+1680\,a^4b^4x^8\log\left(x\right)-672\,a^5b^3x^6-168\,a^6b^2x^4-32\,a^7bx^2-3\,a^8}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^8*x^16 + 32*a*b^7*x^14 + 168*a^2*b^6*x^12 + 672*a^3*b^5*x^10 + 16
80*a^4*b^4*x^8*log(x) - 672*a^5*b^3*x^6 - 168*a^6*b^2*x^4 - 32*a^7*b*x^2 -
3*a^8)/x^8
```

Sympy [A]
time = 0.16, size = 100, normalized size = 1.03

$$70a^4b^4\log{(x)} + 28a^3b^5x^2 + 7a^2b^6x^4 + \frac{4ab^7x^6}{3} + \frac{b^8x^8}{8} + \frac{-3a^8 - 32a^7bx^2 - 168a^6b^2x^4 - 672a^5b^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**9,x)
```

```
[Out] 70*a**4*b**4*log(x) + 28*a**3*b**5*x**2 + 7*a**2*b**6*x**4 + 4*a*b**7*x**6/
3 + b**8*x**8/8 + (-3*a**8 - 32*a**7*b*x**2 - 168*a**6*b**2*x**4 - 672*a**5
*b**3*x**6)/(24*x**8)
```

Giac [A]
time = 1.06, size = 105, normalized size = 1.08

$$\frac{1}{8}b^8x^8 + \frac{4}{3}ab^7x^6 + 7a^2b^6x^4 + 28a^3b^5x^2 + 35a^4b^4\log{(x^2)} - \frac{1750a^4b^4x^8 + 672a^5b^3x^6 + 168a^6b^2x^4 + 32a^7bx^2 + 3a^8}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="giac")
```

```
[Out] 1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*l
og(x^2) - 1/24*(1750*a^4*b^4*x^8 + 672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a
^7*b*x^2 + 3*a^8)/x^8
```

Mupad [B]
time = 0.05, size = 92, normalized size = 0.95

$$\frac{b^8x^8}{8} - \frac{\frac{a^8}{8} + \frac{4a^7bx^2}{3} + 7a^6b^2x^4 + 28a^5b^3x^6}{x^8} + \frac{4ab^7x^6}{3} + 28a^3b^5x^2 + 7a^2b^6x^4 + 70a^4b^4\ln{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^9,x)
```

```
[Out] (b^8*x^8)/8 - (a^8/8 + (4*a^7*b*x^2)/3 + 7*a^6*b^2*x^4 + 28*a^5*b^3*x^6)/x^
8 + (4*a*b^7*x^6)/3 + 28*a^3*b^5*x^2 + 7*a^2*b^6*x^4 + 70*a^4*b^4*log(x)
```

3.97 $\int \frac{(a+bx^2)^8}{x^{11}}\,dx$

Optimal. Leaf size=95

$$-\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}+56a^3b^5\log(x)$$

```
[Out] -1/10*a^8/x^10-a^7*b/x^8-14/3*a^6*b^2/x^6-14*a^5*b^3/x^4-35*a^4*b^4/x^2+14*
a^2*b^6*x^2+2*a*b^7*x^4+1/6*b^8*x^6+56*a^3*b^5*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+56a^3b^5\log(x)+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^11,x]
```

```
[Out] -1/10*a^8/x^10 - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (3
5*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Lo
g[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{11}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^6}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(28a^2b^6+\frac{a^8}{x^6}+\frac{8a^7b}{x^5}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^3}+\frac{70a^4b^4}{x^2}+\frac{56a^3b^5}{x}+8ab^7x+b^8x\right.\right.\\ &= -\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}+56a^3b^5\log(x\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$-\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}+56a^3b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^11,x]
```

```
[Out] -1/10*a^8/x^10 - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (3
5*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Lo
g[x]
```

Maple [A]
time = 0.04, size = 90, normalized size = 0.95

method	result	size
default	$-\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+2a\,b^7x^4+\frac{b^8x^6}{6}+56a^3b^5\ln(x)$	90
norman	$\frac{-\frac{1}{10}a^8+\frac{1}{6}b^8x^{16}+2a\,b^7x^{14}+14a^2b^6x^{12}-35a^4b^4x^8-14a^5b^3x^6-\frac{14}{3}a^6b^2x^4-a^7b\,x^2}{x^{10}}+56a^3b^5\ln(x)$	92
risch	$\frac{b^8x^6}{6}+2a\,b^7x^4+14a^2b^6x^2+\frac{-35a^4b^4x^8-14a^5b^3x^6-\frac{14}{3}a^6b^2x^4-a^7b\,x^2-\frac{1}{10}a^8}{x^{10}}+56a^3b^5\ln(x)$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^8/x^10-a^7*b/x^8-14/3*a^6*b^2/x^6-14*a^5*b^3/x^4-35*a^4*b^4/x^2+14*
a^2*b^6*x^2+2*a*b^7*x^4+1/6*b^8*x^6+56*a^3*b^5*ln(x)
```

Maxima [A]
time = 0.30, size = 94, normalized size = 0.99

$$\frac{1}{6}\,b^8x^6+2\,ab^7x^4+14\,a^2b^6x^2+28\,a^3b^5\log\left(x^2\right)-\frac{1050\,a^4b^4x^8+420\,a^5b^3x^6+140\,a^6b^2x^4+30\,a^7bx^2+3\,a^8}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="maxima")
```

```
[Out] 1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(10
50*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30*a^7*b*x^2 + 3*a^8)/
x^10
```

Fricas [A]
time = 1.43, size = 94, normalized size = 0.99

$$\frac{5\,b^8x^{16}+60\,ab^7x^{14}+420\,a^2b^6x^{12}+1680\,a^3b^5x^{10}\log(x)-1050\,a^4b^4x^8-420\,a^5b^3x^6-140\,a^6b^2x^4-30\,a^7bx^2-3\,a^8}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="fricas")
```

```
[Out] 1/30*(5*b^8*x^16 + 60*a*b^7*x^14 + 420*a^2*b^6*x^12 + 1680*a^3*b^5*x^10*log
(x) - 1050*a^4*b^4*x^8 - 420*a^5*b^3*x^6 - 140*a^6*b^2*x^4 - 30*a^7*b*x^2 -
 3*a^8)/x^10
```

Sympy [A]
time = 0.21, size = 99, normalized size = 1.04

$$56a^3b^5\log(x) + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6} + \frac{-3a^8 - 30a^7bx^2 - 140a^6b^2x^4 - 420a^5b^3x^6 - 1050a^4b^4x^8}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**11,x)
```

```
[Out] 56*a**3*b**5*log(x) + 14*a**2*b**6*x**2 + 2*a*b**7*x**4 + b**8*x**6/6 + (-3
*a**8 - 30*a**7*b*x**2 - 140*a**6*b**2*x**4 - 420*a**5*b**3*x**6 - 1050*a**
4*b**4*x**8)/(30*x**10)
```

Giac [A]
time = 0.88, size = 105, normalized size = 1.11

$$\frac{1}{6}b^8x^6 + 2\,ab^7x^4 + 14\,a^2b^6x^2 + 28\,a^3b^5\log(x^2) - \frac{1918\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 420\,a^5b^3x^6 + 140\,a^6b^2x^4 + 30\,a^7bx^2 + 3\,a^8}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="giac")
```

```
[Out] 1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(19
18*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30
*a^7*b*x^2 + 3*a^8)/x^10
```

Mupad [B]
time = 5.11, size = 91, normalized size = 0.96

$$\frac{b^8\,x^6}{6} - \frac{\frac{a^8}{10} + a^7\,b\,x^2 + \frac{14\,a^6\,b^2\,x^4}{3} + 14\,a^5\,b^3\,x^6 + 35\,a^4\,b^4\,x^8}{x^{10}} + 2\,a\,b^7\,x^4 + 14\,a^2\,b^6\,x^2 + 56\,a^3\,b^5\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^11,x)
```

```
[Out] (b^8*x^6)/6 - (a^8/10 + a^7*b*x^2 + (14*a^6*b^2*x^4)/3 + 14*a^5*b^3*x^6 + 3
5*a^4*b^4*x^8)/x^10 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 56*a^3*b^5*log(x)
```

3.98 $\int \frac{(a+bx^2)^8}{x^{13}} \, dx$

Optimal. Leaf size=101

$$-\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+4ab^7x^2+\frac{b^8x^4}{4}+28a^2b^6\log(x)$$

```
[Out] -1/12*a^8/x^12-4/5*a^7*b/x^10-7/2*a^6*b^2/x^8-28/3*a^5*b^3/x^6-35/2*a^4*b^4
/x^4-28*a^3*b^5/x^2+4*a*b^7*x^2+1/4*b^8*x^4+28*a^2*b^6*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)+4ab^7x^2+\frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^13,x]
```

```
[Out] -1/12*a^8/x^12 - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3
*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)/4
 + 28*a^2*b^6*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{13}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^7} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(8ab^7+\frac{a^8}{x^7}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^5}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^3}+\frac{56a^3b^5}{x^2}+\frac{28a^2b^6}{x}+b^8x\right) \right. \\
&= -\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+4ab^7x^2+\frac{b^8x^4}{4}+28a^2b^6\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 101, normalized size = 1.00

$$-\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+4ab^7x^2+\frac{b^8x^4}{4}+28a^2b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^13,x]
```

```
[Out] -1/12*a^8/x^12 - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3
*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)/4
 + 28*a^2*b^6*Log[x]
```

Maple [A]
time = 0.05, size = 90, normalized size = 0.89

method	result	size
default	$-\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+4a\,b^7x^2+\frac{b^8x^4}{4}+28a^2b^6\ln(x)$	90
norman	$\frac{-\frac{1}{12}a^8+\frac{1}{4}b^8x^{16}+4a\,b^7x^{14}-28a^3b^5x^{10}-\frac{35}{2}a^4b^4x^8-\frac{28}{3}a^5b^3x^6-\frac{7}{2}a^6b^2x^4-\frac{4}{5}a^7b\,x^2}{x^{12}}+28a^2b^6\ln(x)$	92
risch	$\frac{b^8x^4}{4}+4a\,b^7x^2+16a^2b^6+\frac{-28a^3b^5x^{10}-\frac{35}{2}a^4b^4x^8-\frac{28}{3}a^5b^3x^6-\frac{7}{2}a^6b^2x^4-\frac{4}{5}a^7b\,x^2-\frac{1}{12}a^8}{x^{12}}+28a^2b^6\ln(x)$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*a^8/x^12-4/5*a^7*b/x^10-7/2*a^6*b^2/x^8-28/3*a^5*b^3/x^6-35/2*a^4*b^4
/x^4-28*a^3*b^5/x^2+4*a*b^7*x^2+1/4*b^8*x^4+28*a^2*b^6*ln(x)
```

Maxima [A]
time = 0.30, size = 94, normalized size = 0.93

$$\frac{1}{4}\,b^8x^4+4\,ab^7x^2+14\,a^2b^6\log\left(x^2\right)-\frac{1680\,a^3b^5x^{10}+1050\,a^4b^4x^8+560\,a^5b^3x^6+210\,a^6b^2x^4+48\,a^7bx^2+5\,a^8}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="maxima")
```

```
[Out] 1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(1680*a^3*b^5*x^10 +
 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 + 48*a^7*b*x^2 + 5*a^
8)/x^12
```

Fricas [A]
time = 1.28, size = 94, normalized size = 0.93

$$\frac{15\,b^8x^{16}+240\,ab^7x^{14}+1680\,a^2b^6x^{12}\log(x)-1680\,a^3b^5x^{10}-1050\,a^4b^4x^8-560\,a^5b^3x^6-210\,a^6b^2x^4-48\,a^7bx^2-5\,a^8}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="fricas")
```

```
[Out] 1/60*(15*b^8*x^16 + 240*a*b^7*x^14 + 1680*a^2*b^6*x^12*log(x) - 1680*a^3*b^
5*x^10 - 1050*a^4*b^4*x^8 - 560*a^5*b^3*x^6 - 210*a^6*b^2*x^4 - 48*a^7*b*x^
2 - 5*a^8)/x^12
```

Sympy [A]
time = 0.26, size = 99, normalized size = 0.98

$$28a^2b^6\log(x) + 4ab^7x^2 + \frac{b^8x^4}{4} + \frac{-5a^8 - 48a^7bx^2 - 210a^6b^2x^4 - 560a^5b^3x^6 - 1050a^4b^4x^8 - 1680a^3b^5x^{10}}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**13,x)
```

```
[Out] 28*a**2*b**6*log(x) + 4*a*b**7*x**2 + b**8*x**4/4 + (-5*a**8 - 48*a**7*b*x*
*2 - 210*a**6*b**2*x**4 - 560*a**5*b**3*x**6 - 1050*a**4*b**4*x**8 - 1680*a
**3*b**5*x**10)/(60*x**12)
```

Giac [A]
time = 0.94, size = 105, normalized size = 1.04

$$\frac{1}{4}b^8x^4 + 4ab^7x^2 + 14a^2b^6\log(x^2) - \frac{2058a^2b^6x^{12} + 1680a^3b^5x^{10} + 1050a^4b^4x^8 + 560a^5b^3x^6 + 210a^6b^2x^4 + 48a^7bx^2 + 5a^8}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="giac")
```

```
[Out] 1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(2058*a^2*b^6*x^12 +
 1680*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 +
 48*a^7*b*x^2 + 5*a^8)/x^12
```

Mupad [B]
time = 0.06, size = 92, normalized size = 0.91

$$\frac{b^8x^4}{4} - \frac{\frac{a^8}{12} + \frac{4a^7bx^2}{5} + \frac{7a^6b^2x^4}{2} + \frac{28a^5b^3x^6}{3} + \frac{35a^4b^4x^8}{2} + 28a^3b^5x^{10}}{x^{12}} + 4ab^7x^2 + 28a^2b^6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^13,x)
```

```
[Out] (b^8*x^4)/4 - (a^8/12 + (4*a^7*b*x^2)/5 + (7*a^6*b^2*x^4)/2 + (28*a^5*b^3*x
^6)/3 + (35*a^4*b^4*x^8)/2 + 28*a^3*b^5*x^10)/x^12 + 4*a*b^7*x^2 + 28*a^2*b
^6*log(x)
```

3.99 $\int \frac{(a+bx^2)^8}{x^{15}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+\frac{b^8x^2}{2}+8ab^7\log(x)$$

```
[Out] -1/14*a^8/x^14-2/3*a^7*b/x^12-14/5*a^6*b^2/x^10-7*a^5*b^3/x^8-35/3*a^4*b^4/
x^6-14*a^3*b^5/x^4-14*a^2*b^6/x^2+1/2*b^8*x^2+8*a*b^7*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$-\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^15,x]
```

```
[Out] -1/14*a^8/x^14 - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/x
^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)
/2 + 8*a*b^7*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{15}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^8}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^8+\frac{a^8}{x^8}+\frac{8a^7b}{x^7}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^5}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^3}+\frac{28a^2b^6}{x^2}+\frac{8ab^7}{x}\right)\right.\\
&= -\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+\frac{b^8x^2}{2}+8ab^7\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 99, normalized size = 1.00

$$-\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+\frac{b^8x^2}{2}+8ab^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^15,x]
```

```
[Out] -1/14*a^8/x^14 - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/x
^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)
/2 + 8*a*b^7*Log[x]
```

Maple [A]
time = 0.04, size = 90, normalized size = 0.91

method	result	size
default	$-\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+\frac{b^8x^2}{2}+8a\,b^7\ln(x)$	90
norman	$\frac{-\frac{1}{14}a^8+\frac{1}{2}b^8x^{16}-14a^2b^6x^{12}-14a^3b^5x^{10}-\frac{35}{3}a^4b^4x^8-7a^5b^3x^6-\frac{14}{5}a^6b^2x^4-\frac{2}{3}a^7b\,x^2}{x^{14}}+8a\,b^7\ln(x)$	92
risch	$\frac{b^8x^2}{2}+\frac{-14a^2b^6x^{12}-14a^3b^5x^{10}-\frac{35}{3}a^4b^4x^8-7a^5b^3x^6-\frac{14}{5}a^6b^2x^4-\frac{2}{3}a^7b\,x^2-\frac{1}{14}a^8}{x^{14}}+8a\,b^7\ln(x)$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*a^8/x^14-2/3*a^7*b/x^12-14/5*a^6*b^2/x^10-7*a^5*b^3/x^8-35/3*a^4*b^4/
x^6-14*a^3*b^5/x^4-14*a^2*b^6/x^2+1/2*b^8*x^2+8*a*b^7*ln(x)
```

Maxima [A]
time = 0.32, size = 94, normalized size = 0.95

$$\frac{1}{2}\,b^8x^2+4\,ab^7\log\left(x^2\right)-\frac{2940\,a^2b^6x^{12}+2940\,a^3b^5x^{10}+2450\,a^4b^4x^8+1470\,a^5b^3x^6+588\,a^6b^2x^4+140\,a^7bx^2+15\,a^8}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="maxima")
```

```
[Out] 1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2940*a^2*b^6*x^12 + 2940*a^3*b^5*x^
10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^4 + 140*a^7*b*x^2
+ 15*a^8)/x^14
```

Fricas [A]
time = 1.87, size = 94, normalized size = 0.95

$$\frac{105\,b^8x^{16}+1680\,ab^7x^{14}\log\left(x\right)-2940\,a^2b^6x^{12}-2940\,a^3b^5x^{10}-2450\,a^4b^4x^8-1470\,a^5b^3x^6-588\,a^6b^2x^4-140\,a^7bx^2-15\,a^8}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="fricas")
```

```
[Out] 1/210*(105*b^8*x^16 + 1680*a*b^7*x^14*log(x) - 2940*a^2*b^6*x^12 - 2940*a^3
*b^5*x^10 - 2450*a^4*b^4*x^8 - 1470*a^5*b^3*x^6 - 588*a^6*b^2*x^4 - 140*a^7
*b*x^2 - 15*a^8)/x^14
```

Sympy [A]
time = 0.31, size = 99, normalized size = 1.00

$$8ab^7\log(x) + \frac{b^8x^2}{2} + \frac{-15a^8 - 140a^7bx^2 - 588a^6b^2x^4 - 1470a^5b^3x^6 - 2450a^4b^4x^8 - 2940a^3b^5x^{10} - 2940a^2b^6x^{12}}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**15,x)
```

```
[Out] 8*a*b**7*log(x) + b**8*x**2/2 + (-15*a**8 - 140*a**7*b*x**2 - 588*a**6*b**2
*x**4 - 1470*a**5*b**3*x**6 - 2450*a**4*b**4*x**8 - 2940*a**3*b**5*x**10 -
2940*a**2*b**6*x**12)/(210*x**14)
```

Giac [A]
time = 1.51, size = 103, normalized size = 1.04

$$\frac{1}{2}b^8x^2 + 4ab^7\log(x^2) - \frac{2178ab^7x^{14} + 2940a^2b^6x^{12} + 2940a^3b^5x^{10} + 2450a^4b^4x^8 + 1470a^5b^3x^6 + 588a^6b^2x^4 + 140a^7bx^2 + 15a^8}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="giac")
```

```
[Out] 1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2178*a*b^7*x^14 + 2940*a^2*b^6*x^12
 + 2940*a^3*b^5*x^10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^
4 + 140*a^7*b*x^2 + 15*a^8)/x^14
```

Mupad [B]
time = 5.15, size = 94, normalized size = 0.95

$$-\frac{\frac{a^8}{14} - \frac{b^8x^{16}}{2} + \frac{2a^7bx^2}{3} + \frac{14a^6b^2x^4}{5} + 7a^5b^3x^6 + \frac{35a^4b^4x^8}{3} + 14a^3b^5x^{10} + 14a^2b^6x^{12} - 8ab^7x^{14}\ln(x)}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^15,x)
```

```
[Out] -(a^8/14 - (b^8*x^16)/2 + (2*a^7*b*x^2)/3 + (14*a^6*b^2*x^4)/5 + 7*a^5*b^3*
x^6 + (35*a^4*b^4*x^8)/3 + 14*a^3*b^5*x^10 + 14*a^2*b^6*x^12 - 8*a*b^7*x^14
*log(x))/x^14
```

3.100 $\int \frac{(a+bx^2)^8}{x^{17}} dx$

Optimal. Leaf size=100

$$-\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)$$

```
[Out] -1/16*a^8/x^16-4/7*a^7*b/x^14-7/3*a^6*b^2/x^12-28/5*a^5*b^3/x^10-35/4*a^4*b
^4/x^8-28/3*a^3*b^5/x^6-7*a^2*b^6/x^4-4*a*b^7/x^2+b^8*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^17,x]
```

```
[Out] -1/16*a^8/x^16 - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/(
5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (
4*a*b^7)/x^2 + b^8*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{17}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^9}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^7}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^5}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^3}+\frac{8ab^7}{x^2}+\frac{b^8}{x}\right)\right.\\ &= -\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 100, normalized size = 1.00

$$-\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^17,x]
[Out] -1/16*a^8/x^16 - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/(
5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (
4*a*b^7)/x^2 + b^8*Log[x]
```

Maple [A]
time = 0.03, size = 89, normalized size = 0.89

method	result	size
default	$-\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4a\,b^7}{x^2}+b^8\ln(x)$	89
norman	$\frac{-\frac{1}{16}a^8-4a\,b^7x^{14}-7a^2b^6x^{12}-\frac{28}{3}a^3b^5x^{10}-\frac{35}{4}a^4b^4x^8-\frac{28}{5}a^5b^3x^6-\frac{7}{3}a^6b^2x^4-\frac{4}{7}a^7b\,x^2}{x^{16}}+b^8\ln(x)$	91
risch	$\frac{-\frac{1}{16}a^8-4a\,b^7x^{14}-7a^2b^6x^{12}-\frac{28}{3}a^3b^5x^{10}-\frac{35}{4}a^4b^4x^8-\frac{28}{5}a^5b^3x^6-\frac{7}{3}a^6b^2x^4-\frac{4}{7}a^7b\,x^2}{x^{16}}+b^8\ln(x)$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^17,x,method=_RETURNVERBOSE)
[Out] -1/16*a^8/x^16-4/7*a^7*b/x^14-7/3*a^6*b^2/x^12-28/5*a^5*b^3/x^10-35/4*a^4*b
^4/x^8-28/3*a^3*b^5/x^6-7*a^2*b^6/x^4-4*a*b^7/x^2+b^8*ln(x)
```

Maxima [A]
time = 0.30, size = 94, normalized size = 0.94

$$\frac{1}{2}b^8\log\left(x^2\right)-\frac{6720\,ab^7x^{14}+11760\,a^2b^6x^{12}+15680\,a^3b^5x^{10}+14700\,a^4b^4x^8+9408\,a^5b^3x^6+3920\,a^6b^2x^4+960\,a^7bx^2+105\,a^8}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="maxima")
[Out] 1/2*b^8*log(x^2) - 1/1680*(6720*a*b^7*x^14 + 11760*a^2*b^6*x^12 + 15680*a^3
*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6*b^2*x^4 + 960*a
^7*b*x^2 + 105*a^8)/x^16
```

Fricas [A]
time = 1.05, size = 94, normalized size = 0.94

$$\frac{1680\,b^8x^{16}\log(x)-6720\,ab^7x^{14}-11760\,a^2b^6x^{12}-15680\,a^3b^5x^{10}-14700\,a^4b^4x^8-9408\,a^5b^3x^6-3920\,a^6b^2x^4-960\,a^7bx^2-105\,a^8}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="fricas")
```

```
[Out] 1/1680*(1680*b^8*x^16*log(x) - 6720*a*b^7*x^14 - 11760*a^2*b^6*x^12 - 15680
*a^3*b^5*x^10 - 14700*a^4*b^4*x^8 - 9408*a^5*b^3*x^6 - 3920*a^6*b^2*x^4 - 9
60*a^7*b*x^2 - 105*a^8)/x^16
```

Sympy [A]
time = 0.37, size = 97, normalized size = 0.97

$$b^8\log(x)+\frac{-105a^8-960a^7bx^2-3920a^6b^2x^4-9408a^5b^3x^6-14700a^4b^4x^8-15680a^3b^5x^{10}-11760a^2b^6x^{12}-6720ab^7x^{14}}{1680x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**17,x)
```

```
[Out] b**8*log(x) + (-105*a**8 - 960*a**7*b*x**2 - 3920*a**6*b**2*x**4 - 9408*a**
5*b**3*x**6 - 14700*a**4*b**4*x**8 - 15680*a**3*b**5*x**10 - 11760*a**2*b**
6*x**12 - 6720*a*b**7*x**14)/(1680*x**16)
```

Giac [A]
time = 1.59, size = 102, normalized size = 1.02

$$\frac{1}{2}b^8\log(x^2)-\frac{2283\,b^8x^{16}+6720\,ab^7x^{14}+11760\,a^2b^6x^{12}+15680\,a^3b^5x^{10}+14700\,a^4b^4x^8+9408\,a^5b^3x^6+3920\,a^6b^2x^4+960\,a^7bx^2+105\,a^8}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="giac")
```

```
[Out] 1/2*b^8*log(x^2) - 1/1680*(2283*b^8*x^16 + 6720*a*b^7*x^14 + 11760*a^2*b^6*
x^12 + 15680*a^3*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6
*b^2*x^4 + 960*a^7*b*x^2 + 105*a^8)/x^16
```

Mupad [B]
time = 5.09, size = 91, normalized size = 0.91

$$b^8\,\ln(x)-\frac{\frac{a^8}{16}+\frac{4\,a^7\,b\,x^2}{7}+\frac{7\,a^6\,b^2\,x^4}{3}+\frac{28\,a^5\,b^3\,x^6}{5}+\frac{35\,a^4\,b^4\,x^8}{4}+\frac{28\,a^3\,b^5\,x^{10}}{3}+7\,a^2\,b^6\,x^{12}+4\,a\,b^7\,x^{14}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^17,x)
```

```
[Out] b^8*log(x) - (a^8/16 + (4*a^7*b*x^2)/7 + 4*a*b^7*x^14 + (7*a^6*b^2*x^4)/3 +
 (28*a^5*b^3*x^6)/5 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^10)/3 + 7*a^2*b^6*
x^12)/x^16
```

3.101 $\int \frac{(a+bx^2)^8}{x^{19}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

[Out] -1/18*(b*x^2+a)^9/a/x^18

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{270\}$

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^8/x^19,x]

[Out] -1/18*(a + b*x^2)^9/(a*x^18)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx = -\frac{(a+bx^2)^9}{18ax^{18}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 100 vs. 2(19) = 38.
time = 0.00, size = 100, normalized size = 5.26

$$-\frac{a^8}{18x^{18}}-\frac{a^7b}{2x^{16}}-\frac{2a^6b^2}{x^{14}}-\frac{14a^5b^3}{3x^{12}}-\frac{7a^4b^4}{x^{10}}-\frac{7a^3b^5}{x^8}-\frac{14a^2b^6}{3x^6}-\frac{2ab^7}{x^4}-\frac{b^8}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^8/x^19,x]

```
[Out] -1/18*a^8/x^18 - (a^7*b)/(2*x^16) - (2*a^6*b^2)/x^14 - (14*a^5*b^3)/(3*x^12
) - (7*a^4*b^4)/x^10 - (7*a^3*b^5)/x^8 - (14*a^2*b^6)/(3*x^6) - (2*a*b^7)/x
^4 - b^8/(2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.
time = 0.04, size = 91, normalized size = 4.79

method	result	size
gosper	$-\frac{9b^{8}x^{16}+36a\,b^{7}x^{14}+84a^{2}b^{6}x^{12}+126a^{3}b^{5}x^{10}+126a^{4}b^{4}x^{8}+84a^{5}b^{3}x^{6}+36a^{6}b^{2}x^{4}+9a^{7}b\,x^{2}+a^{8}}{18x^{18}}$	91
default	$-\frac{2a^{6}b^{2}}{x^{14}}-\frac{a^{8}}{18x^{18}}-\frac{7a^{4}b^{4}}{x^{10}}-\frac{a^{7}b}{2x^{16}}-\frac{2a\,b^{7}}{x^{4}}-\frac{14a^{2}b^{6}}{3x^{6}}-\frac{b^{8}}{2x^{2}}-\frac{7a^{3}b^{5}}{x^{8}}-\frac{14a^{5}b^{3}}{3x^{12}}$	91
norman	$\frac{-\frac{1}{18}a^{8}-\frac{1}{2}a^{7}b\,x^{2}-2a^{6}b^{2}x^{4}-\frac{14}{3}a^{5}b^{3}x^{6}-7a^{4}b^{4}x^{8}-7a^{3}b^{5}x^{10}-\frac{14}{3}a^{2}b^{6}x^{12}-2a\,b^{7}x^{14}-\frac{1}{2}b^{8}x^{16}}{x^{18}}$	92
risch	$\frac{-\frac{1}{18}a^{8}-\frac{1}{2}a^{7}b\,x^{2}-2a^{6}b^{2}x^{4}-\frac{14}{3}a^{5}b^{3}x^{6}-7a^{4}b^{4}x^{8}-7a^{3}b^{5}x^{10}-\frac{14}{3}a^{2}b^{6}x^{12}-2a\,b^{7}x^{14}-\frac{1}{2}b^{8}x^{16}}{x^{18}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^6*b^2/x^14-1/18*a^8/x^18-7*a^4*b^4/x^10-1/2*a^7*b/x^16-2*a*b^7/x^4-14/
3*a^2*b^6/x^6-1/2*b^8/x^2-7*a^3*b^5/x^8-14/3*a^5*b^3/x^12
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.
time = 0.27, size = 90, normalized size = 4.74

$$-\frac{9\,b^{8}x^{16}+36\,ab^{7}x^{14}+84\,a^{2}b^{6}x^{12}+126\,a^{3}b^{5}x^{10}+126\,a^{4}b^{4}x^{8}+84\,a^{5}b^{3}x^{6}+36\,a^{6}b^{2}x^{4}+9\,a^{7}bx^{2}+a^{8}}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="maxima")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.
time = 1.56, size = 90, normalized size = 4.74

$$-\frac{9\,b^{8}x^{16}+36\,ab^{7}x^{14}+84\,a^{2}b^{6}x^{12}+126\,a^{3}b^{5}x^{10}+126\,a^{4}b^{4}x^{8}+84\,a^{5}b^{3}x^{6}+36\,a^{6}b^{2}x^{4}+9\,a^{7}bx^{2}+a^{8}}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(15) = 30$.
time = 0.39, size = 97, normalized size = 5.11

$$\frac{-a^8 - 9a^7bx^2 - 36a^6b^2x^4 - 84a^5b^3x^6 - 126a^4b^4x^8 - 126a^3b^5x^{10} - 84a^2b^6x^{12} - 36ab^7x^{14} - 9b^8x^{16}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**19,x)
```

```
[Out] (-a**8 - 9*a**7*b*x**2 - 36*a**6*b**2*x**4 - 84*a**5*b**3*x**6 - 126*a**4*b
**4*x**8 - 126*a**3*b**5*x**10 - 84*a**2*b**6*x**12 - 36*a*b**7*x**14 - 9*b
**8*x**16)/(18*x**18)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.
time = 1.29, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{16} + 36\,ab^7x^{14} + 84\,a^2b^6x^{12} + 126\,a^3b^5x^{10} + 126\,a^4b^4x^8 + 84\,a^5b^3x^6 + 36\,a^6b^2x^4 + 9\,a^7bx^2 + a^8}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

Mupad [B]
time = 0.08, size = 92, normalized size = 4.84

$$-\frac{\frac{a^8}{18} + \frac{a^7\,b\,x^2}{2} + 2\,a^6\,b^2\,x^4 + \frac{14\,a^5\,b^3\,x^6}{3} + 7\,a^4\,b^4\,x^8 + 7\,a^3\,b^5\,x^{10} + \frac{14\,a^2\,b^6\,x^{12}}{3} + 2\,a\,b^7\,x^{14} + \frac{b^8\,x^{16}}{2}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^19,x)
```

```
[Out] -(a^8/18 + (b^8*x^16)/2 + (a^7*b*x^2)/2 + 2*a*b^7*x^14 + 2*a^6*b^2*x^4 + (1
4*a^5*b^3*x^6)/3 + 7*a^4*b^4*x^8 + 7*a^3*b^5*x^10 + (14*a^2*b^6*x^12)/3)/x^
18
```

3.102 $\int \frac{(a+bx^2)^8}{x^{21}}\,dx$

Optimal. Leaf size=40

$$-\frac{(a+bx^2)^9}{20ax^{20}}+\frac{b(a+bx^2)^9}{180a^2x^{18}}$$

```
[Out] -1/20*(b*x^2+a)^9/a/x^20+1/180*b*(b*x^2+a)^9/a^2/x^18
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 47, 37}

$$\frac{b(a+bx^2)^9}{180a^2x^{18}}-\frac{(a+bx^2)^9}{20ax^{20}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^21,x]
```

```
[Out] -1/20*(a + b*x^2)^9/(a*x^20) + (b*(a + b*x^2)^9)/(180*a^2*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^9}{20ax^{20}} - \frac{b\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^2\right)}{20a}$$
$$= -\frac{(a+bx^2)^9}{20ax^{20}} + \frac{b(a+bx^2)^9}{180a^2x^{18}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. $2(40) = 80$.
time = 0.00, size = 106, normalized size = 2.65

$$-\frac{a^8}{20x^{20}} - \frac{4a^7b}{9x^{18}} - \frac{7a^6b^2}{4x^{16}} - \frac{4a^5b^3}{x^{14}} - \frac{35a^4b^4}{6x^{12}} - \frac{28a^3b^5}{5x^{10}} - \frac{7a^2b^6}{2x^8} - \frac{4ab^7}{3x^6} - \frac{b^8}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^21,x]
```

```
[Out] -1/20*a^8/x^20 - (4*a^7*b)/(9*x^18) - (7*a^6*b^2)/(4*x^16) - (4*a^5*b^3)/x^
14 - (35*a^4*b^4)/(6*x^12) - (28*a^3*b^5)/(5*x^10) - (7*a^2*b^6)/(2*x^8) -
(4*a*b^7)/(3*x^6) - b^8/(4*x^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(36) = 72$.
time = 0.04, size = 91, normalized size = 2.28

method	result	size
default	$-\frac{a^8}{20x^{20}} - \frac{4a^5b^3}{x^{14}} - \frac{4a^7b}{9x^{18}} - \frac{28a^3b^5}{5x^{10}} - \frac{7a^6b^2}{4x^{16}} - \frac{b^8}{4x^4} - \frac{4a\,b^7}{3x^6} - \frac{7a^2b^6}{2x^8} - \frac{35a^4b^4}{6x^{12}}$	91
norman	$\frac{-\frac{1}{20}a^8-\frac{4}{9}a^7b\,x^2-\frac{7}{4}a^6b^2x^4-4a^5b^3x^6-\frac{35}{6}a^4b^4x^8-\frac{28}{5}a^3b^5x^{10}-\frac{7}{2}a^2b^6x^{12}-\frac{4}{3}a\,b^7x^{14}-\frac{1}{4}b^8x^{16}}{x^{20}}$	92
risch	$\frac{-\frac{1}{20}a^8-\frac{4}{9}a^7b\,x^2-\frac{7}{4}a^6b^2x^4-4a^5b^3x^6-\frac{35}{6}a^4b^4x^8-\frac{28}{5}a^3b^5x^{10}-\frac{7}{2}a^2b^6x^{12}-\frac{4}{3}a\,b^7x^{14}-\frac{1}{4}b^8x^{16}}{x^{20}}$	92
gosper	$-\frac{45b^8x^{16}+240a\,b^7x^{14}+630a^2b^6x^{12}+1008a^3b^5x^{10}+1050a^4b^4x^8+720a^5b^3x^6+315a^6b^2x^4+80a^7b\,x^2+9a^8}{180x^{20}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^21,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*a^8/x^20-4*a^5*b^3/x^14-4/9*a^7*b/x^18-28/5*a^3*b^5/x^10-7/4*a^6*b^2/
x^16-1/4*b^8/x^4-4/3*a*b^7/x^6-7/2*a^2*b^6/x^8-35/6*a^4*b^4/x^12
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.
time = 0.28, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{16} + 240\,ab^7x^{14} + 630\,a^2b^6x^{12} + 1008\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 720\,a^5b^3x^6 + 315\,a^6b^2x^4 + 80\,a^7bx^2 + 9\,a^8}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="maxima")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.
time = 1.17, size = 92, normalized size = 2.30

$$-\frac{45\,b^{8}x^{16}+240\,ab^{7}x^{14}+630\,a^{2}b^{6}x^{12}+1008\,a^{3}b^{5}x^{10}+1050\,a^{4}b^{4}x^{8}+720\,a^{5}b^{3}x^{6}+315\,a^{6}b^{2}x^{4}+80\,a^{7}bx^{2}+9\,a^{8}}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="fricas")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(32) = 64$.
time = 0.41, size = 99, normalized size = 2.48

$$\frac{-9a^{8}-80a^{7}bx^{2}-315a^{6}b^{2}x^{4}-720a^{5}b^{3}x^{6}-1050a^{4}b^{4}x^{8}-1008a^{3}b^{5}x^{10}-630a^{2}b^{6}x^{12}-240ab^{7}x^{14}-45b^{8}x^{16}}{180x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**21,x)
```

```
[Out] (-9*a**8 - 80*a**7*b*x**2 - 315*a**6*b**2*x**4 - 720*a**5*b**3*x**6 - 1050*
a**4*b**4*x**8 - 1008*a**3*b**5*x**10 - 630*a**2*b**6*x**12 - 240*a*b**7*x*
*14 - 45*b**8*x**16)/(180*x**20)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.
time = 0.92, size = 92, normalized size = 2.30

$$-\frac{45\,b^{8}x^{16}+240\,ab^{7}x^{14}+630\,a^{2}b^{6}x^{12}+1008\,a^{3}b^{5}x^{10}+1050\,a^{4}b^{4}x^{8}+720\,a^{5}b^{3}x^{6}+315\,a^{6}b^{2}x^{4}+80\,a^{7}bx^{2}+9\,a^{8}}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="giac")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

Mupad [B]
time = 0.08, size = 92, normalized size = 2.30

$$-\frac{\frac{a^8}{20}+\frac{4\,a^7\,b\,x^2}{9}+\frac{7\,a^6\,b^2\,x^4}{4}+4\,a^5\,b^3\,x^6+\frac{35\,a^4\,b^4\,x^8}{6}+\frac{28\,a^3\,b^5\,x^{10}}{5}+\frac{7\,a^2\,b^6\,x^{12}}{2}+\frac{4\,a\,b^7\,x^{14}}{3}+\frac{b^8\,x^{16}}{4}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^21,x)
```

```
[Out] -(a^8/20 + (b^8*x^16)/4 + (4*a^7*b*x^2)/9 + (4*a*b^7*x^14)/3 + (7*a^6*b^2*x
^4)/4 + 4*a^5*b^3*x^6 + (35*a^4*b^4*x^8)/6 + (28*a^3*b^5*x^10)/5 + (7*a^2*b
^6*x^12)/2)/x^20
```

3.103 $\int \frac{(a+bx^2)^8}{x^{23}}\,dx$

Optimal. Leaf size=62

$$-\frac{(a+bx^2)^9}{22ax^{22}}+\frac{b(a+bx^2)^9}{110a^2x^{20}}-\frac{b^2(a+bx^2)^9}{990a^3x^{18}}$$

```
[Out] -1/22*(b*x^2+a)^9/a/x^22+1/110*b*(b*x^2+a)^9/a^2/x^20-1/990*b^2*(b*x^2+a)^9
/a^3/x^18
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 47, 37}

$$-\frac{b^2(a+bx^2)^9}{990a^3x^{18}}+\frac{b(a+bx^2)^9}{110a^2x^{20}}-\frac{(a+bx^2)^9}{22ax^{22}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^23,x]
```

```
[Out] -1/22*(a + b*x^2)^9/(a*x^22) + (b*(a + b*x^2)^9)/(110*a^2*x^20) - (b^2*(a +
 b*x^2)^9)/(990*a^3*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{23}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx, x, x^2\right)\\ &= -\frac{(a+bx^2)^9}{22ax^{22}} - \frac{b\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^2\right)}{11a}\\ &= -\frac{(a+bx^2)^9}{22ax^{22}} + \frac{b(a+bx^2)^9}{110a^2x^{20}} + \frac{b^2\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^2\right)}{110a^2}\\ &= -\frac{(a+bx^2)^9}{22ax^{22}} + \frac{b(a+bx^2)^9}{110a^2x^{20}} - \frac{b^2(a+bx^2)^9}{990a^3x^{18}}\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 104, normalized size = 1.68

$$-\frac{a^8}{22x^{22}} - \frac{2a^7b}{5x^{20}} - \frac{14a^6b^2}{9x^{18}} - \frac{7a^5b^3}{2x^{16}} - \frac{5a^4b^4}{x^{14}} - \frac{14a^3b^5}{3x^{12}} - \frac{14a^2b^6}{5x^{10}} - \frac{ab^7}{x^8} - \frac{b^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^23,x]
```

```
[Out] -1/22*a^8/x^22 - (2*a^7*b)/(5*x^20) - (14*a^6*b^2)/(9*x^18) - (7*a^5*b^3)/(
2*x^16) - (5*a^4*b^4)/x^14 - (14*a^3*b^5)/(3*x^12) - (14*a^2*b^6)/(5*x^10)
- (a*b^7)/x^8 - b^8/(6*x^6)
```

Maple [**A**]
time = 0.04, size = 91, normalized size = 1.47

method	result	size
default	$-\frac{2a^7b}{5x^{20}} - \frac{5a^4b^4}{x^{14}} - \frac{a^8}{22x^{22}} - \frac{14a^6b^2}{9x^{18}} - \frac{14a^2b^6}{5x^{10}} - \frac{7a^5b^3}{2x^{16}} - \frac{b^8}{6x^6} - \frac{a\,b^7}{x^8} - \frac{14a^3b^5}{3x^{12}}$	91
norman	$\frac{-\frac{1}{22}a^8 - \frac{2}{5}a^7b\,x^2 - \frac{14}{9}a^6b^2x^4 - \frac{7}{2}a^5b^3x^6 - 5a^4b^4x^8 - \frac{14}{3}a^3b^5x^{10} - \frac{14}{5}a^2b^6x^{12} - a\,b^7x^{14} - \frac{1}{6}b^8x^{16}}{x^{22}}$	92
risch	$\frac{-\frac{1}{22}a^8 - \frac{2}{5}a^7b\,x^2 - \frac{14}{9}a^6b^2x^4 - \frac{7}{2}a^5b^3x^6 - 5a^4b^4x^8 - \frac{14}{3}a^3b^5x^{10} - \frac{14}{5}a^2b^6x^{12} - a\,b^7x^{14} - \frac{1}{6}b^8x^{16}}{x^{22}}$	92
gosper	$-\frac{165b^8x^{16} + 990a\,b^7x^{14} + 2772a^2b^6x^{12} + 4620a^3b^5x^{10} + 4950a^4b^4x^8 + 3465a^5b^3x^6 + 1540a^6b^2x^4 + 396a^7b\,x^2 + 45a^8}{990x^{22}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^23,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*a^7*b/x^20-5*a^4*b^4/x^14-1/22*a^8/x^22-14/9*a^6*b^2/x^18-14/5*a^2*b^6
/x^10-7/2*a^5*b^3/x^16-1/6*b^8/x^6-a*b^7/x^8-14/3*a^3*b^5/x^12
```

Maxima [A]
time = 0.29, size = 92, normalized size = 1.48

$$-\frac{165\,b^8x^{16}+990\,ab^7x^{14}+2772\,a^2b^6x^{12}+4620\,a^3b^5x^{10}+4950\,a^4b^4x^8+3465\,a^5b^3x^6+1540\,a^6b^2x^4+396\,a^7bx^2+45\,a^8}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="maxima")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

Fricas [A]
time = 1.06, size = 92, normalized size = 1.48

$$-\frac{165\,b^8x^{16}+990\,ab^7x^{14}+2772\,a^2b^6x^{12}+4620\,a^3b^5x^{10}+4950\,a^4b^4x^8+3465\,a^5b^3x^6+1540\,a^6b^2x^4+396\,a^7bx^2+45\,a^8}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="fricas")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

Sympy [A]
time = 0.44, size = 99, normalized size = 1.60

$$\frac{-45a^8-396a^7bx^2-1540a^6b^2x^4-3465a^5b^3x^6-4950a^4b^4x^8-4620a^3b^5x^{10}-2772a^2b^6x^{12}-990ab^7x^{14}-165b^8x^{16}}{990x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**23,x)
```

```
[Out] (-45*a**8 - 396*a**7*b*x**2 - 1540*a**6*b**2*x**4 - 3465*a**5*b**3*x**6 - 4
950*a**4*b**4*x**8 - 4620*a**3*b**5*x**10 - 2772*a**2*b**6*x**12 - 990*a*b*
*7*x**14 - 165*b**8*x**16)/(990*x**22)
```

Giac [A]
time = 0.79, size = 92, normalized size = 1.48

$$-\frac{165\,b^8x^{16}+990\,ab^7x^{14}+2772\,a^2b^6x^{12}+4620\,a^3b^5x^{10}+4950\,a^4b^4x^8+3465\,a^5b^3x^6+1540\,a^6b^2x^4+396\,a^7bx^2+45\,a^8}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="giac")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

Mupad [B]
time = 0.08, size = 91, normalized size = 1.47

$$-\frac{\frac{a^8}{22}+\frac{2\,a^7\,b\,x^2}{5}+\frac{14\,a^6\,b^2\,x^4}{9}+\frac{7\,a^5\,b^3\,x^6}{2}+5\,a^4\,b^4\,x^8+\frac{14\,a^3\,b^5\,x^{10}}{3}+\frac{14\,a^2\,b^6\,x^{12}}{5}+a\,b^7\,x^{14}+\frac{b^8\,x^{16}}{6}}{x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^23,x)
```

```
[Out] -(a^8/22 + (b^8*x^16)/6 + (2*a^7*b*x^2)/5 + a*b^7*x^14 + (14*a^6*b^2*x^4)/9
 + (7*a^5*b^3*x^6)/2 + 5*a^4*b^4*x^8 + (14*a^3*b^5*x^10)/3 + (14*a^2*b^6*x^
12)/5)/x^22
```

3.104 $\int \frac{(a+bx^2)^8}{x^{25}}\,dx$

Optimal. Leaf size=84

$$-\frac{(a+bx^2)^9}{24ax^{24}}+\frac{b(a+bx^2)^9}{88a^2x^{22}}-\frac{b^2(a+bx^2)^9}{440a^3x^{20}}+\frac{b^3(a+bx^2)^9}{3960a^4x^{18}}$$

```
[Out] -1/24*(b*x^2+a)^9/a/x^24+1/88*b*(b*x^2+a)^9/a^2/x^22-1/440*b^2*(b*x^2+a)^9/
a^3/x^20+1/3960*b^3*(b*x^2+a)^9/a^4/x^18
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 47, 37}

$$\frac{b^3(a+bx^2)^9}{3960a^4x^{18}}-\frac{b^2(a+bx^2)^9}{440a^3x^{20}}+\frac{b(a+bx^2)^9}{88a^2x^{22}}-\frac{(a+bx^2)^9}{24ax^{24}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^25,x]
```

```
[Out] -1/24*(a + b*x^2)^9/(a*x^24) + (b*(a + b*x^2)^9)/(88*a^2*x^22) - (b^2*(a +
b*x^2)^9)/(440*a^3*x^20) + (b^3*(a + b*x^2)^9)/(3960*a^4*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{25}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx,x,x^2\right)\\
&= -\frac{(a+bx^2)^9}{24ax^{24}} - \frac{b\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^2\right)}{8a}\\
&= -\frac{(a+bx^2)^9}{24ax^{24}} + \frac{b(a+bx^2)^9}{88a^2x^{22}} + \frac{b^2\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^2\right)}{44a^2}\\
&= -\frac{(a+bx^2)^9}{24ax^{24}} + \frac{b(a+bx^2)^9}{88a^2x^{22}} - \frac{b^2(a+bx^2)^9}{440a^3x^{20}} - \frac{b^3\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^2\right)}{440a^3}\\
&= -\frac{(a+bx^2)^9}{24ax^{24}} + \frac{b(a+bx^2)^9}{88a^2x^{22}} - \frac{b^2(a+bx^2)^9}{440a^3x^{20}} + \frac{b^3(a+bx^2)^9}{3960a^4x^{18}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 106, normalized size = 1.26

$$-\frac{a^8}{24x^{24}} - \frac{4a^7b}{11x^{22}} - \frac{7a^6b^2}{5x^{20}} - \frac{28a^5b^3}{9x^{18}} - \frac{35a^4b^4}{8x^{16}} - \frac{4a^3b^5}{x^{14}} - \frac{7a^2b^6}{3x^{12}} - \frac{4ab^7}{5x^{10}} - \frac{b^8}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^25,x]
```

```
[Out] -1/24*a^8/x^24 - (4*a^7*b)/(11*x^22) - (7*a^6*b^2)/(5*x^20) - (28*a^5*b^3)/
(9*x^18) - (35*a^4*b^4)/(8*x^16) - (4*a^3*b^5)/x^14 - (7*a^2*b^6)/(3*x^12)
- (4*a*b^7)/(5*x^10) - b^8/(8*x^8)
```

Maple [A]
time = 0.04, size = 91, normalized size = 1.08

method	result	size
default	$-\frac{4a^7b}{11x^{22}} - \frac{28a^5b^3}{9x^{18}} - \frac{4ab^7}{5x^{10}} - \frac{a^8}{24x^{24}} - \frac{35a^4b^4}{8x^{16}} - \frac{b^8}{8x^8} - \frac{4a^3b^5}{x^{14}} - \frac{7a^6b^2}{5x^{20}} - \frac{7a^2b^6}{3x^{12}}$	91
norman	$\frac{-\frac{7}{3}a^2b^6x^{12} - \frac{4}{5}ab^7x^{14} - \frac{1}{8}b^8x^{16} - \frac{4}{11}a^7bx^2 - \frac{7}{5}a^6b^2x^4 - \frac{28}{9}a^5b^3x^6 - \frac{35}{8}a^4b^4x^8 - 4a^3b^5x^{10} - \frac{1}{24}a^8}{x^{24}}$	92
risch	$\frac{-\frac{7}{3}a^2b^6x^{12} - \frac{4}{5}ab^7x^{14} - \frac{1}{8}b^8x^{16} - \frac{4}{11}a^7bx^2 - \frac{7}{5}a^6b^2x^4 - \frac{28}{9}a^5b^3x^6 - \frac{35}{8}a^4b^4x^8 - 4a^3b^5x^{10} - \frac{1}{24}a^8}{x^{24}}$	92
gosper	$-\frac{495b^8x^{16} + 3168ab^7x^{14} + 9240a^2b^6x^{12} + 15840a^3b^5x^{10} + 17325a^4b^4x^8 + 12320a^5b^3x^6 + 5544a^6b^2x^4 + 1440a^7bx^2 + 165a^8}{3960x^{24}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^25,x,method=_RETURNVERBOSE)
```

```
[Out] -4/11*a^7*b/x^22-28/9*a^5*b^3/x^18-4/5*a*b^7/x^10-1/24*a^8/x^24-35/8*a^4*b^
4/x^16-1/8*b^8/x^8-4*a^3*b^5/x^14-7/5*a^6*b^2/x^20-7/3*a^2*b^6/x^12
```

Maxima [A]
time = 0.28, size = 92, normalized size = 1.10

$$-\frac{495\,b^8x^{16} + 3168\,ab^7x^{14} + 9240\,a^2b^6x^{12} + 15840\,a^3b^5x^{10} + 17325\,a^4b^4x^8 + 12320\,a^5b^3x^6 + 5544\,a^6b^2x^4 + 1440\,a^7bx^2 + 165\,a^8}{3960\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="maxima")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

Fricas [A]
time = 1.01, size = 92, normalized size = 1.10

$$-\frac{495\,b^8x^{16} + 3168\,ab^7x^{14} + 9240\,a^2b^6x^{12} + 15840\,a^3b^5x^{10} + 17325\,a^4b^4x^8 + 12320\,a^5b^3x^6 + 5544\,a^6b^2x^4 + 1440\,a^7bx^2 + 165\,a^8}{3960\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="fricas")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

Sympy [A]
time = 0.46, size = 99, normalized size = 1.18

$$\frac{-165a^8 - 1440a^7bx^2 - 5544a^6b^2x^4 - 12320a^5b^3x^6 - 17325a^4b^4x^8 - 15840a^3b^5x^{10} - 9240a^2b^6x^{12} - 3168ab^7x^{14} - 495b^8x^{16}}{3960x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**25,x)
```

```
[Out] (-165*a**8 - 1440*a**7*b*x**2 - 5544*a**6*b**2*x**4 - 12320*a**5*b**3*x**6
- 17325*a**4*b**4*x**8 - 15840*a**3*b**5*x**10 - 9240*a**2*b**6*x**12 - 316
8*a*b**7*x**14 - 495*b**8*x**16)/(3960*x**24)
```

Giac [A]
time = 0.74, size = 92, normalized size = 1.10

$$-\frac{495\,b^8x^{16} + 3168\,ab^7x^{14} + 9240\,a^2b^6x^{12} + 15840\,a^3b^5x^{10} + 17325\,a^4b^4x^8 + 12320\,a^5b^3x^6 + 5544\,a^6b^2x^4 + 1440\,a^7bx^2 + 165\,a^8}{3960\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="giac")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

Mupad [B]
time = 4.97, size = 92, normalized size = 1.10

$$-\frac{\frac{a^8}{24}+\frac{4\,a^7\,b\,x^2}{11}+\frac{7\,a^6\,b^2\,x^4}{5}+\frac{28\,a^5\,b^3\,x^6}{9}+\frac{35\,a^4\,b^4\,x^8}{8}+4\,a^3\,b^5\,x^{10}+\frac{7\,a^2\,b^6\,x^{12}}{3}+\frac{4\,a\,b^7\,x^{14}}{5}+\frac{b^8\,x^{16}}{8}}{x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^25,x)
```

```
[Out] -(a^8/24 + (b^8*x^16)/8 + (4*a^7*b*x^2)/11 + (4*a*b^7*x^14)/5 + (7*a^6*b^2*
x^4)/5 + (28*a^5*b^3*x^6)/9 + (35*a^4*b^4*x^8)/8 + 4*a^3*b^5*x^10 + (7*a^2*
b^6*x^12)/3)/x^24
```

3.105 $\int \frac{(a+bx^2)^8}{x^{27}}\,dx$

Optimal. Leaf size=106

$$-\frac{(a+bx^2)^9}{26ax^{26}}+\frac{b(a+bx^2)^9}{78a^2x^{24}}-\frac{b^2(a+bx^2)^9}{286a^3x^{22}}+\frac{b^3(a+bx^2)^9}{1430a^4x^{20}}-\frac{b^4(a+bx^2)^9}{12870a^5x^{18}}$$

```
[Out] -1/26*(b*x^2+a)^9/a/x^26+1/78*b*(b*x^2+a)^9/a^2/x^24-1/286*b^2*(b*x^2+a)^9/
a^3/x^22+1/1430*b^3*(b*x^2+a)^9/a^4/x^20-1/12870*b^4*(b*x^2+a)^9/a^5/x^18
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 47, 37}

$$-\frac{b^4(a+bx^2)^9}{12870a^5x^{18}}+\frac{b^3(a+bx^2)^9}{1430a^4x^{20}}-\frac{b^2(a+bx^2)^9}{286a^3x^{22}}+\frac{b(a+bx^2)^9}{78a^2x^{24}}-\frac{(a+bx^2)^9}{26ax^{26}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^27,x]
```

```
[Out] -1/26*(a + b*x^2)^9/(a*x^26) + (b*(a + b*x^2)^9)/(78*a^2*x^24) - (b^2*(a +
b*x^2)^9)/(286*a^3*x^22) + (b^3*(a + b*x^2)^9)/(1430*a^4*x^20) - (b^4*(a +
b*x^2)^9)/(12870*a^5*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^8}{x^{27}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{14}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}-\frac{(2b)\text{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx,x,x^2\right)}{13a}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}+\frac{b^2\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^2\right)}{26a^2}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}-\frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}}-\frac{b^3\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^2\right)}{143a^3}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}-\frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}}+\frac{b^3\left(a+bx^2\right)^9}{1430a^4x^{20}}+\frac{b^4\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^2\right)}{1430a^4}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}-\frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}}+\frac{b^3\left(a+bx^2\right)^9}{1430a^4x^{20}}-\frac{b^4\left(a+bx^2\right)^9}{12870a^5x^{18}}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 106, normalized size = 1.00

$$
-\frac{a^8}{26x^{26}}-\frac{a^7b}{3x^{24}}-\frac{14a^6b^2}{11x^{22}}-\frac{14a^5b^3}{5x^{20}}-\frac{35a^4b^4}{9x^{18}}-\frac{7a^3b^5}{2x^{16}}-\frac{2a^2b^6}{x^{14}}-\frac{2ab^7}{3x^{12}}-\frac{b^8}{10x^{10}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^27,x]
[Out] -1/26*a^8/x^26 - (a^7*b)/(3*x^24) - (14*a^6*b^2)/(11*x^22) - (14*a^5*b^3)/(
5*x^20) - (35*a^4*b^4)/(9*x^18) - (7*a^3*b^5)/(2*x^16) - (2*a^2*b^6)/x^14 -
 (2*a*b^7)/(3*x^12) - b^8/(10*x^10)
```

Maple [A]
time = 0.04, size = 91, normalized size = 0.86

method	result	size
default	$-\frac{2a^2b^6}{x^{14}}-\frac{35a^4b^4}{9x^{18}}-\frac{b^8}{10x^{10}}-\frac{a^7b}{3x^{24}}-\frac{7a^3b^5}{2x^{16}}-\frac{14a^5b^3}{5x^{20}}-\frac{14a^6b^2}{11x^{22}}-\frac{2ab^7}{3x^{12}}-\frac{a^8}{26x^{26}}$	91
norman	$\frac{-\frac{1}{10}b^8x^{16}-\frac{14}{11}a^6b^2x^4-\frac{14}{5}a^5b^3x^6-\frac{35}{9}a^4b^4x^8-\frac{7}{2}a^3b^5x^{10}-2a^2b^6x^{12}-\frac{2}{3}ab^7x^{14}-\frac{1}{3}a^7bx^2-\frac{1}{26}a^8}{x^{26}}$	92
risch	$\frac{-\frac{1}{10}b^8x^{16}-\frac{14}{11}a^6b^2x^4-\frac{14}{5}a^5b^3x^6-\frac{35}{9}a^4b^4x^8-\frac{7}{2}a^3b^5x^{10}-2a^2b^6x^{12}-\frac{2}{3}ab^7x^{14}-\frac{1}{3}a^7bx^2-\frac{1}{26}a^8}{x^{26}}$	92

gosper	$-\frac{1287b^8x^{16}+8580a\,b^7x^{14}+25740a^2b^6x^{12}+45045a^3b^5x^{10}+50050a^4b^4x^8+36036a^5b^3x^6+16380a^6b^2x^4+4290a^7b\,x^2+495a^8}{12870x^{26}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^27,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^2*b^6/x^14-35/9*a^4*b^4/x^18-1/10*b^8/x^10-1/3*a^7*b/x^24-7/2*a^3*b^5/
x^16-14/5*a^5*b^3/x^20-14/11*a^6*b^2/x^22-2/3*a*b^7/x^12-1/26*a^8/x^26
```

Maxima [A]
time = 0.41, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4+4290\,a^7bx^2+495\,a^8}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="maxima")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

Fricas [A]
time = 0.63, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4+4290\,a^7bx^2+495\,a^8}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="fricas")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

Sympy [A]
time = 0.49, size = 99, normalized size = 0.93

$$\frac{-495a^8-4290a^7bx^2-16380a^6b^2x^4-36036a^5b^3x^6-50050a^4b^4x^8-45045a^3b^5x^{10}-25740a^2b^6x^{12}-8580ab^7x^{14}-1287b^8x^{16}}{12870x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**27,x)
```

```
[Out] (-495*a**8 - 4290*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 36036*a**5*b**3*x**6
 - 50050*a**4*b**4*x**8 - 45045*a**3*b**5*x**10 - 25740*a**2*b**6*x**12 - 8
580*a*b**7*x**14 - 1287*b**8*x**16)/(12870*x**26)
```

Giac [A]
time = 0.62, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4+4290\,a^7bx^2+495\,a^8}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="giac")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

Mupad [B]
time = 0.08, size = 92, normalized size = 0.87

$$-\frac{\frac{a^8}{26}+\frac{a^7\,b\,x^2}{3}+\frac{14\,a^6\,b^2\,x^4}{11}+\frac{14\,a^5\,b^3\,x^6}{5}+\frac{35\,a^4\,b^4\,x^8}{9}+\frac{7\,a^3\,b^5\,x^{10}}{2}+2\,a^2\,b^6\,x^{12}+\frac{2\,a\,b^7\,x^{14}}{3}+\frac{b^8\,x^{16}}{10}}{x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^27,x)
```

```
[Out] -(a^8/26 + (b^8*x^16)/10 + (a^7*b*x^2)/3 + (2*a*b^7*x^14)/3 + (14*a^6*b^2*x
^4)/11 + (14*a^5*b^3*x^6)/5 + (35*a^4*b^4*x^8)/9 + (7*a^3*b^5*x^10)/2 + 2*a
^2*b^6*x^12)/x^26
```

3.106 $\int \frac{(a+bx^2)^8}{x^{29}} dx$

Optimal. Leaf size=108

$$-\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

```
[Out] -1/28*a^8/x^28-4/13*a^7*b/x^26-7/6*a^6*b^2/x^24-28/11*a^5*b^3/x^22-7/2*a^4*
b^4/x^20-28/9*a^3*b^5/x^18-7/4*a^2*b^6/x^16-4/7*a*b^7/x^14-1/12*b^8/x^12
```

Rubi [A]

time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^29,x]
```

```
[Out] -1/28*a^8/x^28 - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)/
(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4*x
^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{29}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{15}} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{15}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{13}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{11}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^9}+\frac{8ab^7}{x^8}+\frac{b^8}{x^7}\right)\right. \\
&= -\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 108, normalized size = 1.00

$$-\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^29,x]
```

```
[Out] -1/28*a^8/x^28 - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)/
(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4*x
^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

Maple [A]
time = 0.04, size = 91, normalized size = 0.84

method	result
default	$-\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4a\,b^7}{7x^{14}}-\frac{b^8}{12x^{12}}$
norman	$\frac{-\frac{28}{11}a^5b^3x^6-\frac{7}{2}a^4b^4x^8-\frac{28}{9}a^3b^5x^{10}-\frac{7}{4}a^2b^6x^{12}-\frac{4}{7}a\,b^7x^{14}-\frac{1}{12}b^8x^{16}-\frac{4}{13}a^7b\,x^2-\frac{7}{6}a^6b^2x^4-\frac{1}{28}a^8}{x^{28}}$
risch	$\frac{-\frac{28}{11}a^5b^3x^6-\frac{7}{2}a^4b^4x^8-\frac{28}{9}a^3b^5x^{10}-\frac{7}{4}a^2b^6x^{12}-\frac{4}{7}a\,b^7x^{14}-\frac{1}{12}b^8x^{16}-\frac{4}{13}a^7b\,x^2-\frac{7}{6}a^6b^2x^4-\frac{1}{28}a^8}{x^{28}}$
gosper	$-\frac{3003b^8x^{16}+20592a\,b^7x^{14}+63063a^2b^6x^{12}+112112a^3b^5x^{10}+126126a^4b^4x^8+91728a^5b^3x^6+42042a^6b^2x^4+11088a^7b\,x^2+1287a^8}{36036x^{28}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^29,x,method=_RETURNVERBOSE)
```

```
[Out] -1/28*a^8/x^28-4/13*a^7*b/x^26-7/6*a^6*b^2/x^24-28/11*a^5*b^3/x^22-7/2*a^4*
b^4/x^20-28/9*a^3*b^5/x^18-7/4*a^2*b^6/x^16-4/7*a*b^7/x^14-1/12*b^8/x^12
```

Maxima [A]
time = 0.29, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042\,a^6b^2x^4+11088\,a^7bx^2+1287\,a^8}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="maxima")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

Fricas [A]
time = 0.48, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042\,a^6b^2x^4+11088\,a^7bx^2+1287\,a^8}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^8/x^29,x, algorithm="fricas")

[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 11088*a^7*b*x^2 + 1287*a^8)/x^28

Sympy [A]
time = 0.51, size = 99, normalized size = 0.92

$$\frac{-1287a^8 - 11088a^7bx^2 - 42042a^6b^2x^4 - 91728a^5b^3x^6 - 126126a^4b^4x^8 - 112112a^3b^5x^{10} - 63063a^2b^6x^{12} - 20592ab^7x^{14} - 3003b^8x^{16}}{36036x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**8/x**29,x)

[Out] (-1287*a**8 - 11088*a**7*b*x**2 - 42042*a**6*b**2*x**4 - 91728*a**5*b**3*x**6 - 126126*a**4*b**4*x**8 - 112112*a**3*b**5*x**10 - 63063*a**2*b**6*x**12 - 20592*a*b**7*x**14 - 3003*b**8*x**16)/(36036*x**28)

Giac [A]
time = 0.74, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{16} + 20592\,ab^7x^{14} + 63063\,a^2b^6x^{12} + 112112\,a^3b^5x^{10} + 126126\,a^4b^4x^8 + 91728\,a^5b^3x^6 + 42042\,a^6b^2x^4 + 11088\,a^7bx^2 + 1287\,a^8}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^8/x^29,x, algorithm="giac")

[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 11088*a^7*b*x^2 + 1287*a^8)/x^28

Mupad [B]
time = 4.89, size = 92, normalized size = 0.85

$$-\frac{\frac{a^8}{28} + \frac{4\,a^7\,b\,x^2}{13} + \frac{7\,a^6\,b^2\,x^4}{6} + \frac{28\,a^5\,b^3\,x^6}{11} + \frac{7\,a^4\,b^4\,x^8}{2} + \frac{28\,a^3\,b^5\,x^{10}}{9} + \frac{7\,a^2\,b^6\,x^{12}}{4} + \frac{4\,a\,b^7\,x^{14}}{7} + \frac{b^8\,x^{16}}{12}}{x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^8/x^29,x)

[Out] -(a^8/28 + (b^8*x^16)/12 + (4*a^7*b*x^2)/13 + (4*a*b^7*x^14)/7 + (7*a^6*b^2*x^4)/6 + (28*a^5*b^3*x^6)/11 + (7*a^4*b^4*x^8)/2 + (28*a^3*b^5*x^10)/9 + (7*a^2*b^6*x^12)/4)/x^28

3.107 $\int \frac{(a+bx^2)^8}{x^{31}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

```
[Out] -1/30*a^8/x^30-2/7*a^7*b/x^28-14/13*a^6*b^2/x^26-7/3*a^5*b^3/x^24-35/11*a^4
*b^4/x^22-14/5*a^3*b^5/x^20-14/9*a^2*b^6/x^18-1/2*a*b^7/x^16-1/14*b^8/x^14
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^31,x]
```

```
[Out] -1/30*a^8/x^30 - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)/
(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/(9
*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{31}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{16}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{15}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{13}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{11}}+\frac{28a^2b^6}{x^{10}}+\frac{8ab^7}{x^9}+\frac{b^8}{x^8}\right.\right.\\ &= -\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 108, normalized size = 1.00

$$-\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^31,x]
```

```
[Out] -1/30*a^8/x^30 - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)/
(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/(9
*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

Maple [A]
time = 0.05, size = 91, normalized size = 0.84

method	result
default	$-\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{a\,b^7}{2x^{16}}-\frac{b^8}{14x^{14}}$
norman	$\frac{-\frac{7}{3}a^5b^3x^6-\frac{35}{11}a^4b^4x^8-\frac{14}{5}a^3b^5x^{10}-\frac{14}{9}a^2b^6x^{12}-\frac{1}{2}a\,b^7x^{14}-\frac{1}{14}b^8x^{16}-\frac{2}{7}a^7b\,x^2-\frac{14}{13}a^6b^2x^4-\frac{1}{30}a^8}{x^{30}}$
risch	$\frac{-\frac{7}{3}a^5b^3x^6-\frac{35}{11}a^4b^4x^8-\frac{14}{5}a^3b^5x^{10}-\frac{14}{9}a^2b^6x^{12}-\frac{1}{2}a\,b^7x^{14}-\frac{1}{14}b^8x^{16}-\frac{2}{7}a^7b\,x^2-\frac{14}{13}a^6b^2x^4-\frac{1}{30}a^8}{x^{30}}$
gosper	$-\frac{6435b^8x^{16}+45045a\,b^7x^{14}+140140a^2b^6x^{12}+252252a^3b^5x^{10}+286650a^4b^4x^8+210210a^5b^3x^6+97020a^6b^2x^4+25740a^7b\,x^2+3003a^8}{90090x^{30}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^31,x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*a^8/x^30-2/7*a^7*b/x^28-14/13*a^6*b^2/x^26-7/3*a^5*b^3/x^24-35/11*a^4
*b^4/x^22-14/5*a^3*b^5/x^20-14/9*a^2*b^6/x^18-1/2*a*b^7/x^16-1/14*b^8/x^14
```

Maxima [A]
time = 0.29, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{16}+45045\,ab^7x^{14}+140140\,a^2b^6x^{12}+252252\,a^3b^5x^{10}+286650\,a^4b^4x^8+210210\,a^5b^3x^6+97020\,a^6b^2x^4+25740\,a^7bx^2+3003\,a^8}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="maxima")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

Fricas [A]
time = 0.82, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{16}+45045\,ab^7x^{14}+140140\,a^2b^6x^{12}+252252\,a^3b^5x^{10}+286650\,a^4b^4x^8+210210\,a^5b^3x^6+97020\,a^6b^2x^4+25740\,a^7bx^2+3003\,a^8}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="fricas")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

Sympy [A]
time = 0.53, size = 99, normalized size = 0.92

$$\frac{-3003a^8-25740a^7bx^2-97020a^6b^2x^4-210210a^5b^3x^6-286650a^4b^4x^8-252252a^3b^5x^{10}-140140a^2b^6x^{12}-45045ab^7x^{14}-6435b^8x^{16}}{90090x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**31,x)
```

```
[Out] (-3003*a**8 - 25740*a**7*b*x**2 - 97020*a**6*b**2*x**4 - 210210*a**5*b**3*x
**6 - 286650*a**4*b**4*x**8 - 252252*a**3*b**5*x**10 - 140140*a**2*b**6*x**
12 - 45045*a*b**7*x**14 - 6435*b**8*x**16)/(90090*x**30)
```

Giac [A]
time = 0.69, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{16}+45045\,ab^7x^{14}+140140\,a^2b^6x^{12}+252252\,a^3b^5x^{10}+286650\,a^4b^4x^8+210210\,a^5b^3x^6+97020\,a^6b^2x^4+25740\,a^7bx^2+3003\,a^8}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="giac")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

Mupad [B]
time = 4.86, size = 92, normalized size = 0.85

$$-\frac{\frac{a^8}{30}+\frac{2\,a^7\,b\,x^2}{7}+\frac{14\,a^6\,b^2\,x^4}{13}+\frac{7\,a^5\,b^3\,x^6}{3}+\frac{35\,a^4\,b^4\,x^8}{11}+\frac{14\,a^3\,b^5\,x^{10}}{5}+\frac{14\,a^2\,b^6\,x^{12}}{9}+\frac{a\,b^7\,x^{14}}{2}+\frac{b^8\,x^{16}}{14}}{x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^31,x)
```

```
[Out] -(a^8/30 + (b^8*x^16)/14 + (2*a^7*b*x^2)/7 + (a*b^7*x^14)/2 + (14*a^6*b^2*x
^4)/13 + (7*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/11 + (14*a^3*b^5*x^10)/5 + (1
4*a^2*b^6*x^12)/9)/x^30
```

3.108 $\int \frac{(a+bx^2)^8}{x^{33}} dx$

Optimal. Leaf size=106

$$-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

```
[Out] -1/32*a^8/x^32-4/15*a^7*b/x^30-a^6*b^2/x^28-28/13*a^5*b^3/x^26-35/12*a^4*b^
4/x^24-28/11*a^3*b^5/x^22-7/5*a^2*b^6/x^20-4/9*a*b^7/x^18-1/16*b^8/x^16
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^33,x]
```

```
[Out] -1/32*a^8/x^32 - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x^
26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^20
) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{33}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{17}} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{17}}+\frac{8a^7b}{x^{16}}+\frac{28a^6b^2}{x^{15}}+\frac{56a^5b^3}{x^{14}}+\frac{70a^4b^4}{x^{13}}+\frac{56a^3b^5}{x^{12}}+\frac{28a^2b^6}{x^{11}}+\frac{8ab^7}{x^{10}}+\frac{b^8}{x^9}\right)\right. \\
&= -\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 106, normalized size = 1.00

$$-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^33,x]
```

```
[Out] -1/32*a^8/x^32 - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x^
26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^20
) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

Maple [A]
time = 0.05, size = 91, normalized size = 0.86

method	result
default	$-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$
norman	$\frac{-\frac{28}{13}a^5b^3x^6-\frac{35}{12}a^4b^4x^8-\frac{28}{11}a^3b^5x^{10}-\frac{7}{5}a^2b^6x^{12}-\frac{4}{9}a\,b^7x^{14}-\frac{1}{16}b^8x^{16}-\frac{4}{15}a^7b\,x^2-a^6b^2x^4-\frac{1}{32}a^8}{x^{32}}$
risch	$\frac{-\frac{28}{13}a^5b^3x^6-\frac{35}{12}a^4b^4x^8-\frac{28}{11}a^3b^5x^{10}-\frac{7}{5}a^2b^6x^{12}-\frac{4}{9}a\,b^7x^{14}-\frac{1}{16}b^8x^{16}-\frac{4}{15}a^7b\,x^2-a^6b^2x^4-\frac{1}{32}a^8}{x^{32}}$
gosper	$-\frac{12870b^8x^{16}+91520a\,b^7x^{14}+288288a^2b^6x^{12}+524160a^3b^5x^{10}+600600a^4b^4x^8+443520a^5b^3x^6+205920a^6b^2x^4+54912a^7b\,x^2+6435}{205920x^{32}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^33,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*a^8/x^32-4/15*a^7*b/x^30-a^6*b^2/x^28-28/13*a^5*b^3/x^26-35/12*a^4*b^
4/x^24-28/11*a^3*b^5/x^22-7/5*a^2*b^6/x^20-4/9*a*b^7/x^18-1/16*b^8/x^16
```

Maxima [A]
time = 0.31, size = 92, normalized size = 0.87

$$-\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+205920\,a^6b^2x^4+54912\,a^7bx^2+6435}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="maxima")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

Fricas [A]
time = 0.57, size = 92, normalized size = 0.87

$$-\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+205920\,a^6b^2x^4+54912\,a^7bx^2+6435\,a^8}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="fricas")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

Sympy [A]
time = 0.56, size = 99, normalized size = 0.93

$$\frac{-6435a^8-54912a^7bx^2-205920a^6b^2x^4-443520a^5b^3x^6-600600a^4b^4x^8-524160a^3b^5x^{10}-288288a^2b^6x^{12}-91520ab^7x^{14}-12870b^8x^{16}}{205920x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**33,x)
```

```
[Out] (-6435*a**8 - 54912*a**7*b*x**2 - 205920*a**6*b**2*x**4 - 443520*a**5*b**3*
x**6 - 600600*a**4*b**4*x**8 - 524160*a**3*b**5*x**10 - 288288*a**2*b**6*x*
*12 - 91520*a*b**7*x**14 - 12870*b**8*x**16)/(205920*x**32)
```

Giac [A]
time = 0.70, size = 92, normalized size = 0.87

$$-\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+205920\,a^6b^2x^4+54912\,a^7bx^2+6435\,a^8}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="giac")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

Mupad [B]
time = 0.08, size = 91, normalized size = 0.86

$$-\frac{\frac{a^8}{32}+\frac{4\,a^7\,b\,x^2}{15}+a^6\,b^2\,x^4+\frac{28\,a^5\,b^3\,x^6}{13}+\frac{35\,a^4\,b^4\,x^8}{12}+\frac{28\,a^3\,b^5\,x^{10}}{11}+\frac{7\,a^2\,b^6\,x^{12}}{5}+\frac{4\,a\,b^7\,x^{14}}{9}+\frac{b^8\,x^{16}}{16}}{x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^33,x)
```

```
[Out] -(a^8/32 + (b^8*x^16)/16 + (4*a^7*b*x^2)/15 + (4*a*b^7*x^14)/9 + a^6*b^2*x^
4 + (28*a^5*b^3*x^6)/13 + (35*a^4*b^4*x^8)/12 + (28*a^3*b^5*x^10)/11 + (7*a
^2*b^6*x^12)/5)/x^32
```

3.109 $\int x^8 (a+bx^2)^8 \, dx$

Optimal. Leaf size=108

$$\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

```
[Out] 1/9*a^8*x^9+8/11*a^7*b*x^11+28/13*a^6*b^2*x^13+56/15*a^5*b^3*x^15+70/17*a^4
*b^4*x^17+56/19*a^3*b^5*x^19+4/3*a^2*b^6*x^21+8/23*a*b^7*x^23+1/25*b^8*x^25
```

Rubi [A]
time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/
15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (8*
a*b^7*x^23)/23 + (b^8*x^25)/25
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8 (a+bx^2)^8 \, dx &= \int (a^8x^8+8a^7bx^{10}+28a^6b^2x^{12}+56a^5b^3x^{14}+70a^4b^4x^{16}+56a^3b^5x^{18}+28a^2b^6x^{20}+ \\ &= \frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/
15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (8*
a*b^7*x^23)/23 + (b^8*x^25)/25
```

Maple [A]
time = 0.07, size = 91, normalized size = 0.84

method	result
gosper	$\frac{1}{9}a^8x^9+\frac{8}{11}a^7b\,x^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}a\,b^7x^{23}+\frac{1}{25}b^8$
default	$\frac{1}{9}a^8x^9+\frac{8}{11}a^7b\,x^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}a\,b^7x^{23}+\frac{1}{25}b^8$
norman	$\frac{1}{9}a^8x^9+\frac{8}{11}a^7b\,x^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}a\,b^7x^{23}+\frac{1}{25}b^8$
risch	$\frac{1}{9}a^8x^9+\frac{8}{11}a^7b\,x^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}a\,b^7x^{23}+\frac{1}{25}b^8$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^8*x^9+8/11*a^7*b*x^11+28/13*a^6*b^2*x^13+56/15*a^5*b^3*x^15+70/17*a^4
*b^4*x^17+56/19*a^3*b^5*x^19+4/3*a^2*b^6*x^21+8/23*a*b^7*x^23+1/25*b^8*x^25
```

Maxima [A]
time = 0.30, size = 90, normalized size = 0.83

$$\frac{1}{25}\,b^8x^{25}+\frac{8}{23}\,ab^7x^{23}+\frac{4}{3}\,a^2b^6x^{21}+\frac{56}{19}\,a^3b^5x^{19}+\frac{70}{17}\,a^4b^4x^{17}+\frac{56}{15}\,a^5b^3x^{15}+\frac{28}{13}\,a^6b^2x^{13}+\frac{8}{11}\,a^7bx^{11}+\frac{1}{9}\,a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 + 7
0/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*x^
11 + 1/9*a^8*x^9
```

Fricas [A]
time = 0.62, size = 90, normalized size = 0.83

$$\frac{1}{25}\,b^8x^{25}+\frac{8}{23}\,ab^7x^{23}+\frac{4}{3}\,a^2b^6x^{21}+\frac{56}{19}\,a^3b^5x^{19}+\frac{70}{17}\,a^4b^4x^{17}+\frac{56}{15}\,a^5b^3x^{15}+\frac{28}{13}\,a^6b^2x^{13}+\frac{8}{11}\,a^7bx^{11}+\frac{1}{9}\,a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 + 7
0/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*x^
11 + 1/9*a^8*x^9
```

Sympy [A]
time = 0.01, size = 107, normalized size = 0.99

$$\frac{a^8x^9}{9} + \frac{8a^7bx^{11}}{11} + \frac{28a^6b^2x^{13}}{13} + \frac{56a^5b^3x^{15}}{15} + \frac{70a^4b^4x^{17}}{17} + \frac{56a^3b^5x^{19}}{19} + \frac{4a^2b^6x^{21}}{3} + \frac{8ab^7x^{23}}{23} + \frac{b^8x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**9/9 + 8*a**7*b*x**11/11 + 28*a**6*b**2*x**13/13 + 56*a**5*b**3*x**1
5/15 + 70*a**4*b**4*x**17/17 + 56*a**3*b**5*x**19/19 + 4*a**2*b**6*x**21/3
+ 8*a*b**7*x**23/23 + b**8*x**25/25
```

Giac [A]
time = 0.81, size = 90, normalized size = 0.83

$$\frac{1}{25}\,b^8x^{25} + \frac{8}{23}\,ab^7x^{23} + \frac{4}{3}\,a^2b^6x^{21} + \frac{56}{19}\,a^3b^5x^{19} + \frac{70}{17}\,a^4b^4x^{17} + \frac{56}{15}\,a^5b^3x^{15} + \frac{28}{13}\,a^6b^2x^{13} + \frac{8}{11}\,a^7bx^{11} + \frac{1}{9}\,a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 + 7
0/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*x^
11 + 1/9*a^8*x^9
```

Mupad [B]
time = 4.94, size = 90, normalized size = 0.83

$$\frac{a^8\,x^9}{9} + \frac{8\,a^7\,b\,x^{11}}{11} + \frac{28\,a^6\,b^2\,x^{13}}{13} + \frac{56\,a^5\,b^3\,x^{15}}{15} + \frac{70\,a^4\,b^4\,x^{17}}{17} + \frac{56\,a^3\,b^5\,x^{19}}{19} + \frac{4\,a^2\,b^6\,x^{21}}{3} + \frac{8\,a\,b^7\,x^{23}}{23} + \frac{b^8\,x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^9)/9 + (b^8*x^25)/25 + (8*a^7*b*x^11)/11 + (8*a*b^7*x^23)/23 + (28*a
^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5
*x^19)/19 + (4*a^2*b^6*x^21)/3
```

3.110 $\int x^6 (a+bx^2)^8 \, dx$

Optimal. Leaf size=108

$$\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

```
[Out] 1/7*a^8*x^7+8/9*a^7*b*x^9+28/11*a^6*b^2*x^11+56/13*a^5*b^3*x^13+14/3*a^4*b^
4*x^15+56/17*a^3*b^5*x^17+28/19*a^2*b^6*x^19+8/21*a*b^7*x^21+1/23*b^8*x^23
```

Rubi [A]
time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13
 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8*a
*b^7*x^21)/21 + (b^8*x^23)/23
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^6\left(a+bx^2\right)^8\,dx=\int\left(a^8x^6+8a^7bx^8+28a^6b^2x^{10}+56a^5b^3x^{12}+70a^4b^4x^{14}+56a^3b^5x^{16}+28a^2b^6x^{18}+8ab\right.$$
$$=\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a$$

Mathematica [A]
time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13
 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8*a
*b^7*x^21)/21 + (b^8*x^23)/23
```

Maple [A]
time = 0.06, size = 91, normalized size = 0.84

method	result
gosper	$\frac{1}{7}a^8x^7+\frac{8}{9}a^7b\,x^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a\,b^7x^{21}+\frac{1}{23}b$
default	$\frac{1}{7}a^8x^7+\frac{8}{9}a^7b\,x^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a\,b^7x^{21}+\frac{1}{23}b$
norman	$\frac{1}{7}a^8x^7+\frac{8}{9}a^7b\,x^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a\,b^7x^{21}+\frac{1}{23}b$
risch	$\frac{1}{7}a^8x^7+\frac{8}{9}a^7b\,x^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a\,b^7x^{21}+\frac{1}{23}b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^8*x^7+8/9*a^7*b*x^9+28/11*a^6*b^2*x^11+56/13*a^5*b^3*x^13+14/3*a^4*b^
4*x^15+56/17*a^3*b^5*x^17+28/19*a^2*b^6*x^19+8/21*a*b^7*x^21+1/23*b^8*x^23
```

Maxima [A]
time = 0.29, size = 90, normalized size = 0.83

$$\frac{1}{23}\,b^8x^{23}+\frac{8}{21}\,ab^7x^{21}+\frac{28}{19}\,a^2b^6x^{19}+\frac{56}{17}\,a^3b^5x^{17}+\frac{14}{3}\,a^4b^4x^{15}+\frac{56}{13}\,a^5b^3x^{13}+\frac{28}{11}\,a^6b^2x^{11}+\frac{8}{9}\,a^7bx^9+\frac{1}{7}\,a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17 +
 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*x^
9 + 1/7*a^8*x^7
```

Fricas [A]
time = 0.95, size = 90, normalized size = 0.83

$$\frac{1}{23}\,b^8x^{23}+\frac{8}{21}\,ab^7x^{21}+\frac{28}{19}\,a^2b^6x^{19}+\frac{56}{17}\,a^3b^5x^{17}+\frac{14}{3}\,a^4b^4x^{15}+\frac{56}{13}\,a^5b^3x^{13}+\frac{28}{11}\,a^6b^2x^{11}+\frac{8}{9}\,a^7bx^9+\frac{1}{7}\,a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17 +
 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*x^
9 + 1/7*a^8*x^7
```

Sympy [A]
time = 0.01, size = 107, normalized size = 0.99

$$\frac{a^8x^7}{7}+\frac{8a^7bx^9}{9}+\frac{28a^6b^2x^{11}}{11}+\frac{56a^5b^3x^{13}}{13}+\frac{14a^4b^4x^{15}}{3}+\frac{56a^3b^5x^{17}}{17}+\frac{28a^2b^6x^{19}}{19}+\frac{8ab^7x^{21}}{21}+\frac{b^8x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**7/7 + 8*a**7*b*x**9/9 + 28*a**6*b**2*x**11/11 + 56*a**5*b**3*x**13/
13 + 14*a**4*b**4*x**15/3 + 56*a**3*b**5*x**17/17 + 28*a**2*b**6*x**19/19 +
 8*a*b**7*x**21/21 + b**8*x**23/23
```

Giac [A]
time = 0.71, size = 90, normalized size = 0.83

$$\frac{1}{23}b^8x^{23}+\frac{8}{21}ab^7x^{21}+\frac{28}{19}a^2b^6x^{19}+\frac{56}{17}a^3b^5x^{17}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{13}a^5b^3x^{13}+\frac{28}{11}a^6b^2x^{11}+\frac{8}{9}a^7bx^9+\frac{1}{7}a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17 +
 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*x^
9 + 1/7*a^8*x^7
```

Mupad [B]
time = 0.10, size = 90, normalized size = 0.83

$$\frac{a^8\,x^7}{7}+\frac{8\,a^7\,b\,x^9}{9}+\frac{28\,a^6\,b^2\,x^{11}}{11}+\frac{56\,a^5\,b^3\,x^{13}}{13}+\frac{14\,a^4\,b^4\,x^{15}}{3}+\frac{56\,a^3\,b^5\,x^{17}}{17}+\frac{28\,a^2\,b^6\,x^{19}}{19}+\frac{8\,a\,b^7\,x^{21}}{21}+\frac{b^8\,x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^7)/7 + (b^8*x^23)/23 + (8*a^7*b*x^9)/9 + (8*a*b^7*x^21)/21 + (28*a^6
*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^
17)/17 + (28*a^2*b^6*x^19)/19
```

3.111 $\int x^4(a+bx^2)^8 \, dx$

Optimal. Leaf size=108

$$\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

```
[Out] 1/5*a^8*x^5+8/7*a^7*b*x^7+28/9*a^6*b^2*x^9+56/11*a^5*b^3*x^11+70/13*a^4*b^4
*x^13+56/15*a^3*b^5*x^15+28/17*a^2*b^6*x^17+8/19*a*b^7*x^19+1/21*b^8*x^21
```

Rubi [A]
time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 +
 (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*a*
b^7*x^19)/19 + (b^8*x^21)/21
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^8 \, dx = \int \left(a^8x^4+8a^7bx^6+28a^6b^2x^8+56a^5b^3x^{10}+70a^4b^4x^{12}+56a^3b^5x^{14}+28a^2b^6x^{16}+8a\right.$$
$$=\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}$$

Mathematica [A]
time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 +
 (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*a*
b^7*x^19)/19 + (b^8*x^21)/21
```

Maple [A]
time = 0.06, size = 91, normalized size = 0.84

method	result
gosper	$\frac{1}{5}a^8x^5+\frac{8}{7}a^7b\,x^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}a\,b^7x^{19}+\frac{1}{21}b^8x$
default	$\frac{1}{5}a^8x^5+\frac{8}{7}a^7b\,x^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}a\,b^7x^{19}+\frac{1}{21}b^8x$
norman	$\frac{1}{5}a^8x^5+\frac{8}{7}a^7b\,x^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}a\,b^7x^{19}+\frac{1}{21}b^8x$
risch	$\frac{1}{5}a^8x^5+\frac{8}{7}a^7b\,x^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}a\,b^7x^{19}+\frac{1}{21}b^8x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^8*x^5+8/7*a^7*b*x^7+28/9*a^6*b^2*x^9+56/11*a^5*b^3*x^11+70/13*a^4*b^4
*x^13+56/15*a^3*b^5*x^15+28/17*a^2*b^6*x^17+8/19*a*b^7*x^19+1/21*b^8*x^21
```

Maxima [A]
time = 0.31, size = 90, normalized size = 0.83

$$\frac{1}{21}\,b^8x^{21}+\frac{8}{19}\,ab^7x^{19}+\frac{28}{17}\,a^2b^6x^{17}+\frac{56}{15}\,a^3b^5x^{15}+\frac{70}{13}\,a^4b^4x^{13}+\frac{56}{11}\,a^5b^3x^{11}+\frac{28}{9}\,a^6b^2x^9+\frac{8}{7}\,a^7bx^7+\frac{1}{5}\,a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15 +
 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x^7
 + 1/5*a^8*x^5
```

Fricas [A]
time = 0.64, size = 90, normalized size = 0.83

$$\frac{1}{21}\,b^8x^{21}+\frac{8}{19}\,ab^7x^{19}+\frac{28}{17}\,a^2b^6x^{17}+\frac{56}{15}\,a^3b^5x^{15}+\frac{70}{13}\,a^4b^4x^{13}+\frac{56}{11}\,a^5b^3x^{11}+\frac{28}{9}\,a^6b^2x^9+\frac{8}{7}\,a^7bx^7+\frac{1}{5}\,a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15 +
 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x^7
 + 1/5*a^8*x^5
```

Sympy [A]
time = 0.01, size = 107, normalized size = 0.99

$$\frac{a^8x^5}{5} + \frac{8a^7bx^7}{7} + \frac{28a^6b^2x^9}{9} + \frac{56a^5b^3x^{11}}{11} + \frac{70a^4b^4x^{13}}{13} + \frac{56a^3b^5x^{15}}{15} + \frac{28a^2b^6x^{17}}{17} + \frac{8ab^7x^{19}}{19} + \frac{b^8x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**5/5 + 8*a**7*b*x**7/7 + 28*a**6*b**2*x**9/9 + 56*a**5*b**3*x**11/11
 + 70*a**4*b**4*x**13/13 + 56*a**3*b**5*x**15/15 + 28*a**2*b**6*x**17/17 +
8*a*b**7*x**19/19 + b**8*x**21/21
```

Giac [A]
time = 1.37, size = 90, normalized size = 0.83

$$\frac{1}{21}\,b^8x^{21} + \frac{8}{19}\,ab^7x^{19} + \frac{28}{17}\,a^2b^6x^{17} + \frac{56}{15}\,a^3b^5x^{15} + \frac{70}{13}\,a^4b^4x^{13} + \frac{56}{11}\,a^5b^3x^{11} + \frac{28}{9}\,a^6b^2x^9 + \frac{8}{7}\,a^7bx^7 + \frac{1}{5}\,a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15 +
 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x^7
 + 1/5*a^8*x^5
```

Mupad [B]
time = 0.10, size = 90, normalized size = 0.83

$$\frac{a^8\,x^5}{5} + \frac{8\,a^7\,b\,x^7}{7} + \frac{28\,a^6\,b^2\,x^9}{9} + \frac{56\,a^5\,b^3\,x^{11}}{11} + \frac{70\,a^4\,b^4\,x^{13}}{13} + \frac{56\,a^3\,b^5\,x^{15}}{15} + \frac{28\,a^2\,b^6\,x^{17}}{17} + \frac{8\,a\,b^7\,x^{19}}{19} + \frac{b^8\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^5)/5 + (b^8*x^21)/21 + (8*a^7*b*x^7)/7 + (8*a*b^7*x^19)/19 + (28*a^6
*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 + (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^1
5)/15 + (28*a^2*b^6*x^17)/17
```

3.112 $\int x^2(a+bx^2)^8\,dx$

Optimal. Leaf size=106

$$\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

```
[Out] 1/3*a^8*x^3+8/5*a^7*b*x^5+4*a^6*b^2*x^7+56/9*a^5*b^3*x^9+70/11*a^4*b^4*x^11
+56/13*a^3*b^5*x^13+28/15*a^2*b^6*x^15+8/17*a*b^7*x^17+1/19*b^8*x^19
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^
4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x^1
7)/17 + (b^8*x^19)/19
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^8\,dx &= \int \left(a^8x^2+8a^7bx^4+28a^6b^2x^6+56a^5b^3x^8+70a^4b^4x^{10}+56a^3b^5x^{12}+28a^2b^6x^{14}+8ab^7\right.\\ &= \frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7\end{aligned}$$

Mathematica [A]
time = 0.00, size = 106, normalized size = 1.00

$$\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^
4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x^1
7)/17 + (b^8*x^19)/19
```

Maple [A]
time = 0.07, size = 91, normalized size = 0.86

method	result
gosper	$\frac{1}{3}a^8x^3+\frac{8}{5}a^7b\,x^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}a\,b^7x^{17}+\frac{1}{19}b^8x$
default	$\frac{1}{3}a^8x^3+\frac{8}{5}a^7b\,x^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}a\,b^7x^{17}+\frac{1}{19}b^8x$
norman	$\frac{1}{3}a^8x^3+\frac{8}{5}a^7b\,x^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}a\,b^7x^{17}+\frac{1}{19}b^8x$
risch	$\frac{1}{3}a^8x^3+\frac{8}{5}a^7b\,x^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}a\,b^7x^{17}+\frac{1}{19}b^8x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^8*x^3+8/5*a^7*b*x^5+4*a^6*b^2*x^7+56/9*a^5*b^3*x^9+70/11*a^4*b^4*x^11
+56/13*a^3*b^5*x^13+28/15*a^2*b^6*x^15+8/17*a*b^7*x^17+1/19*b^8*x^19
```

Maxima [A]
time = 0.31, size = 90, normalized size = 0.85

$$\frac{1}{19}\,b^8x^{19}+\frac{8}{17}\,ab^7x^{17}+\frac{28}{15}\,a^2b^6x^{15}+\frac{56}{13}\,a^3b^5x^{13}+\frac{70}{11}\,a^4b^4x^{11}+\frac{56}{9}\,a^5b^3x^9+4\,a^6b^2x^7+\frac{8}{5}\,a^7bx^5+\frac{1}{3}\,a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13 +
 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 + 1/
3*a^8*x^3
```

Fricas [A]
time = 0.98, size = 90, normalized size = 0.85

$$\frac{1}{19}\,b^8x^{19}+\frac{8}{17}\,ab^7x^{17}+\frac{28}{15}\,a^2b^6x^{15}+\frac{56}{13}\,a^3b^5x^{13}+\frac{70}{11}\,a^4b^4x^{11}+\frac{56}{9}\,a^5b^3x^9+4\,a^6b^2x^7+\frac{8}{5}\,a^7bx^5+\frac{1}{3}\,a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13 +
 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 + 1/
3*a^8*x^3
```

Sympy [A]
time = 0.01, size = 105, normalized size = 0.99

$$\frac{a^8x^3}{3} + \frac{8a^7bx^5}{5} + 4a^6b^2x^7 + \frac{56a^5b^3x^9}{9} + \frac{70a^4b^4x^{11}}{11} + \frac{56a^3b^5x^{13}}{13} + \frac{28a^2b^6x^{15}}{15} + \frac{8ab^7x^{17}}{17} + \frac{b^8x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**3/3 + 8*a**7*b*x**5/5 + 4*a**6*b**2*x**7 + 56*a**5*b**3*x**9/9 + 70
*a**4*b**4*x**11/11 + 56*a**3*b**5*x**13/13 + 28*a**2*b**6*x**15/15 + 8*a*b
**7*x**17/17 + b**8*x**19/19
```

Giac [A]
time = 1.84, size = 90, normalized size = 0.85

$$\frac{1}{19}\,b^8x^{19} + \frac{8}{17}\,ab^7x^{17} + \frac{28}{15}\,a^2b^6x^{15} + \frac{56}{13}\,a^3b^5x^{13} + \frac{70}{11}\,a^4b^4x^{11} + \frac{56}{9}\,a^5b^3x^9 + 4\,a^6b^2x^7 + \frac{8}{5}\,a^7bx^5 + \frac{1}{3}\,a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13 +
 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 + 1/
3*a^8*x^3
```

Mupad [B]
time = 4.97, size = 90, normalized size = 0.85

$$\frac{a^8\,x^3}{3} + \frac{8\,a^7\,b\,x^5}{5} + 4\,a^6\,b^2\,x^7 + \frac{56\,a^5\,b^3\,x^9}{9} + \frac{70\,a^4\,b^4\,x^{11}}{11} + \frac{56\,a^3\,b^5\,x^{13}}{13} + \frac{28\,a^2\,b^6\,x^{15}}{15} + \frac{8\,a\,b^7\,x^{17}}{17} + \frac{b^8\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^3)/3 + (b^8*x^19)/19 + (8*a^7*b*x^5)/5 + (8*a*b^7*x^17)/17 + 4*a^6*b
^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 +
 (28*a^2*b^6*x^15)/15
```

3.113 $\int (a+bx^2)^8 \, dx$

Optimal. Leaf size=101

$$a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

```
[Out] a^8*x+8/3*a^7*b*x^3+28/5*a^6*b^2*x^5+8*a^5*b^3*x^7+70/9*a^4*b^4*x^9+56/11*a
^3*b^5*x^11+28/13*a^2*b^6*x^13+8/15*a*b^7*x^15+1/17*b^8*x^17
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {200}

$$a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8,x]
```

```
[Out] a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*
x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15 +
(b^8*x^17)/17
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^8 \, dx &= \int \left(a^8+8a^7bx^2+28a^6b^2x^4+56a^5b^3x^6+70a^4b^4x^8+56a^3b^5x^{10}+28a^2b^6x^{12}+8ab^7x^{14}\right. \\ &= a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 101, normalized size = 1.00

$$a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8,x]
```

```
[Out] a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*
x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15 +
(b^8*x^17)/17
```

Maple [A]
time = 0.02, size = 88, normalized size = 0.87

method	result
gosper	$a^{8}x+\frac{8}{3}a^{7}b\,x^{3}+\frac{28}{5}a^{6}b^{2}x^{5}+8a^{5}b^{3}x^{7}+\frac{70}{9}a^{4}b^{4}x^{9}+\frac{56}{11}a^{3}b^{5}x^{11}+\frac{28}{13}a^{2}b^{6}x^{13}+\frac{8}{15}a\,b^{7}x^{15}+\frac{1}{17}b^{8}x^{17}$
default	$a^{8}x+\frac{8}{3}a^{7}b\,x^{3}+\frac{28}{5}a^{6}b^{2}x^{5}+8a^{5}b^{3}x^{7}+\frac{70}{9}a^{4}b^{4}x^{9}+\frac{56}{11}a^{3}b^{5}x^{11}+\frac{28}{13}a^{2}b^{6}x^{13}+\frac{8}{15}a\,b^{7}x^{15}+\frac{1}{17}b^{8}x^{17}$
norman	$a^{8}x+\frac{8}{3}a^{7}b\,x^{3}+\frac{28}{5}a^{6}b^{2}x^{5}+8a^{5}b^{3}x^{7}+\frac{70}{9}a^{4}b^{4}x^{9}+\frac{56}{11}a^{3}b^{5}x^{11}+\frac{28}{13}a^{2}b^{6}x^{13}+\frac{8}{15}a\,b^{7}x^{15}+\frac{1}{17}b^{8}x^{17}$
risch	$a^{8}x+\frac{8}{3}a^{7}b\,x^{3}+\frac{28}{5}a^{6}b^{2}x^{5}+8a^{5}b^{3}x^{7}+\frac{70}{9}a^{4}b^{4}x^{9}+\frac{56}{11}a^{3}b^{5}x^{11}+\frac{28}{13}a^{2}b^{6}x^{13}+\frac{8}{15}a\,b^{7}x^{15}+\frac{1}{17}b^{8}x^{17}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] a^8*x+8/3*a^7*b*x^3+28/5*a^6*b^2*x^5+8*a^5*b^3*x^7+70/9*a^4*b^4*x^9+56/11*a
^3*b^5*x^11+28/13*a^2*b^6*x^13+8/15*a*b^7*x^15+1/17*b^8*x^17
```

Maxima [A]
time = 0.29, size = 87, normalized size = 0.86

$$\frac{1}{17}\,b^{8}x^{17}+\frac{8}{15}\,ab^{7}x^{15}+\frac{28}{13}\,a^{2}b^{6}x^{13}+\frac{56}{11}\,a^{3}b^{5}x^{11}+\frac{70}{9}\,a^{4}b^{4}x^{9}+8\,a^{5}b^{3}x^{7}+\frac{28}{5}\,a^{6}b^{2}x^{5}+\frac{8}{3}\,a^{7}bx^{3}+a^{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11 +
 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*
x
```

Fricas [A]
time = 0.85, size = 87, normalized size = 0.86

$$\frac{1}{17}\,b^{8}x^{17}+\frac{8}{15}\,ab^{7}x^{15}+\frac{28}{13}\,a^{2}b^{6}x^{13}+\frac{56}{11}\,a^{3}b^{5}x^{11}+\frac{70}{9}\,a^{4}b^{4}x^{9}+8\,a^{5}b^{3}x^{7}+\frac{28}{5}\,a^{6}b^{2}x^{5}+\frac{8}{3}\,a^{7}bx^{3}+a^{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11 +
 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*
x
```

Sympy [A]
time = 0.01, size = 102, normalized size = 1.01

$$a^8x+\frac{8a^7bx^3}{3}+\frac{28a^6b^2x^5}{5}+8a^5b^3x^7+\frac{70a^4b^4x^9}{9}+\frac{56a^3b^5x^{11}}{11}+\frac{28a^2b^6x^{13}}{13}+\frac{8ab^7x^{15}}{15}+\frac{b^8x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8,x)
```

```
[Out] a**8*x + 8*a**7*b*x**3/3 + 28*a**6*b**2*x**5/5 + 8*a**5*b**3*x**7 + 70*a**4
*b**4*x**9/9 + 56*a**3*b**5*x**11/11 + 28*a**2*b**6*x**13/13 + 8*a*b**7*x**
15/15 + b**8*x**17/17
```

Giac [A]
time = 1.23, size = 87, normalized size = 0.86

$$\frac{1}{17}b^8x^{17}+\frac{8}{15}ab^7x^{15}+\frac{28}{13}a^2b^6x^{13}+\frac{56}{11}a^3b^5x^{11}+\frac{70}{9}a^4b^4x^9+8a^5b^3x^7+\frac{28}{5}a^6b^2x^5+\frac{8}{3}a^7bx^3+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11 +
 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*
x
```

Mupad [B]
time = 0.05, size = 87, normalized size = 0.86

$$a^8\,x+\frac{8\,a^7\,b\,x^3}{3}+\frac{28\,a^6\,b^2\,x^5}{5}+8\,a^5\,b^3\,x^7+\frac{70\,a^4\,b^4\,x^9}{9}+\frac{56\,a^3\,b^5\,x^{11}}{11}+\frac{28\,a^2\,b^6\,x^{13}}{13}+\frac{8\,a\,b^7\,x^{15}}{15}+\frac{b^8\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8,x)
```

```
[Out] a^8*x + (b^8*x^17)/17 + (8*a^7*b*x^3)/3 + (8*a*b^7*x^15)/15 + (28*a^6*b^2*x
^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2
*b^6*x^13)/13
```

3.114 $\int \frac{(a+bx^2)^8}{x^2} dx$

Optimal. Leaf size=100

$$-\frac{a^8}{x}+8a^7bx+\frac{28}{3}a^6b^2x^3+\frac{56}{5}a^5b^3x^5+10a^4b^4x^7+\frac{56}{9}a^3b^5x^9+\frac{28}{11}a^2b^6x^{11}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

```
[Out] -a^8/x+8*a^7*b*x+28/3*a^6*b^2*x^3+56/5*a^5*b^3*x^5+10*a^4*b^4*x^7+56/9*a^3*
b^5*x^9+28/11*a^2*b^6*x^11+8/13*a*b^7*x^13+1/15*b^8*x^15
```

Rubi [A]
time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^8}{x}+8a^7bx+\frac{28}{3}a^6b^2x^3+\frac{56}{5}a^5b^3x^5+10a^4b^4x^7+\frac{56}{9}a^3b^5x^9+\frac{28}{11}a^2b^6x^{11}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^2,x]
```

```
[Out] -(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4
*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b^8
*x^15)/15
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^2}\,dx = \int \left(8a^7b+\frac{a^8}{x^2}+28a^6b^2x^2+56a^5b^3x^4+70a^4b^4x^6+56a^3b^5x^8+28a^2b^6x^{10}+8ab^7x^{12}+b^8\right.$$
$$= -\frac{a^8}{x}+8a^7bx+\frac{28}{3}a^6b^2x^3+\frac{56}{5}a^5b^3x^5+10a^4b^4x^7+\frac{56}{9}a^3b^5x^9+\frac{28}{11}a^2b^6x^{11}+\frac{8}{13}ab^7x^{13}+$$

Mathematica [A]
time = 0.01, size = 100, normalized size = 1.00

$$-\frac{a^8}{x}+8a^7bx+\frac{28}{3}a^6b^2x^3+\frac{56}{5}a^5b^3x^5+10a^4b^4x^7+\frac{56}{9}a^3b^5x^9+\frac{28}{11}a^2b^6x^{11}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^2,x]
```

```
[Out] -(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4
*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b^8
*x^15)/15
```

Maple [A]
time = 0.10, size = 89, normalized size = 0.89

method	result	si
default	$-\frac{a^{8}}{x}+8a^{7}bx+\frac{28a^{6}b^{2}x^{3}}{3}+\frac{56a^{5}b^{3}x^{5}}{5}+10a^{4}b^{4}x^{7}+\frac{56a^{3}b^{5}x^{9}}{9}+\frac{28a^{2}b^{6}x^{11}}{11}+\frac{8a\,b^{7}x^{13}}{13}+\frac{b^{8}x^{15}}{15}$	8
risch	$-\frac{a^{8}}{x}+8a^{7}bx+\frac{28a^{6}b^{2}x^{3}}{3}+\frac{56a^{5}b^{3}x^{5}}{5}+10a^{4}b^{4}x^{7}+\frac{56a^{3}b^{5}x^{9}}{9}+\frac{28a^{2}b^{6}x^{11}}{11}+\frac{8a\,b^{7}x^{13}}{13}+\frac{b^{8}x^{15}}{15}$	8
norman	$\frac{-a^{8}+8a^{7}b\,x^{2}+\frac{28}{3}a^{6}b^{2}x^{4}+\frac{56}{5}a^{5}b^{3}x^{6}+10a^{4}b^{4}x^{8}+\frac{56}{9}a^{3}b^{5}x^{10}+\frac{28}{11}a^{2}b^{6}x^{12}+\frac{8}{13}a\,b^{7}x^{14}+\frac{1}{15}b^{8}x^{16}}{x}$	9
gosper	$-\frac{-429b^{8}x^{16}-3960a\,b^{7}x^{14}-16380a^{2}b^{6}x^{12}-40040a^{3}b^{5}x^{10}-64350a^{4}b^{4}x^{8}-72072a^{5}b^{3}x^{6}-60060a^{6}b^{2}x^{4}-51480a^{7}b\,x^{2}+6435a^{8}}{6435x}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^8/x+8*a^7*b*x+28/3*a^6*b^2*x^3+56/5*a^5*b^3*x^5+10*a^4*b^4*x^7+56/9*a^3*
b^5*x^9+28/11*a^2*b^6*x^11+8/13*a*b^7*x^13+1/15*b^8*x^15
```

Maxima [A]
time = 0.28, size = 88, normalized size = 0.88

$$\frac{1}{15}\,b^{8}x^{15}+\frac{8}{13}\,ab^{7}x^{13}+\frac{28}{11}\,a^{2}b^{6}x^{11}+\frac{56}{9}\,a^{3}b^{5}x^{9}+10\,a^{4}b^{4}x^{7}+\frac{56}{5}\,a^{5}b^{3}x^{5}+\frac{28}{3}\,a^{6}b^{2}x^{3}+8\,a^{7}bx-\frac{a^{8}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 + 1
0*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

Fricas [A]
time = 0.66, size = 92, normalized size = 0.92

$$\frac{429\,b^{8}x^{16}+3960\,ab^{7}x^{14}+16380\,a^{2}b^{6}x^{12}+40040\,a^{3}b^{5}x^{10}+64350\,a^{4}b^{4}x^{8}+72072\,a^{5}b^{3}x^{6}+60060\,a^{6}b^{2}x^{4}+51480\,a^{7}bx^{2}-6435\,a^{8}}{6435\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="fricas")
```

```
[Out] 1/6435*(429*b^8*x^16 + 3960*a*b^7*x^14 + 16380*a^2*b^6*x^12 + 40040*a^3*b^5
*x^10 + 64350*a^4*b^4*x^8 + 72072*a^5*b^3*x^6 + 60060*a^6*b^2*x^4 + 51480*a
^7*b*x^2 - 6435*a^8)/x
```

Sympy [A]
time = 0.05, size = 99, normalized size = 0.99

$$-\frac{a^8}{x}+8a^7bx+\frac{28a^6b^2x^3}{3}+\frac{56a^5b^3x^5}{5}+10a^4b^4x^7+\frac{56a^3b^5x^9}{9}+\frac{28a^2b^6x^{11}}{11}+\frac{8ab^7x^{13}}{13}+\frac{b^8x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**2,x)
```

```
[Out] -a**8/x + 8*a**7*b*x + 28*a**6*b**2*x**3/3 + 56*a**5*b**3*x**5/5 + 10*a**4*
b**4*x**7 + 56*a**3*b**5*x**9/9 + 28*a**2*b**6*x**11/11 + 8*a*b**7*x**13/13
 + b**8*x**15/15
```

Giac [A]
time = 1.01, size = 88, normalized size = 0.88

$$\frac{1}{15}\,b^8x^{15}+\frac{8}{13}\,ab^7x^{13}+\frac{28}{11}\,a^2b^6x^{11}+\frac{56}{9}\,a^3b^5x^9+10\,a^4b^4x^7+\frac{56}{5}\,a^5b^3x^5+\frac{28}{3}\,a^6b^2x^3+8\,a^7bx-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="giac")
```

```
[Out] 1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 + 1
0*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

Mupad [B]
time = 0.06, size = 88, normalized size = 0.88

$$\frac{b^8\,x^{15}}{15}-\frac{a^8}{x}+\frac{8\,a\,b^7\,x^{13}}{13}+\frac{28\,a^6\,b^2\,x^3}{3}+\frac{56\,a^5\,b^3\,x^5}{5}+10\,a^4\,b^4\,x^7+\frac{56\,a^3\,b^5\,x^9}{9}+\frac{28\,a^2\,b^6\,x^{11}}{11}+8\,a^7\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^2,x)
```

```
[Out] (b^8*x^15)/15 - a^8/x + (8*a*b^7*x^13)/13 + (28*a^6*b^2*x^3)/3 + (56*a^5*b^
3*x^5)/5 + 10*a^4*b^4*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + 8*a
^7*b*x
```

3.115 $\int \frac{(a+bx^2)^8}{x^4}\,dx$

Optimal. Leaf size=98

$$-\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

```
[Out] -1/3*a^8/x^3-8*a^7*b/x+28*a^6*b^2*x+56/3*a^5*b^3*x^3+14*a^4*b^4*x^5+8*a^3*b
^5*x^7+28/9*a^2*b^6*x^9+8/11*a*b^7*x^11+1/13*b^8*x^13
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$-\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^4,x]
```

```
[Out] -1/3*a^8/x^3 - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4
*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13)/
13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^4}\,dx = \int \left(28a^6b^2+\frac{a^8}{x^4}+\frac{8a^7b}{x^2}+56a^5b^3x^2+70a^4b^4x^4+56a^3b^5x^6+28a^2b^6x^8+8ab^7x^{10}+b^8x\right.$$
$$=-\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8}{}$$

Mathematica [A]
time = 0.01, size = 98, normalized size = 1.00

$$-\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^4,x]
```

```
[Out] -1/3*a^8/x^3 - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4
*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13)/
13
```

Maple [A]
time = 0.03, size = 89, normalized size = 0.91

method	result	size
default	$-\frac{a^{8}}{3x^{3}}-\frac{8a^{7}b}{x}+28a^{6}b^{2}x+\frac{56a^{5}b^{3}x^{3}}{3}+14a^{4}b^{4}x^{5}+8a^{3}b^{5}x^{7}+\frac{28a^{2}b^{6}x^{9}}{9}+\frac{8a\,b^{7}x^{11}}{11}+\frac{b^{8}x^{13}}{13}$	89
risch	$\frac{b^{8}x^{13}}{13}+\frac{8a\,b^{7}x^{11}}{11}+\frac{28a^{2}b^{6}x^{9}}{9}+8a^{3}b^{5}x^{7}+14a^{4}b^{4}x^{5}+\frac{56a^{5}b^{3}x^{3}}{3}+28a^{6}b^{2}x+\frac{-8a^{7}b\,x^{2}-\frac{1}{3}a^{8}}{x^{3}}$	91
norman	$\frac{-\frac{1}{3}a^{8}-8a^{7}b\,x^{2}+28a^{6}b^{2}x^{4}+\frac{56}{3}a^{5}b^{3}x^{6}+14a^{4}b^{4}x^{8}+8a^{3}b^{5}x^{10}+\frac{28}{9}a^{2}b^{6}x^{12}+\frac{8}{11}a\,b^{7}x^{14}+\frac{1}{13}b^{8}x^{16}}{x^{3}}$	92
gosper	$-\frac{-99b^{8}x^{16}-936a\,b^{7}x^{14}-4004a^{2}b^{6}x^{12}-10296a^{3}b^{5}x^{10}-18018a^{4}b^{4}x^{8}-24024a^{5}b^{3}x^{6}-36036a^{6}b^{2}x^{4}+10296a^{7}b\,x^{2}+429a^{8}}{1287x^{3}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^8/x^3-8*a^7*b/x+28*a^6*b^2*x+56/3*a^5*b^3*x^3+14*a^4*b^4*x^5+8*a^3*b
^5*x^7+28/9*a^2*b^6*x^9+8/11*a*b^7*x^11+1/13*b^8*x^13
```

Maxima [A]
time = 0.29, size = 89, normalized size = 0.91

$$\frac{1}{13}\,b^{8}x^{13}+\frac{8}{11}\,ab^{7}x^{11}+\frac{28}{9}\,a^{2}b^{6}x^{9}+8\,a^{3}b^{5}x^{7}+14\,a^{4}b^{4}x^{5}+\frac{56}{3}\,a^{5}b^{3}x^{3}+28\,a^{6}b^{2}x-\frac{24\,a^{7}bx^{2}+a^{8}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="maxima")
```

```
[Out] 1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^4
*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

Fricas [A]
time = 1.01, size = 92, normalized size = 0.94

$$\frac{99\,b^{8}x^{16}+936\,ab^{7}x^{14}+4004\,a^{2}b^{6}x^{12}+10296\,a^{3}b^{5}x^{10}+18018\,a^{4}b^{4}x^{8}+24024\,a^{5}b^{3}x^{6}+36036\,a^{6}b^{2}x^{4}-10296\,a^{7}bx^{2}-429\,a^{8}}{1287\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="fricas")
```

```
[Out] 1/1287*(99*b^8*x^16 + 936*a*b^7*x^14 + 4004*a^2*b^6*x^12 + 10296*a^3*b^5*x^
10 + 18018*a^4*b^4*x^8 + 24024*a^5*b^3*x^6 + 36036*a^6*b^2*x^4 - 10296*a^7*
b*x^2 - 429*a^8)/x^3
```

Sympy [A]
time = 0.07, size = 100, normalized size = 1.02

$$28a^6b^2x + \frac{56a^5b^3x^3}{3} + 14a^4b^4x^5 + 8a^3b^5x^7 + \frac{28a^2b^6x^9}{9} + \frac{8ab^7x^{11}}{11} + \frac{b^8x^{13}}{13} + \frac{-a^8 - 24a^7bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**4,x)
```

```
[Out] 28*a**6*b**2*x + 56*a**5*b**3*x**3/3 + 14*a**4*b**4*x**5 + 8*a**3*b**5*x**7
 + 28*a**2*b**6*x**9/9 + 8*a*b**7*x**11/11 + b**8*x**13/13 + (-a**8 - 24*a*
*7*b*x**2)/(3*x**3)
```

Giac [A]
time = 1.18, size = 89, normalized size = 0.91

$$\frac{1}{13}\,b^8x^{13} + \frac{8}{11}\,ab^7x^{11} + \frac{28}{9}\,a^2b^6x^9 + 8\,a^3b^5x^7 + 14\,a^4b^4x^5 + \frac{56}{3}\,a^5b^3x^3 + 28\,a^6b^2x - \frac{24\,a^7bx^2 + a^8}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="giac")
```

```
[Out] 1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^4
*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

Mupad [B]
time = 0.05, size = 91, normalized size = 0.93

$$\frac{b^8\,x^{13}}{13} - \frac{\frac{a^8}{3} + 8\,b\,a^7\,x^2}{x^3} + 28\,a^6\,b^2\,x + \frac{8\,a\,b^7\,x^{11}}{11} + \frac{56\,a^5\,b^3\,x^3}{3} + 14\,a^4\,b^4\,x^5 + 8\,a^3\,b^5\,x^7 + \frac{28\,a^2\,b^6\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^4,x)
```

```
[Out] (b^8*x^13)/13 - (a^8/3 + 8*a^7*b*x^2)/x^3 + 28*a^6*b^2*x + (8*a*b^7*x^11)/1
1 + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/
9
```

3.116 $\int \frac{(a+bx^2)^8}{x^6} dx$

Optimal. Leaf size=100

$$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

```
[Out] -1/5*a^8/x^5-8/3*a^7*b/x^3-28*a^6*b^2/x+56*a^5*b^3*x+70/3*a^4*b^4*x^3+56/5*
a^3*b^5*x^5+4*a^2*b^6*x^7+8/9*a*b^7*x^9+1/11*b^8*x^11
```

Rubi [A]
time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^6,x]
```

```
[Out] -1/5*a^8/x^5 - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*
b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^
11)/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^6}\,dx &= \int \left(56a^5b^3+\frac{a^8}{x^6}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^2}+70a^4b^4x^2+56a^3b^5x^4+28a^2b^6x^6+8ab^7x^8+b^8x^{10}\right)dx\\ &= -\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 100, normalized size = 1.00

$$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^6,x]
```

```
[Out] -1/5*a^8/x^5 - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*
b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^
11)/11
```

Maple [A]
time = 0.03, size = 89, normalized size = 0.89

method	result	size
default	$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70a^4b^4x^3}{3}+\frac{56a^3b^5x^5}{5}+4a^2b^6x^7+\frac{8a\,b^7x^9}{9}+\frac{b^8x^{11}}{11}$	89
risch	$\frac{b^8x^{11}}{11}+\frac{8a\,b^7x^9}{9}+4a^2b^6x^7+\frac{56a^3b^5x^5}{5}+\frac{70a^4b^4x^3}{3}+56a^5b^3x+\frac{-28a^6b^2x^4-\frac{8}{3}a^7b\,x^2-\frac{1}{5}a^8}{x^5}$	91
norman	$\frac{-\frac{1}{5}a^8-\frac{8}{3}a^7b\,x^2-28a^6b^2x^4+56a^5b^3x^6+\frac{70}{3}a^4b^4x^8+\frac{56}{5}a^3b^5x^{10}+4a^2b^6x^{12}+\frac{8}{9}a\,b^7x^{14}+\frac{1}{11}b^8x^{16}}{x^5}$	92
gosper	$-\frac{-45b^8x^{16}-440a\,b^7x^{14}-1980a^2b^6x^{12}-5544a^3b^5x^{10}-11550a^4b^4x^8-27720a^5b^3x^6+13860a^6b^2x^4+1320a^7b\,x^2+99a^8}{495x^5}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^8/x^5-8/3*a^7*b/x^3-28*a^6*b^2/x+56*a^5*b^3*x+70/3*a^4*b^4*x^3+56/5*
a^3*b^5*x^5+4*a^2*b^6*x^7+8/9*a*b^7*x^9+1/11*b^8*x^11
```

Maxima [A]
time = 0.29, size = 91, normalized size = 0.91

$$\frac{1}{11}\,b^8x^{11}+\frac{8}{9}\,ab^7x^9+4\,a^2b^6x^7+\frac{56}{5}\,a^3b^5x^5+\frac{70}{3}\,a^4b^4x^3+56\,a^5b^3x-\frac{420\,a^6b^2x^4+40\,a^7bx^2+3\,a^8}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="maxima")
```

```
[Out] 1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^4
*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x^5
```

Fricas [A]
time = 0.76, size = 92, normalized size = 0.92

$$\frac{45\,b^8x^{16}+440\,ab^7x^{14}+1980\,a^2b^6x^{12}+5544\,a^3b^5x^{10}+11550\,a^4b^4x^8+27720\,a^5b^3x^6-13860\,a^6b^2x^4-1320\,a^7bx^2-99\,a^8}{495\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="fricas")
```

```
[Out] 1/495*(45*b^8*x^16 + 440*a*b^7*x^14 + 1980*a^2*b^6*x^12 + 5544*a^3*b^5*x^10
 + 11550*a^4*b^4*x^8 + 27720*a^5*b^3*x^6 - 13860*a^6*b^2*x^4 - 1320*a^7*b*x
^2 - 99*a^8)/x^5
```

Sympy [A]
time = 0.09, size = 102, normalized size = 1.02

$$56a^5b^3x + \frac{70a^4b^4x^3}{3} + \frac{56a^3b^5x^5}{5} + 4a^2b^6x^7 + \frac{8ab^7x^9}{9} + \frac{b^8x^{11}}{11} + \frac{-3a^8 - 40a^7bx^2 - 420a^6b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**6,x)
```

```
[Out] 56*a**5*b**3*x + 70*a**4*b**4*x**3/3 + 56*a**3*b**5*x**5/5 + 4*a**2*b**6*x*
*7 + 8*a*b**7*x**9/9 + b**8*x**11/11 + (-3*a**8 - 40*a**7*b*x**2 - 420*a**6
*b**2*x**4)/(15*x**5)
```

Giac [A]
time = 0.90, size = 91, normalized size = 0.91

$$\frac{1}{11}\,b^8x^{11} + \frac{8}{9}\,ab^7x^9 + 4\,a^2b^6x^7 + \frac{56}{5}\,a^3b^5x^5 + \frac{70}{3}\,a^4b^4x^3 + 56\,a^5b^3x - \frac{420\,a^6b^2x^4 + 40\,a^7bx^2 + 3\,a^8}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="giac")
```

```
[Out] 1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^4
*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x^5
```

Mupad [B]
time = 4.96, size = 91, normalized size = 0.91

$$\frac{b^8\,x^{11}}{11} - \frac{\frac{a^8}{5} + \frac{8\,a^7\,b\,x^2}{3} + 28\,a^6\,b^2\,x^4}{x^5} + 56\,a^5\,b^3\,x + \frac{8\,a\,b^7\,x^9}{9} + \frac{70\,a^4\,b^4\,x^3}{3} + \frac{56\,a^3\,b^5\,x^5}{5} + 4\,a^2\,b^6\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^6,x)
```

```
[Out] (b^8*x^11)/11 - (a^8/5 + (8*a^7*b*x^2)/3 + 28*a^6*b^2*x^4)/x^5 + 56*a^5*b^3
*x + (8*a*b^7*x^9)/9 + (70*a^4*b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*
x^7
```

3.117 $\int \frac{(a+bx^2)^8}{x^8} dx$

Optimal. Leaf size=102

$$-\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

```
[Out] -1/7*a^8/x^7-8/5*a^7*b/x^5-28/3*a^6*b^2/x^3-56*a^5*b^3/x+70*a^4*b^4*x+56/3*
a^3*b^5*x^3+28/5*a^2*b^6*x^5+8/7*a*b^7*x^7+1/9*b^8*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$-\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^8,x]
```

```
[Out] -1/7*a^8/x^7 - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 +
(b^8*x^9)/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^8}\,dx &= \int \left(70a^4b^4+\frac{a^8}{x^8}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^2}+56a^3b^5x^2+28a^2b^6x^4+8ab^7x^6+b^8x^8\right) dx\\ &= -\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 102, normalized size = 1.00

$$-\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^8,x]
```

```
[Out] -1/7*a^8/x^7 - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 +
(b^8*x^9)/9
```

Maple [A]
time = 0.03, size = 89, normalized size = 0.87

method	result	size
default	$-\frac{a^{8}}{7x^{7}}-\frac{8a^{7}b}{5x^{5}}-\frac{28a^{6}b^{2}}{3x^{3}}-\frac{56a^{5}b^{3}}{x}+70a^{4}b^{4}x+\frac{56a^{3}b^{5}x^{3}}{3}+\frac{28a^{2}b^{6}x^{5}}{5}+\frac{8a\,b^{7}x^{7}}{7}+\frac{b^{8}x^{9}}{9}$	89
risch	$\frac{b^{8}x^{9}}{9}+\frac{8a\,b^{7}x^{7}}{7}+\frac{28a^{2}b^{6}x^{5}}{5}+\frac{56a^{3}b^{5}x^{3}}{3}+70a^{4}b^{4}x+\frac{-56a^{5}b^{3}x^{6}-\frac{28}{3}a^{6}b^{2}x^{4}-\frac{8}{5}a^{7}b\,x^{2}-\frac{1}{7}a^{8}}{x^{7}}$	91
norman	$\frac{-\frac{1}{7}a^{8}-\frac{8}{5}a^{7}b\,x^{2}-\frac{28}{3}a^{6}b^{2}x^{4}-56a^{5}b^{3}x^{6}+70a^{4}b^{4}x^{8}+\frac{56}{3}a^{3}b^{5}x^{10}+\frac{28}{5}a^{2}b^{6}x^{12}+\frac{8}{7}a\,b^{7}x^{14}+\frac{1}{9}b^{8}x^{16}}{x^{7}}$	92
gosper	$-\frac{-35b^{8}x^{16}-360a\,b^{7}x^{14}-1764a^{2}b^{6}x^{12}-5880a^{3}b^{5}x^{10}-22050a^{4}b^{4}x^{8}+17640a^{5}b^{3}x^{6}+2940a^{6}b^{2}x^{4}+504a^{7}b\,x^{2}+45a^{8}}{315x^{7}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^8/x^7-8/5*a^7*b/x^5-28/3*a^6*b^2/x^3-56*a^5*b^3/x+70*a^4*b^4*x+56/3*
a^3*b^5*x^3+28/5*a^2*b^6*x^5+8/7*a*b^7*x^7+1/9*b^8*x^9
```

Maxima [A]
time = 0.28, size = 91, normalized size = 0.89

$$\frac{1}{9}\,b^{8}x^{9}+\frac{8}{7}\,ab^{7}x^{7}+\frac{28}{5}\,a^{2}b^{6}x^{5}+\frac{56}{3}\,a^{3}b^{5}x^{3}+70\,a^{4}b^{4}x-\frac{5880\,a^{5}b^{3}x^{6}+980\,a^{6}b^{2}x^{4}+168\,a^{7}bx^{2}+15\,a^{8}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="maxima")
```

```
[Out] 1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4*
b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^8)
/x^7
```

Fricas [A]
time = 0.82, size = 92, normalized size = 0.90

$$\frac{35\,b^{8}x^{16}+360\,ab^{7}x^{14}+1764\,a^{2}b^{6}x^{12}+5880\,a^{3}b^{5}x^{10}+22050\,a^{4}b^{4}x^{8}-17640\,a^{5}b^{3}x^{6}-2940\,a^{6}b^{2}x^{4}-504\,a^{7}bx^{2}-45\,a^{8}}{315\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^8*x^16 + 360*a*b^7*x^14 + 1764*a^2*b^6*x^12 + 5880*a^3*b^5*x^10
 + 22050*a^4*b^4*x^8 - 17640*a^5*b^3*x^6 - 2940*a^6*b^2*x^4 - 504*a^7*b*x^2
 - 45*a^8)/x^7
```

Sympy [A]
time = 0.12, size = 102, normalized size = 1.00

$$70a^4b^4x + \frac{56a^3b^5x^3}{3} + \frac{28a^2b^6x^5}{5} + \frac{8ab^7x^7}{7} + \frac{b^8x^9}{9} + \frac{-15a^8 - 168a^7bx^2 - 980a^6b^2x^4 - 5880a^5b^3x^6}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**8,x)
```

```
[Out] 70*a**4*b**4*x + 56*a**3*b**5*x**3/3 + 28*a**2*b**6*x**5/5 + 8*a*b**7*x**7/
7 + b**8*x**9/9 + (-15*a**8 - 168*a**7*b*x**2 - 980*a**6*b**2*x**4 - 5880*a
**5*b**3*x**6)/(105*x**7)
```

Giac [A]
time = 0.95, size = 91, normalized size = 0.89

$$\frac{1}{9}b^8x^9 + \frac{8}{7}ab^7x^7 + \frac{28}{5}a^2b^6x^5 + \frac{56}{3}a^3b^5x^3 + 70\,a^4b^4x - \frac{5880\,a^5b^3x^6 + 980\,a^6b^2x^4 + 168\,a^7bx^2 + 15\,a^8}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="giac")
```

```
[Out] 1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4*
b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^8)
/x^7
```

Mupad [B]
time = 4.80, size = 91, normalized size = 0.89

$$\frac{b^8\,x^9}{9} - \frac{\frac{a^8}{7} + \frac{8\,a^7\,b\,x^2}{5} + \frac{28\,a^6\,b^2\,x^4}{3} + 56\,a^5\,b^3\,x^6}{x^7} + 70\,a^4\,b^4\,x + \frac{8\,a\,b^7\,x^7}{7} + \frac{56\,a^3\,b^5\,x^3}{3} + \frac{28\,a^2\,b^6\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^8,x)
```

```
[Out] (b^8*x^9)/9 - (a^8/7 + (8*a^7*b*x^2)/5 + (28*a^6*b^2*x^4)/3 + 56*a^5*b^3*x^
6)/x^7 + 70*a^4*b^4*x + (8*a*b^7*x^7)/7 + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*
x^5)/5
```

3.118 $\int \frac{(a+bx^2)^8}{x^{10}}\,dx$

Optimal. Leaf size=102

$$-\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

```
[Out] -1/9*a^8/x^9-8/7*a^7*b/x^7-28/5*a^6*b^2/x^5-56/3*a^5*b^3/x^3-70*a^4*b^4/x+5
6*a^3*b^5*x+28/3*a^2*b^6*x^3+8/5*a*b^7*x^5+1/7*b^8*x^7
```

Rubi [A]
time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^10,x]
```

```
[Out] -1/9*a^8/x^9 - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x
^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5
+ (b^8*x^7)/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{10}}\,dx &= \int \left(56a^3b^5+\frac{a^8}{x^{10}}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^2}+28a^2b^6x^2+8ab^7x^4+b^8x^6\right)dx\\ &= -\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 102, normalized size = 1.00

$$-\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^10,x]
```

```
[Out] -1/9*a^8/x^9 - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x
^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5
+ (b^8*x^7)/7
```

Maple [A]
time = 0.03, size = 89, normalized size = 0.87

method	result	size
default	$-\frac{a^{8}}{9x^{9}}-\frac{8a^{7}b}{7x^{7}}-\frac{28a^{6}b^{2}}{5x^{5}}-\frac{56a^{5}b^{3}}{3x^{3}}-\frac{70a^{4}b^{4}}{x}+56a^{3}b^{5}x+\frac{28a^{2}b^{6}x^{3}}{3}+\frac{8a\,b^{7}x^{5}}{5}+\frac{b^{8}x^{7}}{7}$	89
risch	$\frac{b^{8}x^{7}}{7}+\frac{8a\,b^{7}x^{5}}{5}+\frac{28a^{2}b^{6}x^{3}}{3}+56a^{3}b^{5}x+\frac{-70a^{4}b^{4}x^{8}-\frac{56}{3}a^{5}b^{3}x^{6}-\frac{28}{5}a^{6}b^{2}x^{4}-\frac{8}{7}a^{7}b\,x^{2}-\frac{1}{9}a^{8}}{x^{9}}$	91
norman	$\frac{-\frac{1}{9}a^{8}-\frac{8}{7}a^{7}b\,x^{2}-\frac{28}{5}a^{6}b^{2}x^{4}-\frac{56}{3}a^{5}b^{3}x^{6}-70a^{4}b^{4}x^{8}+56a^{3}b^{5}x^{10}+\frac{28}{3}a^{2}b^{6}x^{12}+\frac{8}{5}a\,b^{7}x^{14}+\frac{1}{7}b^{8}x^{16}}{x^{9}}$	92
gosper	$-\frac{-45b^{8}x^{16}-504a\,b^{7}x^{14}-2940a^{2}b^{6}x^{12}-17640a^{3}b^{5}x^{10}+22050a^{4}b^{4}x^{8}+5880a^{5}b^{3}x^{6}+1764a^{6}b^{2}x^{4}+360a^{7}b\,x^{2}+35a^{8}}{315x^{9}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^8/x^9-8/7*a^7*b/x^7-28/5*a^6*b^2/x^5-56/3*a^5*b^3/x^3-70*a^4*b^4/x+5
6*a^3*b^5*x+28/3*a^2*b^6*x^3+8/5*a*b^7*x^5+1/7*b^8*x^7
```

Maxima [A]
time = 0.27, size = 91, normalized size = 0.89

$$\frac{1}{7}\,b^{8}x^{7}+\frac{8}{5}\,ab^{7}x^{5}+\frac{28}{3}\,a^{2}b^{6}x^{3}+56\,a^{3}b^{5}x-\frac{22050\,a^{4}b^{4}x^{8}+5880\,a^{5}b^{3}x^{6}+1764\,a^{6}b^{2}x^{4}+360\,a^{7}bx^{2}+35\,a^{8}}{315\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] 1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(2205
0*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*a^
8)/x^9
```

Fricas [A]
time = 1.16, size = 92, normalized size = 0.90

$$\frac{45\,b^{8}x^{16}+504\,ab^{7}x^{14}+2940\,a^{2}b^{6}x^{12}+17640\,a^{3}b^{5}x^{10}-22050\,a^{4}b^{4}x^{8}-5880\,a^{5}b^{3}x^{6}-1764\,a^{6}b^{2}x^{4}-360\,a^{7}bx^{2}-35\,a^{8}}{315\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(45*b^8*x^16 + 504*a*b^7*x^14 + 2940*a^2*b^6*x^12 + 17640*a^3*b^5*x^1
0 - 22050*a^4*b^4*x^8 - 5880*a^5*b^3*x^6 - 1764*a^6*b^2*x^4 - 360*a^7*b*x^2
 - 35*a^8)/x^9
```

Sympy [A]
time = 0.16, size = 100, normalized size = 0.98

$$56a^3b^5x + \frac{28a^2b^6x^3}{3} + \frac{8ab^7x^5}{5} + \frac{b^8x^7}{7} + \frac{-35a^8 - 360a^7bx^2 - 1764a^6b^2x^4 - 5880a^5b^3x^6 - 22050a^4b^4x^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**10,x)
```

```
[Out] 56*a**3*b**5*x + 28*a**2*b**6*x**3/3 + 8*a*b**7*x**5/5 + b**8*x**7/7 + (-35
*a**8 - 360*a**7*b*x**2 - 1764*a**6*b**2*x**4 - 5880*a**5*b**3*x**6 - 22050
*a**4*b**4*x**8)/(315*x**9)
```

Giac [A]
time = 1.14, size = 91, normalized size = 0.89

$$\frac{1}{7}b^8x^7 + \frac{8}{5}ab^7x^5 + \frac{28}{3}a^2b^6x^3 + 56a^3b^5x - \frac{22050a^4b^4x^8 + 5880a^5b^3x^6 + 1764a^6b^2x^4 + 360a^7bx^2 + 35a^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="giac")
```

```
[Out] 1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(2205
0*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*a^
8)/x^9
```

Mupad [B]
time = 0.05, size = 91, normalized size = 0.89

$$\frac{b^8x^7}{7} - \frac{\frac{a^8}{9} + \frac{8a^7bx^2}{7} + \frac{28a^6b^2x^4}{5} + \frac{56a^5b^3x^6}{3} + 70a^4b^4x^8}{x^9} + 56a^3b^5x + \frac{8ab^7x^5}{5} + \frac{28a^2b^6x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^10,x)
```

```
[Out] (b^8*x^7)/7 - (a^8/9 + (8*a^7*b*x^2)/7 + (28*a^6*b^2*x^4)/5 + (56*a^5*b^3*x
^6)/3 + 70*a^4*b^4*x^8)/x^9 + 56*a^3*b^5*x + (8*a*b^7*x^5)/5 + (28*a^2*b^6*
x^3)/3
```

3.119 $\int \frac{(a+bx^2)^8}{x^{12}}\,dx$

Optimal. Leaf size=100

$$-\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

```
[Out] -1/11*a^8/x^11-8/9*a^7*b/x^9-4*a^6*b^2/x^7-56/5*a^5*b^3/x^5-70/3*a^4*b^4/x^
3-56*a^3*b^5/x+28*a^2*b^6*x+8/3*a*b^7*x^3+1/5*b^8*x^5
```

Rubi [A]
time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^12,x]
```

```
[Out] -1/11*a^8/x^11 - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5)
 - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 +
 (b^8*x^5)/5
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{12}}\,dx &= \int \left(28a^2b^6+\frac{a^8}{x^{12}}+\frac{8a^7b}{x^{10}}+\frac{28a^6b^2}{x^8}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^2}+8ab^7x^2+b^8x^4\right)dx\\ &= -\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 100, normalized size = 1.00

$$-\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^12,x]
```

```
[Out] -1/11*a^8/x^11 - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5)
 - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 +
 (b^8*x^5)/5
```

Maple [A]
time = 0.03, size = 89, normalized size = 0.89

method	result	size
default	$-\frac{a^{8}}{11x^{11}}-\frac{8a^{7}b}{9x^{9}}-\frac{4a^{6}b^{2}}{x^{7}}-\frac{56a^{5}b^{3}}{5x^{5}}-\frac{70a^{4}b^{4}}{3x^{3}}-\frac{56a^{3}b^{5}}{x}+28a^{2}b^{6}x+\frac{8a\,b^{7}x^{3}}{3}+\frac{b^{8}x^{5}}{5}$	89
risch	$\frac{b^{8}x^{5}}{5}+\frac{8a\,b^{7}x^{3}}{3}+28a^{2}b^{6}x+\frac{-56a^{3}b^{5}x^{10}-\frac{70}{3}a^{4}b^{4}x^{8}-\frac{56}{5}a^{5}b^{3}x^{6}-4a^{6}b^{2}x^{4}-\frac{8}{9}a^{7}b\,x^{2}-\frac{1}{11}a^{8}}{x^{11}}$	91
norman	$\frac{-\frac{1}{11}a^{8}-\frac{8}{9}a^{7}b\,x^{2}-4a^{6}b^{2}x^{4}-\frac{56}{5}a^{5}b^{3}x^{6}-\frac{70}{3}a^{4}b^{4}x^{8}-56a^{3}b^{5}x^{10}+28a^{2}b^{6}x^{12}+\frac{8}{3}a\,b^{7}x^{14}+\frac{1}{5}b^{8}x^{16}}{x^{11}}$	92
gosper	$-\frac{-99b^{8}x^{16}-1320a\,b^{7}x^{14}-13860a^{2}b^{6}x^{12}+27720a^{3}b^{5}x^{10}+11550a^{4}b^{4}x^{8}+5544a^{5}b^{3}x^{6}+1980a^{6}b^{2}x^{4}+440a^{7}b\,x^{2}+45a^{8}}{495x^{11}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*a^8/x^11-8/9*a^7*b/x^9-4*a^6*b^2/x^7-56/5*a^5*b^3/x^5-70/3*a^4*b^4/x^
3-56*a^3*b^5/x+28*a^2*b^6*x+8/3*a*b^7*x^3+1/5*b^8*x^5
```

Maxima [A]
time = 0.29, size = 91, normalized size = 0.91

$$\frac{1}{5}\,b^{8}x^{5}+\frac{8}{3}\,ab^{7}x^{3}+28\,a^{2}b^{6}x-\frac{27720\,a^{3}b^{5}x^{10}+11550\,a^{4}b^{4}x^{8}+5544\,a^{5}b^{3}x^{6}+1980\,a^{6}b^{2}x^{4}+440\,a^{7}bx^{2}+45\,a^{8}}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="maxima")
```

```
[Out] 1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 11
550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 45*
a^8)/x^11
```

Fricas [A]
time = 1.44, size = 92, normalized size = 0.92

$$\frac{99\,b^{8}x^{16}+1320\,ab^{7}x^{14}+13860\,a^{2}b^{6}x^{12}-27720\,a^{3}b^{5}x^{10}-11550\,a^{4}b^{4}x^{8}-5544\,a^{5}b^{3}x^{6}-1980\,a^{6}b^{2}x^{4}-440\,a^{7}bx^{2}-45\,a^{8}}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="fricas")
```

```
[Out] 1/495*(99*b^8*x^16 + 1320*a*b^7*x^14 + 13860*a^2*b^6*x^12 - 27720*a^3*b^5*x
^10 - 11550*a^4*b^4*x^8 - 5544*a^5*b^3*x^6 - 1980*a^6*b^2*x^4 - 440*a^7*b*x
^2 - 45*a^8)/x^11
```

Sympy [A]
time = 0.21, size = 99, normalized size = 0.99

$$28a^2b^6x + \frac{8ab^7x^3}{3} + \frac{b^8x^5}{5} + \frac{-45a^8 - 440a^7bx^2 - 1980a^6b^2x^4 - 5544a^5b^3x^6 - 11550a^4b^4x^8 - 27720a^3b^5x^{10}}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**12,x)
```

```
[Out] 28*a**2*b**6*x + 8*a*b**7*x**3/3 + b**8*x**5/5 + (-45*a**8 - 440*a**7*b*x**
2 - 1980*a**6*b**2*x**4 - 5544*a**5*b**3*x**6 - 11550*a**4*b**4*x**8 - 2772
0*a**3*b**5*x**10)/(495*x**11)
```

Giac [A]
time = 1.04, size = 91, normalized size = 0.91

$$\frac{1}{5}b^8x^5 + \frac{8}{3}ab^7x^3 + 28a^2b^6x - \frac{27720a^3b^5x^{10} + 11550a^4b^4x^8 + 5544a^5b^3x^6 + 1980a^6b^2x^4 + 440a^7bx^2 + 45a^8}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="giac")
```

```
[Out] 1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 11
550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 45*
a^8)/x^11
```

Mupad [B]
time = 4.58, size = 91, normalized size = 0.91

$$\frac{b^8x^5}{5} - \frac{\frac{a^8}{11} + \frac{8a^7bx^2}{9} + 4a^6b^2x^4 + \frac{56a^5b^3x^6}{5} + \frac{70a^4b^4x^8}{3} + 56a^3b^5x^{10}}{x^{11}} + 28a^2b^6x + \frac{8ab^7x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^12,x)
```

```
[Out] (b^8*x^5)/5 - (a^8/11 + (8*a^7*b*x^2)/9 + 4*a^6*b^2*x^4 + (56*a^5*b^3*x^6)/
5 + (70*a^4*b^4*x^8)/3 + 56*a^3*b^5*x^10)/x^11 + 28*a^2*b^6*x + (8*a*b^7*x^
3)/3
```

3.120 $\int \frac{(a+bx^2)^8}{x^{14}}\,dx$

Optimal. Leaf size=98

$$-\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}$$

```
[Out] -1/13*a^8/x^13-8/11*a^7*b/x^11-28/9*a^6*b^2/x^9-8*a^5*b^3/x^7-14*a^4*b^4/x^
5-56/3*a^3*b^5/x^3-28*a^2*b^6/x+8*a*b^7*x+1/3*b^8*x^3
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^14,x]
```

```
[Out] -1/13*a^8/x^13 - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x
^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x +
 (b^8*x^3)/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{14}}\,dx &= \int \left(8ab^7+\frac{a^8}{x^{14}}+\frac{8a^7b}{x^{12}}+\frac{28a^6b^2}{x^{10}}+\frac{56a^5b^3}{x^8}+\frac{70a^4b^4}{x^6}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^2}+b^8x^2\right)dx\\ &= -\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 98, normalized size = 1.00

$$-\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^14,x]
```

```
[Out] -1/13*a^8/x^13 - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x
^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x +
 (b^8*x^3)/3
```

Maple [A]
time = 0.03, size = 89, normalized size = 0.91

method	result	size
default	$-\frac{a^{8}}{13x^{13}}-\frac{8a^{7}b}{11x^{11}}-\frac{28a^{6}b^{2}}{9x^{9}}-\frac{8a^{5}b^{3}}{x^{7}}-\frac{14a^{4}b^{4}}{x^{5}}-\frac{56a^{3}b^{5}}{3x^{3}}-\frac{28a^{2}b^{6}}{x}+8a\,b^{7}x+\frac{b^{8}x^{3}}{3}$	89
risch	$\frac{b^{8}x^{3}}{3}+8a\,b^{7}x+\frac{-28a^{2}b^{6}x^{12}-\frac{56}{3}a^{3}b^{5}x^{10}-14a^{4}b^{4}x^{8}-8a^{5}b^{3}x^{6}-\frac{28}{9}a^{6}b^{2}x^{4}-\frac{8}{11}a^{7}b\,x^{2}-\frac{1}{13}a^{8}}{x^{13}}$	91
norman	$\frac{-\frac{1}{13}a^{8}-\frac{8}{11}a^{7}b\,x^{2}-\frac{28}{9}a^{6}b^{2}x^{4}-8a^{5}b^{3}x^{6}-14a^{4}b^{4}x^{8}-\frac{56}{3}a^{3}b^{5}x^{10}-28a^{2}b^{6}x^{12}+8a\,b^{7}x^{14}+\frac{1}{3}b^{8}x^{16}}{x^{13}}$	92
gosper	$-\frac{-429b^{8}x^{16}-10296a\,b^{7}x^{14}+36036a^{2}b^{6}x^{12}+24024a^{3}b^{5}x^{10}+18018a^{4}b^{4}x^{8}+10296a^{5}b^{3}x^{6}+4004a^{6}b^{2}x^{4}+936a^{7}b\,x^{2}+99a^{8}}{1287x^{13}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*a^8/x^13-8/11*a^7*b/x^11-28/9*a^6*b^2/x^9-8*a^5*b^3/x^7-14*a^4*b^4/x^
5-56/3*a^3*b^5/x^3-28*a^2*b^6/x+8*a*b^7*x+1/3*b^8*x^3
```

Maxima [A]
time = 0.31, size = 91, normalized size = 0.93

$$\frac{1}{3}\,b^{8}x^{3}+8\,ab^{7}x-\frac{36036\,a^{2}b^{6}x^{12}+24024\,a^{3}b^{5}x^{10}+18018\,a^{4}b^{4}x^{8}+10296\,a^{5}b^{3}x^{6}+4004\,a^{6}b^{2}x^{4}+936\,a^{7}bx^{2}+99\,a^{8}}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="maxima")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10 +
 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2 +
 99*a^8)/x^13
```

Fricas [A]
time = 0.93, size = 92, normalized size = 0.94

$$\frac{429\,b^{8}x^{16}+10296\,ab^{7}x^{14}-36036\,a^{2}b^{6}x^{12}-24024\,a^{3}b^{5}x^{10}-18018\,a^{4}b^{4}x^{8}-10296\,a^{5}b^{3}x^{6}-4004\,a^{6}b^{2}x^{4}-936\,a^{7}bx^{2}-99\,a^{8}}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="fricas")
```

```
[Out] 1/1287*(429*b^8*x^16 + 10296*a*b^7*x^14 - 36036*a^2*b^6*x^12 - 24024*a^3*b^
5*x^10 - 18018*a^4*b^4*x^8 - 10296*a^5*b^3*x^6 - 4004*a^6*b^2*x^4 - 936*a^7
*b*x^2 - 99*a^8)/x^13
```

Sympy [A]
time = 0.24, size = 97, normalized size = 0.99

$$8ab^7x + \frac{b^8x^3}{3} + \frac{-99a^8 - 936a^7bx^2 - 4004a^6b^2x^4 - 10296a^5b^3x^6 - 18018a^4b^4x^8 - 24024a^3b^5x^{10} - 36036a^2b^6x^{12}}{1287x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**14,x)
```

```
[Out] 8*a*b**7*x + b**8*x**3/3 + (-99*a**8 - 936*a**7*b*x**2 - 4004*a**6*b**2*x**
4 - 10296*a**5*b**3*x**6 - 18018*a**4*b**4*x**8 - 24024*a**3*b**5*x**10 - 3
6036*a**2*b**6*x**12)/(1287*x**13)
```

Giac [A]
time = 0.97, size = 91, normalized size = 0.93

$$\frac{1}{3}b^8x^3 + 8ab^7x - \frac{36036a^2b^6x^{12} + 24024a^3b^5x^{10} + 18018a^4b^4x^8 + 10296a^5b^3x^6 + 4004a^6b^2x^4 + 936a^7bx^2 + 99a^8}{1287x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="giac")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10 +
 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2 +
 99*a^8)/x^13
```

Mupad [B]
time = 0.07, size = 92, normalized size = 0.94

$$-\frac{\frac{a^8}{13} + \frac{8a^7bx^2}{11} + \frac{28a^6b^2x^4}{9} + 8a^5b^3x^6 + 14a^4b^4x^8 + \frac{56a^3b^5x^{10}}{3} + 28a^2b^6x^{12} - 8ab^7x^{14} - \frac{b^8x^{16}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^14,x)
```

```
[Out] -(a^8/13 - (b^8*x^16)/3 + (8*a^7*b*x^2)/11 - 8*a*b^7*x^14 + (28*a^6*b^2*x^4
)/9 + 8*a^5*b^3*x^6 + 14*a^4*b^4*x^8 + (56*a^3*b^5*x^10)/3 + 28*a^2*b^6*x^1
2)/x^13
```

3.121 $\int \frac{(a+bx^2)^8}{x^{16}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x$$

```
[Out] -1/15*a^8/x^15-8/13*a^7*b/x^13-28/11*a^6*b^2/x^11-56/9*a^5*b^3/x^9-10*a^4*b
^4/x^7-56/5*a^3*b^5/x^5-28/3*a^2*b^6/x^3-8*a*b^7/x+b^8*x
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {276}

$$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^16,x]
```

```
[Out] -1/15*a^8/x^15 - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^3
)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3)
- (8*a*b^7)/x + b^8*x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{16}}\,dx &= \int \left(b^8+\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{12}}+\frac{56a^5b^3}{x^{10}}+\frac{70a^4b^4}{x^8}+\frac{56a^3b^5}{x^6}+\frac{28a^2b^6}{x^4}+\frac{8ab^7}{x^2}\right)dx\\ &= -\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x\end{aligned}$$

Mathematica [A]
time = 0.00, size = 99, normalized size = 1.00

$$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^16,x]
```

```
[Out] -1/15*a^8/x^15 - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^3
)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3)
- (8*a*b^7)/x + b^8*x
```

Maple [A]
time = 0.03, size = 88, normalized size = 0.89

method	result	size
default	$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8a\,b^7}{x}+b^8x$	88
risch	$b^8x+\frac{-\frac{1}{15}a^8-\frac{8}{13}a^7b\,x^2-\frac{28}{11}a^6b^2x^4-\frac{56}{9}a^5b^3x^6-10a^4b^4x^8-\frac{56}{5}a^3b^5x^{10}-\frac{28}{3}a^2b^6x^{12}-8a\,b^7x^{14}}{x^{15}}$	90
norman	$\frac{-\frac{1}{15}a^8-\frac{8}{13}a^7b\,x^2-\frac{28}{11}a^6b^2x^4-\frac{56}{9}a^5b^3x^6-10a^4b^4x^8-\frac{56}{5}a^3b^5x^{10}-\frac{28}{3}a^2b^6x^{12}-8a\,b^7x^{14}+b^8x^{16}}{x^{15}}$	91
gosper	$-\frac{-6435b^8x^{16}+51480a\,b^7x^{14}+60060a^2b^6x^{12}+72072a^3b^5x^{10}+64350a^4b^4x^8+40040a^5b^3x^6+16380a^6b^2x^4+3960a^7b\,x^2+429a^8}{6435x^{15}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^8/x^15-8/13*a^7*b/x^13-28/11*a^6*b^2/x^11-56/9*a^5*b^3/x^9-10*a^4*b
^4/x^7-56/5*a^3*b^5/x^5-28/3*a^2*b^6/x^3-8*a*b^7/x+b^8*x
```

Maxima [A]
time = 0.32, size = 90, normalized size = 0.91

$$b^8x-\frac{51480\,ab^7x^{14}+60060\,a^2b^6x^{12}+72072\,a^3b^5x^{10}+64350\,a^4b^4x^8+40040\,a^5b^3x^6+16380\,a^6b^2x^4+3960\,a^7bx^2+429\,a^8}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="maxima")
```

```
[Out] b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
+ 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*x^
2 + 429*a^8)/x^15
```

Fricas [A]
time = 1.05, size = 92, normalized size = 0.93

$$\frac{6435\,b^8x^{16}-51480\,ab^7x^{14}-60060\,a^2b^6x^{12}-72072\,a^3b^5x^{10}-64350\,a^4b^4x^8-40040\,a^5b^3x^6-16380\,a^6b^2x^4-3960\,a^7bx^2-429\,a^8}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="fricas")
```

```
[Out] 1/6435*(6435*b^8*x^16 - 51480*a*b^7*x^14 - 60060*a^2*b^6*x^12 - 72072*a^3*b
^5*x^10 - 64350*a^4*b^4*x^8 - 40040*a^5*b^3*x^6 - 16380*a^6*b^2*x^4 - 3960*
a^7*b*x^2 - 429*a^8)/x^15
```

Sympy [A]
time = 0.30, size = 95, normalized size = 0.96

$$b^8x + \frac{-429a^8 - 3960a^7bx^2 - 16380a^6b^2x^4 - 40040a^5b^3x^6 - 64350a^4b^4x^8 - 72072a^3b^5x^{10} - 60060a^2b^6x^{12} - 51480ab^7x^{14}}{6435x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**16,x)
```

```
[Out] b**8*x + (-429*a**8 - 3960*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 40040*a**5*
b**3*x**6 - 64350*a**4*b**4*x**8 - 72072*a**3*b**5*x**10 - 60060*a**2*b**6*
x**12 - 51480*a*b**7*x**14)/(6435*x**15)
```

Giac [A]
time = 0.87, size = 90, normalized size = 0.91

$$b^8x - \frac{51480\,ab^7x^{14} + 60060\,a^2b^6x^{12} + 72072\,a^3b^5x^{10} + 64350\,a^4b^4x^8 + 40040\,a^5b^3x^6 + 16380\,a^6b^2x^4 + 3960\,a^7bx^2 + 429\,a^8}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="giac")
```

```
[Out] b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
+ 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*x^
2 + 429*a^8)/x^15
```

Mupad [B]
time = 4.52, size = 90, normalized size = 0.91

$$b^8\,x - \frac{\frac{a^8}{15} + \frac{8\,a^7\,b\,x^2}{13} + \frac{28\,a^6\,b^2\,x^4}{11} + \frac{56\,a^5\,b^3\,x^6}{9} + 10\,a^4\,b^4\,x^8 + \frac{56\,a^3\,b^5\,x^{10}}{5} + \frac{28\,a^2\,b^6\,x^{12}}{3} + 8\,a\,b^7\,x^{14}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^16,x)
```

```
[Out] b^8*x - (a^8/15 + (8*a^7*b*x^2)/13 + 8*a*b^7*x^14 + (28*a^6*b^2*x^4)/11 + (
56*a^5*b^3*x^6)/9 + 10*a^4*b^4*x^8 + (56*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^12
)/3)/x^15
```

3.122 $\int \frac{(a+bx^2)^8}{x^{18}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

```
[Out] -1/17*a^8/x^17-8/15*a^7*b/x^15-28/13*a^6*b^2/x^13-56/11*a^5*b^3/x^11-70/9*a
^4*b^4/x^9-8*a^3*b^5/x^7-28/5*a^2*b^6/x^5-8/3*a*b^7/x^3-b^8/x
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^18,x]
```

```
[Out] -1/17*a^8/x^17 - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^3
)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5)
 - (8*a*b^7)/(3*x^3) - b^8/x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{18}}\,dx &= \int \left(\frac{a^8}{x^{18}}+\frac{8a^7b}{x^{16}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{10}}+\frac{56a^3b^5}{x^8}+\frac{28a^2b^6}{x^6}+\frac{8ab^7}{x^4}+\frac{b^8}{x^2}\right)dx\\ &= -\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 104, normalized size = 1.00

$$-\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^18,x]
```

```
[Out] -1/17*a^8/x^17 - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^3
)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5)
 - (8*a*b^7)/(3*x^3) - b^8/x
```

Maple [A]
time = 0.03, size = 91, normalized size = 0.88

method	result
default	$-\frac{a^{8}}{17 x^{17}}-\frac{8 a^{7} b}{15 x^{15}}-\frac{28 a^{6} b^{2}}{13 x^{13}}-\frac{56 a^{5} b^{3}}{11 x^{11}}-\frac{70 a^{4} b^{4}}{9 x^{9}}-\frac{8 a^{3} b^{5}}{x^{7}}-\frac{28 a^{2} b^{6}}{5 x^{5}}-\frac{8 a\,b^{7}}{3 x^{3}}-\frac{b^{8}}{x}$
norman	$\frac{-\frac{1}{17} a^{8}-\frac{8}{15} a^{7} b\,x^{2}-\frac{28}{13} a^{6} b^{2} x^{4}-\frac{56}{11} a^{5} b^{3} x^{6}-\frac{70}{9} a^{4} b^{4} x^{8}-8 a^{3} b^{5} x^{10}-\frac{28}{5} a^{2} b^{6} x^{12}-\frac{8}{3} a\,b^{7} x^{14}-b^{8} x^{16}}{x^{17}}$
risch	$\frac{-\frac{1}{17} a^{8}-\frac{8}{15} a^{7} b\,x^{2}-\frac{28}{13} a^{6} b^{2} x^{4}-\frac{56}{11} a^{5} b^{3} x^{6}-\frac{70}{9} a^{4} b^{4} x^{8}-8 a^{3} b^{5} x^{10}-\frac{28}{5} a^{2} b^{6} x^{12}-\frac{8}{3} a\,b^{7} x^{14}-b^{8} x^{16}}{x^{17}}$
gosper	$-\frac{109395 b^{8} x^{16}+291720 a\,b^{7} x^{14}+612612 a^{2} b^{6} x^{12}+875160 a^{3} b^{5} x^{10}+850850 a^{4} b^{4} x^{8}+556920 a^{5} b^{3} x^{6}+235620 a^{6} b^{2} x^{4}+58344 a^{7} b\,x^{2}+643}{109395 x^{17}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -1/17*a^8/x^17-8/15*a^7*b/x^15-28/13*a^6*b^2/x^13-56/11*a^5*b^3/x^11-70/9*a
^4*b^4/x^9-8*a^3*b^5/x^7-28/5*a^2*b^6/x^5-8/3*a*b^7/x^3-b^8/x
```

Maxima [A]
time = 0.29, size = 92, normalized size = 0.88

$$-\frac{109395 \, b^{8} x^{16} + 291720 \, a b^{7} x^{14} + 612612 \, a^{2} b^{6} x^{12} + 875160 \, a^{3} b^{5} x^{10} + 850850 \, a^{4} b^{4} x^{8} + 556920 \, a^{5} b^{3} x^{6} + 235620 \, a^{6} b^{2} x^{4} + 58344 \, a^{7} b x^{2} + 6435 \, a^{8}}{109395 \, x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="maxima")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

Fricas [A]
time = 1.03, size = 92, normalized size = 0.88

$$-\frac{109395 \, b^{8} x^{16} + 291720 \, a b^{7} x^{14} + 612612 \, a^{2} b^{6} x^{12} + 875160 \, a^{3} b^{5} x^{10} + 850850 \, a^{4} b^{4} x^{8} + 556920 \, a^{5} b^{3} x^{6} + 235620 \, a^{6} b^{2} x^{4} + 58344 \, a^{7} b x^{2} + 6435 \, a^{8}}{109395 \, x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="fricas")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

Sympy [A]
time = 0.35, size = 99, normalized size = 0.95

$$\frac{-6435a^8 - 58344a^7bx^2 - 235620a^6b^2x^4 - 556920a^5b^3x^6 - 850850a^4b^4x^8 - 875160a^3b^5x^{10} - 612612a^2b^6x^{12} - 291720ab^7x^{14} - 109395b^8x^{16}}{109395x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**18,x)
```

```
[Out] (-6435*a**8 - 58344*a**7*b*x**2 - 235620*a**6*b**2*x**4 - 556920*a**5*b**3*
x**6 - 850850*a**4*b**4*x**8 - 875160*a**3*b**5*x**10 - 612612*a**2*b**6*x*
*12 - 291720*a*b**7*x**14 - 109395*b**8*x**16)/(109395*x**17)
```

Giac [A]
time = 1.11, size = 92, normalized size = 0.88

$$-\frac{109395\,b^8x^{16} + 291720\,ab^7x^{14} + 612612\,a^2b^6x^{12} + 875160\,a^3b^5x^{10} + 850850\,a^4b^4x^8 + 556920\,a^5b^3x^6 + 235620\,a^6b^2x^4 + 58344\,a^7bx^2 + 6435\,a^8}{109395\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="giac")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

Mupad [B]
time = 0.07, size = 91, normalized size = 0.88

$$-\frac{\frac{a^8}{17} + \frac{8\,a^7\,b\,x^2}{15} + \frac{28\,a^6\,b^2\,x^4}{13} + \frac{56\,a^5\,b^3\,x^6}{11} + \frac{70\,a^4\,b^4\,x^8}{9} + 8\,a^3\,b^5\,x^{10} + \frac{28\,a^2\,b^6\,x^{12}}{5} + \frac{8\,a\,b^7\,x^{14}}{3} + b^8\,x^{16}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^18,x)
```

```
[Out] -(a^8/17 + b^8*x^16 + (8*a^7*b*x^2)/15 + (8*a*b^7*x^14)/3 + (28*a^6*b^2*x^4
)/13 + (56*a^5*b^3*x^6)/11 + (70*a^4*b^4*x^8)/9 + 8*a^3*b^5*x^10 + (28*a^2*
b^6*x^12)/5)/x^17
```

3.123 $\int \frac{(a+bx^2)^8}{x^{20}}\,dx$

Optimal. Leaf size=106

$$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

```
[Out] -1/19*a^8/x^19-8/17*a^7*b/x^17-28/15*a^6*b^2/x^15-56/13*a^5*b^3/x^13-70/11*
a^4*b^4/x^11-56/9*a^3*b^5/x^9-4*a^2*b^6/x^7-8/5*a*b^7/x^5-1/3*b^8/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^20,x]
```

```
[Out] -1/19*a^8/x^19 - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^3
)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x
^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{20}}\,dx &= \int \left(\frac{a^8}{x^{20}}+\frac{8a^7b}{x^{18}}+\frac{28a^6b^2}{x^{16}}+\frac{56a^5b^3}{x^{14}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^8}+\frac{8ab^7}{x^6}+\frac{b^8}{x^4}\right)dx\\ &= -\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 106, normalized size = 1.00

$$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^20,x]
```

```
[Out] -1/19*a^8/x^19 - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^3
)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x
^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

Maple [A]
time = 0.04, size = 91, normalized size = 0.86

method	result
default	$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8a\,b^7}{5x^5}-\frac{b^8}{3x^3}$
norman	$\frac{-\frac{1}{19}a^8-\frac{8}{17}a^7b\,x^2-\frac{28}{15}a^6b^2x^4-\frac{56}{13}a^5b^3x^6-\frac{70}{11}a^4b^4x^8-\frac{56}{9}a^3b^5x^{10}-4a^2b^6x^{12}-\frac{8}{5}a\,b^7x^{14}-\frac{1}{3}b^8x^{16}}{x^{19}}$
risch	$\frac{-\frac{1}{19}a^8-\frac{8}{17}a^7b\,x^2-\frac{28}{15}a^6b^2x^4-\frac{56}{13}a^5b^3x^6-\frac{70}{11}a^4b^4x^8-\frac{56}{9}a^3b^5x^{10}-4a^2b^6x^{12}-\frac{8}{5}a\,b^7x^{14}-\frac{1}{3}b^8x^{16}}{x^{19}}$
gosper	$-\frac{692835b^8x^{16}+3325608a\,b^7x^{14}+8314020a^2b^6x^{12}+12932920a^3b^5x^{10}+13226850a^4b^4x^8+8953560a^5b^3x^6+3879876a^6b^2x^4+978120a^7}{2078505x^{19}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/19*a^8/x^19-8/17*a^7*b/x^17-28/15*a^6*b^2/x^15-56/13*a^5*b^3/x^13-70/11*
a^4*b^4/x^11-56/9*a^3*b^5/x^9-4*a^2*b^6/x^7-8/5*a*b^7/x^5-1/3*b^8/x^3
```

Maxima [A]
time = 0.38, size = 92, normalized size = 0.87

$$-\frac{692835\,b^8x^{16}+3325608\,ab^7x^{14}+8314020\,a^2b^6x^{12}+12932920\,a^3b^5x^{10}+13226850\,a^4b^4x^8+8953560\,a^5b^3x^6+3879876\,a^6b^2x^4+978120\,a^7bx^2+109395\,a^8}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="maxima")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

Fricas [A]
time = 0.74, size = 92, normalized size = 0.87

$$-\frac{692835\,b^8x^{16}+3325608\,ab^7x^{14}+8314020\,a^2b^6x^{12}+12932920\,a^3b^5x^{10}+13226850\,a^4b^4x^8+8953560\,a^5b^3x^6+3879876\,a^6b^2x^4+978120\,a^7bx^2+109395\,a^8}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="fricas")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

Sympy [A]
time = 0.36, size = 99, normalized size = 0.93

$$\frac{-109395a^8 - 978120a^7bx^2 - 3879876a^6b^2x^4 - 8953560a^5b^3x^6 - 13226850a^4b^4x^8 - 12932920a^3b^5x^{10} - 8314020a^2b^6x^{12} - 3325608ab^7x^{14} - 692835b^8x^{16}}{2078505x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**20,x)
```

```
[Out] (-109395*a**8 - 978120*a**7*b*x**2 - 3879876*a**6*b**2*x**4 - 8953560*a**5*
b**3*x**6 - 13226850*a**4*b**4*x**8 - 12932920*a**3*b**5*x**10 - 8314020*a*
*2*b**6*x**12 - 3325608*a*b**7*x**14 - 692835*b**8*x**16)/(2078505*x**19)
```

Giac [A]
time = 1.59, size = 92, normalized size = 0.87

$$-\frac{692835\,b^8x^{16} + 3325608\,ab^7x^{14} + 8314020\,a^2b^6x^{12} + 12932920\,a^3b^5x^{10} + 13226850\,a^4b^4x^8 + 8953560\,a^5b^3x^6 + 3879876\,a^6b^2x^4 + 978120\,a^7bx^2 + 109395\,a^8}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="giac")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

Mupad [B]
time = 0.08, size = 92, normalized size = 0.87

$$-\frac{\frac{a^8}{19} + \frac{8\,a^7\,b\,x^2}{17} + \frac{28\,a^6\,b^2\,x^4}{15} + \frac{56\,a^5\,b^3\,x^6}{13} + \frac{70\,a^4\,b^4\,x^8}{11} + \frac{56\,a^3\,b^5\,x^{10}}{9} + 4\,a^2\,b^6\,x^{12} + \frac{8\,a\,b^7\,x^{14}}{5} + \frac{b^8\,x^{16}}{3}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^20,x)
```

```
[Out] -(a^8/19 + (b^8*x^16)/3 + (8*a^7*b*x^2)/17 + (8*a*b^7*x^14)/5 + (28*a^6*b^2
*x^4)/15 + (56*a^5*b^3*x^6)/13 + (70*a^4*b^4*x^8)/11 + (56*a^3*b^5*x^10)/9
+ 4*a^2*b^6*x^12)/x^19
```

3.124 $\int \frac{x^{11}}{a+bx^2} \, dx$

Optimal. Leaf size=79

$$\frac{a^4x^2}{2b^5} - \frac{a^3x^4}{4b^4} + \frac{a^2x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b} - \frac{a^5\log(a+bx^2)}{2b^6}$$

```
[Out] 1/2*a^4*x^2/b^5-1/4*a^3*x^4/b^4+1/6*a^2*x^6/b^3-1/8*a*x^8/b^2+1/10*x^10/b-1
/2*a^5*ln(b*x^2+a)/b^6
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^5\log(a+bx^2)}{2b^6} + \frac{a^4x^2}{2b^5} - \frac{a^3x^4}{4b^4} + \frac{a^2x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2),x]
```

```
[Out] (a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2)
 + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{a+bx^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{a+bx} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a^5}{b^5(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{a^4x^2}{2b^5} - \frac{a^3x^4}{4b^4} + \frac{a^2x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b} - \frac{a^5\log(a+bx^2)}{2b^6}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 79, normalized size = 1.00

$$\frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}-\frac{a^5\log(a+bx^2)}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2),x]
```

```
[Out] (a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2)
 + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

Maple [A]
time = 0.05, size = 68, normalized size = 0.86

method	result	size
default	$\frac{\frac{1}{5}b^4x^{10}-\frac{1}{4}a\,b^3x^8+\frac{1}{3}a^2b^2x^6-\frac{1}{2}a^3b\,x^4+a^4x^2}{2b^5}-\frac{a^5\ln(b\,x^2+a)}{2b^6}$	68
norman	$\frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{a\,x^8}{8b^2}+\frac{x^{10}}{10b}-\frac{a^5\ln(b\,x^2+a)}{2b^6}$	68
risch	$\frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{a\,x^8}{8b^2}+\frac{x^{10}}{10b}-\frac{a^5\ln(b\,x^2+a)}{2b^6}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^5*(1/5*b^4*x^10-1/4*a*b^3*x^8+1/3*a^2*b^2*x^6-1/2*a^3*b*x^4+a^4*x^2)-
1/2*a^5*ln(b*x^2+a)/b^6
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.86

$$-\frac{a^5\log(bx^2+a)}{2\,b^6}+\frac{12\,b^4x^{10}-15\,ab^3x^8+20\,a^2b^2x^6-30\,a^3bx^4+60\,a^4x^2}{120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a^5*log(b*x^2 + a)/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*a^2*b^
2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

Fricas [A]
time = 0.87, size = 67, normalized size = 0.85

$$\frac{12\,b^5x^{10}-15\,ab^4x^8+20\,a^2b^3x^6-30\,a^3b^2x^4+60\,a^4bx^2-60\,a^5\log(bx^2+a)}{120\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/120*(12*b^5*x^10 - 15*a*b^4*x^8 + 20*a^2*b^3*x^6 - 30*a^3*b^2*x^4 + 60*a^
4*b*x^2 - 60*a^5*log(b*x^2 + a))/b^6
```

Sympy [A]
time = 0.06, size = 68, normalized size = 0.86

$$-\frac{a^5\log\left(a+bx^2\right)}{2b^6}+\frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a),x)
```

```
[Out] -a**5*log(a + b*x**2)/(2*b**6) + a**4*x**2/(2*b**5) - a**3*x**4/(4*b**4) +
a**2*x**6/(6*b**3) - a*x**8/(8*b**2) + x**10/(10*b)
```

Giac [A]
time = 1.19, size = 69, normalized size = 0.87

$$-\frac{a^5\log\left(|bx^2+a|\right)}{2\,b^6}+\frac{12\,b^4x^{10}-15\,ab^3x^8+20\,a^2b^2x^6-30\,a^3bx^4+60\,a^4x^2}{120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*a^5*log(abs(b*x^2 + a))/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*a
^2*b^2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

Mupad [B]
time = 0.06, size = 67, normalized size = 0.85

$$\frac{x^{10}}{10\,b}-\frac{a\,x^8}{8\,b^2}-\frac{a^5\,\ln\left(b\,x^2+a\right)}{2\,b^6}+\frac{a^2\,x^6}{6\,b^3}-\frac{a^3\,x^4}{4\,b^4}+\frac{a^4\,x^2}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2),x)
```

```
[Out] x^10/(10*b) - (a*x^8)/(8*b^2) - (a^5*log(a + b*x^2))/(2*b^6) + (a^2*x^6)/(6
*b^3) - (a^3*x^4)/(4*b^4) + (a^4*x^2)/(2*b^5)
```

3.125 $\int \frac{x^{10}}{a+bx^2} \, dx$

Optimal. Leaf size=81

$$\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}$$

```
[Out] a^4*x/b^5-1/3*a^3*x^3/b^4+1/5*a^2*x^5/b^3-1/7*a*x^7/b^2+1/9*x^9/b-a^(9/2)*a
rctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {308, 211}

$$-\frac{a^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9
/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{a+bx^2}\,dx &= \int \left(\frac{a^4}{b^5}-\frac{a^3x^2}{b^4}+\frac{a^2x^4}{b^3}-\frac{ax^6}{b^2}+\frac{x^8}{b}-\frac{a^5}{b^5\left(a+bx^2\right)}\right)dx\\
&= \frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^5\int \frac{1}{a+bx^2}\,dx}{b^5}\\
&= \frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 81, normalized size = 1.00

$$\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9
/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Maple [A]
time = 0.06, size = 71, normalized size = 0.88

method	result	size
default	$\frac{\frac{1}{9}b^4x^9-\frac{1}{7}a\,b^3x^7+\frac{1}{5}a^2b^2x^5-\frac{1}{3}a^3b\,x^3+a^4x}{b^5}-\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^5\sqrt{ab}}$	71
risch	$\frac{x^9}{9b}-\frac{a\,x^7}{7b^2}+\frac{a^2x^5}{5b^3}-\frac{a^3x^3}{3b^4}+\frac{a^4x}{b^5}+\frac{\sqrt{-ab}\,a^4\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^6}-\frac{\sqrt{-ab}\,a^4\ln\left(\sqrt{-ab}\,x-a\right)}{2b^6}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/9*b^4*x^9-1/7*a*b^3*x^7+1/5*a^2*b^2*x^5-1/3*a^3*b*x^3+a^4*x)-a^5/b
^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 72, normalized size = 0.89

$$-\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5}+\frac{35\,b^4x^9-45\,ab^3x^7+63\,a^2b^2x^5-105\,a^3bx^3+315\,a^4x}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^4*x^9 - 45*a*b^3*x
^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 + 315*a^4*x)/b^5
```

Fricas [A]
time = 0.98, size = 170, normalized size = 2.10

$$\left[\frac{70\,b^4x^9-90\,ab^3x^7+126\,a^2b^2x^5-210\,a^3bx^3+315\,a^4\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+630\,a^4x}{630\,b^5},\ \frac{35\,b^4x^9-45\,ab^3x^7+63\,a^2b^2x^5-105\,a^3bx^3-315\,a^4\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+315\,a^4x}{315\,b^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*x^9 - 90*a*b^3*x^7 + 126*a^2*b^2*x^5 - 210*a^3*b*x^3 + 315*a
^4*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 630*a^4*x)/
b^5, 1/315*(35*b^4*x^9 - 45*a*b^3*x^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 - 31
5*a^4*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 315*a^4*x)/b^5]
```

Sympy [A]
time = 0.07, size = 119, normalized size = 1.47

$$\frac{a^4x}{b^5} - \frac{a^3x^3}{3b^4} + \frac{a^2x^5}{5b^3} - \frac{ax^7}{7b^2} + \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} - \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} + \frac{x^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a),x)
```

```
[Out] a**4*x/b**5 - a**3*x**3/(3*b**4) + a**2*x**5/(5*b**3) - a*x**7/(7*b**2) + s
qrt(-a**9/b**11)*log(x - b**5*sqrt(-a**9/b**11)/a**4)/2 - sqrt(-a**9/b**11)
*log(x + b**5*sqrt(-a**9/b**11)/a**4)/2 + x**9/(9*b)
```

Giac [A]
time = 1.43, size = 77, normalized size = 0.95

$$-\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5} + \frac{35\,b^8x^9 - 45\,ab^7x^7 + 63\,a^2b^6x^5 - 105\,a^3b^5x^3 + 315\,a^4b^4x}{315\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*x^9 - 45*a*b^7*x
^7 + 63*a^2*b^6*x^5 - 105*a^3*b^5*x^3 + 315*a^4*b^4*x)/b^9
```

Mupad [B]
time = 0.06, size = 65, normalized size = 0.80

$$\frac{x^9}{9\,b} - \frac{a\,x^7}{7\,b^2} + \frac{a^4\,x}{b^5} - \frac{a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}} + \frac{a^2\,x^5}{5\,b^3} - \frac{a^3\,x^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2),x)
```

```
[Out] x^9/(9*b) - (a*x^7)/(7*b^2) + (a^4*x)/b^5 - (a^(9/2)*atan((b^(1/2)*x)/a^(1/
2)))/b^(11/2) + (a^2*x^5)/(5*b^3) - (a^3*x^3)/(3*b^4)
```

3.126 $\int \frac{x^9}{a+bx^2}\,dx$

Optimal. Leaf size=66

$$-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}+\frac{a^4\log\left(a+bx^2\right)}{2b^5}$$

```
[Out] -1/2*a^3*x^2/b^4+1/4*a^2*x^4/b^3-1/6*a*x^6/b^2+1/8*x^8/b+1/2*a^4*ln(b*x^2+a
)/b^5
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2),x]
```

```
[Out] -1/2*(a^3*x^2)/b^4 + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4
*Log[a + b*x^2])/(2*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)\,dx,x,x^2\right)\\ &= -\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}+\frac{a^4\log\left(a+bx^2\right)}{2b^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 66, normalized size = 1.00

$$-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}+\frac{a^4\log(a+bx^2)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2),x]
```

```
[Out] -1/2*(a^3*x^2)/b^4 + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4
*Log[a + b*x^2])/(2*b^5)
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.86

method	result	size
default	$-\frac{-\frac{1}{4}b^3x^8+\frac{1}{3}a\,b^2x^6-\frac{1}{2}a^2b\,x^4+a^3x^2}{2b^4}+\frac{a^4\ln(b\,x^2+a)}{2b^5}$	57
norman	$-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{a\,x^6}{6b^2}+\frac{x^8}{8b}+\frac{a^4\ln(b\,x^2+a)}{2b^5}$	57
risch	$-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{a\,x^6}{6b^2}+\frac{x^8}{8b}+\frac{a^4\ln(b\,x^2+a)}{2b^5}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^4*(-1/4*b^3*x^8+1/3*a*b^2*x^6-1/2*a^2*b*x^4+a^3*x^2)+1/2*a^4*ln(b*x^
2+a)/b^5
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.86

$$\frac{a^4\log(bx^2+a)}{2\,b^5}+\frac{3\,b^3x^8-4\,ab^2x^6+6\,a^2bx^4-12\,a^3x^2}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a^4*log(b*x^2 + a)/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x^4 -
12*a^3*x^2)/b^4
```

Fricas [A]
time = 1.52, size = 56, normalized size = 0.85

$$\frac{3\,b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-12\,a^3bx^2+12\,a^4\log(bx^2+a)}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 12*a^3*b*x^2 + 12*a^4*log(b
*x^2 + a))/b^5
```

Sympy [A]
time = 0.06, size = 56, normalized size = 0.85

$$\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a),x)
```

```
[Out] a**4*log(a + b*x**2)/(2*b**5) - a**3*x**2/(2*b**4) + a**2*x**4/(4*b**3) - a
*x**6/(6*b**2) + x**8/(8*b)
```

Giac [A]
time = 1.10, size = 58, normalized size = 0.88

$$\frac{a^4\log\left(|bx^2+a|\right)}{2\,b^5}+\frac{3\,b^3x^8-4\,ab^2x^6+6\,a^2bx^4-12\,a^3x^2}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a^4*log(abs(b*x^2 + a))/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x
^4 - 12*a^3*x^2)/b^4
```

Mupad [B]
time = 0.08, size = 56, normalized size = 0.85

$$\frac{x^8}{8\,b}-\frac{a\,x^6}{6\,b^2}+\frac{a^4\,\ln\left(b\,x^2+a\right)}{2\,b^5}+\frac{a^2\,x^4}{4\,b^3}-\frac{a^3\,x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2),x)
```

```
[Out] x^8/(8*b) - (a*x^6)/(6*b^2) + (a^4*log(a + b*x^2))/(2*b^5) + (a^2*x^4)/(4*b
^3) - (a^3*x^2)/(2*b^4)
```

3.127 $\int \frac{x^8}{a+bx^2}\,dx$

Optimal. Leaf size=68

$$-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

```
[Out] -a^3*x/b^4+1/3*a^2*x^3/b^3-1/5*a*x^5/b^2+1/7*x^7/b+a^(7/2)*arctan(x*b^(1/2)
/a^(1/2))/b^(9/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {308, 211}

$$\frac{a^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{a+bx^2}\,dx &= \int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4\left(a+bx^2\right)}\right)dx\\ &= -\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^4\int \frac{1}{a+bx^2}\,dx}{b^4}\\ &= -\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 1.00

$$-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2),x]
[Out] -((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A]
time = 0.08, size = 60, normalized size = 0.88

method	result	size
default	$-\frac{-\frac{1}{7}b^3x^7+\frac{1}{5}a\,b^2x^5-\frac{1}{3}a^2b\,x^3+a^3x}{b^4}+\frac{a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^4\sqrt{ab}}$	60
risch	$\frac{x^7}{7b}-\frac{a\,x^5}{5b^2}+\frac{a^2x^3}{3b^3}-\frac{a^3x}{b^4}+\frac{\sqrt{-ab}\,a^3\ln\left(-\sqrt{-ab}\,x+a\right)}{2b^5}-\frac{\sqrt{-ab}\,a^3\ln\left(\sqrt{-ab}\,x+a\right)}{2b^5}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/b^4*(-1/7*b^3*x^7+1/5*a*b^2*x^5-1/3*a^2*b*x^3+a^3*x)+a^4/b^4/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 60, normalized size = 0.88

$$\frac{a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^3x^7-21\,ab^2x^5+35\,a^2bx^3-105\,a^3x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="maxima")
[Out] a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*x^7 - 21*a*b^2*x^
5 + 35*a^2*b*x^3 - 105*a^3*x)/b^4
```

Fricas [A]
time = 0.79, size = 148, normalized size = 2.18

$$\left[\frac{30\,b^3x^7-42\,ab^2x^5+70\,a^2bx^3+105\,a^3\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\,a^3x}{210\,b^4},\frac{15\,b^3x^7-21\,ab^2x^5+35\,a^2bx^3+105\,a^3\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-105\,a^3x}{105\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*x^7 - 42*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*sqrt(-a/b)*log((
b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*a^3*x)/b^4, 1/105*(15*b^3*
x^7 - 21*a*b^2*x^5 + 35*a^2*b*x^3 + 105*a^3*sqrt(a/b)*arctan(b*x*sqrt(a/b)/
a) - 105*a^3*x)/b^4]
```

Sympy [A]
time = 0.07, size = 107, normalized size = 1.57

$$-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}-\frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2}+\frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2}+\frac{x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a),x)
```

```
[Out] -a**3*x/b**4 + a**2*x**3/(3*b**3) - a*x**5/(5*b**2) - sqrt(-a**7/b**9)*log(
x - b**4*sqrt(-a**7/b**9)/a**3)/2 + sqrt(-a**7/b**9)*log(x + b**4*sqrt(-a**
7/b**9)/a**3)/2 + x**7/(7*b)
```

Giac [A]
time = 0.81, size = 65, normalized size = 0.96

$$\frac{a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^6x^7-21\,ab^5x^5+35\,a^2b^4x^3-105\,a^3b^3x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*x^7 - 21*a*b^5*x^
5 + 35*a^2*b^4*x^3 - 105*a^3*b^3*x)/b^7
```

Mupad [B]
time = 0.05, size = 54, normalized size = 0.79

$$\frac{x^7}{7\,b}-\frac{a\,x^5}{5\,b^2}-\frac{a^3\,x}{b^4}+\frac{a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{a^2\,x^3}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2),x)
```

```
[Out] x^7/(7*b) - (a*x^5)/(5*b^2) - (a^3*x)/b^4 + (a^(7/2)*atan((b^(1/2)*x)/a^(1/
2)))/b^(9/2) + (a^2*x^3)/(3*b^3)
```

3.128 $\int \frac{x^7}{a+bx^2}\,dx$

Optimal. Leaf size=53

$$\frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}$$

```
[Out] 1/2*a^2*x^2/b^3-1/4*a*x^4/b^2+1/6*x^6/b-1/2*a^3*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 45}

$$-\frac{a^3\log\left(a+bx^2\right)}{2b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2),x]
```

```
[Out] (a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*b
^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)\,dx,x,x^2\right)\\ &= \frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 53, normalized size = 1.00

$$\frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b} - \frac{a^3\log\left(a+bx^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2),x]
```

```
[Out] (a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*b
^4)
```

Maple [A]
time = 0.03, size = 46, normalized size = 0.87

method	result	size
default	$\frac{\frac{1}{3}b^2x^6-\frac{1}{2}ab\,x^4+a^2x^2}{2b^3}-\frac{a^3\ln\left(b\,x^2+a\right)}{2b^4}$	46
norman	$\frac{a^2x^2}{2b^3}-\frac{a\,x^4}{4b^2}+\frac{x^6}{6b}-\frac{a^3\ln\left(b\,x^2+a\right)}{2b^4}$	46
risch	$\frac{a^2x^2}{2b^3}-\frac{a\,x^4}{4b^2}+\frac{x^6}{6b}-\frac{a^3\ln\left(b\,x^2+a\right)}{2b^4}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^3*(1/3*b^2*x^6-1/2*a*b*x^4+a^2*x^2)-1/2*a^3*ln(b*x^2+a)/b^4
```

Maxima [A]
time = 0.26, size = 46, normalized size = 0.87

$$-\frac{a^3\log\left(bx^2+a\right)}{2\,b^4} + \frac{2\,b^2x^6 - 3\,abx^4 + 6\,a^2x^2}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a^3*log(b*x^2 + a)/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3
```

Fricas [A]
time = 1.57, size = 45, normalized size = 0.85

$$\frac{2\,b^3x^6 - 3\,ab^2x^4 + 6\,a^2bx^2 - 6\,a^3\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^6 - 3*a*b^2*x^4 + 6*a^2*b*x^2 - 6*a^3*log(b*x^2 + a))/b^4
```

Sympy [A]
time = 0.05, size = 44, normalized size = 0.83

$$-\frac{a^3 \log \left(a+b x^2\right)}{2 b^4}+\frac{a^2 x^2}{2 b^3}-\frac{a x^4}{4 b^2}+\frac{x^6}{6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a),x)
```

```
[Out] -a**3*log(a + b*x**2)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4/(4*b**2) + x**
6/(6*b)
```

Giac [A]
time = 1.22, size = 47, normalized size = 0.89

$$-\frac{a^3 \log \left(|b x^2+a|\right)}{2 b^4}+\frac{2 b^2 x^6-3 a b x^4+6 a^2 x^2}{12 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*a^3*log(abs(b*x^2 + a))/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)
/b^3
```

Mupad [B]
time = 4.73, size = 45, normalized size = 0.85

$$\frac{x^6}{6 b}-\frac{a x^4}{4 b^2}-\frac{a^3 \ln \left(b x^2+a\right)}{2 b^4}+\frac{a^2 x^2}{2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2),x)
```

```
[Out] x^6/(6*b) - (a*x^4)/(4*b^2) - (a^3*log(a + b*x^2))/(2*b^4) + (a^2*x^2)/(2*b
^3)
```

3.129 $\int \frac{x^6}{a+bx^2} dx$

Optimal. Leaf size=55

$$\frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] a^2*x/b^3-1/3*a*x^3/b^2+1/5*x^5/b-a^(5/2)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {308, 211}

$$-\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}} + \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{a+bx^2}\,dx &= \int \left(\frac{a^2}{b^3} - \frac{ax^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3\,(a+bx^2)}\right) dx\\
&= \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b} - \frac{a^3\int \frac{1}{a+bx^2}\,dx}{b^3}\\
&= \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 1.00

$$\frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(7/2)
```

Maple [A]
time = 0.04, size = 49, normalized size = 0.89

method	result	size
default	$\frac{\frac{1}{5}b^2x^5-\frac{1}{3}ab\,x^3+a^2x}{b^3} - \frac{a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	49
risch	$\frac{x^5}{5b} - \frac{a\,x^3}{3b^2} + \frac{a^2x}{b^3} + \frac{\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^4} - \frac{\sqrt{-ab}\,a^2\ln\left(\sqrt{-ab}\,x-a\right)}{2b^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*b^2*x^5-1/3*a*b*x^3+a^2*x)-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2))
```

Maxima [A]
time = 0.49, size = 50, normalized size = 0.91

$$-\frac{a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^2x^5 - 5\,abx^3 + 15\,a^2x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^2*x^5 - 5*a*b*x^3 +
15*a^2*x)/b^3
```

Fricas [A]
time = 1.17, size = 126, normalized size = 2.29

$$\left[\frac{6\,b^{2}x^{5}-10\,abx^{3}+15\,a^{2}\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)+30\,a^{2}x}{30\,b^{3}},\ \frac{3\,b^{2}x^{5}-5\,abx^{3}-15\,a^{2}\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+15\,a^{2}x}{15\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*x^5 - 10*a*b*x^3 + 15*a^2*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-
a/b) - a)/(b*x^2 + a)) + 30*a^2*x)/b^3, 1/15*(3*b^2*x^5 - 5*a*b*x^3 - 15*a^
2*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*a^2*x)/b^3]
```

Sympy [A]
time = 0.06, size = 95, normalized size = 1.73

$$\frac{a^{2}x}{b^{3}}-\frac{ax^{3}}{3b^{2}}+\frac{\sqrt{-\frac{a^{5}}{b^{7}}}\,\log\left(x-\frac{b^{3}\sqrt{-\frac{a^{5}}{b^{7}}}}{a^{2}}\right)}{2}-\frac{\sqrt{-\frac{a^{5}}{b^{7}}}\,\log\left(x+\frac{b^{3}\sqrt{-\frac{a^{5}}{b^{7}}}}{a^{2}}\right)}{2}+\frac{x^{5}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a),x)
```

```
[Out] a**2*x/b**3 - a*x**3/(3*b**2) + sqrt(-a**5/b**7)*log(x - b**3*sqrt(-a**5/b*
*7)/a**2)/2 - sqrt(-a**5/b**7)*log(x + b**3*sqrt(-a**5/b**7)/a**2)/2 + x**5
/(5*b)
```

Giac [A]
time = 0.83, size = 55, normalized size = 1.00

$$-\frac{a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{3\,b^{4}x^{5}-5\,ab^{3}x^{3}+15\,a^{2}b^{2}x}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*x^5 - 5*a*b^3*x^3
+ 15*a^2*b^2*x)/b^5
```

Mupad [B]
time = 0.07, size = 43, normalized size = 0.78

$$\frac{x^5}{5\,b}-\frac{a\,x^3}{3\,b^2}+\frac{a^2\,x}{b^3}-\frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2),x)
```

```
[Out] x^5/(5*b) - (a*x^3)/(3*b^2) + (a^2*x)/b^3 - (a^(5/2)*atan((b^(1/2)*x)/a^(1/
2)))/b^(7/2)
```

3.130 $\int \frac{x^5}{a+bx^2} \, dx$

Optimal. Leaf size=40

$$-\frac{ax^2}{2b^2}+\frac{x^4}{4b}+\frac{a^2\log(a+bx^2)}{2b^3}$$

```
[Out] -1/2*a*x^2/b^2+1/4*x^4/b+1/2*a^2*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^2\log(a+bx^2)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2),x]
```

```
[Out] -1/2*(a*x^2)/b^2 + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx, x, x^2\right)\\ &= -\frac{ax^2}{2b^2}+\frac{x^4}{4b}+\frac{a^2\log(a+bx^2)}{2b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.00

$$-\frac{ax^2}{2b^2}+\frac{x^4}{4b}+\frac{a^2\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2),x]
[Out] -1/2*(a*x^2)/b^2 + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

Maple [A]
time = 0.04, size = 35, normalized size = 0.88

method	result	size
default	$-\frac{-\frac{1}{2}b\,x^4+a\,x^2}{2b^2}+\frac{a^2\ln\left(b\,x^2+a\right)}{2b^3}$	35
norman	$-\frac{a\,x^2}{2b^2}+\frac{x^4}{4b}+\frac{a^2\ln\left(b\,x^2+a\right)}{2b^3}$	35
risch	$\frac{x^4}{4b}-\frac{a\,x^2}{2b^2}+\frac{a^2}{4b^3}+\frac{a^2\ln\left(b\,x^2+a\right)}{2b^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/2/b^2*(-1/2*b*x^4+a*x^2)+1/2*a^2*ln(b*x^2+a)/b^3
```

Maxima [A]
time = 0.27, size = 34, normalized size = 0.85

$$\frac{a^2\log\left(bx^2+a\right)}{2\,b^3}+\frac{bx^4-2\,ax^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*a^2*log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

Fricas [A]
time = 0.98, size = 33, normalized size = 0.82

$$\frac{b^2x^4-2\,abx^2+2\,a^2\log\left(bx^2+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="fricas")
[Out] 1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

Sympy [A]
time = 0.05, size = 32, normalized size = 0.80

$$\frac{a^2 \log{(a + bx^2)}}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a),x)
```

```
[Out] a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

Giac [A]
time = 0.75, size = 35, normalized size = 0.88

$$\frac{a^2 \log(|bx^2 + a|)}{2\,b^3} + \frac{bx^4 - 2\,ax^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(b*x^2 + a))/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

Mupad [B]
time = 4.65, size = 33, normalized size = 0.82

$$\frac{2\,a^2\,\ln(b\,x^2 + a) + b^2\,x^4 - 2\,a\,b\,x^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2),x)
```

```
[Out] (2*a^2*log(a + b*x^2) + b^2*x^4 - 2*a*b*x^2)/(4*b^3)
```

3.131 $\int \frac{x^4}{a+bx^2}\,dx$

Optimal. Leaf size=42

$$-\frac{ax}{b^2}+\frac{x^3}{3b}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] -a*x/b^2+1/3*x^3/b+a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {308, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2),x]
```

```
[Out] -((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+bx^2}\,dx &= \int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^3}{3b}+\frac{a^2\int \frac{1}{a+bx^2}\,dx}{b^2}\\ &= -\frac{ax}{b^2}+\frac{x^3}{3b}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$-\frac{ax}{b^2}+\frac{x^3}{3b}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2),x]
[Out] -((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Maple [A]
time = 0.04, size = 38, normalized size = 0.90

method	result	size
default	$-\frac{-\frac{1}{3}b\,x^3+ax}{b^2}+\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	38
risch	$\frac{x^3}{3b}-\frac{ax}{b^2}+\frac{\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)}{2b^3}-\frac{\sqrt{-ab}\,a\ln\left(\sqrt{-ab}\,x+a\right)}{2b^3}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/3*b*x^3+a*x)+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.57, size = 37, normalized size = 0.88

$$\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{bx^3-3\,ax}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="maxima")
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*x^3 - 3*a*x)/b^2
```

Fricas [A]

time = 1.10, size = 99, normalized size = 2.36

$$\left[\frac{2\,bx^{3}+3\,a\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-6\,ax}{6\,b^{2}},\ \frac{bx^{3}+3\,a\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,ax}{3\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*a*
x)/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(36) = 72.
time = 0.06, size = 80, normalized size = 1.90

$$-\frac{ax}{b^{2}}-\frac{\sqrt{-\frac{a^{3}}{b^{5}}}\,\log\left(x-\frac{b^{2}\sqrt{-\frac{a^{3}}{b^{5}}}}{a}\right)}{2}+\frac{\sqrt{-\frac{a^{3}}{b^{5}}}\,\log\left(x+\frac{b^{2}\sqrt{-\frac{a^{3}}{b^{5}}}}{a}\right)}{2}+\frac{x^{3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a),x)
```

```
[Out] -a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 + sqrt(-a**
3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

Giac [A]
time = 0.60, size = 40, normalized size = 0.95

$$\frac{a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{b^{2}x^{3}-3\,abx}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3 - 3*a*b*x)/b^3
```

Mupad [B]
time = 0.07, size = 32, normalized size = 0.76

$$\frac{x^3}{3\,b}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2),x)
[Out] x^3/(3*b) + (a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(5/2) - (a*x)/b^2
```

3.132 $\int \frac{x^3}{a+bx^2} dx$

Optimal. Leaf size=27

$$\frac{x^2}{2b} - \frac{a\log(a+bx^2)}{2b^2}$$

[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{x^2}{2b} - \frac{a\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*x^2),x]

[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+bx^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{x^2}{2b} - \frac{a\log(a+bx^2)}{2b^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2b} - \frac{a\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.02, size = 24, normalized size = 0.89

method	result	size
default	$\frac{x^2}{2b} - \frac{a\ln(bx^2+a)}{2b^2}$	24
norman	$\frac{x^2}{2b} - \frac{a\ln(bx^2+a)}{2b^2}$	24
risch	$\frac{x^2}{2b} - \frac{a\ln(bx^2+a)}{2b^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

Maxima [A]
time = 0.34, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,b} - \frac{a\log(bx^2+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 1.28, size = 22, normalized size = 0.81

$$\frac{bx^2 - a\log(bx^2+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

Giac [A]
time = 0.56, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,b}-\frac{a\log\left(|bx^2+a|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.81

$$-\frac{a\,\ln\left(b\,x^2+a\right)-b\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2),x)
```

```
[Out] -(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

3.133 $\int \frac{x^2}{a+bx^2} \, dx$

Optimal. Leaf size=31

$$\frac{x}{b} - \frac{\sqrt{a}\, \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] x/b-arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {327, 211}

$$\frac{x}{b} - \frac{\sqrt{a}\, \text{ArcTan}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a+bx^2} \, dx &= \frac{x}{b} - \frac{a \int \frac{1}{a+bx^2} \, dx}{b} \\
&= \frac{x}{b} - \frac{\sqrt{a}\, \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{b}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2),x]
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Maple [A]
time = 0.03, size = 27, normalized size = 0.87

method	result	size
default	$\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	27
risch	$\frac{x}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^{2}}-\frac{\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{2b^{2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.55, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="maxima")
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Fricas [A]

time = 1.07, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},\,-\frac{\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.
time = 0.05, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}}\,\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\,\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

Giac [A]
time = 0.66, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.74

$$\frac{x}{b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2),x)
[Out] x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

3.134 $\int \frac{x}{a+bx^2} \, dx$

Optimal. Leaf size=15

$$\frac{\log(a+bx^2)}{2b}$$

```
[Out] 1/2*ln(b*x^2+a)/b
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {266}

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2),x]
[Out] Log[a + b*x^2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{a+bx^2} \, dx = \frac{\log(a+bx^2)}{2b}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2),x]
[Out] Log[a + b*x^2]/(2*b)
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\ln(b\,x^2+a)}{2b}$	14
default	$\frac{\ln(b\,x^2+a)}{2b}$	14
norman	$\frac{\ln(b\,x^2+a)}{2b}$	14
risch	$\frac{\ln(b\,x^2+a)}{2b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(b*x^2+a)/b
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

Fricas [A]
time = 1.45, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a),x)
```

```
[Out] log(a + b*x**2)/(2*b)
```

Giac [A]
time = 0.64, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b
```

Mupad [B]
time = 4.63, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2),x)
```

```
[Out] log(a + b*x^2)/(2*b)
```

3.135 $\int \frac{1}{a+bx^2} dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 1.04, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\,\frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.
time = 0.04, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Giac [**A**]
time = 0.54, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [**B**]
time = 4.70, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.136 $\int \frac{1}{x(a+bx^2)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

[Out] ln(x)/a-1/2*ln(b*x^2+a)/a

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {272, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^2)),x]

[Out] Log[x]/a - Log[a + b*x^2]/(2*a)

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^2\right)}{2a}$$
$$= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)),x]
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.95

method	result	size
default	$\frac{\ln(x)}{a} - \frac{\ln(b\,x^2+a)}{2a}$	21
norman	$\frac{\ln(x)}{a} - \frac{\ln(b\,x^2+a)}{2a}$	21
risch	$\frac{\ln(x)}{a} - \frac{\ln(b\,x^2+a)}{2a}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

Maxima [A]
time = 0.40, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^2+a\right)}{2\,a} + \frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + a)/a + 1/2*log(x^2)/a
```

Fricas [A]
time = 0.84, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^2+a\right)-2\,\log\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

Giac [A]
time = 0.49, size = 24, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,a}-\frac{\log\left(|bx^2+a|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

Mupad [B]
time = 0.08, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^2+a\right)-2\,\ln\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)),x)
```

```
[Out] -(log(a + b*x^2) - 2*log(x))/(2*a)
```

3.137 $\int \frac{1}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{ax}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -1/a/x-arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {331, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)}\,dx &= -\frac{1}{ax}-\frac{b\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 34, normalized size = 1.00

$$-\frac{1}{ax}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)),x]
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Maple [**A**]
time = 0.05, size = 30, normalized size = 0.88

method	result	size
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}$	30
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^3\_Z^2+b\right)}\_R\ln\left(\left(3\_R^2a^3+2b\right)x+a^2\_R\right)\right)}{2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x
```

Maxima [**A**]
time = 0.49, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="maxima")
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Fricas [**A**]

time = 0.71, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 0.06, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

Giac [A]
time = 0.50, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Mupad [B]
time = 4.62, size = 26, normalized size = 0.76

$$-\frac{1}{a\,x}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)),x)
```

```
[Out] - 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.138 $\int \frac{1}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 46}

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^2\right)}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)\,dx,x,x^2\right)\\&=-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx^2)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)),x]
[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Maple [A]
time = 0.04, size = 32, normalized size = 0.91

method	result	size
default	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln(b\,x^2+a)}{2a^2}$	32
norman	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln(b\,x^2+a)}{2a^2}$	32
risch	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln(-b\,x^2-a)}{2a^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.94

$$\frac{b\log(bx^2+a)}{2\,a^2}-\frac{b\log(x^2)}{2\,a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

Fricas [A]
time = 0.95, size = 33, normalized size = 0.94

$$\frac{bx^2\log(bx^2+a)-2\,bx^2\log(x)-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [A]
time = 0.11, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

Giac [A]
time = 0.49, size = 43, normalized size = 1.23

$$-\frac{b\log(x^2)}{2\,a^2} + \frac{b\log(|bx^2 + a|)}{2\,a^2} + \frac{bx^2 - a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*
x^2)
```

Mupad [B]
time = 0.07, size = 31, normalized size = 0.89

$$\frac{b\,\ln\left(b\,x^2 + a\right)}{2\,a^2} - \frac{1}{2\,a\,x^2} - \frac{b\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)),x)
```

```
[Out] (b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

3.139 $\int \frac{1}{x^4(a+bx^2)} dx$

Optimal. Leaf size=43

$$-\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -1/3/a/x^3+b/a^2/x+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {331, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)}\,dx &= -\frac{1}{3ax^3}-\frac{b\int \frac{1}{x^2(a+bx^2)}\,dx}{a}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a^2}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.00

$$-\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)),x]
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A]
time = 0.05, size = 39, normalized size = 0.91

method	result	size
default	$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}-\frac{1}{3a\,x^3}+\frac{b}{a^2x}$	39
risch	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{3a}}{x^3}+\frac{\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{2a^3}-\frac{\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{2a^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/3/a/x^3+b/a^2/x
```

Maxima [A]
time = 0.58, size = 40, normalized size = 0.93

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="maxima")
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Fricas [A]

time = 0.92, size = 106, normalized size = 2.47

$$\left[\frac{3\,bx^{3}\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+6\,bx^{2}-2\,a}{6\,a^{2}x^{3}},\ \frac{3\,bx^{3}\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^{2}-a}{3\,a^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(37) = 74$.
time = 0.08, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^{3}}{a^{5}}}\log\left(-\frac{a^{3}\sqrt{-\frac{b^{3}}{a^{5}}}}{b^{2}}+x\right)}{2}+\frac{\sqrt{-\frac{b^{3}}{a^{5}}}\log\left(\frac{a^{3}\sqrt{-\frac{b^{3}}{a^{5}}}}{b^{2}}+x\right)}{2}+\frac{-a+3bx^{2}}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

Giac [A]
time = 0.46, size = 40, normalized size = 0.93

$$\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{3\,bx^{2}-a}{3\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Mupad [B]
time = 4.67, size = 37, normalized size = 0.86

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{1}{3\,a}-\frac{b\,x^2}{a^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)),x)
[Out] (b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

3.140 $\int \frac{1}{x^5(a+bx^2)} dx$

Optimal. Leaf size=49

$$-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^2)}{2a^3}$$

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$-\frac{b^2\log(a+bx^2)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^2)}{2a^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^2)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)),x]
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

Maple [A]
time = 0.04, size = 44, normalized size = 0.90

method	result	size
default	$-\frac{1}{4a\,x^4}+\frac{b}{2a^2x^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(b\,x^2+a)}{2a^3}$	44
norman	$\frac{-\frac{1}{4a}+\frac{b\,x^2}{2a^2}}{x^4}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(b\,x^2+a)}{2a^3}$	46
risch	$\frac{-\frac{1}{4a}+\frac{b\,x^2}{2a^2}}{x^4}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(b\,x^2+a)}{2a^3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.96

$$-\frac{b^2\log(bx^2+a)}{2\,a^3}+\frac{b^2\log(x^2)}{2\,a^3}+\frac{2\,bx^2-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="maxima")
[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + 1/2*b^2*log(x^2)/a^3 + 1/4*(2*b*x^2 - a)/(a^2
*x^4)
```

Fricas [A]
time = 1.29, size = 45, normalized size = 0.92

$$-\frac{2\,b^2x^4\log(bx^2+a)-4\,b^2x^4\log(x)-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x
^4)
```

Sympy [A]
time = 0.13, size = 42, normalized size = 0.86

$$\frac{-a+2bx^2}{4a^2x^4}+\frac{b^2\log\left(x\right)}{a^3}-\frac{b^2\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a),x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*
a**3)
```

Giac [A]
time = 0.44, size = 57, normalized size = 1.16

$$\frac{b^2\log\left(x^2\right)}{2\,a^3}-\frac{b^2\log\left(|bx^2+a|\right)}{2\,a^3}-\frac{3\,b^2x^4-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2
*a*b*x^2 + a^2)/(a^3*x^4)
```

Mupad [B]
time = 0.08, size = 46, normalized size = 0.94

$$\frac{b^2\,\ln\left(x\right)}{a^3}-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,a^3}-\frac{\frac{1}{4\,a}-\frac{b\,x^2}{2\,a^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)),x)
```

```
[Out] (b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^2
))/x^4
```

3.141 $\int \frac{1}{x^6(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$-\frac{1}{5ax^5}+\frac{b}{3a^2x^3}-\frac{b^2}{a^3x}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -1/5/a/x^5+1/3*b/a^2/x^3-b^2/a^3/x-b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2
)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {331, 211}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}-\frac{b^2}{a^3x}+\frac{b}{3a^2x^3}-\frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/5*1/(a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6(a+bx^2)}\,dx = -\frac{1}{5ax^5} - \frac{b\int \frac{1}{x^4(a+bx^2)}\,dx}{a}$$
$$= -\frac{1}{5ax^5} + \frac{b}{3a^2x^3} + \frac{b^2\int \frac{1}{x^2(a+bx^2)}\,dx}{a^2}$$
$$= -\frac{1}{5ax^5} + \frac{b}{3a^2x^3} - \frac{b^2}{a^3x} - \frac{b^3\int \frac{1}{a+bx^2}\,dx}{a^3}$$
$$= -\frac{1}{5ax^5} + \frac{b}{3a^2x^3} - \frac{b^2}{a^3x} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 1.00

$$-\frac{1}{5ax^5} + \frac{b}{3a^2x^3} - \frac{b^2}{a^3x} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)),x]
[Out] -1/5*1/(a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(7/2)
```

Maple [A]
time = 0.05, size = 52, normalized size = 0.90

method	result	size
default	$-\frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^3\sqrt{ab}} - \frac{1}{5a\,x^5} - \frac{b^2}{a^3x} + \frac{b}{3a^2x^3}$	52
risch	$\frac{-\frac{b^2x^4}{a^3} + \frac{b\,x^2}{3a^2} - \frac{1}{5a}}{x^5} + \frac{\sqrt{-ab}\,b^2\ln\left(-bx+\sqrt{-ab}\right)}{2a^4} - \frac{\sqrt{-ab}\,b^2\ln\left(-bx-\sqrt{-ab}\right)}{2a^4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -b^3/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/5/a/x^5-b^2/a^3/x+1/3*b/a^2/
x^3
```

Maxima [A]
time = 0.53, size = 52, normalized size = 0.90

$$-\frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} - \frac{15\,b^2x^4 - 5\,abx^2 + 3\,a^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5*a*b*x^2 +
 3*a^2)/(a^3*x^5)
```

Fricas [A]
time = 1.18, size = 132, normalized size = 2.28

$$\left[\frac{15\, b^2 x^5 \sqrt{-\frac{b}{a}} \log\left(\frac{b x^2 - 2\, a x \sqrt{-\frac{b}{a}} - a}{b x^2 + a}\right) - 30\, b^2 x^4 + 10\, a b x^2 - 6\, a^2}{30\, a^3 x^5}, -\frac{15\, b^2 x^5 \sqrt{\frac{b}{a}} \arctan\left(x \sqrt{\frac{b}{a}}\right) + 15\, b^2 x^4 - 5\, a b x^2 + 3\, a^2}{15\, a^3 x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^2*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)
) - 30*b^2*x^4 + 10*a*b*x^2 - 6*a^2)/(a^3*x^5), -1/15*(15*b^2*x^5*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 15*b^2*x^4 - 5*a*b*x^2 + 3*a^2)/(a^3*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.
time = 0.10, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{b^5}{a^7}} \log\left(-\frac{a^4 \sqrt{-\frac{b^5}{a^7}}}{b^3} + x\right)}{2} - \frac{\sqrt{-\frac{b^5}{a^7}} \log\left(\frac{a^4 \sqrt{-\frac{b^5}{a^7}}}{b^3} + x\right)}{2} + \frac{-3a^2 + 5abx^2 - 15b^2x^4}{15a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**7)*log(-a**4*sqrt(-b**5/a**7)/b**3 + x)/2 - sqrt(-b**5/a**7)*
log(a**4*sqrt(-b**5/a**7)/b**3 + x)/2 + (-3*a**2 + 5*a*b*x**2 - 15*b**2*x**
4)/(15*a**3*x**5)
```

Giac [A]
time = 0.45, size = 52, normalized size = 0.90

$$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^3} - \frac{15\, b^2 x^4 - 5\, a b x^2 + 3\, a^2}{15\, a^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5*a*b*x^2 +
 3*a^2)/(a^3*x^5)
```

Mupad [B]
time = 0.06, size = 48, normalized size = 0.83

$$-\frac{\frac{1}{5\,a}-\frac{b\,x^2}{3\,a^2}+\frac{b^2\,x^4}{a^3}}{x^5}-\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)),x)
```

```
[Out] - (1/(5*a) - (b*x^2)/(3*a^2) + (b^2*x^4)/a^3)/x^5 - (b^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/a^(7/2)
```

3.142 $\int \frac{1}{x^7(a+bx^2)}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{6ax^6}+\frac{b}{4a^2x^4}-\frac{b^2}{2a^3x^2}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx^2)}{2a^4}$$

```
[Out] -1/6/a/x^6+1/4*b/a^2/x^4-1/2*b^2/a^3/x^2-b^3*ln(x)/a^4+1/2*b^3*ln(b*x^2+a)/
a^4
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 46}

$$\frac{b^3\log(a+bx^2)}{2a^4}-\frac{b^3\log(x)}{a^4}-\frac{b^2}{2a^3x^2}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/6*1/(a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3*
Log[a + b*x^2])/(2*a^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7(a+bx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{6ax^6}+\frac{b}{4a^2x^4}-\frac{b^2}{2a^3x^2}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx^2)}{2a^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 1.00

$$-\frac{1}{6ax^6}+\frac{b}{4a^2x^4}-\frac{b^2}{2a^3x^2}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx^2)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/6*1/(a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3*
Log[a + b*x^2])/(2*a^4)
```

Maple [A]
time = 0.04, size = 56, normalized size = 0.89

method	result	size
default	$-\frac{1}{6a\,x^6}+\frac{b}{4a^2x^4}-\frac{b^2}{2a^3x^2}-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln(b\,x^2+a)}{2a^4}$	56
norman	$\frac{-\frac{1}{6a}+\frac{b\,x^2}{4a^2}-\frac{b^2x^4}{2a^3}}{x^6}-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln(b\,x^2+a)}{2a^4}$	58
risch	$\frac{-\frac{1}{6a}+\frac{b\,x^2}{4a^2}-\frac{b^2x^4}{2a^3}}{x^6}-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln(-b\,x^2-a)}{2a^4}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/x^6+1/4*b/a^2/x^4-1/2*b^2/a^3/x^2-b^3*ln(x)/a^4+1/2*b^3*ln(b*x^2+a)/
a^4
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.92

$$\frac{b^3\log(bx^2+a)}{2\,a^4}-\frac{b^3\log(x^2)}{2\,a^4}-\frac{6\,b^2x^4-3\,abx^2+2\,a^2}{12\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(b*x^2 + a)/a^4 - 1/2*b^3*log(x^2)/a^4 - 1/12*(6*b^2*x^4 - 3*a*b
*x^2 + 2*a^2)/(a^3*x^6)
```

Fricas [A]
time = 1.16, size = 58, normalized size = 0.92

$$\frac{6\,b^3x^6\log(bx^2+a)-12\,b^3x^6\log(x)-6\,ab^2x^4+3\,a^2bx^2-2\,a^3}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^3*x^6*log(b*x^2 + a) - 12*b^3*x^6*log(x) - 6*a*b^2*x^4 + 3*a^2*b*
x^2 - 2*a^3)/(a^4*x^6)
```

Sympy [A]
time = 0.15, size = 56, normalized size = 0.89

$$\frac{-2a^2 + 3abx^2 - 6b^2x^4}{12a^3x^6} - \frac{b^3 \log(x)}{a^4} + \frac{b^3 \log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a),x)
```

```
[Out] (-2*a**2 + 3*a*b*x**2 - 6*b**2*x**4)/(12*a**3*x**6) - b**3*log(x)/a**4 + b*
*3*log(a/b + x**2)/(2*a**4)
```

Giac [A]
time = 0.41, size = 70, normalized size = 1.11

$$-\frac{b^3 \log(x^2)}{2\,a^4} + \frac{b^3 \log(|bx^2 + a|)}{2\,a^4} + \frac{11\,b^3x^6 - 6\,ab^2x^4 + 3\,a^2bx^2 - 2\,a^3}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(x^2)/a^4 + 1/2*b^3*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6
 - 6*a*b^2*x^4 + 3*a^2*b*x^2 - 2*a^3)/(a^4*x^6)
```

Mupad [B]
time = 4.64, size = 58, normalized size = 0.92

$$\frac{b^3\,\ln\left(b\,x^2 + a\right)}{2\,a^4} - \frac{\frac{1}{6\,a} - \frac{b\,x^2}{4\,a^2} + \frac{b^2\,x^4}{2\,a^3}}{x^6} - \frac{b^3\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)),x)
```

```
[Out] (b^3*log(a + b*x^2))/(2*a^4) - (1/(6*a) - (b*x^2)/(4*a^2) + (b^2*x^4)/(2*a^
3))/x^6 - (b^3*log(x))/a^4
```

3.143 $\int \frac{1}{x^8(a+bx^2)}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[Out] -1/7/a/x^7+1/5*b/a^2/x^5-1/3*b^2/a^3/x^3+b^3/a^4/x+b^(7/2)*arctan(x*b^(1/2)
/a^(1/2))/a^(9/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {331, 211}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b^3}{a^4x}-\frac{b^2}{3a^3x^3}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*1/(a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^8(a+bx^2)}\,dx &= -\frac{1}{7ax^7}-\frac{b\int \frac{1}{x^6(a+bx^2)}\,dx}{a}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}+\frac{b^2\int \frac{1}{x^4(a+bx^2)}\,dx}{a^2}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}-\frac{b^3\int \frac{1}{x^2(a+bx^2)}\,dx}{a^3}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^4\int \frac{1}{a+bx^2}\,dx}{a^4}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 69, normalized size = 1.00

$$
-\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*1/(a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Maple [A]
time = 0.04, size = 61, normalized size = 0.88

method	result	size
default	$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}-\frac{1}{7a\,x^7}-\frac{b^2}{3a^3x^3}+\frac{b}{5a^2x^5}+\frac{b^3}{a^4x}$	61
risch	$\frac{\frac{b^3x^6}{a^4}-\frac{b^2x^4}{3a^3}+\frac{b\,x^2}{5a^2}-\frac{1}{7a}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^2+b^7\right)}\_R\ln\left(\left(3\_R^2a^9+2b^7\right)x-a^5b^3\_R\right)\right)}{2}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] b^4/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/7/a/x^7-1/3*b^2/a^3/x^3+1/5*b
/a^2/x^5+b^3/a^4/x
```

Maxima [A]
time = 0.53, size = 62, normalized size = 0.90

$$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^4} + \frac{105\, b^3 x^6 - 35\, a b^2 x^4 + 21\, a^2 b x^2 - 15\, a^3}{105\, a^4 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 - 35*a*b^2*x
^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```

Fricas [A]
time = 1.59, size = 154, normalized size = 2.23

$$\left[\frac{105\, b^3 x^7 \sqrt{-\frac{b}{a}} \log\left(\frac{b x^2 + 2\, a x \sqrt{-\frac{b}{a}} - a}{b x^2 + a}\right) + 210\, b^3 x^6 - 70\, a b^2 x^4 + 42\, a^2 b x^2 - 30\, a^3}{210\, a^4 x^7}, \frac{105\, b^3 x^7 \sqrt{\frac{b}{a}} \arctan\left(x \sqrt{\frac{b}{a}}\right) + 105\, b^3 x^6 - 35\, a b^2 x^4 + 21\, a^2 b x^2 - 15\, a^3}{105\, a^4 x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(105*b^3*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
a)) + 210*b^3*x^6 - 70*a*b^2*x^4 + 42*a^2*b*x^2 - 30*a^3)/(a^4*x^7), 1/105*
(105*b^3*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*b^3*x^6 - 35*a*b^2*x^4 + 2
1*a^2*b*x^2 - 15*a^3)/(a^4*x^7)]
```

Sympy [A]
time = 0.12, size = 112, normalized size = 1.62

$$-\frac{\sqrt{-\frac{b^7}{a^9}} \log\left(-\frac{a^5 \sqrt{-\frac{b^7}{a^9}}}{b^4} + x\right)}{2} + \frac{\sqrt{-\frac{b^7}{a^9}} \log\left(\frac{a^5 \sqrt{-\frac{b^7}{a^9}}}{b^4} + x\right)}{2} + \frac{-15a^3 + 21a^2bx^2 - 35ab^2x^4 + 105b^3x^6}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**7/a**9)*log(-a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + sqrt(-b**7/a**9)
*log(a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + (-15*a**3 + 21*a**2*b*x**2 - 35*a*
b**2*x**4 + 105*b**3*x**6)/(105*a**4*x**7)
```

Giac [A]
time = 0.49, size = 62, normalized size = 0.90

$$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^4} + \frac{105\, b^3x^6 - 35\, ab^2x^4 + 21\, a^2bx^2 - 15\, a^3}{105\, a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 - 35*a*b^2*x
^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```

Mupad [B]
time = 0.06, size = 59, normalized size = 0.86

$$\frac{b^{7/2}\, \mathrm{atan}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{a^{9/2}} - \frac{\frac{1}{7\,a} - \frac{b\,x^2}{5\,a^2} + \frac{b^2\,x^4}{3\,a^3} - \frac{b^3\,x^6}{a^4}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^2)),x)
```

```
[Out] (b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(9/2) - (1/(7*a) - (b*x^2)/(5*a^2) +
(b^2*x^4)/(3*a^3) - (b^3*x^6)/a^4)/x^7
```

3.144 $\int \frac{1}{x^9(a+bx^2)}\,dx$

Optimal. Leaf size=75

$$-\frac{1}{8ax^8}+\frac{b}{6a^2x^6}-\frac{b^2}{4a^3x^4}+\frac{b^3}{2a^4x^2}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log\left(a+bx^2\right)}{2a^5}$$

```
[Out] -1/8/a/x^8+1/6*b/a^2/x^6-1/4*b^2/a^3/x^4+1/2*b^3/a^4/x^2+b^4*ln(x)/a^5-1/2*
b^4*ln(b*x^2+a)/a^5
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 46}

$$-\frac{b^4\log\left(a+bx^2\right)}{2a^5}+\frac{b^4\log(x)}{a^5}+\frac{b^3}{2a^4x^2}-\frac{b^2}{4a^3x^4}+\frac{b}{6a^2x^6}-\frac{1}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)),x]
```

```
[Out] -1/8*1/(a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*L
og[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^5(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^5}-\frac{b}{a^2x^4}+\frac{b^2}{a^3x^3}-\frac{b^3}{a^4x^2}+\frac{b^4}{a^5x}-\frac{b^5}{a^5(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{8ax^8}+\frac{b}{6a^2x^6}-\frac{b^2}{4a^3x^4}+\frac{b^3}{2a^4x^2}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log\left(a+bx^2\right)}{2a^5}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 75, normalized size = 1.00

$$-\frac{1}{8ax^8}+\frac{b}{6a^2x^6}-\frac{b^2}{4a^3x^4}+\frac{b^3}{2a^4x^2}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log\left(a+bx^2\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)),x]
```

```
[Out] -1/8*1/(a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*L
og[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

Maple [A]
time = 0.04, size = 66, normalized size = 0.88

method	result	size
default	$-\frac{1}{8a\,x^8}+\frac{b}{6a^2x^6}-\frac{b^2}{4a^3x^4}+\frac{b^3}{2a^4x^2}+\frac{b^4\ln(x)}{a^5}-\frac{b^4\ln\left(b\,x^2+a\right)}{2a^5}$	66
norman	$\frac{-\frac{1}{8a}+\frac{b\,x^2}{6a^2}-\frac{b^2x^4}{4a^3}+\frac{b^3x^6}{2a^4}}{x^8}+\frac{b^4\ln(x)}{a^5}-\frac{b^4\ln\left(b\,x^2+a\right)}{2a^5}$	68
risch	$\frac{-\frac{1}{8a}+\frac{b\,x^2}{6a^2}-\frac{b^2x^4}{4a^3}+\frac{b^3x^6}{2a^4}}{x^8}+\frac{b^4\ln(x)}{a^5}-\frac{b^4\ln\left(b\,x^2+a\right)}{2a^5}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a/x^8+1/6*b/a^2/x^6-1/4*b^2/a^3/x^4+1/2*b^3/a^4/x^2+b^4*ln(x)/a^5-1/2*
b^4*ln(b*x^2+a)/a^5
```

Maxima [A]
time = 0.35, size = 69, normalized size = 0.92

$$-\frac{b^4\log\left(bx^2+a\right)}{2\,a^5}+\frac{b^4\log\left(x^2\right)}{2\,a^5}+\frac{12\,b^3x^6-6\,ab^2x^4+4\,a^2bx^2-3\,a^3}{24\,a^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b^4*log(b*x^2 + a)/a^5 + 1/2*b^4*log(x^2)/a^5 + 1/24*(12*b^3*x^6 - 6*a
*b^2*x^4 + 4*a^2*b*x^2 - 3*a^3)/(a^4*x^8)
```

Fricas [A]
time = 1.23, size = 69, normalized size = 0.92

$$-\frac{12\,b^4x^8\log\left(bx^2+a\right)-24\,b^4x^8\log\left(x\right)-12\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+3\,a^4}{24\,a^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/24*(12*b^4*x^8*log(b*x^2 + a) - 24*b^4*x^8*log(x) - 12*a*b^3*x^6 + 6*a^2
*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

Sympy [A]
time = 0.16, size = 68, normalized size = 0.91

$$\frac{-3a^3 + 4a^2bx^2 - 6ab^2x^4 + 12b^3x^6}{24a^4x^8} + \frac{b^4 \log(x)}{a^5} - \frac{b^4 \log\left(\frac{a}{b} + x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a),x)
```

```
[Out] (-3*a**3 + 4*a**2*b*x**2 - 6*a*b**2*x**4 + 12*b**3*x**6)/(24*a**4*x**8) + b
**4*log(x)/a**5 - b**4*log(a/b + x**2)/(2*a**5)
```

Giac [A]
time = 0.47, size = 81, normalized size = 1.08

$$\frac{b^4 \log(x^2)}{2\,a^5} - \frac{b^4 \log(|bx^2 + a|)}{2\,a^5} - \frac{25\,b^4x^8 - 12\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + 3\,a^4}{24\,a^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(x^2)/a^5 - 1/2*b^4*log(abs(b*x^2 + a))/a^5 - 1/24*(25*b^4*x^8 -
 12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

Mupad [B]
time = 4.67, size = 68, normalized size = 0.91

$$\frac{b^4\,\ln(x)}{a^5} - \frac{b^4\,\ln(b\,x^2 + a)}{2\,a^5} - \frac{\frac{1}{8\,a} - \frac{b\,x^2}{6\,a^2} + \frac{b^2\,x^4}{4\,a^3} - \frac{b^3\,x^6}{2\,a^4}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^2)),x)
```

```
[Out] (b^4*log(x))/a^5 - (b^4*log(a + b*x^2))/(2*a^5) - (1/(8*a) - (b*x^2)/(6*a^2
) + (b^2*x^4)/(4*a^3) - (b^3*x^6)/(2*a^4))/x^8
```

3.145 $\int \frac{x^{13}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=94

$$\frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}-\frac{a^6}{2b^7(a+bx^2)}-\frac{3a^5\log(a+bx^2)}{b^7}$$

```
[Out] 5/2*a^4*x^2/b^6-a^3*x^4/b^5+1/2*a^2*x^6/b^4-1/4*a*x^8/b^3+1/10*x^10/b^2-1/2
*a^6/b^7/(b*x^2+a)-3*a^5*ln(b*x^2+a)/b^7
```

Rubi [A]

time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$-\frac{a^6}{2b^7(a+bx^2)}-\frac{3a^5\log(a+bx^2)}{b^7}+\frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^2,x]
```

```
[Out] (5*a^4*x^2)/(2*b^6) - (a^3*x^4)/b^5 + (a^2*x^6)/(2*b^4) - (a*x^8)/(4*b^3) +
 x^10/(10*b^2) - a^6/(2*b^7*(a + b*x^2)) - (3*a^5*Log[a + b*x^2])/b^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{13}}{(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{5a^4}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{b^4}-\frac{2ax^3}{b^3}+\frac{x^4}{b^2}+\frac{a^6}{b^6(a+bx)^2}-\frac{6a^5}{b^6(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}-\frac{a^6}{2b^7(a+bx^2)}-\frac{3a^5\log(a+bx^2)}{b^7}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 83, normalized size = 0.88

$$\frac{50a^4bx^2-20a^3b^2x^4+10a^2b^3x^6-5ab^4x^8+2b^5x^{10}-\frac{10a^6}{a+bx^2}-60a^5\log(a+bx^2)}{20b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^2,x]
```

```
[Out] (50*a^4*b*x^2 - 20*a^3*b^2*x^4 + 10*a^2*b^3*x^6 - 5*a*b^4*x^8 + 2*b^5*x^10
- (10*a^6)/(a + b*x^2) - 60*a^5*Log[a + b*x^2])/(20*b^7)
```

Maple [**A**]
time = 0.04, size = 88, normalized size = 0.94

method	result	size
risch	$\frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{a\,x^8}{4b^3}+\frac{x^{10}}{10b^2}-\frac{a^6}{2b^7(b\,x^2+a)}-\frac{3a^5\ln(b\,x^2+a)}{b^7}$	85
norman	$\frac{\frac{x^{12}}{10b}-\frac{3a\,x^{10}}{20b^2}-\frac{3a^6}{b^7}+\frac{a^2x^8}{4b^3}-\frac{a^3x^6}{2b^4}+\frac{3a^4x^4}{2b^5}}{b\,x^2+a}-\frac{3a^5\ln(b\,x^2+a)}{b^7}$	87
default	$\frac{\frac{1}{10}b^4x^{10}-\frac{1}{4}a\,b^3x^8+\frac{1}{2}a^2b^2x^6-a^3b\,x^4+\frac{5}{2}a^4x^2}{b^6}-\frac{a^5\left(\frac{a}{b(b\,x^2+a)}+\frac{6\ln(b\,x^2+a)}{b}\right)}{2b^6}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/10*b^4*x^10-1/4*a*b^3*x^8+1/2*a^2*b^2*x^6-a^3*b*x^4+5/2*a^4*x^2)-1
/2*a^5/b^6*(a/b/(b*x^2+a)+6*ln(b*x^2+a)/b)
```

Maxima [**A**]
time = 0.38, size = 88, normalized size = 0.94

$$-\frac{a^6}{2\,(b^8x^2+ab^7)}-\frac{3\,a^5\log(bx^2+a)}{b^7}+\frac{2\,b^4x^{10}-5\,ab^3x^8+10\,a^2b^2x^6-20\,a^3bx^4+50\,a^4x^2}{20\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^6/(b^8*x^2 + a*b^7) - 3*a^5*log(b*x^2 + a)/b^7 + 1/20*(2*b^4*x^10 -
5*a*b^3*x^8 + 10*a^2*b^2*x^6 - 20*a^3*b*x^4 + 50*a^4*x^2)/b^6
```

Fricas [**A**]
time = 1.32, size = 104, normalized size = 1.11

$$\frac{2\,b^6x^{12}-3\,ab^5x^{10}+5\,a^2b^4x^8-10\,a^3b^3x^6+30\,a^4b^2x^4+50\,a^5bx^2-10\,a^6-60\,(a^5bx^2+a^6)\log(bx^2+a)}{20\,(b^8x^2+ab^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^6*x^12 - 3*a*b^5*x^10 + 5*a^2*b^4*x^8 - 10*a^3*b^3*x^6 + 30*a^4*b
^2*x^4 + 50*a^5*b*x^2 - 10*a^6 - 60*(a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^8*
x^2 + a*b^7)
```

Sympy [A]
time = 0.12, size = 88, normalized size = 0.94

$$-\frac{a^6}{2ab^7+2b^8x^2}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}+\frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**2,x)
```

```
[Out] -a**6/(2*a*b**7 + 2*b**8*x**2) - 3*a**5*log(a + b*x**2)/b**7 + 5*a**4*x**2/
(2*b**6) - a**3*x**4/b**5 + a**2*x**6/(2*b**4) - a*x**8/(4*b**3) + x**10/(1
0*b**2)
```

Giac [A]
time = 0.53, size = 103, normalized size = 1.10

$$-\frac{3\,a^5\log\left(|bx^2+a|\right)}{b^7}+\frac{6\,a^5bx^2+5\,a^6}{2\,(bx^2+a)b^7}+\frac{2\,b^8x^{10}-5\,ab^7x^8+10\,a^2b^6x^6-20\,a^3b^5x^4+50\,a^4b^4x^2}{20\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3*a^5*log(abs(b*x^2 + a))/b^7 + 1/2*(6*a^5*b*x^2 + 5*a^6)/((b*x^2 + a)*b^7
) + 1/20*(2*b^8*x^10 - 5*a*b^7*x^8 + 10*a^2*b^6*x^6 - 20*a^3*b^5*x^4 + 50*a
^4*b^4*x^2)/b^10
```

Mupad [B]
time = 0.09, size = 90, normalized size = 0.96

$$\frac{x^{10}}{10\,b^2}-\frac{a^6}{2\,b\,\left(b^7\,x^2+a\,b^6\right)}-\frac{a\,x^8}{4\,b^3}-\frac{3\,a^5\,\ln\left(b\,x^2+a\right)}{b^7}+\frac{a^2\,x^6}{2\,b^4}-\frac{a^3\,x^4}{b^5}+\frac{5\,a^4\,x^2}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^2)^2,x)
```

```
[Out] x^10/(10*b^2) - a^6/(2*b*(a*b^6 + b^7*x^2)) - (a*x^8)/(4*b^3) - (3*a^5*log(
a + b*x^2))/b^7 + (a^2*x^6)/(2*b^4) - (a^3*x^4)/b^5 + (5*a^4*x^2)/(2*b^6)
```

3.146 $\int \frac{x^{12}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=105

$$\frac{11a^4x}{2b^6}-\frac{11a^3x^3}{6b^5}+\frac{11a^2x^5}{10b^4}-\frac{11ax^7}{14b^3}+\frac{11x^9}{18b^2}-\frac{x^{11}}{2b\,(a+bx^2)}-\frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}$$

```
[Out] 11/2*a^4*x/b^6-11/6*a^3*x^3/b^5+11/10*a^2*x^5/b^4-11/14*a*x^7/b^3+11/18*x^9
/b^2-1/2*x^11/b/(b*x^2+a)-11/2*a^(9/2)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)
```

Rubi [A]
time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 308, 211}

$$-\frac{11a^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}+\frac{11a^4x}{2b^6}-\frac{11a^3x^3}{6b^5}+\frac{11a^2x^5}{10b^4}-\frac{11ax^7}{14b^3}-\frac{x^{11}}{2b\,(a+bx^2)}+\frac{11x^9}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^2,x]
```

```
[Out] (11*a^4*x)/(2*b^6) - (11*a^3*x^3)/(6*b^5) + (11*a^2*x^5)/(10*b^4) - (11*a*x
^7)/(14*b^3) + (11*x^9)/(18*b^2) - x^11/(2*b*(a + b*x^2)) - (11*a^(9/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}}{(a+bx^2)^2}\,dx &= -\frac{x^{11}}{2b(a+bx^2)} + \frac{11\int \frac{x^{10}}{a+bx^2}\,dx}{2b} \\
&= -\frac{x^{11}}{2b(a+bx^2)} + \frac{11\int \left(\frac{a^4}{b^5} - \frac{a^3x^2}{b^4} + \frac{a^2x^4}{b^3} - \frac{ax^6}{b^2} + \frac{x^8}{b} - \frac{a^5}{b^5(a+bx^2)}\right)dx}{2b} \\
&= \frac{11a^4x}{2b^6} - \frac{11a^3x^3}{6b^5} + \frac{11a^2x^5}{10b^4} - \frac{11ax^7}{14b^3} + \frac{11x^9}{18b^2} - \frac{x^{11}}{2b(a+bx^2)} - \frac{(11a^5)\int \frac{1}{a+bx^2}\,dx}{2b^6} \\
&= \frac{11a^4x}{2b^6} - \frac{11a^3x^3}{6b^5} + \frac{11a^2x^5}{10b^4} - \frac{11ax^7}{14b^3} + \frac{11x^9}{18b^2} - \frac{x^{11}}{2b(a+bx^2)} - \frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 0.89

$$\frac{x\left(3150a^4 - 840a^3bx^2 + 378a^2b^2x^4 - 180ab^3x^6 + 70b^4x^8 + \frac{315a^5}{a+bx^2}\right)}{630b^6} - \frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^2,x]
```

```
[Out] (x*(3150*a^4 - 840*a^3*b*x^2 + 378*a^2*b^2*x^4 - 180*a*b^3*x^6 + 70*b^4*x^8
 + (315*a^5)/(a + b*x^2)))/(630*b^6) - (11*a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(13/2))
```

Maple [A]
time = 0.05, size = 87, normalized size = 0.83

method	result
default	$\frac{\frac{1}{9}b^4x^9 - \frac{2}{7}a\,b^3x^7 + \frac{3}{5}a^2b^2x^5 - \frac{4}{3}a^3b\,x^3 + 5a^4x}{b^6} - \frac{a^5\left(-\frac{x}{2\left(b\,x^2+a\right)} + \frac{11\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^6}$
risch	$\frac{x^9}{9b^2} - \frac{2a\,x^7}{7b^3} + \frac{3a^2x^5}{5b^4} - \frac{4a^3x^3}{3b^5} + \frac{5a^4x}{b^6} + \frac{a^5x}{2b^6(b\,x^2+a)} + \frac{11\sqrt{-ab}\,a^4\ln\left(-\sqrt{-ab}\,x-a\right)}{4b^7} - \frac{11\sqrt{-ab}\,a^4\ln\left(\sqrt{-a\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/b^6*(1/9*b^4*x^9-2/7*a*b^3*x^7+3/5*a^2*b^2*x^5-4/3*a^3*b*x^3+5*a^4*x)-a^5
/b^6*(-1/2*x/(b*x^2+a)+11/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))

Maxima [A]
time = 0.49, size = 93, normalized size = 0.89

$$\frac{a^5x}{2\,(b^7x^2+ab^6)}-\frac{11\,a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^6}+\frac{35\,b^4x^9-90\,ab^3x^7+189\,a^2b^2x^5-420\,a^3bx^3+1575\,a^4x}{315\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*a^5*x/(b^7*x^2 + a*b^6) - 11/2*a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6
) + 1/315*(35*b^4*x^9 - 90*a*b^3*x^7 + 189*a^2*b^2*x^5 - 420*a^3*b*x^3 + 15
75*a^4*x)/b^6

Fricas [A]
time = 1.18, size = 234, normalized size = 2.23

$$\left[\frac{140\,b^5x^{11}-220\,ab^4x^9+396\,a^2b^3x^7-924\,a^3b^2x^5+4620\,a^4bx^3+6930\,a^5x+3465\,(a^4bx^2+a^5)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{1260\,(b^7x^2+ab^6)},\ \frac{70\,b^5x^{11}-110\,ab^4x^9+198\,a^2b^3x^7-462\,a^3b^2x^5+2310\,a^4bx^3+3465\,a^5x-3465\,(a^4bx^2+a^5)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{630\,(b^7x^2+ab^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/1260*(140*b^5*x^11 - 220*a*b^4*x^9 + 396*a^2*b^3*x^7 - 924*a^3*b^2*x^5 +
 4620*a^4*b*x^3 + 6930*a^5*x + 3465*(a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2
 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^2 + a*b^6), 1/630*(70*b^5*x^1
1 - 110*a*b^4*x^9 + 198*a^2*b^3*x^7 - 462*a^3*b^2*x^5 + 2310*a^4*b*x^3 + 34
65*a^5*x - 3465*(a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x
^2 + a*b^6)]

Sympy [A]
time = 0.14, size = 151, normalized size = 1.44

$$\frac{a^5x}{2ab^6+2b^7x^2}+\frac{5a^4x}{b^6}-\frac{4a^3x^3}{3b^5}+\frac{3a^2x^5}{5b^4}-\frac{2ax^7}{7b^3}+\frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x-\frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4}-\frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x+\frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4}+\frac{x^9}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**12/(b*x**2+a)**2,x)

[Out] a**5*x/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*x/b**6 - 4*a**3*x**3/(3*b**5) + 3*
a**2*x**5/(5*b**4) - 2*a*x**7/(7*b**3) + 11*sqrt(-a**9/b**13)*log(x - b**6*

```
sqrt(-a**9/b**13)/a**4)/4 - 11*sqrt(-a**9/b**13)*log(x + b**6*sqrt(-a**9/b*
*13)/a**4)/4 + x**9/(9*b**2)
```

Giac [A]
time = 0.49, size = 95, normalized size = 0.90

$$-\frac{11\,a^{5}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{6}}+\frac{a^{5}x}{2\,(bx^{2}+a)b^{6}}+\frac{35\,b^{16}x^{9}-90\,ab^{15}x^{7}+189\,a^{2}b^{14}x^{5}-420\,a^{3}b^{13}x^{3}+1575\,a^{4}b^{12}x}{315\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -11/2*a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/2*a^5*x/((b*x^2 + a)*b^
6) + 1/315*(35*b^16*x^9 - 90*a*b^15*x^7 + 189*a^2*b^14*x^5 - 420*a^3*b^13*x
^3 + 1575*a^4*b^12*x)/b^18
```

Mupad [B]
time = 0.07, size = 88, normalized size = 0.84

$$\frac{x^{9}}{9\,b^{2}}-\frac{2\,a\,x^{7}}{7\,b^{3}}+\frac{5\,a^{4}\,x}{b^{6}}-\frac{11\,a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{13/2}}+\frac{3\,a^{2}\,x^{5}}{5\,b^{4}}-\frac{4\,a^{3}\,x^{3}}{3\,b^{5}}+\frac{a^{5}\,x}{2\,\left(b^{7}\,x^{2}+a\,b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^2)^2,x)
```

```
[Out] x^9/(9*b^2) - (2*a*x^7)/(7*b^3) + (5*a^4*x)/b^6 - (11*a^(9/2)*atan((b^(1/2)
*x)/a^(1/2)))/(2*b^(13/2)) + (3*a^2*x^5)/(5*b^4) - (4*a^3*x^3)/(3*b^5) + (a
^5*x)/(2*(a*b^6 + b^7*x^2))
```

3.147 $\int \frac{x^{11}}{(a+bx^2)^2} dx$

Optimal. Leaf size=83

$$-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6\,(a+bx^2)}+\frac{5a^4\log\,(a+bx^2)}{2b^6}$$

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^5}{2b^6\,(a+bx^2)}+\frac{5a^4\log\,(a+bx^2)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^2,x]
```

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6\,(a+bx^2)}+\frac{5a^4\log\,(a+bx^2)}{2b^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 72, normalized size = 0.87

$$\frac{-48a^3bx^2 + 18a^2b^2x^4 - 8ab^3x^6 + 3b^4x^8 + \frac{12a^5}{a+bx^2} + 60a^4\log(a+bx^2)}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^2,x]
```

```
[Out] (-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a + b
*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

Maple [A]
time = 0.04, size = 78, normalized size = 0.94

method	result	size
risch	$-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{a\,x^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6(b\,x^2+a)}+\frac{5a^4\ln(b\,x^2+a)}{2b^6}$	74
norman	$\frac{\frac{x^{10}}{8b}-\frac{5a\,x^8}{24b^2}+\frac{5a^5}{2b^6}+\frac{5a^2x^6}{12b^3}-\frac{5a^3x^4}{4b^4}}{b\,x^2+a}+\frac{5a^4\ln(b\,x^2+a)}{2b^6}$	76
default	$-\frac{-\frac{1}{8}b^3x^8+\frac{1}{3}a\,b^2x^6-\frac{3}{4}a^2b\,x^4+2a^3x^2}{b^5}+\frac{a^4\left(\frac{a}{b(b\,x^2+a)}+\frac{5\ln(b\,x^2+a)}{b}\right)}{2b^5}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/8*b^3*x^8+1/3*a*b^2*x^6-3/4*a^2*b*x^4+2*a^3*x^2)+1/2*a^4/b^5*(a/
b/(b*x^2+a)+5*ln(b*x^2+a)/b)
```

Maxima [A]
time = 0.27, size = 77, normalized size = 0.93

$$\frac{a^5}{2\,(b^7x^2+ab^6)} + \frac{5\,a^4\log(bx^2+a)}{2\,b^6} + \frac{3\,b^3x^8 - 8\,ab^2x^6 + 18\,a^2bx^4 - 48\,a^3x^2}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

Fricas [A]
time = 1.46, size = 93, normalized size = 1.12

$$\frac{3\,b^5x^{10} - 5\,ab^4x^8 + 10\,a^2b^3x^6 - 30\,a^3b^2x^4 - 48\,a^4bx^2 + 12\,a^5 + 60\,(a^4bx^2 + a^5)\log(bx^2+a)}{24\,(b^7x^2+ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*b
*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

Sympy [A]
time = 0.12, size = 80, normalized size = 0.96

$$\frac{a^5}{2ab^6 + 2b^7x^2} + \frac{5a^4\log\left(a + bx^2\right)}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**2,x)
```

```
[Out] a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x*
*2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

Giac [A]
time = 1.00, size = 92, normalized size = 1.11

$$\frac{5\,a^4\log\left(|bx^2 + a|\right)}{2\,b^6} - \frac{5\,a^4bx^2 + 4\,a^5}{2\,(bx^2 + a)b^6} + \frac{3\,b^6x^8 - 8\,ab^5x^6 + 18\,a^2b^4x^4 - 48\,a^3b^3x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b^
6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

Mupad [B]
time = 4.48, size = 79, normalized size = 0.95

$$\frac{x^8}{8\,b^2} + \frac{a^5}{2\,b\,\left(b^6\,x^2 + a\,b^5\right)} - \frac{a\,x^6}{3\,b^3} + \frac{5\,a^4\,\ln\left(b\,x^2 + a\right)}{2\,b^6} + \frac{3\,a^2\,x^4}{4\,b^4} - \frac{2\,a^3\,x^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2)^2,x)
```

```
[Out] x^8/(8*b^2) + a^5/(2*b*(a*b^5 + b^6*x^2)) - (a*x^6)/(3*b^3) + (5*a^4*log(a
+ b*x^2))/(2*b^6) + (3*a^2*x^4)/(4*b^4) - (2*a^3*x^2)/b^5
```

3.148 $\int \frac{x^{10}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=92

$$-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

```
[Out] -9/2*a^3*x/b^5+3/2*a^2*x^3/b^4-9/10*a*x^5/b^3+9/14*x^7/b^2-1/2*x^9/b/(b*x^2
+a)+9/2*a^(7/2)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 308, 211}

$$\frac{9a^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^2,x]
```

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{(a+bx^2)^2}\,dx &= -\frac{x^9}{2b(a+bx^2)}+\frac{9\int \frac{x^8}{a+bx^2}\,dx}{2b}\\ &= -\frac{x^9}{2b(a+bx^2)}+\frac{9\int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4(a+bx^2)}\right)\,dx}{2b}\\ &= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b(a+bx^2)}+\frac{(9a^4)\int \frac{1}{a+bx^2}\,dx}{2b^5}\\ &= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b(a+bx^2)}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 82, normalized size = 0.89

$$\frac{x\left(-280a^3+70a^2bx^2-28ab^2x^4+10b^3x^6-\frac{35a^4}{a+bx^2}\right)}{70b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^2,x]
[Out] (x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*x
^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Maple [A]
time = 0.06, size = 76, normalized size = 0.83

method	result
default	$-\frac{-\frac{1}{7}b^3x^7+\frac{2}{5}a\,b^2x^5-a^2b\,x^3+4a^3x}{b^5}+\frac{a^4\left(-\frac{x}{2\left(b\,x^2+a\right)}+\frac{9\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^5}$
risch	$\frac{x^7}{7b^2}-\frac{2a\,x^5}{5b^3}+\frac{a^2x^3}{b^4}-\frac{4a^3x}{b^5}-\frac{a^4x}{2b^5(b\,x^2+a)}+\frac{9\sqrt{-ab}\,a^3\ln\left(-\sqrt{-ab}\,x+a\right)}{4b^6}-\frac{9\sqrt{-ab}\,a^3\ln\left(\sqrt{-ab}\,x+a\right)}{4b^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/b^5*(-1/7*b^3*x^7+2/5*a*b^2*x^5-a^2*b*x^3+4*a^3*x)+a^4/b^5*(-1/2*x/(b*x^
2+a)+9/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 82, normalized size = 0.89

$$-\frac{a^4x}{2\,(b^6x^2+ab^5)}+\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{5\,b^3x^7-14\,ab^2x^5+35\,a^2bx^3-140\,a^3x}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^4*x/(b^6*x^2 + a*b^5) + 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5
) + 1/35*(5*b^3*x^7 - 14*a*b^2*x^5 + 35*a^2*b*x^3 - 140*a^3*x)/b^5
```

Fricas [A]
time = 0.92, size = 212, normalized size = 2.30

$$\left[\frac{20\,b^4x^9-36\,ab^3x^7+84\,a^2b^2x^5-420\,a^3bx^3-630\,a^4x+315\,(a^3bx^2+a^4)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{140\,(b^6x^2+ab^5)},\ \frac{10\,b^4x^9-18\,ab^3x^7+42\,a^2b^2x^5-210\,a^3bx^3-315\,a^4x+315\,(a^3bx^2+a^4)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{70\,(b^6x^2+ab^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a^
4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2*b^
2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

Sympy [A]
time = 0.14, size = 134, normalized size = 1.46

$$-\frac{a^4x}{2ab^5+2b^6x^2}-\frac{4a^3x}{b^5}+\frac{a^2x^3}{b^4}-\frac{2ax^5}{5b^3}-\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**2,x)
```

```
[Out] -a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x**
5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 + 9
*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2)
```

Giac [A]
time = 1.46, size = 84, normalized size = 0.91

$$\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}-\frac{a^4x}{2\,(bx^2+a)b^5}+\frac{5\,b^{12}x^7-14\,ab^{11}x^5+35\,a^2b^{10}x^3-140\,a^3b^9x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5)
 + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^14
```

Mupad [B]
time = 4.56, size = 77, normalized size = 0.84

$$\frac{x^7}{7\,b^2}-\frac{2\,a\,x^5}{5\,b^3}-\frac{4\,a^3\,x}{b^5}+\frac{9\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{11/2}}+\frac{a^2\,x^3}{b^4}-\frac{a^4\,x}{2\,\left(b^6\,x^2+a\,b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^2,x)
```

```
[Out] x^7/(7*b^2) - (2*a*x^5)/(5*b^3) - (4*a^3*x)/b^5 + (9*a^(7/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(11/2)) + (a^2*x^3)/b^4 - (a^4*x)/(2*(a*b^5 + b^6*x^2))
```

3.149 $\int \frac{x^9}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=70

$$\frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2} - \frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3\log(a+bx^2)}{b^5}$$

[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*x^2+a)/b^5

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3\log(a+bx^2)}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a + b*x^2)^2,x]

[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2)) - (2*a^3*Log[a + b*x^2])/b^5

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(a+bx^2)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2} - \frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3\log(a+bx^2)}{b^5}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 0.86

$$\frac{9a^2bx^2-3ab^2x^4+b^3x^6-\frac{3a^4}{a+bx^2}-12a^3\log\left(a+bx^2\right)}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^2,x]
```

```
[Out] (9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a +
 b*x^2])/(6*b^5)
```

Maple [A]
time = 0.04, size = 66, normalized size = 0.94

method	result	size
risch	$\frac{3a^2x^2}{2b^4}-\frac{a\,x^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5\left(b\,x^2+a\right)}-\frac{2a^3\ln\left(b\,x^2+a\right)}{b^5}$	63
norman	$\frac{\frac{a^2x^4}{b^3}+\frac{x^8}{6b}-\frac{a\,x^6}{3b^2}-\frac{2a^4}{b^5}}{b\,x^2+a}-\frac{2a^3\ln\left(b\,x^2+a\right)}{b^5}$	64
default	$\frac{\frac{1}{6}b^2x^6-\frac{1}{2}ab\,x^4+\frac{3}{2}a^2x^2}{b^4}-\frac{a^3\left(\frac{a}{b\left(b\,x^2+a\right)}+\frac{4\ln\left(b\,x^2+a\right)}{b}\right)}{2b^4}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/6*b^2*x^6-1/2*a*b*x^4+3/2*a^2*x^2)-1/2*a^3/b^4*(a/b/(b*x^2+a)+4*ln
(b*x^2+a)/b)
```

Maxima [A]
time = 0.29, size = 65, normalized size = 0.93

$$-\frac{a^4}{2\left(b^6x^2+ab^5\right)}-\frac{2\,a^3\log\left(bx^2+a\right)}{b^5}+\frac{b^2x^6-3\,abx^4+9\,a^2x^2}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a*
b*x^4 + 9*a^2*x^2)/b^4
```

Fricas [A]
time = 0.64, size = 81, normalized size = 1.16

$$\frac{b^4x^8-2\,ab^3x^6+6\,a^2b^2x^4+9\,a^3bx^2-3\,a^4-12\,\left(a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{6\,\left(b^6x^2+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3*
b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

Sympy [A]
time = 0.11, size = 66, normalized size = 0.94

$$-\frac{a^4}{2ab^5+2b^6x^2}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**2,x)
```

```
[Out] -a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2/
(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

Giac [A]
time = 0.73, size = 80, normalized size = 1.14

$$-\frac{2\,a^3\log\left(|bx^2+a|\right)}{b^5}+\frac{b^4x^6-3\,ab^3x^4+9\,a^2b^2x^2}{6\,b^6}+\frac{4\,a^3bx^2+3\,a^4}{2\,(bx^2+a)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^2
)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

Mupad [B]
time = 0.07, size = 68, normalized size = 0.97

$$\frac{x^6}{6\,b^2}-\frac{a^4}{2\,b\,\left(b^5\,x^2+a\,b^4\right)}-\frac{a\,x^4}{2\,b^3}-\frac{2\,a^3\,\ln\left(b\,x^2+a\right)}{b^5}+\frac{3\,a^2\,x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^2,x)
```

```
[Out] x^6/(6*b^2) - a^4/(2*b*(a*b^4 + b^5*x^2)) - (a*x^4)/(2*b^3) - (2*a^3*log(a
+ b*x^2))/b^5 + (3*a^2*x^2)/(2*b^4)
```

3.150 $\int \frac{x^8}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=79

$$\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\,(a+bx^2)}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

```
[Out] 7/2*a^2*x/b^4-7/6*a*x^3/b^3+7/10*x^5/b^2-1/2*x^7/b/(b*x^2+a)-7/2*a^(5/2)*ar
ctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {294, 308, 211}

$$-\frac{7a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\,(a+bx^2)}+\frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^2,x]
```

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^2)^2}\,dx = -\frac{x^7}{2b(a+bx^2)} + \frac{7\int \frac{x^6}{a+bx^2}\,dx}{2b}$$
$$= -\frac{x^7}{2b(a+bx^2)} + \frac{7\int \left(\frac{a^2}{b^3} - \frac{ax^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3(a+bx^2)}\right) dx}{2b}$$
$$= \frac{7a^2x}{2b^4} - \frac{7ax^3}{6b^3} + \frac{7x^5}{10b^2} - \frac{x^7}{2b(a+bx^2)} - \frac{(7a^3)\int \frac{1}{a+bx^2}\,dx}{2b^4}$$
$$= \frac{7a^2x}{2b^4} - \frac{7ax^3}{6b^3} + \frac{7x^5}{10b^2} - \frac{x^7}{2b(a+bx^2)} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 0.90

$$\frac{x\left(90a^2 - 20abx^2 + 6b^2x^4 + \frac{15a^3}{a+bx^2}\right)}{30b^4} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^2,x]
```

```
[Out] (x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7*
a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A]
time = 0.05, size = 65, normalized size = 0.82

method	result	size
default	$\frac{\frac{1}{5}b^2x^5-\frac{2}{3}ab\,x^3+3a^2x}{b^4} - \frac{a^3\left(-\frac{x}{2\left(b\,x^2+a\right)}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$	65
risch	$\frac{x^5}{5b^2} - \frac{2a\,x^3}{3b^3} + \frac{3a^2x}{b^4} + \frac{a^3x}{2b^4(b\,x^2+a)} + \frac{7\sqrt{-ab}\;a^2\ln\left(-\sqrt{-ab}\;x-a\right)}{4b^5} - \frac{7\sqrt{-ab}\;a^2\ln\left(\sqrt{-ab}\;x-a\right)}{4b^5}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/5*b^2*x^5-2/3*a*b*x^3+3*a^2*x)-a^3/b^4*(-1/2*x/(b*x^2+a)+7/2/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 71, normalized size = 0.90

$$\frac{a^{3}x}{2\,(b^{5}x^{2}+ab^{4})}-\frac{7\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{4}}+\frac{3\,b^{2}x^{5}-10\,abx^{3}+45\,a^{2}x}{15\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x/(b^5*x^2 + a*b^4) - 7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/15*(3*b^2*x^5 - 10*a*b*x^3 + 45*a^2*x)/b^4
```

Fricas [A]
time = 1.29, size = 190, normalized size = 2.41

$$\left[\frac{12\,b^{3}x^{7}-28\,ab^{2}x^{5}+140\,a^{2}bx^{3}+210\,a^{3}x+105\,(a^{2}bx^{2}+a^{3})\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)}{60\,(b^{5}x^{2}+ab^{4})},\ \frac{6\,b^{3}x^{7}-14\,ab^{2}x^{5}+70\,a^{2}bx^{3}+105\,a^{3}x-105\,(a^{2}bx^{2}+a^{3})\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{30\,(b^{5}x^{2}+ab^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*x
^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*
x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x - 1
05*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^4)]
```

Sympy [A]
time = 0.13, size = 124, normalized size = 1.57

$$\frac{a^{3}x}{2ab^{4}+2b^{5}x^{2}}+\frac{3a^{2}x}{b^{4}}-\frac{2ax^{3}}{3b^{3}}+\frac{7\sqrt{-\frac{a^{5}}{b^{9}}}\log\left(x-\frac{b^{4}\sqrt{-\frac{a^{5}}{b^{9}}}}{a^{2}}\right)}{4}-\frac{7\sqrt{-\frac{a^{5}}{b^{9}}}\log\left(x+\frac{b^{4}\sqrt{-\frac{a^{5}}{b^{9}}}}{a^{2}}\right)}{4}+\frac{x^{5}}{5b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**2,x)
```

```
[Out] a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sqr
t(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*lo
g(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

Giac [A]
time = 0.85, size = 73, normalized size = 0.92

$$-\frac{7\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{4}}+\frac{a^{3}x}{2\,(bx^{2}+a)b^{4}}+\frac{3\,b^{8}x^{5}-10\,ab^{7}x^{3}+45\,a^{2}b^{6}x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

Mupad [B]
time = 4.59, size = 66, normalized size = 0.84

$$\frac{x^{5}}{5\,b^{2}}-\frac{2\,a\,x^{3}}{3\,b^{3}}+\frac{3\,a^{2}\,x}{b^{4}}-\frac{7\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{9/2}}+\frac{a^{3}\,x}{2\,\left(b^{5}\,x^{2}+a\,b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^2,x)
```

```
[Out] x^5/(5*b^2) - (2*a*x^3)/(3*b^3) + (3*a^2*x)/b^4 - (7*a^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(9/2)) + (a^3*x)/(2*(a*b^4 + b^5*x^2))
```

3.151 $\int \frac{x^7}{(a+bx^2)^2} dx$

Optimal. Leaf size=57

$$-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}$$

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^2,x]
```

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.86

$$\frac{-4abx^2+b^2x^4+\frac{2a^3}{a+bx^2}+6a^2\log\left(a+bx^2\right)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^2,x]
[Out] (-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4)
```

Maple [A]
time = 0.06, size = 55, normalized size = 0.96

method	result	size
norman	$\frac{\frac{x^6}{4b}-\frac{3a\,x^4}{4b^2}+\frac{3a^3}{2b^4}}{b\,x^2+a}+\frac{3a^2\ln\left(b\,x^2+a\right)}{2b^4}$	54
default	$\frac{\left(-b\,x^2+2a\right)^2}{4b^4}+\frac{a^2\left(\frac{a}{b\left(b\,x^2+a\right)}+\frac{3\ln\left(b\,x^2+a\right)}{b}\right)}{2b^3}$	55
risch	$\frac{x^4}{4b^2}-\frac{a\,x^2}{b^3}+\frac{a^2}{b^4}+\frac{a^3}{2b^4\left(b\,x^2+a\right)}+\frac{3a^2\ln\left(b\,x^2+a\right)}{2b^4}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(-b*x^2+2*a)^2/b^4+1/2*a^2/b^3*(a/b/(b*x^2+a)+3*ln(b*x^2+a)/b)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.95

$$\frac{a^3}{2\left(b^5x^2+ab^4\right)}+\frac{3\,a^2\log\left(bx^2+a\right)}{2\,b^4}+\frac{bx^4-4\,ax^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*x
^2)/b^3
```

Fricas [A]
time = 0.65, size = 70, normalized size = 1.23

$$\frac{b^3x^6-3\,ab^2x^4-4\,a^2bx^2+2\,a^3+6\left(a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log(
b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A]
time = 0.10, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4 + 2b^5x^2} + \frac{3a^2\log\left(a + bx^2\right)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**2,x)
```

```
[Out] a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b*
*3 + x**4/(4*b**2)
```

Giac [A]
time = 0.79, size = 67, normalized size = 1.18

$$\frac{3\,a^2\log\left(|bx^2 + a|\right)}{2\,b^4} + \frac{b^2x^4 - 4\,abx^2}{4\,b^4} - \frac{3\,a^2bx^2 + 2\,a^3}{2\,(bx^2 + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a^
2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

Mupad [B]
time = 0.08, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,b^2} + \frac{a^3}{2\,b\,\left(b^4\,x^2 + a\,b^3\right)} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,\ln\left(b\,x^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^2,x)
```

```
[Out] x^4/(4*b^2) + a^3/(2*b*(a*b^3 + b^4*x^2)) - (a*x^2)/b^3 + (3*a^2*log(a + b*
x^2))/(2*b^4)
```

3.152 $\int \frac{x^6}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b\,(a+bx^2)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

```
[Out] -5/2*a*x/b^3+5/6*x^3/b^2-1/2*x^5/b/(b*x^2+a)+5/2*a^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/b^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 308, 211}

$$\frac{5a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\,(a+bx^2)}+\frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^2,x]
```

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx^2)^2}\,dx &= -\frac{x^5}{2b(a+bx^2)}+\frac{5\int \frac{x^4}{a+bx^2}\,dx}{2b}\\ &= -\frac{x^5}{2b(a+bx^2)}+\frac{5\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right)dx}{2b}\\ &= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b(a+bx^2)}+\frac{(5a^2)\int \frac{1}{a+bx^2}\,dx}{2b^3}\\ &= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b(a+bx^2)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.91

$$\frac{x\left(-12a+2bx^2-\frac{3a^2}{a+bx^2}\right)}{6b^3}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^2,x]
```

```
[Out] (x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Maple [A]
time = 0.05, size = 54, normalized size = 0.82

method	result	size
default	$-\frac{-\frac{1}{3}b\,x^3+2ax}{b^3}+\frac{a^2\left(-\frac{x}{2\left(b\,x^2+a\right)}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$	54
risch	$\frac{x^3}{3b^2}-\frac{2ax}{b^3}-\frac{a^2x}{2b^3(b\,x^2+a)}+\frac{5\sqrt{-ab}\;a\ln\left(-\sqrt{-ab}\;x+a\right)}{4b^4}-\frac{5\sqrt{-ab}\;a\ln\left(\sqrt{-ab}\;x+a\right)}{4b^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/3*b*x^3+2*a*x)+a^2/b^3*(-1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.63, size = 59, normalized size = 0.89

$$-\frac{a^2x}{2\,(b^4x^2+ab^3)}+\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{bx^3-6\,ax}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x/(b^4*x^2 + a*b^3) + 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3
) + 1/3*(b*x^3 - 6*a*x)/b^3
```

Fricas [A]
time = 0.62, size = 164, normalized size = 2.48

$$\left[\frac{4\,b^2x^5-20\,abx^3-30\,a^2x+15\,(abx^2+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\,(b^4x^2+ab^3)},\ \frac{2\,b^2x^5-10\,abx^3-15\,a^2x+15\,(abx^2+a^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{6\,(b^4x^2+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*lo
g((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*b^
2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a))/(b^4*x^2 + a*b^3)]
```

Sympy [A]
time = 0.12, size = 107, normalized size = 1.62

$$-\frac{a^2x}{2ab^3+2b^4x^2}-\frac{2ax}{b^3}-\frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**2,x)
```

```
[Out] -a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b**
7)/a)/4 + x**3/(3*b**2)
```

Giac [A]
time = 0.76, size = 61, normalized size = 0.92

$$\frac{5 a^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab}\, b^{3}} - \frac{a^{2} x}{2 \left(bx^{2} + a\right) b^{3}} + \frac{b^{4} x^{3} - 6 a b^{3} x}{3 b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3)
 + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

Mupad [B]
time = 0.09, size = 56, normalized size = 0.85

$$\frac{x^{3}}{3 b^{2}} + \frac{5 a^{3/2} \operatorname{atan}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{2 b^{7/2}} - \frac{a^{2} x}{2 \left(b^{4} x^{2} + a b^{3}\right)} - \frac{2 a x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^2,x)
```

```
[Out] x^3/(3*b^2) + (5*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(7/2)) - (a^2*x)/(
2*(a*b^3 + b^4*x^2)) - (2*a*x)/b^3
```

3.153 $\int \frac{x^5}{(a+bx^2)^2} dx$

Optimal. Leaf size=44

$$\frac{x^2}{2b^2} - \frac{a^2}{2b^3 (a + bx^2)} - \frac{a \log (a + bx^2)}{b^3}$$

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^2}{2b^3 (a + bx^2)} - \frac{a \log (a + bx^2)}{b^3} + \frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^2,x]
```

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a + bx^2)^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a + bx)^2} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^2} + \frac{a^2}{b^2(a + bx)^2} - \frac{2a}{b^2(a + bx)}\right) dx, x, x^2\right) \\
&= \frac{x^2}{2b^2} - \frac{a^2}{2b^3 (a + bx^2)} - \frac{a \log (a + bx^2)}{b^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.86

$$\frac{bx^2 - \frac{a^2}{a+bx^2} - 2a\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^2,x]
[Out] (b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

Maple [A]
time = 0.04, size = 44, normalized size = 1.00

method	result	size
risch	$\frac{x^2}{2b^2} - \frac{a^2}{2b^3\left(b\,x^2+a\right)} - \frac{a\ln\left(b\,x^2+a\right)}{b^3}$	41
norman	$\frac{\frac{x^4}{2b}-\frac{a^2}{b^3}}{b\,x^2+a} - \frac{a\ln\left(b\,x^2+a\right)}{b^3}$	43
default	$\frac{x^2}{2b^2} - \frac{a\left(\frac{a}{b\left(b\,x^2+a\right)}+\frac{2\ln\left(b\,x^2+a\right)}{b}\right)}{2b^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/b^2-1/2*a/b^2*(a/b/(b*x^2+a)+2*ln(b*x^2+a)/b)
```

Maxima [A]
time = 0.30, size = 43, normalized size = 0.98

$$-\frac{a^2}{2\left(b^4x^2+ab^3\right)} + \frac{x^2}{2\,b^2} - \frac{a\log\left(bx^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

Fricas [A]
time = 0.77, size = 56, normalized size = 1.27

$$\frac{b^2x^4 + abx^2 - a^2 - 2\left(abx^2 + a^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2 +
 a*b^3)
```

Sympy [A]
time = 0.09, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3+2b^4x^2}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**2,x)
```

```
[Out] -a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

Giac [A]
time = 0.72, size = 49, normalized size = 1.11

$$\frac{x^2}{2\,b^2}-\frac{a\log\left(|bx^2+a|\right)}{b^3}+\frac{2\,abx^2+a^2}{2\,(bx^2+a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 + a
)*b^3)
```

Mupad [B]
time = 0.05, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,b^2}-\frac{a^2}{2\,\left(b^4\,x^2+a\,b^3\right)}-\frac{a\,\ln\left(b\,x^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^2,x)
```

```
[Out] x^2/(2*b^2) - a^2/(2*(a*b^3 + b^4*x^2)) - (a*log(a + b*x^2))/b^3
```

3.154 $\int \frac{x^4}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=55

$$\frac{3x}{2b^2}-\frac{x^3}{2b\,(a+bx^2)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}$$

```
[Out] 3/2*x/b^2-1/2*x^3/b/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {294, 327, 211}

$$-\frac{3\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\,(a+bx^2)}+\frac{3x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^2,x]
```

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^2\right)^2}\,dx &= -\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3\int \frac{x^2}{a+bx^2}\,dx}{2b}\\ &= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{(3a)\int \frac{1}{a+bx^2}\,dx}{2b^2}\\ &= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 51, normalized size = 0.93

$$\frac{x}{b^2}+\frac{ax}{2b^2\left(a+bx^2\right)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^2,x]
```

```
[Out] x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(5/2))
```

Maple [**A**]
time = 0.05, size = 42, normalized size = 0.76

method	result	size
default	$\frac{x}{b^2}-\frac{a\left(-\frac{x}{2\left(b\,x^2+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	42
risch	$\frac{x}{b^2}+\frac{ax}{2b^2\left(b\,x^2+a\right)}+\frac{3\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{4b^3}-\frac{3\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{4b^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^2-a/b^2*(-1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 45, normalized size = 0.82

$$\frac{ax}{2\,(b^3x^2+ab^2)}-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*x/(b^3*x^2 + a*b^2) - 3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + x
/b^2
```

Fricas [A]
time = 1.00, size = 136, normalized size = 2.47

$$\left[\frac{4\,bx^3+3\,(bx^2+a)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,ax}{4\,(b^3x^2+ab^2)},\ \frac{2\,bx^3-3\,(bx^2+a)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,ax}{2\,(b^3x^2+ab^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

Sympy [A]
time = 0.10, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2+2b^3x^2}+\frac{3\sqrt{-\frac{a}{b^5}}\,\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{a}{b^5}}\,\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2,x)
```

```
[Out] a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x)
/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

Giac [A]
time = 1.30, size = 42, normalized size = 0.76

$$-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{ax}{2\,(bx^2+a)b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
x/b^2
```

Mupad [B]
time = 4.59, size = 43, normalized size = 0.78

$$\frac{x}{b^2}+\frac{a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)}-\frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^2,x)
```

```
[Out] x/b^2 + (a*x)/(2*(a*b^2 + b^3*x^2)) - (3*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))
/(2*b^(5/2))
```

3.155 $\int \frac{x^3}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=33

$$\frac{a}{2b^2(a+bx^2)} + \frac{\log(a+bx^2)}{2b^2}$$

```
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a}{2b^2(a+bx^2)} + \frac{\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^2,x]
```

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^2)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) \, dx, x, x^2\right)\\
&= \frac{a}{2b^2(a+bx^2)} + \frac{\log(a+bx^2)}{2b^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^2}+\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^2,x]
```

```
[Out] (a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.03, size = 30, normalized size = 0.91

method	result	size
default	$\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}$	30
norman	$\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}$	30
risch	$\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.97

$$\frac{a}{2\left(b^3x^2+ab^2\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 1.09, size = 35, normalized size = 1.06

$$\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)+a}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

Sympy [A]
time = 0.07, size = 29, normalized size = 0.88

$$\frac{a}{2ab^2 + 2b^3x^2} + \frac{\log(a + bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2,x)
[Out] a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

Giac [A]
time = 1.20, size = 48, normalized size = 1.45

$$-\frac{\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b} - \frac{a}{(bx^2+a)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="giac")
[Out] -1/2*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
```

Mupad [B]
time = 0.05, size = 29, normalized size = 0.88

$$\frac{\ln(b\,x^2 + a)}{2\,b^2} + \frac{a}{2\,b^2\,(b\,x^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^2,x)
[Out] log(a + b*x^2)/(2*b^2) + a/(2*b^2*(a + b*x^2))
```

3.156 $\int \frac{x^2}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=45

$$-\frac{x}{2b(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

```
[Out] -1/2*x/b/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {294, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^2,x]
[Out] -1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)^2}\,dx &= -\frac{x}{2b(a+bx^2)}+\frac{\int \frac{1}{a+bx^2}\,dx}{2b}\\ &= -\frac{x}{2b(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 1.00

$$-\frac{x}{2b(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^2,x]
[Out] -1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.80

method	result	size
default	$-\frac{x}{2b(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}$	36
risch	$-\frac{x}{2b(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 36, normalized size = 0.80

$$-\frac{x}{2(b^2x^2+ab)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*x/(b^2*x^2 + a*b) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 1.57, size = 120, normalized size = 2.67

$$\left[-\frac{2\,abx+(bx^2+a)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,(ab^3x^2+a^2b^2)},-\frac{abx-(bx^2+a)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(ab^3x^2+a^2b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.
time = 0.08, size = 78, normalized size = 1.73

$$-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2,x)
```

```
[Out] -x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) + x
)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

Giac [A]
time = 0.94, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}-\frac{x}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

Mupad [B]
time = 4.76, size = 33, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^2,x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2)) - x/(2*b*(a + b*x^2))
```

3.157 $\int \frac{x}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2b(a+bx^2)}$$

```
[Out] -1/2/b/(b*x^2+a)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$-\frac{1}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^2,x]
```

```
[Out] -1/2*1/(b*(a + b*x^2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^2} \, dx = -\frac{1}{2b(a+bx^2)}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^2,x]
```

```
[Out] -1/2*1/(b*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 15, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{2b(b\,x^2+a)}$	15
derivativedivides	$-\frac{1}{2b(b\,x^2+a)}$	15
default	$-\frac{1}{2b(b\,x^2+a)}$	15
norman	$-\frac{1}{2b(b\,x^2+a)}$	15
risch	$-\frac{1}{2b(b\,x^2+a)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.88

$$-\frac{1}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

Fricas [A]
time = 1.79, size = 15, normalized size = 0.94

$$-\frac{1}{2\,(b^2x^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.94

$$-\frac{1}{2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2,x)
```

```
[Out] -1/(2*a*b + 2*b**2*x**2)
```

Giac [A]
time = 0.72, size = 14, normalized size = 0.88

$$-\frac{1}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.88

$$-\frac{1}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^2,x)
```

```
[Out] -1/(2*b*(a + b*x^2))
```

3.158 $\int \frac{1}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=45

$$\frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

```
[Out] 1/2*x/a/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^2} \, dx &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2} \, dx}{2a}\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-2),x]
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.80

method	result	size
default	$\frac{x}{2a(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$\frac{x}{2a(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 35, normalized size = 0.78

$$\frac{x}{2\left(abx^2+a^2\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Fricas [A]
time = 1.62, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx-\left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)},\frac{abx+\left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.
time = 0.08, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Giac [A]
time = 1.06, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a} + \frac{x}{2(bx^2 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

Mupad [B]
time = 4.74, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(b\,x^2 + a\right)} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^2,x)
```

```
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

3.159 $\int \frac{1}{x(a+bx^2)^2} \, dx$

Optimal. Leaf size=38

$$\frac{1}{2a(a+bx^2)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx^2)}{2a^2}$$

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$-\frac{\log(a+bx^2)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2),x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^2)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) \, dx, x, x^2\right) \\
&= \frac{1}{2a(a+bx^2)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx^2)}{2a^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2}+2\log(x)-\log(a+bx^2)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2),x]
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Maple [A]
time = 0.04, size = 42, normalized size = 1.11

method	result	size
risch	$\frac{1}{2a(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	35
norman	$-\frac{b\,x^2}{2a^2(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	39
default	$-\frac{b\left(-\frac{a}{b\left(b\,x^2+a\right)}+\frac{\ln\left(b\,x^2+a\right)}{b}\right)}{2a^2}+\frac{\ln(x)}{a^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*b/a^2*(-a/b/(b*x^2+a)+ln(b*x^2+a)/b)+ln(x)/a^2
```

Maxima [A]
time = 0.29, size = 37, normalized size = 0.97

$$\frac{1}{2\,(abx^2+a^2)}-\frac{\log(bx^2+a)}{2\,a^2}+\frac{\log(x^2)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Fricas [A]
time = 1.11, size = 47, normalized size = 1.24

$$-\frac{(bx^2+a)\log(bx^2+a)-2\,(bx^2+a)\log(x)-a}{2\,(a^2bx^2+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="fricas")
```

[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a^3)

Sympy [A]
time = 0.12, size = 34, normalized size = 0.89

$$\frac{1}{2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**2+a)**2,x)

[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)

Giac [A]
time = 1.03, size = 47, normalized size = 1.24

$$\frac{\log(x^2)}{2\,a^2} - \frac{\log(|bx^2 + a|)}{2\,a^2} + \frac{bx^2 + 2\,a}{2\,(bx^2 + a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="giac")

[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2 + a)*a^2)

Mupad [B]
time = 4.70, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2} + \frac{1}{2\,a\,\left(b\,x^2 + a\right)} - \frac{\ln\left(b\,x^2 + a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^2)^2),x)

[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)

3.160 $\int \frac{1}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] -3/2/a^2/x+1/2/a/x/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {296, 331, 211}

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{1}{2ax\left(a+bx^2\right)}+\frac{3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a}\\
&= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{(3b)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\
&= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.95

$$
-\frac{1}{a^2x}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

Maple [A]
time = 0.06, size = 45, normalized size = 0.79

method	result	size
default	$-\frac{b\left(\frac{x}{2b\,x^{2}+2a}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{1}{a^{2}x}$	45
risch	$\frac{-\frac{3b\,x^{2}}{2a^{2}}-\frac{1}{a}}{x\left(b\,x^{2}+a\right)}+\frac{3\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{4a^{3}}-\frac{3\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{4a^{3}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/a^2/x
```

Maxima [A]
time = 0.49, size = 49, normalized size = 0.86

$$-\frac{3\,bx^2+2\,a}{2\,(a^2bx^3+a^3x)}-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

Fricas [A]
time = 1.35, size = 136, normalized size = 2.39

$$\left[-\frac{6\,bx^2-3\,(bx^3+ax)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\,(a^2bx^3+a^3x)},-\frac{3\,bx^2+3\,(bx^3+ax)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\,(a^2bx^3+a^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A]
time = 0.12, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{b}{a^5}}\,\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\,\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2,x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

Giac [A]
time = 0.91, size = 47, normalized size = 0.82

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bx^2+2\,a}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

Mupad [B]
time = 0.07, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2),x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

3.161 $\int \frac{1}{x^3(a+bx^2)^2} dx$

Optimal. Leaf size=49

$$-\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}$$

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*1/(a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*
x^2])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{1}{x^2}+\frac{b}{a+bx^2}\right)+4b\log(x)-2b\log\left(a+bx^2\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

Maple [A]
time = 0.04, size = 55, normalized size = 1.12

method	result	size
norman	$\frac{\frac{b^{2}x^{4}}{a^{3}}-\frac{1}{2a}}{x^{2}\left(b\,x^{2}+a\right)}+\frac{b\ln\left(b\,x^{2}+a\right)}{a^{3}}-\frac{2b\ln\left(x\right)}{a^{3}}$	52
risch	$\frac{-\frac{b\,x^{2}}{a^{2}}-\frac{1}{2a}}{x^{2}\left(b\,x^{2}+a\right)}-\frac{2b\ln\left(x\right)}{a^{3}}+\frac{b\ln\left(-b\,x^{2}-a\right)}{a^{3}}$	54
default	$\frac{b^{2}\left(-\frac{a}{b\left(b\,x^{2}+a\right)}+\frac{2\ln\left(b\,x^{2}+a\right)}{b}\right)}{2a^{3}}-\frac{1}{2a^{2}x^{2}}-\frac{2b\ln\left(x\right)}{a^{3}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^3*(-a/b/(b*x^2+a)+2*ln(b*x^2+a)/b)-1/2/a^2/x^2-2*b*ln(x)/a^3
```

Maxima [A]
time = 0.30, size = 52, normalized size = 1.06

$$-\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^2
)/a^3
```

Fricas [A]
time = 1.50, size = 73, normalized size = 1.49

$$-\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log\left(x\right)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A]
time = 0.16, size = 51, normalized size = 1.04

$$\frac{-a - 2bx^2}{2a^3x^2 + 2a^2bx^4} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(\frac{a}{b} + x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

Giac [A]
time = 0.98, size = 51, normalized size = 1.04

$$-\frac{b\log(x^2)}{a^3} + \frac{b\log(|bx^2 + a|)}{a^3} - \frac{2\,bx^2 + a}{2\,(bx^4 + ax^2)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

Mupad [B]
time = 0.08, size = 51, normalized size = 1.04

$$\frac{b\,\ln\left(b\,x^2 + a\right)}{a^3} - \frac{\frac{1}{2\,a} + \frac{b\,x^2}{a^2}}{b\,x^4 + a\,x^2} - \frac{2\,b\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

3.162 $\int \frac{1}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] -5/6/a^2/x^3+5/2*b/a^3/x+1/2/a/x^3/(b*x^2+a)+5/2*b^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/a^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {296, 331, 211}

$$\frac{5b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\int \frac{1}{x^4 (a+bx^2)^2} \, dx = \frac{1}{2ax^3 (a+bx^2)} + \frac{5 \int \frac{1}{x^4 (a+bx^2)} \, dx}{2a}$$
$$= -\frac{5}{6a^2x^3} + \frac{1}{2ax^3 (a+bx^2)} - \frac{(5b) \int \frac{1}{x^2 (a+bx^2)} \, dx}{2a^2}$$
$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3 (a+bx^2)} + \frac{(5b^2) \int \frac{1}{a+bx^2} \, dx}{2a^3}$$
$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3 (a+bx^2)} + \frac{5b^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.99

$$-\frac{1}{3a^2x^3} + \frac{2b}{a^3x} + \frac{b^2x}{2a^3 (a+bx^2)} + \frac{5b^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Maple [A]
time = 0.05, size = 55, normalized size = 0.81

method	result	size
default	$\frac{b^2\left(\frac{x}{2b\,x^2+2a}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}$	55
risch	$\frac{\frac{5b^2x^4}{2a^3}+\frac{5b\,x^2}{3a^2}-\frac{1}{3a}}{x^3(b\,x^2+a)}+\frac{5\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{4a^4}-\frac{5\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{4a^4}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^3*(1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3/a^2/x
^3+2*b/a^3/x
```

Maxima [A]
time = 0.48, size = 64, normalized size = 0.94

$$\frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\,(a^3bx^5+a^4x^3)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 1.24, size = 172, normalized size = 2.53

$$\left[\frac{30\,b^2x^4+20\,abx^2+15\,(b^2x^5+abx^3)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\,(a^3bx^5+a^4x^3)},\frac{15\,b^2x^4+10\,abx^2+15\,(b^2x^5+abx^3)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)-2\,a^2}{6\,(a^3bx^5+a^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [A]
time = 0.15, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\,\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\,\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2,x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Giac [A]
time = 1.14, size = 59, normalized size = 0.87

$$\frac{5\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}}+\frac{b^{2}x}{2\,(bx^{2}+a)a^{3}}+\frac{6\,bx^{2}-a}{3\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

Mupad [B]
time = 4.73, size = 58, normalized size = 0.85

$$\frac{\frac{5\,b\,x^{2}}{3\,a^{2}}-\frac{1}{3\,a}+\frac{5\,b^{2}\,x^{4}}{2\,a^{3}}}{b\,x^{5}+a\,x^{3}}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

3.163 $\int \frac{1}{x^5(a+bx^2)^2} dx$

Optimal. Leaf size=66

$$-\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3(a+bx^2)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx^2)}{2a^4}$$

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 46}

$$-\frac{3b^2\log(a+bx^2)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{2a^3(a+bx^2)}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -1/4*1/(a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a
^4 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3(a+bx^2)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx^2)}{2a^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.86

$$\frac{a\left(-\frac{a}{x^4}+\frac{4b}{x^2}+\frac{2b^2}{a+bx^2}\right)+12b^2\log(x)-6b^2\log\left(a+bx^2\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^2),x]
```

```
[Out] (a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Log
[a + b*x^2])/(4*a^4)
```

Maple [A]
time = 0.04, size = 65, normalized size = 0.98

method	result	size
default	$-\frac{b^3\left(-\frac{a}{b(b\,x^2+a)}+\frac{3\ln(b\,x^2+a)}{b}\right)}{2a^4}-\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{3b^2\ln(x)}{a^4}$	65
norman	$\frac{-\frac{1}{4a}+\frac{3b\,x^2}{4a^2}-\frac{3b^3x^6}{2a^4}}{x^4(b\,x^2+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(b\,x^2+a)}{2a^4}$	67
risch	$\frac{\frac{3b^2x^4}{2a^3}+\frac{3b\,x^2}{4a^2}-\frac{1}{4a}}{x^4(b\,x^2+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(b\,x^2+a)}{2a^4}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3/a^4*(-a/b/(b*x^2+a)+3*ln(b*x^2+a)/b)-1/4/a^2/x^4+b/a^3/x^2+3*b^2*l
n(x)/a^4
```

Maxima [A]
time = 0.27, size = 70, normalized size = 1.06

$$\frac{6\,b^2x^4+3\,abx^2-a^2}{4\,(a^3bx^6+a^4x^4)}-\frac{3\,b^2\log\left(bx^2+a\right)}{2\,a^4}+\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^2
 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

Fricas [A]
time = 1.38, size = 90, normalized size = 1.36

$$\frac{6\,ab^2x^4+3\,a^2bx^2-a^3-6\,(b^3x^6+ab^2x^4)\log\left(bx^2+a\right)+12\,(b^3x^6+ab^2x^4)\log\left(x\right)}{4\,(a^4bx^6+a^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

Sympy [A]
time = 0.19, size = 68, normalized size = 1.03

$$\frac{-a^2+3abx^2+6b^2x^4}{4a^4x^4+4a^3bx^6}+\frac{3b^2\log\left(x\right)}{a^4}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*l
og(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

Giac [A]
time = 1.17, size = 86, normalized size = 1.30

$$\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}-\frac{3\,b^2\log\left(|bx^2+a|\right)}{2\,a^4}+\frac{3\,b^3x^2+4\,ab^2}{2\,(bx^2+a)a^4}-\frac{9\,b^2x^4-4\,abx^2+a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 + 4
*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

Mupad [B]
time = 4.80, size = 67, normalized size = 1.02

$$\frac{\frac{3\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{3\,b^2\,x^4}{2\,a^3}}{b\,x^6+a\,x^4}-\frac{3\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^4}+\frac{3\,b^2\,\ln\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^2),x)
```

```
[Out] ((3*b*x^2)/(4*a^2) - 1/(4*a) + (3*b^2*x^4)/(2*a^3))/(a*x^4 + b*x^6) - (3*b^
2*log(a + b*x^2))/(2*a^4) + (3*b^2*log(x))/a^4
```

3.164 $\int \frac{1}{x^6(a+bx^2)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}$$

```
[Out] -7/10/a^2/x^5+7/6*b/a^3/x^3-7/2*b^2/a^4/x+1/2/a/x^5/(b*x^2+a)-7/2*b^(5/2)*a
rctan(x*b^(1/2)/a^(1/2))/a^(9/2)
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {296, 331, 211}

$$-\frac{7b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{7b^2}{2a^4x}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\int \frac{1}{x^6 (a+bx^2)^2} dx = \frac{1}{2ax^5 (a+bx^2)} + \frac{7 \int \frac{1}{x^6 (a+bx^2)} dx}{2a}$$
$$= -\frac{7}{10a^2x^5} + \frac{1}{2ax^5 (a+bx^2)} - \frac{(7b) \int \frac{1}{x^4 (a+bx^2)} dx}{2a^2}$$
$$= -\frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} + \frac{1}{2ax^5 (a+bx^2)} + \frac{(7b^2) \int \frac{1}{x^2 (a+bx^2)} dx}{2a^3}$$
$$= -\frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} - \frac{7b^2}{2a^4x} + \frac{1}{2ax^5 (a+bx^2)} - \frac{(7b^3) \int \frac{1}{a+bx^2} dx}{2a^4}$$
$$= -\frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} - \frac{7b^2}{2a^4x} + \frac{1}{2ax^5 (a+bx^2)} - \frac{7b^{5/2} \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{2a^{9/2}}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.99

$$-\frac{1}{5a^2x^5} + \frac{2b}{3a^3x^3} - \frac{3b^2}{a^4x} - \frac{b^3x}{2a^4 (a+bx^2)} - \frac{7b^{5/2} \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{2a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^2),x]
[Out] -1/5*1/(a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a
+ b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Maple [A]
time = 0.04, size = 67, normalized size = 0.83

method	result	size
default	$-\frac{b^3 \left(\frac{x}{2b\,x^2+2a}+\frac{7 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}-\frac{1}{5a^2x^5}+\frac{2b}{3a^3x^3}-\frac{3b^2}{a^4x}$	67

risch	$\frac{-\frac{7b^3x^6}{2a^4}-\frac{7b^2x^4}{3a^3}+\frac{7b\,x^2}{15a^2}-\frac{1}{5a}}{x^5(b\,x^2+a)}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^2+b^5\right)}\_R\ln\left(\left(3\_R^2a^9+2b^5\right)x+a^5b^2\_R\right)\right)}{4}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/a^4*(1/2*x/(b*x^2+a)+7/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/5/a^2/
x^5+2/3*b/a^3/x^3-3*b^2/a^4/x
```

Maxima [A]
time = 0.50, size = 75, normalized size = 0.93

$$-\frac{105\,b^3x^6+70\,ab^2x^4-14\,a^2bx^2+6\,a^3}{30\,(a^4bx^7+a^5x^5)}-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3)/(a^4*b*x^7 + a^5*
x^5) - 7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 1.42, size = 198, normalized size = 2.44

$$\left[-\frac{210\,b^3x^6+140\,ab^2x^4-28\,a^2bx^2+12\,a^3-105\,(b^3x^7+ab^2x^5)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{60\,(a^4bx^7+a^5x^5)},-\frac{105\,b^3x^6+70\,ab^2x^4-14\,a^2bx^2+6\,a^3+105\,(b^3x^7+ab^2x^5)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)}{30\,(a^4bx^7+a^5x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
+ a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a
^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a
^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^7 +
a^5*x^5)]
```

Sympy [A]
time = 0.17, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^5}{a^9}}\,\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}-\frac{7\sqrt{-\frac{b^5}{a^9}}\,\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}+\frac{-6a^3+14a^2bx^2-70ab^2x^4-105b^3x^6}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**2,x)
```

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a*
*9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 + (-6*a**3 + 14*a**2*b*x**2 - 70*
a*b**2*x**4 - 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

Giac [A]
time = 1.10, size = 70, normalized size = 0.86

$$-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}-\frac{b^3x}{2\,(bx^2+a)a^4}-\frac{45\,b^2x^4-10\,abx^2+3\,a^2}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^4
) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

Mupad [B]
time = 4.85, size = 70, normalized size = 0.86

$$-\frac{\frac{1}{5\,a}-\frac{7\,b\,x^2}{15\,a^2}+\frac{7\,b^2\,x^4}{3\,a^3}+\frac{7\,b^3\,x^6}{2\,a^4}}{b\,x^7+a\,x^5}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^2),x)
```

```
[Out] - (1/(5*a) - (7*b*x^2)/(15*a^2) + (7*b^2*x^4)/(3*a^3) + (7*b^3*x^6)/(2*a^4)
)/(a*x^5 + b*x^7) - (7*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(9/2))
```

3.165 $\int \frac{1}{x^7(a+bx^2)^2} dx$

Optimal. Leaf size=80

$$-\frac{1}{6a^2x^6}+\frac{b}{2a^3x^4}-\frac{3b^2}{2a^4x^2}-\frac{b^3}{2a^4\left(a+bx^2\right)}-\frac{4b^3\log(x)}{a^5}+\frac{2b^3\log\left(a+bx^2\right)}{a^5}$$

```
[Out] -1/6/a^2/x^6+1/2*b/a^3/x^4-3/2*b^2/a^4/x^2-1/2*b^3/a^4/(b*x^2+a)-4*b^3*ln(x
)/a^5+2*b^3*ln(b*x^2+a)/a^5
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\frac{2b^3\log\left(a+bx^2\right)}{a^5}-\frac{4b^3\log(x)}{a^5}-\frac{b^3}{2a^4\left(a+bx^2\right)}-\frac{3b^2}{2a^4x^2}+\frac{b}{2a^3x^4}-\frac{1}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^2),x]
```

```
[Out] -1/6*1/(a^2*x^6) + b/(2*a^3*x^4) - (3*b^2)/(2*a^4*x^2) - b^3/(2*a^4*(a + b*
x^2)) - (4*b^3*Log[x])/a^5 + (2*b^3*Log[a + b*x^2])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{6a^2x^6}+\frac{b}{2a^3x^4}-\frac{3b^2}{2a^4x^2}-\frac{b^3}{2a^4\left(a+bx^2\right)}-\frac{4b^3\log(x)}{a^5}+\frac{2b^3\log\left(a+bx^2\right)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.85

$$\frac{a\left(-\frac{a^2}{x^6}+\frac{3ab}{x^4}-\frac{9b^2}{x^2}-\frac{3b^3}{a+bx^2}\right)-24b^3\log(x)+12b^3\log\left(a+bx^2\right)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^2),x]
```

```
[Out] (a*(-(a^2/x^6) + (3*a*b)/x^4 - (9*b^2)/x^2 - (3*b^3)/(a + b*x^2)) - 24*b^3*
Log[x] + 12*b^3*Log[a + b*x^2])/(6*a^5)
```

Maple [A]
time = 0.05, size = 77, normalized size = 0.96

method	result	size
default	$\frac{b^4\left(-\frac{a}{b\left(b\,x^2+a\right)}+\frac{4\ln\left(b\,x^2+a\right)}{b}\right)}{2a^5}-\frac{1}{6a^2x^6}+\frac{b}{2a^3x^4}-\frac{3b^2}{2a^4x^2}-\frac{4b^3\ln(x)}{a^5}$	77
norman	$\frac{\frac{2b^4x^8}{a^5}-\frac{1}{6a}+\frac{b\,x^2}{3a^2}-\frac{b^2x^4}{a^3}}{x^6\left(b\,x^2+a\right)}-\frac{4b^3\ln(x)}{a^5}+\frac{2b^3\ln\left(b\,x^2+a\right)}{a^5}$	78
risch	$\frac{-\frac{2b^3x^6}{a^4}-\frac{b^2x^4}{a^3}+\frac{b\,x^2}{3a^2}-\frac{1}{6a}}{x^6\left(b\,x^2+a\right)}-\frac{4b^3\ln(x)}{a^5}+\frac{2b^3\ln\left(-b\,x^2-a\right)}{a^5}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^4/a^5*(-a/b/(b*x^2+a)+4*ln(b*x^2+a)/b)-1/6/a^2/x^6+1/2*b/a^3/x^4-3/2*
b^2/a^4/x^2-4*b^3*ln(x)/a^5
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.99

$$-\frac{12\,b^3x^6+6\,ab^2x^4-2\,a^2bx^2+a^3}{6\left(a^4bx^8+a^5x^6\right)}+\frac{2\,b^3\log\left(bx^2+a\right)}{a^5}-\frac{2\,b^3\log\left(x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(12*b^3*x^6 + 6*a*b^2*x^4 - 2*a^2*b*x^2 + a^3)/(a^4*b*x^8 + a^5*x^6) +
 2*b^3*log(b*x^2 + a)/a^5 - 2*b^3*log(x^2)/a^5
```

Fricas [A]
time = 1.21, size = 99, normalized size = 1.24

$$-\frac{12\,ab^3x^6+6\,a^2b^2x^4-2\,a^3bx^2+a^4-12\left(b^4x^8+ab^3x^6\right)\log\left(bx^2+a\right)+24\left(b^4x^8+ab^3x^6\right)\log\left(x\right)}{6\left(a^5bx^8+a^6x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4 - 12*(b^4*x^8 + a*b^
3*x^6)*log(b*x^2 + a) + 24*(b^4*x^8 + a*b^3*x^6)*log(x))/(a^5*b*x^8 + a^6*x
^6)
```

Sympy [A]
time = 0.21, size = 78, normalized size = 0.98

$$\frac{-a^3 + 2a^2bx^2 - 6ab^2x^4 - 12b^3x^6}{6a^5x^6 + 6a^4bx^8} - \frac{4b^3\log(x)}{a^5} + \frac{2b^3\log\left(\frac{a}{b} + x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x**2 - 6*a*b**2*x**4 - 12*b**3*x**6)/(6*a**5*x**6 + 6*a**
4*b*x**8) - 4*b**3*log(x)/a**5 + 2*b**3*log(a/b + x**2)/a**5
```

Giac [A]
time = 1.18, size = 99, normalized size = 1.24

$$-\frac{2\,b^3\log(x^2)}{a^5} + \frac{2\,b^3\log(|bx^2 + a|)}{a^5} - \frac{4\,b^4x^2 + 5\,ab^3}{2\,(bx^2 + a)a^5} + \frac{22\,b^3x^6 - 9\,ab^2x^4 + 3\,a^2bx^2 - a^3}{6\,a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*b^3*log(x^2)/a^5 + 2*b^3*log(abs(b*x^2 + a))/a^5 - 1/2*(4*b^4*x^2 + 5*a*
b^3)/((b*x^2 + a)*a^5) + 1/6*(22*b^3*x^6 - 9*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)
/(a^5*x^6)
```

Mupad [B]
time = 0.12, size = 78, normalized size = 0.98

$$\frac{2\,b^3\,\ln(b\,x^2 + a)}{a^5} - \frac{\frac{1}{6\,a} - \frac{b\,x^2}{3\,a^2} + \frac{b^2\,x^4}{a^3} + \frac{2\,b^3\,x^6}{a^4}}{b\,x^8 + a\,x^6} - \frac{4\,b^3\,\ln(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^2),x)
```

```
[Out] (2*b^3*log(a + b*x^2))/a^5 - (1/(6*a) - (b*x^2)/(3*a^2) + (b^2*x^4)/a^3 + (
2*b^3*x^6)/a^4)/(a*x^6 + b*x^8) - (4*b^3*log(x))/a^5
```

3.166 $\int \frac{1}{x^8(a+bx^2)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{9}{14a^2x^7}+\frac{9b}{10a^3x^5}-\frac{3b^2}{2a^4x^3}+\frac{9b^3}{2a^5x}+\frac{1}{2ax^7\left(a+bx^2\right)}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}$$

```
[Out] -9/14/a^2/x^7+9/10*b/a^3/x^5-3/2*b^2/a^4/x^3+9/2*b^3/a^5/x+1/2/a/x^7/(b*x^2
+a)+9/2*b^(7/2)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A]
time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {296, 331, 211}

$$\frac{9b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{9b^3}{2a^5x}-\frac{3b^2}{2a^4x^3}+\frac{9b}{10a^3x^5}-\frac{9}{14a^2x^7}+\frac{1}{2ax^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -9/(14*a^2*x^7) + (9*b)/(10*a^3*x^5) - (3*b^2)/(2*a^4*x^3) + (9*b^3)/(2*a^5
*x) + 1/(2*a*x^7*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
a^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^7\left(a+bx^2\right)}+\frac{9\int \frac{1}{x^8(a+bx^2)}\,dx}{2a}$$
$$=-\frac{9}{14a^2x^7}+\frac{1}{2ax^7\left(a+bx^2\right)}-\frac{(9b)\int \frac{1}{x^6(a+bx^2)}\,dx}{2a^2}$$
$$=-\frac{9}{14a^2x^7}+\frac{9b}{10a^3x^5}+\frac{1}{2ax^7\left(a+bx^2\right)}+\frac{\left(9b^2\right)\int \frac{1}{x^4(a+bx^2)}\,dx}{2a^3}$$
$$=-\frac{9}{14a^2x^7}+\frac{9b}{10a^3x^5}-\frac{3b^2}{2a^4x^3}+\frac{1}{2ax^7\left(a+bx^2\right)}-\frac{\left(9b^3\right)\int \frac{1}{x^2(a+bx^2)}\,dx}{2a^4}$$
$$=-\frac{9}{14a^2x^7}+\frac{9b}{10a^3x^5}-\frac{3b^2}{2a^4x^3}+\frac{9b^3}{2a^5x}+\frac{1}{2ax^7\left(a+bx^2\right)}+\frac{\left(9b^4\right)\int \frac{1}{a+bx^2}\,dx}{2a^5}$$
$$=-\frac{9}{14a^2x^7}+\frac{9b}{10a^3x^5}-\frac{3b^2}{2a^4x^3}+\frac{9b^3}{2a^5x}+\frac{1}{2ax^7\left(a+bx^2\right)}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.97

$$-\frac{1}{7a^2x^7}+\frac{2b}{5a^3x^5}-\frac{b^2}{a^4x^3}+\frac{4b^3}{a^5x}+\frac{b^4x}{2a^5\left(a+bx^2\right)}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/7*1/(a^2*x^7) + (2*b)/(5*a^3*x^5) - b^2/(a^4*x^3) + (4*b^3)/(a^5*x) + (b
^4*x)/(2*a^5*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(1
1/2))
```

Maple [A]
time = 0.05, size = 77, normalized size = 0.82

method	result	size

default	$\frac{b^4\left(\frac{x}{2b\,x^2+2a}+\frac{9\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^5}-\frac{1}{7a^2x^7}+\frac{2b}{5a^3x^5}-\frac{b^2}{a^4x^3}+\frac{4b^3}{a^5x}$	77
risch	$\frac{\frac{9b^4x^8}{2a^5}+\frac{3b^3x^6}{a^4}-\frac{3b^2x^4}{5a^3}+\frac{9b\,x^2}{35a^2}-\frac{1}{7a}}{x^7(b\,x^2+a)}+\frac{9\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^2+b^7\right)}\_R\ln\left(\left(3\_R^2a^{11}+2b^7\right)x-a^6b^3\_R\right)\right)}{4}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^4/a^5*(1/2*x/(b*x^2+a)+9/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/7/a^2/x
^7+2/5*b/a^3/x^5-b^2/a^4/x^3+4*b^3/a^5/x
```

Maxima [A]
time = 0.49, size = 86, normalized size = 0.91

$$\frac{315\,b^4x^8+210\,ab^3x^6-42\,a^2b^2x^4+18\,a^3bx^2-10\,a^4}{70\,(a^5bx^9+a^6x^7)}+\frac{9\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/70*(315*b^4*x^8 + 210*a*b^3*x^6 - 42*a^2*b^2*x^4 + 18*a^3*b*x^2 - 10*a^4)
/(a^5*b*x^9 + a^6*x^7) + 9/2*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Fricas [A]
time = 1.37, size = 220, normalized size = 2.34

$$\left[\frac{630\,b^4x^8+420\,ab^3x^6-84\,a^2b^2x^4+36\,a^3bx^2-20\,a^4+315\,(b^4x^9+ab^3x^7)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{140\,(a^5bx^9+a^6x^7)},\ \frac{315\,b^4x^8+210\,ab^3x^6-42\,a^2b^2x^4+18\,a^3bx^2-10\,a^4+315\,(b^4x^9+ab^3x^7)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{70\,(a^5bx^9+a^6x^7)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(630*b^4*x^8 + 420*a*b^3*x^6 - 84*a^2*b^2*x^4 + 36*a^3*b*x^2 - 20*a^
4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)
/(b*x^2 + a)))/(a^5*b*x^9 + a^6*x^7), 1/70*(315*b^4*x^8 + 210*a*b^3*x^6 - 4
2*a^2*b^2*x^4 + 18*a^3*b*x^2 - 10*a^4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(b/a)
*arctan(x*sqrt(b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

Sympy [A]
time = 0.20, size = 138, normalized size = 1.47

$$-\frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4}+x\right)}{4}+\frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4}+x\right)}{4}+\frac{-10a^4+18a^3bx^2-42a^2b^2x^4+210ab^3x^6+315b^4x^8}{70a^6x^7+70a^5bx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a)**2,x)
```

```
[Out] -9*sqrt(-b**7/a**11)*log(-a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + 9*sqrt(-b**7
/a**11)*log(a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + (-10*a**4 + 18*a**3*b*x**2
 - 42*a**2*b**2*x**4 + 210*a*b**3*x**6 + 315*b**4*x**8)/(70*a**6*x**7 + 70*
a**5*b*x**9)
```

Giac [A]
time = 2.37, size = 81, normalized size = 0.86

$$\frac{9\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^5}+\frac{b^4x}{2\,(bx^2+a)a^5}+\frac{140\,b^3x^6-35\,ab^2x^4+14\,a^2bx^2-5\,a^3}{35\,a^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 9/2*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/2*b^4*x/((b*x^2 + a)*a^5)
 + 1/35*(140*b^3*x^6 - 35*a*b^2*x^4 + 14*a^2*b*x^2 - 5*a^3)/(a^5*x^7)
```

Mupad [B]
time = 4.56, size = 80, normalized size = 0.85

$$\frac{\frac{9\,b\,x^2}{35\,a^2}-\frac{1}{7\,a}-\frac{3\,b^2\,x^4}{5\,a^3}+\frac{3\,b^3\,x^6}{a^4}+\frac{9\,b^4\,x^8}{2\,a^5}}{b\,x^9+a\,x^7}+\frac{9\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^2)^2),x)
```

```
[Out] ((9*b*x^2)/(35*a^2) - 1/(7*a) - (3*b^2*x^4)/(5*a^3) + (3*b^3*x^6)/a^4 + (9*
b^4*x^8)/(2*a^5))/(a*x^7 + b*x^9) + (9*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/(
2*a^(11/2))
```

3.167 $\int \frac{1}{x^9(a+bx^2)^2} dx$

Optimal. Leaf size=93

$$-\frac{1}{8a^2x^8}+\frac{b}{3a^3x^6}-\frac{3b^2}{4a^4x^4}+\frac{2b^3}{a^5x^2}+\frac{b^4}{2a^5(a+bx^2)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx^2)}{2a^6}$$

```
[Out] -1/8/a^2/x^8+1/3*b/a^3/x^6-3/4*b^2/a^4/x^4+2*b^3/a^5/x^2+1/2*b^4/a^5/(b*x^2
+a)+5*b^4*ln(x)/a^6-5/2*b^4*ln(b*x^2+a)/a^6
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$-\frac{5b^4\log(a+bx^2)}{2a^6}+\frac{5b^4\log(x)}{a^6}+\frac{b^4}{2a^5(a+bx^2)}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}+\frac{b}{3a^3x^6}-\frac{1}{8a^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)^2),x]
```

```
[Out] -1/8*1/(a^2*x^8) + b/(3*a^3*x^6) - (3*b^2)/(4*a^4*x^4) + (2*b^3)/(a^5*x^2)
+ b^4/(2*a^5*(a + b*x^2)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x^2])/(2*
a^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^9(a+bx^2)^2}dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^5(a+bx)^2}dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx, x\right.\\ &= -\frac{1}{8a^2x^8}+\frac{b}{3a^3x^6}-\frac{3b^2}{4a^4x^4}+\frac{2b^3}{a^5x^2}+\frac{b^4}{2a^5(a+bx^2)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx^2)}{2a^6}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 79, normalized size = 0.85

$$\frac{a\left(-\frac{3a^3}{x^8}+\frac{8a^2b}{x^6}-\frac{18ab^2}{x^4}+12b^3\left(\frac{4}{x^2}+\frac{b}{a+bx^2}\right)\right)+120b^4\log(x)-60b^4\log\left(a+bx^2\right)}{24a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)^2),x]
```

```
[Out] (a*((-3*a^3)/x^8 + (8*a^2*b)/x^6 - (18*a*b^2)/x^4 + 12*b^3*(4/x^2 + b/(a +
b*x^2))) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/(24*a^6)
```

Maple [A]
time = 0.05, size = 88, normalized size = 0.95

method	result	size
default	$-\frac{b^5\left(-\frac{a}{b\left(b\,x^2+a\right)}+\frac{5\ln\left(b\,x^2+a\right)}{b}\right)}{2a^6}-\frac{1}{8a^2x^8}+\frac{5b^4\ln(x)}{a^6}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}+\frac{b}{3a^3x^6}$	88
norman	$\frac{-\frac{1}{8a}+\frac{5b\,x^2}{24a^2}-\frac{5b^2x^4}{12a^3}+\frac{5b^3x^6}{4a^4}-\frac{5b^5x^{10}}{2a^6}}{x^8\left(b\,x^2+a\right)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln\left(b\,x^2+a\right)}{2a^6}$	89
risch	$\frac{\frac{5b^4x^8}{2a^5}+\frac{5b^3x^6}{4a^4}-\frac{5b^2x^4}{12a^3}+\frac{5b\,x^2}{24a^2}-\frac{1}{8a}}{x^8\left(b\,x^2+a\right)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln\left(b\,x^2+a\right)}{2a^6}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^5/a^6*(-a/b/(b*x^2+a)+5*ln(b*x^2+a)/b)-1/8/a^2/x^8+5*b^4*ln(x)/a^6+2
*b^3/a^5/x^2-3/4*b^2/a^4/x^4+1/3*b/a^3/x^6
```

Maxima [A]
time = 0.28, size = 92, normalized size = 0.99

$$\frac{60\,b^4x^8+30\,ab^3x^6-10\,a^2b^2x^4+5\,a^3bx^2-3\,a^4}{24\left(a^5bx^{10}+a^6x^8\right)}-\frac{5\,b^4\log\left(bx^2+a\right)}{2\,a^6}+\frac{5\,b^4\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(60*b^4*x^8 + 30*a*b^3*x^6 - 10*a^2*b^2*x^4 + 5*a^3*b*x^2 - 3*a^4)/(a^
5*b*x^10 + a^6*x^8) - 5/2*b^4*log(b*x^2 + a)/a^6 + 5/2*b^4*log(x^2)/a^6
```

Fricas [A]
time = 1.19, size = 112, normalized size = 1.20

$$\frac{60\,ab^4x^8+30\,a^2b^3x^6-10\,a^3b^2x^4+5\,a^4bx^2-3\,a^5-60\left(b^5x^{10}+ab^4x^8\right)\log\left(bx^2+a\right)+120\left(b^5x^{10}+ab^4x^8\right)\log\left(x\right)}{24\left(a^6bx^{10}+a^7x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(60*a*b^4*x^8 + 30*a^2*b^3*x^6 - 10*a^3*b^2*x^4 + 5*a^4*b*x^2 - 3*a^5
- 60*(b^5*x^10 + a*b^4*x^8)*log(b*x^2 + a) + 120*(b^5*x^10 + a*b^4*x^8)*log
(x))/(a^6*b*x^10 + a^7*x^8)
```

Sympy [A]
time = 0.24, size = 94, normalized size = 1.01

$$\frac{-3a^4+5a^3bx^2-10a^2b^2x^4+30ab^3x^6+60b^4x^8}{24a^6x^8+24a^5bx^{10}}+\frac{5b^4\log{(x)}}{a^6}-\frac{5b^4\log{\left(\frac{a}{b}+x^2\right)}}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x**2 - 10*a**2*b**2*x**4 + 30*a*b**3*x**6 + 60*b**4*x**
8)/(24*a**6*x**8 + 24*a**5*b*x**10) + 5*b**4*log(x)/a**6 - 5*b**4*log(a/b +
 x**2)/(2*a**6)
```

Giac [A]
time = 1.79, size = 110, normalized size = 1.18

$$\frac{5\,b^4\log\left(x^2\right)}{2\,a^6}-\frac{5\,b^4\log\left(|bx^2+a|\right)}{2\,a^6}+\frac{5\,b^5x^2+6\,ab^4}{2\,(bx^2+a)a^6}-\frac{125\,b^4x^8-48\,ab^3x^6+18\,a^2b^2x^4-8\,a^3bx^2+3\,a^4}{24\,a^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^4*log(x^2)/a^6 - 5/2*b^4*log(abs(b*x^2 + a))/a^6 + 1/2*(5*b^5*x^2 + 6
*a*b^4)/((b*x^2 + a)*a^6) - 1/24*(125*b^4*x^8 - 48*a*b^3*x^6 + 18*a^2*b^2*x
^4 - 8*a^3*b*x^2 + 3*a^4)/(a^6*x^8)
```

Mupad [B]
time = 4.73, size = 89, normalized size = 0.96

$$\frac{\frac{5\,b\,x^2}{24\,a^2}-\frac{1}{8\,a}-\frac{5\,b^2\,x^4}{12\,a^3}+\frac{5\,b^3\,x^6}{4\,a^4}+\frac{5\,b^4\,x^8}{2\,a^5}}{b\,x^{10}+a\,x^8}-\frac{5\,b^4\,\ln\left(b\,x^2+a\right)}{2\,a^6}+\frac{5\,b^4\,\ln\left(x\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^2)^2),x)
```

```
[Out] ((5*b*x^2)/(24*a^2) - 1/(8*a) - (5*b^2*x^4)/(12*a^3) + (5*b^3*x^6)/(4*a^4)
+ (5*b^4*x^8)/(2*a^5))/(a*x^8 + b*x^10) - (5*b^4*log(a + b*x^2))/(2*a^6) +
(5*b^4*log(x))/a^6
```

3.168 $\int \frac{x^{15}}{(a+bx^2)^3} dx$

Optimal. Leaf size=114

$$\frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3} + \frac{a^7}{4b^8(a+bx^2)^2} - \frac{7a^6}{2b^8(a+bx^2)} - \frac{21a^5\log(a+bx^2)}{2b^8}$$

```
[Out] 15/2*a^4*x^2/b^7-5/2*a^3*x^4/b^6+a^2*x^6/b^5-3/8*a*x^8/b^4+1/10*x^10/b^3+1/
4*a^7/b^8/(b*x^2+a)^2-7/2*a^6/b^8/(b*x^2+a)-21/2*a^5*ln(b*x^2+a)/b^8
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^7}{4b^8(a+bx^2)^2} - \frac{7a^6}{2b^8(a+bx^2)} - \frac{21a^5\log(a+bx^2)}{2b^8} + \frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^2)^3,x]
```

```
[Out] (15*a^4*x^2)/(2*b^7) - (5*a^3*x^4)/(2*b^6) + (a^2*x^6)/b^5 - (3*a*x^8)/(8*b
^4) + x^10/(10*b^3) + a^7/(4*b^8*(a + b*x^2)^2) - (7*a^6)/(2*b^8*(a + b*x^2
)) - (21*a^5*Log[a + b*x^2])/(2*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{(a+bx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^7}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{15a^4}{b^7} - \frac{10a^3x}{b^6} + \frac{6a^2x^2}{b^5} - \frac{3ax^3}{b^4} + \frac{x^4}{b^3} - \frac{a^7}{b^7(a+bx)^3} + \frac{7a^6}{b^7(a+bx)^2} - \frac{2}{b^7(a}\right.\right.$$
$$= \frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3} + \frac{a^7}{4b^8(a+bx^2)^2} - \frac{7a^6}{2b^8(a+bx^2)} - \frac{21a^5\log(a}{2b^8}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 0.85

$$\frac{300a^4bx^2 - 100a^3b^2x^4 + 40a^2b^3x^6 - 15ab^4x^8 + 4b^5x^{10} + \frac{10a^7}{(a+bx^2)^2} - \frac{140a^6}{a+bx^2} - 420a^5\log(a+bx^2)}{40b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^2)^3,x]
```

```
[Out] (300*a^4*b*x^2 - 100*a^3*b^2*x^4 + 40*a^2*b^3*x^6 - 15*a*b^4*x^8 + 4*b^5*x^
10 + (10*a^7)/(a + b*x^2)^2 - (140*a^6)/(a + b*x^2) - 420*a^5*Log[a + b*x^2
])/(40*b^8)
```

Maple [A]
time = 0.05, size = 105, normalized size = 0.92

method	result	size
risch	$\frac{x^{10}}{10b^3} - \frac{3a\,x^8}{8b^4} + \frac{a^2x^6}{b^5} - \frac{5a^3x^4}{2b^6} + \frac{15a^4x^2}{2b^7} + \frac{-\frac{7a^6x^2}{2} - \frac{13a^7}{4b}}{b^7(b\,x^2+a)^2} - \frac{21a^5\ln(b\,x^2+a)}{2b^8}$	97
norman	$\frac{\frac{x^{14}}{10b} - \frac{7a\,x^{12}}{40b^2} + \frac{7a^2x^{10}}{20b^3} - \frac{63a^7}{4b^8} - \frac{7a^3x^8}{8b^4} + \frac{7a^4x^6}{2b^5} - \frac{21a^6x^2}{b^7}}{(b\,x^2+a)^2} - \frac{21a^5\ln(b\,x^2+a)}{2b^8}$	98
default	$\frac{\frac{1}{10}b^4x^{10} - \frac{3}{8}a\,b^3x^8 + a^2b^2x^6 - \frac{5}{2}a^3b\,x^4 + \frac{15}{2}a^4x^2}{b^7} - \frac{a^5\left(\frac{7a}{b(b\,x^2+a)} + \frac{21\ln(b\,x^2+a)}{b} - \frac{a^2}{2b(b\,x^2+a)^2}\right)}{2b^7}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/10*b^4*x^10-3/8*a*b^3*x^8+a^2*b^2*x^6-5/2*a^3*b*x^4+15/2*a^4*x^2)-
1/2*a^5/b^7*(7*a/b/(b*x^2+a)+21*ln(b*x^2+a)/b-1/2/b*a^2/(b*x^2+a)^2)
```

Maxima [A]
time = 0.28, size = 111, normalized size = 0.97

$$-\frac{14\,a^6bx^2 + 13\,a^7}{4\,(b^{10}x^4 + 2\,ab^9x^2 + a^2b^8)} - \frac{21\,a^5\log(bx^2+a)}{2\,b^8} + \frac{4\,b^4x^{10} - 15\,ab^3x^8 + 40\,a^2b^2x^6 - 100\,a^3bx^4 + 300\,a^4x^2}{40\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(14*a^6*b*x^2 + 13*a^7)/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8) - 21/2*a^5*
log(b*x^2 + a)/b^8 + 1/40*(4*b^4*x^10 - 15*a*b^3*x^8 + 40*a^2*b^2*x^6 - 100
*a^3*b*x^4 + 300*a^4*x^2)/b^7
```

Fricas [A]
time = 1.50, size = 137, normalized size = 1.20

$$\frac{4\,b^7x^{14}-7\,ab^6x^{12}+14\,a^2b^5x^{10}-35\,a^3b^4x^8+140\,a^4b^3x^6+500\,a^5b^2x^4+160\,a^6bx^2-130\,a^7-420\,(a^5b^2x^4+2\,a^6bx^2+a^7)\log\left(bx^2+a\right)}{40\,(b^{10}x^4+2\,ab^9x^2+a^2b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(4*b^7*x^14 - 7*a*b^6*x^12 + 14*a^2*b^5*x^10 - 35*a^3*b^4*x^8 + 140*a^
4*b^3*x^6 + 500*a^5*b^2*x^4 + 160*a^6*b*x^2 - 130*a^7 - 420*(a^5*b^2*x^4 +
2*a^6*b*x^2 + a^7)*log(b*x^2 + a))/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8)
```

Sympy [A]
time = 0.21, size = 119, normalized size = 1.04

$$-\frac{21a^5\log\left(a+bx^2\right)}{2b^8}+\frac{15a^4x^2}{2b^7}-\frac{5a^3x^4}{2b^6}+\frac{a^2x^6}{b^5}-\frac{3ax^8}{8b^4}+\frac{-13a^7-14a^6bx^2}{4a^2b^8+8ab^9x^2+4b^{10}x^4}+\frac{x^{10}}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**2+a)**3,x)
```

```
[Out] -21*a**5*log(a + b*x**2)/(2*b**8) + 15*a**4*x**2/(2*b**7) - 5*a**3*x**4/(2*
b**6) + a**2*x**6/b**5 - 3*a*x**8/(8*b**4) + (-13*a**7 - 14*a**6*b*x**2)/(4
*a**2*b**8 + 8*a*b**9*x**2 + 4*b**10*x**4) + x**10/(10*b**3)
```

Giac [A]
time = 1.31, size = 114, normalized size = 1.00

$$-\frac{21\,a^5\log\left(|bx^2+a|\right)}{2\,b^8}+\frac{63\,a^5b^2x^4+112\,a^6bx^2+50\,a^7}{4\,(bx^2+a)^2b^8}+\frac{4\,b^{12}x^{10}-15\,ab^{11}x^8+40\,a^2b^{10}x^6-100\,a^3b^9x^4+300\,a^4b^8x^2}{40\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -21/2*a^5*log(abs(b*x^2 + a))/b^8 + 1/4*(63*a^5*b^2*x^4 + 112*a^6*b*x^2 + 5
0*a^7)/((b*x^2 + a)^2*b^8) + 1/40*(4*b^12*x^10 - 15*a*b^11*x^8 + 40*a^2*b^1
0*x^6 - 100*a^3*b^9*x^4 + 300*a^4*b^8*x^2)/b^15
```

Mupad [B]
time = 4.72, size = 111, normalized size = 0.97

$$\frac{x^{10}}{10\,b^3}-\frac{\frac{13\,a^7}{4\,b}+\frac{7\,a^6\,x^2}{2}}{a^2\,b^7+2\,a\,b^8\,x^2+b^9\,x^4}-\frac{3\,a\,x^8}{8\,b^4}-\frac{21\,a^5\,\ln\left(b\,x^2+a\right)}{2\,b^8}+\frac{a^2\,x^6}{b^5}-\frac{5\,a^3\,x^4}{2\,b^6}+\frac{15\,a^4\,x^2}{2\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a + b*x^2)^3,x)
```

```
[Out] x^10/(10*b^3) - ((13*a^7)/(4*b) + (7*a^6*x^2)/2)/(a^2*b^7 + b^9*x^4 + 2*a*b
^8*x^2) - (3*a*x^8)/(8*b^4) - (21*a^5*log(a + b*x^2))/(2*b^8) + (a^2*x^6)/b
^5 - (5*a^3*x^4)/(2*b^6) + (15*a^4*x^2)/(2*b^7)
```

3.169 $\int \frac{x^{13}}{(a+bx^2)^3} dx$

Optimal. Leaf size=100

$$-\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}-\frac{a^6}{4b^7(a+bx^2)^2}+\frac{3a^5}{b^7(a+bx^2)}+\frac{15a^4\log(a+bx^2)}{2b^7}$$

```
[Out] -5*a^3*x^2/b^6+3/2*a^2*x^4/b^5-1/2*a*x^6/b^4+1/8*x^8/b^3-1/4*a^6/b^7/(b*x^2
+a)^2+3*a^5/b^7/(b*x^2+a)+15/2*a^4*ln(b*x^2+a)/b^7
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^6}{4b^7(a+bx^2)^2}+\frac{3a^5}{b^7(a+bx^2)}+\frac{15a^4\log(a+bx^2)}{2b^7}-\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^3,x]
```

```
[Out] (-5*a^3*x^2)/b^6 + (3*a^2*x^4)/(2*b^5) - (a*x^6)/(2*b^4) + x^8/(8*b^3) - a^
6/(4*b^7*(a + b*x^2)^2) + (3*a^5)/(b^7*(a + b*x^2)) + (15*a^4*Log[a + b*x^2
])/(2*b^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{13}}{(a+bx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{10a^3}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{b^4}+\frac{x^3}{b^3}+\frac{a^6}{b^6(a+bx)^3}-\frac{6a^5}{b^6(a+bx)^2}+\frac{15a^4}{b^6(a+bx)}\right)\right)$$
$$= -\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}-\frac{a^6}{4b^7(a+bx^2)^2}+\frac{3a^5}{b^7(a+bx^2)}+\frac{15a^4\log(a+bx^2)}{2b^7}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.85

$$\frac{-40a^3bx^2+12a^2b^2x^4-4ab^3x^6+b^4x^8-\frac{2a^6}{(a+bx^2)^2}+\frac{24a^5}{a+bx^2}+60a^4\log(a+bx^2)}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^3,x]
```

```
[Out] (-40*a^3*b*x^2 + 12*a^2*b^2*x^4 - 4*a*b^3*x^6 + b^4*x^8 - (2*a^6)/(a + b*x^
2)^2 + (24*a^5)/(a + b*x^2) + 60*a^4*Log[a + b*x^2])/(8*b^7)
```

Maple [A]
time = 0.05, size = 96, normalized size = 0.96

method	result	size
norman	$\frac{\frac{15a^5x^2}{b^6}+\frac{x^{12}}{8b}-\frac{a\,x^{10}}{4b^2}+\frac{5a^2x^8}{8b^3}+\frac{45a^6}{4b^7}-\frac{5a^3x^6}{2b^4}}{(b\,x^2+a)^2}+\frac{15a^4\ln(b\,x^2+a)}{2b^7}$	87
risch	$\frac{x^8}{8b^3}-\frac{a\,x^6}{2b^4}+\frac{3a^2x^4}{2b^5}-\frac{5a^3x^2}{b^6}+\frac{3a^5x^2+\frac{11a^6}{4b}}{b^6(b\,x^2+a)^2}+\frac{15a^4\ln(b\,x^2+a)}{2b^7}$	87
default	$-\frac{-\frac{1}{8}b^3x^8+\frac{1}{2}a\,b^2x^6-\frac{3}{2}a^2b\,x^4+5a^3x^2}{b^6}+\frac{a^4\left(\frac{6a}{b\left(b\,x^2+a\right)}+\frac{15\ln\left(b\,x^2+a\right)}{b}-\frac{a^2}{2b\left(b\,x^2+a\right)^2}\right)}{2b^6}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/8*b^3*x^8+1/2*a*b^2*x^6-3/2*a^2*b*x^4+5*a^3*x^2)+1/2*a^4/b^6*(6*
a/b/(b*x^2+a)+15*ln(b*x^2+a)/b-1/2/b*a^2/(b*x^2+a)^2)
```

Maxima [A]
time = 0.32, size = 99, normalized size = 0.99

$$\frac{12\,a^5bx^2+11\,a^6}{4\,(b^9x^4+2\,ab^8x^2+a^2b^7)}+\frac{15\,a^4\log(bx^2+a)}{2\,b^7}+\frac{b^3x^8-4\,ab^2x^6+12\,a^2bx^4-40\,a^3x^2}{8\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*a^5*b*x^2 + 11*a^6)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7) + 15/2*a^4*lo
g(b*x^2 + a)/b^7 + 1/8*(b^3*x^8 - 4*a*b^2*x^6 + 12*a^2*b*x^4 - 40*a^3*x^2)/
b^6
```

Fricas [A]
time = 1.43, size = 125, normalized size = 1.25

$$\frac{b^6x^{12} - 2\,ab^5x^{10} + 5\,a^2b^4x^8 - 20\,a^3b^3x^6 - 68\,a^4b^2x^4 - 16\,a^5bx^2 + 22\,a^6 + 60\,(a^4b^2x^4 + 2\,a^5bx^2 + a^6)\log\left(bx^2 + a\right)}{8\,(b^9x^4 + 2\,ab^8x^2 + a^2b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(b^6*x^12 - 2*a*b^5*x^10 + 5*a^2*b^4*x^8 - 20*a^3*b^3*x^6 - 68*a^4*b^2*
x^4 - 16*a^5*b*x^2 + 22*a^6 + 60*(a^4*b^2*x^4 + 2*a^5*b*x^2 + a^6)*log(b*x^
2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

Sympy [A]
time = 0.20, size = 104, normalized size = 1.04

$$\frac{15a^4\log\left(a + bx^2\right)}{2b^7} - \frac{5a^3x^2}{b^6} + \frac{3a^2x^4}{2b^5} - \frac{ax^6}{2b^4} + \frac{11a^6 + 12a^5bx^2}{4a^2b^7 + 8ab^8x^2 + 4b^9x^4} + \frac{x^8}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**3,x)
```

```
[Out] 15*a**4*log(a + b*x**2)/(2*b**7) - 5*a**3*x**2/b**6 + 3*a**2*x**4/(2*b**5)
- a*x**6/(2*b**4) + (11*a**6 + 12*a**5*b*x**2)/(4*a**2*b**7 + 8*a*b**8*x**2
 + 4*b**9*x**4) + x**8/(8*b**3)
```

Giac [A]
time = 1.24, size = 102, normalized size = 1.02

$$\frac{15\,a^4\log\left(|bx^2 + a|\right)}{2\,b^7} - \frac{45\,a^4b^2x^4 + 78\,a^5bx^2 + 34\,a^6}{4\,(bx^2 + a)^2b^7} + \frac{b^9x^8 - 4\,ab^8x^6 + 12\,a^2b^7x^4 - 40\,a^3b^6x^2}{8\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 15/2*a^4*log(abs(b*x^2 + a))/b^7 - 1/4*(45*a^4*b^2*x^4 + 78*a^5*b*x^2 + 34*
a^6)/((b*x^2 + a)^2*b^7) + 1/8*(b^9*x^8 - 4*a*b^8*x^6 + 12*a^2*b^7*x^4 - 40
*a^3*b^6*x^2)/b^12
```

Mupad [B]
time = 0.08, size = 100, normalized size = 1.00

$$\frac{\frac{11\,a^6}{4\,b}+3\,a^5\,x^2}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}+\frac{x^8}{8\,b^3}-\frac{a\,x^6}{2\,b^4}+\frac{15\,a^4\,\ln\left(b\,x^2+a\right)}{2\,b^7}+\frac{3\,a^2\,x^4}{2\,b^5}-\frac{5\,a^3\,x^2}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^2)^3,x)
```

```
[Out] ((11*a^6)/(4*b) + 3*a^5*x^2)/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) + x^8/(8*b^3
) - (a*x^6)/(2*b^4) + (15*a^4*log(a + b*x^2))/(2*b^7) + (3*a^2*x^4)/(2*b^5)
 - (5*a^3*x^2)/b^6
```

3.170 $\int \frac{x^{11}}{(a+bx^2)^3} dx$

Optimal. Leaf size=87

$$\frac{3a^2x^2}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^6}{6b^3} + \frac{a^5}{4b^6(a+bx^2)^2} - \frac{5a^4}{2b^6(a+bx^2)} - \frac{5a^3\log(a+bx^2)}{b^6}$$

```
[Out] 3*a^2*x^2/b^5-3/4*a*x^4/b^4+1/6*x^6/b^3+1/4*a^5/b^6/(b*x^2+a)^2-5/2*a^4/b^6
/(b*x^2+a)-5*a^3*ln(b*x^2+a)/b^6
```

Rubi [A]

time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^5}{4b^6(a+bx^2)^2} - \frac{5a^4}{2b^6(a+bx^2)} - \frac{5a^3\log(a+bx^2)}{b^6} + \frac{3a^2x^2}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^3,x]
```

```
[Out] (3*a^2*x^2)/b^5 - (3*a*x^4)/(4*b^4) + x^6/(6*b^3) + a^5/(4*b^6*(a + b*x^2)^
2) - (5*a^4)/(2*b^6*(a + b*x^2)) - (5*a^3*Log[a + b*x^2])/b^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(a+bx^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{(a+bx)^3} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{6a^2}{b^5} - \frac{3ax}{b^4} + \frac{x^2}{b^3} - \frac{a^5}{b^5(a+bx)^3} + \frac{5a^4}{b^5(a+bx)^2} - \frac{10a^3}{b^5(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{3a^2x^2}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^6}{6b^3} + \frac{a^5}{4b^6(a+bx^2)^2} - \frac{5a^4}{2b^6(a+bx^2)} - \frac{5a^3\log(a+bx^2)}{b^6}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 75, normalized size = 0.86

$$\frac{36a^2bx^2 - 9ab^2x^4 + 2b^3x^6 + \frac{3a^5}{(a+bx^2)^2} - \frac{30a^4}{a+bx^2} - 60a^3\log(a+bx^2)}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^3,x]
```

```
[Out] (36*a^2*b*x^2 - 9*a*b^2*x^4 + 2*b^3*x^6 + (3*a^5)/(a + b*x^2)^2 - (30*a^4)/
(a + b*x^2) - 60*a^3*Log[a + b*x^2])/(12*b^6)
```

Maple [A]
time = 0.04, size = 84, normalized size = 0.97

method	result	size
norman	$\frac{\frac{x^{10}}{6b}-\frac{5a\,x^8}{12b^2}+\frac{5a^2x^6}{3b^3}-\frac{15a^5}{2b^6}-\frac{10a^4x^2}{b^5}}{(b\,x^2+a)^2}-\frac{5a^3\ln(b\,x^2+a)}{b^6}$	76
risch	$\frac{x^6}{6b^3}-\frac{3a\,x^4}{4b^4}+\frac{3a^2x^2}{b^5}+\frac{-\frac{5a^4x^2}{2}-\frac{9a^5}{4b}}{b^5(b\,x^2+a)^2}-\frac{5a^3\ln(b\,x^2+a)}{b^6}$	76
default	$\frac{\frac{1}{6}b^2x^6-\frac{3}{4}ab\,x^4+3a^2x^2}{b^5}-\frac{a^3\left(\frac{5a}{b\left(b\,x^2+a\right)}+\frac{10\ln\left(b\,x^2+a\right)}{b}-\frac{a^2}{2b\left(b\,x^2+a\right)^2}\right)}{2b^5}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/6*b^2*x^6-3/4*a*b*x^4+3*a^2*x^2)-1/2*a^3/b^5*(5*a/b/(b*x^2+a)+10*l
n(b*x^2+a)/b-1/2/b*a^2/(b*x^2+a)^2)
```

Maxima [A]
time = 0.28, size = 89, normalized size = 1.02

$$-\frac{10\,a^4bx^2 + 9\,a^5}{4\,(b^8x^4 + 2\,ab^7x^2 + a^2b^6)} - \frac{5\,a^3\log(bx^2 + a)}{b^6} + \frac{2\,b^2x^6 - 9\,abx^4 + 36\,a^2x^2}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(10*a^4*b*x^2 + 9*a^5)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) - 5*a^3*log(b
*x^2 + a)/b^6 + 1/12*(2*b^2*x^6 - 9*a*b*x^4 + 36*a^2*x^2)/b^5
```

Fricas [A]
time = 1.57, size = 115, normalized size = 1.32

$$\frac{2\,b^5x^{10} - 5\,ab^4x^8 + 20\,a^2b^3x^6 + 63\,a^3b^2x^4 + 6\,a^4bx^2 - 27\,a^5 - 60\,(a^3b^2x^4 + 2\,a^4bx^2 + a^5)\log(bx^2 + a)}{12\,(b^8x^4 + 2\,ab^7x^2 + a^2b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 - 5*a*b^4*x^8 + 20*a^2*b^3*x^6 + 63*a^3*b^2*x^4 + 6*a^4*b*
x^2 - 27*a^5 - 60*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^8*x^
4 + 2*a*b^7*x^2 + a^2*b^6)
```

Sympy [A]
time = 0.19, size = 92, normalized size = 1.06

$$-\frac{5a^3\log\left(a+bx^2\right)}{b^6}+\frac{3a^2x^2}{b^5}-\frac{3ax^4}{4b^4}+\frac{-9a^5-10a^4bx^2}{4a^2b^6+8ab^7x^2+4b^8x^4}+\frac{x^6}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**3,x)
```

```
[Out] -5*a**3*log(a + b*x**2)/b**6 + 3*a**2*x**2/b**5 - 3*a*x**4/(4*b**4) + (-9*a
**5 - 10*a**4*b*x**2)/(4*a**2*b**6 + 8*a*b**7*x**2 + 4*b**8*x**4) + x**6/(6
*b**3)
```

Giac [A]
time = 1.71, size = 92, normalized size = 1.06

$$-\frac{5\,a^3\log\left(|bx^2+a|\right)}{b^6}+\frac{30\,a^3b^2x^4+50\,a^4bx^2+21\,a^5}{4\,(bx^2+a)^2b^6}+\frac{2\,b^6x^6-9\,ab^5x^4+36\,a^2b^4x^2}{12\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5*a^3*log(abs(b*x^2 + a))/b^6 + 1/4*(30*a^3*b^2*x^4 + 50*a^4*b*x^2 + 21*a^
5)/((b*x^2 + a)^2*b^6) + 1/12*(2*b^6*x^6 - 9*a*b^5*x^4 + 36*a^2*b^4*x^2)/b^
9
```

Mupad [B]
time = 4.49, size = 90, normalized size = 1.03

$$\frac{x^6}{6\,b^3}-\frac{\frac{9\,a^5}{4\,b}+\frac{5\,a^4\,x^2}{2}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}-\frac{3\,a\,x^4}{4\,b^4}-\frac{5\,a^3\,\ln\left(b\,x^2+a\right)}{b^6}+\frac{3\,a^2\,x^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2)^3,x)
```

```
[Out] x^6/(6*b^3) - ((9*a^5)/(4*b) + (5*a^4*x^2)/2)/(a^2*b^5 + b^7*x^4 + 2*a*b^6*
x^2) - (3*a*x^4)/(4*b^4) - (5*a^3*log(a + b*x^2))/b^6 + (3*a^2*x^2)/b^5
```

3.171 $\int \frac{x^9}{(a+bx^2)^3} dx$

Optimal. Leaf size=74

$$-\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}-\frac{a^4}{4b^5(a+bx^2)^2}+\frac{2a^3}{b^5(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{b^5}$$

```
[Out] -3/2*a*x^2/b^4+1/4*x^4/b^3-1/4*a^4/b^5/(b*x^2+a)^2+2*a^3/b^5/(b*x^2+a)+3*a^
2*ln(b*x^2+a)/b^5
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$-\frac{a^4}{4b^5(a+bx^2)^2}+\frac{2a^3}{b^5(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*x^2)/(2*b^4) + x^4/(4*b^3) - a^4/(4*b^5*(a + b*x^2)^2) + (2*a^3)/(b^5
*(a + b*x^2)) + (3*a^2*Log[a + b*x^2])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(a+bx^2)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{3a}{b^4}+\frac{x}{b^3}+\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}-\frac{a^4}{4b^5(a+bx^2)^2}+\frac{2a^3}{b^5(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{b^5}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 0.85

$$\frac{-6abx^2+b^2x^4-\frac{a^4}{(a+bx^2)^2}+\frac{8a^3}{a+bx^2}+12a^2\log(a+bx^2)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^3,x]
```

```
[Out] (-6*a*b*x^2 + b^2*x^4 - a^4/(a + b*x^2)^2 + (8*a^3)/(a + b*x^2) + 12*a^2*Lo
g[a + b*x^2])/(4*b^5)
```

Maple [A]
time = 0.06, size = 73, normalized size = 0.99

method	result	size
norman	$\frac{\frac{x^8}{4b}-\frac{a\,x^6}{b^2}+\frac{9a^4}{2b^5}+\frac{6a^3x^2}{b^4}}{(b\,x^2+a)^2}+\frac{3a^2\ln(b\,x^2+a)}{b^5}$	65
default	$\frac{(-b\,x^2+3a)^2}{4b^5}+\frac{a^2\left(\frac{4a}{b(b\,x^2+a)}+\frac{6\ln(b\,x^2+a)}{b}-\frac{a^2}{2b(b\,x^2+a)^2}\right)}{2b^4}$	73
risch	$\frac{x^4}{4b^3}-\frac{3a\,x^2}{2b^4}+\frac{9a^2}{4b^5}+\frac{2a^3x^2+\frac{7a^4}{4b}}{b^4(b\,x^2+a)^2}+\frac{3a^2\ln(b\,x^2+a)}{b^5}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-b*x^2+3*a)^2/b^5+1/2*a^2/b^4*(4*a/b/(b*x^2+a)+6*ln(b*x^2+a)/b-1/2/b*a
^2/(b*x^2+a)^2)
```

Maxima [A]
time = 0.31, size = 77, normalized size = 1.04

$$\frac{8\,a^3bx^2+7\,a^4}{4\,(b^7x^4+2\,ab^6x^2+a^2b^5)}+\frac{3\,a^2\log(bx^2+a)}{b^5}+\frac{bx^4-6\,ax^2}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*b*x^2 + 7*a^4)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) + 3*a^2*log(b*x
^2 + a)/b^5 + 1/4*(b*x^4 - 6*a*x^2)/b^4
```

Fricas [A]
time = 1.32, size = 103, normalized size = 1.39

$$\frac{b^4x^8-4\,ab^3x^6-11\,a^2b^2x^4+2\,a^3bx^2+7\,a^4+12\,(a^2b^2x^4+2\,a^3bx^2+a^4)\log(bx^2+a)}{4\,(b^7x^4+2\,ab^6x^2+a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^8 - 4*a*b^3*x^6 - 11*a^2*b^2*x^4 + 2*a^3*b*x^2 + 7*a^4 + 12*(a^2
*b^2*x^4 + 2*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*
b^5)
```

Sympy [A]
time = 0.18, size = 78, normalized size = 1.05

$$\frac{3a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{2b^4}+\frac{7a^4+8a^3bx^2}{4a^2b^5+8ab^6x^2+4b^7x^4}+\frac{x^4}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**3,x)
```

```
[Out] 3*a**2*log(a + b*x**2)/b**5 - 3*a*x**2/(2*b**4) + (7*a**4 + 8*a**3*b*x**2)/
(4*a**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4) + x**4/(4*b**3)
```

Giac [A]
time = 1.40, size = 80, normalized size = 1.08

$$\frac{3\,a^2\log\left(|bx^2+a|\right)}{b^5}+\frac{b^3x^4-6\,ab^2x^2}{4\,b^6}-\frac{18\,a^2b^2x^4+28\,a^3bx^2+11\,a^4}{4\,(bx^2+a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x^2 + a))/b^5 + 1/4*(b^3*x^4 - 6*a*b^2*x^2)/b^6 - 1/4*(18*a
^2*b^2*x^4 + 28*a^3*b*x^2 + 11*a^4)/((b*x^2 + a)^2*b^5)
```

Mupad [B]
time = 0.08, size = 78, normalized size = 1.05

$$\frac{\frac{7\,a^4}{4\,b}+2\,a^3\,x^2}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{x^4}{4\,b^3}-\frac{3\,a\,x^2}{2\,b^4}+\frac{3\,a^2\,\ln\left(b\,x^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^3,x)
```

```
[Out] ((7*a^4)/(4*b) + 2*a^3*x^2)/(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + x^4/(4*b^3)
 - (3*a*x^2)/(2*b^4) + (3*a^2*log(a + b*x^2))/b^5
```

3.172 $\int \frac{x^7}{(a+bx^2)^3} dx$

Optimal. Leaf size=65

$$\frac{x^2}{2b^3} + \frac{a^3}{4b^4 (a + bx^2)^2} - \frac{3a^2}{2b^4 (a + bx^2)} - \frac{3a \log (a + bx^2)}{2b^4}$$

```
[Out] 1/2*x^2/b^3+1/4*a^3/b^4/(b*x^2+a)^2-3/2*a^2/b^4/(b*x^2+a)-3/2*a*ln(b*x^2+a)
/b^4
```

Rubi [A]

time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^3}{4b^4 (a + bx^2)^2} - \frac{3a^2}{2b^4 (a + bx^2)} - \frac{3a \log (a + bx^2)}{2b^4} + \frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^3,x]
```

```
[Out] x^2/(2*b^3) + a^3/(4*b^4*(a + b*x^2)^2) - (3*a^2)/(2*b^4*(a + b*x^2)) - (3*
a*Log[a + b*x^2])/(2*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a + bx^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a + bx)^3} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^3} - \frac{a^3}{b^3(a + bx)^3} + \frac{3a^2}{b^3(a + bx)^2} - \frac{3a}{b^3(a + bx)}\right) dx, x, x^2\right) \\
&= \frac{x^2}{2b^3} + \frac{a^3}{4b^4 (a + bx^2)^2} - \frac{3a^2}{2b^4 (a + bx^2)} - \frac{3a \log (a + bx^2)}{2b^4}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.74

$$-\frac{-2bx^2+\frac{a^2(5a+6bx^2)}{(a+bx^2)^2}+6a\log(a+bx^2)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(-2*b*x^2 + (a^2*(5*a + 6*b*x^2))/(a + b*x^2)^2 + 6*a*Log[a + b*x^2])/
b^4
```

Maple [A]
time = 0.04, size = 62, normalized size = 0.95

method	result	size
norman	$\frac{\frac{x^6}{2b}-\frac{9a^3}{4b^4}-\frac{3a^2x^2}{b^3}}{(b\,x^2+a)^2}-\frac{3a\ln(b\,x^2+a)}{2b^4}$	54
risch	$\frac{x^2}{2b^3}+\frac{-\frac{3a^2x^2}{2}-\frac{5a^3}{4b}}{b^3(b\,x^2+a)^2}-\frac{3a\ln(b\,x^2+a)}{2b^4}$	54
default	$\frac{x^2}{2b^3}-\frac{a\left(\frac{3a}{b(b\,x^2+a)}+\frac{3\ln(b\,x^2+a)}{b}-\frac{a^2}{2b(b\,x^2+a)^2}\right)}{2b^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b^3-1/2*a/b^3*(3*a/b/(b*x^2+a)+3*ln(b*x^2+a)/b-1/2/b*a^2/(b*x^2+a)^
2)
```

Maxima [A]
time = 0.30, size = 66, normalized size = 1.02

$$-\frac{6\,a^2bx^2+5\,a^3}{4\,(b^6x^4+2\,ab^5x^2+a^2b^4)}+\frac{x^2}{2\,b^3}-\frac{3\,a\log(bx^2+a)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3
- 3/2*a*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 1.87, size = 91, normalized size = 1.40

$$\frac{2\,b^3x^6+4\,ab^2x^4-4\,a^2bx^2-5\,a^3-6\,(ab^2x^4+2\,a^2bx^2+a^3)\log(bx^2+a)}{4\,(b^6x^4+2\,ab^5x^2+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*b
*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [A]
time = 0.17, size = 68, normalized size = 1.05

$$-\frac{3a\log\left(a+bx^2\right)}{2b^4}+\frac{-5a^3-6a^2bx^2}{4a^2b^4+8ab^5x^2+4b^6x^4}+\frac{x^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**3,x)
```

```
[Out] -3*a*log(a + b*x**2)/(2*b**4) + (-5*a**3 - 6*a**2*b*x**2)/(4*a**2*b**4 + 8*
a*b**5*x**2 + 4*b**6*x**4) + x**2/(2*b**3)
```

Giac [A]
time = 1.11, size = 62, normalized size = 0.95

$$\frac{x^2}{2\,b^3}-\frac{3\,a\log\left(|bx^2+a|\right)}{2\,b^4}+\frac{9\,ab^2x^4+12\,a^2bx^2+4\,a^3}{4\,(bx^2+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^3 - 3/2*a*log(abs(b*x^2 + a))/b^4 + 1/4*(9*a*b^2*x^4 + 12*a^2*b*x
^2 + 4*a^3)/((b*x^2 + a)^2*b^4)
```

Mupad [B]
time = 4.75, size = 68, normalized size = 1.05

$$\frac{x^2}{2\,b^3}-\frac{\frac{5\,a^3}{4\,b}+\frac{3\,a^2\,x^2}{2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}-\frac{3\,a\,\ln\left(b\,x^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^3,x)
```

```
[Out] x^2/(2*b^3) - ((5*a^3)/(4*b) + (3*a^2*x^2)/2)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*
x^2) - (3*a*log(a + b*x^2))/(2*b^4)
```

3.173 $\int \frac{x^5}{(a+bx^2)^3} dx$

Optimal. Leaf size=49

$$-\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] -1/4*a^2/b^3/(b*x^2+a)^2+a/b^3/(b*x^2+a)+1/2*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$-\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^3,x]
```

```
[Out] -1/4*a^2/(b^3*(a + b*x^2)^2) + a/(b^3*(a + b*x^2)) + Log[a + b*x^2]/(2*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)\,dx,x,x^2\right)\\
&= -\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.80

$$\frac{\frac{a(3a+4bx^2)}{(a+bx^2)^2}+2\log(a+bx^2)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^3,x]
[Out] ((a*(3*a + 4*b*x^2))/(a + b*x^2)^2 + 2*Log[a + b*x^2])/(4*b^3)
```

Maple [A]
time = 0.03, size = 46, normalized size = 0.94

method	result	size
norman	$\frac{\frac{a\,x^2}{b^2}+\frac{3a^2}{4b^3}}{(b\,x^2+a)^2}+\frac{\ln(b\,x^2+a)}{2b^3}$	42
risch	$\frac{\frac{a\,x^2}{b^2}+\frac{3a^2}{4b^3}}{(b\,x^2+a)^2}+\frac{\ln(b\,x^2+a)}{2b^3}$	42
default	$-\frac{a^2}{4b^3(b\,x^2+a)^2}+\frac{a}{b^3(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/4*a^2/b^3/(b*x^2+a)^2+a/b^3/(b*x^2+a)+1/2*ln(b*x^2+a)/b^3
```

Maxima [A]
time = 0.28, size = 55, normalized size = 1.12

$$\frac{4\,abx^2+3\,a^2}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)}+\frac{\log(bx^2+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*log(b*x^2 +
 a)/b^3
```

Fricas [A]
time = 1.46, size = 69, normalized size = 1.41

$$\frac{4\,abx^2+3\,a^2+2\,(b^2x^4+2\,abx^2+a^2)\log(bx^2+a)}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^5
*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [**A**]
time = 0.13, size = 53, normalized size = 1.08

$$\frac{3a^2 + 4abx^2}{4a^2b^3 + 8ab^4x^2 + 4b^5x^4} + \frac{\log(a + bx^2)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x**2)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + log(a +
 b*x**2)/(2*b**3)
```

Giac [**A**]
time = 1.48, size = 42, normalized size = 0.86

$$\frac{\log(|bx^2 + a|)}{2\,b^3} - \frac{3\,bx^4 + 2\,ax^2}{4\,(bx^2 + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^3 - 1/4*(3*b*x^4 + 2*a*x^2)/((b*x^2 + a)^2*b^2)
```

Mupad [**B**]
time = 0.06, size = 52, normalized size = 1.06

$$\frac{\frac{3\,a^2}{4\,b^3} + \frac{a\,x^2}{b^2}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4} + \frac{\ln(b\,x^2 + a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^3,x)
```

```
[Out] ((3*a^2)/(4*b^3) + (a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) + log(a + b*x^2
)/(2*b^3)
```

3.174 $\int \frac{x^3}{(a+bx^2)^3} dx$

Optimal. Leaf size=19

$$\frac{x^4}{4a(a+bx^2)^2}$$

```
[Out] 1/4*x^4/a/(b*x^2+a)^2
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{x^4}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^3,x]
```

```
[Out] x^4/(4*a*(a + b*x^2)^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^3} dx = \frac{x^4}{4a(a+bx^2)^2}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.26

$$-\frac{a+2bx^2}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(a + 2*b*x^2)/(b^2*(a + b*x^2)^2)
```

Maple [A]
time = 0.03, size = 31, normalized size = 1.63

method	result	size
gosper	$-\frac{2 b \,x^{2}+a}{4 b^{2} \left(b \,x^{2}+a \right)^{2}}$	23
norman	$\frac{-\frac{x^{2}}{2 b}-\frac{a}{4 b^{2}}}{\left(b \,x^{2}+a \right)^{2}}$	26
risch	$\frac{-\frac{x^{2}}{2 b}-\frac{a}{4 b^{2}}}{\left(b \,x^{2}+a \right)^{2}}$	26
default	$-\frac{1}{2 b^{2} \left(b \,x^{2}+a \right)}+\frac{a}{4 b^{2} \left(b \,x^{2}+a \right)^{2}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b^2/(b*x^2+a)+1/4*a/b^2/(b*x^2+a)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 0.27, size = 36, normalized size = 1.89

$$-\frac{2 \, b x^{2} + a}{4 \, {\left(b^{4} x^{4} + 2 \, a b^{3} x^{2} + a^{2} b^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 1.13, size = 36, normalized size = 1.89

$$-\frac{2 \, b x^{2} + a}{4 \, {\left(b^{4} x^{4} + 2 \, a b^{3} x^{2} + a^{2} b^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="fricas")
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.
time = 0.11, size = 36, normalized size = 1.89

$$\frac{- a - 2 b x^{2}}{4 a^{2} b^{2} + 8 a b^{3} x^{2} + 4 b^{4} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**3,x)
```

```
[Out] (-a - 2*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

Giac [A]
time = 2.39, size = 22, normalized size = 1.16

$$-\frac{2\,bx^2+a}{4\,{\left(bx^2+a\right)}^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2)
```

Mupad [B]
time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{a}{4\,b^2}+\frac{x^2}{2\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^3,x)
```

```
[Out] -(a/(4*b^2) + x^2/(2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.175 $\int \frac{x}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{4b(a+bx^2)^2}$$

[Out] -1/4/b/(b*x^2+a)^2

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$-\frac{1}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^2)^3,x]

[Out] -1/4*1/(b*(a + b*x^2)^2)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^3} \, dx = -\frac{1}{4b(a+bx^2)^2}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^2)^3,x]

[Out] -1/4*1/(b*(a + b*x^2)^2)

Maple [A]
time = 0.02, size = 15, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{4b(b\,x^2+a)^2}$	15
derivativedivides	$-\frac{1}{4b(b\,x^2+a)^2}$	15
default	$-\frac{1}{4b(b\,x^2+a)^2}$	15
norman	$-\frac{1}{4b(b\,x^2+a)^2}$	15
risch	$-\frac{1}{4b(b\,x^2+a)^2}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/4/b/(b*x^2+a)^2
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.88

$$-\frac{1}{4\,(bx^2+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="maxima")
[Out] -1/4/((b*x^2 + a)^2*b)
```

Fricas [A]
time = 1.32, size = 26, normalized size = 1.62

$$-\frac{1}{4\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="fricas")
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [A]
time = 0.09, size = 27, normalized size = 1.69

$$-\frac{1}{4a^2b+8ab^2x^2+4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**3,x)
```

```
[Out] -1/(4*a**2*b + 8*a*b**2*x**2 + 4*b**3*x**4)
```

Giac [A]
time = 1.11, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(bx^2+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4/((b*x^2 + a)^2*b)
```

Mupad [B]
time = 4.62, size = 28, normalized size = 1.75

$$-\frac{1}{4\,a^2\,b+8\,a\,b^2\,x^2+4\,b^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^3,x)
```

```
[Out] -1/(4*a^2*b + 4*b^3*x^4 + 8*a*b^2*x^2)
```

3.176 $\int \frac{1}{x(a+bx^2)^3} dx$

Optimal. Leaf size=54

$$\frac{1}{4a\left(a+bx^2\right)^2}+\frac{1}{2a^2\left(a+bx^2\right)}+\frac{\log(x)}{a^3}-\frac{\log\left(a+bx^2\right)}{2a^3}$$

```
[Out] 1/4/a/(b*x^2+a)^2+1/2/a^2/(b*x^2+a)+ln(x)/a^3-1/2*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 46}

$$-\frac{\log\left(a+bx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a^2\left(a+bx^2\right)}+\frac{1}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^3),x]
```

```
[Out] 1/(4*a*(a + b*x^2)^2) + 1/(2*a^2*(a + b*x^2)) + Log[x]/a^3 - Log[a + b*x^2]
/(2*a^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)\,dx,x,x^2\right)\\
&= \frac{1}{4a\left(a+bx^2\right)^2}+\frac{1}{2a^2\left(a+bx^2\right)}+\frac{\log(x)}{a^3}-\frac{\log\left(a+bx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.80

$$\frac{\frac{a(3a+2bx^2)}{(a+bx^2)^2}+4\log(x)-2\log(a+bx^2)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x^2))/(a + b*x^2)^2 + 4*Log[x] - 2*Log[a + b*x^2])/(4*a^3)
```

Maple [A]
time = 0.04, size = 59, normalized size = 1.09

method	result	size
risch	$\frac{\frac{b\,x^2}{2a^2}+\frac{3}{4a}}{(b\,x^2+a)^2}+\frac{\ln(x)}{a^3}-\frac{\ln(b\,x^2+a)}{2a^3}$	46
norman	$\frac{-\frac{b\,x^2}{a^2}-\frac{3b^2x^4}{4a^3}}{(b\,x^2+a)^2}+\frac{\ln(x)}{a^3}-\frac{\ln(b\,x^2+a)}{2a^3}$	52
default	$-\frac{b\left(-\frac{a}{b(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{b}-\frac{a^2}{2b(b\,x^2+a)^2}\right)}{2a^3}+\frac{\ln(x)}{a^3}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*b*(-a/b/(b*x^2+a)+ln(b*x^2+a)/b-1/2/b*a^2/(b*x^2+a)^2)+ln(x)/a^3
```

Maxima [A]
time = 0.28, size = 60, normalized size = 1.11

$$\frac{2\,bx^2+3\,a}{4\,(a^2b^2x^4+2\,a^3bx^2+a^4)}-\frac{\log(bx^2+a)}{2\,a^3}+\frac{\log(x^2)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 + a)/
a^3 + 1/2*log(x^2)/a^3
```

Fricas [A]
time = 0.97, size = 90, normalized size = 1.67

$$\frac{2\,abx^2+3\,a^2-2\,(b^2x^4+2\,abx^2+a^2)\log(bx^2+a)+4\,(b^2x^4+2\,abx^2+a^2)\log(x)}{4\,(a^3b^2x^4+2\,a^4bx^2+a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*(
b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)
```

Sympy [A]
time = 0.18, size = 56, normalized size = 1.04

$$\frac{3a + 2bx^2}{4a^4 + 8a^3bx^2 + 4a^2b^2x^4} + \frac{\log(x)}{a^3} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**3,x)
```

```
[Out] (3*a + 2*b*x**2)/(4*a**4 + 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a**3
- log(a/b + x**2)/(2*a**3)
```

Giac [A]
time = 1.33, size = 59, normalized size = 1.09

$$\frac{\log(x^2)}{2\,a^3} - \frac{\log(|bx^2 + a|)}{2\,a^3} + \frac{3\,b^2x^4 + 8\,abx^2 + 6\,a^2}{4\,(bx^2 + a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 + a))/a^3 + 1/4*(3*b^2*x^4 + 8*a*b*x^2
 + 6*a^2)/((b*x^2 + a)^2*a^3)
```

Mupad [B]
time = 4.68, size = 56, normalized size = 1.04

$$\frac{\ln(x)}{a^3} + \frac{\frac{3}{4\,a} + \frac{b\,x^2}{2\,a^2}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4} - \frac{\ln(b\,x^2 + a)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^3),x)
```

```
[Out] log(x)/a^3 + (3/(4*a) + (b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - log(
a + b*x^2)/(2*a^3)
```

3.177 $\int \frac{1}{x^3(a+bx^2)^3} \, dx$

Optimal. Leaf size=67

$$-\frac{1}{2a^3x^2}-\frac{b}{4a^2\left(a+bx^2\right)^2}-\frac{b}{a^3\left(a+bx^2\right)}-\frac{3b\log(x)}{a^4}+\frac{3b\log\left(a+bx^2\right)}{2a^4}$$

```
[Out] -1/2/a^3/x^2-1/4*b/a^2/(b*x^2+a)^2-b/a^3/(b*x^2+a)-3*b*ln(x)/a^4+3/2*b*ln(b
*x^2+a)/a^4
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\frac{3b\log\left(a+bx^2\right)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{b}{a^3\left(a+bx^2\right)}-\frac{1}{2a^3x^2}-\frac{b}{4a^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*1/(a^3*x^2) - b/(4*a^2*(a + b*x^2)^2) - b/(a^3*(a + b*x^2)) - (3*b*Log
[x])/a^4 + (3*b*Log[a + b*x^2])/(2*a^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2a^3x^2}-\frac{b}{4a^2\left(a+bx^2\right)^2}-\frac{b}{a^3\left(a+bx^2\right)}-\frac{3b\log(x)}{a^4}+\frac{3b\log\left(a+bx^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.88

$$-\frac{\frac{a(2a^2+9abx^2+6b^2x^4)}{x^2(a+bx^2)^2}+12b\log(x)-6b\log(a+bx^2)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^3),x]
[Out] -1/4*((a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4))/(x^2*(a + b*x^2)^2) + 12*b*Log[x]
 - 6*b*Log[a + b*x^2])/a^4
```

Maple [A]
time = 0.05, size = 72, normalized size = 1.07

method	result	size
norman	$\frac{\frac{3b^2x^4}{a^3}-\frac{1}{2a}+\frac{9b^3x^6}{4a^4}}{x^2(bx^2+a)^2}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln(bx^2+a)}{2a^4}$	65
risch	$\frac{-\frac{3b^2x^4}{2a^3}-\frac{9bx^2}{4a^2}-\frac{1}{2a}}{x^2(bx^2+a)^2}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln(-bx^2-a)}{2a^4}$	66
default	$\frac{b^2\left(-\frac{2a}{b(bx^2+a)}+\frac{3\ln(bx^2+a)}{b}-\frac{a^2}{2b(bx^2+a)^2}\right)}{2a^4}-\frac{1}{2a^3x^2}-\frac{3b\ln(x)}{a^4}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/2/a^4*b^2*(-2*a/b/(b*x^2+a)+3*ln(b*x^2+a)/b-1/2/b*a^2/(b*x^2+a)^2)-1/2/a^
3/x^2-3*b*ln(x)/a^4
```

Maxima [A]
time = 0.28, size = 77, normalized size = 1.15

$$-\frac{6\,b^2x^4+9\,abx^2+2\,a^2}{4\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)}+\frac{3\,b\log(bx^2+a)}{2\,a^4}-\frac{3\,b\log(x^2)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="maxima")
[Out] -1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
+ 3/2*b*log(b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^4
```

Fricas [A]
time = 1.02, size = 119, normalized size = 1.78

$$-\frac{6\,ab^2x^4+9\,a^2bx^2+2\,a^3-6\,(b^3x^6+2\,ab^2x^4+a^2bx^2)\log(bx^2+a)+12\,(b^3x^6+2\,ab^2x^4+a^2bx^2)\log(x)}{4\,(a^4b^2x^6+2\,a^5bx^4+a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [A]
time = 0.23, size = 80, normalized size = 1.19

$$\frac{-2a^2 - 9abx^2 - 6b^2x^4}{4a^5x^2 + 8a^4bx^4 + 4a^3b^2x^6} - \frac{3b\log(x)}{a^4} + \frac{3b\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*x**2 - 6*b**2*x**4)/(4*a**5*x**2 + 8*a**4*b*x**4 + 4*a**3*
b**2*x**6) - 3*b*log(x)/a**4 + 3*b*log(a/b + x**2)/(2*a**4)
```

Giac [A]
time = 2.07, size = 82, normalized size = 1.22

$$-\frac{3\,b\log\left(x^2\right)}{2\,a^4} + \frac{3\,b\log\left(|bx^2 + a|\right)}{2\,a^4} - \frac{9\,b^3x^4 + 22\,ab^2x^2 + 14\,a^2b}{4\,(bx^2 + a)^2a^4} + \frac{3\,bx^2 - a}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*b*log(x^2)/a^4 + 3/2*b*log(abs(b*x^2 + a))/a^4 - 1/4*(9*b^3*x^4 + 22*a
*b^2*x^2 + 14*a^2*b)/((b*x^2 + a)^2*a^4) + 1/2*(3*b*x^2 - a)/(a^4*x^2)
```

Mupad [B]
time = 0.08, size = 75, normalized size = 1.12

$$\frac{3\,b\,\ln\left(b\,x^2 + a\right)}{2\,a^4} - \frac{\frac{1}{2\,a} + \frac{9\,b\,x^2}{4\,a^2} + \frac{3\,b^2\,x^4}{2\,a^3}}{a^2\,x^2 + 2\,a\,b\,x^4 + b^2\,x^6} - \frac{3\,b\,\ln\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^3),x)
```

```
[Out] (3*b*log(a + b*x^2))/(2*a^4) - (1/(2*a) + (9*b*x^2)/(4*a^2) + (3*b^2*x^4)/(
2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4) - (3*b*log(x))/a^4
```

3.178 $\int \frac{1}{x^5(a+bx^2)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{1}{4a^3x^4}+\frac{3b}{2a^4x^2}+\frac{b^2}{4a^3\left(a+bx^2\right)^2}+\frac{3b^2}{2a^4\left(a+bx^2\right)}+\frac{6b^2\log(x)}{a^5}-\frac{3b^2\log\left(a+bx^2\right)}{a^5}$$

```
[Out] -1/4/a^3/x^4+3/2*b/a^4/x^2+1/4*b^2/a^3/(b*x^2+a)^2+3/2*b^2/a^4/(b*x^2+a)+6*
b^2*ln(x)/a^5-3*b^2*ln(b*x^2+a)/a^5
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$-\frac{3b^2\log\left(a+bx^2\right)}{a^5}+\frac{6b^2\log(x)}{a^5}+\frac{3b^2}{2a^4\left(a+bx^2\right)}+\frac{3b}{2a^4x^2}+\frac{b^2}{4a^3\left(a+bx^2\right)^2}-\frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^3),x]
```

```
[Out] -1/4*1/(a^3*x^4) + (3*b)/(2*a^4*x^2) + b^2/(4*a^3*(a + b*x^2)^2) + (3*b^2)/
(2*a^4*(a + b*x^2)) + (6*b^2*Log[x])/a^5 - (3*b^2*Log[a + b*x^2])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^3}-\frac{3b}{a^4x^2}+\frac{6b^2}{a^5x}-\frac{b^3}{a^3(a+bx)^3}-\frac{3b^3}{a^4(a+bx)^2}-\frac{6b^3}{a^5(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{4a^3x^4}+\frac{3b}{2a^4x^2}+\frac{b^2}{4a^3\left(a+bx^2\right)^2}+\frac{3b^2}{2a^4\left(a+bx^2\right)}+\frac{6b^2\log(x)}{a^5}-\frac{3b^2\log\left(a+bx^2\right)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.86

$$\frac{\frac{a\left(-a^3+4a^2bx^2+18ab^2x^4+12b^3x^6\right)}{x^4(a+bx^2)^2}+24b^2\log(x)-12b^2\log\left(a+bx^2\right)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x^2 + 18*a*b^2*x^4 + 12*b^3*x^6))/(x^4*(a + b*x^2)^2) +
 24*b^2*Log[x] - 12*b^2*Log[a + b*x^2])/(4*a^5)
```

Maple [A]
time = 0.05, size = 83, normalized size = 0.97

method	result	size
norman	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{4a}-\frac{6b^3x^6}{a^4}-\frac{9b^4x^8}{2a^5}}{x^4\left(b\,x^2+a\right)^2}+\frac{6b^2\ln(x)}{a^5}-\frac{3b^2\ln\left(b\,x^2+a\right)}{a^5}$	77
risch	$\frac{\frac{3b^3x^6}{a^4}+\frac{9b^2x^4}{2a^3}+\frac{b\,x^2}{a^2}-\frac{1}{4a}}{x^4\left(b\,x^2+a\right)^2}+\frac{6b^2\ln(x)}{a^5}-\frac{3b^2\ln\left(b\,x^2+a\right)}{a^5}$	77
default	$-\frac{b^3\left(-\frac{3a}{b\left(b\,x^2+a\right)}+\frac{6\ln\left(b\,x^2+a\right)}{b}-\frac{a^2}{2b\left(b\,x^2+a\right)^2}\right)}{2a^5}-\frac{1}{4a^3x^4}+\frac{3b}{2a^4x^2}+\frac{6b^2\ln(x)}{a^5}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^5*b^3*(-3*a/b/(b*x^2+a)+6*ln(b*x^2+a)/b-1/2/b*a^2/(b*x^2+a)^2)-1/4/a
^3/x^4+3/2*b/a^4/x^2+6*b^2*ln(x)/a^5
```

Maxima [A]
time = 0.29, size = 92, normalized size = 1.07

$$\frac{12\,b^3x^6+18\,ab^2x^4+4\,a^2bx^2-a^3}{4\left(a^4b^2x^8+2\,a^5bx^6+a^6x^4\right)}-\frac{3\,b^2\log\left(bx^2+a\right)}{a^5}+\frac{3\,b^2\log\left(x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*b^3*x^6 + 18*a*b^2*x^4 + 4*a^2*b*x^2 - a^3)/(a^4*b^2*x^8 + 2*a^5*b*
x^6 + a^6*x^4) - 3*b^2*log(b*x^2 + a)/a^5 + 3*b^2*log(x^2)/a^5
```

Fricas [A]
time = 1.00, size = 134, normalized size = 1.56

$$\frac{12\,ab^3x^6+18\,a^2b^2x^4+4\,a^3bx^2-a^4-12\left(b^4x^8+2\,ab^3x^6+a^2b^2x^4\right)\log\left(bx^2+a\right)+24\left(b^4x^8+2\,ab^3x^6+a^2b^2x^4\right)\log\left(x\right)}{4\left(a^5b^2x^8+2\,a^6bx^6+a^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*a*b^3*x^6 + 18*a^2*b^2*x^4 + 4*a^3*b*x^2 - a^4 - 12*(b^4*x^8 + 2*a*
b^3*x^6 + a^2*b^2*x^4)*log(b*x^2 + a) + 24*(b^4*x^8 + 2*a*b^3*x^6 + a^2*b^2
*x^4)*log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)
```

Sympy [A]
time = 0.24, size = 90, normalized size = 1.05

$$\frac{-a^3+4a^2bx^2+18ab^2x^4+12b^3x^6}{4a^6x^4+8a^5bx^6+4a^4b^2x^8}+\frac{6b^2\log(x)}{a^5}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x**2 + 18*a*b**2*x**4 + 12*b**3*x**6)/(4*a**6*x**4 + 8*a*
*5*b*x**6 + 4*a**4*b**2*x**8) + 6*b**2*log(x)/a**5 - 3*b**2*log(a/b + x**2)
/a**5
```

Giac [A]
time = 1.92, size = 80, normalized size = 0.93

$$\frac{3\,b^2\log\left(x^2\right)}{a^5}-\frac{3\,b^2\log\left(|bx^2+a|\right)}{a^5}+\frac{12\,b^3x^6+18\,ab^2x^4+4\,a^2bx^2-a^3}{4\,\left(bx^4+ax^2\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*b^2*log(x^2)/a^5 - 3*b^2*log(abs(b*x^2 + a))/a^5 + 1/4*(12*b^3*x^6 + 18*a
*b^2*x^4 + 4*a^2*b*x^2 - a^3)/((b*x^4 + a*x^2)^2*a^4)
```

Mupad [B]
time = 4.67, size = 88, normalized size = 1.02

$$\frac{\frac{b\,x^2}{a^2}-\frac{1}{4\,a}+\frac{9\,b^2\,x^4}{2\,a^3}+\frac{3\,b^3\,x^6}{a^4}}{a^2\,x^4+2\,a\,b\,x^6+b^2\,x^8}-\frac{3\,b^2\,\ln\left(b\,x^2+a\right)}{a^5}+\frac{6\,b^2\,\ln\left(x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^3),x)
```

```
[Out] ((b*x^2)/a^2 - 1/(4*a) + (9*b^2*x^4)/(2*a^3) + (3*b^3*x^6)/a^4)/(a^2*x^4 +
b^2*x^8 + 2*a*b*x^6) - (3*b^2*log(a + b*x^2))/a^5 + (6*b^2*log(x))/a^5
```

3.179 $\int \frac{1}{x^7(a+bx^2)^3} dx$

Optimal. Leaf size=95

$$-\frac{1}{6a^3x^6}+\frac{3b}{4a^4x^4}-\frac{3b^2}{a^5x^2}-\frac{b^3}{4a^4(a+bx^2)^2}-\frac{2b^3}{a^5(a+bx^2)}-\frac{10b^3\log(x)}{a^6}+\frac{5b^3\log(a+bx^2)}{a^6}$$

```
[Out] -1/6/a^3/x^6+3/4*b/a^4/x^4-3*b^2/a^5/x^2-1/4*b^3/a^4/(b*x^2+a)^2-2*b^3/a^5/
(b*x^2+a)-10*b^3*ln(x)/a^6+5*b^3*ln(b*x^2+a)/a^6
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\frac{5b^3\log(a+bx^2)}{a^6}-\frac{10b^3\log(x)}{a^6}-\frac{2b^3}{a^5(a+bx^2)}-\frac{3b^2}{a^5x^2}-\frac{b^3}{4a^4(a+bx^2)^2}+\frac{3b}{4a^4x^4}-\frac{1}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -1/6*1/(a^3*x^6) + (3*b)/(4*a^4*x^4) - (3*b^2)/(a^5*x^2) - b^3/(4*a^4*(a +
b*x^2)^2) - (2*b^3)/(a^5*(a + b*x^2)) - (10*b^3*Log[x])/a^6 + (5*b^3*Log[a
+ b*x^2])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7(a+bx^2)^3}dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)^3}dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right.\right.\\ &= -\frac{1}{6a^3x^6}+\frac{3b}{4a^4x^4}-\frac{3b^2}{a^5x^2}-\frac{b^3}{4a^4(a+bx^2)^2}-\frac{2b^3}{a^5(a+bx^2)}-\frac{10b^3\log(x)}{a^6}+\frac{5b^3\log(a}{a^6}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 85, normalized size = 0.89

$$-\frac{\frac{a(2a^4-5a^3bx^2+20a^2b^2x^4+90ab^3x^6+60b^4x^8)}{x^6(a+bx^2)^2}+120b^3\log(x)-60b^3\log(a+bx^2)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -1/12*((a*(2*a^4 - 5*a^3*b*x^2 + 20*a^2*b^2*x^4 + 90*a*b^3*x^6 + 60*b^4*x^8
))/(x^6*(a + b*x^2)^2) + 120*b^3*Log[x] - 60*b^3*Log[a + b*x^2])/a^6
```

Maple [A]
time = 0.06, size = 94, normalized size = 0.99

method	result	size
norman	$\frac{-\frac{1}{6a}+\frac{5b\,x^2}{12a^2}-\frac{5b^2x^4}{3a^3}+\frac{10b^4x^8}{a^5}+\frac{15b^5x^{10}}{2a^6}}{x^6(b\,x^2+a)^2}-\frac{10b^3\ln(x)}{a^6}+\frac{5b^3\ln(b\,x^2+a)}{a^6}$	89
risch	$\frac{-\frac{5b^4x^8}{a^5}-\frac{15b^3x^6}{2a^4}-\frac{5b^2x^4}{3a^3}+\frac{5b\,x^2}{12a^2}-\frac{1}{6a}}{x^6(b\,x^2+a)^2}-\frac{10b^3\ln(x)}{a^6}+\frac{5b^3\ln(-b\,x^2-a)}{a^6}$	92
default	$\frac{b^4\left(-\frac{4a}{b(b\,x^2+a)}+\frac{10\ln(b\,x^2+a)}{b}-\frac{a^2}{2b(b\,x^2+a)^2}\right)}{2a^6}-\frac{1}{6a^3x^6}+\frac{3b}{4a^4x^4}-\frac{3b^2}{a^5x^2}-\frac{10b^3\ln(x)}{a^6}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^6*b^4*(-4*a/b/(b*x^2+a)+10*ln(b*x^2+a)/b-1/2/b*a^2/(b*x^2+a)^2)-1/6/a
^3/x^6+3/4*b/a^4/x^4-3*b^2/a^5/x^2-10*b^3*ln(x)/a^6
```

Maxima [A]
time = 0.28, size = 103, normalized size = 1.08

$$-\frac{60\,b^4x^8+90\,ab^3x^6+20\,a^2b^2x^4-5\,a^3bx^2+2\,a^4}{12\,(a^5b^2x^{10}+2\,a^6bx^8+a^7x^6)}+\frac{5\,b^3\log(bx^2+a)}{a^6}-\frac{5\,b^3\log(x^2)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*b^4*x^8 + 90*a*b^3*x^6 + 20*a^2*b^2*x^4 - 5*a^3*b*x^2 + 2*a^4)/(a
^5*b^2*x^10 + 2*a^6*b*x^8 + a^7*x^6) + 5*b^3*log(b*x^2 + a)/a^6 - 5*b^3*log
(x^2)/a^6
```

Fricas [A]
time = 1.38, size = 145, normalized size = 1.53

$$-\frac{60\,ab^4x^8+90\,a^2b^3x^6+20\,a^3b^2x^4-5\,a^4bx^2+2\,a^5-60\,(b^5x^{10}+2\,ab^4x^8+a^2b^3x^6)\log(bx^2+a)+120\,(b^5x^{10}+2\,ab^4x^8+a^2b^3x^6)\log(x)}{12\,(a^6b^2x^{10}+2\,a^7bx^8+a^8x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*a*b^4*x^8 + 90*a^2*b^3*x^6 + 20*a^3*b^2*x^4 - 5*a^4*b*x^2 + 2*a^5
 - 60*(b^5*x^10 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(b*x^2 + a) + 120*(b^5*x^10
 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(x))/(a^6*b^2*x^10 + 2*a^7*b*x^8 + a^8*x^6
)
```

Sympy [A]
time = 0.27, size = 104, normalized size = 1.09

$$\frac{-2a^4+5a^3bx^2-20a^2b^2x^4-90ab^3x^6-60b^4x^8}{12a^7x^6+24a^6bx^8+12a^5b^2x^{10}}-\frac{10b^3\log(x)}{a^6}+\frac{5b^3\log\left(\frac{a}{b}+x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**3,x)
```

```
[Out] (-2*a**4 + 5*a**3*b*x**2 - 20*a**2*b**2*x**4 - 90*a*b**3*x**6 - 60*b**4*x**
8)/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) - 10*b**3*log(x)/a*
*6 + 5*b**3*log(a/b + x**2)/a**6
```

Giac [A]
time = 1.42, size = 110, normalized size = 1.16

$$-\frac{5\,b^3\log(x^2)}{a^6}+\frac{5\,b^3\log(|bx^2+a|)}{a^6}-\frac{30\,b^5x^4+68\,ab^4x^2+39\,a^2b^3}{4\,(bx^2+a)^2a^6}+\frac{110\,b^3x^6-36\,ab^2x^4+9\,a^2bx^2-2\,a^3}{12\,a^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5*b^3*log(x^2)/a^6 + 5*b^3*log(abs(b*x^2 + a))/a^6 - 1/4*(30*b^5*x^4 + 68*
a*b^4*x^2 + 39*a^2*b^3)/((b*x^2 + a)^2*a^6) + 1/12*(110*b^3*x^6 - 36*a*b^2*
x^4 + 9*a^2*b*x^2 - 2*a^3)/(a^6*x^6)
```

Mupad [B]
time = 4.69, size = 101, normalized size = 1.06

$$\frac{5\,b^3\,\ln(b\,x^2+a)}{a^6}-\frac{\frac{1}{6\,a}-\frac{5\,b\,x^2}{12\,a^2}+\frac{5\,b^2\,x^4}{3\,a^3}+\frac{15\,b^3\,x^6}{2\,a^4}+\frac{5\,b^4\,x^8}{a^5}}{a^2\,x^6+2\,a\,b\,x^8+b^2\,x^{10}}-\frac{10\,b^3\,\ln(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^3),x)
```

```
[Out] (5*b^3*log(a + b*x^2))/a^6 - (1/(6*a) - (5*b*x^2)/(12*a^2) + (5*b^2*x^4)/(3
*a^3) + (15*b^3*x^6)/(2*a^4) + (5*b^4*x^8)/a^5)/(a^2*x^6 + b^2*x^10 + 2*a*b
*x^8) - (10*b^3*log(x))/a^6
```

3.180 $\int \frac{1}{x^9(a+bx^2)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{8a^3x^8}+\frac{b}{2a^4x^6}-\frac{3b^2}{2a^5x^4}+\frac{5b^3}{a^6x^2}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}+\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log\left(a+bx^2\right)}{2a^7}$$

```
[Out] -1/8/a^3/x^8+1/2*b/a^4/x^6-3/2*b^2/a^5/x^4+5*b^3/a^6/x^2+1/4*b^4/a^5/(b*x^2
+a)^2+5/2*b^4/a^6/(b*x^2+a)+15*b^4*ln(x)/a^7-15/2*b^4*ln(b*x^2+a)/a^7
```

Rubi [A]
time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$-\frac{15b^4\log\left(a+bx^2\right)}{2a^7}+\frac{15b^4\log(x)}{a^7}+\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{5b^3}{a^6x^2}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}-\frac{3b^2}{2a^5x^4}+\frac{b}{2a^4x^6}-\frac{1}{8a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)^3),x]
```

```
[Out] -1/8*1/(a^3*x^8) + b/(2*a^4*x^6) - (3*b^2)/(2*a^5*x^4) + (5*b^3)/(a^6*x^2)
+ b^4/(4*a^5*(a + b*x^2)^2) + (5*b^4)/(2*a^6*(a + b*x^2)) + (15*b^4*Log[x])
/a^7 - (15*b^4*Log[a + b*x^2])/(2*a^7)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^5(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^5}-\frac{3b}{a^4x^4}+\frac{6b^2}{a^5x^3}-\frac{10b^3}{a^6x^2}+\frac{15b^4}{a^7x}-\frac{b^5}{a^5(a+bx)^3}-\frac{5b^5}{a^6(a+bx)^2}-\cdots\right.\right.$$
$$= -\frac{1}{8a^3x^8}+\frac{b}{2a^4x^6}-\frac{3b^2}{2a^5x^4}+\frac{5b^3}{a^6x^2}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}+\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{15b^4\log(x)}{a^7}-\cdots$$

Mathematica [A]
time = 0.04, size = 96, normalized size = 0.86

$$\frac{\frac{a(-a^5+2a^4bx^2-5a^3b^2x^4+20a^2b^3x^6+90ab^4x^8+60b^5x^{10})}{x^8(a+bx^2)^2}+120b^4\log(x)-60b^4\log(a+bx^2)}{8a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*x^2 - 5*a^3*b^2*x^4 + 20*a^2*b^3*x^6 + 90*a*b^4*x^8 + 6
0*b^5*x^10))/(x^8*(a + b*x^2)^2) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/
(8*a^7)
```

Maple [A]
time = 0.07, size = 105, normalized size = 0.94

method	result	size
norman	$\frac{-\frac{15b^5x^{10}}{a^6}-\frac{1}{8a}+\frac{b\,x^2}{4a^2}-\frac{5b^2x^4}{8a^3}+\frac{5b^3x^6}{2a^4}-\frac{45b^6x^{12}}{4a^7}}{x^8(b\,x^2+a)^2}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln(b\,x^2+a)}{2a^7}$	100
risch	$\frac{\frac{15b^5x^{10}}{2a^6}+\frac{45b^4x^8}{4a^5}+\frac{5b^3x^6}{2a^4}-\frac{5b^2x^4}{8a^3}+\frac{b\,x^2}{4a^2}-\frac{1}{8a}}{x^8(b\,x^2+a)^2}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln(b\,x^2+a)}{2a^7}$	100
default	$-\frac{b^5\left(-\frac{5a}{b(b\,x^2+a)}+\frac{15\ln(b\,x^2+a)}{b}-\frac{a^2}{2b(b\,x^2+a)^2}\right)}{2a^7}-\frac{1}{8a^3x^8}+\frac{15b^4\ln(x)}{a^7}+\frac{5b^3}{a^6x^2}-\frac{3b^2}{2a^5x^4}+\frac{b}{2a^4x^6}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^5/a^7*(-5*a/b/(b*x^2+a)+15*ln(b*x^2+a)/b-1/2/b*a^2/(b*x^2+a)^2)-1/8/
a^3/x^8+15*b^4*ln(x)/a^7+5*b^3/a^6/x^2-3/2*b^2/a^5/x^4+1/2*b/a^4/x^6
```

Maxima [A]
time = 0.31, size = 114, normalized size = 1.02

$$\frac{60\,b^5x^{10}+90\,ab^4x^8+20\,a^2b^3x^6-5\,a^3b^2x^4+2\,a^4bx^2-a^5}{8\,(a^6b^2x^{12}+2\,a^7bx^{10}+a^8x^8)}-\frac{15\,b^4\log(bx^2+a)}{2\,a^7}+\frac{15\,b^4\log(x^2)}{2\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(60*b^5*x^10 + 90*a*b^4*x^8 + 20*a^2*b^3*x^6 - 5*a^3*b^2*x^4 + 2*a^4*b*
x^2 - a^5)/(a^6*b^2*x^12 + 2*a^7*b*x^10 + a^8*x^8) - 15/2*b^4*log(b*x^2 + a
)/a^7 + 15/2*b^4*log(x^2)/a^7
```

Fricas [A]
time = 1.00, size = 156, normalized size = 1.39

$$\frac{60\,ab^5x^{10}+90\,a^2b^4x^8+20\,a^3b^3x^6-5\,a^4b^2x^4+2\,a^5bx^2-a^6-60\,(b^6x^{12}+2\,ab^5x^{10}+a^2b^4x^8)\log(bx^2+a)+120\,(b^6x^{12}+2\,ab^5x^{10}+a^2b^4x^8)\log(x)}{8\,(a^7b^2x^{12}+2\,a^8bx^{10}+a^9x^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(60*a*b^5*x^10 + 90*a^2*b^4*x^8 + 20*a^3*b^3*x^6 - 5*a^4*b^2*x^4 + 2*a^
5*b*x^2 - a^6 - 60*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(b*x^2 + a) +
 120*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(x))/(a^7*b^2*x^12 + 2*a^8*
b*x^10 + a^9*x^8)
```

Sympy [A]
time = 0.30, size = 116, normalized size = 1.04

$$\frac{-a^5+2a^4bx^2-5a^3b^2x^4+20a^2b^3x^6+90ab^4x^8+60b^5x^{10}}{8a^8x^8+16a^7bx^{10}+8a^6b^2x^{12}}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log\left(\frac{a}{b}+x^2\right)}{2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a)**3,x)
```

```
[Out] (-a**5 + 2*a**4*b*x**2 - 5*a**3*b**2*x**4 + 20*a**2*b**3*x**6 + 90*a*b**4*x
**8 + 60*b**5*x**10)/(8*a**8*x**8 + 16*a**7*b*x**10 + 8*a**6*b**2*x**12) +
15*b**4*log(x)/a**7 - 15*b**4*log(a/b + x**2)/(2*a**7)
```

Giac [A]
time = 1.63, size = 119, normalized size = 1.06

$$\frac{15\,b^4\log(x^2)}{2\,a^7}-\frac{15\,b^4\log(|bx^2+a|)}{2\,a^7}+\frac{45\,b^6x^4+100\,ab^5x^2+56\,a^2b^4}{4\,(bx^2+a)^2a^7}-\frac{125\,b^4x^8-40\,ab^3x^6+12\,a^2b^2x^4-4\,a^3bx^2+a^4}{8\,a^7x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 15/2*b^4*log(x^2)/a^7 - 15/2*b^4*log(abs(b*x^2 + a))/a^7 + 1/4*(45*b^6*x^4
+ 100*a*b^5*x^2 + 56*a^2*b^4)/((b*x^2 + a)^2*a^7) - 1/8*(125*b^4*x^8 - 40*a
*b^3*x^6 + 12*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)/(a^7*x^8)
```

Mupad [B]
time = 4.86, size = 111, normalized size = 0.99

$$\frac{\frac{b\,x^2}{4\,a^2}-\frac{1}{8\,a}-\frac{5\,b^2\,x^4}{8\,a^3}+\frac{5\,b^3\,x^6}{2\,a^4}+\frac{45\,b^4\,x^8}{4\,a^5}+\frac{15\,b^5\,x^{10}}{2\,a^6}}{a^2\,x^8+2\,a\,b\,x^{10}+b^2\,x^{12}}-\frac{15\,b^4\,\ln\left(b\,x^2+a\right)}{2\,a^7}+\frac{15\,b^4\,\ln\left(x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^2)^3),x)
```

```
[Out] ((b*x^2)/(4*a^2) - 1/(8*a) - (5*b^2*x^4)/(8*a^3) + (5*b^3*x^6)/(2*a^4) + (4
5*b^4*x^8)/(4*a^5) + (15*b^5*x^10)/(2*a^6))/(a^2*x^8 + b^2*x^12 + 2*a*b*x^1
0) - (15*b^4*log(a + b*x^2))/(2*a^7) + (15*b^4*log(x))/a^7
```

3.181 $\int \frac{x^{12}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=111

$$-\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}+\frac{99x^7}{56b^3}-\frac{x^{11}}{4b\,(a+bx^2)^2}-\frac{11x^9}{8b^2\,(a+bx^2)}+\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}$$

```
[Out] -99/8*a^3*x/b^6+33/8*a^2*x^3/b^5-99/40*a*x^5/b^4+99/56*x^7/b^3-1/4*x^11/b/(
b*x^2+a)^2-11/8*x^9/b^2/(b*x^2+a)+99/8*a^(7/2)*arctan(x*b^(1/2)/a^(1/2))/b^
(13/2)
```

Rubi [A]
time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 308, 211}

$$\frac{99a^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}-\frac{11x^9}{8b^2\,(a+bx^2)}-\frac{x^{11}}{4b\,(a+bx^2)^2}+\frac{99x^7}{56b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^3,x]
```

```
[Out] (-99*a^3*x)/(8*b^6) + (33*a^2*x^3)/(8*b^5) - (99*a*x^5)/(40*b^4) + (99*x^7)
/(56*b^3) - x^11/(4*b*(a + b*x^2)^2) - (11*x^9)/(8*b^2*(a + b*x^2)) + (99*a
^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

`Q[m, 2*n - 1]`

Rubi steps

$$\int \frac{x^{12}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^{11}}{4b\left(a+bx^2\right)^2}+\frac{11\int \frac{x^{10}}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{99\int \frac{x^8}{a+bx^2}\,dx}{8b^2}$$

$$= -\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{99\int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4(a+bx^2)}\right)\,dx}{8b^2}$$

$$= -\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}+\frac{99x^7}{56b^3}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{\left(99a^4\right)\int \frac{1}{a+bx^2}\,dx}{8b^6}$$

$$= -\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}+\frac{99x^7}{56b^3}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}$$

Mathematica [A]
time = 0.04, size = 99, normalized size = 0.89

$$-\frac{3465a^5x+5775a^4bx^3+1848a^3b^2x^5-264a^2b^3x^7+88ab^4x^9-40b^5x^{11}}{280b^6\left(a+bx^2\right)^2}+\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^3,x]
```

```
[Out] -1/280*(3465*a^5*x + 5775*a^4*b*x^3 + 1848*a^3*b^2*x^5 - 264*a^2*b^3*x^7 +
88*a*b^4*x^9 - 40*b^5*x^11)/(b^6*(a + b*x^2)^2) + (99*a^(7/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Maple [A]
time = 0.05, size = 85, normalized size = 0.77

method	result
default	$-\frac{-\frac{1}{7}b^3x^7+\frac{3}{5}a\,b^2x^5-2a^2b\,x^3+10a^3x}{b^6}+\frac{a^4\left(\frac{-\frac{21}{8}b\,x^3-\frac{19}{8}ax}{\left(b\,x^2+a\right)^2}+\frac{99\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^6}$

risch	$\frac{x^7}{7b^3}-\frac{3a\,x^5}{5b^4}+\frac{2a^2x^3}{b^5}-\frac{10a^3x}{b^6}+\frac{-\frac{21}{8}a^4b\,x^3-\frac{19}{8}a^5x}{b^6(b\,x^2+a)^2}+\frac{99\sqrt{-ab}\;a^3\ln\left(-\sqrt{-ab}\;x+a\right)}{16b^7}-\frac{99\sqrt{-ab}\;a^3\ln\left(\sqrt{-ab}\right.}{16b^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/7*b^3*x^7+3/5*a*b^2*x^5-2*a^2*b*x^3+10*a^3*x)+a^4/b^6*((-21/8*b*
x^3-19/8*a*x)/(b*x^2+a)^2+99/8/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 105, normalized size = 0.95

$$-\frac{21\,a^4bx^3+19\,a^5x}{8\,(b^8x^4+2\,ab^7x^2+a^2b^6)}+\frac{99\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}+\frac{5\,b^3x^7-21\,ab^2x^5+70\,a^2bx^3-350\,a^3x}{35\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(21*a^4*b*x^3 + 19*a^5*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) + 99/8*a^4
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/35*(5*b^3*x^7 - 21*a*b^2*x^5 + 7
0*a^2*b*x^3 - 350*a^3*x)/b^6
```

Fricas [A]
time = 1.08, size = 278, normalized size = 2.50

$$\left[\frac{80\,b^5x^{11}-176\,ab^4x^9+528\,a^2b^3x^7-3696\,a^3b^2x^5-11550\,a^4bx^3-6930\,a^5x+3465\,(a^3b^2x^4+2\,a^4bx^2+a^5)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{560\,(b^8x^4+2\,ab^7x^2+a^2b^6)},\ \frac{40\,b^5x^{11}-88\,ab^4x^9+264\,a^2b^3x^7-1848\,a^3b^2x^5-5775\,a^4bx^3-3465\,a^5x+3465\,(a^3b^2x^4+2\,a^4bx^2+a^5)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{280\,(b^8x^4+2\,ab^7x^2+a^2b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(80*b^5*x^11 - 176*a*b^4*x^9 + 528*a^2*b^3*x^7 - 3696*a^3*b^2*x^5 -
11550*a^4*b*x^3 - 6930*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^4 + 2*a*b^7*x
^2 + a^2*b^6), 1/280*(40*b^5*x^11 - 88*a*b^4*x^9 + 264*a^2*b^3*x^7 - 1848*a
^3*b^2*x^5 - 5775*a^4*b*x^3 - 3465*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b*x^2
+ a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)
]
```

Sympy [A]
time = 0.21, size = 162, normalized size = 1.46

$$-\frac{10a^3x}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^5}{5b^4}-\frac{99\sqrt{-\frac{a^7}{b^{13}}}\log\left(x-\frac{b^6\sqrt{-\frac{a^7}{b^{13}}}}{a^3}\right)}{16}+\frac{99\sqrt{-\frac{a^7}{b^{13}}}\log\left(x+\frac{b^6\sqrt{-\frac{a^7}{b^{13}}}}{a^3}\right)}{16}+\frac{-19a^5x-21a^4bx^3}{8a^2b^6+16ab^7x^2+8b^8x^4}+\frac{x^7}{7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**3,x)
```

```
[Out] -10*a**3*x/b**6 + 2*a**2*x**3/b**5 - 3*a*x**5/(5*b**4) - 99*sqrt(-a**7/b**1
3)*log(x - b**6*sqrt(-a**7/b**13)/a**3)/16 + 99*sqrt(-a**7/b**13)*log(x + b
**6*sqrt(-a**7/b**13)/a**3)/16 + (-19*a**5*x - 21*a**4*b*x**3)/(8*a**2*b**6
 + 16*a*b**7*x**2 + 8*b**8*x**4) + x**7/(7*b**3)
```

Giac [A]
time = 1.34, size = 96, normalized size = 0.86

$$\frac{99\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}-\frac{21\,a^4bx^3+19\,a^5x}{8\,(bx^2+a)^2b^6}+\frac{5\,b^{18}x^7-21\,ab^{17}x^5+70\,a^2b^{16}x^3-350\,a^3b^{15}x}{35\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 99/8*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21*a^4*b*x^3 + 19*a^5
*x)/((b*x^2 + a)^2*b^6) + 1/35*(5*b^18*x^7 - 21*a*b^17*x^5 + 70*a^2*b^16*x^
3 - 350*a^3*b^15*x)/b^21
```

Mupad [B]
time = 0.08, size = 99, normalized size = 0.89

$$\frac{x^7}{7\,b^3}-\frac{\frac{19\,a^5\,x}{8}+\frac{21\,b\,a^4\,x^3}{8}}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}-\frac{3\,a\,x^5}{5\,b^4}-\frac{10\,a^3\,x}{b^6}+\frac{99\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{13/2}}+\frac{2\,a^2\,x^3}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^2)^3,x)
```

```
[Out] x^7/(7*b^3) - ((19*a^5*x)/8 + (21*a^4*b*x^3)/8)/(a^2*b^6 + b^8*x^4 + 2*a*b^
7*x^2) - (3*a*x^5)/(5*b^4) - (10*a^3*x)/b^6 + (99*a^(7/2)*atan((b^(1/2)*x)/
a^(1/2)))/(8*b^(13/2)) + (2*a^2*x^3)/b^5
```

3.182 $\int \frac{x^{10}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=98

$$\frac{63a^2x}{8b^5}-\frac{21ax^3}{8b^4}+\frac{63x^5}{40b^3}-\frac{x^9}{4b\,(a+bx^2)^2}-\frac{9x^7}{8b^2\,(a+bx^2)}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}$$

```
[Out] 63/8*a^2*x/b^5-21/8*a*x^3/b^4+63/40*x^5/b^3-1/4*x^9/b/(b*x^2+a)^2-9/8*x^7/b
^2/(b*x^2+a)-63/8*a^(5/2)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {294, 308, 211}

$$-\frac{63a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{63a^2x}{8b^5}-\frac{21ax^3}{8b^4}-\frac{9x^7}{8b^2\,(a+bx^2)}-\frac{x^9}{4b\,(a+bx^2)^2}+\frac{63x^5}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^3,x]
```

```
[Out] (63*a^2*x)/(8*b^5) - (21*a*x^3)/(8*b^4) + (63*x^5)/(40*b^3) - x^9/(4*b*(a +
 b*x^2)^2) - (9*x^7)/(8*b^2*(a + b*x^2)) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{\left(a+bx^2\right)^3}\,dx &= -\frac{x^9}{4b\left(a+bx^2\right)^2}+\frac{9\int \frac{x^8}{\left(a+bx^2\right)^2}\,dx}{4b} \\
&= -\frac{x^9}{4b\left(a+bx^2\right)^2}-\frac{9x^7}{8b^2\left(a+bx^2\right)}+\frac{63\int \frac{x^6}{a+bx^2}\,dx}{8b^2} \\
&= -\frac{x^9}{4b\left(a+bx^2\right)^2}-\frac{9x^7}{8b^2\left(a+bx^2\right)}+\frac{63\int \left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3(a+bx^2)}\right)dx}{8b^2} \\
&= \frac{63a^2x}{8b^5}-\frac{21ax^3}{8b^4}+\frac{63x^5}{40b^3}-\frac{x^9}{4b\left(a+bx^2\right)^2}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{\left(63a^3\right)\int \frac{1}{a+bx^2}\,dx}{8b^5} \\
&= \frac{63a^2x}{8b^5}-\frac{21ax^3}{8b^4}+\frac{63x^5}{40b^3}-\frac{x^9}{4b\left(a+bx^2\right)^2}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.90

$$\frac{315a^4x+525a^3bx^3+168a^2b^2x^5-24ab^3x^7+8b^4x^9}{40b^5\left(a+bx^2\right)^2}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^3,x]
```

```
[Out] (315*a^4*x + 525*a^3*b*x^3 + 168*a^2*b^2*x^5 - 24*a*b^3*x^7 + 8*b^4*x^9)/(4
0*b^5*(a + b*x^2)^2) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2)
)
```

Maple [A]
time = 0.05, size = 74, normalized size = 0.76

method	result	size
default	$\frac{\frac{1}{5}b^2x^5-ab\,x^3+6a^2x}{b^5}-\frac{a^3\left(\frac{-\frac{17}{8}b\,x^3-\frac{15}{8}ax}{\left(b\,x^2+a\right)^2}+\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^5}$	74
risch	$\frac{x^5}{5b^3}-\frac{a\,x^3}{b^4}+\frac{6a^2x}{b^5}+\frac{\frac{17}{8}a^3b\,x^3+\frac{15}{8}a^4x}{b^5\left(b\,x^2+a\right)^2}+\frac{63\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-a\right)}{16b^6}-\frac{63\sqrt{-ab}\,a^2\ln\left(\sqrt{-ab}\,x-a\right)}{16b^6}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*b^2*x^5-a*b*x^3+6*a^2*x)-a^3/b^5*((-17/8*b*x^3-15/8*a*x)/(b*x^2+
a)^2+63/8/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 93, normalized size = 0.95

$$\frac{17\,a^3bx^3+15\,a^4x}{8\,(b^7x^4+2\,ab^6x^2+a^2b^5)}-\frac{63\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^5}+\frac{b^2x^5-5\,abx^3+30\,a^2x}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(17*a^3*b*x^3 + 15*a^4*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) - 63/8*a^3*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/5*(b^2*x^5 - 5*a*b*x^3 + 30*a^2*x
)/b^5
```

Fricas [A]
time = 1.20, size = 256, normalized size = 2.61

$$\left[\frac{16\,b^4x^9-48\,ab^3x^7+336\,a^2b^2x^5+1050\,a^3bx^3+630\,a^4x+315\,(a^2b^2x^4+2\,a^3bx^2+a^4)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{80\,(b^7x^4+2\,ab^6x^2+a^2b^5)},\ \frac{8\,b^4x^9-24\,ab^3x^7+168\,a^2b^2x^5+525\,a^3bx^3+315\,a^4x-315\,(a^2b^2x^4+2\,a^3bx^2+a^4)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{40\,(b^7x^4+2\,ab^6x^2+a^2b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/80*(16*b^4*x^9 - 48*a*b^3*x^7 + 336*a^2*b^2*x^5 + 1050*a^3*b*x^3 + 630*a
^4*x + 315*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 - 2*b*x*
sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5), 1/40*(8*b^
4*x^9 - 24*a*b^3*x^7 + 168*a^2*b^2*x^5 + 525*a^3*b*x^3 + 315*a^4*x - 315*(a
^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x^4
 + 2*a*b^6*x^2 + a^2*b^5)]
```

Sympy [A]
time = 0.21, size = 144, normalized size = 1.47

$$\frac{6a^2x}{b^5}-\frac{ax^3}{b^4}+\frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16}-\frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16}+\frac{15a^4x+17a^3bx^3}{8a^2b^5+16ab^6x^2+8b^7x^4}+\frac{x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**3,x)
```

```
[Out] 6*a**2*x/b**5 - a*x**3/b**4 + 63*sqrt(-a**5/b**11)*log(x - b**5*sqrt(-a**5/
b**11)/a**2)/16 - 63*sqrt(-a**5/b**11)*log(x + b**5*sqrt(-a**5/b**11)/a**2)
/16 + (15*a**4*x + 17*a**3*b*x**3)/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x
**4) + x**5/(5*b**3)
```

Giac [A]
time = 1.93, size = 84, normalized size = 0.86

$$-\frac{63\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{5}}+\frac{17\,a^{3}bx^{3}+15\,a^{4}x}{8\,(bx^{2}+a)^{2}b^{5}}+\frac{b^{12}x^{5}-5\,ab^{11}x^{3}+30\,a^{2}b^{10}x}{5\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -63/8*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/8*(17*a^3*b*x^3 + 15*a^
4*x)/((b*x^2 + a)^2*b^5) + 1/5*(b^12*x^5 - 5*a*b^11*x^3 + 30*a^2*b^10*x)/b^
15
```

Mupad [B]
time = 0.07, size = 87, normalized size = 0.89

$$\frac{\frac{15\,a^{4}\,x}{8}+\frac{17\,b\,a^{3}\,x^{3}}{8}}{a^{2}\,b^{5}+2\,a\,b^{6}\,x^{2}+b^{7}\,x^{4}}+\frac{x^{5}}{5\,b^{3}}-\frac{a\,x^{3}}{b^{4}}+\frac{6\,a^{2}\,x}{b^{5}}-\frac{63\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^3,x)
```

```
[Out] ((15*a^4*x)/8 + (17*a^3*b*x^3)/8)/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) + x^5/(
5*b^3) - (a*x^3)/b^4 + (6*a^2*x)/b^5 - (63*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)
))/(8*b^(11/2))
```

3.183 $\int \frac{x^8}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=85

$$-\frac{35ax}{8b^4}+\frac{35x^3}{24b^3}-\frac{x^7}{4b\left(a+bx^2\right)^2}-\frac{7x^5}{8b^2\left(a+bx^2\right)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}$$

```
[Out] -35/8*a*x/b^4+35/24*x^3/b^3-1/4*x^7/b/(b*x^2+a)^2-7/8*x^5/b^2/(b*x^2+a)+35/
8*a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

Rubi [A]

time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 308, 211}

$$\frac{35a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}-\frac{35ax}{8b^4}-\frac{7x^5}{8b^2\left(a+bx^2\right)}-\frac{x^7}{4b\left(a+bx^2\right)^2}+\frac{35x^3}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^3,x]
```

```
[Out] (-35*a*x)/(8*b^4) + (35*x^3)/(24*b^3) - x^7/(4*b*(a + b*x^2)^2) - (7*x^5)/(
8*b^2*(a + b*x^2)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^2)^3}\,dx = -\frac{x^7}{4b(a+bx^2)^2} + \frac{7\int \frac{x^6}{(a+bx^2)^2}\,dx}{4b}$$
$$= -\frac{x^7}{4b(a+bx^2)^2} - \frac{7x^5}{8b^2(a+bx^2)} + \frac{35\int \frac{x^4}{a+bx^2}\,dx}{8b^2}$$
$$= -\frac{x^7}{4b(a+bx^2)^2} - \frac{7x^5}{8b^2(a+bx^2)} + \frac{35\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right)dx}{8b^2}$$
$$= -\frac{35ax}{8b^4} + \frac{35x^3}{24b^3} - \frac{x^7}{4b(a+bx^2)^2} - \frac{7x^5}{8b^2(a+bx^2)} + \frac{(35a^2)\int \frac{1}{a+bx^2}\,dx}{8b^4}$$
$$= -\frac{35ax}{8b^4} + \frac{35x^3}{24b^3} - \frac{x^7}{4b(a+bx^2)^2} - \frac{7x^5}{8b^2(a+bx^2)} + \frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.91

$$-\frac{105a^3x+175a^2bx^3+56ab^2x^5-8b^3x^7}{24b^4(a+bx^2)^2} + \frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^3,x]
[Out] -1/24*(105*a^3*x + 175*a^2*b*x^3 + 56*a*b^2*x^5 - 8*b^3*x^7)/(b^4*(a + b*x^
2)^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Maple [A]
time = 0.04, size = 63, normalized size = 0.74

method	result	size
default	$-\frac{-\frac{1}{3}b\,x^3+3ax}{b^4}+\frac{a^2\left(\frac{-\frac{13}{8}b\,x^3-\frac{11}{8}ax}{\left(b\,x^2+a\right)^2}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	63
risch	$\frac{x^3}{3b^3}-\frac{3ax}{b^4}+\frac{-\frac{13}{8}a^2b\,x^3-\frac{11}{8}a^3x}{b^4(b\,x^2+a)^2}+\frac{35\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)}{16b^5}-\frac{35\sqrt{-ab}\,a\ln\left(\sqrt{-ab}\,x+a\right)}{16b^5}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/3*b*x^3+3*a*x)+a^2/b^4*((-13/8*b*x^3-11/8*a*x)/(b*x^2+a)^2+35/8/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 82, normalized size = 0.96

$$-\frac{13\,a^2bx^3+11\,a^3x}{8\,(b^6x^4+2\,ab^5x^2+a^2b^4)}+\frac{35\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^4}+\frac{bx^3-9\,ax}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(13*a^2*b*x^3 + 11*a^3*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 35/8*a^2
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/3*(b*x^3 - 9*a*x)/b^4
```

Fricas [A]
time = 1.43, size = 230, normalized size = 2.71

$$\left[\frac{16\,b^3x^7-112\,ab^2x^5-350\,a^2bx^3-210\,a^3x+105\,(ab^2x^4+2\,a^2bx^2+a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{48\,(b^6x^4+2\,ab^5x^2+a^2b^4)},\ \frac{8\,b^3x^7-56\,ab^2x^5-175\,a^2bx^3-105\,a^3x+105\,(ab^2x^4+2\,a^2bx^2+a^3)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{24\,(b^6x^4+2\,ab^5x^2+a^2b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*b^3*x^7 - 112*a*b^2*x^5 - 350*a^2*b*x^3 - 210*a^3*x + 105*(a*b^2*
x^4 + 2*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/24*(8*b^3*x^7 - 56*a*b^2*x^5
 - 175*a^2*b*x^3 - 105*a^3*x + 105*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

Sympy [A]
time = 0.19, size = 133, normalized size = 1.56

$$-\frac{3ax}{b^4}-\frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16}+\frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16}+\frac{-11a^3x-13a^2bx^3}{8a^2b^4+16ab^5x^2+8b^6x^4}+\frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**3,x)
```

```
[Out] -3*a*x/b**4 - 35*sqrt(-a**3/b**9)*log(x - b**4*sqrt(-a**3/b**9)/a)/16 + 35*
sqrt(-a**3/b**9)*log(x + b**4*sqrt(-a**3/b**9)/a)/16 + (-11*a**3*x - 13*a**
2*b*x**3)/(8*a**2*b**4 + 16*a*b**5*x**2 + 8*b**6*x**4) + x**3/(3*b**3)
```

Giac [A]
time = 1.38, size = 73, normalized size = 0.86

$$\frac{35\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^4}-\frac{13\,a^2bx^3+11\,a^3x}{8\,(bx^2+a)^2b^4}+\frac{b^6x^3-9\,ab^5x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 35/8*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/8*(13*a^2*b*x^3 + 11*a^3
*x)/((b*x^2 + a)^2*b^4) + 1/3*(b^6*x^3 - 9*a*b^5*x)/b^9
```

Mupad [B]
time = 4.72, size = 77, normalized size = 0.91

$$\frac{x^3}{3\,b^3}-\frac{\frac{11\,a^3\,x}{8}+\frac{13\,b\,a^2\,x^3}{8}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{9/2}}-\frac{3\,a\,x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^3,x)
```

```
[Out] x^3/(3*b^3) - ((11*a^3*x)/8 + (13*a^2*b*x^3)/8)/(a^2*b^4 + b^6*x^4 + 2*a*b^
5*x^2) + (35*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(9/2)) - (3*a*x)/b^4
```

3.184 $\int \frac{x^6}{(a+bx^2)^3} dx$

Optimal. Leaf size=74

$$\frac{15x}{8b^3} - \frac{x^5}{4b(a+bx^2)^2} - \frac{5x^3}{8b^2(a+bx^2)} - \frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}$$

```
[Out] 15/8*x/b^3-1/4*x^5/b/(b*x^2+a)^2-5/8*x^3/b^2/(b*x^2+a)-15/8*arctan(x*b^(1/2
)/a^(1/2))*a^(1/2)/b^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 327, 211}

$$-\frac{15\sqrt{a}\text{ArcTan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}} - \frac{5x^3}{8b^2(a+bx^2)} - \frac{x^5}{4b(a+bx^2)^2} + \frac{15x}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^3,x]
```

```
[Out] (15*x)/(8*b^3) - x^5/(4*b*(a + b*x^2)^2) - (5*x^3)/(8*b^2*(a + b*x^2)) - (1
5*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx^2)^3}\,dx &= -\frac{x^5}{4b\,(a+bx^2)^2}+\frac{5\int \frac{x^4}{(a+bx^2)^2}\,dx}{4b}\\ &= -\frac{x^5}{4b\,(a+bx^2)^2}-\frac{5x^3}{8b^2\,(a+bx^2)}+\frac{15\int \frac{x^2}{a+bx^2}\,dx}{8b^2}\\ &= \frac{15x}{8b^3}-\frac{x^5}{4b\,(a+bx^2)^2}-\frac{5x^3}{8b^2\,(a+bx^2)}-\frac{(15a)\int \frac{1}{a+bx^2}\,dx}{8b^3}\\ &= \frac{15x}{8b^3}-\frac{x^5}{4b\,(a+bx^2)^2}-\frac{5x^3}{8b^2\,(a+bx^2)}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.89

$$\frac{15a^2x+25abx^3+8b^2x^5}{8b^3\,(a+bx^2)^2}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^3,x]
```

```
[Out] (15*a^2*x + 25*a*b*x^3 + 8*b^2*x^5)/(8*b^3*(a + b*x^2)^2) - (15*Sqrt[a]*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

Maple [A]
time = 0.04, size = 51, normalized size = 0.69

method	result	size
default	$\frac{x}{b^3}-\frac{a\left(\frac{-\frac{9}{8}b\,x^3-\frac{7}{8}ax}{\left(b\,x^2+a\right)^2}+\frac{15\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	51
risch	$\frac{x}{b^3}+\frac{\frac{9}{8}ab\,x^3+\frac{7}{8}a^2x}{b^3(b\,x^2+a)^2}+\frac{15\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{16b^4}-\frac{15\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{16b^4}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^3-a/b^3*((-9/8*b*x^3-7/8*a*x)/(b*x^2+a)^2+15/8/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 68, normalized size = 0.92

$$\frac{9\,abx^3+7\,a^2x}{8\,(b^5x^4+2\,ab^4x^2+a^2b^3)}-\frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(9*a*b*x^3 + 7*a^2*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) - 15/8*a*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3
```

Fricas [A]
time = 1.13, size = 202, normalized size = 2.73

$$\left[\frac{16\,b^2x^5+50\,abx^3+30\,a^2x+15\,(b^2x^4+2\,abx^2+a^2)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{16\,(b^5x^4+2\,ab^4x^2+a^2b^3)},\ \frac{8\,b^2x^5+25\,abx^3+15\,a^2x-15\,(b^2x^4+2\,abx^2+a^2)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{8\,(b^5x^4+2\,ab^4x^2+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^2*x^5 + 50*a*b*x^3 + 30*a^2*x + 15*(b^2*x^4 + 2*a*b*x^2 + a^2)*
sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*x^4 + 2*a*
b^4*x^2 + a^2*b^3), 1/8*(8*b^2*x^5 + 25*a*b*x^3 + 15*a^2*x - 15*(b^2*x^4 +
2*a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^4 + 2*a*b^4*x^2
+ a^2*b^3)]
```

Sympy [A]
time = 0.18, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{a}{b^7}}\,\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{a}{b^7}}\,\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{16}+\frac{7a^2x+9abx^3}{8a^2b^3+16ab^4x^2+8b^5x^4}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**3,x)
```

```
[Out] 15*sqrt(-a/b**7)*log(-b**3*sqrt(-a/b**7) + x)/16 - 15*sqrt(-a/b**7)*log(b**
3*sqrt(-a/b**7) + x)/16 + (7*a**2*x + 9*a*b*x**3)/(8*a**2*b**3 + 16*a*b**4*
x**2 + 8*b**5*x**4) + x/b**3
```

Giac [A]
time = 1.61, size = 54, normalized size = 0.73

$$-\frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{x}{b^3}+\frac{9\,abx^3+7\,a^2x}{8\,(bx^2+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3 + 1/8*(9*a*b*x^3 + 7*
a^2*x)/((b*x^2 + a)^2*b^3)
```

Mupad [B]
time = 4.75, size = 64, normalized size = 0.86

$$\frac{\frac{7\,a^2\,x}{8}+\frac{9\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{x}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^3,x)
```

```
[Out] ((7*a^2*x)/8 + (9*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + x/b^3 - (
15*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

3.185 $\int \frac{x^4}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=64

$$-\frac{x^3}{4b(a+bx^2)^2}-\frac{3x}{8b^2(a+bx^2)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}$$

```
[Out] -1/4*x^3/b/(b*x^2+a)^2-3/8*x/b^2/(b*x^2+a)+3/8*arctan(x*b^(1/2)/a^(1/2))/b^
(5/2)/a^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {294, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}-\frac{3x}{8b^2(a+bx^2)}-\frac{x^3}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^3,x]
```

```
[Out] -1/4*x^3/(b*(a + b*x^2)^2) - (3*x)/(8*b^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^3}\,dx = -\frac{x^3}{4b(a+bx^2)^2} + \frac{3\int \frac{x^2}{(a+bx^2)^2}\,dx}{4b}$$
$$= -\frac{x^3}{4b(a+bx^2)^2} - \frac{3x}{8b^2(a+bx^2)} + \frac{3\int \frac{1}{a+bx^2}\,dx}{8b^2}$$
$$= -\frac{x^3}{4b(a+bx^2)^2} - \frac{3x}{8b^2(a+bx^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.86

$$-\frac{3ax+5bx^3}{8b^2(a+bx^2)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^3,x]
[Out] -1/8*(3*a*x + 5*b*x^3)/(b^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*Sqrt[a]*b^(5/2))
```

Maple [A]
time = 0.04, size = 47, normalized size = 0.73

method	result	size
default	$\frac{-\frac{5x^3}{8b}-\frac{3ax}{8b^2}}{(b\,x^2+a)^2} + \frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^2\sqrt{ab}}$	47
risch	$\frac{-\frac{5x^3}{8b}-\frac{3ax}{8b^2}}{(b\,x^2+a)^2} - \frac{3\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,b^2} + \frac{3\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,b^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] (-5/8*x^3/b-3/8*a*x/b^2)/(b*x^2+a)^2+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))
```

Maxima [A]
time = 0.49, size = 59, normalized size = 0.92

$$-\frac{5\,bx^3+3\,ax}{8\,(b^4x^4+2\,ab^3x^2+a^2b^2)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 3/8*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.92, size = 188, normalized size = 2.94

$$\left[-\frac{10\,ab^2x^3+6\,a^2bx+3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)}, -\frac{5\,ab^2x^3+3\,a^2bx-3\,(b^2x^4+2\,abx^2+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^2
+ a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [A]
time = 0.14, size = 110, normalized size = 1.72

$$-\frac{3\sqrt{-\frac{1}{ab^5}}\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{ab^5}}\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16}+\frac{-3ax-5bx^3}{8a^2b^2+16ab^3x^2+8b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a*b**5))*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/16 + 3*sqrt(-1/(a*b
**5))*log(a*b**2*sqrt(-1/(a*b**5)) + x)/16 + (-3*a*x - 5*b*x**3)/(8*a**2*b*
*2 + 16*a*b**3*x**2 + 8*b**4*x**4)
```

Giac [A]
time = 1.26, size = 45, normalized size = 0.70

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}-\frac{5\,bx^3+3\,ax}{8\,(bx^2+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/8*(5*b*x^3 + 3*a*x)/((b*x^2 +
 a)^2*b^2)
```

Mupad [B]
time = 4.77, size = 56, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}-\frac{\frac{5\,x^3}{8\,b}+\frac{3\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2)) - ((5*x^3)/(8*b) + (3*a*x
)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.186 $\int \frac{x^2}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=65

$$-\frac{x}{4b\left(a+bx^2\right)^2}+\frac{x}{8ab\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

```
[Out] -1/4*x/b/(b*x^2+a)^2+1/8*x/a/b/(b*x^2+a)+1/8*arctan(x*b^(1/2)/a^(1/2))/a^(3
/2)/b^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}+\frac{x}{8ab\left(a+bx^2\right)}-\frac{x}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^3,x]
```

```
[Out] -1/4*x/(b*(a + b*x^2)^2) + x/(8*a*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/(8*a^(3/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)^3}\,dx &= -\frac{x}{4b(a+bx^2)^2}+\frac{\int \frac{1}{(a+bx^2)^2}\,dx}{4b}\\ &= -\frac{x}{4b(a+bx^2)^2}+\frac{x}{8ab(a+bx^2)}+\frac{\int \frac{1}{a+bx^2}\,dx}{8ab}\\ &= -\frac{x}{4b(a+bx^2)^2}+\frac{x}{8ab(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{a}\sqrt{b}\,x(-a+bx^2)}{(a+bx^2)^2}+\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^3,x]
[Out] ((Sqrt[a]*Sqrt[b]*x*(-a + b*x^2))/(a + b*x^2)^2 + ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(3/2)*b^(3/2))
```

Maple [A]
time = 0.04, size = 49, normalized size = 0.75

method	result	size
default	$\frac{\frac{x^3}{8a}-\frac{x}{8b}}{(b\,x^2+a)^2}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8ba\sqrt{ab}}$	49
risch	$\frac{\frac{x^3}{8a}-\frac{x}{8b}}{(b\,x^2+a)^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,ba}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,ba}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] (1/8/a*x^3-1/8*x/b)/(b*x^2+a)^2+1/8/b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 62, normalized size = 0.95

$$\frac{bx^3 - ax}{8\,(ab^3x^4 + 2\,a^2b^2x^2 + a^3b)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 1/8*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.94, size = 190, normalized size = 2.92

$$\left[\frac{2\,ab^2x^3 - 2\,a^2bx - (b^2x^4 + 2\,abx^2 + a^2)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{16\,(a^2b^4x^4 + 2\,a^3b^3x^2 + a^4b^2)}, \frac{ab^2x^3 - a^2bx + (b^2x^4 + 2\,abx^2 + a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\,(a^2b^4x^4 + 2\,a^3b^3x^2 + a^4b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*log
((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 +
a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.
time = 0.13, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{a^3b^3}}\,\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^3}}\,\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{16} + \frac{-ax + bx^3}{8a^3b + 16a^2b^2x^2 + 8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**3))*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + sqrt(-1/(a
**3*b**3))*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + (-a*x + b*x**3)/(8*a**
3*b + 16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

Giac [A]
time = 1.35, size = 50, normalized size = 0.77

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab} + \frac{bx^3 - ax}{8\,(bx^2 + a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/8*(b*x^3 - a*x)/((b*x^2 + a)^
2*a*b)
```

Mupad [B]
time = 4.74, size = 55, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}-\frac{\frac{x}{8\,b}-\frac{x^3}{8\,a}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^3,x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(8*a^(3/2)*b^(3/2)) - (x/(8*b) - x^3/(8*a))/(a^2
+ b^2*x^4 + 2*a*b*x^2)
```

3.187 $\int \frac{1}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=62

$$\frac{x}{4a\left(a+bx^2\right)^2}+\frac{3x}{8a^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

```
[Out] 1/4*x/a/(b*x^2+a)^2+3/8*x/a^2/(b*x^2+a)+3/8*arctan(x*b^(1/2)/a^(1/2))/a^(5/
2)/b^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{3x}{8a^2\left(a+bx^2\right)}+\frac{x}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a + b*x^2)^2) + (3*x)/(8*a^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^3} dx = \frac{x}{4a(a+bx^2)^2} + \frac{3\int \frac{1}{(a+bx^2)^2} dx}{4a}$$
$$= \frac{x}{4a(a+bx^2)^2} + \frac{3x}{8a^2(a+bx^2)} + \frac{3\int \frac{1}{a+bx^2} dx}{8a^2}$$
$$= \frac{x}{4a(a+bx^2)^2} + \frac{3x}{8a^2(a+bx^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.89

$$\frac{5ax+3bx^3}{8a^2(a+bx^2)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3),x]
[Out] (5*a*x + 3*b*x^3)/(8*a^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(5/2)*Sqrt[b])
```

Maple [A]
time = 0.05, size = 57, normalized size = 0.92

method	result	size
default	$\frac{x}{4a(b\,x^2+a)^2} + \frac{\frac{3x}{8a(b\,x^2+a)} + \frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{a}$	57
risch	$\frac{\frac{3b\,x^3}{8a^2} + \frac{5x}{8a}}{(b\,x^2+a)^2} - \frac{3\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2} + \frac{3\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*x/a/(b*x^2+a)^2+3/4/a*(1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 58, normalized size = 0.94

$$\frac{3\,bx^3+5\,ax}{8\,(a^2b^2x^4+2\,a^3bx^2+a^4)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3/8*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.79, size = 188, normalized size = 3.03

$$\left[\frac{6\,ab^2x^3+10\,a^2bx-3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)}, \frac{3\,ab^2x^3+5\,a^2bx+3\,(b^2x^4+2\,abx^2+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2
 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [A]
time = 0.13, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{5ax+3bx^3}{8a^4+16a^3bx^2+8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*b))*log(-a**3*sqrt(-1/(a**5*b)) + x)/16 + 3*sqrt(-1/(a**5*
b))*log(a**3*sqrt(-1/(a**5*b)) + x)/16 + (5*a*x + 3*b*x**3)/(8*a**4 + 16*a*
*3*b*x**2 + 8*a**2*b**2*x**4)
```

Giac [A]
time = 1.35, size = 45, normalized size = 0.73

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}+\frac{3\,bx^3+5\,ax}{8\,(bx^2+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/8*(3*b*x^3 + 5*a*x)/((b*x^2 +
 a)^2*a^2)
```

Mupad [B]
time = 4.66, size = 55, normalized size = 0.89

$$\frac{\frac{5\,x}{8\,a}+\frac{3\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^3,x)
```

```
[Out] ((5*x)/(8*a) + (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*atan((b^
(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

3.188 $\int \frac{1}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=76

$$-\frac{15}{8a^3x}+\frac{1}{4ax\left(a+bx^2\right)^2}+\frac{5}{8a^2x\left(a+bx^2\right)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}$$

```
[Out] -15/8/a^3/x+1/4/a/x/(b*x^2+a)^2+5/8/a^2/x/(b*x^2+a)-15/8*arctan(x*b^(1/2)/a
^(1/2))*b^(1/2)/a^(7/2)
```

Rubi [A]

time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {296, 331, 211}

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{5}{8a^2x\left(a+bx^2\right)}+\frac{1}{4ax\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a + b*x^2)^2) + 5/(8*a^2*x*(a + b*x^2)) - (15*Sqr
t[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx &= \frac{1}{4ax\left(a+bx^2\right)^2}+\frac{5\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx}{4a}\\ &= \frac{1}{4ax\left(a+bx^2\right)^2}+\frac{5}{8a^2x\left(a+bx^2\right)}+\frac{15\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^2}\\ &= -\frac{15}{8a^3x}+\frac{1}{4ax\left(a+bx^2\right)^2}+\frac{5}{8a^2x\left(a+bx^2\right)}-\frac{(15b)\int \frac{1}{a+bx^2}\,dx}{8a^3}\\ &= -\frac{15}{8a^3x}+\frac{1}{4ax\left(a+bx^2\right)^2}+\frac{5}{8a^2x\left(a+bx^2\right)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.89

$$-\frac{8a^2+25abx^2+15b^2x^4}{8a^3x\left(a+bx^2\right)^2}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -1/8*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)/(a^3*x*(a + b*x^2)^2) - (15*Sqrt[b]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Maple [A]
time = 0.05, size = 54, normalized size = 0.71

method	result	size
default	$-\frac{b\left(\frac{\frac{7}{8}bx^3+\frac{9}{8}ax}{\left(bx^2+a\right)^2}+\frac{15\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3x}$	54
risch	$\frac{-\frac{15b^2x^4}{8a^3}-\frac{25bx^2}{8a^2}-\frac{1}{a}}{x\left(bx^2+a\right)^2}+\frac{15\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{16a^4}-\frac{15\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{16a^4}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*b*((7/8*b*x^3+9/8*a*x)/(b*x^2+a)^2+15/8/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2)))-1/a^3/x
```

Maxima [A]
time = 0.50, size = 71, normalized size = 0.93

$$-\frac{15\,b^2x^4+25\,abx^2+8\,a^2}{8\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
- 15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.93, size = 202, normalized size = 2.66

$$\left[-\frac{30\,b^2x^4+50\,abx^2-15\,(b^2x^5+2\,abx^3+a^2x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+16\,a^2}{16\,(a^3b^2x^5+2\,a^4bx^3+a^5x)},-\frac{15\,b^2x^4+25\,abx^2+15\,(b^2x^5+2\,abx^3+a^2x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+8\,a^2}{8\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5 +
 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a*b*
x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*
x^3 + a^5*x)]
```

Sympy [A]
time = 0.18, size = 116, normalized size = 1.53

$$\frac{15\sqrt{-\frac{b}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}}{b}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}}{b}+x\right)}{16}+\frac{-8a^2-25abx^2-15b^2x^4}{8a^5x+16a^4bx^3+8a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**3,x)
```

```
[Out] 15*sqrt(-b/a**7)*log(-a**4*sqrt(-b/a**7)/b + x)/16 - 15*sqrt(-b/a**7)*log(a
**4*sqrt(-b/a**7)/b + x)/16 + (-8*a**2 - 25*a*b*x**2 - 15*b**2*x**4)/(8*a**
5*x + 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

Giac [A]
time = 1.36, size = 57, normalized size = 0.75

$$-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}-\frac{7\,b^2x^3+9\,abx}{8\,(bx^2+a)^2a^3}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/8*(7*b^2*x^3 + 9*a*b*x)/(
(b*x^2 + a)^2*a^3) - 1/(a^3*x)
```

Mupad [B]
time = 4.67, size = 66, normalized size = 0.87

$$-\frac{\frac{1}{a}+\frac{25\,b\,x^2}{8\,a^2}+\frac{15\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^3),x)
```

```
[Out] - (1/a + (25*b*x^2)/(8*a^2) + (15*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*
b*x^3) - (15*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

3.189 $\int \frac{1}{x^4(a+bx^2)^3}\,dx$

Optimal. Leaf size=87

$$-\frac{35}{24a^3x^3}+\frac{35b}{8a^4x}+\frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}$$

```
[Out] -35/24/a^3/x^3+35/8*b/a^4/x+1/4/a/x^3/(b*x^2+a)^2+7/8/a^2/x^3/(b*x^2+a)+35/
8*b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {296, 331, 211}

$$\frac{35b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{35b}{8a^4x}-\frac{35}{24a^3x^3}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{1}{4ax^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -35/(24*a^3*x^3) + (35*b)/(8*a^4*x) + 1/(4*a*x^3*(a + b*x^2)^2) + 7/(8*a^2*
x^3*(a + b*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)^3}\,dx &= \frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7\int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx}{4a}\\ &= \frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{35\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{8a^2}\\ &= -\frac{35}{24a^3x^3}+\frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7}{8a^2x^3\left(a+bx^2\right)}-\frac{(35b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^3}\\ &= -\frac{35}{24a^3x^3}+\frac{35b}{8a^4x}+\frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{\left(35b^2\right)\int \frac{1}{a+bx^2}\,dx}{8a^4}\\ &= -\frac{35}{24a^3x^3}+\frac{35b}{8a^4x}+\frac{1}{4ax^3\left(a+bx^2\right)^2}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.91

$$\frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^4x^3\left(a+bx^2\right)^2}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^3),x]
[Out] (-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6)/(24*a^4*x^3*(a + b*x^
2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Maple [A]
time = 0.06, size = 64, normalized size = 0.74

method	result	size
default	$\frac{b^2\left(\frac{\frac{11}{8}bx^3+\frac{13}{8}ax}{\left(bx^2+a\right)^2}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}-\frac{1}{3a^3x^3}+\frac{3b}{a^4x}$	64

risch	$\frac{\frac{35b^3x^6}{8a^4}+\frac{175b^2x^4}{24a^3}+\frac{7b\,x^2}{3a^2}-\frac{1}{3a}}{x^3(b\,x^2+a)^2}+\frac{35\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{16a^5}-\frac{35\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{16a^5}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*b^2*((11/8*b*x^3+13/8*a*x)/(b*x^2+a)^2+35/8/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2)))-1/3/a^3/x^3+3*b/a^4/x
```

Maxima [A]
time = 0.62, size = 86, normalized size = 0.99

$$\frac{105\,b^3x^6+175\,ab^2x^4+56\,a^2bx^2-8\,a^3}{24\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}+\frac{35\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(105*b^3*x^6 + 175*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3)/(a^4*b^2*x^7 + 2*
a^5*b*x^5 + a^6*x^3) + 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 0.85, size = 238, normalized size = 2.74

$$\left[\frac{210\,b^3x^6+350\,ab^2x^4+112\,a^2bx^2-16\,a^3+105\,(b^3x^7+2\,ab^2x^5+a^2bx^3)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)},\ \frac{105\,b^3x^6+175\,ab^2x^4+56\,a^2bx^2-8\,a^3+105\,(b^3x^7+2\,ab^2x^5+a^2bx^3)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{24\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
+ 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 175*
a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [A]
time = 0.21, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16}+\frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16}+\frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^6x^3+48a^5bx^5+24a^4b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**3,x)
```

```
[Out] -35*sqrt(-b**3/a**9)*log(-a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + 35*sqrt(-b**
3/a**9)*log(a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + (-8*a**3 + 56*a**2*b*x**2
+ 175*a*b**2*x**4 + 105*b**3*x**6)/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a**4
*b**2*x**7)
```

Giac [A]
time = 0.93, size = 71, normalized size = 0.82

$$\frac{35\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}+\frac{11\,b^3x^3+13\,ab^2x}{8\,(bx^2+a)^2a^4}+\frac{9\,bx^2-a}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/8*(11*b^3*x^3 + 13*a*b^2
*x)/((b*x^2 + a)^2*a^4) + 1/3*(9*b*x^2 - a)/(a^4*x^3)
```

Mupad [B]
time = 4.67, size = 80, normalized size = 0.92

$$\frac{\frac{7\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{175\,b^2\,x^4}{24\,a^3}+\frac{35\,b^3\,x^6}{8\,a^4}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^3),x)
```

```
[Out] ((7*b*x^2)/(3*a^2) - 1/(3*a) + (175*b^2*x^4)/(24*a^3) + (35*b^3*x^6)/(8*a^4
))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5) + (35*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))
/(8*a^(9/2))
```

3.190 $\int \frac{1}{x^6\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{63}{40a^3x^5}+\frac{21b}{8a^4x^3}-\frac{63b^2}{8a^5x}+\frac{1}{4ax^5\left(a+bx^2\right)^2}+\frac{9}{8a^2x^5\left(a+bx^2\right)}-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}$$

```
[Out] -63/40/a^3/x^5+21/8*b/a^4/x^3-63/8*b^2/a^5/x+1/4/a/x^5/(b*x^2+a)^2+9/8/a^2/
x^5/(b*x^2+a)-63/8*b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A]
time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {296, 331, 211}

$$-\frac{63b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}-\frac{63b^2}{8a^5x}+\frac{21b}{8a^4x^3}-\frac{63}{40a^3x^5}+\frac{9}{8a^2x^5\left(a+bx^2\right)}+\frac{1}{4ax^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -63/(40*a^3*x^5) + (21*b)/(8*a^4*x^3) - (63*b^2)/(8*a^5*x) + 1/(4*a*x^5*(a
 + b*x^2)^2) + 9/(8*a^2*x^5*(a + b*x^2)) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*a^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\int \frac{1}{x^6 (a+bx^2)^3}\,dx = \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9\int \frac{1}{x^6(a+bx^2)^2}\,dx}{4a}$$
$$= \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} + \frac{63\int \frac{1}{x^6(a+bx^2)}\,dx}{8a^2}$$
$$= -\frac{63}{40a^3x^5} + \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} - \frac{(63b)\int \frac{1}{x^4(a+bx^2)}\,dx}{8a^3}$$
$$= -\frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} + \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} + \frac{(63b^2)\int \frac{1}{x^2(a+bx^2)}\,dx}{8a^4}$$
$$= -\frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} - \frac{63b^2}{8a^5x} + \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} - \frac{(63b^3)\int \frac{1}{a+bx^2}\,dx}{8a^5}$$
$$= -\frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} - \frac{63b^2}{8a^5x} + \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} - \frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 0.90

$$-\frac{8a^4 - 24a^3bx^2 + 168a^2b^2x^4 + 525ab^3x^6 + 315b^4x^8}{40a^5x^5 (a+bx^2)^2} - \frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^3),x]
[Out] -1/40*(8*a^4 - 24*a^3*b*x^2 + 168*a^2*b^2*x^4 + 525*a*b^3*x^6 + 315*b^4*x^8
)/(a^5*x^5*(a + b*x^2)^2) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
11/2))
```

Maple [A]
time = 0.06, size = 75, normalized size = 0.75

method	result	size

default	$-\frac{b^3\left(\frac{\frac{15}{8}b\,x^3+\frac{17}{8}ax}{\left(b\,x^2+a\right)^2}+\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^5}-\frac{1}{5a^3x^5}+\frac{b}{a^4x^3}-\frac{6b^2}{a^5x}$	75
risch	$\frac{-\frac{63b^4x^8}{8a^5}-\frac{105b^3x^6}{8a^4}-\frac{21b^2x^4}{5a^3}+\frac{3b\,x^2}{5a^2}-\frac{1}{5a}}{x^5(b\,x^2+a)^2}+\frac{63\sqrt{-ab}\,b^2\ln\left(-bx+\sqrt{-ab}\right)}{16a^6}-\frac{63\sqrt{-ab}\,b^2\ln\left(-bx-\sqrt{-ab}\right)}{16a^6}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^5*b^3*((15/8*b*x^3+17/8*a*x)/(b*x^2+a)^2+63/8/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))-1/5/a^3/x^5+b/a^4/x^3-6*b^2/a^5/x
```

Maxima [A]
time = 0.50, size = 97, normalized size = 0.97

$$-\frac{315\,b^4x^8+525\,ab^3x^6+168\,a^2b^2x^4-24\,a^3bx^2+8\,a^4}{40\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}-\frac{63\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(315*b^4*x^8 + 525*a*b^3*x^6 + 168*a^2*b^2*x^4 - 24*a^3*b*x^2 + 8*a^4
)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5) - 63/8*b^3*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*a^5)
```

Fricas [A]
time = 1.29, size = 264, normalized size = 2.64

$$\left[-\frac{630\,b^4x^8+1050\,ab^3x^6+336\,a^2b^2x^4-48\,a^3bx^2+16\,a^4-315\,(b^4x^9+2\,ab^3x^7+a^2b^2x^5)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{80\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)},\,-\frac{315\,b^4x^8+525\,ab^3x^6+168\,a^2b^2x^4-24\,a^3bx^2+8\,a^4+315\,(b^4x^9+2\,ab^3x^7+a^2b^2x^5)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{40\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(630*b^4*x^8 + 1050*a*b^3*x^6 + 336*a^2*b^2*x^4 - 48*a^3*b*x^2 + 16*
a^4 - 315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5), -1/
40*(315*b^4*x^8 + 525*a*b^3*x^6 + 168*a^2*b^2*x^4 - 24*a^3*b*x^2 + 8*a^4 +
315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a
^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)]
```

Sympy [A]
time = 0.23, size = 150, normalized size = 1.50

$$\frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3}+x\right)}{16}-\frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3}+x\right)}{16}+\frac{-8a^4+24a^3bx^2-168a^2b^2x^4-525ab^3x^6-315b^4x^8}{40a^7x^5+80a^6bx^7+40a^5b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**3,x)
```

```
[Out] 63*sqrt(-b**5/a**11)*log(-a**6*sqrt(-b**5/a**11)/b**3 + x)/16 - 63*sqrt(-b*
*5/a**11)*log(a**6*sqrt(-b**5/a**11)/b**3 + x)/16 + (-8*a**4 + 24*a**3*b*x*
*2 - 168*a**2*b**2*x**4 - 525*a*b**3*x**6 - 315*b**4*x**8)/(40*a**7*x**5 +
80*a**6*b*x**7 + 40*a**5*b**2*x**9)
```

Giac [A]
time = 1.78, size = 80, normalized size = 0.80

$$-\frac{63\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}-\frac{15\,b^4x^3+17\,ab^3x}{8\,(bx^2+a)^2a^5}-\frac{30\,b^2x^4-5\,abx^2+a^2}{5\,a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -63/8*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/8*(15*b^4*x^3 + 17*a*b^
3*x)/((b*x^2 + a)^2*a^5) - 1/5*(30*b^2*x^4 - 5*a*b*x^2 + a^2)/(a^5*x^5)
```

Mupad [B]
time = 5.02, size = 92, normalized size = 0.92

$$-\frac{\frac{1}{5\,a}-\frac{3\,b\,x^2}{5\,a^2}+\frac{21\,b^2\,x^4}{5\,a^3}+\frac{105\,b^3\,x^6}{8\,a^4}+\frac{63\,b^4\,x^8}{8\,a^5}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9}-\frac{63\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^3),x)
```

```
[Out] - (1/(5*a) - (3*b*x^2)/(5*a^2) + (21*b^2*x^4)/(5*a^3) + (105*b^3*x^6)/(8*a^
4) + (63*b^4*x^8)/(8*a^5))/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (63*b^(5/2)*at
an((b^(1/2)*x)/a^(1/2)))/(8*a^(11/2))
```

3.191 $\int \frac{1}{x^8(a+bx^2)^3}\,dx$

Optimal. Leaf size=113

$$-\frac{99}{56a^3x^7}+\frac{99b}{40a^4x^5}-\frac{33b^2}{8a^5x^3}+\frac{99b^3}{8a^6x}+\frac{1}{4ax^7(a+bx^2)^2}+\frac{11}{8a^2x^7(a+bx^2)}+\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{13/2}}$$

```
[Out] -99/56/a^3/x^7+99/40*b/a^4/x^5-33/8*b^2/a^5/x^3+99/8*b^3/a^6/x+1/4/a/x^7/(b
*x^2+a)^2+11/8/a^2/x^7/(b*x^2+a)+99/8*b^(7/2)*arctan(x*b^(1/2)/a^(1/2))/a^(
13/2)
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {296, 331, 211}

$$\frac{99b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{13/2}}+\frac{99b^3}{8a^6x}-\frac{33b^2}{8a^5x^3}+\frac{99b}{40a^4x^5}-\frac{99}{56a^3x^7}+\frac{11}{8a^2x^7(a+bx^2)}+\frac{1}{4ax^7(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -99/(56*a^3*x^7) + (99*b)/(40*a^4*x^5) - (33*b^2)/(8*a^5*x^3) + (99*b^3)/(8
*a^6*x) + 1/(4*a*x^7*(a + b*x^2)^2) + 11/(8*a^2*x^7*(a + b*x^2)) + (99*b^(7
/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^8\left(a+bx^2\right)^3}\,dx &= \frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11\int \frac{1}{x^8\left(a+bx^2\right)^2}\,dx}{4a}\\
&= \frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{99\int \frac{1}{x^8\left(a+bx^2\right)}\,dx}{8a^2}\\
&= -\frac{99}{56a^3x^7}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}-\frac{(99b)\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{8a^3}\\
&= -\frac{99}{56a^3x^7}+\frac{99b}{40a^4x^5}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{\left(99b^2\right)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{8a^4}\\
&= -\frac{99}{56a^3x^7}+\frac{99b}{40a^4x^5}-\frac{33b^2}{8a^5x^3}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}-\frac{\left(99b^3\right)\int \frac{1}{x^2\left(a+bx^2\right)}}{8a^5}\\
&= -\frac{99}{56a^3x^7}+\frac{99b}{40a^4x^5}-\frac{33b^2}{8a^5x^3}+\frac{99b^3}{8a^6x}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{\left(99b^4\right)\int}{8a}\\
&= -\frac{99}{56a^3x^7}+\frac{99b}{40a^4x^5}-\frac{33b^2}{8a^5x^3}+\frac{99b^3}{8a^6x}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{99b^{7/2}\tan}{8}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 0.89

$$
\frac{-40a^5+88a^4bx^2-264a^3b^2x^4+1848a^2b^3x^6+5775ab^4x^8+3465b^5x^{10}}{280a^6x^7\left(a+bx^2\right)^2}+\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{13/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)^3),x]
```

```
[Out] (-40*a^5 + 88*a^4*b*x^2 - 264*a^3*b^2*x^4 + 1848*a^2*b^3*x^6 + 5775*a*b^4*x
^8 + 3465*b^5*x^10)/(280*a^6*x^7*(a + b*x^2)^2) + (99*b^(7/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Maple [A]
time = 0.07, size = 86, normalized size = 0.76

method	result
default	$\frac{b^4\left(\frac{\frac{19}{8}b\,x^3+\frac{21}{8}ax}{\left(b\,x^2+a\right)^2}+\frac{99\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^6}-\frac{1}{7a^3x^7}+\frac{3b}{5a^4x^5}-\frac{2b^2}{a^5x^3}+\frac{10b^3}{a^6x}$
risch	$\frac{\frac{99b^5x^{10}}{8a^6}+\frac{165b^4x^8}{8a^5}+\frac{33b^3x^6}{5a^4}-\frac{33b^2x^4}{35a^3}+\frac{11b\,x^2}{35a^2}-\frac{1}{7a}}{x^7\left(b\,x^2+a\right)^2}+\frac{99\sqrt{-ab}\,b^3\ln\left(-bx-\sqrt{-ab}\right)}{16a^7}-\frac{99\sqrt{-ab}\,b^3\ln\left(-bx+\sqrt{-ab}\right)}{16a^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*b^4*((19/8*b*x^3+21/8*a*x)/(b*x^2+a)^2+99/8/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2)))-1/7/a^3/x^7+3/5*b/a^4/x^5-2*b^2/a^5/x^3+10*b^3/a^6/x
```

Maxima [A]
time = 0.51, size = 108, normalized size = 0.96

$$\frac{3465\,b^5x^{10}+5775\,ab^4x^8+1848\,a^2b^3x^6-264\,a^3b^2x^4+88\,a^4bx^2-40\,a^5}{280\,(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7)}+\frac{99\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/280*(3465*b^5*x^10 + 5775*a*b^4*x^8 + 1848*a^2*b^3*x^6 - 264*a^3*b^2*x^4
+ 88*a^4*b*x^2 - 40*a^5)/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7) + 99/8*b^4*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

Fricas [A]
time = 1.26, size = 286, normalized size = 2.53

$$\left[\frac{6930\,b^5x^{10}+11550\,ab^4x^8+3696\,a^2b^3x^6-528\,a^3b^2x^4+176\,a^4bx^2-80\,a^5+3465\,(b^5x^{11}+2\,ab^4x^9+a^2b^3x^7)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{560\,(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7)},\ \frac{3465\,b^5x^{10}+5775\,ab^4x^8+1848\,a^2b^3x^6-264\,a^3b^2x^4+88\,a^4bx^2-40\,a^5+3465\,(b^5x^{11}+2\,ab^4x^9+a^2b^3x^7)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{280\,(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(6930*b^5*x^10 + 11550*a*b^4*x^8 + 3696*a^2*b^3*x^6 - 528*a^3*b^2*x^
4 + 176*a^4*b*x^2 - 80*a^5 + 3465*(b^5*x^11 + 2*a*b^4*x^9 + a^2*b^3*x^7)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2
*a^7*b*x^9 + a^8*x^7), 1/280*(3465*b^5*x^10 + 5775*a*b^4*x^8 + 1848*a^2*b^3
*x^6 - 264*a^3*b^2*x^4 + 88*a^4*b*x^2 - 40*a^5 + 3465*(b^5*x^11 + 2*a*b^4*x
```

```
^9 + a^2*b^3*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11 + 2*a^7*b*x^
9 + a^8*x^7)]
```

Sympy [A]
time = 0.26, size = 162, normalized size = 1.43

$$-\frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4}+x\right)}{16}+\frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4}+x\right)}{16}+\frac{-40a^5+88a^4bx^2-264a^3b^2x^4+1848a^2b^3x^6+5775ab^4x^8+3465b^5x^{10}}{280a^8x^7+560a^7bx^9+280a^6b^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a)**3,x)
```

```
[Out] -99*sqrt(-b**7/a**13)*log(-a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + 99*sqrt(-b
**7/a**13)*log(a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + (-40*a**5 + 88*a**4*b*
x**2 - 264*a**3*b**2*x**4 + 1848*a**2*b**3*x**6 + 5775*a*b**4*x**8 + 3465*b
**5*x**10)/(280*a**8*x**7 + 560*a**7*b*x**9 + 280*a**6*b**2*x**11)
```

Giac [A]
time = 1.24, size = 93, normalized size = 0.82

$$\frac{99\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}+\frac{19\,b^5x^3+21\,ab^4x}{8\,(bx^2+a)^2a^6}+\frac{350\,b^3x^6-70\,ab^2x^4+21\,a^2bx^2-5\,a^3}{35\,a^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 99/8*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*x^3 + 21*a*b^4
*x)/((b*x^2 + a)^2*a^6) + 1/35*(350*b^3*x^6 - 70*a*b^2*x^4 + 21*a^2*b*x^2 -
 5*a^3)/(a^6*x^7)
```

Mupad [B]
time = 4.98, size = 102, normalized size = 0.90

$$\frac{\frac{11\,b\,x^2}{35\,a^2}-\frac{1}{7\,a}-\frac{33\,b^2\,x^4}{35\,a^3}+\frac{33\,b^3\,x^6}{5\,a^4}+\frac{165\,b^4\,x^8}{8\,a^5}+\frac{99\,b^5\,x^{10}}{8\,a^6}}{a^2\,x^7+2\,a\,b\,x^9+b^2\,x^{11}}+\frac{99\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^2)^3),x)
```

```
[Out] ((11*b*x^2)/(35*a^2) - 1/(7*a) - (33*b^2*x^4)/(35*a^3) + (33*b^3*x^6)/(5*a^
4) + (165*b^4*x^8)/(8*a^5) + (99*b^5*x^10)/(8*a^6))/(a^2*x^7 + b^2*x^11 + 2
*a*b*x^9) + (99*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(13/2))
```

3.192 $\int \frac{x^{25}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=216

$$\frac{55a^2x^2}{2b^{12}}-\frac{5ax^4}{2b^{11}}+\frac{x^6}{6b^{10}}-\frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9}+\frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8}-\frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7}+\frac{55a^9}{3b^{13}\left(a+bx^2\right)^6}-\frac{99a^8}{2b^{13}\left(a+bx^2\right)^5}-$$

```
[Out] 55/2*a^2*x^2/b^12-5/2*a*x^4/b^11+1/6*x^6/b^10-1/18*a^12/b^13/(b*x^2+a)^9+3/
4*a^11/b^13/(b*x^2+a)^8-33/7*a^10/b^13/(b*x^2+a)^7+55/3*a^9/b^13/(b*x^2+a)^
6-99/2*a^8/b^13/(b*x^2+a)^5+99*a^7/b^13/(b*x^2+a)^4-154*a^6/b^13/(b*x^2+a)^
3+198*a^5/b^13/(b*x^2+a)^2-495/2*a^4/b^13/(b*x^2+a)-110*a^3*ln(b*x^2+a)/b^1
3
```

Rubi [A]

time = 0.18, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9}+\frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8}-\frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7}+\frac{55a^9}{3b^{13}\left(a+bx^2\right)^6}-\frac{99a^8}{2b^{13}\left(a+bx^2\right)^5}+\frac{99a^7}{b^{13}\left(a+bx^2\right)^4}-\frac{154a^6}{b^{13}\left(a+bx^2\right)^3}+\frac{198a^5}{b^{13}\left(a+bx^2\right)^2}-\frac{495a^4}{2b^{13}\left(a+bx^2\right)}-\frac{110a^3\log\left(a+bx^2\right)}{b^{13}}+\frac{55a^2x^2}{2b^{12}}-\frac{5ax^4}{2b^{11}}+\frac{x^6}{6b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^25/(a + b*x^2)^10,x]
```

```
[Out] (55*a^2*x^2)/(2*b^12) - (5*a*x^4)/(2*b^11) + x^6/(6*b^10) - a^12/(18*b^13*(
a + b*x^2)^9) + (3*a^11)/(4*b^13*(a + b*x^2)^8) - (33*a^10)/(7*b^13*(a + b*
x^2)^7) + (55*a^9)/(3*b^13*(a + b*x^2)^6) - (99*a^8)/(2*b^13*(a + b*x^2)^5)
 + (99*a^7)/(b^13*(a + b*x^2)^4) - (154*a^6)/(b^13*(a + b*x^2)^3) + (198*a^
5)/(b^13*(a + b*x^2)^2) - (495*a^4)/(2*b^13*(a + b*x^2)) - (110*a^3*Log[a +
 b*x^2])/b^13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^{12}}{(a+bx)^{10}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{55a^2}{b^{12}} - \frac{10ax}{b^{11}} + \frac{x^2}{b^{10}} + \frac{a^{12}}{b^{12}(a+bx)^{10}} - \frac{12a^{11}}{b^{12}(a+bx)^9} + \frac{66a^{10}}{b^{12}(a+bx)^8} - \frac{2}{b^{12}(a}\right.\right. \\
&= \frac{55a^2x^2}{2b^{12}} - \frac{5ax^4}{2b^{11}} + \frac{x^6}{6b^{10}} - \frac{a^{12}}{18b^{13}(a+bx^2)^9} + \frac{3a^{11}}{4b^{13}(a+bx^2)^8} - \frac{33a^{10}}{7b^{13}(a+bx^2)^7} + \frac{55}{3b^{13}(a}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 169, normalized size = 0.78

$$
-\frac{35201a^{12} + 289089a^{11}bx^2 + 1031616a^{10}b^2x^4 + 2074464a^9b^3x^6 + 2529576a^8b^4x^8 + 1831032a^7b^5x^{10} + 638568a^6b^6x^{12} - 58968a^5b^7x^{14} - 139482a^4b^8x^{16} - 43218a^3b^9x^{18} - 2772a^2b^{10}x^{20} + 252ab^{11}x^{22} - 42b^{12}x^{24} + 27720a^3(a+bx^2)^9\log(a+bx^2)}{252b^{13}(a+bx^2)^9}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^25/(a + b*x^2)^10,x]
```

```
[Out] -1/252*(35201*a^12 + 289089*a^11*b*x^2 + 1031616*a^10*b^2*x^4 + 2074464*a^9
*b^3*x^6 + 2529576*a^8*b^4*x^8 + 1831032*a^7*b^5*x^10 + 638568*a^6*b^6*x^12
 - 58968*a^5*b^7*x^14 - 139482*a^4*b^8*x^16 - 43218*a^3*b^9*x^18 - 2772*a^2
*b^10*x^20 + 252*a*b^11*x^22 - 42*b^12*x^24 + 27720*a^3*(a + b*x^2)^9*Log[a
 + b*x^2])/(b^13*(a + b*x^2)^9)
```

Maple [A]
time = 0.12, size = 203, normalized size = 0.94

method	result
risch	$\frac{x^6}{6b^{10}} - \frac{5a\,x^4}{2b^{11}} + \frac{55a^2x^2}{2b^{12}} + \frac{-\frac{35201a^{12}}{252b} - \frac{32891a^{11}x^2}{28} - \frac{30371a^{10}b\,x^4}{7} - \frac{27599b^2a^9x^6}{3} - \frac{24519a^8b^3x^8}{2} - 10527a^7b^4x^{10} - 5698a^6b^5x^{12} - 178}{b^{12}(b\,x^2+a)^9}$
norman	$\frac{\frac{x^{24}}{6b} - \frac{a\,x^{22}}{b^2} + \frac{11a^2x^{20}}{b^3} - \frac{78419a^{12}}{252b^{13}} - \frac{990a^4x^{16}}{b^5} - \frac{5940a^5x^{14}}{b^6} - \frac{16940a^6x^{12}}{b^7} - \frac{28875a^7x^{10}}{b^8} - \frac{31647a^8x^8}{b^9} - \frac{22638a^9x^6}{b^{10}} - \frac{71874a^{10}x^4}{7b^{11}} - \frac{75339a^{11}x^2}{28b^{12}}}{(b\,x^2+a)^9}$
default	$\frac{\frac{1}{6}b^2x^6 - \frac{5}{2}ab\,x^4 + \frac{55}{2}a^2x^2}{b^{12}} - \frac{a^3\left(-\frac{198a^4}{b(b\,x^2+a)^4} - \frac{3a^8}{2b(b\,x^2+a)^8} - \frac{110a^6}{3b(b\,x^2+a)^6} + \frac{495a}{b(b\,x^2+a)} + \frac{308a^3}{b(b\,x^2+a)^3} + \frac{220\ln(b\,x^2+a)}{b} - \frac{396a^2}{b(b\,x^2+a)^2} + \frac{}{9b(}\right.}{2b^{12}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^25/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^12*(1/6*b^2*x^6-5/2*a*b*x^4+55/2*a^2*x^2)-1/2*a^3/b^12*(-198/b*a^4/(b*x
^2+a)^4-3/2*a^8/b/(b*x^2+a)^8-110/3/b*a^6/(b*x^2+a)^6+495*a/b/(b*x^2+a)+308
/b*a^3/(b*x^2+a)^3+220*ln(b*x^2+a)/b-396/b*a^2/(b*x^2+a)^2+1/9/b*a^9/(b*x^2
+a)^9+66/7*a^7/b/(b*x^2+a)^7+99*a^5/b/(b*x^2+a)^5)
```

Maxima [A]
time = 0.30, size = 242, normalized size = 1.12

$$-\frac{62370\,a^{4}b^{8}x^{16}+449064\,a^{5}b^{7}x^{14}+1435896\,a^{6}b^{6}x^{12}+2652804\,a^{7}b^{5}x^{10}+3089394\,a^{8}b^{4}x^{8}+2318316\,a^{9}b^{3}x^{6}+1093356\,a^{10}b^{2}x^{4}+296019\,a^{11}bx^{2}+35201\,a^{12}}{252\,(b^{22}x^{18}+9\,ab^{21}x^{16}+36\,a^{2}b^{20}x^{14}+84\,a^{3}b^{19}x^{12}+126\,a^{4}b^{18}x^{10}+126\,a^{5}b^{17}x^{8}+84\,a^{6}b^{16}x^{6}+36\,a^{7}b^{15}x^{4}+9\,a^{8}b^{14}x^{2}+a^{9}b^{13})}-\frac{110\,a^{3}\log(bx^{2}+a)}{b^{13}}+\frac{b^{2}x^{6}-15\,abx^{4}+165\,a^{2}x^{2}}{6\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(62370*a^4*b^8*x^16 + 449064*a^5*b^7*x^14 + 1435896*a^6*b^6*x^12 + 2
652804*a^7*b^5*x^10 + 3089394*a^8*b^4*x^8 + 2318316*a^9*b^3*x^6 + 1093356*a
^10*b^2*x^4 + 296019*a^11*b*x^2 + 35201*a^12)/(b^22*x^18 + 9*a*b^21*x^16 +
36*a^2*b^20*x^14 + 84*a^3*b^19*x^12 + 126*a^4*b^18*x^10 + 126*a^5*b^17*x^8
+ 84*a^6*b^16*x^6 + 36*a^7*b^15*x^4 + 9*a^8*b^14*x^2 + a^9*b^13) - 110*a^3*
log(b*x^2 + a)/b^13 + 1/6*(b^2*x^6 - 15*a*b*x^4 + 165*a^2*x^2)/b^12
```

Fricas [A]
time = 1.29, size = 346, normalized size = 1.60

$$\frac{42\,b^{12}x^{24}-252\,ab^{11}x^{22}+2772\,a^{2}b^{10}x^{20}+43218\,a^{3}b^{9}x^{18}+139482\,a^{4}b^{8}x^{16}+58968\,a^{5}b^{7}x^{14}-638568\,a^{6}b^{6}x^{12}-1831032\,a^{7}b^{5}x^{10}-2529576\,a^{8}b^{4}x^{8}-2074464\,a^{9}b^{3}x^{6}-1031616\,a^{10}b^{2}x^{4}-289089\,a^{11}bx^{2}-35201\,a^{12}-27720\,(a^{3}b^{9}x^{18}+9\,a^{4}b^{8}x^{16}+36\,a^{5}b^{7}x^{14}+84\,a^{6}b^{6}x^{12}+126\,a^{7}b^{5}x^{10}+126\,a^{8}b^{4}x^{8}+84\,a^{9}b^{3}x^{6}+36\,a^{10}b^{2}x^{4}+9\,a^{11}bx^{2}+a^{12})\log(bx^{2}+a)}{252\,(b^{22}x^{18}+9\,ab^{21}x^{16}+36\,a^{2}b^{20}x^{14}+84\,a^{3}b^{19}x^{12}+126\,a^{4}b^{18}x^{10}+126\,a^{5}b^{17}x^{8}+84\,a^{6}b^{16}x^{6}+36\,a^{7}b^{15}x^{4}+9\,a^{8}b^{14}x^{2}+a^{9}b^{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(42*b^12*x^24 - 252*a*b^11*x^22 + 2772*a^2*b^10*x^20 + 43218*a^3*b^9*
x^18 + 139482*a^4*b^8*x^16 + 58968*a^5*b^7*x^14 - 638568*a^6*b^6*x^12 - 183
1032*a^7*b^5*x^10 - 2529576*a^8*b^4*x^8 - 2074464*a^9*b^3*x^6 - 1031616*a^1
0*b^2*x^4 - 289089*a^11*b*x^2 - 35201*a^12 - 27720*(a^3*b^9*x^18 + 9*a^4*b^
8*x^16 + 36*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 126*a^7*b^5*x^10 + 126*a^8*b^4
*x^8 + 84*a^9*b^3*x^6 + 36*a^10*b^2*x^4 + 9*a^11*b*x^2 + a^12)*log(b*x^2 +
a))/(b^22*x^18 + 9*a*b^21*x^16 + 36*a^2*b^20*x^14 + 84*a^3*b^19*x^12 + 126*
a^4*b^18*x^10 + 126*a^5*b^17*x^8 + 84*a^6*b^16*x^6 + 36*a^7*b^15*x^4 + 9*a^
8*b^14*x^2 + a^9*b^13)
```

Sympy [A]
time = 0.96, size = 260, normalized size = 1.20

$$-\frac{110a^{3}\log(a+bx^{2})}{b^{13}}+\frac{55a^{2}x^{2}}{2b^{12}}-\frac{5ax^{4}}{2b^{11}}+\frac{-35201a^{12}-296019a^{11}bx^{2}-1093356a^{10}b^{2}x^{4}-2318316a^{9}b^{3}x^{6}-3089394a^{8}b^{4}x^{8}-2652804a^{7}b^{5}x^{10}-1435896a^{6}b^{6}x^{12}-449064a^{5}b^{7}x^{14}-62370a^{4}b^{8}x^{16}}{252a^{9}b^{13}+2268a^{8}b^{14}x^{2}+9072a^{7}b^{15}x^{4}+21168a^{6}b^{16}x^{6}+31752a^{5}b^{17}x^{8}+31752a^{4}b^{18}x^{10}+21168a^{3}b^{19}x^{12}+9072a^{2}b^{20}x^{14}+2268ab^{21}x^{16}+252b^{22}x^{18}}+\frac{x^{6}}{6b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**25/(b*x**2+a)**10,x)
```

```
[Out] -110*a**3*log(a + b*x**2)/b**13 + 55*a**2*x**2/(2*b**12) - 5*a*x**4/(2*b**1
1) + (-35201*a**12 - 296019*a**11*b*x**2 - 1093356*a**10*b**2*x**4 - 231831
6*a**9*b**3*x**6 - 3089394*a**8*b**4*x**8 - 2652804*a**7*b**5*x**10 - 14358
96*a**6*b**6*x**12 - 449064*a**5*b**7*x**14 - 62370*a**4*b**8*x**16)/(252*a
```

```
**9*b**13 + 2268*a**8*b**14*x**2 + 9072*a**7*b**15*x**4 + 21168*a**6*b**16*
x**6 + 31752*a**5*b**17*x**8 + 31752*a**4*b**18*x**10 + 21168*a**3*b**19*x*
*12 + 9072*a**2*b**20*x**14 + 2268*a*b**21*x**16 + 252*b**22*x**18) + x**6/
(6*b**10)
```

Giac [A]
time = 1.43, size = 168, normalized size = 0.78

$$-\frac{110\,a^3\log\left(|bx^2+a|\right)}{b^{13}}+\frac{78419\,a^3b^9x^{18}+643401\,a^4b^8x^{16}+2374020\,a^5b^7x^{14}+5151300\,a^6b^6x^{12}+7227990\,a^7b^5x^{10}+6791400\,a^8b^4x^8+4268880\,a^9b^3x^6+1729728\,a^{10}b^2x^4+409752\,a^{11}bx^2+43218\,a^{12}}{252\,(bx^2+a)^9b^{13}}+\frac{b^{20}x^6-15\,ab^{19}x^4+165\,a^2b^{18}x^2}{6\,b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -110*a^3*log(abs(b*x^2 + a))/b^13 + 1/252*(78419*a^3*b^9*x^18 + 643401*a^4*
b^8*x^16 + 2374020*a^5*b^7*x^14 + 5151300*a^6*b^6*x^12 + 7227990*a^7*b^5*x^
10 + 6791400*a^8*b^4*x^8 + 4268880*a^9*b^3*x^6 + 1729728*a^10*b^2*x^4 + 409
752*a^11*b*x^2 + 43218*a^12)/((b*x^2 + a)^9*b^13) + 1/6*(b^20*x^6 - 15*a*b^
19*x^4 + 165*a^2*b^18*x^2)/b^30
```

Mupad [B]
time = 5.29, size = 242, normalized size = 1.12

$$\frac{x^6}{6\,b^{10}}-\frac{\frac{35201\,a^{12}}{252\,b}+\frac{32891\,a^{11}\,x^2}{28}+\frac{30371\,a^{10}\,b\,x^4}{7}+\frac{27599\,a^9\,b^2\,x^6}{3}+\frac{24519\,a^8\,b^3\,x^8}{2}+10527\,a^7\,b^4\,x^{10}+5698\,a^6\,b^5\,x^{12}+1782\,a^5\,b^6\,x^{14}+\frac{495\,a^4\,b^7\,x^{16}}{2}}{a^9\,b^{12}+9\,a^8\,b^{13}\,x^2+36\,a^7\,b^{14}\,x^4+84\,a^6\,b^{15}\,x^6+126\,a^5\,b^{16}\,x^8+126\,a^4\,b^{17}\,x^{10}+84\,a^3\,b^{18}\,x^{12}+36\,a^2\,b^{19}\,x^{14}+9\,a\,b^{20}\,x^{16}+b^{21}\,x^{18}}-\frac{5\,a\,x^4}{2\,b^{11}}-\frac{110\,a^3\,\ln\left(b\,x^2+a\right)}{b^{13}}+\frac{55\,a^2\,x^2}{2\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^25/(a + b*x^2)^10,x)
```

```
[Out] x^6/(6*b^10) - ((35201*a^12)/(252*b) + (32891*a^11*x^2)/28 + (30371*a^10*b*
x^4)/7 + (27599*a^9*b^2*x^6)/3 + (24519*a^8*b^3*x^8)/2 + 10527*a^7*b^4*x^10
 + 5698*a^6*b^5*x^12 + 1782*a^5*b^6*x^14 + (495*a^4*b^7*x^16)/2)/(a^9*b^12
+ b^21*x^18 + 9*a*b^20*x^16 + 9*a^8*b^13*x^2 + 36*a^7*b^14*x^4 + 84*a^6*b^1
5*x^6 + 126*a^5*b^16*x^8 + 126*a^4*b^17*x^10 + 84*a^3*b^18*x^12 + 36*a^2*b^
19*x^14) - (5*a*x^4)/(2*b^11) - (110*a^3*log(a + b*x^2))/b^13 + (55*a^2*x^2
)/(2*b^12)
```

3.193 $\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=205

$$-\frac{5ax^2}{b^{11}}+\frac{x^4}{4b^{10}}+\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}+\frac{33a^7}{b^{12}\left(a+bx^2\right)^5}-\frac{2}{4b^{12}\,(}$$

```
[Out] -5*a*x^2/b^11+1/4*x^4/b^10+1/18*a^11/b^12/(b*x^2+a)^9-11/16*a^10/b^12/(b*x^
2+a)^8+55/14*a^9/b^12/(b*x^2+a)^7-55/4*a^8/b^12/(b*x^2+a)^6+33*a^7/b^12/(b*
x^2+a)^5-231/4*a^6/b^12/(b*x^2+a)^4+77*a^5/b^12/(b*x^2+a)^3-165/2*a^4/b^12/
(b*x^2+a)^2+165/2*a^3/b^12/(b*x^2+a)+55/2*a^2*ln(b*x^2+a)/b^12
```

Rubi [A]

time = 0.15, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}+\frac{33a^7}{b^{12}\left(a+bx^2\right)^5}-\frac{231a^6}{4b^{12}\left(a+bx^2\right)^4}+\frac{77a^5}{b^{12}\left(a+bx^2\right)^3}-\frac{165a^4}{2b^{12}\left(a+bx^2\right)^2}+\frac{165a^3}{2b^{12}\left(a+bx^2\right)}+\frac{55a^2\log\left(a+bx^2\right)}{2b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^4}{4b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/(a + b*x^2)^10,x]
```

```
[Out] (-5*a*x^2)/b^11 + x^4/(4*b^10) + a^11/(18*b^12*(a + b*x^2)^9) - (11*a^10)/(
16*b^12*(a + b*x^2)^8) + (55*a^9)/(14*b^12*(a + b*x^2)^7) - (55*a^8)/(4*b^1
2*(a + b*x^2)^6) + (33*a^7)/(b^12*(a + b*x^2)^5) - (231*a^6)/(4*b^12*(a + b
*x^2)^4) + (77*a^5)/(b^12*(a + b*x^2)^3) - (165*a^4)/(2*b^12*(a + b*x^2)^2)
 + (165*a^3)/(2*b^12*(a + b*x^2)) + (55*a^2*Log[a + b*x^2])/(2*b^12)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^{11}}{(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{10a}{b^{11}} + \frac{x}{b^{10}} - \frac{a^{11}}{b^{11}(a+bx)^{10}} + \frac{11a^{10}}{b^{11}(a+bx)^9} - \frac{55a^9}{b^{11}(a+bx)^8} + \frac{165a^8}{b^{11}(a+bx)^7}\right.\right.$$

$$= -\frac{5ax^2}{b^{11}} + \frac{x^4}{4b^{10}} + \frac{a^{11}}{18b^{12}(a+bx^2)^9} - \frac{11a^{10}}{16b^{12}(a+bx^2)^8} + \frac{55a^9}{14b^{12}(a+bx^2)^7} - \frac{55a^8}{4b^{12}(a+bx^2)}$$

Mathematica [A]
time = 0.02, size = 158, normalized size = 0.77

$$\frac{42131a^{11}+351459a^{10}bx^2+1281096a^9b^2x^4+2656584a^8b^3x^6+3402756a^7b^4x^8+2704212a^6b^5x^{10}+1220688a^5b^6x^{12}+190512a^4b^7x^{14}-77112a^3b^8x^{16}-36288a^2b^9x^{18}-2772ab^{10}x^{20}+252b^{11}x^{22}+27720a^2(a+bx^2)^9\log(a+bx^2)}{1008b^{12}(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/(a + b*x^2)^10,x]
```

```
[Out] (42131*a^11 + 351459*a^10*b*x^2 + 1281096*a^9*b^2*x^4 + 2656584*a^8*b^3*x^6
 + 3402756*a^7*b^4*x^8 + 2704212*a^6*b^5*x^10 + 1220688*a^5*b^6*x^12 + 1905
12*a^4*b^7*x^14 - 77112*a^3*b^8*x^16 - 36288*a^2*b^9*x^18 - 2772*a*b^10*x^2
0 + 252*b^11*x^22 + 27720*a^2*(a + b*x^2)^9*Log[a + b*x^2])/(1008*b^12*(a +
 b*x^2)^9)
```

Maple [A]
time = 0.14, size = 192, normalized size = 0.94

method	result
norman	$\frac{\frac{x^{22}}{4b}-\frac{11a\,x^{20}}{4b^2}+\frac{78419a^{11}}{1008b^{12}}+\frac{495a^3x^{16}}{2b^4}+\frac{1485a^4x^{14}}{b^5}+\frac{4235a^5x^{12}}{b^6}+\frac{28875a^6x^{10}}{4b^7}+\frac{31647a^7x^8}{4b^8}+\frac{11319a^8x^6}{2b^9}+\frac{35937a^9x^4}{14b^{10}}+\frac{75339a^{10}x^2}{112b^{11}}}{(b\,x^2+a)^9}+\frac{55a^2\ln}{2}$
risch	$\frac{x^4}{4b^{10}}-\frac{5a\,x^2}{b^{11}}+\frac{25a^2}{b^{12}}+\frac{\frac{42131a^{11}}{1008b}+\frac{39611a^{10}x^2}{112}+\frac{36839a^9b\,x^4}{28}+\frac{11253a^8b^2x^6}{4}+\frac{15147a^7b^3x^8}{4}+\frac{13167a^6b^4x^{10}}{4}+\frac{3619a^5b^5x^{12}}{2}+\frac{1155a^4b^6x}{2}}{b^{11}(b\,x^2+a)^9}$
default	$\frac{(-b\,x^2+10a)^2}{4b^{12}}+\frac{a^2\left(\frac{66a^5}{b(b\,x^2+a)^5}-\frac{11a^8}{8b(b\,x^2+a)^8}+\frac{165a}{b(b\,x^2+a)}+\frac{55a^7}{7b(b\,x^2+a)^7}+\frac{55\ln(b\,x^2+a)}{b}-\frac{165a^2}{b(b\,x^2+a)^2}+\frac{a^9}{9b(b\,x^2+a)^9}+\frac{154a^3}{b(b\,x^2+a)^3}-\right.}{2b^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-b*x^2+10*a)^2/b^12+1/2*a^2/b^11*(66*a^5/b/(b*x^2+a)^5-11/8*a^8/b/(b*x
^2+a)^8+165*a/b/(b*x^2+a)+55/7*a^7/b/(b*x^2+a)^7+55*ln(b*x^2+a)/b-165/b*a^2
/(b*x^2+a)^2+1/9/b*a^9/(b*x^2+a)^9+154/b*a^3/(b*x^2+a)^3-231/2/b*a^4/(b*x^2
+a)^4-55/2/b*a^6/(b*x^2+a)^6)
```

Maxima [A]
time = 0.41, size = 231, normalized size = 1.13

$$\frac{83160\,a^3b^8x^{16}+582120\,a^4b^7x^{14}+1823976\,a^5b^6x^{12}+3318084\,a^6b^5x^{10}+3817044\,a^7b^4x^8+2835756\,a^8b^3x^6+1326204\,a^9b^2x^4+356499\,a^{10}bx^2+42131\,a^{11}}{1008\,(b^{21}x^{18}+9\,ab^{20}x^{16}+36\,a^2b^{19}x^{14}+84\,a^3b^{18}x^{12}+126\,a^4b^{17}x^{10}+126\,a^5b^{16}x^8+84\,a^6b^{15}x^6+36\,a^7b^{14}x^4+9\,a^8b^{13}x^2+a^9b^{12})}+\frac{55\,a^2\log\left(bx^2+a\right)}{2\,b^{12}}+\frac{bx^4-20\,ax^2}{4\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/1008*(83160*a^3*b^8*x^16 + 582120*a^4*b^7*x^14 + 1823976*a^5*b^6*x^12 + 3
318084*a^6*b^5*x^10 + 3817044*a^7*b^4*x^8 + 2835756*a^8*b^3*x^6 + 1326204*a
^9*b^2*x^4 + 356499*a^10*b*x^2 + 42131*a^11)/(b^21*x^18 + 9*a*b^20*x^16 + 3
6*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 +
 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12) + 55/2*a^2*
log(b*x^2 + a)/b^12 + 1/4*(b*x^4 - 20*a*x^2)/b^11
```

Fricas [A]
time = 1.31, size = 335, normalized size = 1.63

$$\frac{252\,b^{11}x^{22}-2772\,ab^{10}x^{20}-36288\,a^2b^9x^{18}-77112\,a^3b^8x^{16}+190512\,a^4b^7x^{14}+1220688\,a^5b^6x^{12}+2704212\,a^6b^5x^{10}+3402756\,a^7b^4x^8+2656584\,a^8b^3x^6+1281096\,a^9b^2x^4+351459\,a^{10}bx^2+42131\,a^{11}+27720\,(a^2b^9x^{18}+9\,a^3b^8x^{16}+36\,a^4b^7x^{14}+84\,a^5b^6x^{12}+126\,a^6b^5x^{10}+126\,a^7b^4x^8+84\,a^8b^3x^6+36\,a^9b^2x^4+9\,a^{10}bx^2+a^{11})\log\left(bx^2+a\right)}{1008\,(b^{21}x^{18}+9\,ab^{20}x^{16}+36\,a^2b^{19}x^{14}+84\,a^3b^{18}x^{12}+126\,a^4b^{17}x^{10}+126\,a^5b^{16}x^8+84\,a^6b^{15}x^6+36\,a^7b^{14}x^4+9\,a^8b^{13}x^2+a^9b^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/1008*(252*b^11*x^22 - 2772*a*b^10*x^20 - 36288*a^2*b^9*x^18 - 77112*a^3*b
^8*x^16 + 190512*a^4*b^7*x^14 + 1220688*a^5*b^6*x^12 + 2704212*a^6*b^5*x^10
 + 3402756*a^7*b^4*x^8 + 2656584*a^8*b^3*x^6 + 1281096*a^9*b^2*x^4 + 351459
*a^10*b*x^2 + 42131*a^11 + 27720*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^
7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*
x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*log(b*x^2 + a))/(b^21*x^18 + 9*
a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126
*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^
12)
```

Sympy [A]
time = 0.93, size = 245, normalized size = 1.20

$$\frac{55a^2\log\left(a+bx^2\right)}{2b^{12}}-\frac{5ax^2}{b^{11}}+\frac{42131a^{11}+356499a^{10}bx^2+1326204a^9b^2x^4+2835756a^8b^3x^6+3817044a^7b^4x^8+3318084a^6b^5x^{10}+1823976a^5b^6x^{12}+582120a^4b^7x^{14}+83160a^3b^8x^{16}}{1008a^9b^{12}+9072a^8b^{13}x^2+36288a^7b^{14}x^4+84672a^6b^{15}x^6+127008a^5b^{16}x^8+127008a^4b^{17}x^{10}+84672a^3b^{18}x^{12}+36288a^2b^{19}x^{14}+9072ab^{20}x^{16}+1008b^{21}x^{18}}+\frac{x^4}{4b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**2+a)**10,x)
```

```
[Out] 55*a**2*log(a + b*x**2)/(2*b**12) - 5*a*x**2/b**11 + (42131*a**11 + 356499*
a**10*b*x**2 + 1326204*a**9*b**2*x**4 + 2835756*a**8*b**3*x**6 + 3817044*a*
*7*b**4*x**8 + 3318084*a**6*b**5*x**10 + 1823976*a**5*b**6*x**12 + 582120*a
**4*b**7*x**14 + 83160*a**3*b**8*x**16)/(1008*a**9*b**12 + 9072*a**8*b**13*
```

```
x**2 + 36288*a**7*b**14*x**4 + 84672*a**6*b**15*x**6 + 127008*a**5*b**16*x*
*8 + 127008*a**4*b**17*x**10 + 84672*a**3*b**18*x**12 + 36288*a**2*b**19*x*
*14 + 9072*a*b**20*x**16 + 1008*b**21*x**18) + x**4/(4*b**10)
```

Giac [A]
time = 1.21, size = 157, normalized size = 0.77

$$\frac{55\,a^2\log\left(|bx^2+a|\right)}{2\,b^{12}}+\frac{b^{10}x^4-20\,ab^9x^2}{4\,b^{20}}-\frac{78419\,a^2b^9x^{18}+622611\,a^3b^8x^{16}+2240964\,a^4b^7x^{14}+4763220\,a^5b^6x^{12}+6562710\,a^6b^5x^{10}+6063750\,a^7b^4x^8+3751440\,a^8b^3x^6+1496880\,a^9b^2x^4+349272\,a^{10}bx^2+36288\,a^{11}}{1008\,(bx^2+a)^9b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/2*a^2*log(abs(b*x^2 + a))/b^12 + 1/4*(b^10*x^4 - 20*a*b^9*x^2)/b^20 - 1/
1008*(78419*a^2*b^9*x^18 + 622611*a^3*b^8*x^16 + 2240964*a^4*b^7*x^14 + 476
3220*a^5*b^6*x^12 + 6562710*a^6*b^5*x^10 + 6063750*a^7*b^4*x^8 + 3751440*a^
8*b^3*x^6 + 1496880*a^9*b^2*x^4 + 349272*a^10*b*x^2 + 36288*a^11)/((b*x^2 +
 a)^9*b^12)
```

Mupad [B]
time = 0.40, size = 230, normalized size = 1.12

$$\frac{\frac{42131\,a^{11}}{1008\,b}+\frac{39611\,a^{10}\,x^2}{112}+\frac{36839\,a^9\,b\,x^4}{28}+\frac{11253\,a^8\,b^2\,x^6}{4}+\frac{15147\,a^7\,b^3\,x^8}{4}+\frac{13167\,a^6\,b^4\,x^{10}}{4}+\frac{3619\,a^5\,b^5\,x^{12}}{2}+\frac{1155\,a^4\,b^6\,x^{14}}{2}+\frac{165\,a^3\,b^7\,x^{16}}{2}}{a^9\,b^{11}+9\,a^8\,b^{12}\,x^2+36\,a^7\,b^{13}\,x^4+84\,a^6\,b^{14}\,x^6+126\,a^5\,b^{15}\,x^8+126\,a^4\,b^{16}\,x^{10}+84\,a^3\,b^{17}\,x^{12}+36\,a^2\,b^{18}\,x^{14}+9\,a\,b^{19}\,x^{16}+b^{20}\,x^{18}}+\frac{x^4}{4\,b^{10}}-\frac{5\,a\,x^2}{b^{11}}+\frac{55\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(a + b*x^2)^10,x)
```

```
[Out] ((42131*a^11)/(1008*b) + (39611*a^10*x^2)/112 + (36839*a^9*b*x^4)/28 + (112
53*a^8*b^2*x^6)/4 + (15147*a^7*b^3*x^8)/4 + (13167*a^6*b^4*x^10)/4 + (3619*
a^5*b^5*x^12)/2 + (1155*a^4*b^6*x^14)/2 + (165*a^3*b^7*x^16)/2)/(a^9*b^11 +
 b^20*x^18 + 9*a*b^19*x^16 + 9*a^8*b^12*x^2 + 36*a^7*b^13*x^4 + 84*a^6*b^14
*x^6 + 126*a^5*b^15*x^8 + 126*a^4*b^16*x^10 + 84*a^3*b^17*x^12 + 36*a^2*b^1
8*x^14) + x^4/(4*b^10) - (5*a*x^2)/b^11 + (55*a^2*log(a + b*x^2))/(2*b^12)
```

3.194 $\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=188

$$\frac{x^2}{2b^{10}}-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21a^6}{b^{11}\left(a+bx^2\right)^5}+\frac{63a^5}{2b^{11}\left(a+bx^2\right)^4}-$$

```
[Out] 1/2*x^2/b^10-1/18*a^10/b^11/(b*x^2+a)^9+5/8*a^9/b^11/(b*x^2+a)^8-45/14*a^8/
b^11/(b*x^2+a)^7+10*a^7/b^11/(b*x^2+a)^6-21*a^6/b^11/(b*x^2+a)^5+63/2*a^5/b
^11/(b*x^2+a)^4-35*a^4/b^11/(b*x^2+a)^3+30*a^3/b^11/(b*x^2+a)^2-45/2*a^2/b^
11/(b*x^2+a)-5*a*ln(b*x^2+a)/b^11
```

Rubi [A]
time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21a^6}{b^{11}\left(a+bx^2\right)^5}+\frac{63a^5}{2b^{11}\left(a+bx^2\right)^4}-\frac{35a^4}{b^{11}\left(a+bx^2\right)^3}+\frac{30a^3}{b^{11}\left(a+bx^2\right)^2}-\frac{45a^2}{2b^{11}\left(a+bx^2\right)}-\frac{5a\log\left(a+bx^2\right)}{b^{11}}+\frac{x^2}{2b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^21/(a + b*x^2)^10,x]
```

```
[Out] x^2/(2*b^10) - a^10/(18*b^11*(a + b*x^2)^9) + (5*a^9)/(8*b^11*(a + b*x^2)^8
) - (45*a^8)/(14*b^11*(a + b*x^2)^7) + (10*a^7)/(b^11*(a + b*x^2)^6) - (21*
a^6)/(b^11*(a + b*x^2)^5) + (63*a^5)/(2*b^11*(a + b*x^2)^4) - (35*a^4)/(b^1
1*(a + b*x^2)^3) + (30*a^3)/(b^11*(a + b*x^2)^2) - (45*a^2)/(2*b^11*(a + b*
x^2)) - (5*a*Log[a + b*x^2])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^{10}}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^{10}} + \frac{a^{10}}{b^{10}(a+bx)^{10}} - \frac{10a^9}{b^{10}(a+bx)^9} + \frac{45a^8}{b^{10}(a+bx)^8} - \frac{120a^7}{b^{10}(a+bx)^7} + \frac{21\ldots}{b^{10}(a\ldots}\right.\right.$$
$$= \frac{x^2}{2b^{10}} - \frac{a^{10}}{18b^{11}(a+bx^2)^9} + \frac{5a^9}{8b^{11}(a+bx^2)^8} - \frac{45a^8}{14b^{11}(a+bx^2)^7} + \frac{10a^7}{b^{11}(a+bx^2)^6} - \frac{21a\ldots}{b^{11}(a+\ldots}$$

Mathematica [A]
time = 0.02, size = 145, normalized size = 0.77

$$-\frac{4861a^{10}+41229a^9bx^2+153576a^8b^2x^4+328104a^7b^3x^6+439236a^6b^4x^8+375732a^5b^5x^{10}+197568a^4b^6x^{12}+54432a^3b^7x^{14}+2268a^2b^8x^{16}-2268ab^9x^{18}-252b^{10}x^{20}+2520a(a+bx^2)^9\log(a+bx^2)}{504b^{11}(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^21/(a + b*x^2)^10,x]
```

```
[Out] -1/504*(4861*a^10 + 41229*a^9*b*x^2 + 153576*a^8*b^2*x^4 + 328104*a^7*b^3*x
^6 + 439236*a^6*b^4*x^8 + 375732*a^5*b^5*x^10 + 197568*a^4*b^6*x^12 + 54432
*a^3*b^7*x^14 + 2268*a^2*b^8*x^16 - 2268*a*b^9*x^18 - 252*b^10*x^20 + 2520*
a*(a + b*x^2)^9*Log[a + b*x^2])/(b^11*(a + b*x^2)^9)
```

Maple [A]
time = 0.10, size = 181, normalized size = 0.96

method	result
risch	$\frac{x^2}{2b^{10}}+\frac{-\frac{4861a^{10}}{504b}-\frac{4609a^9x^2}{56}-\frac{4329a^8b\,x^4}{14}-669a^7b^2x^6-\frac{1827a^6b^3x^8}{2}-\frac{1617a^5b^4x^{10}}{2}-455a^4b^5x^{12}-150a^3b^6x^{14}-\frac{45a^2b^7x^{16}}{2}}{b^{10}(b\,x^2+a)^9}-\frac{5a\ln(b\ldots}{b^{1\ldots}}$
norman	$\frac{\frac{x^{20}}{2b}-\frac{7129a^{10}}{504b^{11}}-\frac{45a^2x^{16}}{b^3}-\frac{270a^3x^{14}}{b^4}-\frac{770a^4x^{12}}{b^5}-\frac{2625a^5x^{10}}{2b^6}-\frac{2877a^6x^8}{2b^7}-\frac{1029a^7x^6}{b^8}-\frac{3267a^8x^4}{7b^9}-\frac{6849a^9x^2}{56b^{10}}}{(b\,x^2+a)^9}-\frac{5a\ln(b\,x^2+a)}{b^{11}}$
default	$\frac{x^2}{2b^{10}}-\frac{a\left(-\frac{63a^4}{b(b\,x^2+a)^4}+\frac{45a}{b(b\,x^2+a)}+\frac{45a^7}{7b(b\,x^2+a)^7}+\frac{10\ln(b\,x^2+a)}{b}+\frac{42a^5}{b(b\,x^2+a)^5}-\frac{60a^2}{b(b\,x^2+a)^2}-\frac{5a^8}{4b(b\,x^2+a)^8}-\frac{20a^6}{b(b\,x^2+a)^6}+\frac{a^9}{9b(b\,x^2+a)^9}\ldots}{2b^{10}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^21/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b^10-1/2*a/b^10*(-63/b*a^4/(b*x^2+a)^4+45*a/b/(b*x^2+a)+45/7*a^7/b/
(b*x^2+a)^7+10*ln(b*x^2+a)/b+42*a^5/b/(b*x^2+a)^5-60/b*a^2/(b*x^2+a)^2-5/4*
a^8/b/(b*x^2+a)^8-20/b*a^6/(b*x^2+a)^6+1/9/b*a^9/(b*x^2+a)^9+70/b*a^3/(b*x^
2+a)^3)
```

Maxima [A]
time = 0.40, size = 220, normalized size = 1.17

$$-\frac{11340\,a^2b^8x^{16}+75600\,a^3b^7x^{14}+229320\,a^4b^6x^{12}+407484\,a^5b^5x^{10}+460404\,a^6b^4x^8+337176\,a^7b^3x^6+155844\,a^8b^2x^4+41481\,a^9bx^2+4861\,a^{10}}{504\,(b^{20}x^{18}+9\,ab^{19}x^{16}+36\,a^2b^{18}x^{14}+84\,a^3b^{17}x^{12}+126\,a^4b^{16}x^{10}+126\,a^5b^{15}x^8+84\,a^6b^{14}x^6+36\,a^7b^{13}x^4+9\,a^8b^{12}x^2+a^9b^{11})}+\frac{x^2}{2\,b^{10}}-\frac{5\,a\log(bx^2+a)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(11340*a^2*b^8*x^16 + 75600*a^3*b^7*x^14 + 229320*a^4*b^6*x^12 + 407
484*a^5*b^5*x^10 + 460404*a^6*b^4*x^8 + 337176*a^7*b^3*x^6 + 155844*a^8*b^2
*x^4 + 41481*a^9*b*x^2 + 4861*a^10)/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^1
8*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b
^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11) + 1/2*x^2/b^10 - 5*a
*log(b*x^2 + a)/b^11
```

Fricas [A]
time = 0.96, size = 322, normalized size = 1.71

$$\frac{252\,b^{10}x^{20} + 2268\,ab^{9}x^{18} - 2268\,a^{2}b^{8}x^{16} - 54432\,a^{3}b^{7}x^{14} - 197568\,a^{4}b^{6}x^{12} - 375732\,a^{5}b^{5}x^{10} - 439236\,a^{6}b^{4}x^{8} - 328104\,a^{7}b^{3}x^{6} - 153576\,a^{8}b^{2}x^{4} - 41229\,a^{9}bx^{2} - 4861\,a^{10} - 2520\,(ab^{9}x^{18} + 9\,a^{2}b^{8}x^{16} + 36\,a^{3}b^{7}x^{14} + 84\,a^{4}b^{6}x^{12} + 126\,a^{5}b^{5}x^{10} + 126\,a^{6}b^{4}x^{8} + 84\,a^{7}b^{3}x^{6} + 36\,a^{8}b^{2}x^{4} + 9\,a^{9}bx^{2} + a^{10})\log\left(bx^{2} + a\right)}{504\,(b^{20}x^{18} + 9\,ab^{19}x^{16} + 36\,a^{2}b^{18}x^{14} + 84\,a^{3}b^{17}x^{12} + 126\,a^{4}b^{16}x^{10} + 126\,a^{5}b^{15}x^{8} + 84\,a^{6}b^{14}x^{6} + 36\,a^{7}b^{13}x^{4} + 9\,a^{8}b^{12}x^{2} + a^{9}b^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^10*x^20 + 2268*a*b^9*x^18 - 2268*a^2*b^8*x^16 - 54432*a^3*b^7*
x^14 - 197568*a^4*b^6*x^12 - 375732*a^5*b^5*x^10 - 439236*a^6*b^4*x^8 - 328
104*a^7*b^3*x^6 - 153576*a^8*b^2*x^4 - 41229*a^9*b*x^2 - 4861*a^10 - 2520*(
a*b^9*x^18 + 9*a^2*b^8*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b
^5*x^10 + 126*a^6*b^4*x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 +
 a^10)*log(b*x^2 + a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a
^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*
a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11)
```

Sympy [A]
time = 0.88, size = 233, normalized size = 1.24

$$-\frac{5a\log\left(a + bx^{2}\right)}{b^{11}} + \frac{-4861a^{10} - 41481a^{9}bx^{2} - 155844a^{8}b^{2}x^{4} - 337176a^{7}b^{3}x^{6} - 460404a^{6}b^{4}x^{8} - 407484a^{5}b^{5}x^{10} - 229320a^{4}b^{6}x^{12} - 75600a^{3}b^{7}x^{14} - 11340a^{2}b^{8}x^{16}}{504a^{9}b^{11} + 4536a^{8}b^{12}x^{2} + 18144a^{7}b^{13}x^{4} + 42336a^{6}b^{14}x^{6} + 63504a^{5}b^{15}x^{8} + 63504a^{4}b^{16}x^{10} + 42336a^{3}b^{17}x^{12} + 18144a^{2}b^{18}x^{14} + 4536ab^{19}x^{16} + 504b^{20}x^{18}} + \frac{x^{2}}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**21/(b*x**2+a)**10,x)
```

```
[Out] -5*a*log(a + b*x**2)/b**11 + (-4861*a**10 - 41481*a**9*b*x**2 - 155844*a**8
*b**2*x**4 - 337176*a**7*b**3*x**6 - 460404*a**6*b**4*x**8 - 407484*a**5*b*
*5*x**10 - 229320*a**4*b**6*x**12 - 75600*a**3*b**7*x**14 - 11340*a**2*b**8
*x**16)/(504*a**9*b**11 + 4536*a**8*b**12*x**2 + 18144*a**7*b**13*x**4 + 42
336*a**6*b**14*x**6 + 63504*a**5*b**15*x**8 + 63504*a**4*b**16*x**10 + 4233
6*a**3*b**17*x**12 + 18144*a**2*b**18*x**14 + 4536*a*b**19*x**16 + 504*b**2
0*x**18) + x**2/(2*b**10)
```

Giac [A]
time = 1.16, size = 139, normalized size = 0.74

$$\frac{x^{2}}{2\,b^{10}} - \frac{5\,a\log\left(|bx^{2} + a|\right)}{b^{11}} + \frac{7129\,ab^{9}x^{18} + 52821\,a^{2}b^{8}x^{16} + 181044\,a^{3}b^{7}x^{14} + 369516\,a^{4}b^{6}x^{12} + 490770\,a^{5}b^{5}x^{10} + 437850\,a^{6}b^{4}x^{8} + 261660\,a^{7}b^{3}x^{6} + 100800\,a^{8}b^{2}x^{4} + 22680\,a^{9}bx^{2} + 2268\,a^{10}}{504\,(bx^{2} + a)^{9}b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^10 - 5*a*log(abs(b*x^2 + a))/b^11 + 1/504*(7129*a*b^9*x^18 + 5282
1*a^2*b^8*x^16 + 181044*a^3*b^7*x^14 + 369516*a^4*b^6*x^12 + 490770*a^5*b^5
*x^10 + 437850*a^6*b^4*x^8 + 261660*a^7*b^3*x^6 + 100800*a^8*b^2*x^4 + 2268
0*a^9*b*x^2 + 2268*a^10)/((b*x^2 + a)^9*b^11)
```

Mupad [B]
time = 0.43, size = 220, normalized size = 1.17

$$\frac{x^2}{2\,b^{10}}-\frac{\frac{4861\,a^{10}}{504\,b}+\frac{4609\,a^9\,x^2}{56}+\frac{4329\,a^8\,b\,x^4}{14}+669\,a^7\,b^2\,x^6+\frac{1827\,a^6\,b^3\,x^8}{2}+\frac{1617\,a^5\,b^4\,x^{10}}{2}+455\,a^4\,b^5\,x^{12}+150\,a^3\,b^6\,x^{14}+\frac{45\,a^2\,b^7\,x^{16}}{2}}{a^9\,b^{10}+9\,a^8\,b^{11}\,x^2+36\,a^7\,b^{12}\,x^4+84\,a^6\,b^{13}\,x^6+126\,a^5\,b^{14}\,x^8+126\,a^4\,b^{15}\,x^{10}+84\,a^3\,b^{16}\,x^{12}+36\,a^2\,b^{17}\,x^{14}+9\,a\,b^{18}\,x^{16}+b^{19}\,x^{18}}-\frac{5\,a\,\ln\left(b\,x^2+a\right)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^21/(a + b*x^2)^10,x)
```

```
[Out] x^2/(2*b^10) - ((4861*a^10)/(504*b) + (4609*a^9*x^2)/56 + (4329*a^8*b*x^4)/
14 + 669*a^7*b^2*x^6 + (1827*a^6*b^3*x^8)/2 + (1617*a^5*b^4*x^10)/2 + 455*a
^4*b^5*x^12 + 150*a^3*b^6*x^14 + (45*a^2*b^7*x^16)/2)/(a^9*b^10 + b^19*x^18
 + 9*a*b^18*x^16 + 9*a^8*b^11*x^2 + 36*a^7*b^12*x^4 + 84*a^6*b^13*x^6 + 126
*a^5*b^14*x^8 + 126*a^4*b^15*x^10 + 84*a^3*b^16*x^12 + 36*a^2*b^17*x^14) -
(5*a*log(a + b*x^2))/b^11
```

3.195 $\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=179

$$\frac{a^9}{18b^{10}(a+bx^2)^9}-\frac{9a^8}{16b^{10}(a+bx^2)^8}+\frac{18a^7}{7b^{10}(a+bx^2)^7}-\frac{7a^6}{b^{10}(a+bx^2)^6}+\frac{63a^5}{5b^{10}(a+bx^2)^5}-\frac{63a^4}{4b^{10}(a+bx^2)^4}+\frac{}{b^{10}(}$$

```
[Out] 1/18*a^9/b^10/(b*x^2+a)^9-9/16*a^8/b^10/(b*x^2+a)^8+18/7*a^7/b^10/(b*x^2+a)
^7-7*a^6/b^10/(b*x^2+a)^6+63/5*a^5/b^10/(b*x^2+a)^5-63/4*a^4/b^10/(b*x^2+a)
^4+14*a^3/b^10/(b*x^2+a)^3-9*a^2/b^10/(b*x^2+a)^2+9/2*a/b^10/(b*x^2+a)+1/2*
ln(b*x^2+a)/b^10
```

Rubi [A]
time = 0.11, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a^9}{18b^{10}(a+bx^2)^9}-\frac{9a^8}{16b^{10}(a+bx^2)^8}+\frac{18a^7}{7b^{10}(a+bx^2)^7}-\frac{7a^6}{b^{10}(a+bx^2)^6}+\frac{63a^5}{5b^{10}(a+bx^2)^5}-\frac{63a^4}{4b^{10}(a+bx^2)^4}+\frac{14a^3}{b^{10}(a+bx^2)^3}-\frac{9a^2}{b^{10}(a+bx^2)^2}+\frac{9a}{2b^{10}(a+bx^2)}+\frac{\log(a+bx^2)}{2b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a + b*x^2)^10,x]
```

```
[Out] a^9/(18*b^10*(a + b*x^2)^9) - (9*a^8)/(16*b^10*(a + b*x^2)^8) + (18*a^7)/(7
*b^10*(a + b*x^2)^7) - (7*a^6)/(b^10*(a + b*x^2)^6) + (63*a^5)/(5*b^10*(a +
 b*x^2)^5) - (63*a^4)/(4*b^10*(a + b*x^2)^4) + (14*a^3)/(b^10*(a + b*x^2)^3
) - (9*a^2)/(b^10*(a + b*x^2)^2) + (9*a)/(2*b^10*(a + b*x^2)) + Log[a + b*x
^2]/(2*b^10)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^9}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^9}{b^9(a+bx)^{10}} + \frac{9a^8}{b^9(a+bx)^9} - \frac{36a^7}{b^9(a+bx)^8} + \frac{84a^6}{b^9(a+bx)^7} - \frac{126a^5}{b^9(a+bx)^6} +\right.\right.$$
$$= \frac{a^9}{18b^{10}(a+bx^2)^9} - \frac{9a^8}{16b^{10}(a+bx^2)^8} + \frac{18a^7}{7b^{10}(a+bx^2)^7} - \frac{7a^6}{b^{10}(a+bx^2)^6} + \frac{63a^5}{5b^{10}(a+bx^2)^5}$$

Mathematica [A]
time = 0.02, size = 116, normalized size = 0.65

$$\frac{\frac{a(7129a^8+61641a^7bx^2+235224a^6b^2x^4+518616a^5b^3x^6+725004a^4b^4x^8+661500a^3b^5x^{10}+388080a^2b^6x^{12}+136080ab^7x^{14}+22680b^8x^{16})}{(a+bx^2)^9} + 2520\log(a+bx^2)}{5040b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a + b*x^2)^10,x]
```

```
[Out] ((a*(7129*a^8 + 61641*a^7*b*x^2 + 235224*a^6*b^2*x^4 + 518616*a^5*b^3*x^6 +
 725004*a^4*b^4*x^8 + 661500*a^3*b^5*x^10 + 388080*a^2*b^6*x^12 + 136080*a*
b^7*x^14 + 22680*b^8*x^16))/(a + b*x^2)^9 + 2520*Log[a + b*x^2])/(5040*b^10
)
```

Maple [A]
time = 0.10, size = 166, normalized size = 0.93

method	result
norman	$\frac{\frac{7129a^9}{5040b^{10}}+\frac{9a\,x^{16}}{2b^2}+\frac{27a^2x^{14}}{b^3}+\frac{77a^3x^{12}}{b^4}+\frac{525a^4x^{10}}{4b^5}+\frac{2877a^5x^8}{20b^6}+\frac{1029a^6x^6}{10b^7}+\frac{3267a^7x^4}{70b^8}+\frac{6849a^8x^2}{560b^9}}{(b\,x^2+a)^9}+\frac{\ln(b\,x^2+a)}{2b^{10}}$
risch	$\frac{\frac{7129a^9}{5040b^{10}}+\frac{9a\,x^{16}}{2b^2}+\frac{27a^2x^{14}}{b^3}+\frac{77a^3x^{12}}{b^4}+\frac{525a^4x^{10}}{4b^5}+\frac{2877a^5x^8}{20b^6}+\frac{1029a^6x^6}{10b^7}+\frac{3267a^7x^4}{70b^8}+\frac{6849a^8x^2}{560b^9}}{(b\,x^2+a)^9}+\frac{\ln(b\,x^2+a)}{2b^{10}}$
default	$\frac{a^9}{18b^{10}(b\,x^2+a)^9}-\frac{9a^8}{16b^{10}(b\,x^2+a)^8}+\frac{18a^7}{7b^{10}(b\,x^2+a)^7}-\frac{7a^6}{b^{10}(b\,x^2+a)^6}+\frac{63a^5}{5b^{10}(b\,x^2+a)^5}-\frac{63a^4}{4b^{10}(b\,x^2+a)^4}+\frac{14a^3}{b^{10}(b\,x^2+a)^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*a^9/b^10/(b*x^2+a)^9-9/16*a^8/b^10/(b*x^2+a)^8+18/7*a^7/b^10/(b*x^2+a)
^7-7*a^6/b^10/(b*x^2+a)^6+63/5*a^5/b^10/(b*x^2+a)^5-63/4*a^4/b^10/(b*x^2+a)
^4+14*a^3/b^10/(b*x^2+a)^3-9*a^2/b^10/(b*x^2+a)^2+9/2*a/b^10/(b*x^2+a)+1/2*
ln(b*x^2+a)/b^10
```

Maxima [A]
time = 0.30, size = 209, normalized size = 1.17

$$\frac{22680\,ab^8x^{16}+136080\,a^2b^7x^{14}+388080\,a^3b^6x^{12}+661500\,a^4b^5x^{10}+725004\,a^5b^4x^8+518616\,a^6b^3x^6+235224\,a^7b^2x^4+61641\,a^8bx^2+7129\,a^9}{5040\,(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+84\,a^3b^{16}x^{12}+126\,a^4b^{15}x^{10}+126\,a^5b^{14}x^8+84\,a^6b^{13}x^6+36\,a^7b^{12}x^4+9\,a^8b^{11}x^2+a^9b^{10})}+\frac{\log(bx^2+a)}{2\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="maxima")

[Out] 1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 661500*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^2*x^4 + 61641*a^8*b*x^2 + 7129*a^9)/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10) + 1/2*log(b*x^2 + a)/b^10

Fricas [A]
time = 1.47, size = 300, normalized size = 1.68

$$\frac{22680\,ab^8x^{16}+136080\,a^2b^7x^{14}+388080\,a^3b^6x^{12}+661500\,a^4b^5x^{10}+725004\,a^5b^4x^8+518616\,a^6b^3x^6+235224\,a^7b^2x^4+61641\,a^8bx^2+7129\,a^9+2520\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\log\left(bx^2+a\right)}{5040\,(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+84\,a^3b^{16}x^{12}+126\,a^4b^{15}x^{10}+126\,a^5b^{14}x^8+84\,a^6b^{13}x^6+36\,a^7b^{12}x^4+9\,a^8b^{11}x^2+a^9b^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="fricas")

[Out] 1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 661500*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^2*x^4 + 61641*a^8*b*x^2 + 7129*a^9 + 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10)

Sympy [A]
time = 0.75, size = 219, normalized size = 1.22

$$\frac{7129a^9+61641a^8bx^2+235224a^7b^2x^4+518616a^6b^3x^6+725004a^5b^4x^8+661500a^4b^5x^{10}+388080a^3b^6x^{12}+136080a^2b^7x^{14}+22680ab^8x^{16}}{5040a^9b^{10}+45360a^8b^{11}x^2+181440a^7b^{12}x^4+423360a^6b^{13}x^6+635040a^5b^{14}x^8+635040a^4b^{15}x^{10}+423360a^3b^{16}x^{12}+181440a^2b^{17}x^{14}+45360ab^{18}x^{16}+5040b^{19}x^{18}}+\frac{\log\left(a+bx^2\right)}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**19/(b*x**2+a)**10,x)

[Out] (7129*a**9 + 61641*a**8*b*x**2 + 235224*a**7*b**2*x**4 + 518616*a**6*b**3*x**6 + 725004*a**5*b**4*x**8 + 661500*a**4*b**5*x**10 + 388080*a**3*b**6*x**12 + 136080*a**2*b**7*x**14 + 22680*a*b**8*x**16)/(5040*a**9*b**10 + 45360*a**8*b**11*x**2 + 181440*a**7*b**12*x**4 + 423360*a**6*b**13*x**6 + 635040*a**5*b**14*x**8 + 635040*a**4*b**15*x**10 + 423360*a**3*b**16*x**12 + 181440*a**2*b**17*x**14 + 45360*a*b**18*x**16 + 5040*b**19*x**18) + log(a + b*x**2)/(2*b**10)

Giac [A]
time = 1.73, size = 119, normalized size = 0.66

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^{10}}-\frac{7129\,b^8x^{18}+41481\,ab^7x^{16}+120564\,a^2b^6x^{14}+210756\,a^3b^5x^{12}+236754\,a^4b^4x^{10}+173250\,a^5b^3x^8+80220\,a^6b^2x^6+21420\,a^7bx^4+2520\,a^8x^2}{5040\,\left(bx^2+a\right)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^10 - 1/5040*(7129*b^8*x^18 + 41481*a*b^7*x^16 + 1
20564*a^2*b^6*x^14 + 210756*a^3*b^5*x^12 + 236754*a^4*b^4*x^10 + 173250*a^5
*b^3*x^8 + 80220*a^6*b^2*x^6 + 21420*a^7*b*x^4 + 2520*a^8*x^2)/((b*x^2 + a)
^9*b^9)
```

Mupad [B]
time = 5.36, size = 207, normalized size = 1.16

$$\frac{\frac{7129\,a^9}{5040\,b^{10}}+\frac{9\,a\,x^{16}}{2\,b^2}+\frac{27\,a^2\,x^{14}}{b^3}+\frac{77\,a^3\,x^{12}}{b^4}+\frac{525\,a^4\,x^{10}}{4\,b^5}+\frac{2877\,a^5\,x^8}{20\,b^6}+\frac{1029\,a^6\,x^6}{10\,b^7}+\frac{3267\,a^7\,x^4}{70\,b^8}+\frac{6849\,a^8\,x^2}{560\,b^9}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{\ln\left(b\,x^2+a\right)}{2\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a + b*x^2)^10,x)
```

```
[Out] ((7129*a^9)/(5040*b^10) + (9*a*x^16)/(2*b^2) + (27*a^2*x^14)/b^3 + (77*a^3*
x^12)/b^4 + (525*a^4*x^10)/(4*b^5) + (2877*a^5*x^8)/(20*b^6) + (1029*a^6*x^
6)/(10*b^7) + (3267*a^7*x^4)/(70*b^8) + (6849*a^8*x^2)/(560*b^9))/(a^9 + b^
9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126
*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + log(
a + b*x^2)/(2*b^10)
```

3.196 $\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=19

$$\frac{x^{18}}{18a\left(a+bx^2\right)^9}$$

[Out] 1/18*x^18/a/(b*x^2+a)^9

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{x^{18}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

[In] Int[x^17/(a + b*x^2)^10,x]

[Out] x^18/(18*a*(a + b*x^2)^9)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{17}}{\left(a+bx^2\right)^{10}}\,dx = \frac{x^{18}}{18a\left(a+bx^2\right)^9}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 101 vs. 2(19) = 38.
time = 0.01, size = 101, normalized size = 5.32

$$-\frac{a^8+9a^7bx^2+36a^6b^2x^4+84a^5b^3x^6+126a^4b^4x^8+126a^3b^5x^{10}+84a^2b^6x^{12}+36ab^7x^{14}+9b^8x^{16}}{18b^9\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^17/(a + b*x^2)^10,x]

```
[Out] -1/18*(a^8 + 9*a^7*b*x^2 + 36*a^6*b^2*x^4 + 84*a^5*b^3*x^6 + 126*a^4*b^4*x^
8 + 126*a^3*b^5*x^10 + 84*a^2*b^6*x^12 + 36*a*b^7*x^14 + 9*b^8*x^16)/(b^9*(
a + b*x^2)^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(17) = 34$.
time = 0.09, size = 150, normalized size = 7.89

method	result
gosper	$-\frac{9b^8x^{16}+36a\,b^7x^{14}+84a^2b^6x^{12}+126a^3b^5x^{10}+126a^4b^4x^8+84a^5b^3x^6+36a^6b^2x^4+9a^7b\,x^2+a^8}{18(b\,x^2+a)^9b^9}$
norman	$\frac{-\frac{a^8}{18b^9}-\frac{a^7x^2}{2b^8}-\frac{2a^6x^4}{b^7}-\frac{14a^5x^6}{3b^6}-\frac{7a^4x^8}{b^5}-\frac{7a^3x^{10}}{b^4}-\frac{14a^2x^{12}}{3b^3}-\frac{2a\,x^{14}}{b^2}-\frac{x^{16}}{2b}}{(b\,x^2+a)^9}$
risch	$\frac{-\frac{a^8}{18b^9}-\frac{a^7x^2}{2b^8}-\frac{2a^6x^4}{b^7}-\frac{14a^5x^6}{3b^6}-\frac{7a^4x^8}{b^5}-\frac{7a^3x^{10}}{b^4}-\frac{14a^2x^{12}}{3b^3}-\frac{2a\,x^{14}}{b^2}-\frac{x^{16}}{2b}}{(b\,x^2+a)^9}$
default	$\frac{7a^3}{b^9(b\,x^2+a)^4}+\frac{14a^5}{3b^9(b\,x^2+a)^6}-\frac{7a^4}{b^9(b\,x^2+a)^5}-\frac{1}{2b^9(b\,x^2+a)}+\frac{2a}{b^9(b\,x^2+a)^2}-\frac{a^8}{18b^9(b\,x^2+a)^9}-\frac{2a^6}{b^9(b\,x^2+a)^7}-\frac{14a^2}{3b^9(b\,x^2+a)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 7*a^3/b^9/(b*x^2+a)^4+14/3*a^5/b^9/(b*x^2+a)^6-7*a^4/b^9/(b*x^2+a)^5-1/2/b^
9/(b*x^2+a)+2*a/b^9/(b*x^2+a)^2-1/18*a^8/b^9/(b*x^2+a)^9-2*a^6/b^9/(b*x^2+a
)^7-14/3*a^2/b^9/(b*x^2+a)^3+1/2*a^7/b^9/(b*x^2+a)^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(17) = 34$.
time = 0.31, size = 190, normalized size = 10.00

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,(b^{18}x^{18}+9\,ab^{17}x^{16}+36\,a^2b^{16}x^{14}+84\,a^3b^{15}x^{12}+126\,a^4b^{14}x^{10}+126\,a^5b^{13}x^8+84\,a^6b^{12}x^6+36\,a^7b^{11}x^4+9\,a^8b^{10}x^2+a^9b^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^18*
x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x
^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2
 + a^9*b^9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(17) = 34$.
time = 0.98, size = 190, normalized size = 10.00

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,(b^{18}x^{18}+9\,ab^{17}x^{16}+36\,a^2b^{16}x^{14}+84\,a^3b^{15}x^{12}+126\,a^4b^{14}x^{10}+126\,a^5b^{13}x^8+84\,a^6b^{12}x^6+36\,a^7b^{11}x^4+9\,a^8b^{10}x^2+a^9b^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^18*
x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x
^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2
 + a^9*b^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(14) = 28$.
time = 0.68, size = 202, normalized size = 10.63

$$\frac{-a^8 - 9a^7bx^2 - 36a^6b^2x^4 - 84a^5b^3x^6 - 126a^4b^4x^8 - 126a^3b^5x^{10} - 84a^2b^6x^{12} - 36ab^7x^{14} - 9b^8x^{16}}{18a^9b^9 + 162a^8b^{10}x^2 + 648a^7b^{11}x^4 + 1512a^6b^{12}x^6 + 2268a^5b^{13}x^8 + 2268a^4b^{14}x^{10} + 1512a^3b^{15}x^{12} + 648a^2b^{16}x^{14} + 162ab^{17}x^{16} + 18b^{18}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**2+a)**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x**2 - 36*a**6*b**2*x**4 - 84*a**5*b**3*x**6 - 126*a**4*b
**4*x**8 - 126*a**3*b**5*x**10 - 84*a**2*b**6*x**12 - 36*a*b**7*x**14 - 9*b
**8*x**16)/(18*a**9*b**9 + 162*a**8*b**10*x**2 + 648*a**7*b**11*x**4 + 1512
*a**6*b**12*x**6 + 2268*a**5*b**13*x**8 + 2268*a**4*b**14*x**10 + 1512*a**3
*b**15*x**12 + 648*a**2*b**16*x**14 + 162*a*b**17*x**16 + 18*b**18*x**18)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(17) = 34$.
time = 0.96, size = 99, normalized size = 5.21

$$-\frac{9\,b^8x^{16} + 36\,ab^7x^{14} + 84\,a^2b^6x^{12} + 126\,a^3b^5x^{10} + 126\,a^4b^4x^8 + 84\,a^5b^3x^6 + 36\,a^6b^2x^4 + 9\,a^7bx^2 + a^8}{18\,(bx^2 + a)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/((b*x^
2 + a)^9*b^9)
```

Mupad [B]
time = 0.12, size = 192, normalized size = 10.11

$$-\frac{a^8 + 9\,a^7\,b\,x^2 + 36\,a^6\,b^2\,x^4 + 84\,a^5\,b^3\,x^6 + 126\,a^4\,b^4\,x^8 + 126\,a^3\,b^5\,x^{10} + 84\,a^2\,b^6\,x^{12} + 36\,a\,b^7\,x^{14} + 9\,b^8\,x^{16}}{18\,a^9\,b^9 + 162\,a^8\,b^{10}\,x^2 + 648\,a^7\,b^{11}\,x^4 + 1512\,a^6\,b^{12}\,x^6 + 2268\,a^5\,b^{13}\,x^8 + 2268\,a^4\,b^{14}\,x^{10} + 1512\,a^3\,b^{15}\,x^{12} + 648\,a^2\,b^{16}\,x^{14} + 162\,a\,b^{17}\,x^{16} + 18\,b^{18}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(a + b*x^2)^10,x)
```

```
[Out] -(a^8 + 9*b^8*x^16 + 9*a^7*b*x^2 + 36*a*b^7*x^14 + 36*a^6*b^2*x^4 + 84*a^5*
b^3*x^6 + 126*a^4*b^4*x^8 + 126*a^3*b^5*x^10 + 84*a^2*b^6*x^12)/(18*a^9*b^9
 + 18*b^18*x^18 + 162*a*b^17*x^16 + 162*a^8*b^10*x^2 + 648*a^7*b^11*x^4 + 1
512*a^6*b^12*x^6 + 2268*a^5*b^13*x^8 + 2268*a^4*b^14*x^10 + 1512*a^3*b^15*x
^12 + 648*a^2*b^16*x^14)
```

3.197 $\int \frac{x^{15}}{(a+bx^2)^{10}}\, dx$

Optimal. Leaf size=39

$$\frac{x^{16}}{18a\,(a+bx^2)^9}+\frac{x^{16}}{144a^2\,(a+bx^2)^8}$$

```
[Out] 1/18*x^16/a/(b*x^2+a)^9+1/144*x^16/a^2/(b*x^2+a)^8
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 47, 37}

$$\frac{x^{16}}{144a^2\,(a+bx^2)^8}+\frac{x^{16}}{18a\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^2)^10,x]
```

```
[Out] x^16/(18*a*(a + b*x^2)^9) + x^16/(144*a^2*(a + b*x^2)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^7}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{x^{16}}{18a(a+bx^2)^9} + \frac{\text{Subst}\left(\int \frac{x^7}{(a+bx)^9}\,dx, x, x^2\right)}{18a}$$
$$= \frac{x^{16}}{18a(a+bx^2)^9} + \frac{x^{16}}{144a^2(a+bx^2)^8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 90 vs. $2(39) = 78$.
time = 0.01, size = 90, normalized size = 2.31

$$-\frac{a^7 + 9a^6bx^2 + 36a^5b^2x^4 + 84a^4b^3x^6 + 126a^3b^4x^8 + 126a^2b^5x^{10} + 84ab^6x^{12} + 36b^7x^{14}}{144b^8(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^2)^10,x]
```

```
[Out] -1/144*(a^7 + 9*a^6*b*x^2 + 36*a^5*b^2*x^4 + 84*a^4*b^3*x^6 + 126*a^3*b^4*x
^8 + 126*a^2*b^5*x^10 + 84*a*b^6*x^12 + 36*b^7*x^14)/(b^8*(a + b*x^2)^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(35) = 70$.
time = 0.09, size = 133, normalized size = 3.41

method	result
gosper	$-\frac{36x^{14}b^7+84a\,x^{12}b^6+126a^2x^{10}b^5+126a^3x^8b^4+84a^4x^6b^3+36a^5x^4b^2+9a^6x^2b+a^7}{144(b\,x^2+a)^9b^8}$
norman	$\frac{-\frac{a^7}{144b^8}-\frac{a^6x^2}{16b^7}-\frac{a^5x^4}{4b^6}-\frac{7a^4x^6}{12b^5}-\frac{7a^3x^8}{8b^4}-\frac{7a^2x^{10}}{8b^3}-\frac{7a\,x^{12}}{12b^2}-\frac{x^{14}}{4b}}{(b\,x^2+a)^9}$
risch	$\frac{-\frac{a^7}{144b^8}-\frac{a^6x^2}{16b^7}-\frac{a^5x^4}{4b^6}-\frac{7a^4x^6}{12b^5}-\frac{7a^3x^8}{8b^4}-\frac{7a^2x^{10}}{8b^3}-\frac{7a\,x^{12}}{12b^2}-\frac{x^{14}}{4b}}{(b\,x^2+a)^9}$
default	$-\frac{1}{4b^8(b\,x^2+a)^2}-\frac{21a^2}{8b^8(b\,x^2+a)^4}-\frac{7a^6}{16b^8(b\,x^2+a)^8}+\frac{7a^3}{2b^8(b\,x^2+a)^5}-\frac{35a^4}{12b^8(b\,x^2+a)^6}+\frac{a^7}{18b^8(b\,x^2+a)^9}+\frac{7a}{6b^8(b\,x^2+a)^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^8/(b*x^2+a)^2-21/8*a^2/b^8/(b*x^2+a)^4-7/16*a^6/b^8/(b*x^2+a)^8+7/2*
a^3/b^8/(b*x^2+a)^5-35/12*a^4/b^8/(b*x^2+a)^6+1/18*a^7/b^8/(b*x^2+a)^9+7/6*
a/b^8/(b*x^2+a)^3+3/2*a^5/b^8/(b*x^2+a)^7
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(35) = 70.
time = 0.29, size = 179, normalized size = 4.59

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\,(b^{17}x^{18}+9\,ab^{16}x^{16}+36\,a^2b^{15}x^{14}+84\,a^3b^{14}x^{12}+126\,a^4b^{13}x^{10}+126\,a^5b^{12}x^8+84\,a^6b^{11}x^6+36\,a^7b^{10}x^4+9\,a^8b^9x^2+a^9b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^16*
x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5*b^
12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(35) = 70.
time = 1.13, size = 179, normalized size = 4.59

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\,(b^{17}x^{18}+9\,ab^{16}x^{16}+36\,a^2b^{15}x^{14}+84\,a^3b^{14}x^{12}+126\,a^4b^{13}x^{10}+126\,a^5b^{12}x^8+84\,a^6b^{11}x^6+36\,a^7b^{10}x^4+9\,a^8b^9x^2+a^9b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^16*
x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5*b^
12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(31) = 62.
time = 0.62, size = 190, normalized size = 4.87

$$\frac{-a^7-9a^6bx^2-36a^5b^2x^4-84a^4b^3x^6-126a^3b^4x^8-126a^2b^5x^{10}-84ab^6x^{12}-36b^7x^{14}}{144a^9b^8+1296a^8b^9x^2+5184a^7b^{10}x^4+12096a^6b^{11}x^6+18144a^5b^{12}x^8+18144a^4b^{13}x^{10}+12096a^3b^{14}x^{12}+5184a^2b^{15}x^{14}+1296ab^{16}x^{16}+144b^{17}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**2+a)**10,x)
```

```
[Out] (-a**7 - 9*a**6*b*x**2 - 36*a**5*b**2*x**4 - 84*a**4*b**3*x**6 - 126*a**3*b
**4*x**8 - 126*a**2*b**5*x**10 - 84*a*b**6*x**12 - 36*b**7*x**14)/(144*a**9
*b**8 + 1296*a**8*b**9*x**2 + 5184*a**7*b**10*x**4 + 12096*a**6*b**11*x**6
+ 18144*a**5*b**12*x**8 + 18144*a**4*b**13*x**10 + 12096*a**3*b**14*x**12 +
 5184*a**2*b**15*x**14 + 1296*a*b**16*x**16 + 144*b**17*x**18)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(35) = 70.
time = 1.02, size = 88, normalized size = 2.26

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\,(bx^2+a)^9b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/((b*x^2 + a)^9*b^8)
```

Mupad [B]
time = 4.89, size = 181, normalized size = 4.64

$$-\frac{a^7+9\,a^6\,b\,x^2+36\,a^5\,b^2\,x^4+84\,a^4\,b^3\,x^6+126\,a^3\,b^4\,x^8+126\,a^2\,b^5\,x^{10}+84\,a\,b^6\,x^{12}+36\,b^7\,x^{14}}{144\,a^9\,b^8+1296\,a^8\,b^9\,x^2+5184\,a^7\,b^{10}\,x^4+12096\,a^6\,b^{11}\,x^6+18144\,a^5\,b^{12}\,x^8+18144\,a^4\,b^{13}\,x^{10}+12096\,a^3\,b^{14}\,x^{12}+5184\,a^2\,b^{15}\,x^{14}+1296\,a\,b^{16}\,x^{16}+144\,b^{17}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a + b*x^2)^10,x)
```

```
[Out] -(a^7 + 36*b^7*x^14 + 9*a^6*b*x^2 + 84*a*b^6*x^12 + 36*a^5*b^2*x^4 + 84*a^4
*b^3*x^6 + 126*a^3*b^4*x^8 + 126*a^2*b^5*x^10)/(144*a^9*b^8 + 144*b^17*x^18
 + 1296*a*b^16*x^16 + 1296*a^8*b^9*x^2 + 5184*a^7*b^10*x^4 + 12096*a^6*b^11
*x^6 + 18144*a^5*b^12*x^8 + 18144*a^4*b^13*x^10 + 12096*a^3*b^14*x^12 + 518
4*a^2*b^15*x^14)
```

3.198 $\int \frac{x^{13}}{(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=58

$$\frac{x^{14}}{18a\left(a+bx^2\right)^9}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}$$

```
[Out] 1/18*x^14/a/(b*x^2+a)^9+1/72*x^14/a^2/(b*x^2+a)^8+1/504*x^14/a^3/(b*x^2+a)^
7
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 47, 37}

$$\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^10,x]
```

```
[Out] x^14/(18*a*(a + b*x^2)^9) + x^14/(72*a^2*(a + b*x^2)^8) + x^14/(504*a^3*(a
+ b*x^2)^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx)^{10}}\,dx, x, x^2\right)\\ &= \frac{x^{14}}{18a\,(a+bx^2)^9} + \frac{\text{Subst}\left(\int \frac{x^6}{(a+bx)^9}\,dx, x, x^2\right)}{9a}\\ &= \frac{x^{14}}{18a\,(a+bx^2)^9} + \frac{x^{14}}{72a^2\,(a+bx^2)^8} + \frac{\text{Subst}\left(\int \frac{x^6}{(a+bx)^8}\,dx, x, x^2\right)}{72a^2}\\ &= \frac{x^{14}}{18a\,(a+bx^2)^9} + \frac{x^{14}}{72a^2\,(a+bx^2)^8} + \frac{x^{14}}{504a^3\,(a+bx^2)^7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 79, normalized size = 1.36

$$-\frac{a^6+9a^5bx^2+36a^4b^2x^4+84a^3b^3x^6+126a^2b^4x^8+126ab^5x^{10}+84b^6x^{12}}{504b^7\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^10,x]
```

```
[Out] -1/504*(a^6 + 9*a^5*b*x^2 + 36*a^4*b^2*x^4 + 84*a^3*b^3*x^6 + 126*a^2*b^4*x
^8 + 126*a*b^5*x^10 + 84*b^6*x^12)/(b^7*(a + b*x^2)^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(52) = 104.
time = 0.09, size = 116, normalized size = 2.00

method	result	si
gosper	$-\frac{84x^{12}b^6+126a\,x^{10}b^5+126a^2x^8b^4+84a^3x^6b^3+36a^4x^4b^2+9a^5x^2b+a^6}{504(b\,x^2+a)^9b^7}$	7
norman	$\frac{-\frac{a^6}{504b^7}-\frac{a^5x^2}{56b^6}-\frac{a^4x^4}{14b^5}-\frac{a^3x^6}{6b^4}-\frac{a^2x^8}{4b^3}-\frac{a\,x^{10}}{4b^2}-\frac{x^{12}}{6b}}{(b\,x^2+a)^9}$	8
risch	$\frac{-\frac{a^6}{504b^7}-\frac{a^5x^2}{56b^6}-\frac{a^4x^4}{14b^5}-\frac{a^3x^6}{6b^4}-\frac{a^2x^8}{4b^3}-\frac{a\,x^{10}}{4b^2}-\frac{x^{12}}{6b}}{(b\,x^2+a)^9}$	8
default	$-\frac{3a^2}{2b^7(b\,x^2+a)^5}+\frac{5a^3}{3b^7(b\,x^2+a)^6}-\frac{1}{6b^7(b\,x^2+a)^3}-\frac{a^6}{18b^7(b\,x^2+a)^9}+\frac{3a^5}{8b^7(b\,x^2+a)^8}+\frac{3a}{4b^7(b\,x^2+a)^4}-\frac{15a^4}{14b^7(b\,x^2+a)^7}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*a^2/b^7/(b*x^2+a)^5+5/3*a^3/b^7/(b*x^2+a)^6-1/6/b^7/(b*x^2+a)^3-1/18*a
^6/b^7/(b*x^2+a)^9+3/8*a^5/b^7/(b*x^2+a)^8+3/4*a/b^7/(b*x^2+a)^4-15/14*a^4/
b^7/(b*x^2+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(52) = 104$.
time = 0.33, size = 168, normalized size = 2.90

$$-\frac{84\,b^{6}x^{12}+126\,ab^{5}x^{10}+126\,a^{2}b^{4}x^{8}+84\,a^{3}b^{3}x^{6}+36\,a^{4}b^{2}x^{4}+9\,a^{5}bx^{2}+a^{6}}{504\,(b^{16}x^{18}+9\,ab^{15}x^{16}+36\,a^{2}b^{14}x^{14}+84\,a^{3}b^{13}x^{12}+126\,a^{4}b^{12}x^{10}+126\,a^{5}b^{11}x^{8}+84\,a^{6}b^{10}x^{6}+36\,a^{7}b^{9}x^{4}+9\,a^{8}b^{8}x^{2}+a^{9}b^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14
*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^
10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(52) = 104$.
time = 0.87, size = 168, normalized size = 2.90

$$-\frac{84\,b^{6}x^{12}+126\,ab^{5}x^{10}+126\,a^{2}b^{4}x^{8}+84\,a^{3}b^{3}x^{6}+36\,a^{4}b^{2}x^{4}+9\,a^{5}bx^{2}+a^{6}}{504\,(b^{16}x^{18}+9\,ab^{15}x^{16}+36\,a^{2}b^{14}x^{14}+84\,a^{3}b^{13}x^{12}+126\,a^{4}b^{12}x^{10}+126\,a^{5}b^{11}x^{8}+84\,a^{6}b^{10}x^{6}+36\,a^{7}b^{9}x^{4}+9\,a^{8}b^{8}x^{2}+a^{9}b^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14
*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^
10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(48) = 96$.
time = 0.57, size = 178, normalized size = 3.07

$$\frac{-a^{6}-9a^{5}bx^{2}-36a^{4}b^{2}x^{4}-84a^{3}b^{3}x^{6}-126a^{2}b^{4}x^{8}-126ab^{5}x^{10}-84b^{6}x^{12}}{504a^{9}b^{7}+4536a^{8}b^{8}x^{2}+18144a^{7}b^{9}x^{4}+42336a^{6}b^{10}x^{6}+63504a^{5}b^{11}x^{8}+63504a^{4}b^{12}x^{10}+42336a^{3}b^{13}x^{12}+18144a^{2}b^{14}x^{14}+4536ab^{15}x^{16}+504b^{16}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**10,x)
```

```
[Out] (-a**6 - 9*a**5*b*x**2 - 36*a**4*b**2*x**4 - 84*a**3*b**3*x**6 - 126*a**2*b
**4*x**8 - 126*a*b**5*x**10 - 84*b**6*x**12)/(504*a**9*b**7 + 4536*a**8*b**
8*x**2 + 18144*a**7*b**9*x**4 + 42336*a**6*b**10*x**6 + 63504*a**5*b**11*x**
```

```
*8 + 63504*a**4*b**12*x**10 + 42336*a**3*b**13*x**12 + 18144*a**2*b**14*x**
14 + 4536*a*b**15*x**16 + 504*b**16*x**18)
```

Giac [**A**]
time = 1.73, size = 77, normalized size = 1.33

$$-\frac{84\,b^6x^{12}+126\,ab^5x^{10}+126\,a^2b^4x^8+84\,a^3b^3x^6+36\,a^4b^2x^4+9\,a^5bx^2+a^6}{504\,(bx^2+a)^9b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/((b*x^2 + a)^9*b^7)
```

Mupad [**B**]
time = 0.10, size = 170, normalized size = 2.93

$$-\frac{a^6+9\,a^5\,b\,x^2+36\,a^4\,b^2\,x^4+84\,a^3\,b^3\,x^6+126\,a^2\,b^4\,x^8+126\,a\,b^5\,x^{10}+84\,b^6\,x^{12}}{504\,a^9\,b^7+4536\,a^8\,b^8\,x^2+18144\,a^7\,b^9\,x^4+42336\,a^6\,b^{10}\,x^6+63504\,a^5\,b^{11}\,x^8+63504\,a^4\,b^{12}\,x^{10}+42336\,a^3\,b^{13}\,x^{12}+18144\,a^2\,b^{14}\,x^{14}+4536\,a\,b^{15}\,x^{16}+504\,b^{16}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^2)^10,x)
```

```
[Out] -(a^6 + 84*b^6*x^12 + 9*a^5*b*x^2 + 126*a*b^5*x^10 + 36*a^4*b^2*x^4 + 84*a^
3*b^3*x^6 + 126*a^2*b^4*x^8)/(504*a^9*b^7 + 504*b^16*x^18 + 4536*a*b^15*x^1
6 + 4536*a^8*b^8*x^2 + 18144*a^7*b^9*x^4 + 42336*a^6*b^10*x^6 + 63504*a^5*b
^11*x^8 + 63504*a^4*b^12*x^10 + 42336*a^3*b^13*x^12 + 18144*a^2*b^14*x^14)
```

3.199 $\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=77

$$\frac{x^{12}}{18a\left(a+bx^2\right)^9}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}$$

```
[Out] 1/18*x^12/a/(b*x^2+a)^9+1/48*x^12/a^2/(b*x^2+a)^8+1/168*x^12/a^3/(b*x^2+a)^
7+1/1008*x^12/a^4/(b*x^2+a)^6
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {272, 47, 37}

$$\frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^10,x]
```

```
[Out] x^12/(18*a*(a + b*x^2)^9) + x^12/(48*a^2*(a + b*x^2)^8) + x^12/(168*a^3*(a
+ b*x^2)^7) + x^12/(1008*a^4*(a + b*x^2)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{(a+bx)^{10}}\,dx, x, x^2\right)\\
&= \frac{x^{12}}{18a(a+bx^2)^9} + \frac{\text{Subst}\left(\int \frac{x^5}{(a+bx)^9}\,dx, x, x^2\right)}{6a}\\
&= \frac{x^{12}}{18a(a+bx^2)^9} + \frac{x^{12}}{48a^2(a+bx^2)^8} + \frac{\text{Subst}\left(\int \frac{x^5}{(a+bx)^8}\,dx, x, x^2\right)}{24a^2}\\
&= \frac{x^{12}}{18a(a+bx^2)^9} + \frac{x^{12}}{48a^2(a+bx^2)^8} + \frac{x^{12}}{168a^3(a+bx^2)^7} + \frac{\text{Subst}\left(\int \frac{x^5}{(a+bx)^7}\,dx, x, x^2\right)}{168a^3}\\
&= \frac{x^{12}}{18a(a+bx^2)^9} + \frac{x^{12}}{48a^2(a+bx^2)^8} + \frac{x^{12}}{168a^3(a+bx^2)^7} + \frac{x^{12}}{1008a^4(a+bx^2)^6}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 68, normalized size = 0.88

$$-\frac{a^5+9a^4bx^2+36a^3b^2x^4+84a^2b^3x^6+126ab^4x^8+126b^5x^{10}}{1008b^6(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^10,x]
[Out] -1/1008*(a^5 + 9*a^4*b*x^2 + 36*a^3*b^2*x^4 + 84*a^2*b^3*x^6 + 126*a*b^4*x^
8 + 126*b^5*x^10)/(b^6*(a + b*x^2)^9)
```

Maple [A]
time = 0.09, size = 99, normalized size = 1.29

method	result	size
gosper	$-\frac{126b^5x^{10}+126ab^4x^8+84a^2b^3x^6+36a^3b^2x^4+9a^4bx^2+a^5}{1008(bx^2+a)^9b^6}$	67
norman	$\frac{-\frac{a^5}{1008b^6}-\frac{a^4x^2}{112b^5}-\frac{a^3x^4}{28b^4}-\frac{a^2x^6}{12b^3}-\frac{a\,x^8}{8b^2}-\frac{x^{10}}{8b}}{(bx^2+a)^9}$	70
risch	$\frac{-\frac{a^5}{1008b^6}-\frac{a^4x^2}{112b^5}-\frac{a^3x^4}{28b^4}-\frac{a^2x^6}{12b^3}-\frac{a\,x^8}{8b^2}-\frac{x^{10}}{8b}}{(bx^2+a)^9}$	70
default	$-\frac{1}{8b^6(bx^2+a)^4}-\frac{5a^4}{16b^6(bx^2+a)^8}+\frac{a^5}{18b^6(bx^2+a)^9}-\frac{5a^2}{6b^6(bx^2+a)^6}+\frac{5a^3}{7b^6(bx^2+a)^7}+\frac{a}{2b^6(bx^2+a)^5}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b^6/(b*x^2+a)^4-5/16*a^4/b^6/(b*x^2+a)^8+1/18*a^5/b^6/(b*x^2+a)^9-5/6*
a^2/b^6/(b*x^2+a)^6+5/7*a^3/b^6/(b*x^2+a)^7+1/2*a/b^6/(b*x^2+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(69) = 138$.
time = 0.31, size = 157, normalized size = 2.04

$$-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\,(b^{15}x^{18}+9\,ab^{14}x^{16}+36\,a^2b^{13}x^{14}+84\,a^3b^{12}x^{12}+126\,a^4b^{11}x^{10}+126\,a^5b^{10}x^8+84\,a^6b^9x^6+36\,a^7b^8x^4+9\,a^8b^7x^2+a^9b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*b^
12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7*b^
8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(69) = 138$.
time = 0.95, size = 157, normalized size = 2.04

$$-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\,(b^{15}x^{18}+9\,ab^{14}x^{16}+36\,a^2b^{13}x^{14}+84\,a^3b^{12}x^{12}+126\,a^4b^{11}x^{10}+126\,a^5b^{10}x^8+84\,a^6b^9x^6+36\,a^7b^8x^4+9\,a^8b^7x^2+a^9b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*b^
12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7*b^
8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(65) = 130$.
time = 0.52, size = 167, normalized size = 2.17

$$\frac{-a^5-9a^4bx^2-36a^3b^2x^4-84a^2b^3x^6-126ab^4x^8-126b^5x^{10}}{1008a^9b^6+9072a^8b^7x^2+36288a^7b^8x^4+84672a^6b^9x^6+127008a^5b^{10}x^8+127008a^4b^{11}x^{10}+84672a^3b^{12}x^{12}+36288a^2b^{13}x^{14}+9072ab^{14}x^{16}+1008b^{15}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**10,x)
```

```
[Out] (-a**5 - 9*a**4*b*x**2 - 36*a**3*b**2*x**4 - 84*a**2*b**3*x**6 - 126*a*b**4
*x**8 - 126*b**5*x**10)/(1008*a**9*b**6 + 9072*a**8*b**7*x**2 + 36288*a**7*
b**8*x**4 + 84672*a**6*b**9*x**6 + 127008*a**5*b**10*x**8 + 127008*a**4*b**
11*x**10 + 84672*a**3*b**12*x**12 + 36288*a**2*b**13*x**14 + 9072*a*b**14*x
**16 + 1008*b**15*x**18)
```

Giac [A]
time = 1.79, size = 66, normalized size = 0.86

$$-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\,(bx^2+a)^9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/((b*x^2 + a)^9*b^6)
```

Mupad [B]
time = 0.10, size = 159, normalized size = 2.06

$$-\frac{a^5+9\,a^4\,b\,x^2+36\,a^3\,b^2\,x^4+84\,a^2\,b^3\,x^6+126\,a\,b^4\,x^8+126\,b^5\,x^{10}}{1008\,a^9\,b^6+9072\,a^8\,b^7\,x^2+36288\,a^7\,b^8\,x^4+84672\,a^6\,b^9\,x^6+127008\,a^5\,b^{10}\,x^8+127008\,a^4\,b^{11}\,x^{10}+84672\,a^3\,b^{12}\,x^{12}+36288\,a^2\,b^{13}\,x^{14}+9072\,a\,b^{14}\,x^{16}+1008\,b^{15}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2)^10,x)
```

```
[Out] -(a^5 + 126*b^5*x^10 + 9*a^4*b*x^2 + 126*a*b^4*x^8 + 36*a^3*b^2*x^4 + 84*a^
2*b^3*x^6)/(1008*a^9*b^6 + 1008*b^15*x^18 + 9072*a*b^14*x^16 + 9072*a^8*b^7
*x^2 + 36288*a^7*b^8*x^4 + 84672*a^6*b^9*x^6 + 127008*a^5*b^10*x^8 + 127008
*a^4*b^11*x^10 + 84672*a^3*b^12*x^12 + 36288*a^2*b^13*x^14)
```

3.200 $\int \frac{x^9}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=91

$$-\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}$$

```
[Out] -1/18*a^4/b^5/(b*x^2+a)^9+1/4*a^3/b^5/(b*x^2+a)^8-3/7*a^2/b^5/(b*x^2+a)^7+1
/3*a/b^5/(b*x^2+a)^6-1/10/b^5/(b*x^2+a)^5
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$-\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^10,x]
```

```
[Out] -1/18*a^4/(b^5*(a + b*x^2)^9) + a^3/(4*b^5*(a + b*x^2)^8) - (3*a^2)/(7*b^5*
(a + b*x^2)^7) + a/(3*b^5*(a + b*x^2)^6) - 1/(10*b^5*(a + b*x^2)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(a+bx^2\right)^{10}}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{10}}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{10}}-\frac{4a^3}{b^4(a+bx)^9}+\frac{6a^2}{b^4(a+bx)^8}-\frac{4a}{b^4(a+bx)^7}+\frac{1}{b^4(a+bx)^6}\right)\,d\\&=-\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 0.63

$$-\frac{a^4+9a^3bx^2+36a^2b^2x^4+84ab^3x^6+126b^4x^8}{1260b^5\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^10,x]
```

```
[Out] -1/1260*(a^4 + 9*a^3*b*x^2 + 36*a^2*b^2*x^4 + 84*a*b^3*x^6 + 126*b^4*x^8)/(
b^5*(a + b*x^2)^9)
```

Maple [A]
time = 0.09, size = 82, normalized size = 0.90

method	result	size
gosper	$-\frac{126x^8b^4+84a\,x^6b^3+36a^2x^4b^2+9a^3x^2b+a^4}{1260\left(b\,x^2+a\right)^9b^5}$	56
norman	$\frac{-\frac{a^4}{1260b^5}-\frac{a^3x^2}{140b^4}-\frac{a^2x^4}{35b^3}-\frac{a\,x^6}{15b^2}-\frac{x^8}{10b}}{\left(b\,x^2+a\right)^9}$	59
risch	$\frac{-\frac{a^4}{1260b^5}-\frac{a^3x^2}{140b^4}-\frac{a^2x^4}{35b^3}-\frac{a\,x^6}{15b^2}-\frac{x^8}{10b}}{\left(b\,x^2+a\right)^9}$	59
default	$-\frac{a^4}{18b^5\left(b\,x^2+a\right)^9}+\frac{a^3}{4b^5\left(b\,x^2+a\right)^8}-\frac{3a^2}{7b^5\left(b\,x^2+a\right)^7}+\frac{a}{3b^5\left(b\,x^2+a\right)^6}-\frac{1}{10b^5\left(b\,x^2+a\right)^5}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*a^4/b^5/(b*x^2+a)^9+1/4*a^3/b^5/(b*x^2+a)^8-3/7*a^2/b^5/(b*x^2+a)^7+1
/3*a/b^5/(b*x^2+a)^6-1/10/b^5/(b*x^2+a)^5
```

Maxima [A]
time = 0.30, size = 146, normalized size = 1.60

$$-\frac{126\,b^4x^8+84\,ab^3x^6+36\,a^2b^2x^4+9\,a^3bx^2+a^4}{1260\,\left(b^{14}x^{18}+9\,ab^{13}x^{16}+36\,a^2b^{12}x^{14}+84\,a^3b^{11}x^{12}+126\,a^4b^{10}x^{10}+126\,a^5b^9x^8+84\,a^6b^8x^6+36\,a^7b^7x^4+9\,a^8b^6x^2+a^9b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4*b
^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^
2 + a^9*b^5)
```

Fricas [A]
time = 1.23, size = 146, normalized size = 1.60

$$-\frac{126\,b^4x^8+84\,ab^3x^6+36\,a^2b^2x^4+9\,a^3bx^2+a^4}{1260\,\left(b^{14}x^{18}+9\,ab^{13}x^{16}+36\,a^2b^{12}x^{14}+84\,a^3b^{11}x^{12}+126\,a^4b^{10}x^{10}+126\,a^5b^9x^8+84\,a^6b^8x^6+36\,a^7b^7x^4+9\,a^8b^6x^2+a^9b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4*b
^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^
2 + a^9*b^5)
```

Sympy [A]
time = 0.49, size = 155, normalized size = 1.70

$$\frac{-a^4-9a^3bx^2-36a^2b^2x^4-84ab^3x^6-126b^4x^8}{1260a^9b^5+11340a^8b^6x^2+45360a^7b^7x^4+105840a^6b^8x^6+158760a^5b^9x^8+158760a^4b^{10}x^{10}+105840a^3b^{11}x^{12}+45360a^2b^{12}x^{14}+11340ab^{13}x^{16}+1260b^{14}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**10,x)
```

```
[Out] (-a**4 - 9*a**3*b*x**2 - 36*a**2*b**2*x**4 - 84*a*b**3*x**6 - 126*b**4*x**8
)/(1260*a**9*b**5 + 11340*a**8*b**6*x**2 + 45360*a**7*b**7*x**4 + 105840*a*
*6*b**8*x**6 + 158760*a**5*b**9*x**8 + 158760*a**4*b**10*x**10 + 105840*a**
3*b**11*x**12 + 45360*a**2*b**12*x**14 + 11340*a*b**13*x**16 + 1260*b**14*x
**18)
```

Giac [A]
time = 1.33, size = 55, normalized size = 0.60

$$-\frac{126\,b^4x^8+84\,ab^3x^6+36\,a^2b^2x^4+9\,a^3bx^2+a^4}{1260\,(bx^2+a)^9b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
(b*x^2 + a)^9*b^5)
```

Mupad [B]
time = 4.82, size = 148, normalized size = 1.63

$$-\frac{a^4+9\,a^3\,b\,x^2+36\,a^2\,b^2\,x^4+84\,a\,b^3\,x^6+126\,b^4\,x^8}{1260\,a^9\,b^5+11340\,a^8\,b^6\,x^2+45360\,a^7\,b^7\,x^4+105840\,a^6\,b^8\,x^6+158760\,a^5\,b^9\,x^8+158760\,a^4\,b^{10}\,x^{10}+105840\,a^3\,b^{11}\,x^{12}+45360\,a^2\,b^{12}\,x^{14}+11340\,a\,b^{13}\,x^{16}+1260\,b^{14}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^10,x)
```

```
[Out] -(a^4 + 126*b^4*x^8 + 9*a^3*b*x^2 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4)/(1260*a^
9*b^5 + 1260*b^14*x^18 + 11340*a*b^13*x^16 + 11340*a^8*b^6*x^2 + 45360*a^7*
b^7*x^4 + 105840*a^6*b^8*x^6 + 158760*a^5*b^9*x^8 + 158760*a^4*b^10*x^10 +
105840*a^3*b^11*x^12 + 45360*a^2*b^12*x^14)
```

3.201 $\int \frac{x^7}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=72

$$\frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}$$

```
[Out] 1/18*a^3/b^4/(b*x^2+a)^9-3/16*a^2/b^4/(b*x^2+a)^8+3/14*a/b^4/(b*x^2+a)^7-1/
12/b^4/(b*x^2+a)^6
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^10,x]
```

```
[Out] a^3/(18*b^4*(a + b*x^2)^9) - (3*a^2)/(16*b^4*(a + b*x^2)^8) + (3*a)/(14*b^4
*(a + b*x^2)^7) - 1/(12*b^4*(a + b*x^2)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{10}}+\frac{3a^2}{b^3(a+bx)^9}-\frac{3a}{b^3(a+bx)^8}+\frac{1}{b^3(a+bx)^7}\right)dx,x,x^2\right)\\ &= \frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.64

$$-\frac{a^3+9a^2bx^2+36ab^2x^4+84b^3x^6}{1008b^4\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^10,x]
[Out] -1/1008*(a^3 + 9*a^2*b*x^2 + 36*a*b^2*x^4 + 84*b^3*x^6)/(b^4*(a + b*x^2)^9)
```

Maple [A]
time = 0.10, size = 65, normalized size = 0.90

method	result	size
gosper	$-\frac{84b^3x^6+36a\,b^2x^4+9a^2b\,x^2+a^3}{1008(b\,x^2+a)^9b^4}$	45
norman	$\frac{-\frac{a^3}{1008b^4}-\frac{a^2x^2}{112b^3}-\frac{a\,x^4}{28b^2}-\frac{x^6}{12b}}{(b\,x^2+a)^9}$	48
risch	$\frac{-\frac{a^3}{1008b^4}-\frac{a^2x^2}{112b^3}-\frac{a\,x^4}{28b^2}-\frac{x^6}{12b}}{(b\,x^2+a)^9}$	48
default	$\frac{a^3}{18b^4(b\,x^2+a)^9}-\frac{3a^2}{16b^4(b\,x^2+a)^8}+\frac{3a}{14b^4(b\,x^2+a)^7}-\frac{1}{12b^4(b\,x^2+a)^6}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
[Out] 1/18*a^3/b^4/(b*x^2+a)^9-3/16*a^2/b^4/(b*x^2+a)^8+3/14*a/b^4/(b*x^2+a)^7-1/
12/b^4/(b*x^2+a)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(64) = 128.
time = 0.29, size = 135, normalized size = 1.88

$$-\frac{84\,b^3x^6+36\,ab^2x^4+9\,a^2bx^2+a^3}{1008\,(b^{13}x^{18}+9\,ab^{12}x^{16}+36\,a^2b^{11}x^{14}+84\,a^3b^{10}x^{12}+126\,a^4b^9x^{10}+126\,a^5b^8x^8+84\,a^6b^7x^6+36\,a^7b^6x^4+9\,a^8b^5x^2+a^9b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="maxima")
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b^
12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^5*
b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(64) = 128.
time = 0.84, size = 135, normalized size = 1.88

$$-\frac{84\,b^3x^6+36\,ab^2x^4+9\,a^2bx^2+a^3}{1008\,(b^{13}x^{18}+9\,ab^{12}x^{16}+36\,a^2b^{11}x^{14}+84\,a^3b^{10}x^{12}+126\,a^4b^9x^{10}+126\,a^5b^8x^8+84\,a^6b^7x^6+36\,a^7b^6x^4+9\,a^8b^5x^2+a^9b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b^
12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^5*
b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(66) = 132$.
time = 0.46, size = 143, normalized size = 1.99

$$\frac{-a^3 - 9a^2bx^2 - 36ab^2x^4 - 84b^3x^6}{1008a^9b^4 + 9072a^8b^5x^2 + 36288a^7b^6x^4 + 84672a^6b^7x^6 + 127008a^5b^8x^8 + 127008a^4b^9x^{10} + 84672a^3b^{10}x^{12} + 36288a^2b^{11}x^{14} + 9072ab^{12}x^{16} + 1008b^{13}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**10,x)
```

```
[Out] (-a**3 - 9*a**2*b*x**2 - 36*a*b**2*x**4 - 84*b**3*x**6)/(1008*a**9*b**4 + 9
072*a**8*b**5*x**2 + 36288*a**7*b**6*x**4 + 84672*a**6*b**7*x**6 + 127008*a
**5*b**8*x**8 + 127008*a**4*b**9*x**10 + 84672*a**3*b**10*x**12 + 36288*a**
2*b**11*x**14 + 9072*a*b**12*x**16 + 1008*b**13*x**18)
```

Giac [A]
time = 1.26, size = 44, normalized size = 0.61

$$-\frac{84\,b^3x^6 + 36\,ab^2x^4 + 9\,a^2bx^2 + a^3}{1008\,(bx^2 + a)^9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/((b*x^2 + a)^9*b^4)
```

Mupad [B]
time = 0.10, size = 136, normalized size = 1.89

$$-\frac{\frac{a^3}{1008\,b^4} + \frac{x^6}{12\,b} + \frac{a\,x^4}{28\,b^2} + \frac{a^2\,x^2}{112\,b^3}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a\,b^8\,x^{16} + b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^10,x)
```

```
[Out] -(a^3/(1008*b^4) + x^6/(12*b) + (a*x^4)/(28*b^2) + (a^2*x^2)/(112*b^3))/(a^
9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6
 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

3.202 $\int \frac{x^5}{(a+bx^2)^{10}} dx$

Optimal. Leaf size=53

$$-\frac{a^2}{18b^3(a+bx^2)^9}+\frac{a}{8b^3(a+bx^2)^8}-\frac{1}{14b^3(a+bx^2)^7}$$

```
[Out] -1/18*a^2/b^3/(b*x^2+a)^9+1/8*a/b^3/(b*x^2+a)^8-1/14/b^3/(b*x^2+a)^7
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{a^2}{18b^3(a+bx^2)^9}+\frac{a}{8b^3(a+bx^2)^8}-\frac{1}{14b^3(a+bx^2)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^10,x]
```

```
[Out] -1/18*a^2/(b^3*(a + b*x^2)^9) + a/(8*b^3*(a + b*x^2)^8) - 1/(14*b^3*(a + b*
x^2)^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^2)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{18b^3(a+bx^2)^9}+\frac{a}{8b^3(a+bx^2)^8}-\frac{1}{14b^3(a+bx^2)^7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.66

$$-\frac{a^2+9abx^2+36b^2x^4}{504b^3\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^10,x]
[Out] -1/504*(a^2 + 9*a*b*x^2 + 36*b^2*x^4)/(b^3*(a + b*x^2)^9)
```

Maple [A]
time = 0.09, size = 48, normalized size = 0.91

method	result	size
gosper	$-\frac{36b^2x^4+9ab\,x^2+a^2}{504(b\,x^2+a)^9b^3}$	34
norman	$\frac{-\frac{a^2}{504b^3}-\frac{a\,x^2}{56b^2}-\frac{x^4}{14b}}{(b\,x^2+a)^9}$	37
risch	$\frac{-\frac{a^2}{504b^3}-\frac{a\,x^2}{56b^2}-\frac{x^4}{14b}}{(b\,x^2+a)^9}$	37
default	$-\frac{a^2}{18b^3(b\,x^2+a)^9}+\frac{a}{8b^3(b\,x^2+a)^8}-\frac{1}{14b^3(b\,x^2+a)^7}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
[Out] -1/18*a^2/b^3/(b*x^2+a)^9+1/8*a/b^3/(b*x^2+a)^8-1/14/b^3/(b*x^2+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(47) = 94.
time = 0.30, size = 124, normalized size = 2.34

$$-\frac{36\,b^2x^4+9\,abx^2+a^2}{504\,(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^2b^{10}x^{14}+84\,a^3b^9x^{12}+126\,a^4b^8x^{10}+126\,a^5b^7x^8+84\,a^6b^6x^6+36\,a^7b^5x^4+9\,a^8b^4x^2+a^9b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="maxima")
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*b
^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*b^
6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(47) = 94.
time = 1.20, size = 124, normalized size = 2.34

$$-\frac{36\,b^2x^4+9\,abx^2+a^2}{504\,(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^2b^{10}x^{14}+84\,a^3b^9x^{12}+126\,a^4b^8x^{10}+126\,a^5b^7x^8+84\,a^6b^6x^6+36\,a^7b^5x^4+9\,a^8b^4x^2+a^9b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*b
^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*b^
6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(46) = 92$.
time = 0.43, size = 131, normalized size = 2.47

$$\frac{-a^2 - 9abx^2 - 36b^2x^4}{504a^9b^3 + 4536a^8b^4x^2 + 18144a^7b^5x^4 + 42336a^6b^6x^6 + 63504a^5b^7x^8 + 63504a^4b^8x^{10} + 42336a^3b^9x^{12} + 18144a^2b^{10}x^{14} + 4536ab^{11}x^{16} + 504b^{12}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**10,x)
```

```
[Out] (-a**2 - 9*a*b*x**2 - 36*b**2*x**4)/(504*a**9*b**3 + 4536*a**8*b**4*x**2 +
18144*a**7*b**5*x**4 + 42336*a**6*b**6*x**6 + 63504*a**5*b**7*x**8 + 63504*
a**4*b**8*x**10 + 42336*a**3*b**9*x**12 + 18144*a**2*b**10*x**14 + 4536*a*b
**11*x**16 + 504*b**12*x**18)
```

Giac [A]
time = 1.14, size = 33, normalized size = 0.62

$$-\frac{36\,b^2x^4 + 9\,abx^2 + a^2}{504\,(bx^2 + a)^9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/((b*x^2 + a)^9*b^3)
```

Mupad [B]
time = 4.83, size = 125, normalized size = 2.36

$$-\frac{\frac{a^2}{504\,b^3} + \frac{x^4}{14\,b} + \frac{a\,x^2}{56\,b^2}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a\,b^8\,x^{16} + b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^10,x)
```

```
[Out] -(a^2/(504*b^3) + x^4/(14*b) + (a*x^2)/(56*b^2))/(a^9 + b^9*x^18 + 9*a^8*b*
x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 12
6*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

3.203 $\int \frac{x^3}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=34

$$\frac{a}{18b^2\left(a+bx^2\right)^9}-\frac{1}{16b^2\left(a+bx^2\right)^8}$$

```
[Out] 1/18*a/b^2/(b*x^2+a)^9-1/16/b^2/(b*x^2+a)^8
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{a}{18b^2\left(a+bx^2\right)^9}-\frac{1}{16b^2\left(a+bx^2\right)^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^10,x]
```

```
[Out] a/(18*b^2*(a + b*x^2)^9) - 1/(16*b^2*(a + b*x^2)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{10}}+\frac{1}{b(a+bx)^9}\right)\,dx,x,x^2\right)\\ &= \frac{a}{18b^2\left(a+bx^2\right)^9}-\frac{1}{16b^2\left(a+bx^2\right)^8}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.71

$$-\frac{a+9bx^2}{144b^2\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^10,x]
```

```
[Out] -1/144*(a + 9*b*x^2)/(b^2*(a + b*x^2)^9)
```

Maple [A]
time = 0.09, size = 31, normalized size = 0.91

method	result	size
gosper	$-\frac{9b\,x^2+a}{144\left(b\,x^2+a\right)^9 b^2}$	23
norman	$\frac{-\frac{a}{144b^2}-\frac{x^2}{16b}}{\left(b\,x^2+a\right)^9}$	26
risch	$\frac{-\frac{a}{144b^2}-\frac{x^2}{16b}}{\left(b\,x^2+a\right)^9}$	26
default	$\frac{a}{18b^2\left(b\,x^2+a\right)^9}-\frac{1}{16b^2\left(b\,x^2+a\right)^8}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*a/b^2/(b*x^2+a)^9-1/16/b^2/(b*x^2+a)^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(30) = 60.
time = 0.30, size = 113, normalized size = 3.32

$$-\frac{9\,bx^2+a}{144\,(b^{11}x^{18}+9\,ab^{10}x^{16}+36\,a^2b^9x^{14}+84\,a^3b^8x^{12}+126\,a^4b^7x^{10}+126\,a^5b^6x^8+84\,a^6b^5x^6+36\,a^7b^4x^4+9\,a^8b^3x^2+a^9b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3*
b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b^4
*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(30) = 60.
time = 1.53, size = 113, normalized size = 3.32

$$-\frac{9\,bx^2+a}{144\,(b^{11}x^{18}+9\,ab^{10}x^{16}+36\,a^2b^9x^{14}+84\,a^3b^8x^{12}+126\,a^4b^7x^{10}+126\,a^5b^6x^8+84\,a^6b^5x^6+36\,a^7b^4x^4+9\,a^8b^3x^2+a^9b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3*
b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b^4
*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(29) = 58$.
time = 0.41, size = 119, normalized size = 3.50

$$\frac{-a - 9bx^2}{144a^9b^2 + 1296a^8b^3x^2 + 5184a^7b^4x^4 + 12096a^6b^5x^6 + 18144a^5b^6x^8 + 18144a^4b^7x^{10} + 12096a^3b^8x^{12} + 5184a^2b^9x^{14} + 1296ab^{10}x^{16} + 144b^{11}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**10,x)
```

```
[Out] (-a - 9*b*x**2)/(144*a**9*b**2 + 1296*a**8*b**3*x**2 + 5184*a**7*b**4*x**4
+ 12096*a**6*b**5*x**6 + 18144*a**5*b**6*x**8 + 18144*a**4*b**7*x**10 + 120
96*a**3*b**8*x**12 + 5184*a**2*b**9*x**14 + 1296*a*b**10*x**16 + 144*b**11*
x**18)
```

Giac [A]
time = 1.25, size = 22, normalized size = 0.65

$$-\frac{9\,bx^2 + a}{144\,(bx^2 + a)^9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/144*(9*b*x^2 + a)/((b*x^2 + a)^9*b^2)
```

Mupad [B]
time = 0.11, size = 114, normalized size = 3.35

$$-\frac{\frac{a}{144\,b^2} + \frac{x^2}{16\,b}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a\,b^8\,x^{16} + b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^10,x)
```

```
[Out] -(a/(144*b^2) + x^2/(16*b))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 +
36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a
^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

3.204 $\int \frac{x}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{18b\,(a+bx^2)^9}$$

```
[Out] -1/18/b/(b*x^2+a)^9
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$-\frac{1}{18b\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^10,x]
```

```
[Out] -1/18*1/(b*(a + b*x^2)^9)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{10}}\,dx = -\frac{1}{18b\,(a+bx^2)^9}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{18b\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^10,x]
```

```
[Out] -1/18*1/(b*(a + b*x^2)^9)
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{18b(b\,x^2+a)^9}$	15
derivativedivides	$-\frac{1}{18b(b\,x^2+a)^9}$	15
default	$-\frac{1}{18b(b\,x^2+a)^9}$	15
norman	$-\frac{1}{18b(b\,x^2+a)^9}$	15
risch	$-\frac{1}{18b(b\,x^2+a)^9}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18/b/(b*x^2+a)^9
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.88

$$-\frac{1}{18\,(bx^2+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/18/((b*x^2 + a)^9*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(14) = 28.
time = 1.12, size = 103, normalized size = 6.44

$$-\frac{1}{18\,(b^{10}x^{18}+9\,ab^9x^{16}+36\,a^2b^8x^{14}+84\,a^3b^7x^{12}+126\,a^4b^6x^{10}+126\,a^5b^5x^8+84\,a^6b^4x^6+36\,a^7b^3x^4+9\,a^8b^2x^2+a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/18/(b^10*x^18 + 9*a*b^9*x^16 + 36*a^2*b^8*x^14 + 84*a^3*b^7*x^12 + 126*a
^4*b^6*x^10 + 126*a^5*b^5*x^8 + 84*a^6*b^4*x^6 + 36*a^7*b^3*x^4 + 9*a^8*b^2
*x^2 + a^9*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(14) = 28.
time = 0.40, size = 110, normalized size = 6.88

$$-\frac{1}{18a^9b+162a^8b^2x^2+648a^7b^3x^4+1512a^6b^4x^6+2268a^5b^5x^8+2268a^4b^6x^{10}+1512a^3b^7x^{12}+648a^2b^8x^{14}+162ab^9x^{16}+18b^{10}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**10,x)
```

```
[Out] -1/(18*a**9*b + 162*a**8*b**2*x**2 + 648*a**7*b**3*x**4 + 1512*a**6*b**4*x*
*6 + 2268*a**5*b**5*x**8 + 2268*a**4*b**6*x**10 + 1512*a**3*b**7*x**12 + 64
8*a**2*b**8*x**14 + 162*a*b**9*x**16 + 18*b**10*x**18)
```

Giac [A]
time = 1.39, size = 14, normalized size = 0.88

$$-\frac{1}{18\,(bx^2+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/18/((b*x^2 + a)^9*b)
```

Mupad [B]
time = 0.13, size = 14, normalized size = 0.88

$$-\frac{1}{18\,b\,(b\,x^2+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^10,x)
```

```
[Out] -1/(18*b*(a + b*x^2)^9)
```

3.205 $\int \frac{1}{x(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=166

$$\frac{1}{18a\,(a+bx^2)^9}+\frac{1}{16a^2\,(a+bx^2)^8}+\frac{1}{14a^3\,(a+bx^2)^7}+\frac{1}{12a^4\,(a+bx^2)^6}+\frac{1}{10a^5\,(a+bx^2)^5}+\frac{1}{8a^6\,(a+bx^2)^4}+\frac{}{6a^7\,(}$$

```
[Out] 1/18/a/(b*x^2+a)^9+1/16/a^2/(b*x^2+a)^8+1/14/a^3/(b*x^2+a)^7+1/12/a^4/(b*x^
2+a)^6+1/10/a^5/(b*x^2+a)^5+1/8/a^6/(b*x^2+a)^4+1/6/a^7/(b*x^2+a)^3+1/4/a^8
/(b*x^2+a)^2+1/2/a^9/(b*x^2+a)+ln(x)/a^10-1/2*ln(b*x^2+a)/a^10
```

Rubi [A]

time = 0.09, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$-\frac{\log(a+bx^2)}{2a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{2a^9\,(a+bx^2)}+\frac{1}{4a^8\,(a+bx^2)^2}+\frac{1}{6a^7\,(a+bx^2)^3}+\frac{1}{8a^6\,(a+bx^2)^4}+\frac{1}{10a^5\,(a+bx^2)^5}+\frac{1}{12a^4\,(a+bx^2)^6}+\frac{1}{14a^3\,(a+bx^2)^7}+\frac{1}{16a^2\,(a+bx^2)^8}+\frac{1}{18a\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^10),x]
```

```
[Out] 1/(18*a*(a + b*x^2)^9) + 1/(16*a^2*(a + b*x^2)^8) + 1/(14*a^3*(a + b*x^2)^7
) + 1/(12*a^4*(a + b*x^2)^6) + 1/(10*a^5*(a + b*x^2)^5) + 1/(8*a^6*(a + b*x
^2)^4) + 1/(6*a^7*(a + b*x^2)^3) + 1/(4*a^8*(a + b*x^2)^2) + 1/(2*a^9*(a +
b*x^2)) + Log[x]/a^10 - Log[a + b*x^2]/(2*a^10)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{10}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^{10}x} - \frac{b}{a(a+bx)^{10}} - \frac{b}{a^2(a+bx)^9} - \frac{b}{a^3(a+bx)^8} - \frac{b}{a^4(a+bx)^7} - \frac{}{a^5(a+\cdots}\right.\right. \\
&= \frac{1}{18a\left(a+bx^2\right)^9} + \frac{1}{16a^2\left(a+bx^2\right)^8} + \frac{1}{14a^3\left(a+bx^2\right)^7} + \frac{1}{12a^4\left(a+bx^2\right)^6} + \frac{1}{10a^5\left(a+bx^2\cdots\right.}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 120, normalized size = 0.72

$$
\frac{\frac{a\left(7129a^8+41481a^7bx^2+120564a^6b^2x^4+210756a^5b^3x^6+236754a^4b^4x^8+173250a^3b^5x^{10}+80220a^2b^6x^{12}+21420ab^7x^{14}+2520b^8x^{16}\right)}{\left(a+bx^2\right)^9}+5040\log(x)-2520\log\left(a+bx^2\right)}{5040a^{10}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^10),x]
```

```
[Out] ((a*(7129*a^8 + 41481*a^7*b*x^2 + 120564*a^6*b^2*x^4 + 210756*a^5*b^3*x^6 +
 236754*a^4*b^4*x^8 + 173250*a^3*b^5*x^10 + 80220*a^2*b^6*x^12 + 21420*a*b^
7*x^14 + 2520*b^8*x^16))/(a + b*x^2)^9 + 5040*Log[x] - 2520*Log[a + b*x^2])
/(5040*a^10)
```

Maple [A]
time = 0.12, size = 178, normalized size = 1.07

method	result
risch	$\frac{\frac{7129}{5040a}+\frac{4609b\,x^2}{560a^2}+\frac{3349b^2x^4}{140a^3}+\frac{2509b^3x^6}{60a^4}+\frac{1879b^4x^8}{40a^5}+\frac{275b^5x^{10}}{8a^6}+\frac{191b^6x^{12}}{12a^7}+\frac{17b^7x^{14}}{4a^8}+\frac{b^8x^{16}}{2a^9}}{\left(b\,x^2+a\right)^9}+\frac{\ln(x)}{a^{10}}-\frac{\ln\left(b\,x^2+a\right)}{2a^{10}}$
norman	$\frac{-\frac{9b\,x^2}{2a^2}-\frac{27b^2x^4}{a^3}-\frac{77b^3x^6}{a^4}-\frac{525b^4x^8}{4a^5}-\frac{2877b^5x^{10}}{20a^6}-\frac{1029b^6x^{12}}{10a^7}-\frac{3267b^7x^{14}}{70a^8}-\frac{6849b^8x^{16}}{560a^9}-\frac{7129b^9x^{18}}{5040a^{10}}}{\left(b\,x^2+a\right)^9}+\frac{\ln(x)}{a^{10}}-\frac{\ln\left(b\,x^2+a\right)}{2a^{10}}$
default	$-\frac{b\left(-\frac{a^8}{8b\left(b\,x^2+a\right)^8}-\frac{a^2}{2b\left(b\,x^2+a\right)^2}-\frac{a}{b\left(b\,x^2+a\right)}-\frac{a^3}{3b\left(b\,x^2+a\right)^3}+\frac{\ln\left(b\,x^2+a\right)}{b}-\frac{a^7}{7b\left(b\,x^2+a\right)^7}-\frac{a^6}{6b\left(b\,x^2+a\right)^6}-\frac{a^9}{9b\left(b\,x^2+a\right)^9}-\frac{a^5}{5b\left(b\,x^2+a\right)^5}-\frac{}{4b\left(\cdots\right.}\right.}{2a^{10}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a^10*(-1/8*a^8/b/(b*x^2+a)^8-1/2/b*a^2/(b*x^2+a)^2-a/b/(b*x^2+a)-1/3
/b*a^3/(b*x^2+a)^3+ln(b*x^2+a)/b-1/7*a^7/b/(b*x^2+a)^7-1/6/b*a^6/(b*x^2+a)^
6-1/9/b*a^9/(b*x^2+a)^9-1/5*a^5/b/(b*x^2+a)^5-1/4/b*a^4/(b*x^2+a)^4)+ln(x)/
a^10
```

Maxima [**A**]
time = 0.31, size = 214, normalized size = 1.29

$$\frac{2520\,b^8x^{16}+21420\,ab^7x^{14}+80220\,a^2b^6x^{12}+173250\,a^3b^5x^{10}+236754\,a^4b^4x^8+210756\,a^5b^3x^6+120564\,a^6b^2x^4+41481\,a^7bx^2+7129\,a^8}{5040\,(a^9b^9x^{18}+9\,a^{10}b^8x^{16}+36\,a^{11}b^7x^{14}+84\,a^{12}b^6x^{12}+126\,a^{13}b^5x^{10}+126\,a^{14}b^4x^8+84\,a^{15}b^3x^6+36\,a^{16}b^2x^4+9\,a^{17}bx^2+a^{18})}-\frac{\log\left(bx^2+a\right)}{2\,a^{10}}+\frac{\log\left(x^2\right)}{2\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/5040*(2520*b^8*x^16 + 21420*a*b^7*x^14 + 80220*a^2*b^6*x^12 + 173250*a^3*
b^5*x^10 + 236754*a^4*b^4*x^8 + 210756*a^5*b^3*x^6 + 120564*a^6*b^2*x^4 + 4
1481*a^7*b*x^2 + 7129*a^8)/(a^9*b^9*x^18 + 9*a^10*b^8*x^16 + 36*a^11*b^7*x^
14 + 84*a^12*b^6*x^12 + 126*a^13*b^5*x^10 + 126*a^14*b^4*x^8 + 84*a^15*b^3*
x^6 + 36*a^16*b^2*x^4 + 9*a^17*b*x^2 + a^18) - 1/2*log(b*x^2 + a)/a^10 + 1/
2*log(x^2)/a^10
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(146) = 292.
time = 1.28, size = 398, normalized size = 2.40

$$\frac{2520\,ab^8x^{16}+21420\,a^2b^7x^{14}+80220\,a^3b^6x^{12}+173250\,a^4b^5x^{10}+236754\,a^5b^4x^8+210756\,a^6b^3x^6+120564\,a^7b^2x^4+41481\,a^8bx^2+7129\,a^9-2520\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\log\left(bx^2+a\right)+5040\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\log\left(x\right)}{5040\,(a^{10}b^9x^{18}+9\,a^{11}b^8x^{16}+36\,a^{12}b^7x^{14}+84\,a^{13}b^6x^{12}+126\,a^{14}b^5x^{10}+126\,a^{15}b^4x^8+84\,a^{16}b^3x^6+36\,a^{17}b^2x^4+9\,a^{18}bx^2+a^{19})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/5040*(2520*a*b^8*x^16 + 21420*a^2*b^7*x^14 + 80220*a^3*b^6*x^12 + 173250*
a^4*b^5*x^10 + 236754*a^5*b^4*x^8 + 210756*a^6*b^3*x^6 + 120564*a^7*b^2*x^4
 + 41481*a^8*b*x^2 + 7129*a^9 - 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*
x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^
6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a) + 5040*(b^9*x^18 + 9
*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^
5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(x))/(a
^10*b^9*x^18 + 9*a^11*b^8*x^16 + 36*a^12*b^7*x^14 + 84*a^13*b^6*x^12 + 126*
a^14*b^5*x^10 + 126*a^15*b^4*x^8 + 84*a^16*b^3*x^6 + 36*a^17*b^2*x^4 + 9*a^
18*b*x^2 + a^19)
```

Sympy [**A**]
time = 0.58, size = 223, normalized size = 1.34

$$\frac{7129a^8+41481a^7bx^2+120564a^6b^2x^4+210756a^5b^3x^6+236754a^4b^4x^8+173250a^3b^5x^{10}+80220a^2b^6x^{12}+21420ab^7x^{14}+2520b^8x^{16}}{5040a^{18}+45360a^{17}bx^2+181440a^{16}b^2x^4+423360a^{15}b^3x^6+635040a^{14}b^4x^8+635040a^{13}b^5x^{10}+423360a^{12}b^6x^{12}+181440a^{11}b^7x^{14}+45360a^{10}b^8x^{16}+5040a^9b^9x^{18}}+\frac{\log\left(x\right)}{a^{10}}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**10,x)
```

```
[Out] (7129*a**8 + 41481*a**7*b*x**2 + 120564*a**6*b**2*x**4 + 210756*a**5*b**3*x
**6 + 236754*a**4*b**4*x**8 + 173250*a**3*b**5*x**10 + 80220*a**2*b**6*x**1
```

```
2 + 21420*a*b**7*x**14 + 2520*b**8*x**16)/(5040*a**18 + 45360*a**17*b*x**2
+ 181440*a**16*b**2*x**4 + 423360*a**15*b**3*x**6 + 635040*a**14*b**4*x**8
+ 635040*a**13*b**5*x**10 + 423360*a**12*b**6*x**12 + 181440*a**11*b**7*x**
14 + 45360*a**10*b**8*x**16 + 5040*a**9*b**9*x**18) + log(x)/a**10 - log(a/
b + x**2)/(2*a**10)
```

Giac [A]
time = 1.14, size = 136, normalized size = 0.82

$$\frac{\log\left(x^{2}\right)}{2\,a^{10}} - \frac{\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{10}} + \frac{7129\,b^{9}x^{18} + 66681\,ab^{8}x^{16} + 278064\,a^{2}b^{7}x^{14} + 679056\,a^{3}b^{6}x^{12} + 1071504\,a^{4}b^{5}x^{10} + 1135008\,a^{5}b^{4}x^{8} + 809592\,a^{6}b^{3}x^{6} + 377208\,a^{7}b^{2}x^{4} + 105642\,a^{8}bx^{2} + 14258\,a^{9}}{5040\,\left(bx^{2}+a\right)^{9}a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^10 - 1/2*log(abs(b*x^2 + a))/a^10 + 1/5040*(7129*b^9*x^18 +
66681*a*b^8*x^16 + 278064*a^2*b^7*x^14 + 679056*a^3*b^6*x^12 + 1071504*a^4*
b^5*x^10 + 1135008*a^5*b^4*x^8 + 809592*a^6*b^3*x^6 + 377208*a^7*b^2*x^4 +
105642*a^8*b*x^2 + 14258*a^9)/((b*x^2 + a)^9*a^10)
```

Mupad [B]
time = 5.46, size = 210, normalized size = 1.27

$$\frac{\ln\left(x\right)}{a^{10}} + \frac{\frac{7129}{5040\,a} + \frac{4609\,b\,x^{2}}{560\,a^{2}} + \frac{3349\,b^{2}\,x^{4}}{140\,a^{3}} + \frac{2509\,b^{3}\,x^{6}}{60\,a^{4}} + \frac{1879\,b^{4}\,x^{8}}{40\,a^{5}} + \frac{275\,b^{5}\,x^{10}}{8\,a^{6}} + \frac{191\,b^{6}\,x^{12}}{12\,a^{7}} + \frac{17\,b^{7}\,x^{14}}{4\,a^{8}} + \frac{b^{8}\,x^{16}}{2\,a^{9}}}{a^{9} + 9\,a^{8}\,b\,x^{2} + 36\,a^{7}\,b^{2}\,x^{4} + 84\,a^{6}\,b^{3}\,x^{6} + 126\,a^{5}\,b^{4}\,x^{8} + 126\,a^{4}\,b^{5}\,x^{10} + 84\,a^{3}\,b^{6}\,x^{12} + 36\,a^{2}\,b^{7}\,x^{14} + 9\,a\,b^{8}\,x^{16} + b^{9}\,x^{18}} - \frac{\ln\left(b\,x^{2}+a\right)}{2\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^10),x)
```

```
[Out] log(x)/a^10 + (7129/(5040*a) + (4609*b*x^2)/(560*a^2) + (3349*b^2*x^4)/(140
*a^3) + (2509*b^3*x^6)/(60*a^4) + (1879*b^4*x^8)/(40*a^5) + (275*b^5*x^10)/
(8*a^6) + (191*b^6*x^12)/(12*a^7) + (17*b^7*x^14)/(4*a^8) + (b^8*x^16)/(2*a
^9))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6
*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^
7*x^14) - log(a + b*x^2)/(2*a^10)
```

3.206 $\int \frac{1}{x^3(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=184

$$-\frac{1}{2a^{10}x^2}-\frac{b}{18a^2(a+bx^2)^9}-\frac{b}{8a^3(a+bx^2)^8}-\frac{3b}{14a^4(a+bx^2)^7}-\frac{b}{3a^5(a+bx^2)^6}-\frac{b}{2a^6(a+bx^2)^5}-\frac{3b}{4a^7(a+bx^2}$$

```
[Out] -1/2/a^10/x^2-1/18*b/a^2/(b*x^2+a)^9-1/8*b/a^3/(b*x^2+a)^8-3/14*b/a^4/(b*x^
2+a)^7-1/3*b/a^5/(b*x^2+a)^6-1/2*b/a^6/(b*x^2+a)^5-3/4*b/a^7/(b*x^2+a)^4-7/
6*b/a^8/(b*x^2+a)^3-2*b/a^9/(b*x^2+a)^2-9/2*b/a^10/(b*x^2+a)-10*b*ln(x)/a^1
1+5*b*ln(b*x^2+a)/a^11
```

Rubi [A]
time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\frac{5b\log(a+bx^2)}{a^{11}}-\frac{10b\log(x)}{a^{11}}-\frac{9b}{2a^{10}(a+bx^2)}-\frac{1}{2a^{10}x^2}-\frac{2b}{a^9(a+bx^2)^2}-\frac{7b}{6a^8(a+bx^2)^3}-\frac{3b}{4a^7(a+bx^2)^4}-\frac{b}{2a^6(a+bx^2)^5}-\frac{b}{3a^5(a+bx^2)^6}-\frac{3b}{14a^4(a+bx^2)^7}-\frac{b}{8a^3(a+bx^2)^8}-\frac{b}{18a^2(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^10),x]
```

```
[Out] -1/2*1/(a^10*x^2) - b/(18*a^2*(a + b*x^2)^9) - b/(8*a^3*(a + b*x^2)^8) - (3
*b)/(14*a^4*(a + b*x^2)^7) - b/(3*a^5*(a + b*x^2)^6) - b/(2*a^6*(a + b*x^2)
^5) - (3*b)/(4*a^7*(a + b*x^2)^4) - (7*b)/(6*a^8*(a + b*x^2)^3) - (2*b)/(a^
9*(a + b*x^2)^2) - (9*b)/(2*a^10*(a + b*x^2)) - (10*b*Log[x])/a^11 + (5*b*L
og[a + b*x^2])/a^11
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^{10}x^2} - \frac{10b}{a^{11}x} + \frac{b^2}{a^2(a+bx)^{10}} + \frac{2b^2}{a^3(a+bx)^9} + \frac{3b^2}{a^4(a+bx)^8} + \frac{4b^2}{a^5(a+bx}\right.\right.$$
$$= -\frac{1}{2a^{10}x^2} - \frac{b}{18a^2\left(a+bx^2\right)^9} - \frac{b}{8a^3\left(a+bx^2\right)^8} - \frac{3b}{14a^4\left(a+bx^2\right)^7} - \frac{b}{3a^5\left(a+bx^2\right)^6} - \frac{}{2a}$$

Mathematica [A]
time = 0.08, size = 136, normalized size = 0.74

$$-\frac{\frac{a\left(252a^9+7129a^8bx^2+41481a^7b^2x^4+120564a^6b^3x^6+210756a^5b^4x^8+236754a^4b^5x^{10}+173250a^3b^6x^{12}+80220a^2b^7x^{14}+21420ab^8x^{16}+2520b^9x^{18}\right)}{x^2\left(a+bx^2\right)^9}+5040b\log(x)-2520b\log\left(a+bx^2\right)}{504a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^10),x]
```

```
[Out] -1/504*((a*(252*a^9 + 7129*a^8*b*x^2 + 41481*a^7*b^2*x^4 + 120564*a^6*b^3*x
^6 + 210756*a^5*b^4*x^8 + 236754*a^4*b^5*x^10 + 173250*a^3*b^6*x^12 + 80220
*a^2*b^7*x^14 + 21420*a*b^8*x^16 + 2520*b^9*x^18))/(x^2*(a + b*x^2)^9) + 50
40*b*Log[x] - 2520*b*Log[a + b*x^2])/a^11
```

Maple [A]
time = 0.13, size = 191, normalized size = 1.04

method	result
norman	$\frac{-\frac{1}{2a}+\frac{45b^2x^4}{a^3}+\frac{270b^3x^6}{a^4}+\frac{770b^4x^8}{a^5}+\frac{2625b^5x^{10}}{2a^6}+\frac{2877b^6x^{12}}{2a^7}+\frac{1029b^7x^{14}}{a^8}+\frac{3267b^8x^{16}}{7a^9}+\frac{6849b^9x^{18}}{56a^{10}}+\frac{7129b^{10}x^{20}}{504a^{11}}}{x^2\left(bx^2+a\right)^9}-\frac{10b\ln(x)}{a^{11}}+\frac{5b\ln\left(bx^2\right.}{a^{11}}$
risch	$\frac{-\frac{1}{2a}-\frac{7129bx^2}{504a^2}-\frac{4609b^2x^4}{56a^3}-\frac{3349b^3x^6}{14a^4}-\frac{2509b^4x^8}{6a^5}-\frac{1879b^5x^{10}}{4a^6}-\frac{1375b^6x^{12}}{4a^7}-\frac{955b^7x^{14}}{6a^8}-\frac{85b^8x^{16}}{2a^9}-\frac{5b^9x^{18}}{a^{10}}}{x^2\left(bx^2+a\right)^9}-\frac{10b\ln(x)}{a^{11}}+\frac{5b\ln\left(-bx^2-a\right.}{a^{11}}$
default	$\frac{b^2\left(-\frac{a^5}{b\left(bx^2+a\right)^5}-\frac{3a^7}{7b\left(bx^2+a\right)^7}-\frac{4a^2}{b\left(bx^2+a\right)^2}-\frac{9a}{b\left(bx^2+a\right)}-\frac{7a^3}{3b\left(bx^2+a\right)^3}+\frac{10\ln\left(bx^2+a\right)}{b}-\frac{a^9}{9b\left(bx^2+a\right)^9}-\frac{3a^4}{2b\left(bx^2+a\right)^4}-\frac{a^8}{4b\left(bx^2+a\right)^8}-\frac{}{3b(}\right.}{2a^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^11*(-a^5/b/(b*x^2+a)^5-3/7*a^7/b/(b*x^2+a)^7-4/b*a^2/(b*x^2+a)^2-
9*a/b/(b*x^2+a)-7/3/b*a^3/(b*x^2+a)^3+10*ln(b*x^2+a)/b-1/9/b*a^9/(b*x^2+a)^
9-3/2/b*a^4/(b*x^2+a)^4-1/4*a^8/b/(b*x^2+a)^8-2/3/b*a^6/(b*x^2+a)^6)-1/2/a^
10/x^2-10*b*ln(x)/a^11
```

Maxima [A]
time = 0.32, size = 231, normalized size = 1.26

$$-\frac{2520\,b^{9}x^{18}+21420\,ab^{8}x^{16}+80220\,a^{2}b^{7}x^{14}+173250\,a^{3}b^{6}x^{12}+236754\,a^{4}b^{5}x^{10}+210756\,a^{5}b^{4}x^{8}+120564\,a^{6}b^{3}x^{6}+41481\,a^{7}b^{2}x^{4}+7129\,a^{8}bx^{2}+252\,a^{9}}{504\,\left(a^{10}b^{9}x^{20}+9\,a^{11}b^{8}x^{18}+36\,a^{12}b^{7}x^{16}+84\,a^{13}b^{6}x^{14}+126\,a^{14}b^{5}x^{12}+126\,a^{15}b^{4}x^{10}+84\,a^{16}b^{3}x^{8}+36\,a^{17}b^{2}x^{6}+9\,a^{18}bx^{4}+a^{19}x^{2}\right)}+\frac{5\,b\log\left(bx^{2}+a\right)}{a^{11}}-\frac{5\,b\log\left(x^{2}\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(2520*b^9*x^18 + 21420*a*b^8*x^16 + 80220*a^2*b^7*x^14 + 173250*a^3*
b^6*x^12 + 236754*a^4*b^5*x^10 + 210756*a^5*b^4*x^8 + 120564*a^6*b^3*x^6 +
41481*a^7*b^2*x^4 + 7129*a^8*b*x^2 + 252*a^9)/(a^10*b^9*x^20 + 9*a^11*b^8*x
^18 + 36*a^12*b^7*x^16 + 84*a^13*b^6*x^14 + 126*a^14*b^5*x^12 + 126*a^15*b^
4*x^10 + 84*a^16*b^3*x^8 + 36*a^17*b^2*x^6 + 9*a^18*b*x^4 + a^19*x^2) + 5*b
*log(b*x^2 + a)/a^11 - 5*b*log(x^2)/a^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(166) = 332.
time = 1.42, size = 427, normalized size = 2.32

$$-\frac{2520\,ab^{9}x^{18}+21420\,a^{2}b^{8}x^{16}+80220\,a^{3}b^{7}x^{14}+173250\,a^{4}b^{6}x^{12}+236754\,a^{5}b^{5}x^{10}+210756\,a^{6}b^{4}x^{8}+120564\,a^{7}b^{3}x^{6}+41481\,a^{8}b^{2}x^{4}+7129\,a^{9}bx^{2}+252\,a^{10}-2520\,\left(b^{10}x^{20}+9\,ab^{9}x^{18}+36\,a^{2}b^{8}x^{16}+84\,a^{3}b^{7}x^{14}+126\,a^{4}b^{6}x^{12}+126\,a^{5}b^{5}x^{10}+84\,a^{6}b^{4}x^{8}+36\,a^{7}b^{3}x^{6}+9\,a^{8}b^{2}x^{4}+a^{9}bx^{2}\right)\log\left(bx^{2}+a\right)+5040\,\left(b^{10}x^{20}+9\,ab^{9}x^{18}+36\,a^{2}b^{8}x^{16}+84\,a^{3}b^{7}x^{14}+126\,a^{4}b^{6}x^{12}+126\,a^{5}b^{5}x^{10}+84\,a^{6}b^{4}x^{8}+36\,a^{7}b^{3}x^{6}+9\,a^{8}b^{2}x^{4}+a^{9}bx^{2}\right)\log\left(x\right)}{504\,\left(a^{11}b^{9}x^{20}+9\,a^{12}b^{8}x^{18}+36\,a^{13}b^{7}x^{16}+84\,a^{14}b^{6}x^{14}+126\,a^{15}b^{5}x^{12}+126\,a^{16}b^{4}x^{10}+84\,a^{17}b^{3}x^{8}+36\,a^{18}b^{2}x^{6}+9\,a^{19}bx^{4}+a^{20}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(2520*a*b^9*x^18 + 21420*a^2*b^8*x^16 + 80220*a^3*b^7*x^14 + 173250*
a^4*b^6*x^12 + 236754*a^5*b^5*x^10 + 210756*a^6*b^4*x^8 + 120564*a^7*b^3*x^
6 + 41481*a^8*b^2*x^4 + 7129*a^9*b*x^2 + 252*a^10 - 2520*(b^10*x^20 + 9*a*b
^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3*b^7*x^14 + 126*a^4*b^6*x^12 + 126*a^5*b^
5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*b^3*x^6 + 9*a^8*b^2*x^4 + a^9*b*x^2)*log(b
*x^2 + a) + 5040*(b^10*x^20 + 9*a*b^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3*b^7*x
^14 + 126*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*b^3*x^6
 + 9*a^8*b^2*x^4 + a^9*b*x^2)*log(x))/(a^11*b^9*x^20 + 9*a^12*b^8*x^18 + 36
*a^13*b^7*x^16 + 84*a^14*b^6*x^14 + 126*a^15*b^5*x^12 + 126*a^16*b^4*x^10 +
 84*a^17*b^3*x^8 + 36*a^18*b^2*x^6 + 9*a^19*b*x^4 + a^20*x^2)
```

Sympy [A]
time = 0.67, size = 245, normalized size = 1.33

$$\frac{-252a^{9}-7129a^{8}bx^{2}-41481a^{7}b^{2}x^{4}-120564a^{6}b^{3}x^{6}-210756a^{5}b^{4}x^{8}-236754a^{4}b^{5}x^{10}-173250a^{3}b^{6}x^{12}-80220a^{2}b^{7}x^{14}-21420ab^{8}x^{16}-2520b^{9}x^{18}}{504a^{19}x^{2}+4536a^{18}bx^{4}+18144a^{17}b^{2}x^{6}+42336a^{16}b^{3}x^{8}+63504a^{15}b^{4}x^{10}+63504a^{14}b^{5}x^{12}+42336a^{13}b^{6}x^{14}+18144a^{12}b^{7}x^{16}+4536a^{11}b^{8}x^{18}+504a^{10}b^{9}x^{20}}-\frac{10b\log\left(x\right)}{a^{11}}+\frac{5b\log\left(\frac{a}{b}+x^{2}\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**10,x)
```

```
[Out] (-252*a**9 - 7129*a**8*b*x**2 - 41481*a**7*b**2*x**4 - 120564*a**6*b**3*x**
6 - 210756*a**5*b**4*x**8 - 236754*a**4*b**5*x**10 - 173250*a**3*b**6*x**12
```

```
 - 80220*a**2*b**7*x**14 - 21420*a*b**8*x**16 - 2520*b**9*x**18)/(504*a**19
*x**2 + 4536*a**18*b*x**4 + 18144*a**17*b**2*x**6 + 42336*a**16*b**3*x**8 +
 63504*a**15*b**4*x**10 + 63504*a**14*b**5*x**12 + 42336*a**13*b**6*x**14 +
 18144*a**12*b**7*x**16 + 4536*a**11*b**8*x**18 + 504*a**10*b**9*x**20) - 1
0*b*log(x)/a**11 + 5*b*log(a/b + x**2)/a**11
```

Giac [A]
time = 1.56, size = 159, normalized size = 0.86

$$-\frac{5\,b\log\left(x^2\right)}{a^{11}}+\frac{5\,b\log\left(|bx^2+a|\right)}{a^{11}}+\frac{10\,bx^2-a}{2\,a^{11}x^2}-\frac{7129\,b^{10}x^{18}+66429\,ab^9x^{16}+275796\,a^2b^8x^{14}+669984\,a^3b^7x^{12}+1050336\,a^4b^6x^{10}+1103256\,a^5b^5x^8+777840\,a^6b^4x^6+356040\,a^7b^3x^4+96570\,a^8b^2x^2+11990\,a^9b}{504\,(bx^2+a)^9a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -5*b*log(x^2)/a^11 + 5*b*log(abs(b*x^2 + a))/a^11 + 1/2*(10*b*x^2 - a)/(a^1
1*x^2) - 1/504*(7129*b^10*x^18 + 66429*a*b^9*x^16 + 275796*a^2*b^8*x^14 + 6
69984*a^3*b^7*x^12 + 1050336*a^4*b^6*x^10 + 1103256*a^5*b^5*x^8 + 777840*a^
6*b^4*x^6 + 356040*a^7*b^3*x^4 + 96570*a^8*b^2*x^2 + 11990*a^9*b)/((b*x^2 +
 a)^9*a^11)
```

Mupad [B]
time = 0.52, size = 229, normalized size = 1.24

$$\frac{5\,b\,\ln\left(b\,x^2+a\right)}{a^{11}}-\frac{\frac{1}{2\,a}+\frac{7129\,b\,x^2}{504\,a^2}+\frac{4609\,b^2\,x^4}{56\,a^3}+\frac{3349\,b^3\,x^6}{14\,a^4}+\frac{2509\,b^4\,x^8}{6\,a^5}+\frac{1879\,b^5\,x^{10}}{4\,a^6}+\frac{1375\,b^6\,x^{12}}{4\,a^7}+\frac{955\,b^7\,x^{14}}{6\,a^8}+\frac{85\,b^8\,x^{16}}{2\,a^9}+\frac{5\,b^9\,x^{18}}{a^{10}}}{a^9\,x^2+9\,a^8\,b\,x^4+36\,a^7\,b^2\,x^6+84\,a^6\,b^3\,x^8+126\,a^5\,b^4\,x^{10}+126\,a^4\,b^5\,x^{12}+84\,a^3\,b^6\,x^{14}+36\,a^2\,b^7\,x^{16}+9\,a\,b^8\,x^{18}+b^9\,x^{20}}-\frac{10\,b\,\ln\left(x\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^10),x)
```

```
[Out] (5*b*log(a + b*x^2))/a^11 - (1/(2*a) + (7129*b*x^2)/(504*a^2) + (4609*b^2*x
^4)/(56*a^3) + (3349*b^3*x^6)/(14*a^4) + (2509*b^4*x^8)/(6*a^5) + (1879*b^5
*x^10)/(4*a^6) + (1375*b^6*x^12)/(4*a^7) + (955*b^7*x^14)/(6*a^8) + (85*b^8
*x^16)/(2*a^9) + (5*b^9*x^18)/a^10)/(a^9*x^2 + b^9*x^20 + 9*a^8*b*x^4 + 9*a
*b^8*x^18 + 36*a^7*b^2*x^6 + 84*a^6*b^3*x^8 + 126*a^5*b^4*x^10 + 126*a^4*b^
5*x^12 + 84*a^3*b^6*x^14 + 36*a^2*b^7*x^16) - (10*b*log(x))/a^11
```

3.207 $\int \frac{1}{x^5\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=217

$$-\frac{1}{4a^{10}x^4}+\frac{5b}{a^{11}x^2}+\frac{b^2}{18a^3\left(a+bx^2\right)^9}+\frac{3b^2}{16a^4\left(a+bx^2\right)^8}+\frac{3b^2}{7a^5\left(a+bx^2\right)^7}+\frac{5b^2}{6a^6\left(a+bx^2\right)^6}+\frac{3b^2}{2a^7\left(a+bx^2\right)^5}+\frac{}{8a^8(}$$

```
[Out] -1/4/a^10/x^4+5*b/a^11/x^2+1/18*b^2/a^3/(b*x^2+a)^9+3/16*b^2/a^4/(b*x^2+a)^
8+3/7*b^2/a^5/(b*x^2+a)^7+5/6*b^2/a^6/(b*x^2+a)^6+3/2*b^2/a^7/(b*x^2+a)^5+2
1/8*b^2/a^8/(b*x^2+a)^4+14/3*b^2/a^9/(b*x^2+a)^3+9*b^2/a^10/(b*x^2+a)^2+45/
2*b^2/a^11/(b*x^2+a)+55*b^2*ln(x)/a^12-55/2*b^2*ln(b*x^2+a)/a^12
```

Rubi [A]
time = 0.15, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$-\frac{55b^2\log\left(a+bx^2\right)}{2a^{12}}+\frac{55b^2\log(x)}{a^{12}}+\frac{45b^2}{2a^{11}\left(a+bx^2\right)}+\frac{5b}{a^{11}x^2}+\frac{9b^2}{a^{10}\left(a+bx^2\right)^2}-\frac{1}{4a^{10}x^4}+\frac{14b^2}{3a^9\left(a+bx^2\right)^3}+\frac{21b^2}{8a^8\left(a+bx^2\right)^4}+\frac{3b^2}{2a^7\left(a+bx^2\right)^5}+\frac{5b^2}{6a^6\left(a+bx^2\right)^6}+\frac{3b^2}{7a^5\left(a+bx^2\right)^7}+\frac{3b^2}{16a^4\left(a+bx^2\right)^8}+\frac{b^2}{18a^3\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^10),x]
```

```
[Out] -1/4*1/(a^10*x^4) + (5*b)/(a^11*x^2) + b^2/(18*a^3*(a + b*x^2)^9) + (3*b^2)
/(16*a^4*(a + b*x^2)^8) + (3*b^2)/(7*a^5*(a + b*x^2)^7) + (5*b^2)/(6*a^6*(a
 + b*x^2)^6) + (3*b^2)/(2*a^7*(a + b*x^2)^5) + (21*b^2)/(8*a^8*(a + b*x^2)^
4) + (14*b^2)/(3*a^9*(a + b*x^2)^3) + (9*b^2)/(a^10*(a + b*x^2)^2) + (45*b^
2)/(2*a^11*(a + b*x^2)) + (55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x^2])/(2
*a^12)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^2)^{10}}\, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{10}}\, dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^{10}x^3} - \frac{10b}{a^{11}x^2} + \frac{55b^2}{a^{12}x} - \frac{b^3}{a^3(a+bx)^{10}} - \frac{3b^3}{a^4(a+bx)^9} - \frac{6b^3}{a^5(a+bx)^8} - \cdots\right.\right.$$

$$= -\frac{1}{4a^{10}x^4} + \frac{5b}{a^{11}x^2} + \frac{b^2}{18a^3 (a+bx^2)^9} + \frac{3b^2}{16a^4 (a+bx^2)^8} + \frac{3b^2}{7a^5 (a+bx^2)^7} + \frac{5b^2}{6a^6 (a+b\cdots}$$

Mathematica [A]
time = 0.07, size = 151, normalized size = 0.70

$$\frac{\frac{a(-252a^{10}+2772a^9bx^2+78419a^8b^2x^4+456291a^7b^3x^6+1326204a^6b^4x^8+2318316a^5b^5x^{10}+2604294a^4b^6x^{12}+1905750a^3b^7x^{14}+882420a^2b^8x^{16}+235620ab^9x^{18}+27720b^{10}x^{20})}{x^4(a+bx^2)^9} + 55440b^2\log(x) - 27720b^2\log(a+bx^2)}{1008a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^10),x]
```

```
[Out] ((a*(-252*a^10 + 2772*a^9*b*x^2 + 78419*a^8*b^2*x^4 + 456291*a^7*b^3*x^6 +
1326204*a^6*b^4*x^8 + 2318316*a^5*b^5*x^10 + 2604294*a^4*b^6*x^12 + 1905750
*a^3*b^7*x^14 + 882420*a^2*b^8*x^16 + 235620*a*b^9*x^18 + 27720*b^10*x^20))
/(x^4*(a + b*x^2)^9) + 55440*b^2*Log[x] - 27720*b^2*Log[a + b*x^2])/(1008*a
^12)
```

Maple [A]
time = 0.13, size = 202, normalized size = 0.93

method	result
norman	$\frac{-\frac{1}{4a}+\frac{11b\,x^2}{4a^2}-\frac{495b^3x^6}{2a^4}-\frac{1485b^4x^8}{a^5}-\frac{4235b^5x^{10}}{a^6}-\frac{28875b^6x^{12}}{4a^7}-\frac{31647b^7x^{14}}{4a^8}-\frac{11319b^8x^{16}}{2a^9}-\frac{35937b^9x^{18}}{14a^{10}}-\frac{75339b^{10}x^{20}}{112a^{11}}-\frac{78419b^{11}x^{22}}{1008a^{12}}}{x^4(b\,x^2+a)^9} + \frac{55b^2\cdots}{\cdots}$
risch	$\frac{-\frac{1}{4a}+\frac{11b\,x^2}{4a^2}+\frac{78419b^2x^4}{1008a^3}+\frac{50699b^3x^6}{112a^4}+\frac{36839b^4x^8}{28a^5}+\frac{27599b^5x^{10}}{12a^6}+\frac{20669b^6x^{12}}{8a^7}+\frac{15125b^7x^{14}}{8a^8}+\frac{10505b^8x^{16}}{12a^9}+\frac{935b^9x^{18}}{4a^{10}}+\frac{55b^{10}x^{20}}{2a^{11}}}{x^4(b\,x^2+a)^9} + \frac{55b^2\ln\cdots}{a^{12}}$
default	$-\frac{b^3\left(-\frac{6a^7}{7b(b\,x^2+a)^7}-\frac{45a}{b(b\,x^2+a)}-\frac{5a^6}{3b(b\,x^2+a)^6}+\frac{55\ln(b\,x^2+a)}{b}-\frac{28a^3}{3b(b\,x^2+a)^3}-\frac{a^9}{9b(b\,x^2+a)^9}-\frac{21a^4}{4b(b\,x^2+a)^4}-\frac{3a^5}{b(b\,x^2+a)^5}-\frac{18a^2}{b(b\,x^2+a)^2}-\cdots\right.}{2a^{12}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3/a^12*(-6/7*a^7/b/(b*x^2+a)^7-45*a/b/(b*x^2+a)-5/3/b*a^6/(b*x^2+a)^
6+55*ln(b*x^2+a)/b-28/3/b*a^3/(b*x^2+a)^3-1/9/b*a^9/(b*x^2+a)^9-21/4/b*a^4/
(b*x^2+a)^4-3*a^5/b/(b*x^2+a)^5-18/b*a^2/(b*x^2+a)^2-3/8*a^8/b/(b*x^2+a)^8)
-1/4/a^10/x^4+5*b/a^11/x^2+55*b^2*ln(x)/a^12
```

Maxima [**A**]
time = 0.33, size = 246, normalized size = 1.13

$$\frac{27720\,b^{10}x^{20}+235620\,ab^{9}x^{18}+882420\,a^{2}b^{8}x^{16}+1905750\,a^{3}b^{7}x^{14}+2604294\,a^{4}b^{6}x^{12}+2318316\,a^{5}b^{5}x^{10}+1326204\,a^{6}b^{4}x^{8}+456291\,a^{7}b^{3}x^{6}+78419\,a^{8}b^{2}x^{4}+2772\,a^{9}bx^{2}-252\,a^{10}}{1008\,(a^{11}b^{9}x^{22}+9\,a^{12}b^{8}x^{20}+36\,a^{13}b^{7}x^{18}+84\,a^{14}b^{6}x^{16}+126\,a^{15}b^{5}x^{14}+126\,a^{16}b^{4}x^{12}+84\,a^{17}b^{3}x^{10}+36\,a^{18}b^{2}x^{8}+9\,a^{19}bx^{6}+a^{20}x^{4})}-\frac{55\,b^{2}\log\left(bx^{2}+a\right)}{2\,a^{12}}+\frac{55\,b^{2}\log\left(x^{2}\right)}{2\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/1008*(27720*b^10*x^20 + 235620*a*b^9*x^18 + 882420*a^2*b^8*x^16 + 1905750
*a^3*b^7*x^14 + 2604294*a^4*b^6*x^12 + 2318316*a^5*b^5*x^10 + 1326204*a^6*b
^4*x^8 + 456291*a^7*b^3*x^6 + 78419*a^8*b^2*x^4 + 2772*a^9*b*x^2 - 252*a^10
)/(a^11*b^9*x^22 + 9*a^12*b^8*x^20 + 36*a^13*b^7*x^18 + 84*a^14*b^6*x^16 +
126*a^15*b^5*x^14 + 126*a^16*b^4*x^12 + 84*a^17*b^3*x^10 + 36*a^18*b^2*x^8
+ 9*a^19*b*x^6 + a^20*x^4) - 55/2*b^2*log(b*x^2 + a)/a^12 + 55/2*b^2*log(x^
2)/a^12
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(197) = 394$.
time = 1.25, size = 442, normalized size = 2.04

$$\frac{27720\,ab^{10}x^{20}+235620\,a^{2}b^{9}x^{18}+882420\,a^{3}b^{8}x^{16}+1905750\,a^{4}b^{7}x^{14}+2604294\,a^{5}b^{6}x^{12}+2318316\,a^{6}b^{5}x^{10}+1326204\,a^{7}b^{4}x^{8}+456291\,a^{8}b^{3}x^{6}+78419\,a^{9}b^{2}x^{4}+2772\,a^{10}bx^{2}-252\,a^{11}-27720\,(b^{11}x^{22}+9\,ab^{10}x^{20}+36\,a^{2}b^{9}x^{18}+84\,a^{3}b^{8}x^{16}+126\,a^{4}b^{7}x^{14}+126\,a^{5}b^{6}x^{12}+84\,a^{6}b^{5}x^{10}+36\,a^{7}b^{4}x^{8}+9\,a^{8}b^{3}x^{6}+a^{9}b^{2}x^{4})\log\left(bx^{2}+a\right)+55440\,(b^{11}x^{22}+9\,ab^{10}x^{20}+36\,a^{2}b^{9}x^{18}+84\,a^{3}b^{8}x^{16}+126\,a^{4}b^{7}x^{14}+126\,a^{5}b^{6}x^{12}+84\,a^{6}b^{5}x^{10}+36\,a^{7}b^{4}x^{8}+9\,a^{8}b^{3}x^{6}+a^{9}b^{2}x^{4})\log\left(x\right)}{1008\,(a^{12}b^{9}x^{22}+9\,a^{13}b^{8}x^{20}+36\,a^{14}b^{7}x^{18}+84\,a^{15}b^{6}x^{16}+126\,a^{16}b^{5}x^{14}+126\,a^{17}b^{4}x^{12}+84\,a^{18}b^{3}x^{10}+36\,a^{19}b^{2}x^{8}+9\,a^{20}bx^{6}+a^{21}x^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/1008*(27720*a*b^10*x^20 + 235620*a^2*b^9*x^18 + 882420*a^3*b^8*x^16 + 190
5750*a^4*b^7*x^14 + 2604294*a^5*b^6*x^12 + 2318316*a^6*b^5*x^10 + 1326204*a
^7*b^4*x^8 + 456291*a^8*b^3*x^6 + 78419*a^9*b^2*x^4 + 2772*a^10*b*x^2 - 252
*a^11 - 27720*(b^11*x^22 + 9*a*b^10*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8*x^1
6 + 126*a^4*b^7*x^14 + 126*a^5*b^6*x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4*x^8
+ 9*a^8*b^3*x^6 + a^9*b^2*x^4)*log(b*x^2 + a) + 55440*(b^11*x^22 + 9*a*b^10
*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8*x^16 + 126*a^4*b^7*x^14 + 126*a^5*b^6*
x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4*x^8 + 9*a^8*b^3*x^6 + a^9*b^2*x^4)*log(
x))/(a^12*b^9*x^22 + 9*a^13*b^8*x^20 + 36*a^14*b^7*x^18 + 84*a^15*b^6*x^16
+ 126*a^16*b^5*x^14 + 126*a^17*b^4*x^12 + 84*a^18*b^3*x^10 + 36*a^19*b^2*x^
8 + 9*a^20*b*x^6 + a^21*x^4)
```

Sympy [**A**]
time = 0.70, size = 260, normalized size = 1.20

$$\frac{-252a^{10}+2772a^{9}bx^{2}+78419a^{8}b^{2}x^{4}+456291a^{7}b^{3}x^{6}+1326204a^{6}b^{4}x^{8}+2318316a^{5}b^{5}x^{10}+2604294a^{4}b^{6}x^{12}+1905750a^{3}b^{7}x^{14}+882420a^{2}b^{8}x^{16}+235620ab^{9}x^{18}+27720b^{10}x^{20}}{1008a^{20}x^{4}+9072a^{19}bx^{6}+36288a^{18}b^{2}x^{8}+84672a^{17}b^{3}x^{10}+127008a^{16}b^{4}x^{12}+127008a^{15}b^{5}x^{14}+84672a^{14}b^{6}x^{16}+36288a^{13}b^{7}x^{18}+9072a^{12}b^{8}x^{20}+1008a^{11}b^{9}x^{22}}+\frac{55b^{2}\log\left(x\right)}{a^{12}}-\frac{55b^{2}\log\left(\frac{a}{b}+x^{2}\right)}{2a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**10,x)
```

```
[Out] (-252*a**10 + 2772*a**9*b*x**2 + 78419*a**8*b**2*x**4 + 456291*a**7*b**3*x**6 + 1326204*a**6*b**4*x**8 + 2318316*a**5*b**5*x**10 + 2604294*a**4*b**6*x**12 + 1905750*a**3*b**7*x**14 + 882420*a**2*b**8*x**16 + 235620*a*b**9*x**18 + 27720*b**10*x**20)/(1008*a**20*x**4 + 9072*a**19*b*x**6 + 36288*a**18*b**2*x**8 + 84672*a**17*b**3*x**10 + 127008*a**16*b**4*x**12 + 127008*a**15*b**5*x**14 + 84672*a**14*b**6*x**16 + 36288*a**13*b**7*x**18 + 9072*a**12*b**8*x**20 + 1008*a**11*b**9*x**22) + 55*b**2*log(x)/a**12 - 55*b**2*log(a/b + x**2)/(2*a**12)
```

Giac [A]
time = 1.37, size = 174, normalized size = 0.80

$$\frac{55\,b^2\log\left(x^2\right)}{2\,a^{12}}-\frac{55\,b^2\log\left(|bx^2+a|\right)}{2\,a^{12}}-\frac{165\,b^2x^4-20\,abx^2+a^2}{4\,a^{12}x^4}+\frac{78419\,b^{11}x^{18}+728451\,ab^{10}x^{16}+3013596\,a^2b^9x^{14}+7290444\,a^3b^8x^{12}+11372256\,a^4b^7x^{10}+11871216\,a^5b^6x^8+8302224\,a^6b^5x^6+3757680\,a^7b^4x^4+1001790\,a^8b^3x^2+120550\,a^9b^2}{1008\,(bx^2+a)^9a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/2*b^2*log(x^2)/a^12 - 55/2*b^2*log(abs(b*x^2 + a))/a^12 - 1/4*(165*b^2*x^4 - 20*a*b*x^2 + a^2)/(a^12*x^4) + 1/1008*(78419*b^11*x^18 + 728451*a*b^10*x^16 + 3013596*a^2*b^9*x^14 + 7290444*a^3*b^8*x^12 + 11372256*a^4*b^7*x^10 + 11871216*a^5*b^6*x^8 + 8302224*a^6*b^5*x^6 + 3757680*a^7*b^4*x^4 + 1001790*a^8*b^3*x^2 + 120550*a^9*b^2)/((b*x^2 + a)^9*a^12)
```

Mupad [B]
time = 5.78, size = 243, normalized size = 1.12

$$\frac{\frac{11\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{78419\,b^2\,x^4}{1008\,a^3}+\frac{50699\,b^3\,x^6}{112\,a^4}+\frac{36839\,b^4\,x^8}{28\,a^5}+\frac{27599\,b^5\,x^{10}}{12\,a^6}+\frac{20669\,b^6\,x^{12}}{8\,a^7}+\frac{15125\,b^7\,x^{14}}{8\,a^8}+\frac{10505\,b^8\,x^{16}}{12\,a^9}+\frac{935\,b^9\,x^{18}}{4\,a^{10}}+\frac{55\,b^{10}\,x^{20}}{2\,a^{11}}}{a^9\,x^4+9\,a^8\,b\,x^6+36\,a^7\,b^2\,x^8+84\,a^6\,b^3\,x^{10}+126\,a^5\,b^4\,x^{12}+126\,a^4\,b^5\,x^{14}+84\,a^3\,b^6\,x^{16}+36\,a^2\,b^7\,x^{18}+9\,a\,b^8\,x^{20}+b^9\,x^{22}}-\frac{55\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^{12}}+\frac{55\,b^2\,\ln\left(x\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^10),x)
```

```
[Out] ((11*b*x^2)/(4*a^2) - 1/(4*a) + (78419*b^2*x^4)/(1008*a^3) + (50699*b^3*x^6)/(112*a^4) + (36839*b^4*x^8)/(28*a^5) + (27599*b^5*x^10)/(12*a^6) + (20669*b^6*x^12)/(8*a^7) + (15125*b^7*x^14)/(8*a^8) + (10505*b^8*x^16)/(12*a^9) + (935*b^9*x^18)/(4*a^10) + (55*b^10*x^20)/(2*a^11))/(a^9*x^4 + b^9*x^22 + 9*a^8*b*x^6 + 9*a*b^8*x^20 + 36*a^7*b^2*x^8 + 84*a^6*b^3*x^10 + 126*a^5*b^4*x^12 + 126*a^4*b^5*x^14 + 84*a^3*b^6*x^16 + 36*a^2*b^7*x^18) - (55*b^2*log(a + b*x^2))/(2*a^12) + (55*b^2*log(x))/a^12
```

3.208 $\int \frac{1}{x^7(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=226

$$-\frac{1}{6a^{10}x^6}+\frac{5b}{2a^{11}x^4}-\frac{55b^2}{2a^{12}x^2}-\frac{b^3}{18a^4\left(a+bx^2\right)^9}-\frac{b^3}{4a^5\left(a+bx^2\right)^8}-\frac{5b^3}{7a^6\left(a+bx^2\right)^7}-\frac{5b^3}{3a^7\left(a+bx^2\right)^6}-\frac{7b^3}{2a^8\left(a+bx^2\right.}$$

```
[Out] -1/6/a^10/x^6+5/2*b/a^11/x^4-55/2*b^2/a^12/x^2-1/18*b^3/a^4/(b*x^2+a)^9-1/4
*b^3/a^5/(b*x^2+a)^8-5/7*b^3/a^6/(b*x^2+a)^7-5/3*b^3/a^7/(b*x^2+a)^6-7/2*b^
3/a^8/(b*x^2+a)^5-7*b^3/a^9/(b*x^2+a)^4-14*b^3/a^10/(b*x^2+a)^3-30*b^3/a^11
/(b*x^2+a)^2-165/2*b^3/a^12/(b*x^2+a)-220*b^3*ln(x)/a^13+110*b^3*ln(b*x^2+a
)/a^13
```

Rubi [A]
time = 0.16, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\frac{110b^3\log\left(a+bx^2\right)}{a^{13}}-\frac{220b^3\log(x)}{a^{13}}-\frac{165b^3}{2a^{12}\left(a+bx^2\right)}-\frac{55b^2}{2a^{12}x^2}-\frac{30b^3}{a^{11}\left(a+bx^2\right)^2}+\frac{5b}{2a^{11}x^4}-\frac{14b^3}{a^{10}\left(a+bx^2\right)^3}-\frac{1}{6a^{10}x^6}-\frac{7b^3}{a^9\left(a+bx^2\right)^4}-\frac{7b^3}{2a^8\left(a+bx^2\right)^5}-\frac{5b^3}{3a^7\left(a+bx^2\right)^6}-\frac{5b^3}{7a^6\left(a+bx^2\right)^7}-\frac{b^3}{4a^5\left(a+bx^2\right)^8}-\frac{b^3}{18a^4\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^10),x]
```

```
[Out] -1/6*1/(a^10*x^6) + (5*b)/(2*a^11*x^4) - (55*b^2)/(2*a^12*x^2) - b^3/(18*a^
4*(a + b*x^2)^9) - b^3/(4*a^5*(a + b*x^2)^8) - (5*b^3)/(7*a^6*(a + b*x^2)^7
) - (5*b^3)/(3*a^7*(a + b*x^2)^6) - (7*b^3)/(2*a^8*(a + b*x^2)^5) - (7*b^3)
/(a^9*(a + b*x^2)^4) - (14*b^3)/(a^10*(a + b*x^2)^3) - (30*b^3)/(a^11*(a +
b*x^2)^2) - (165*b^3)/(2*a^12*(a + b*x^2)) - (220*b^3*Log[x])/a^13 + (110*b
^3*Log[a + b*x^2])/a^13
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7 (a+bx^2)^{10}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^{10}x^4} - \frac{10b}{a^{11}x^3} + \frac{55b^2}{a^{12}x^2} - \frac{220b^3}{a^{13}x} + \frac{b^4}{a^4(a+bx)^{10}} + \frac{4b^4}{a^5(a+bx)^9} + \frac{10}{a^6(a+\cdots}\right.\right.$$
$$= -\frac{1}{6a^{10}x^6} + \frac{5b}{2a^{11}x^4} - \frac{55b^2}{2a^{12}x^2} - \frac{b^3}{18a^4(a+bx^2)^9} - \frac{b^3}{4a^5(a+bx^2)^8} - \frac{5b^3}{7a^6(a+bx^2)^7} - \cdots$$

Mathematica [A]
time = 0.08, size = 162, normalized size = 0.72

$$-\frac{\frac{a(42a^{11}-252a^{10}bx^2+2772a^9b^2x^4+78419a^8b^3x^6+456291a^7b^4x^8+1326204a^6b^5x^{10}+2318316a^5b^6x^{12}+2604294a^4b^7x^{14}+1905750a^3b^8x^{16}+882420a^2b^9x^{18}+235620ab^{10}x^{20}+27720b^{11}x^{22})}{x^6(a+bx^2)^9}+55440b^3\log(x)-27720b^3\log(a+bx^2)}{252a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^10),x]
```

```
[Out] -1/252*((a*(42*a^11 - 252*a^10*b*x^2 + 2772*a^9*b^2*x^4 + 78419*a^8*b^3*x^6
 + 456291*a^7*b^4*x^8 + 1326204*a^6*b^5*x^10 + 2318316*a^5*b^6*x^12 + 26042
94*a^4*b^7*x^14 + 1905750*a^3*b^8*x^16 + 882420*a^2*b^9*x^18 + 235620*a*b^1
0*x^20 + 27720*b^11*x^22))/(x^6*(a + b*x^2)^9) + 55440*b^3*Log[x] - 27720*b
^3*Log[a + b*x^2])/a^13
```

Maple [A]
time = 0.14, size = 213, normalized size = 0.94

method	result
norman	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{6a}-\frac{11b^2x^4}{a^3}+\frac{990b^4x^8}{a^5}+\frac{5940b^5x^{10}}{a^6}+\frac{16940b^6x^{12}}{a^7}+\frac{28875b^7x^{14}}{a^8}+\frac{31647b^8x^{16}}{a^9}+\frac{22638b^9x^{18}}{a^{10}}+\frac{71874b^{10}x^{20}}{7a^{11}}+\frac{75339b^{11}x^{22}}{28a^{12}}+\frac{78419b^{12}x^{24}}{252a^{13}}}{x^6(b\,x^2+a)^9}$
risch	$\frac{-\frac{1}{6a}+\frac{b\,x^2}{a^2}-\frac{11b^2x^4}{a^3}-\frac{78419b^3x^6}{252a^4}-\frac{50699b^4x^8}{28a^5}-\frac{36839b^5x^{10}}{7a^6}-\frac{27599b^6x^{12}}{3a^7}-\frac{20669b^7x^{14}}{2a^8}-\frac{15125b^8x^{16}}{2a^9}-\frac{10505b^9x^{18}}{3a^{10}}-\frac{935b^{10}x^{20}}{a^{11}}-\frac{110b^{11}x^{22}}{a^{12}}}{x^6(b\,x^2+a)^9}$
default	$\frac{b^4\left(-\frac{10a^7}{7b(b\,x^2+a)^7}-\frac{165a}{b(b\,x^2+a)}-\frac{7a^5}{b(b\,x^2+a)^5}-\frac{a^8}{2b(b\,x^2+a)^8}+\frac{220\ln(b\,x^2+a)}{b}-\frac{60a^2}{b(b\,x^2+a)^2}-\frac{28a^3}{b(b\,x^2+a)^3}-\frac{a^9}{9b(b\,x^2+a)^9}-\frac{10a^6}{3b(b\,x^2+a)^6}-\cdots\right.}{2a^{13}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^4/a^13*(-10/7*a^7/b/(b*x^2+a)^7-165*a/b/(b*x^2+a)-7*a^5/b/(b*x^2+a)^5
-1/2*a^8/b/(b*x^2+a)^8+220*ln(b*x^2+a)/b-60/b*a^2/(b*x^2+a)^2-28/b*a^3/(b*x
^2+a)^3-1/9/b*a^9/(b*x^2+a)^9-10/3/b*a^6/(b*x^2+a)^6-14/b*a^4/(b*x^2+a)^4)-
1/6/a^10/x^6-220*b^3*ln(x)/a^13-55/2*b^2/a^12/x^2+5/2*b/a^11/x^4
```

Maxima [A]
time = 0.35, size = 257, normalized size = 1.14

$$-\frac{27720\,b^{11}x^{22}+235620\,ab^{10}x^{20}+882420\,a^{2}b^{9}x^{18}+1905750\,a^{3}b^{8}x^{16}+2604294\,a^{4}b^{7}x^{14}+2318316\,a^{5}b^{6}x^{12}+1326204\,a^{6}b^{5}x^{10}+456291\,a^{7}b^{4}x^{8}+78419\,a^{8}b^{3}x^{6}+2772\,a^{9}b^{2}x^{4}-252\,a^{10}bx^{2}+42\,a^{11}}{252\,\left(a^{12}b^{9}x^{24}+9\,a^{13}b^{8}x^{22}+36\,a^{14}b^{7}x^{20}+84\,a^{15}b^{6}x^{18}+126\,a^{16}b^{5}x^{16}+126\,a^{17}b^{4}x^{14}+84\,a^{18}b^{3}x^{12}+36\,a^{19}b^{2}x^{10}+9\,a^{20}bx^{8}+a^{21}x^{6}\right)}+\frac{110\,b^{3}\log\left(bx^{2}+a\right)}{a^{13}}-\frac{110\,b^{3}\log\left(x^{2}\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(27720*b^11*x^22 + 235620*a*b^10*x^20 + 882420*a^2*b^9*x^18 + 190575
0*a^3*b^8*x^16 + 2604294*a^4*b^7*x^14 + 2318316*a^5*b^6*x^12 + 1326204*a^6*
b^5*x^10 + 456291*a^7*b^4*x^8 + 78419*a^8*b^3*x^6 + 2772*a^9*b^2*x^4 - 252*
a^10*b*x^2 + 42*a^11)/(a^12*b^9*x^24 + 9*a^13*b^8*x^22 + 36*a^14*b^7*x^20 +
 84*a^15*b^6*x^18 + 126*a^16*b^5*x^16 + 126*a^17*b^4*x^14 + 84*a^18*b^3*x^1
2 + 36*a^19*b^2*x^10 + 9*a^20*b*x^8 + a^21*x^6) + 110*b^3*log(b*x^2 + a)/a^
13 - 110*b^3*log(x^2)/a^13
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(208) = 416$.
time = 1.43, size = 453, normalized size = 2.00

$$-\frac{27720\,ab^{11}x^{22}+235620\,a^{2}b^{10}x^{20}+882420\,a^{3}b^{9}x^{18}+1905750\,a^{4}b^{8}x^{16}+2604294\,a^{5}b^{7}x^{14}+2318316\,a^{6}b^{6}x^{12}+1326204\,a^{7}b^{5}x^{10}+456291\,a^{8}b^{4}x^{8}+78419\,a^{9}b^{3}x^{6}+2772\,a^{10}b^{2}x^{4}-252\,a^{11}bx^{2}+42\,a^{12}-27720\,\left(b^{12}x^{24}+9\,ab^{11}x^{22}+36\,a^{2}b^{10}x^{20}+84\,a^{3}b^{9}x^{18}+126\,a^{4}b^{8}x^{16}+126\,a^{5}b^{7}x^{14}+84\,a^{6}b^{6}x^{12}+36\,a^{7}b^{5}x^{10}+9\,a^{8}b^{4}x^{8}+a^{9}b^{3}x^{6}\right)\log\left(bx^{2}+a\right)+55440\,\left(b^{12}x^{24}+9\,ab^{11}x^{22}+36\,a^{2}b^{10}x^{20}+84\,a^{3}b^{9}x^{18}+126\,a^{4}b^{8}x^{16}+126\,a^{5}b^{7}x^{14}+84\,a^{6}b^{6}x^{12}+36\,a^{7}b^{5}x^{10}+9\,a^{8}b^{4}x^{8}+a^{9}b^{3}x^{6}\right)\log\left(x\right)}{252\,\left(a^{13}b^{9}x^{24}+9\,a^{14}b^{8}x^{22}+36\,a^{15}b^{7}x^{20}+84\,a^{16}b^{6}x^{18}+126\,a^{17}b^{5}x^{16}+126\,a^{18}b^{4}x^{14}+84\,a^{19}b^{3}x^{12}+36\,a^{20}b^{2}x^{10}+9\,a^{21}bx^{8}+a^{22}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(27720*a*b^11*x^22 + 235620*a^2*b^10*x^20 + 882420*a^3*b^9*x^18 + 19
05750*a^4*b^8*x^16 + 2604294*a^5*b^7*x^14 + 2318316*a^6*b^6*x^12 + 1326204*
a^7*b^5*x^10 + 456291*a^8*b^4*x^8 + 78419*a^9*b^3*x^6 + 2772*a^10*b^2*x^4 -
 252*a^11*b*x^2 + 42*a^12 - 27720*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b^10*
x^20 + 84*a^3*b^9*x^18 + 126*a^4*b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*b^6*x
^12 + 36*a^7*b^5*x^10 + 9*a^8*b^4*x^8 + a^9*b^3*x^6)*log(b*x^2 + a) + 55440
*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b^10*x^20 + 84*a^3*b^9*x^18 + 126*a^4*
b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 36*a^7*b^5*x^10 + 9*a^8*b^4
*x^8 + a^9*b^3*x^6)*log(x))/(a^13*b^9*x^24 + 9*a^14*b^8*x^22 + 36*a^15*b^7*
x^20 + 84*a^16*b^6*x^18 + 126*a^17*b^5*x^16 + 126*a^18*b^4*x^14 + 84*a^19*b
^3*x^12 + 36*a^20*b^2*x^10 + 9*a^21*b*x^8 + a^22*x^6)
```

Sympy [A]
time = 0.72, size = 270, normalized size = 1.19

$$\frac{-42a^{11}+252a^{10}bx^{2}-2772a^{9}b^{2}x^{4}-78419a^{8}b^{3}x^{6}-456291a^{7}b^{4}x^{8}-1326204a^{6}b^{5}x^{10}-2318316a^{5}b^{6}x^{12}-2604294a^{4}b^{7}x^{14}-1905750a^{3}b^{8}x^{16}-882420a^{2}b^{9}x^{18}-235620ab^{10}x^{20}-27720b^{11}x^{22}}{252a^{21}x^{6}+2268a^{20}bx^{8}+9072a^{19}b^{2}x^{10}+21168a^{18}b^{3}x^{12}+31752a^{17}b^{4}x^{14}+31752a^{16}b^{5}x^{16}+21168a^{15}b^{6}x^{18}+9072a^{14}b^{7}x^{20}+2268a^{13}b^{8}x^{22}+252a^{12}b^{9}x^{24}}-\frac{220b^{3}\log\left(x\right)}{a^{13}}+\frac{110b^{3}\log\left(\frac{a}{b}+x^{2}\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**10,x)
```

```
[Out] (-42*a**11 + 252*a**10*b*x**2 - 2772*a**9*b**2*x**4 - 78419*a**8*b**3*x**6
- 456291*a**7*b**4*x**8 - 1326204*a**6*b**5*x**10 - 2318316*a**5*b**6*x**12
 - 2604294*a**4*b**7*x**14 - 1905750*a**3*b**8*x**16 - 882420*a**2*b**9*x**
18 - 235620*a*b**10*x**20 - 27720*b**11*x**22)/(252*a**21*x**6 + 2268*a**20
*b*x**8 + 9072*a**19*b**2*x**10 + 21168*a**18*b**3*x**12 + 31752*a**17*b**4
*x**14 + 31752*a**16*b**5*x**16 + 21168*a**15*b**6*x**18 + 9072*a**14*b**7*
x**20 + 2268*a**13*b**8*x**22 + 252*a**12*b**9*x**24) - 220*b**3*log(x)/a**
13 + 110*b**3*log(a/b + x**2)/a**13
```

Giac [A]
time = 1.59, size = 187, normalized size = 0.83

$$-\frac{110\,b^3\log\left(x^2\right)}{a^{13}}+\frac{110\,b^3\log\left(|bx^2+a|\right)}{a^{13}}+\frac{1210\,b^3x^6-165\,ab^2x^4+15\,a^2bx^2-a^3}{6\,a^{13}x^6}-\frac{78419\,b^{12}x^{18}+726561\,ab^{11}x^{16}+2996964\,a^2b^{10}x^{14}+7225764\,a^3b^9x^{12}+11226726\,a^4b^8x^{10}+11663316\,a^5b^7x^8+8108184\,a^6b^6x^6+3641256\,a^7b^5x^4+960210\,a^8b^4x^2+113620\,a^9b^3}{252\,\left(bx^2+a\right)^9a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -110*b^3*log(x^2)/a^13 + 110*b^3*log(abs(b*x^2 + a))/a^13 + 1/6*(1210*b^3*x
^6 - 165*a*b^2*x^4 + 15*a^2*b*x^2 - a^3)/(a^13*x^6) - 1/252*(78419*b^12*x^1
8 + 726561*a*b^11*x^16 + 2996964*a^2*b^10*x^14 + 7225764*a^3*b^9*x^12 + 112
26726*a^4*b^8*x^10 + 11663316*a^5*b^7*x^8 + 8108184*a^6*b^6*x^6 + 3641256*a
^7*b^5*x^4 + 960210*a^8*b^4*x^2 + 113620*a^9*b^3)/((b*x^2 + a)^9*a^13)
```

Mupad [B]
time = 1.09, size = 255, normalized size = 1.13

$$\frac{110\,b^3\,\ln\left(b\,x^2+a\right)}{a^{13}}-\frac{\frac{1}{6\,a}-\frac{b\,x^2}{a^2}+\frac{11\,b^2\,x^4}{a^3}+\frac{78419\,b^3\,x^6}{252\,a^4}+\frac{50699\,b^4\,x^8}{28\,a^5}+\frac{36839\,b^5\,x^{10}}{7\,a^6}+\frac{27599\,b^6\,x^{12}}{3\,a^7}+\frac{20669\,b^7\,x^{14}}{2\,a^8}+\frac{15125\,b^8\,x^{16}}{2\,a^9}+\frac{10505\,b^9\,x^{18}}{3\,a^{10}}+\frac{935\,b^{10}\,x^{20}}{a^{11}}+\frac{110\,b^{11}\,x^{22}}{a^{12}}}{a^9\,x^6+9\,a^8\,b\,x^8+36\,a^7\,b^2\,x^{10}+84\,a^6\,b^3\,x^{12}+126\,a^5\,b^4\,x^{14}+126\,a^4\,b^5\,x^{16}+84\,a^3\,b^6\,x^{18}+36\,a^2\,b^7\,x^{20}+9\,a\,b^8\,x^{22}+b^9\,x^{24}}-\frac{220\,b^3\,\ln\left(x\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^10),x)
```

```
[Out] (110*b^3*log(a + b*x^2))/a^13 - (1/(6*a) - (b*x^2)/a^2 + (11*b^2*x^4)/a^3 +
 (78419*b^3*x^6)/(252*a^4) + (50699*b^4*x^8)/(28*a^5) + (36839*b^5*x^10)/(7
*a^6) + (27599*b^6*x^12)/(3*a^7) + (20669*b^7*x^14)/(2*a^8) + (15125*b^8*x^
16)/(2*a^9) + (10505*b^9*x^18)/(3*a^10) + (935*b^10*x^20)/a^11 + (110*b^11*
x^22)/a^12)/(a^9*x^6 + b^9*x^24 + 9*a^8*b*x^8 + 9*a*b^8*x^22 + 36*a^7*b^2*x
^10 + 84*a^6*b^3*x^12 + 126*a^5*b^4*x^14 + 126*a^4*b^5*x^16 + 84*a^3*b^6*x^
18 + 36*a^2*b^7*x^20) - (220*b^3*log(x))/a^13
```

3.209 $\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=231

$$\frac{7436429a^2x}{65536b^{12}}-\frac{7436429ax^3}{196608b^{11}}+\frac{7436429x^5}{327680b^{10}}-\frac{x^{23}}{18b\,(a+bx^2)^9}-\frac{23x^{21}}{288b^2\,(a+bx^2)^8}-\frac{23x^{19}}{192b^3\,(a+bx^2)^7}-\frac{437x^{17}}{2304b^4\,(a+b}$$

```
[Out] 7436429/65536*a^2*x/b^12-7436429/196608*a*x^3/b^11+7436429/327680*x^5/b^10-
1/18*x^23/b/(b*x^2+a)^9-23/288*x^21/b^2/(b*x^2+a)^8-23/192*x^19/b^3/(b*x^2+
a)^7-437/2304*x^17/b^4/(b*x^2+a)^6-7429/23040*x^15/b^5/(b*x^2+a)^5-7429/122
88*x^13/b^6/(b*x^2+a)^4-96577/73728*x^11/b^7/(b*x^2+a)^3-1062347/294912*x^9
/b^8/(b*x^2+a)^2-1062347/65536*x^7/b^9/(b*x^2+a)-7436429/65536*a^(5/2)*arct
an(x*b^(1/2)/a^(1/2))/b^(25/2)
```

Rubi [A]
time = 0.11, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 308, 211}

$$-\frac{7436429a^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{25/2}}+\frac{7436429a^2x}{65536b^{12}}-\frac{7436429ax^3}{196608b^{11}}-\frac{1062347x^7}{65536b^9\,(a+bx^2)}-\frac{1062347x^9}{294912b^8\,(a+bx^2)^2}-\frac{96577x^{11}}{73728b^7\,(a+bx^2)^3}-\frac{7429x^{13}}{12288b^6\,(a+bx^2)^4}-\frac{7429x^{15}}{23040b^5\,(a+bx^2)^5}-\frac{437x^{17}}{2304b^4\,(a+bx^2)^6}-\frac{23x^{19}}{192b^3\,(a+bx^2)^7}-\frac{23x^{21}}{288b^2\,(a+bx^2)^8}-\frac{x^{23}}{18b\,(a+bx^2)^9}+\frac{7436429x^5}{327680b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^24/(a + b*x^2)^10,x]
```

```
[Out] (7436429*a^2*x)/(65536*b^12) - (7436429*a*x^3)/(196608*b^11) + (7436429*x^5
)/(327680*b^10) - x^23/(18*b*(a + b*x^2)^9) - (23*x^21)/(288*b^2*(a + b*x^2
)^8) - (23*x^19)/(192*b^3*(a + b*x^2)^7) - (437*x^17)/(2304*b^4*(a + b*x^2)
^6) - (7429*x^15)/(23040*b^5*(a + b*x^2)^5) - (7429*x^13)/(12288*b^6*(a + b
*x^2)^4) - (96577*x^11)/(73728*b^7*(a + b*x^2)^3) - (1062347*x^9)/(294912*b
^8*(a + b*x^2)^2) - (1062347*x^7)/(65536*b^9*(a + b*x^2)) - (7436429*a^(5/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(25/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx = -\frac{x^{23}}{18b\,(a+bx^2)^9} + \frac{23\int \frac{x^{22}}{(a+bx^2)^9}\,dx}{18b}$$

$$= -\frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8} + \frac{161\int \frac{x^{20}}{(a+bx^2)^8}\,dx}{96b^2}$$

$$= -\frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8} - \frac{23x^{19}}{192b^3\,(a+bx^2)^7} + \frac{437\int \frac{x^{18}}{(a+bx^2)^7}\,dx}{192b^3}$$

$$= -\frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8} - \frac{23x^{19}}{192b^3\,(a+bx^2)^7} - \frac{437x^{17}}{2304b^4\,(a+bx^2)^6}$$

$$= -\frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8} - \frac{23x^{19}}{192b^3\,(a+bx^2)^7} - \frac{437x^{17}}{2304b^4\,(a+bx^2)^6}$$

$$= -\frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8} - \frac{23x^{19}}{192b^3\,(a+bx^2)^7} - \frac{437x^{17}}{2304b^4\,(a+bx^2)^6}$$

$$= -\frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8} - \frac{23x^{19}}{192b^3\,(a+bx^2)^7} - \frac{437x^{17}}{2304b^4\,(a+bx^2)^6}$$

$$= -\frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8} - \frac{23x^{19}}{192b^3\,(a+bx^2)^7} - \frac{437x^{17}}{2304b^4\,(a+bx^2)^6}$$

$$= -\frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8} - \frac{23x^{19}}{192b^3\,(a+bx^2)^7} - \frac{437x^{17}}{2304b^4\,(a+bx^2)^6}$$

$$= -\frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8} - \frac{23x^{19}}{192b^3\,(a+bx^2)^7} - \frac{437x^{17}}{2304b^4\,(a+bx^2)^6}$$

$$= \frac{7436429a^2x}{65536b^{12}} - \frac{7436429ax^3}{196608b^{11}} + \frac{7436429x^5}{327680b^{10}} - \frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8}$$

$$= \frac{7436429a^2x}{65536b^{12}} - \frac{7436429ax^3}{196608b^{11}} + \frac{7436429x^5}{327680b^{10}} - \frac{x^{23}}{18b\,(a+bx^2)^9} - \frac{23x^{21}}{288b^2\,(a+bx^2)^8}$$

Mathematica [A]

time = 0.06, size = 166, normalized size = 0.72

$$\frac{\frac{\sqrt{b}\,x(334639305a^{11}+2900207310a^{10}bx^2+11110024926a^9b^2x^4+24648575094a^8b^3x^6+34810986496a^7b^4x^8+32314857354a^6b^5x^{10}+19562592546a^5b^6x^{12}+7323998514a^4b^7x^{14}+1469632311a^3b^8x^{16}+94961664a^2b^9x^{18}-4521984ab^{10}x^{20}+589824b^{11}x^{22})}{(a+bx^2)^9}-334639305a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2949120b^{25/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(334639305*a^11 + 2900207310*a^10*b*x^2 + 11110024926*a^9*b^2*x
^4 + 24648575094*a^8*b^3*x^6 + 34810986496*a^7*b^4*x^8 + 32314857354*a^6*b^
5*x^10 + 19562592546*a^5*b^6*x^12 + 7323998514*a^4*b^7*x^14 + 1469632311*a^
3*b^8*x^16 + 94961664*a^2*b^9*x^18 - 4521984*a*b^10*x^20 + 589824*b^11*x^22
))/(a + b*x^2)^9 - 334639305*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2949120*
b^(25/2))
```

Maple [A]
time = 0.12, size = 151, normalized size = 0.65

method	result
default	$\frac{\frac{1}{5}b^2x^5-\frac{10}{3}ab\,x^3+55a^2x}{b^{12}}-\frac{a^3\left(\frac{-\frac{3831949}{65536}a^8x-\frac{48340777}{98304}a^7b\,x^3-\frac{297702839}{163840}a^6b^2x^5-\frac{631790371}{163840}a^5b^3x^7-\frac{463199}{90}a^4b^4x^9-\frac{725918941}{163840}a^3b^5x^{11}}{\left(b\,x^2+a\right)^9}\right.}{b^{12}}$
risch	$\frac{x^5}{5b^{10}}-\frac{10a\,x^3}{3b^{11}}+\frac{55a^2x}{b^{12}}+\frac{\frac{3831949}{65536}a^{11}x+\frac{48340777}{98304}a^{10}b\,x^3+\frac{297702839}{163840}b^2a^9x^5+\frac{631790371}{163840}a^8b^3x^7+\frac{463199}{90}a^7b^4x^9+\frac{725918941}{163840}a^6b^5x}{b^{12}(b\,x^2+a)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^12*(1/5*b^2*x^5-10/3*a*b*x^3+55*a^2*x)-1/b^12*a^3*((-3831949/65536*a^8*
x-48340777/98304*a^7*b*x^3-297702839/163840*a^6*b^2*x^5-631790371/163840*a^
5*b^3*x^7-463199/90*a^4*b^4*x^9-725918941/163840*a^3*b^5*x^11-394553929/163
840*a^2*b^6*x^13-74539223/98304*a*b^7*x^15-6981491/65536*b^8*x^17)/(b*x^2+a
)^9+7436429/65536/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.59, size = 248, normalized size = 1.07

$$\frac{314167095\,a^3b^8x^{17}+2236176690\,a^4b^7x^{15}+7101970722\,a^5b^6x^{13}+13066540938\,a^6b^5x^{11}+15178104832\,a^7b^4x^9+11372226678\,a^8b^3x^7+5358651102\,a^9b^2x^5+1450223310\,a^{10}bx^3+172437705\,a^{11}x}{2949120\,(b^{21}x^{18}+9\,ab^{20}x^{16}+36\,a^2b^{19}x^{14}+84\,a^3b^{18}x^{12}+126\,a^4b^{17}x^{10}+126\,a^5b^{16}x^8+84\,a^6b^{15}x^6+36\,a^7b^{14}x^4+9\,a^8b^{13}x^2+a^9b^{12})}-\frac{7436429\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^{12}}+\frac{3\,b^2x^5-50\,abx^3+825\,a^2x}{15\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2949120*(314167095*a^3*b^8*x^17 + 2236176690*a^4*b^7*x^15 + 7101970722*a^
5*b^6*x^13 + 13066540938*a^6*b^5*x^11 + 15178104832*a^7*b^4*x^9 + 113722266
```

```
78*a^8*b^3*x^7 + 5358651102*a^9*b^2*x^5 + 1450223310*a^10*b*x^3 + 172437705
*a^11*x)/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 +
 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 +
 9*a^8*b^13*x^2 + a^9*b^12) - 7436429/65536*a^3*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^12) + 1/15*(3*b^2*x^5 - 50*a*b*x^3 + 825*a^2*x)/b^12
```

Fricas [A]
time = 1.21, size = 718, normalized size = 3.11

$$\left[\frac{1179648 b^{11} x^{23} - 9043968 a b^{10} x^{21} + 189923328 a^{2} b^{9} x^{19} + 2939264622 a^{3} b^{8} x^{17} + 14647997028 a^{4} b^{7} x^{15} + 39125185092 a^{5} b^{6} x^{13} + 64629714708 a^{6} b^{5} x^{11} + 69621972992 a^{7} b^{4} x^{9} + 49297150188 a^{8} b^{3} x^{7} + 22220049852 a^{9} b^{2} x^{5} + 5800414620 a^{10} b x^{3} + 669278610 a^{11} x + 334639305 \left(a^{2} b^{9} x^{18} + 9 a^{3} b^{8} x^{16} + 36 a^{4} b^{7} x^{14} + 84 a^{5} b^{6} x^{12} + 126 a^{6} b^{5} x^{10} + 126 a^{7} b^{4} x^{8} + 84 a^{8} b^{3} x^{6} + 36 a^{9} b^{2} x^{4} + 9 a^{10} b x^{2} + a^{11}\right) \sqrt{-\frac{a}{b}} \log\left(\frac{b x^{2} - 2 b x \sqrt{-\frac{a}{b}} - a}{b x^{2} + a}\right)}{5898240 \left(b^{21} x^{18} + 9 a b^{20} x^{16} + 36 a^{2} b^{19} x^{14} + 84 a^{3} b^{18} x^{12} + 126 a^{4} b^{17} x^{10} + 126 a^{5} b^{16} x^{8} + 84 a^{6} b^{15} x^{6} + 36 a^{7} b^{14} x^{4} + 9 a^{8} b^{13} x^{2} + a^{9} b^{12}\right)}, \frac{589824 b^{11} x^{23} - 4521984 a b^{10} x^{21} + 94961664 a^{2} b^{9} x^{19} + 1469632311 a^{3} b^{8} x^{17} + 7323998514 a^{4} b^{7} x^{15} + 19562592546 a^{5} b^{6} x^{13} + 32314857354 a^{6} b^{5} x^{11} + 34810986496 a^{7} b^{4} x^{9} + 24648575094 a^{8} b^{3} x^{7} + 11110024926 a^{9} b^{2} x^{5} + 2900207310 a^{10} b x^{3} + 334639305 a^{11} x - 334639305 \left(a^{2} b^{9} x^{18} + 9 a^{3} b^{8} x^{16} + 36 a^{4} b^{7} x^{14} + 84 a^{5} b^{6} x^{12} + 126 a^{6} b^{5} x^{10} + 126 a^{7} b^{4} x^{8} + 84 a^{8} b^{3} x^{6} + 36 a^{9} b^{2} x^{4} + 9 a^{10} b x^{2} + a^{11}\right) \sqrt{\frac{a}{b}} \arctan\left(\frac{b x \sqrt{\frac{a}{b}}}{a}\right)}{2949120 \left(b^{21} x^{18} + 9 a b^{20} x^{16} + 36 a^{2} b^{19} x^{14} + 84 a^{3} b^{18} x^{12} + 126 a^{4} b^{17} x^{10} + 126 a^{5} b^{16} x^{8} + 84 a^{6} b^{15} x^{6} + 36 a^{7} b^{14} x^{4} + 9 a^{8} b^{13} x^{2} + a^{9} b^{12}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/5898240*(1179648*b^11*x^23 - 9043968*a*b^10*x^21 + 189923328*a^2*b^9*x^1
9 + 2939264622*a^3*b^8*x^17 + 14647997028*a^4*b^7*x^15 + 39125185092*a^5*b^
6*x^13 + 64629714708*a^6*b^5*x^11 + 69621972992*a^7*b^4*x^9 + 49297150188*a
^8*b^3*x^7 + 22220049852*a^9*b^2*x^5 + 5800414620*a^10*b*x^3 + 669278610*a^
11*x + 334639305*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^7*x^14 + 84*a^5*
b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*x^6 + 36*a^9*b^2
*x^4 + 9*a^10*b*x^2 + a^11)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^
12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x
^4 + 9*a^8*b^13*x^2 + a^9*b^12), 1/2949120*(589824*b^11*x^23 - 4521984*a*b^
10*x^21 + 94961664*a^2*b^9*x^19 + 1469632311*a^3*b^8*x^17 + 7323998514*a^4*
b^7*x^15 + 19562592546*a^5*b^6*x^13 + 32314857354*a^6*b^5*x^11 + 3481098649
6*a^7*b^4*x^9 + 24648575094*a^8*b^3*x^7 + 11110024926*a^9*b^2*x^5 + 2900207
310*a^10*b*x^3 + 334639305*a^11*x - 334639305*(a^2*b^9*x^18 + 9*a^3*b^8*x^1
6 + 36*a^4*b^7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8
+ 84*a^8*b^3*x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*sqrt(a/b)*arctan(b
*x*sqrt(a/b)/a))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^1
8*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^
14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12)]
```

Sympy [A]
time = 0.92, size = 314, normalized size = 1.36

$$\frac{55 a^{2} x}{b^{12}} - \frac{10 a x^{3}}{3 b^{11}} + \frac{7436429 \sqrt{- \frac{a^{5}}{b^{25}}} \log{\left(x - \frac{b^{12} \sqrt{- \frac{a^{5}}{b^{25}}}}{a^{2}} \right)}}{131072} - \frac{7436429 \sqrt{- \frac{a^{5}}{b^{25}}} \log{\left(x + \frac{b^{12} \sqrt{- \frac{a^{5}}{b^{25}}}}{a^{2}} \right)}}{131072} + \frac{172437705 a^{11} x + 1450223310 a^{10} b x^{3} + 5358651102 a^{9} b^{2} x^{5} + 11372226678 a^{8} b^{3} x^{7} + 15178104832 a^{7} b^{4} x^{9} + 13066540938 a^{6} b^{5} x^{11} + 7101970722 a^{5} b^{6} x^{13} + 2236176690 a^{4} b^{7} x^{15} + 314167095 a^{3} b^{8} x^{17}}{2949120 a^{9} b^{12} + 26542080 a^{8} b^{13} x^{2} + 106168320 a^{7} b^{14} x^{4} + 247726080 a^{6} b^{15} x^{6} + 371589120 a^{5} b^{16} x^{8} + 371589120 a^{4} b^{17} x^{10} + 247726080 a^{3} b^{18} x^{12} + 106168320 a^{2} b^{19} x^{14} + 26542080 a b^{20} x^{16} + 2949120 b^{21} x^{18}} + \frac{x^{5}}{5 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24/(b*x**2+a)**10,x)
```

```
[Out] 55*a**2*x/b**12 - 10*a*x**3/(3*b**11) + 7436429*sqrt(-a**5/b**25)*log(x - b
**12*sqrt(-a**5/b**25)/a**2)/131072 - 7436429*sqrt(-a**5/b**25)*log(x + b**
12*sqrt(-a**5/b**25)/a**2)/131072 + (172437705*a**11*x + 1450223310*a**10*b
```

```
*x**3 + 5358651102*a**9*b**2*x**5 + 11372226678*a**8*b**3*x**7 + 1517810483
2*a**7*b**4*x**9 + 13066540938*a**6*b**5*x**11 + 7101970722*a**5*b**6*x**13
 + 2236176690*a**4*b**7*x**15 + 314167095*a**3*b**8*x**17)/(2949120*a**9*b*
*12 + 26542080*a**8*b**13*x**2 + 106168320*a**7*b**14*x**4 + 247726080*a**6
*b**15*x**6 + 371589120*a**5*b**16*x**8 + 371589120*a**4*b**17*x**10 + 2477
26080*a**3*b**18*x**12 + 106168320*a**2*b**19*x**14 + 26542080*a*b**20*x**1
6 + 2949120*b**21*x**18) + x**5/(5*b**10)
```

Giac [A]
time = 1.44, size = 162, normalized size = 0.70

$$-\frac{7436429\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^{12}}+\frac{314167095\,a^3b^8x^{17}+2236176690\,a^4b^7x^{15}+7101970722\,a^5b^6x^{13}+13066540938\,a^6b^5x^{11}+15178104832\,a^7b^4x^9+11372226678\,a^8b^3x^7+5358651102\,a^9b^2x^5+1450223310\,a^{10}bx^3+172437705\,a^{11}x}{2949120\,(bx^2+a)^9b^{12}}+\frac{3\,b^{40}x^5-50\,ab^{39}x^3+825\,a^2b^{38}x}{15\,b^{50}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -7436429/65536*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^12) + 1/2949120*(3141
67095*a^3*b^8*x^17 + 2236176690*a^4*b^7*x^15 + 7101970722*a^5*b^6*x^13 + 13
066540938*a^6*b^5*x^11 + 15178104832*a^7*b^4*x^9 + 11372226678*a^8*b^3*x^7
+ 5358651102*a^9*b^2*x^5 + 1450223310*a^10*b*x^3 + 172437705*a^11*x)/((b*x^
2 + a)^9*b^12) + 1/15*(3*b^40*x^5 - 50*a*b^39*x^3 + 825*a^2*b^38*x)/b^50
```

Mupad [B]
time = 4.90, size = 241, normalized size = 1.04

$$\frac{\frac{3831949\,a^{11}\,x}{65536}+\frac{48340777\,a^{10}\,b\,x^3}{98304}+\frac{297702839\,a^9\,b^2\,x^5}{163840}+\frac{631790371\,a^8\,b^3\,x^7}{163840}+\frac{463199\,a^7\,b^4\,x^9}{90}+\frac{725918941\,a^6\,b^5\,x^{11}}{163840}+\frac{394553929\,a^5\,b^6\,x^{13}}{163840}+\frac{74539223\,a^4\,b^7\,x^{15}}{98304}+\frac{6981491\,a^3\,b^8\,x^{17}}{65536}}{a^9\,b^{12}+9\,a^8\,b^{13}\,x^2+36\,a^7\,b^{14}\,x^4+84\,a^6\,b^{15}\,x^6+126\,a^5\,b^{16}\,x^8+126\,a^4\,b^{17}\,x^{10}+84\,a^3\,b^{18}\,x^{12}+36\,a^2\,b^{19}\,x^{14}+9\,a\,b^{20}\,x^{16}+b^{21}\,x^{18}}+\frac{x^5}{5\,b^{10}}-\frac{10\,a\,x^3}{3\,b^{11}}+\frac{55\,a^2\,x}{b^{12}}-\frac{7436429\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,b^{25/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(a + b*x^2)^10,x)
```

```
[Out] ((3831949*a^11*x)/65536 + (48340777*a^10*b*x^3)/98304 + (297702839*a^9*b^2*
x^5)/163840 + (631790371*a^8*b^3*x^7)/163840 + (463199*a^7*b^4*x^9)/90 + (7
25918941*a^6*b^5*x^11)/163840 + (394553929*a^5*b^6*x^13)/163840 + (74539223
*a^4*b^7*x^15)/98304 + (6981491*a^3*b^8*x^17)/65536)/(a^9*b^12 + b^21*x^18
+ 9*a*b^20*x^16 + 9*a^8*b^13*x^2 + 36*a^7*b^14*x^4 + 84*a^6*b^15*x^6 + 126*
a^5*b^16*x^8 + 126*a^4*b^17*x^10 + 84*a^3*b^18*x^12 + 36*a^2*b^19*x^14) + x
^5/(5*b^10) - (10*a*x^3)/(3*b^11) + (55*a^2*x)/b^12 - (7436429*a^(5/2)*atan
((b^(1/2)*x)/a^(1/2)))/(65536*b^(25/2))
```

3.210 $\int \frac{x^{22}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=218

$$-\frac{1616615ax}{65536b^{11}}+\frac{1616615x^3}{196608b^{10}}-\frac{x^{21}}{18b\left(a+bx^2\right)^9}-\frac{7x^{19}}{96b^2\left(a+bx^2\right)^8}-\frac{19x^{17}}{192b^3\left(a+bx^2\right)^7}-\frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6}-\frac{323}{1536b^5\left(a\right.}$$

```
[Out] -1616615/65536*a*x/b^11+1616615/196608*x^3/b^10-1/18*x^21/b/(b*x^2+a)^9-7/9
6*x^19/b^2/(b*x^2+a)^8-19/192*x^17/b^3/(b*x^2+a)^7-323/2304*x^15/b^4/(b*x^2
+a)^6-323/1536*x^13/b^5/(b*x^2+a)^5-4199/12288*x^11/b^6/(b*x^2+a)^4-46189/7
3728*x^9/b^7/(b*x^2+a)^3-46189/32768*x^7/b^8/(b*x^2+a)^2-323323/65536*x^5/b
^9/(b*x^2+a)+1616615/65536*a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(23/2)
```

Rubi [A]

time = 0.10, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 308, 211}

$$\frac{1616615a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{23/2}}-\frac{1616615ax}{65536b^{11}}-\frac{323323x^5}{65536b^9\left(a+bx^2\right)}-\frac{46189x^7}{32768b^8\left(a+bx^2\right)^2}-\frac{46189x^9}{73728b^7\left(a+bx^2\right)^3}-\frac{4199x^{11}}{12288b^6\left(a+bx^2\right)^4}-\frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5}-\frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6}-\frac{19x^{17}}{192b^3\left(a+bx^2\right)^7}-\frac{7x^{19}}{96b^2\left(a+bx^2\right)^8}-\frac{x^{21}}{18b\left(a+bx^2\right)^9}+\frac{1616615x^3}{196608b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^22/(a + b*x^2)^10,x]
```

```
[Out] (-1616615*a*x)/(65536*b^11) + (1616615*x^3)/(196608*b^10) - x^21/(18*b*(a +
 b*x^2)^9) - (7*x^19)/(96*b^2*(a + b*x^2)^8) - (19*x^17)/(192*b^3*(a + b*x^
2)^7) - (323*x^15)/(2304*b^4*(a + b*x^2)^6) - (323*x^13)/(1536*b^5*(a + b*x
^2)^5) - (4199*x^11)/(12288*b^6*(a + b*x^2)^4) - (46189*x^9)/(73728*b^7*(a
+ b*x^2)^3) - (46189*x^7)/(32768*b^8*(a + b*x^2)^2) - (323323*x^5)/(65536*b
^9*(a + b*x^2)) + (1616615*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(2
3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx = -\frac{x^{21}}{18b(a+bx^2)^9}+\frac{7\int \frac{x^{20}}{(a+bx^2)^9}\,dx}{6b}$$

$$= -\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}+\frac{133\int \frac{x^{18}}{(a+bx^2)^8}\,dx}{96b^2}$$

$$= -\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}+\frac{323\int \frac{x^{16}}{(a+bx^2)^7}\,dx}{192b^3}$$

$$= -\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}-\frac{323x^{15}}{2304b^4(a+bx^2)^6}+\frac{1615\int \cdots}{768\cdots}$$

$$= -\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}-\frac{323x^{15}}{2304b^4(a+bx^2)^6}-\frac{323\cdots}{1536b^5(a\cdots}$$

$$= -\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}-\frac{323x^{15}}{2304b^4(a+bx^2)^6}-\frac{323\cdots}{1536b^5(a\cdots}$$

$$= -\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}-\frac{323x^{15}}{2304b^4(a+bx^2)^6}-\frac{323\cdots}{1536b^5(a\cdots}$$

$$= -\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}-\frac{323x^{15}}{2304b^4(a+bx^2)^6}-\frac{323\cdots}{1536b^5(a\cdots}$$

$$= -\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}-\frac{323x^{15}}{2304b^4(a+bx^2)^6}-\frac{323\cdots}{1536b^5(a\cdots}$$

$$= -\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}-\frac{323x^{15}}{2304b^4(a+bx^2)^6}-\frac{323\cdots}{1536b^5(a\cdots}$$

$$= -\frac{1616615ax}{65536b^{11}}+\frac{1616615x^3}{196608b^{10}}-\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}-\cdots$$

$$= -\frac{1616615ax}{65536b^{11}}+\frac{1616615x^3}{196608b^{10}}-\frac{x^{21}}{18b(a+bx^2)^9}-\frac{7x^{19}}{96b^2(a+bx^2)^8}-\frac{19x^{17}}{192b^3(a+bx^2)^7}-\cdots$$

Mathematica [A]

time = 0.05, size = 155, normalized size = 0.71

$$\frac{\frac{\sqrt{b}\,x(-14549535a^{10}-126095970a^9bx^2-483044562a^8b^2x^4-1071677178a^7b^3x^6-1513521152a^6b^4x^8-1404993798a^5b^5x^{10}-850547502a^4b^6x^{12}-318434718a^3b^7x^{14}-63897057a^2b^8x^{16}-4128768ab^9x^{18}+196608b^{10}x^{20})}{(a+bx^2)^9}+14549535a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{589824b^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^22/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(-14549535*a^10 - 126095970*a^9*b*x^2 - 483044562*a^8*b^2*x^4 -
 1071677178*a^7*b^3*x^6 - 1513521152*a^6*b^4*x^8 - 1404993798*a^5*b^5*x^10
- 850547502*a^4*b^6*x^12 - 318434718*a^3*b^7*x^14 - 63897057*a^2*b^8*x^16 -
 4128768*a*b^9*x^18 + 196608*b^10*x^20))/(a + b*x^2)^9 + 14549535*a^(3/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*b^(23/2))
```

Maple [A]
time = 0.10, size = 140, normalized size = 0.64

method	result
default	$-\frac{-\frac{1}{3}b\,x^3+10ax}{b^{11}}+\frac{a^2\left(\frac{-\frac{961255}{65536}a^8x-\frac{12201403}{98304}a^7b\,x^3-\frac{15137633}{32768}a^6b^2x^5-\frac{32405717}{32768}a^5b^3x^7-\frac{24013}{18}a^4b^4x^9-\frac{38143787}{32768}a^3b^5x^{11}-\frac{21103775}{32768}a^2b}{\left(b\,x^2+a\right)^9}\right.}{b^{11}}$
risch	$\frac{x^3}{3b^{10}}-\frac{10ax}{b^{11}}+\frac{-\frac{961255}{65536}a^{10}x-\frac{12201403}{98304}a^9b\,x^3-\frac{15137633}{32768}a^8b^2x^5-\frac{32405717}{32768}a^7b^3x^7-\frac{24013}{18}a^6b^4x^9-\frac{38143787}{32768}a^5b^5x^{11}-\frac{21103775}{32768}a^4b^6x}{b^{11}(b\,x^2+a)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^22/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^11*(-1/3*b*x^3+10*a*x)+1/b^11*a^2*((-961255/65536*a^8*x-12201403/98304
*a^7*b*x^3-15137633/32768*a^6*b^2*x^5-32405717/32768*a^5*b^3*x^7-24013/18*a
^4*b^4*x^9-38143787/32768*a^3*b^5*x^11-21103775/32768*a^2*b^6*x^13-20435525
/98304*a*b^7*x^15-1987865/65536*b^8*x^17)/(b*x^2+a)^9+1616615/65536/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 236, normalized size = 1.08

$$-\frac{17890785\,a^2b^8x^{17}+122613150\,a^3b^7x^{15}+379867950\,a^4b^6x^{13}+686588166\,a^5b^5x^{11}+786857984\,a^6b^4x^9+583302906\,a^7b^3x^7+272477394\,a^8b^2x^5+73208418\,a^9bx^3+8651295\,a^{10}x}{589824\,(b^{20}x^{18}+9\,ab^{19}x^{16}+36\,a^2b^{18}x^{14}+84\,a^3b^{17}x^{12}+126\,a^4b^{16}x^{10}+126\,a^5b^{15}x^8+84\,a^6b^{14}x^6+36\,a^7b^{13}x^4+9\,a^8b^{12}x^2+a^9b^{11})}+\frac{1616615\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^{11}}+\frac{bx^3-30\,ax}{3\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/589824*(17890785*a^2*b^8*x^17 + 122613150*a^3*b^7*x^15 + 379867950*a^4*b
^6*x^13 + 686588166*a^5*b^5*x^11 + 786857984*a^6*b^4*x^9 + 583302906*a^7*b^
3*x^7 + 272477394*a^8*b^2*x^5 + 73208418*a^9*b*x^3 + 8651295*a^10*x)/(b^20*
```

```
x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x
^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2
 + a^9*b^11) + 1616615/65536*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^11) + 1
/3*(b*x^3 - 30*a*x)/b^11
```

Fricas [A]
time = 1.50, size = 692, normalized size = 3.17

$$\left[\frac{393216\,b^{10}x^{21} - 8257536\,ab^{9}x^{19} - 127794114\,a^{2}b^{8}x^{17} - 636869436\,a^{3}b^{7}x^{15} - 1701095004\,a^{4}b^{6}x^{13} - 2809987596\,a^{5}b^{5}x^{11} - 3027042304\,a^{6}b^{4}x^{9} - 2143354356\,a^{7}b^{3}x^{7} - 966089124\,a^{8}b^{2}x^{5} - 252191940\,a^{9}bx^{3} - 29099070\,a^{10}x + 14549535\,(ab^{9}x^{18} + 9\,a^{2}b^{8}x^{16} + 36\,a^{3}b^{7}x^{14} + 84\,a^{4}b^{6}x^{12} + 126\,a^{5}b^{5}x^{10} + 126\,a^{6}b^{4}x^{8} + 84\,a^{7}b^{3}x^{6} + 36\,a^{8}b^{2}x^{4} + 9\,a^{9}bx^{2} + a^{10})\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2} + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^{2} + a}\right)}{1179648\,(b^{20}x^{18} + 9\,ab^{19}x^{16} + 36\,a^{2}b^{18}x^{14} + 84\,a^{3}b^{17}x^{12} + 126\,a^{4}b^{16}x^{10} + 126\,a^{5}b^{15}x^{8} + 84\,a^{6}b^{14}x^{6} + 36\,a^{7}b^{13}x^{4} + 9\,a^{8}b^{12}x^{2} + a^{9}b^{11})}, \frac{196608\,b^{10}x^{21} - 4128768\,ab^{9}x^{19} - 63897057\,a^{2}b^{8}x^{17} - 318434718\,a^{3}b^{7}x^{15} - 850547502\,a^{4}b^{6}x^{13} - 1404993798\,a^{5}b^{5}x^{11} - 1513521152\,a^{6}b^{4}x^{9} - 1071677178\,a^{7}b^{3}x^{7} - 483044562\,a^{8}b^{2}x^{5} - 126095970\,a^{9}bx^{3} - 14549535\,a^{10}x + 14549535\,(ab^{9}x^{18} + 9\,a^{2}b^{8}x^{16} + 36\,a^{3}b^{7}x^{14} + 84\,a^{4}b^{6}x^{12} + 126\,a^{5}b^{5}x^{10} + 126\,a^{6}b^{4}x^{8} + 84\,a^{7}b^{3}x^{6} + 36\,a^{8}b^{2}x^{4} + 9\,a^{9}bx^{2} + a^{10})\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{589824\,(b^{20}x^{18} + 9\,ab^{19}x^{16} + 36\,a^{2}b^{18}x^{14} + 84\,a^{3}b^{17}x^{12} + 126\,a^{4}b^{16}x^{10} + 126\,a^{5}b^{15}x^{8} + 84\,a^{6}b^{14}x^{6} + 36\,a^{7}b^{13}x^{4} + 9\,a^{8}b^{12}x^{2} + a^{9}b^{11})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(393216*b^10*x^21 - 8257536*a*b^9*x^19 - 127794114*a^2*b^8*x^17
- 636869436*a^3*b^7*x^15 - 1701095004*a^4*b^6*x^13 - 2809987596*a^5*b^5*x^1
1 - 3027042304*a^6*b^4*x^9 - 2143354356*a^7*b^3*x^7 - 966089124*a^8*b^2*x^5
 - 252191940*a^9*b*x^3 - 29099070*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2*b^8
*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6*b^4*
x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(-a/b)*log(
(b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^20*x^18 + 9*a*b^19*x^16 + 3
6*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 +
 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11), 1/589824*(
196608*b^10*x^21 - 4128768*a*b^9*x^19 - 63897057*a^2*b^8*x^17 - 318434718*a
^3*b^7*x^15 - 850547502*a^4*b^6*x^13 - 1404993798*a^5*b^5*x^11 - 1513521152
*a^6*b^4*x^9 - 1071677178*a^7*b^3*x^7 - 483044562*a^8*b^2*x^5 - 126095970*a
^9*b*x^3 - 14549535*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2*b^8*x^16 + 36*a^3
*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6*b^4*x^8 + 84*a^7*b
^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(a/b)*arctan(b*x*sqrt(a/b
)/a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 12
6*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*
a^8*b^12*x^2 + a^9*b^11)]
```

Sympy [A]
time = 0.88, size = 299, normalized size = 1.37

$$- \frac{10ax}{b^{11}} - \frac{1616615\sqrt{- \frac{a^{3}}{b^{23}}} \log{\left(x - \frac{b^{11}\sqrt{- \frac{a^{3}}{b^{23}}}}{a} \right)}}{131072} + \frac{1616615\sqrt{- \frac{a^{3}}{b^{23}}} \log{\left(x + \frac{b^{11}\sqrt{- \frac{a^{3}}{b^{23}}}}{a} \right)}}{131072} + \frac{-8651295a^{10}x - 73208418a^{9}bx^{3} - 272477394a^{8}b^{2}x^{5} - 583302906a^{7}b^{3}x^{7} - 786857984a^{6}b^{4}x^{9} - 686588166a^{5}b^{5}x^{11} - 379867950a^{4}b^{6}x^{13} - 122613150a^{3}b^{7}x^{15} - 17890785a^{2}b^{8}x^{17}}{589824a^{9}b^{11} + 5308416a^{8}b^{12}x^{2} + 21233664a^{7}b^{13}x^{4} + 49545216a^{6}b^{14}x^{6} + 74317824a^{5}b^{15}x^{8} + 74317824a^{4}b^{16}x^{10} + 49545216a^{3}b^{17}x^{12} + 21233664a^{2}b^{18}x^{14} + 5308416ab^{19}x^{16} + 589824b^{20}x^{18}} + \frac{x^{3}}{3b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**22/(b*x**2+a)**10,x)
```

```
[Out] -10*a*x/b**11 - 1616615*sqrt(-a**3/b**23)*log(x - b**11*sqrt(-a**3/b**23)/a
)/131072 + 1616615*sqrt(-a**3/b**23)*log(x + b**11*sqrt(-a**3/b**23)/a)/131
072 + (-8651295*a**10*x - 73208418*a**9*b*x**3 - 272477394*a**8*b**2*x**5 -
 583302906*a**7*b**3*x**7 - 786857984*a**6*b**4*x**9 - 686588166*a**5*b**5*
x**11 - 379867950*a**4*b**6*x**13 - 122613150*a**3*b**7*x**15 - 17890785*a*
```

```
*2*b**8*x**17)/(589824*a**9*b**11 + 5308416*a**8*b**12*x**2 + 21233664*a**7
*b**13*x**4 + 49545216*a**6*b**14*x**6 + 74317824*a**5*b**15*x**8 + 7431782
4*a**4*b**16*x**10 + 49545216*a**3*b**17*x**12 + 21233664*a**2*b**18*x**14
+ 5308416*a*b**19*x**16 + 589824*b**20*x**18) + x**3/(3*b**10)
```

Giac [A]
time = 1.14, size = 150, normalized size = 0.69

$$\frac{1616615\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^{11}}-\frac{17890785\,a^2b^8x^{17}+122613150\,a^3b^7x^{15}+379867950\,a^4b^6x^{13}+686588166\,a^5b^5x^{11}+786857984\,a^6b^4x^9+583302906\,a^7b^3x^7+272477394\,a^8b^2x^5+73208418\,a^9bx^3+8651295\,a^{10}x}{589824\,(bx^2+a)^9b^{11}}+\frac{b^{20}x^3-30\,ab^{19}x}{3\,b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1616615/65536*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^11) - 1/589824*(178907
85*a^2*b^8*x^17 + 122613150*a^3*b^7*x^15 + 379867950*a^4*b^6*x^13 + 6865881
66*a^5*b^5*x^11 + 786857984*a^6*b^4*x^9 + 583302906*a^7*b^3*x^7 + 272477394
*a^8*b^2*x^5 + 73208418*a^9*b*x^3 + 8651295*a^10*x)/((b*x^2 + a)^9*b^11) +
1/3*(b^20*x^3 - 30*a*b^19*x)/b^30
```

Mupad [B]
time = 0.40, size = 231, normalized size = 1.06

$$\frac{x^3}{3\,b^{10}}-\frac{\frac{961255\,a^{10}\,x}{65536}+\frac{12201403\,a^9\,b\,x^3}{98304}+\frac{15137633\,a^8\,b^2\,x^5}{32768}+\frac{32405717\,a^7\,b^3\,x^7}{32768}+\frac{24013\,a^6\,b^4\,x^9}{18}+\frac{38143787\,a^5\,b^5\,x^{11}}{32768}+\frac{21103775\,a^4\,b^6\,x^{13}}{32768}+\frac{20435525\,a^3\,b^7\,x^{15}}{98304}+\frac{1987865\,a^2\,b^8\,x^{17}}{65536}}{a^9\,b^{11}+9\,a^8\,b^{12}\,x^2+36\,a^7\,b^{13}\,x^4+84\,a^6\,b^{14}\,x^6+126\,a^5\,b^{15}\,x^8+126\,a^4\,b^{16}\,x^{10}+84\,a^3\,b^{17}\,x^{12}+36\,a^2\,b^{18}\,x^{14}+9\,a\,b^{19}\,x^{16}+b^{20}\,x^{18}}+\frac{1616615\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,b^{23/2}}-\frac{10\,a\,x}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^22/(a + b*x^2)^10,x)
```

```
[Out] x^3/(3*b^10) - ((961255*a^10*x)/65536 + (12201403*a^9*b*x^3)/98304 + (15137
633*a^8*b^2*x^5)/32768 + (32405717*a^7*b^3*x^7)/32768 + (24013*a^6*b^4*x^9)
/18 + (38143787*a^5*b^5*x^11)/32768 + (21103775*a^4*b^6*x^13)/32768 + (2043
5525*a^3*b^7*x^15)/98304 + (1987865*a^2*b^8*x^17)/65536)/(a^9*b^11 + b^20*x
^18 + 9*a*b^19*x^16 + 9*a^8*b^12*x^2 + 36*a^7*b^13*x^4 + 84*a^6*b^14*x^6 +
126*a^5*b^15*x^8 + 126*a^4*b^16*x^10 + 84*a^3*b^17*x^12 + 36*a^2*b^18*x^14)
 + (1616615*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*b^(23/2)) - (10*a*x)/
b^11
```

3.211 $\int \frac{x^{20}}{(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=207

$$\frac{230945x}{65536b^{10}} - \frac{x^{19}}{18b\,(a+bx^2)^9} - \frac{19x^{17}}{288b^2\,(a+bx^2)^8} - \frac{323x^{15}}{4032b^3\,(a+bx^2)^7} - \frac{1615x^{13}}{16128b^4\,(a+bx^2)^6} - \frac{4199x^{11}}{32256b^5\,(a+bx^2)^5} -$$

```
[Out] 230945/65536*x/b^10-1/18*x^19/b/(b*x^2+a)^9-19/288*x^17/b^2/(b*x^2+a)^8-323
/4032*x^15/b^3/(b*x^2+a)^7-1615/16128*x^13/b^4/(b*x^2+a)^6-4199/32256*x^11/
b^5/(b*x^2+a)^5-46189/258048*x^9/b^6/(b*x^2+a)^4-46189/172032*x^7/b^7/(b*x^
2+a)^3-46189/98304*x^5/b^8/(b*x^2+a)^2-230945/196608*x^3/b^9/(b*x^2+a)-2309
45/65536*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(21/2)
```

Rubi [A]
time = 0.08, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 327, 211}

$$-\frac{230945\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{21/2}} - \frac{230945x^3}{196608b^9\,(a+bx^2)} - \frac{46189x^5}{98304b^8\,(a+bx^2)^2} - \frac{46189x^7}{172032b^7\,(a+bx^2)^3} - \frac{46189x^9}{258048b^6\,(a+bx^2)^4} - \frac{4199x^{11}}{32256b^5\,(a+bx^2)^5} - \frac{1615x^{13}}{16128b^4\,(a+bx^2)^6} - \frac{323x^{15}}{4032b^3\,(a+bx^2)^7} - \frac{19x^{17}}{288b^2\,(a+bx^2)^8} - \frac{x^{19}}{18b\,(a+bx^2)^9} + \frac{230945x}{65536b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^20/(a + b*x^2)^10,x]
```

```
[Out] (230945*x)/(65536*b^10) - x^19/(18*b*(a + b*x^2)^9) - (19*x^17)/(288*b^2*(a
 + b*x^2)^8) - (323*x^15)/(4032*b^3*(a + b*x^2)^7) - (1615*x^13)/(16128*b^4
*(a + b*x^2)^6) - (4199*x^11)/(32256*b^5*(a + b*x^2)^5) - (46189*x^9)/(2580
48*b^6*(a + b*x^2)^4) - (46189*x^7)/(172032*b^7*(a + b*x^2)^3) - (46189*x^5
)/(98304*b^8*(a + b*x^2)^2) - (230945*x^3)/(196608*b^9*(a + b*x^2)) - (2309
45*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(21/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx = -\frac{x^{19}}{18b(a+bx^2)^9} + \frac{19\int \frac{x^{18}}{(a+bx^2)^9}\,dx}{18b}$$

$$= -\frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} + \frac{323\int \frac{x^{16}}{(a+bx^2)^8}\,dx}{288b^2}$$

$$= -\frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} + \frac{1615\int \frac{x^{14}}{(a+bx^2)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6} + \frac{20995}{1}$$

$$= -\frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6} - \frac{4}{32256b}$$

$$= -\frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6} - \frac{4}{32256b}$$

$$= -\frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6} - \frac{4}{32256b}$$

$$= -\frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6} - \frac{4}{32256b}$$

$$= -\frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6} - \frac{4}{32256b}$$

$$= \frac{230945x}{65536b^{10}} - \frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6}$$

$$= \frac{230945x}{65536b^{10}} - \frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6}$$

Mathematica [**A**]

time = 0.05, size = 144, normalized size = 0.70

$$\frac{\frac{\sqrt{b}\,x\left(14549535a^9+126095970a^8bx^2+483044562a^7b^2x^4+1071677178a^6b^3x^6+1513521152a^5b^4x^8+1404993798a^4b^5x^{10}+850547502a^3b^6x^{12}+318434718a^2b^7x^{14}+63897057ab^8x^{16}+4128768b^9x^{18}\right)}{(a+bx^2)^9}-14549535\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{4128768b^{21/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^20/(a + b*x^2)^10,x]

[Out] ((Sqrt[b]*x*(14549535*a^9 + 126095970*a^8*b*x^2 + 483044562*a^7*b^2*x^4 + 1071677178*a^6*b^3*x^6 + 1513521152*a^5*b^4*x^8 + 1404993798*a^4*b^5*x^10 + 850547502*a^3*b^6*x^12 + 318434718*a^2*b^7*x^14 + 63897057*a*b^8*x^16 + 4128768*b^9*x^18))/(a + b*x^2)^9 - 14549535*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*b^(21/2))

Maple [A]
time = 0.10, size = 128, normalized size = 0.62

method	result
default	$\frac{x}{b^{10}}-\frac{a\left(\frac{-\frac{165409}{65536}a^8x-\frac{2117549}{98304}a^7b\,x^3-\frac{2654039}{32768}a^6b^2x^5-\frac{40270037}{229376}a^5b^3x^7-\frac{30313}{126}a^4b^4x^9-\frac{49153835}{229376}a^3b^5x^{11}-\frac{3997865}{32768}a^2b^6x^{13}-\frac{4042835}{98304}a\,\cdots}{\left(b\,x^2+a\right)^9}\right.}{b^{10}}$
risch	$\frac{x}{b^{10}}+\frac{\frac{165409}{65536}a^9x+\frac{2117549}{98304}a^8b\,x^3+\frac{2654039}{32768}a^7b^2x^5+\frac{40270037}{229376}a^6b^3x^7+\frac{30313}{126}a^5b^4x^9+\frac{49153835}{229376}a^4b^5x^{11}+\frac{3997865}{32768}a^3b^6x^{13}+\frac{4042835}{98304}a^2\cdots}{b^{10}\left(b\,x^2+a\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^20/(b*x^2+a)^10,x,method=_RETURNVERBOSE)

[Out] x/b^10-1/b^10*a*((-165409/65536*a^8*x-2117549/98304*a^7*b*x^3-2654039/32768*a^6*b^2*x^5-40270037/229376*a^5*b^3*x^7-30313/126*a^4*b^4*x^9-49153835/229376*a^3*b^5*x^11-3997865/32768*a^2*b^6*x^13-4042835/98304*a*b^7*x^15-424415/65536*b^8*x^17)/(b*x^2+a)^9+230945/65536/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))

Maxima [A]
time = 0.53, size = 222, normalized size = 1.07

$$\frac{26738145\,ab^8x^{17}+169799070\,a^2b^7x^{15}+503730990\,a^3b^6x^{13}+884769030\,a^4b^5x^{11}+993296384\,a^5b^4x^9+724860666\,a^6b^3x^7+334408914\,a^7b^2x^5+88937058\,a^8bx^3+10420767\,a^9x}{4128768\left(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+84\,a^3b^{16}x^{12}+126\,a^4b^{15}x^{10}+126\,a^5b^{14}x^8+84\,a^6b^{13}x^6+36\,a^7b^{12}x^4+9\,a^8b^{11}x^2+a^9b^{10}\right)}-\frac{230945\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,b^{10}}+\frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="maxima")

[Out] 1/4128768*(26738145*a*b^8*x^17 + 169799070*a^2*b^7*x^15 + 503730990*a^3*b^6*x^13 + 884769030*a^4*b^5*x^11 + 993296384*a^5*b^4*x^9 + 724860666*a^6*b^3*

```
x^7 + 334408914*a^7*b^2*x^5 + 88937058*a^8*b*x^3 + 10420767*a^9*x)/(b^19*x^
18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^1
0 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 +
 a^9*b^10) - 230945/65536*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^10) + x/b^10
```

Fricas [A]
time = 1.49, size = 664, normalized size = 3.21

$$\left[\frac{8257536\,b^{9}x^{19}+127794114\,ab^{8}x^{17}+636869436\,a^{2}b^{7}x^{15}+1701095004\,a^{3}b^{6}x^{13}+2809987596\,a^{4}b^{5}x^{11}+3027042304\,a^{5}b^{4}x^{9}+2143354356\,a^{6}b^{3}x^{7}+966089124\,a^{7}b^{2}x^{5}+252191940\,a^{8}bx^{3}+29099070\,a^{9}x+14549535\,(b^{9}x^{18}+9\,ab^{8}x^{16}+36\,a^{2}b^{7}x^{14}+84\,a^{3}b^{6}x^{12}+126\,a^{4}b^{5}x^{10}+126\,a^{5}b^{4}x^{8}+84\,a^{6}b^{3}x^{6}+36\,a^{7}b^{2}x^{4}+9\,a^{8}bx^{2}+a^{9})\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)}{8257536\,(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^{2}b^{17}x^{14}+84\,a^{3}b^{16}x^{12}+126\,a^{4}b^{15}x^{10}+126\,a^{5}b^{14}x^{8}+84\,a^{6}b^{13}x^{6}+36\,a^{7}b^{12}x^{4}+9\,a^{8}b^{11}x^{2}+a^{9}b^{10})},\ \frac{4128768\,b^{9}x^{19}+63897057\,ab^{8}x^{17}+318434718\,a^{2}b^{7}x^{15}+850547502\,a^{3}b^{6}x^{13}+1404993798\,a^{4}b^{5}x^{11}+1513521152\,a^{5}b^{4}x^{9}+1071677178\,a^{6}b^{3}x^{7}+483044562\,a^{7}b^{2}x^{5}+126095970\,a^{8}bx^{3}+14549535\,a^{9}x-14549535\,(b^{9}x^{18}+9\,ab^{8}x^{16}+36\,a^{2}b^{7}x^{14}+84\,a^{3}b^{6}x^{12}+126\,a^{4}b^{5}x^{10}+126\,a^{5}b^{4}x^{8}+84\,a^{6}b^{3}x^{6}+36\,a^{7}b^{2}x^{4}+9\,a^{8}bx^{2}+a^{9})\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{4128768\,(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^{2}b^{17}x^{14}+84\,a^{3}b^{16}x^{12}+126\,a^{4}b^{15}x^{10}+126\,a^{5}b^{14}x^{8}+84\,a^{6}b^{13}x^{6}+36\,a^{7}b^{12}x^{4}+9\,a^{8}b^{11}x^{2}+a^{9}b^{10})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(8257536*b^9*x^19 + 127794114*a*b^8*x^17 + 636869436*a^2*b^7*x^1
5 + 1701095004*a^3*b^6*x^13 + 2809987596*a^4*b^5*x^11 + 3027042304*a^5*b^4*
x^9 + 2143354356*a^6*b^3*x^7 + 966089124*a^7*b^2*x^5 + 252191940*a^8*b*x^3
+ 29099070*a^9*x + 14549535*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84
*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^
7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16
*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^1
2*x^4 + 9*a^8*b^11*x^2 + a^9*b^10), 1/4128768*(4128768*b^9*x^19 + 63897057*
a*b^8*x^17 + 318434718*a^2*b^7*x^15 + 850547502*a^3*b^6*x^13 + 1404993798*a
^4*b^5*x^11 + 1513521152*a^5*b^4*x^9 + 1071677178*a^6*b^3*x^7 + 483044562*a
^7*b^2*x^5 + 126095970*a^8*b*x^3 + 14549535*a^9*x - 14549535*(b^9*x^18 + 9*
a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5
*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a/b)*a
rctan(b*x*sqrt(a/b)/a))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*
a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36
*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10)]
```

Sympy [A]
time = 0.83, size = 274, normalized size = 1.32

$$\frac{230945\sqrt{-\frac{a}{b^{21}}}\log\left(-b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072}-\frac{230945\sqrt{-\frac{a}{b^{21}}}\log\left(b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072}+\frac{10420767a^{9}x+88937058a^{8}bx^{3}+334408914a^{7}b^{2}x^{5}+724860666a^{6}b^{3}x^{7}+993296384a^{5}b^{4}x^{9}+884769030a^{4}b^{5}x^{11}+503730990a^{3}b^{6}x^{13}+169799070a^{2}b^{7}x^{15}+26738145ab^{8}x^{17}}{4128768a^{9}b^{10}+37158912a^{8}b^{11}x^{2}+148635648a^{7}b^{12}x^{4}+346816512a^{6}b^{13}x^{6}+520224768a^{5}b^{14}x^{8}+520224768a^{4}b^{15}x^{10}+346816512a^{3}b^{16}x^{12}+148635648a^{2}b^{17}x^{14}+37158912ab^{18}x^{16}+4128768b^{19}x^{18}}+\frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**20/(b*x**2+a)**10,x)
```

```
[Out] 230945*sqrt(-a/b**21)*log(-b**10*sqrt(-a/b**21) + x)/131072 - 230945*sqrt(-
a/b**21)*log(b**10*sqrt(-a/b**21) + x)/131072 + (10420767*a**9*x + 88937058
*a**8*b*x**3 + 334408914*a**7*b**2*x**5 + 724860666*a**6*b**3*x**7 + 993296
384*a**5*b**4*x**9 + 884769030*a**4*b**5*x**11 + 503730990*a**3*b**6*x**13
+ 169799070*a**2*b**7*x**15 + 26738145*a*b**8*x**17)/(4128768*a**9*b**10 +
37158912*a**8*b**11*x**2 + 148635648*a**7*b**12*x**4 + 346816512*a**6*b**13
```

```
*x**6 + 520224768*a**5*b**14*x**8 + 520224768*a**4*b**15*x**10 + 346816512*
a**3*b**16*x**12 + 148635648*a**2*b**17*x**14 + 37158912*a*b**18*x**16 + 41
28768*b**19*x**18) + x/b**10
```

Giac [A]
time = 2.23, size = 131, normalized size = 0.63

$$-\frac{230945\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^{10}}+\frac{x}{b^{10}}+\frac{26738145\,ab^{8}x^{17}+169799070\,a^{2}b^{7}x^{15}+503730990\,a^{3}b^{6}x^{13}+884769030\,a^{4}b^{5}x^{11}+993296384\,a^{5}b^{4}x^{9}+724860666\,a^{6}b^{3}x^{7}+334408914\,a^{7}b^{2}x^{5}+88937058\,a^{8}bx^{3}+10420767\,a^{9}x}{4128768\,(bx^{2}+a)^{9}b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -230945/65536*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^10) + x/b^10 + 1/4128768
*(26738145*a*b^8*x^17 + 169799070*a^2*b^7*x^15 + 503730990*a^3*b^6*x^13 + 8
84769030*a^4*b^5*x^11 + 993296384*a^5*b^4*x^9 + 724860666*a^6*b^3*x^7 + 334
408914*a^7*b^2*x^5 + 88937058*a^8*b*x^3 + 10420767*a^9*x)/((b*x^2 + a)^9*b^
10)
```

Mupad [B]
time = 0.44, size = 218, normalized size = 1.05

$$\frac{\frac{165409\,a^{9}\,x}{65536}+\frac{2117549\,a^{8}\,b\,x^{3}}{98304}+\frac{2654039\,a^{7}\,b^{2}\,x^{5}}{32768}+\frac{40270037\,a^{6}\,b^{3}\,x^{7}}{229376}+\frac{30313\,a^{5}\,b^{4}\,x^{9}}{126}+\frac{49153835\,a^{4}\,b^{5}\,x^{11}}{229376}+\frac{3997865\,a^{3}\,b^{6}\,x^{13}}{32768}+\frac{4042835\,a^{2}\,b^{7}\,x^{15}}{98304}+\frac{424415\,a\,b^{8}\,x^{17}}{65536}}{a^{9}\,b^{10}+9\,a^{8}\,b^{11}\,x^{2}+36\,a^{7}\,b^{12}\,x^{4}+84\,a^{6}\,b^{13}\,x^{6}+126\,a^{5}\,b^{14}\,x^{8}+126\,a^{4}\,b^{15}\,x^{10}+84\,a^{3}\,b^{16}\,x^{12}+36\,a^{2}\,b^{17}\,x^{14}+9\,a\,b^{18}\,x^{16}+b^{19}\,x^{18}}+\frac{x}{b^{10}}-\frac{230945\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,b^{21/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(a + b*x^2)^10,x)
```

```
[Out] ((165409*a^9*x)/65536 + (2117549*a^8*b*x^3)/98304 + (424415*a*b^8*x^17)/655
36 + (2654039*a^7*b^2*x^5)/32768 + (40270037*a^6*b^3*x^7)/229376 + (30313*a
^5*b^4*x^9)/126 + (49153835*a^4*b^5*x^11)/229376 + (3997865*a^3*b^6*x^13)/3
2768 + (4042835*a^2*b^7*x^15)/98304)/(a^9*b^10 + b^19*x^18 + 9*a*b^18*x^16
+ 9*a^8*b^11*x^2 + 36*a^7*b^12*x^4 + 84*a^6*b^13*x^6 + 126*a^5*b^14*x^8 + 1
26*a^4*b^15*x^10 + 84*a^3*b^16*x^12 + 36*a^2*b^17*x^14) + x/b^10 - (230945*
a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*b^(21/2))
```

3.212 $\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=197

$$-\frac{x^{17}}{18b\,(a+bx^2)^9}-\frac{17x^{15}}{288b^2\,(a+bx^2)^8}-\frac{85x^{13}}{1344b^3\,(a+bx^2)^7}-\frac{1105x^{11}}{16128b^4\,(a+bx^2)^6}-\frac{2431x^9}{32256b^5\,(a+bx^2)^5}-\frac{2431}{28672b^6\,(a}$$

```
[Out] -1/18*x^17/b/(b*x^2+a)^9-17/288*x^15/b^2/(b*x^2+a)^8-85/1344*x^13/b^3/(b*x^
2+a)^7-1105/16128*x^11/b^4/(b*x^2+a)^6-2431/32256*x^9/b^5/(b*x^2+a)^5-2431/
28672*x^7/b^6/(b*x^2+a)^4-2431/24576*x^5/b^7/(b*x^2+a)^3-12155/98304*x^3/b^
8/(b*x^2+a)^2-12155/65536*x/b^9/(b*x^2+a)+12155/65536*arctan(x*b^(1/2)/a^(1
/2))/b^(19/2)/a^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {294, 211}

$$\frac{12155\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\sqrt{a}\,b^{19/2}}-\frac{12155x}{65536b^9\,(a+bx^2)}-\frac{12155x^3}{98304b^8\,(a+bx^2)^2}-\frac{2431x^5}{24576b^7\,(a+bx^2)^3}-\frac{2431x^7}{28672b^6\,(a+bx^2)^4}-\frac{2431x^9}{32256b^5\,(a+bx^2)^5}-\frac{1105x^{11}}{16128b^4\,(a+bx^2)^6}-\frac{85x^{13}}{1344b^3\,(a+bx^2)^7}-\frac{17x^{15}}{288b^2\,(a+bx^2)^8}-\frac{x^{17}}{18b\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^18/(a + b*x^2)^10,x]
```

```
[Out] -1/18*x^17/(b*(a + b*x^2)^9) - (17*x^15)/(288*b^2*(a + b*x^2)^8) - (85*x^13
)/(1344*b^3*(a + b*x^2)^7) - (1105*x^11)/(16128*b^4*(a + b*x^2)^6) - (2431*
x^9)/(32256*b^5*(a + b*x^2)^5) - (2431*x^7)/(28672*b^6*(a + b*x^2)^4) - (24
31*x^5)/(24576*b^7*(a + b*x^2)^3) - (12155*x^3)/(98304*b^8*(a + b*x^2)^2) -
 (12155*x)/(65536*b^9*(a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(6
5536*Sqrt[a]*b^(19/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx = -\frac{x^{17}}{18b\,(a+bx^2)^9} + \frac{17\int \frac{x^{16}}{(a+bx^2)^9}\,dx}{18b}$$

$$= -\frac{x^{17}}{18b\,(a+bx^2)^9} - \frac{17x^{15}}{288b^2\,(a+bx^2)^8} + \frac{85\int \frac{x^{14}}{(a+bx^2)^8}\,dx}{96b^2}$$

$$= -\frac{x^{17}}{18b\,(a+bx^2)^9} - \frac{17x^{15}}{288b^2\,(a+bx^2)^8} - \frac{85x^{13}}{1344b^3\,(a+bx^2)^7} + \frac{1105\int \frac{x^{12}}{(a+bx^2)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{17}}{18b\,(a+bx^2)^9} - \frac{17x^{15}}{288b^2\,(a+bx^2)^8} - \frac{85x^{13}}{1344b^3\,(a+bx^2)^7} - \frac{1105x^{11}}{16128b^4\,(a+bx^2)^6} + 12155$$

$$= -\frac{x^{17}}{18b\,(a+bx^2)^9} - \frac{17x^{15}}{288b^2\,(a+bx^2)^8} - \frac{85x^{13}}{1344b^3\,(a+bx^2)^7} - \frac{1105x^{11}}{16128b^4\,(a+bx^2)^6} - 32256$$

$$= -\frac{x^{17}}{18b\,(a+bx^2)^9} - \frac{17x^{15}}{288b^2\,(a+bx^2)^8} - \frac{85x^{13}}{1344b^3\,(a+bx^2)^7} - \frac{1105x^{11}}{16128b^4\,(a+bx^2)^6} - 32256$$

$$= -\frac{x^{17}}{18b\,(a+bx^2)^9} - \frac{17x^{15}}{288b^2\,(a+bx^2)^8} - \frac{85x^{13}}{1344b^3\,(a+bx^2)^7} - \frac{1105x^{11}}{16128b^4\,(a+bx^2)^6} - 32256$$

$$= -\frac{x^{17}}{18b\,(a+bx^2)^9} - \frac{17x^{15}}{288b^2\,(a+bx^2)^8} - \frac{85x^{13}}{1344b^3\,(a+bx^2)^7} - \frac{1105x^{11}}{16128b^4\,(a+bx^2)^6} - 32256$$

$$= -\frac{x^{17}}{18b\,(a+bx^2)^9} - \frac{17x^{15}}{288b^2\,(a+bx^2)^8} - \frac{85x^{13}}{1344b^3\,(a+bx^2)^7} - \frac{1105x^{11}}{16128b^4\,(a+bx^2)^6} - 32256$$

$$= -\frac{x^{17}}{18b\,(a+bx^2)^9} - \frac{17x^{15}}{288b^2\,(a+bx^2)^8} - \frac{85x^{13}}{1344b^3\,(a+bx^2)^7} - \frac{1105x^{11}}{16128b^4\,(a+bx^2)^6} - 32256$$

Mathematica [A]
time = 0.05, size = 134, normalized size = 0.68

$$\frac{-\frac{\sqrt{b}\,x(765765a^8+6636630a^7bx^2+25423398a^6b^2x^4+56404062a^5b^3x^6+79659008a^4b^4x^8+73947042a^3b^5x^{10}+44765658a^2b^6x^{12}+16759722ab^7x^{14}+3363003b^8x^{16})}{(a+bx^2)^9}+\frac{765765\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}}{4128768b^{19/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^18/(a + b*x^2)^10,x]
```

```
[Out] (-((Sqrt[b]*x*(765765*a^8 + 6636630*a^7*b*x^2 + 25423398*a^6*b^2*x^4 + 5640
4062*a^5*b^3*x^6 + 79659008*a^4*b^4*x^8 + 73947042*a^3*b^5*x^10 + 44765658*
a^2*b^6*x^12 + 16759722*a*b^7*x^14 + 3363003*b^8*x^16))/(a + b*x^2)^9) + (7
65765*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a])/(4128768*b^(19/2))
```

Maple [A]
time = 0.12, size = 124, normalized size = 0.63

method	result
default	$\frac{-\frac{12155a^8x}{65536b^9}-\frac{158015a^7x^3}{98304b^8}-\frac{201773a^6x^5}{32768b^7}-\frac{3133559a^5x^7}{229376b^6}-\frac{2431a^4x^9}{126b^5}-\frac{4108169a^3x^{11}}{229376b^4}-\frac{355283a^2x^{13}}{32768b^3}-\frac{399041a\,x^{15}}{98304b^2}-\frac{53381x^{17}}{65536b}}{(b\,x^2+a)^9}+\frac{12155\arctan}{65536b^9}$
risch	$\frac{-\frac{12155a^8x}{65536b^9}-\frac{158015a^7x^3}{98304b^8}-\frac{201773a^6x^5}{32768b^7}-\frac{3133559a^5x^7}{229376b^6}-\frac{2431a^4x^9}{126b^5}-\frac{4108169a^3x^{11}}{229376b^4}-\frac{355283a^2x^{13}}{32768b^3}-\frac{399041a\,x^{15}}{98304b^2}-\frac{53381x^{17}}{65536b}}{(b\,x^2+a)^9}-\frac{12155\ln\left(bx\right.}{131072\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^18/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] (-12155/65536*a^8/b^9*x-158015/98304*a^7/b^8*x^3-201773/32768*a^6/b^7*x^5-3
133559/229376*a^5/b^6*x^7-2431/126*a^4/b^5*x^9-4108169/229376*a^3/b^4*x^11-
355283/32768*a^2/b^3*x^13-399041/98304*a/b^2*x^15-53381/65536/b*x^17)/(b*x^
2+a)^9+12155/65536/b^9/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.55, size = 213, normalized size = 1.08

$$-\frac{3363003\,b^8x^{17}+16759722\,ab^7x^{15}+44765658\,a^2b^6x^{13}+73947042\,a^3b^5x^{11}+79659008\,a^4b^4x^9+56404062\,a^5b^3x^7+25423398\,a^6b^2x^5+6636630\,a^7bx^3+765765\,a^8x}{4128768\,(b^{18}x^{18}+9\,ab^{17}x^{16}+36\,a^2b^{16}x^{14}+84\,a^3b^{15}x^{12}+126\,a^4b^{14}x^{10}+126\,a^5b^{13}x^8+84\,a^6b^{12}x^6+36\,a^7b^{11}x^4+9\,a^8b^{10}x^2+a^9b^9)}+\frac{12155\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/4128768*(3363003*b^8*x^17 + 16759722*a*b^7*x^15 + 44765658*a^2*b^6*x^13
+ 73947042*a^3*b^5*x^11 + 79659008*a^4*b^4*x^9 + 56404062*a^5*b^3*x^7 + 254
23398*a^6*b^2*x^5 + 6636630*a^7*b*x^3 + 765765*a^8*x)/(b^18*x^18 + 9*a*b^17
*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x^10 + 126*a^5*b
^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2 + a^9*b^9) + 1
2155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^9)
```

Fricas [A]
time = 1.28, size = 650, normalized size = 3.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/8257536*(6726006*a*b^9*x^17 + 33519444*a^2*b^8*x^15 + 89531316*a^3*b^7*
x^13 + 147894084*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 112808124*a^6*b^4*x
^7 + 50846796*a^7*b^3*x^5 + 13273260*a^8*b^2*x^3 + 1531530*a^9*b*x + 765765
```

```
*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5
*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a
^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^19*x^18
+ 9*a^2*b^18*x^16 + 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^15*x^10
 + 126*a^6*b^14*x^8 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^11*x^2 +
a^10*b^10), -1/4128768*(3363003*a*b^9*x^17 + 16759722*a^2*b^8*x^15 + 447656
58*a^3*b^7*x^13 + 73947042*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 + 56404062*a
^6*b^4*x^7 + 25423398*a^7*b^3*x^5 + 6636630*a^8*b^2*x^3 + 765765*a^9*b*x -
765765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a
^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x
^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^19*x^18 + 9*a^2*b^18*x^16 +
 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^15*x^10 + 126*a^6*b^14*x^8
 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^11*x^2 + a^10*b^10)]
```

Sympy [A]
time = 0.68, size = 277, normalized size = 1.41

$$-\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(-ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}+\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}+\frac{-765765a^8x-6636630a^7bx^3-25423398a^6b^2x^5-56404062a^5b^3x^7-79659008a^4b^4x^9-73947042a^3b^5x^{11}-44765658a^2b^6x^{13}-16759722ab^7x^{15}-3363003b^8x^{17}}{4128768a^9b^9+37158912a^8b^{10}x^2+148635648a^7b^{11}x^4+346816512a^6b^{12}x^6+520224768a^5b^{13}x^8+520224768a^4b^{14}x^{10}+346816512a^3b^{15}x^{12}+148635648a^2b^{16}x^{14}+37158912ab^{17}x^{16}+4128768b^{18}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**18/(b*x**2+a)**10,x)
```

```
[Out] -12155*sqrt(-1/(a*b**19))*log(-a*b**9*sqrt(-1/(a*b**19)) + x)/131072 + 1215
5*sqrt(-1/(a*b**19))*log(a*b**9*sqrt(-1/(a*b**19)) + x)/131072 + (-765765*a
**8*x - 6636630*a**7*b*x**3 - 25423398*a**6*b**2*x**5 - 56404062*a**5*b**3*
x**7 - 79659008*a**4*b**4*x**9 - 73947042*a**3*b**5*x**11 - 44765658*a**2*b
**6*x**13 - 16759722*a*b**7*x**15 - 3363003*b**8*x**17)/(4128768*a**9*b**9
+ 37158912*a**8*b**10*x**2 + 148635648*a**7*b**11*x**4 + 346816512*a**6*b**
12*x**6 + 520224768*a**5*b**13*x**8 + 520224768*a**4*b**14*x**10 + 34681651
2*a**3*b**15*x**12 + 148635648*a**2*b**16*x**14 + 37158912*a*b**17*x**16 +
4128768*b**18*x**18)
```

Giac [A]
time = 1.23, size = 122, normalized size = 0.62

$$\frac{12155\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,b^9}-\frac{3363003\,b^8x^{17}+16759722\,ab^7x^{15}+44765658\,a^2b^6x^{13}+73947042\,a^3b^5x^{11}+79659008\,a^4b^4x^9+56404062\,a^5b^3x^7+25423398\,a^6b^2x^5+6636630\,a^7bx^3+765765\,a^8x}{4128768\,(bx^2+a)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^9) - 1/4128768*(3363003*b^8*
x^17 + 16759722*a*b^7*x^15 + 44765658*a^2*b^6*x^13 + 73947042*a^3*b^5*x^11
+ 79659008*a^4*b^4*x^9 + 56404062*a^5*b^3*x^7 + 25423398*a^6*b^2*x^5 + 6636
630*a^7*b*x^3 + 765765*a^8*x)/((b*x^2 + a)^9*b^9)
```

Mupad [B]
time = 4.96, size = 210, normalized size = 1.07

$$\frac{12155\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,\sqrt{a}\,b^{19/2}}-\frac{\frac{53381\,x^{17}}{65536\,b}+\frac{399041\,a\,x^{15}}{98304\,b^2}+\frac{12155\,a^8\,x}{65536\,b^9}+\frac{355283\,a^2\,x^{13}}{32768\,b^3}+\frac{4108169\,a^3\,x^{11}}{229376\,b^4}+\frac{2431\,a^4\,x^9}{126\,b^5}+\frac{3133559\,a^5\,x^7}{229376\,b^6}+\frac{201773\,a^6\,x^5}{32768\,b^7}+\frac{158015\,a^7\,x^3}{98304\,b^8}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^18/(a + b*x^2)^10,x)
```

```
[Out] (12155*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(1/2)*b^(19/2)) - ((53381*x^17)/
(65536*b) + (399041*a*x^15)/(98304*b^2) + (12155*a^8*x)/(65536*b^9) + (3552
83*a^2*x^13)/(32768*b^3) + (4108169*a^3*x^11)/(229376*b^4) + (2431*a^4*x^9)
/(126*b^5) + (3133559*a^5*x^7)/(229376*b^6) + (201773*a^6*x^5)/(32768*b^7)
+ (158015*a^7*x^3)/(98304*b^8))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^1
6 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 +
84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

3.213 $\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=198

$$-\frac{x^{15}}{18b\left(a+bx^2\right)^9}-\frac{5x^{13}}{96b^2\left(a+bx^2\right)^8}-\frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7}-\frac{715x^9}{16128b^4\left(a+bx^2\right)^6}-\frac{143x^7}{3584b^5\left(a+bx^2\right)^5}-\frac{143x^5}{4096b^6\left(a+\right.}$$

```
[Out] -1/18*x^15/b/(b*x^2+a)^9-5/96*x^13/b^2/(b*x^2+a)^8-65/1344*x^11/b^3/(b*x^2+
a)^7-715/16128*x^9/b^4/(b*x^2+a)^6-143/3584*x^7/b^5/(b*x^2+a)^5-143/4096*x^
5/b^6/(b*x^2+a)^4-715/24576*x^3/b^7/(b*x^2+a)^3-715/32768*x/b^8/(b*x^2+a)^2
+715/65536*x/a/b^8/(b*x^2+a)+715/65536*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^
(17/2)
```

Rubi [A]
time = 0.08, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 205, 211}

$$\frac{715\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}}+\frac{715x}{65536ab^8\left(a+bx^2\right)}-\frac{715x}{32768b^8\left(a+bx^2\right)^2}-\frac{715x^3}{24576b^7\left(a+bx^2\right)^3}-\frac{143x^5}{4096b^6\left(a+bx^2\right)^4}-\frac{143x^7}{3584b^5\left(a+bx^2\right)^5}-\frac{715x^9}{16128b^4\left(a+bx^2\right)^6}-\frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7}-\frac{5x^{13}}{96b^2\left(a+bx^2\right)^8}-\frac{x^{15}}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^16/(a + b*x^2)^10,x]
```

```
[Out] -1/18*x^15/(b*(a + b*x^2)^9) - (5*x^13)/(96*b^2*(a + b*x^2)^8) - (65*x^11)/
(1344*b^3*(a + b*x^2)^7) - (715*x^9)/(16128*b^4*(a + b*x^2)^6) - (143*x^7)/
(3584*b^5*(a + b*x^2)^5) - (143*x^5)/(4096*b^6*(a + b*x^2)^4) - (715*x^3)/(
24576*b^7*(a + b*x^2)^3) - (715*x)/(32768*b^8*(a + b*x^2)^2) + (715*x)/(655
36*a*b^8*(a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(3/2)*b^
(17/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx = -\frac{x^{15}}{18b\,(a+bx^2)^9} + \frac{5\int \frac{x^{14}}{(a+bx^2)^9}\,dx}{6b}$$

$$= -\frac{x^{15}}{18b\,(a+bx^2)^9} - \frac{5x^{13}}{96b^2\,(a+bx^2)^8} + \frac{65\int \frac{x^{12}}{(a+bx^2)^8}\,dx}{96b^2}$$

$$= -\frac{x^{15}}{18b\,(a+bx^2)^9} - \frac{5x^{13}}{96b^2\,(a+bx^2)^8} - \frac{65x^{11}}{1344b^3\,(a+bx^2)^7} + \frac{715\int \frac{x^{10}}{(a+bx^2)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{15}}{18b\,(a+bx^2)^9} - \frac{5x^{13}}{96b^2\,(a+bx^2)^8} - \frac{65x^{11}}{1344b^3\,(a+bx^2)^7} - \frac{715x^9}{16128b^4\,(a+bx^2)^6} + \frac{715\int \frac{\cdots}{(a\cdots}}{179\cdots}$$

$$= -\frac{x^{15}}{18b\,(a+bx^2)^9} - \frac{5x^{13}}{96b^2\,(a+bx^2)^8} - \frac{65x^{11}}{1344b^3\,(a+bx^2)^7} - \frac{715x^9}{16128b^4\,(a+bx^2)^6} - \frac{14\cdots}{3584b^5\,(\cdots}$$

$$= -\frac{x^{15}}{18b\,(a+bx^2)^9} - \frac{5x^{13}}{96b^2\,(a+bx^2)^8} - \frac{65x^{11}}{1344b^3\,(a+bx^2)^7} - \frac{715x^9}{16128b^4\,(a+bx^2)^6} - \frac{14\cdots}{3584b^5\,(\cdots}$$

$$= -\frac{x^{15}}{18b\,(a+bx^2)^9} - \frac{5x^{13}}{96b^2\,(a+bx^2)^8} - \frac{65x^{11}}{1344b^3\,(a+bx^2)^7} - \frac{715x^9}{16128b^4\,(a+bx^2)^6} - \frac{14\cdots}{3584b^5\,(\cdots}$$

$$= -\frac{x^{15}}{18b\,(a+bx^2)^9} - \frac{5x^{13}}{96b^2\,(a+bx^2)^8} - \frac{65x^{11}}{1344b^3\,(a+bx^2)^7} - \frac{715x^9}{16128b^4\,(a+bx^2)^6} - \frac{14\cdots}{3584b^5\,(\cdots}$$

$$= -\frac{x^{15}}{18b\,(a+bx^2)^9} - \frac{5x^{13}}{96b^2\,(a+bx^2)^8} - \frac{65x^{11}}{1344b^3\,(a+bx^2)^7} - \frac{715x^9}{16128b^4\,(a+bx^2)^6} - \frac{14\cdots}{3584b^5\,(\cdots}$$

$$= -\frac{x^{15}}{18b\,(a+bx^2)^9} - \frac{5x^{13}}{96b^2\,(a+bx^2)^8} - \frac{65x^{11}}{1344b^3\,(a+bx^2)^7} - \frac{715x^9}{16128b^4\,(a+bx^2)^6} - \frac{14\cdots}{3584b^5\,(\cdots}$$

Mathematica [A]
time = 0.05, size = 138, normalized size = 0.70

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x\left(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}-985866ab^7x^{14}+45045b^8x^{16}\right)}{(a+bx^2)^9}+45045\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{4128768a^{3/2}b^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^16/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 2633274*
a^2*b^6*x^12 - 985866*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(3/2)*b^(17/2))
```

Maple [A]
time = 0.10, size = 124, normalized size = 0.63

method	result
default	$\frac{-\frac{715a^7x}{65536b^8}-\frac{9295a^6x^3}{98304b^7}-\frac{11869a^5x^5}{32768b^6}-\frac{184327a^4x^7}{229376b^5}-\frac{143a^3x^9}{126b^4}-\frac{241657a^2x^{11}}{229376b^3}-\frac{20899a\,x^{13}}{32768b^2}-\frac{23473x^{15}}{98304b}+\frac{715x^{17}}{65536a}}{\left(b\,x^2+a\right)^9}+\frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a\,b^8\sqrt{ab}}$
risch	$\frac{-\frac{715a^7x}{65536b^8}-\frac{9295a^6x^3}{98304b^7}-\frac{11869a^5x^5}{32768b^6}-\frac{184327a^4x^7}{229376b^5}-\frac{143a^3x^9}{126b^4}-\frac{241657a^2x^{11}}{229376b^3}-\frac{20899a\,x^{13}}{32768b^2}-\frac{23473x^{15}}{98304b}+\frac{715x^{17}}{65536a}}{\left(b\,x^2+a\right)^9}-\frac{715\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] (-715/65536*a^7/b^8*x-9295/98304*a^6/b^7*x^3-11869/32768*a^5/b^6*x^5-184327
/229376*a^4/b^5*x^7-143/126*a^3/b^4*x^9-241657/229376*a^2/b^3*x^11-20899/32
768*a/b^2*x^13-23473/98304/b*x^15+715/65536/a*x^17)/(b*x^2+a)^9+715/65536/a
/b^8/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.69, size = 219, normalized size = 1.11

$$\frac{45045\,b^8x^{17}-985866\,ab^7x^{15}-2633274\,a^2b^6x^{13}-4349826\,a^3b^5x^{11}-4685824\,a^4b^4x^9-3317886\,a^5b^3x^7-1495494\,a^6b^2x^5-390390\,a^7bx^3-45045\,a^8x}{4128768\left(ab^{17}x^{18}+9\,a^2b^{16}x^{16}+36\,a^3b^{15}x^{14}+84\,a^4b^{14}x^{12}+126\,a^5b^{13}x^{10}+126\,a^6b^{12}x^8+84\,a^7b^{11}x^6+36\,a^8b^{10}x^4+9\,a^9b^9x^2+a^{10}b^8\right)}+\frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,ab^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(45045*b^8*x^17 - 985866*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349
826*a^3*b^5*x^11 - 4685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*
b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a*b^17*x^18 + 9*a^2*b^16*x^16 +
36*a^3*b^15*x^14 + 84*a^4*b^14*x^12 + 126*a^5*b^13*x^10 + 126*a^6*b^12*x^8
+ 84*a^7*b^11*x^6 + 36*a^8*b^10*x^4 + 9*a^9*b^9*x^2 + a^10*b^8) + 715/65536
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^8)
```

Fricas [A]
time = 1.85, size = 654, normalized size = 3.30

$$\left[\frac{90090\,ab^9x^{17}-1971732\,a^2b^8x^{15}-5266548\,a^3b^7x^{13}-8699652\,a^4b^6x^{11}-9371648\,a^5b^5x^9-6635772\,a^6b^4x^7-2990988\,a^7b^3x^5-780780\,a^8b^2x^3-90090\,a^9bx-45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{8257536\left(a^2b^{18}x^{18}+9\,a^3b^{17}x^{16}+36\,a^4b^{16}x^{14}+84\,a^5b^{15}x^{12}+126\,a^6b^{14}x^{10}+126\,a^7b^{13}x^8+84\,a^8b^{12}x^6+36\,a^9b^{11}x^4+9\,a^{10}b^{10}x^2+a^{11}b^9\right)},\ \frac{45045\,ab^9x^{17}-985866\,a^2b^8x^{15}-2633274\,a^3b^7x^{13}-4349826\,a^4b^6x^{11}-4685824\,a^5b^5x^9-3317886\,a^6b^4x^7-1495494\,a^7b^3x^5-390390\,a^8b^2x^3-45045\,a^9bx+45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{4128768\left(a^2b^{18}x^{18}+9\,a^3b^{17}x^{16}+36\,a^4b^{16}x^{14}+84\,a^5b^{15}x^{12}+126\,a^6b^{14}x^{10}+126\,a^7b^{13}x^8+84\,a^8b^{12}x^6+36\,a^9b^{11}x^4+9\,a^{10}b^{10}x^2+a^{11}b^9\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(90090*a*b^9*x^17 - 1971732*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
- 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 299098
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^18*x^18 + 9*a^3*b^17*x^
16 + 36*a^4*b^16*x^14 + 84*a^5*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a^7*b^13
*x^8 + 84*a^8*b^12*x^6 + 36*a^9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b^9), 1/4
128768*(45045*a*b^9*x^17 - 985866*a^2*b^8*x^15 - 2633274*a^3*b^7*x^13 - 434
9826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 - 1495494*a^7
*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*x
^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^
8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a))/(a^2*b^18*x^18 + 9*a^3*b^17*x^16 + 36*a^4*b^16*x^14 + 84*a^5
*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a^7*b^13*x^8 + 84*a^8*b^12*x^6 + 36*a^
9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b^9)]
```

Sympy [A]
time = 0.64, size = 289, normalized size = 1.46

$$-\frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(-a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072}+\frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072}+\frac{-45045a^8x-390390a^7bx^3-1495494a^6b^2x^5-3317886a^5b^3x^7-4685824a^4b^4x^9-4349826a^3b^5x^{11}-2633274a^2b^6x^{13}-985866ab^7x^{15}+45045b^8x^{17}}{4128768a^{10}b^8+37158912a^9b^9x^2+148635648a^8b^{10}x^4+346816512a^7b^{11}x^6+520224768a^6b^{12}x^8+520224768a^5b^{13}x^{10}+346816512a^4b^{14}x^{12}+148635648a^3b^{15}x^{14}+37158912a^2b^{16}x^{16}+4128768ab^{17}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**16/(b*x**2+a)**10,x)
```

```
[Out] -715*sqrt(-1/(a**3*b**17))*log(-a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/131072
 + 715*sqrt(-1/(a**3*b**17))*log(a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 3317886*
a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 2633274
*a**2*b**6*x**13 - 985866*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a**10*b
**8 + 37158912*a**9*b**9*x**2 + 148635648*a**8*b**10*x**4 + 346816512*a**7*
b**11*x**6 + 520224768*a**6*b**12*x**8 + 520224768*a**5*b**13*x**10 + 34681
6512*a**4*b**14*x**12 + 148635648*a**3*b**15*x**14 + 37158912*a**2*b**16*x*
*16 + 4128768*a*b**17*x**18)
```

Giac [A]
time = 1.19, size = 128, normalized size = 0.65

$$\frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,ab^8}+\frac{45045\,b^8x^{17}-985866\,ab^7x^{15}-2633274\,a^2b^6x^{13}-4349826\,a^3b^5x^{11}-4685824\,a^4b^4x^9-3317886\,a^5b^3x^7-1495494\,a^6b^2x^5-390390\,a^7bx^3-45045\,a^8x}{4128768\,(bx^2+a)^9ab^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^8) + 1/4128768*(45045*b^8*x^
17 - 985866*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 - 4685
824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*a^7*b*
x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a*b^8)
```

Mupad [B]
time = 4.76, size = 207, normalized size = 1.05

$$\frac{715\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{3/2}\,b^{17/2}}-\frac{\frac{23473\,x^{15}}{98304\,b}-\frac{715\,x^{17}}{65536\,a}+\frac{20899\,a\,x^{13}}{32768\,b^2}+\frac{715\,a^7\,x}{65536\,b^8}+\frac{241657\,a^2\,x^{11}}{229376\,b^3}+\frac{143\,a^3\,x^9}{126\,b^4}+\frac{184327\,a^4\,x^7}{229376\,b^5}+\frac{11869\,a^5\,x^5}{32768\,b^6}+\frac{9295\,a^6\,x^3}{98304\,b^7}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(a + b*x^2)^10,x)
```

```
[Out] (715*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(3/2)*b^(17/2)) - ((23473*x^15)/(9
8304*b) - (715*x^17)/(65536*a) + (20899*a*x^13)/(32768*b^2) + (715*a^7*x)/(
65536*b^8) + (241657*a^2*x^11)/(229376*b^3) + (143*a^3*x^9)/(126*b^4) + (18
4327*a^4*x^7)/(229376*b^5) + (11869*a^5*x^5)/(32768*b^6) + (9295*a^6*x^3)/(
98304*b^7))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 +
 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36
*a^2*b^7*x^14)
```

3.214 $\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=199

$$-\frac{x^{13}}{18b\,(a+bx^2)^9}-\frac{13x^{11}}{288b^2\,(a+bx^2)^8}-\frac{143x^9}{4032b^3\,(a+bx^2)^7}-\frac{143x^7}{5376b^4\,(a+bx^2)^6}-\frac{143x^5}{7680b^5\,(a+bx^2)^5}-\frac{143x^3}{12288b^6\,(a+$$

```
[Out] -1/18*x^13/b/(b*x^2+a)^9-13/288*x^11/b^2/(b*x^2+a)^8-143/4032*x^9/b^3/(b*x^
2+a)^7-143/5376*x^7/b^4/(b*x^2+a)^6-143/7680*x^5/b^5/(b*x^2+a)^5-143/12288*
x^3/b^6/(b*x^2+a)^4-143/24576*x/b^7/(b*x^2+a)^3+143/98304*x/a/b^7/(b*x^2+a)
^2+143/65536*x/a^2/b^7/(b*x^2+a)+143/65536*arctan(x*b^(1/2)/a^(1/2))/a^(5/2
)/b^(15/2)
```

Rubi [A]
time = 0.08, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 205, 211}

$$\frac{143\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}}+\frac{143x}{65536a^2b^7\,(a+bx^2)}+\frac{143x}{98304ab^7\,(a+bx^2)^2}-\frac{143x}{24576b^7\,(a+bx^2)^3}-\frac{143x^3}{12288b^6\,(a+bx^2)^4}-\frac{143x^5}{7680b^5\,(a+bx^2)^5}-\frac{143x^7}{5376b^4\,(a+bx^2)^6}-\frac{143x^9}{4032b^3\,(a+bx^2)^7}-\frac{13x^{11}}{288b^2\,(a+bx^2)^8}-\frac{x^{13}}{18b\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^2)^10,x]
```

```
[Out] -1/18*x^13/(b*(a + b*x^2)^9) - (13*x^11)/(288*b^2*(a + b*x^2)^8) - (143*x^9
)/(4032*b^3*(a + b*x^2)^7) - (143*x^7)/(5376*b^4*(a + b*x^2)^6) - (143*x^5)
/(7680*b^5*(a + b*x^2)^5) - (143*x^3)/(12288*b^6*(a + b*x^2)^4) - (143*x)/(
24576*b^7*(a + b*x^2)^3) + (143*x)/(98304*a*b^7*(a + b*x^2)^2) + (143*x)/(6
5536*a^2*b^7*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(5/2
)*b^(15/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx = -\frac{x^{13}}{18b\,(a+bx^2)^9} + \frac{13\int \frac{x^{12}}{(a+bx^2)^9}\,dx}{18b}$$

$$= -\frac{x^{13}}{18b\,(a+bx^2)^9} - \frac{13x^{11}}{288b^2\,(a+bx^2)^8} + \frac{143\int \frac{x^{10}}{(a+bx^2)^8}\,dx}{288b^2}$$

$$= -\frac{x^{13}}{18b\,(a+bx^2)^9} - \frac{13x^{11}}{288b^2\,(a+bx^2)^8} - \frac{143x^9}{4032b^3\,(a+bx^2)^7} + \frac{143\int \frac{x^8}{(a+bx^2)^7}\,dx}{448b^3}$$

$$= -\frac{x^{13}}{18b\,(a+bx^2)^9} - \frac{13x^{11}}{288b^2\,(a+bx^2)^8} - \frac{143x^9}{4032b^3\,(a+bx^2)^7} - \frac{143x^7}{5376b^4\,(a+bx^2)^6} + \frac{143\int \cdots}{7\cdots}$$

$$= -\frac{x^{13}}{18b\,(a+bx^2)^9} - \frac{13x^{11}}{288b^2\,(a+bx^2)^8} - \frac{143x^9}{4032b^3\,(a+bx^2)^7} - \frac{143x^7}{5376b^4\,(a+bx^2)^6} - \frac{1\cdots}{7680b^5\cdots}$$

$$= -\frac{x^{13}}{18b\,(a+bx^2)^9} - \frac{13x^{11}}{288b^2\,(a+bx^2)^8} - \frac{143x^9}{4032b^3\,(a+bx^2)^7} - \frac{143x^7}{5376b^4\,(a+bx^2)^6} - \frac{1\cdots}{7680b^5\cdots}$$

$$= -\frac{x^{13}}{18b\,(a+bx^2)^9} - \frac{13x^{11}}{288b^2\,(a+bx^2)^8} - \frac{143x^9}{4032b^3\,(a+bx^2)^7} - \frac{143x^7}{5376b^4\,(a+bx^2)^6} - \frac{1\cdots}{7680b^5\cdots}$$

$$= -\frac{x^{13}}{18b\,(a+bx^2)^9} - \frac{13x^{11}}{288b^2\,(a+bx^2)^8} - \frac{143x^9}{4032b^3\,(a+bx^2)^7} - \frac{143x^7}{5376b^4\,(a+bx^2)^6} - \frac{1\cdots}{7680b^5\cdots}$$

$$= -\frac{x^{13}}{18b\,(a+bx^2)^9} - \frac{13x^{11}}{288b^2\,(a+bx^2)^8} - \frac{143x^9}{4032b^3\,(a+bx^2)^7} - \frac{143x^7}{5376b^4\,(a+bx^2)^6} - \frac{1\cdots}{7680b^5\cdots}$$

$$= -\frac{x^{13}}{18b\,(a+bx^2)^9} - \frac{13x^{11}}{288b^2\,(a+bx^2)^8} - \frac{143x^9}{4032b^3\,(a+bx^2)^7} - \frac{143x^7}{5376b^4\,(a+bx^2)^6} - \frac{1\cdots}{7680b^5\cdots}$$

Mathematica [A]
time = 0.04, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x\left(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16}\right)}{(a+bx^2)^9}+45045\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{20643840a^{5/2}b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 2633274*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(5/2)*b^(15/2))
```

Maple [A]
time = 0.11, size = 122, normalized size = 0.61

method	result
default	$\frac{-\frac{143a^{6}x}{65536b^{7}}-\frac{1859a^{5}x^{3}}{98304b^{6}}-\frac{11869a^{4}x^{5}}{163840b^{5}}-\frac{184327a^{3}x^{7}}{1146880b^{4}}-\frac{143a^{2}x^{9}}{630b^{3}}-\frac{241657a\,x^{11}}{1146880b^{2}}-\frac{20899x^{13}}{163840b}+\frac{1859x^{15}}{98304a}+\frac{143b\,x^{17}}{65536a^{2}}}{\left(b\,x^{2}+a\right)^{9}}+\frac{143\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^{2}b^{7}\sqrt{ab}}$
risch	$\frac{-\frac{143a^{6}x}{65536b^{7}}-\frac{1859a^{5}x^{3}}{98304b^{6}}-\frac{11869a^{4}x^{5}}{163840b^{5}}-\frac{184327a^{3}x^{7}}{1146880b^{4}}-\frac{143a^{2}x^{9}}{630b^{3}}-\frac{241657a\,x^{11}}{1146880b^{2}}-\frac{20899x^{13}}{163840b}+\frac{1859x^{15}}{98304a}+\frac{143b\,x^{17}}{65536a^{2}}}{\left(b\,x^{2}+a\right)^{9}}-\frac{143\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^{7}a^{2}}+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] (-143/65536*a^6/b^7*x-1859/98304*a^5/b^6*x^3-11869/163840*a^4/b^5*x^5-18432
7/1146880*a^3/b^4*x^7-143/630*a^2/b^3*x^9-241657/1146880*a/b^2*x^11-20899/1
63840/b*x^13+1859/98304/a*x^15+143/65536*b/a^2*x^17)/(b*x^2+a)^9+143/65536/
a^2/b^7/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 221, normalized size = 1.11

$$\frac{45045\,b^{8}x^{17}+390390\,ab^{7}x^{15}-2633274\,a^{2}b^{6}x^{13}-4349826\,a^{3}b^{5}x^{11}-4685824\,a^{4}b^{4}x^{9}-3317886\,a^{5}b^{3}x^{7}-1495494\,a^{6}b^{2}x^{5}-390390\,a^{7}bx^{3}-45045\,a^{8}x}{20643840\,(a^{2}b^{16}x^{18}+9\,a^{3}b^{15}x^{16}+36\,a^{4}b^{14}x^{14}+84\,a^{5}b^{13}x^{12}+126\,a^{6}b^{12}x^{10}+126\,a^{7}b^{11}x^{8}+84\,a^{8}b^{10}x^{6}+36\,a^{9}b^{9}x^{4}+9\,a^{10}b^{8}x^{2}+a^{11}b^{7})}+\frac{143\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{2}b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20643840*(45045*b^8*x^17 + 390390*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 434
9826*a^3*b^5*x^11 - 4685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6
*b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a^2*b^16*x^18 + 9*a^3*b^15*x^16
 + 36*a^4*b^14*x^14 + 84*a^5*b^13*x^12 + 126*a^6*b^12*x^10 + 126*a^7*b^11*x
^8 + 84*a^8*b^10*x^6 + 36*a^9*b^9*x^4 + 9*a^10*b^8*x^2 + a^11*b^7) + 143/65
536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^7)
```

Fricas [A]
time = 1.12, size = 654, normalized size = 3.29

$$\left[\frac{90090\,ab^{9}x^{17}+780780\,a^{2}b^{8}x^{15}-5266548\,a^{3}b^{7}x^{13}-8699652\,a^{4}b^{6}x^{11}-9371648\,a^{5}b^{5}x^{9}-6635772\,a^{6}b^{4}x^{7}-2990988\,a^{7}b^{3}x^{5}-780780\,a^{8}b^{2}x^{3}-90090\,a^{9}bx-45045\,(b^{9}x^{18}+9\,ab^{8}x^{16}+36\,a^{2}b^{7}x^{14}+84\,a^{3}b^{6}x^{12}+126\,a^{4}b^{5}x^{10}+126\,a^{5}b^{4}x^{8}+84\,a^{6}b^{3}x^{6}+36\,a^{7}b^{2}x^{4}+9\,a^{8}bx^{2}+a^{9})\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{41287680\,(a^{3}b^{17}x^{18}+9\,a^{4}b^{16}x^{16}+36\,a^{5}b^{15}x^{14}+84\,a^{6}b^{14}x^{12}+126\,a^{7}b^{13}x^{10}+126\,a^{8}b^{12}x^{8}+84\,a^{9}b^{11}x^{6}+36\,a^{10}b^{10}x^{4}+9\,a^{11}b^{9}x^{2}+a^{12}b^{8})},\ \frac{45045\,ab^{9}x^{17}+390390\,a^{2}b^{8}x^{15}-2633274\,a^{3}b^{7}x^{13}-4349826\,a^{4}b^{6}x^{11}-4685824\,a^{5}b^{5}x^{9}-3317886\,a^{6}b^{4}x^{7}-1495494\,a^{7}b^{3}x^{5}-390390\,a^{8}b^{2}x^{3}-45045\,a^{9}bx+45045\,(b^{9}x^{18}+9\,ab^{8}x^{16}+36\,a^{2}b^{7}x^{14}+84\,a^{3}b^{6}x^{12}+126\,a^{4}b^{5}x^{10}+126\,a^{5}b^{4}x^{8}+84\,a^{6}b^{3}x^{6}+36\,a^{7}b^{2}x^{4}+9\,a^{8}bx^{2}+a^{9})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{20643840\,(a^{3}b^{17}x^{18}+9\,a^{4}b^{16}x^{16}+36\,a^{5}b^{15}x^{14}+84\,a^{6}b^{14}x^{12}+126\,a^{7}b^{13}x^{10}+126\,a^{8}b^{12}x^{8}+84\,a^{9}b^{11}x^{6}+36\,a^{10}b^{10}x^{4}+9\,a^{11}b^{9}x^{2}+a^{12}b^{8})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
- 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 299098
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^17*x^18 + 9*a^4*b^16*x^
16 + 36*a^5*b^15*x^14 + 84*a^6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a^8*b^12
*x^8 + 84*a^9*b^11*x^6 + 36*a^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b^8), 1/2
0643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 - 2633274*a^3*b^7*x^13 - 43
49826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 - 1495494*a^
7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*
x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x
^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a))/(a^3*b^17*x^18 + 9*a^4*b^16*x^16 + 36*a^5*b^15*x^14 + 84*a^
6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a^8*b^12*x^8 + 84*a^9*b^11*x^6 + 36*a
^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b^8)]
```

Sympy [A]
time = 0.61, size = 291, normalized size = 1.46

$$-\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(-a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072}+\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072}+\frac{-45045a^8x-390390a^7bx^3-1495494a^6b^2x^5-3317886a^5b^3x^7-4685824a^4b^4x^9-4349826a^3b^5x^{11}-2633274a^2b^6x^{13}+390390ab^7x^{15}+45045b^8x^{17}}{20643840a^{11}b^7+185794560a^{10}b^8x^2+743178240a^9b^9x^4+1734082560a^8b^{10}x^6+2601123840a^7b^{11}x^8+2601123840a^6b^{12}x^{10}+1734082560a^5b^{13}x^{12}+743178240a^4b^{14}x^{14}+185794560a^3b^{15}x^{16}+20643840a^2b^{16}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**2+a)**10,x)
```

```
[Out] -143*sqrt(-1/(a**5*b**15))*log(-a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/131072
 + 143*sqrt(-1/(a**5*b**15))*log(a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 3317886*
a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 2633274
*a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a**11*
b**7 + 185794560*a**10*b**8*x**2 + 743178240*a**9*b**9*x**4 + 1734082560*a*
*8*b**10*x**6 + 2601123840*a**7*b**11*x**8 + 2601123840*a**6*b**12*x**10 +
1734082560*a**5*b**13*x**12 + 743178240*a**4*b**14*x**14 + 185794560*a**3*b
**15*x**16 + 20643840*a**2*b**16*x**18)
```

Giac [A]
time = 1.48, size = 128, normalized size = 0.64

$$\frac{143\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^2b^7}+\frac{45045\,b^8x^{17}+390390\,ab^7x^{15}-2633274\,a^2b^6x^{13}-4349826\,a^3b^5x^{11}-4685824\,a^4b^4x^9-3317886\,a^5b^3x^7-1495494\,a^6b^2x^5-390390\,a^7bx^3-45045\,a^8x}{20643840\,(bx^2+a)^9a^2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^7) + 1/20643840*(45045*b^8
*x^17 + 390390*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 - 4
685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^2*b^7)
```

Mupad [B]
time = 4.75, size = 205, normalized size = 1.03

$$\frac{143\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{5/2}\,b^{15/2}}-\frac{\frac{20899\,x^{13}}{163840\,b}-\frac{1859\,x^{15}}{98304\,a}+\frac{241657\,a\,x^{11}}{1146880\,b^2}+\frac{143\,a^6\,x}{65536\,b^7}-\frac{143\,b\,x^{17}}{65536\,a^2}+\frac{143\,a^2\,x^9}{630\,b^3}+\frac{184327\,a^3\,x^7}{1146880\,b^4}+\frac{11869\,a^4\,x^5}{163840\,b^5}+\frac{1859\,a^5\,x^3}{98304\,b^6}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^2)^10,x)
```

```
[Out] (143*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(5/2)*b^(15/2)) - ((20899*x^13)/(1
63840*b) - (1859*x^15)/(98304*a) + (241657*a*x^11)/(1146880*b^2) + (143*a^6
*x)/(65536*b^7) - (143*b*x^17)/(65536*a^2) + (143*a^2*x^9)/(630*b^3) + (184
327*a^3*x^7)/(1146880*b^4) + (11869*a^4*x^5)/(163840*b^5) + (1859*a^5*x^3)/
(98304*b^6))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4
+ 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 3
6*a^2*b^7*x^14)
```

3.215 $\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=200

$$-\frac{x^{11}}{18b\,(a+bx^2)^9}-\frac{11x^9}{288b^2\,(a+bx^2)^8}-\frac{11x^7}{448b^3\,(a+bx^2)^7}-\frac{11x^5}{768b^4\,(a+bx^2)^6}-\frac{11x^3}{1536b^5\,(a+bx^2)^5}-\frac{11x}{4096b^6\,(a+bx}$$

```
[Out] -1/18*x^11/b/(b*x^2+a)^9-11/288*x^9/b^2/(b*x^2+a)^8-11/448*x^7/b^3/(b*x^2+a
)^7-11/768*x^5/b^4/(b*x^2+a)^6-11/1536*x^3/b^5/(b*x^2+a)^5-11/4096*x/b^6/(b
*x^2+a)^4+11/24576*x/a/b^6/(b*x^2+a)^3+55/98304*x/a^2/b^6/(b*x^2+a)^2+55/65
536*x/a^3/b^6/(b*x^2+a)+55/65536*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(13/2)
```

Rubi [A]
time = 0.08, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 205, 211}

$$\frac{55\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}}+\frac{55x}{65536a^3b^6\,(a+bx^2)}+\frac{55x}{98304a^2b^6\,(a+bx^2)^2}+\frac{11x}{24576ab^6\,(a+bx^2)^3}-\frac{11x}{4096b^6\,(a+bx^2)^4}-\frac{11x^3}{1536b^5\,(a+bx^2)^5}-\frac{11x^5}{768b^4\,(a+bx^2)^6}-\frac{11x^7}{448b^3\,(a+bx^2)^7}-\frac{11x^9}{288b^2\,(a+bx^2)^8}-\frac{x^{11}}{18b\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^10,x]
```

```
[Out] -1/18*x^11/(b*(a + b*x^2)^9) - (11*x^9)/(288*b^2*(a + b*x^2)^8) - (11*x^7)/
(448*b^3*(a + b*x^2)^7) - (11*x^5)/(768*b^4*(a + b*x^2)^6) - (11*x^3)/(1536
*b^5*(a + b*x^2)^5) - (11*x)/(4096*b^6*(a + b*x^2)^4) + (11*x)/(24576*a*b^6
*(a + b*x^2)^3) + (55*x)/(98304*a^2*b^6*(a + b*x^2)^2) + (55*x)/(65536*a^3*
b^6*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(7/2)*b^(13/2)
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx = -\frac{x^{11}}{18b\,(a+bx^2)^9} + \frac{11\int \frac{x^{10}}{(a+bx^2)^9}\,dx}{18b}$$

$$= -\frac{x^{11}}{18b\,(a+bx^2)^9} - \frac{11x^9}{288b^2\,(a+bx^2)^8} + \frac{11\int \frac{x^8}{(a+bx^2)^8}\,dx}{32b^2}$$

$$= -\frac{x^{11}}{18b\,(a+bx^2)^9} - \frac{11x^9}{288b^2\,(a+bx^2)^8} - \frac{11x^7}{448b^3\,(a+bx^2)^7} + \frac{11\int \frac{x^6}{(a+bx^2)^7}\,dx}{64b^3}$$

$$= -\frac{x^{11}}{18b\,(a+bx^2)^9} - \frac{11x^9}{288b^2\,(a+bx^2)^8} - \frac{11x^7}{448b^3\,(a+bx^2)^7} - \frac{11x^5}{768b^4\,(a+bx^2)^6} + \frac{55\int \frac{x^4}{(a+bx^2\cdots}}{768b^4\cdots}$$

$$= -\frac{x^{11}}{18b\,(a+bx^2)^9} - \frac{11x^9}{288b^2\,(a+bx^2)^8} - \frac{11x^7}{448b^3\,(a+bx^2)^7} - \frac{11x^5}{768b^4\,(a+bx^2)^6} - \frac{11x\cdots}{1536b^5\,(a\cdots}$$

$$= -\frac{x^{11}}{18b\,(a+bx^2)^9} - \frac{11x^9}{288b^2\,(a+bx^2)^8} - \frac{11x^7}{448b^3\,(a+bx^2)^7} - \frac{11x^5}{768b^4\,(a+bx^2)^6} - \frac{11x\cdots}{1536b^5\,(a\cdots}$$

$$= -\frac{x^{11}}{18b\,(a+bx^2)^9} - \frac{11x^9}{288b^2\,(a+bx^2)^8} - \frac{11x^7}{448b^3\,(a+bx^2)^7} - \frac{11x^5}{768b^4\,(a+bx^2)^6} - \frac{11x\cdots}{1536b^5\,(a\cdots}$$

$$= -\frac{x^{11}}{18b\,(a+bx^2)^9} - \frac{11x^9}{288b^2\,(a+bx^2)^8} - \frac{11x^7}{448b^3\,(a+bx^2)^7} - \frac{11x^5}{768b^4\,(a+bx^2)^6} - \frac{11x\cdots}{1536b^5\,(a\cdots}$$

$$= -\frac{x^{11}}{18b\,(a+bx^2)^9} - \frac{11x^9}{288b^2\,(a+bx^2)^8} - \frac{11x^7}{448b^3\,(a+bx^2)^7} - \frac{11x^5}{768b^4\,(a+bx^2)^6} - \frac{11x\cdots}{1536b^5\,(a\cdots}$$

$$= -\frac{x^{11}}{18b\,(a+bx^2)^9} - \frac{11x^9}{288b^2\,(a+bx^2)^8} - \frac{11x^7}{448b^3\,(a+bx^2)^7} - \frac{11x^5}{768b^4\,(a+bx^2)^6} - \frac{11x\cdots}{1536b^5\,(a\cdots}$$

Mathematica [A]
time = 0.05, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x\left(-3465a^8-30030a^7bx^2-115038a^6b^2x^4-255222a^5b^3x^6-360448a^4b^4x^8-334602a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16}\right)}{(a+bx^2)^9}+3465\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{4128768a^{7/2}b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 - 255
222*a^5*b^3*x^6 - 360448*a^4*b^4*x^8 - 334602*a^3*b^5*x^10 + 115038*a^2*b^6
*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(4128768*a^(7/2)*b^(13/2))
```

Maple [A]
time = 0.10, size = 122, normalized size = 0.61

method	result
default	$\frac{-\frac{55a^5x}{65536b^6}-\frac{715a^4x^3}{98304b^5}-\frac{913a^3x^5}{32768b^4}-\frac{14179a^2x^7}{229376b^3}-\frac{11a\,x^9}{126b^2}-\frac{18589x^{11}}{229376b}+\frac{913x^{13}}{32768a}+\frac{715b\,x^{15}}{98304a^2}+\frac{55b^2x^{17}}{65536a^3}}{(b\,x^2+a)^9}+\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^3b^6\sqrt{ab}}$
risch	$\frac{-\frac{55a^5x}{65536b^6}-\frac{715a^4x^3}{98304b^5}-\frac{913a^3x^5}{32768b^4}-\frac{14179a^2x^7}{229376b^3}-\frac{11a\,x^9}{126b^2}-\frac{18589x^{11}}{229376b}+\frac{913x^{13}}{32768a}+\frac{715b\,x^{15}}{98304a^2}+\frac{55b^2x^{17}}{65536a^3}}{(b\,x^2+a)^9}-\frac{55\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^6a^3}+\frac{55\ln\left(-bx\right.}{131072\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] (-55/65536*a^5/b^6*x-715/98304*a^4/b^5*x^3-913/32768*a^3/b^4*x^5-14179/2293
76*a^2/b^3*x^7-11/126*a/b^2*x^9-18589/229376/b*x^11+913/32768/a*x^13+715/98
304*b/a^2*x^15+55/65536*b^2/a^3*x^17)/(b*x^2+a)^9+55/65536/a^3/b^6/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.52, size = 221, normalized size = 1.10

$$\frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}-334602\,a^3b^5x^{11}-360448\,a^4b^4x^9-255222\,a^5b^3x^7-115038\,a^6b^2x^5-30030\,a^7bx^3-3465\,a^8x}{4128768\,(a^3b^{15}x^{18}+9\,a^4b^{14}x^{16}+36\,a^5b^{13}x^{14}+84\,a^6b^{12}x^{12}+126\,a^7b^{11}x^{10}+126\,a^8b^{10}x^8+84\,a^9b^9x^6+36\,a^{10}b^8x^4+9\,a^{11}b^7x^2+a^{12}b^6)}+\frac{55\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(3465*b^8*x^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 - 334602*
a^3*b^5*x^11 - 360448*a^4*b^4*x^9 - 255222*a^5*b^3*x^7 - 115038*a^6*b^2*x^5
 - 30030*a^7*b*x^3 - 3465*a^8*x)/(a^3*b^15*x^18 + 9*a^4*b^14*x^16 + 36*a^5*
b^13*x^14 + 84*a^6*b^12*x^12 + 126*a^7*b^11*x^10 + 126*a^8*b^10*x^8 + 84*a^
9*b^9*x^6 + 36*a^10*b^8*x^4 + 9*a^11*b^7*x^2 + a^12*b^6) + 55/65536*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^6)
```

Fricas [A]
time = 1.54, size = 654, normalized size = 3.27

$$\left[\frac{6930\,ab^9x^{17}+60060\,a^2b^8x^{15}+230076\,a^3b^7x^{13}-669204\,a^4b^6x^{11}-720896\,a^5b^5x^9-510444\,a^6b^4x^7-230076\,a^7b^3x^5-60060\,a^8b^2x^3-6930\,a^9bx-3465\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{8257536\,(a^4b^{16}x^{18}+9\,a^5b^{15}x^{16}+36\,a^6b^{14}x^{14}+84\,a^7b^{13}x^{12}+126\,a^8b^{12}x^{10}+126\,a^9b^{11}x^8+84\,a^{10}b^{10}x^6+36\,a^{11}b^9x^4+9\,a^{12}b^8x^2+a^{13}b^7)},\ \frac{3465\,ab^9x^{17}+30030\,a^2b^8x^{15}+115038\,a^3b^7x^{13}-334602\,a^4b^6x^{11}-360448\,a^5b^5x^9-255222\,a^6b^4x^7-115038\,a^7b^3x^5-30030\,a^8b^2x^3-3465\,a^9bx+3465\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{4128768\,(a^4b^{16}x^{18}+9\,a^5b^{15}x^{16}+36\,a^6b^{14}x^{14}+84\,a^7b^{13}x^{12}+126\,a^8b^{12}x^{10}+126\,a^9b^{11}x^8+84\,a^{10}b^{10}x^6+36\,a^{11}b^9x^4+9\,a^{12}b^8x^2+a^{13}b^7)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 - 66
9204*a^4*b^6*x^11 - 720896*a^5*b^5*x^9 - 510444*a^6*b^4*x^7 - 230076*a^7*b^
3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16 +
36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84
*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^16*x^18 + 9*a^5*b^15*x^16 + 36*a^6
*b^14*x^14 + 84*a^7*b^13*x^12 + 126*a^8*b^12*x^10 + 126*a^9*b^11*x^8 + 84*a
^10*b^10*x^6 + 36*a^11*b^9*x^4 + 9*a^12*b^8*x^2 + a^13*b^7), 1/4128768*(346
5*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 - 334602*a^4*b^6*x^
11 - 360448*a^5*b^5*x^9 - 255222*a^6*b^4*x^7 - 115038*a^7*b^3*x^5 - 30030*a
^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14
 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 +
36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b
^16*x^18 + 9*a^5*b^15*x^16 + 36*a^6*b^14*x^14 + 84*a^7*b^13*x^12 + 126*a^8*
b^12*x^10 + 126*a^9*b^11*x^8 + 84*a^10*b^10*x^6 + 36*a^11*b^9*x^4 + 9*a^12*
b^8*x^2 + a^13*b^7)]
```

Sympy [A]
time = 0.57, size = 291, normalized size = 1.46

$$-\frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(-a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072}+\frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072}+\frac{-3465a^8x-30030a^7bx^3-115038a^6b^2x^5-255222a^5b^3x^7-360448a^4b^4x^9-334602a^3b^5x^{11}+115038a^2b^6x^{13}+30030ab^7x^{15}+3465b^8x^{17}}{4128768a^{12}b^6+37158912a^{11}b^7x^2+148635648a^{10}b^8x^4+346816512a^9b^9x^6+520224768a^8b^{10}x^8+520224768a^7b^{11}x^{10}+346816512a^6b^{12}x^{12}+148635648a^5b^{13}x^{14}+37158912a^4b^{14}x^{16}+4128768a^3b^{15}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**10,x)
```

```
[Out] -55*sqrt(-1/(a**7*b**13))*log(-a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/131072
+ 55*sqrt(-1/(a**7*b**13))*log(a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/131072
+ (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 - 255222*a**5*b
**3*x**7 - 360448*a**4*b**4*x**9 - 334602*a**3*b**5*x**11 + 115038*a**2*b**
6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**12*b**6 + 37158
912*a**11*b**7*x**2 + 148635648*a**10*b**8*x**4 + 346816512*a**9*b**9*x**6
+ 520224768*a**8*b**10*x**8 + 520224768*a**7*b**11*x**10 + 346816512*a**6*b
**12*x**12 + 148635648*a**5*b**13*x**14 + 37158912*a**4*b**14*x**16 + 41287
68*a**3*b**15*x**18)
```

Giac [A]
time = 1.24, size = 128, normalized size = 0.64

$$\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^3b^6}+\frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}-334602\,a^3b^5x^{11}-360448\,a^4b^4x^9-255222\,a^5b^3x^7-115038\,a^6b^2x^5-30030\,a^7bx^3-3465\,a^8x}{4128768\,(bx^2+a)^9a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^6) + 1/4128768*(3465*b^8*x^
17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 - 334602*a^3*b^5*x^11 - 360448*
a^4*b^4*x^9 - 255222*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3 - 3
465*a^8*x)/((b*x^2 + a)^9*a^3*b^6)
```

Mupad [B]
time = 0.15, size = 205, normalized size = 1.02

$$\frac{55\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{7/2}\,b^{13/2}}-\frac{\frac{18589\,x^{11}}{229376\,b}-\frac{913\,x^{13}}{32768\,a}+\frac{11\,a\,x^9}{126\,b^2}+\frac{55\,a^5\,x}{65536\,b^6}-\frac{715\,b\,x^{15}}{98304\,a^2}+\frac{14179\,a^2\,x^7}{229376\,b^3}+\frac{913\,a^3\,x^5}{32768\,b^4}+\frac{715\,a^4\,x^3}{98304\,b^5}-\frac{55\,b^2\,x^{17}}{65536\,a^3}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^2)^10,x)
```

```
[Out] (55*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(7/2)*b^(13/2)) - ((18589*x^11)/(22
9376*b) - (913*x^13)/(32768*a) + (11*a*x^9)/(126*b^2) + (55*a^5*x)/(65536*b
^6) - (715*b*x^15)/(98304*a^2) + (14179*a^2*x^7)/(229376*b^3) + (913*a^3*x^
5)/(32768*b^4) + (715*a^4*x^3)/(98304*b^5) - (55*b^2*x^17)/(65536*a^3))/(a^
9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6
 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

3.216 $\int \frac{x^{10}}{\left(a+bx^2\right)^{10}} \, dx$

Optimal. Leaf size=201

$$-\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}-\frac{x}{512b^5\left(a+bx^2\right)^5}+\frac{x}{4096ab^5\left(a+bx^2\right)^4}$$

```
[Out] -1/18*x^9/b/(b*x^2+a)^9-1/32*x^7/b^2/(b*x^2+a)^8-1/64*x^5/b^3/(b*x^2+a)^7-5
/768*x^3/b^4/(b*x^2+a)^6-1/512*x/b^5/(b*x^2+a)^5+1/4096*x/a/b^5/(b*x^2+a)^4
+7/24576*x/a^2/b^5/(b*x^2+a)^3+35/98304*x/a^3/b^5/(b*x^2+a)^2+35/65536*x/a^
4/b^5/(b*x^2+a)+35/65536*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(11/2)
```

Rubi [A]
time = 0.08, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 205, 211}

$$\frac{35\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}}+\frac{35x}{65536a^4b^5\left(a+bx^2\right)}+\frac{35x}{98304a^3b^5\left(a+bx^2\right)^2}+\frac{7x}{24576a^2b^5\left(a+bx^2\right)^3}+\frac{x}{4096ab^5\left(a+bx^2\right)^4}-\frac{x}{512b^5\left(a+bx^2\right)^5}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^9}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^10,x]
```

```
[Out] -1/18*x^9/(b*(a + b*x^2)^9) - x^7/(32*b^2*(a + b*x^2)^8) - x^5/(64*b^3*(a +
 b*x^2)^7) - (5*x^3)/(768*b^4*(a + b*x^2)^6) - x/(512*b^5*(a + b*x^2)^5) +
x/(4096*a*b^5*(a + b*x^2)^4) + (7*x)/(24576*a^2*b^5*(a + b*x^2)^3) + (35*x)
/(98304*a^3*b^5*(a + b*x^2)^2) + (35*x)/(65536*a^4*b^5*(a + b*x^2)) + (35*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(9/2)*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx = -\frac{x^9}{18b\,(a+bx^2)^9} + \frac{\int \frac{x^8}{(a+bx^2)^9}\,dx}{2b}$$

$$= -\frac{x^9}{18b\,(a+bx^2)^9} - \frac{x^7}{32b^2\,(a+bx^2)^8} + \frac{7\int \frac{x^6}{(a+bx^2)^8}\,dx}{32b^2}$$

$$= -\frac{x^9}{18b\,(a+bx^2)^9} - \frac{x^7}{32b^2\,(a+bx^2)^8} - \frac{x^5}{64b^3\,(a+bx^2)^7} + \frac{5\int \frac{x^4}{(a+bx^2)^7}\,dx}{64b^3}$$

$$= -\frac{x^9}{18b\,(a+bx^2)^9} - \frac{x^7}{32b^2\,(a+bx^2)^8} - \frac{x^5}{64b^3\,(a+bx^2)^7} - \frac{5x^3}{768b^4\,(a+bx^2)^6} + \frac{5\int \frac{x^2}{(a+bx^2)^6}}{256b^4}$$

$$= -\frac{x^9}{18b\,(a+bx^2)^9} - \frac{x^7}{32b^2\,(a+bx^2)^8} - \frac{x^5}{64b^3\,(a+bx^2)^7} - \frac{5x^3}{768b^4\,(a+bx^2)^6} - \frac{x}{512b^5\,(a+}$$

$$= -\frac{x^9}{18b\,(a+bx^2)^9} - \frac{x^7}{32b^2\,(a+bx^2)^8} - \frac{x^5}{64b^3\,(a+bx^2)^7} - \frac{5x^3}{768b^4\,(a+bx^2)^6} - \frac{x}{512b^5\,(a+}$$

$$= -\frac{x^9}{18b\,(a+bx^2)^9} - \frac{x^7}{32b^2\,(a+bx^2)^8} - \frac{x^5}{64b^3\,(a+bx^2)^7} - \frac{5x^3}{768b^4\,(a+bx^2)^6} - \frac{x}{512b^5\,(a+}$$

$$= -\frac{x^9}{18b\,(a+bx^2)^9} - \frac{x^7}{32b^2\,(a+bx^2)^8} - \frac{x^5}{64b^3\,(a+bx^2)^7} - \frac{5x^3}{768b^4\,(a+bx^2)^6} - \frac{x}{512b^5\,(a+}$$

$$= -\frac{x^9}{18b\,(a+bx^2)^9} - \frac{x^7}{32b^2\,(a+bx^2)^8} - \frac{x^5}{64b^3\,(a+bx^2)^7} - \frac{5x^3}{768b^4\,(a+bx^2)^6} - \frac{x}{512b^5\,(a+}$$

$$= -\frac{x^9}{18b\,(a+bx^2)^9} - \frac{x^7}{32b^2\,(a+bx^2)^8} - \frac{x^5}{64b^3\,(a+bx^2)^7} - \frac{5x^3}{768b^4\,(a+bx^2)^6} - \frac{x}{512b^5\,(a+}$$

Mathematica [A]
time = 0.05, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x\left(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6-32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16}\right)}{(a+bx^2)^9}+315\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{589824a^{9/2}b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202*
a^5*b^3*x^6 - 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12 +
 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(589824*a^(9/2)*b^(11/2))
```

Maple [A]
time = 0.10, size = 122, normalized size = 0.61

method	result
default	$\frac{-\frac{35a^4x}{65536b^5}-\frac{455a^3x^3}{98304b^4}-\frac{581a^2x^5}{32768b^3}-\frac{1289a\,x^7}{32768b^2}-\frac{x^9}{18b}+\frac{1289x^{11}}{32768a}+\frac{581b\,x^{13}}{32768a^2}+\frac{455b^2x^{15}}{98304a^3}+\frac{35b^3x^{17}}{65536a^4}}{(b\,x^2+a)^9}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^4b^5\sqrt{ab}}$
risch	$\frac{-\frac{35a^4x}{65536b^5}-\frac{455a^3x^3}{98304b^4}-\frac{581a^2x^5}{32768b^3}-\frac{1289a\,x^7}{32768b^2}-\frac{x^9}{18b}+\frac{1289x^{11}}{32768a}+\frac{581b\,x^{13}}{32768a^2}+\frac{455b^2x^{15}}{98304a^3}+\frac{35b^3x^{17}}{65536a^4}}{(b\,x^2+a)^9}-\frac{35\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^5a^4}+\frac{35\ln\left(-bx+\sqrt{\cdots}\right.}{131072\sqrt{-a\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] (-35/65536*a^4*x/b^5-455/98304*a^3*x^3/b^4-581/32768*a^2*x^5/b^3-1289/32768
*a*x^7/b^2-1/18*x^9/b+1289/32768/a*x^11+581/32768*b/a^2*x^13+455/98304*b^2/
a^3*x^15+35/65536*b^3/a^4*x^17)/(b*x^2+a)^9+35/65536/a^4/b^5/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.53, size = 221, normalized size = 1.10

$$\frac{315\,b^8x^{17}+2730\,ab^7x^{15}+10458\,a^2b^6x^{13}+23202\,a^3b^5x^{11}-32768\,a^4b^4x^9-23202\,a^5b^3x^7-10458\,a^6b^2x^5-2730\,a^7bx^3-315\,a^8x}{589824\,(a^4b^{14}x^{18}+9\,a^5b^{13}x^{16}+36\,a^6b^{12}x^{14}+84\,a^7b^{11}x^{12}+126\,a^8b^{10}x^{10}+126\,a^9b^9x^8+84\,a^{10}b^8x^6+36\,a^{11}b^7x^4+9\,a^{12}b^6x^2+a^{13}b^5)}+\frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/589824*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b
^5*x^11 - 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*
a^7*b*x^3 - 315*a^8*x)/(a^4*b^14*x^18 + 9*a^5*b^13*x^16 + 36*a^6*b^12*x^14
+ 84*a^7*b^11*x^12 + 126*a^8*b^10*x^10 + 126*a^9*b^9*x^8 + 84*a^10*b^8*x^6
+ 36*a^11*b^7*x^4 + 9*a^12*b^6*x^2 + a^13*b^5) + 35/65536*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^4*b^5)
```

Fricas [A]
time = 0.94, size = 654, normalized size = 3.25

$$\left[\frac{630\,ab^9x^{17}+5460\,a^2b^8x^{15}+20916\,a^3b^7x^{13}+46404\,a^4b^6x^{11}-65536\,a^5b^5x^9-46404\,a^6b^4x^7-20916\,a^7b^3x^5-5460\,a^8b^2x^3-630\,a^9bx-315\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{1179648\,(a^5b^{15}x^{18}+9\,a^6b^{14}x^{16}+36\,a^7b^{13}x^{14}+84\,a^8b^{12}x^{12}+126\,a^9b^{11}x^{10}+126\,a^{10}b^{10}x^8+84\,a^{11}b^9x^6+36\,a^{12}b^8x^4+9\,a^{13}b^7x^2+a^{14}b^6)},\ \frac{315\,ab^9x^{17}+2730\,a^2b^8x^{15}+10458\,a^3b^7x^{13}+23202\,a^4b^6x^{11}-32768\,a^5b^5x^9-23202\,a^6b^4x^7-10458\,a^7b^3x^5-2730\,a^8b^2x^3-315\,a^9bx+315\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{589824\,(a^5b^{15}x^{18}+9\,a^6b^{14}x^{16}+36\,a^7b^{13}x^{14}+84\,a^8b^{12}x^{12}+126\,a^9b^{11}x^{10}+126\,a^{10}b^{10}x^8+84\,a^{11}b^9x^6+36\,a^{12}b^8x^4+9\,a^{13}b^7x^2+a^{14}b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 46404
*a^4*b^6*x^11 - 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5 -
 5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7
*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x
^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^5*b^15*x^18 + 9*a^6*b^14*x^16 + 36*a^7*b^13*x^14
 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^10 + 126*a^10*b^10*x^8 + 84*a^11*b^9*x
^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^2 + a^14*b^6), 1/589824*(315*a*b^9*x^17
 + 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 - 32768*a^5*
b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^3 - 315*a^
9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 +
126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^
8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^15*x^18 + 9*a^6*b^14
*x^16 + 36*a^7*b^13*x^14 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^10 + 126*a^10*
b^10*x^8 + 84*a^11*b^9*x^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^2 + a^14*b^6)]
```

Sympy [A]
time = 0.54, size = 291, normalized size = 1.45

$$-\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(-a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072}+\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072}+\frac{-315a^8x-2730a^7bx^3-10458a^6b^2x^5-23202a^5b^3x^7-32768a^4b^4x^9+23202a^3b^5x^{11}+10458a^2b^6x^{13}+2730ab^7x^{15}+315b^8x^{17}}{589824a^{13}b^5+5308416a^{12}b^6x^2+21233664a^{11}b^7x^4+49545216a^{10}b^8x^6+74317824a^9b^9x^8+74317824a^8b^{10}x^{10}+49545216a^7b^{11}x^{12}+21233664a^6b^{12}x^{14}+5308416a^5b^{13}x^{16}+589824a^4b^{14}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**10,x)
```

```
[Out] -35*sqrt(-1/(a**9*b**11))*log(-a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/131072
+ 35*sqrt(-1/(a**9*b**11))*log(a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/131072
+ (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b**3*
x**7 - 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x**13
 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**13*b**5 + 5308416*a**12*b
**6*x**2 + 21233664*a**11*b**7*x**4 + 49545216*a**10*b**8*x**6 + 74317824*a
**9*b**9*x**8 + 74317824*a**8*b**10*x**10 + 49545216*a**7*b**11*x**12 + 212
33664*a**6*b**12*x**14 + 5308416*a**5*b**13*x**16 + 589824*a**4*b**14*x**18
)
```

Giac [A]
time = 1.00, size = 128, normalized size = 0.64

$$\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^4b^5}+\frac{315\,b^8x^{17}+2730\,ab^7x^{15}+10458\,a^2b^6x^{13}+23202\,a^3b^5x^{11}-32768\,a^4b^4x^9-23202\,a^5b^3x^7-10458\,a^6b^2x^5-2730\,a^7bx^3-315\,a^8x}{589824\,(bx^2+a)^9a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b^5) + 1/589824*(315*b^8*x^17
 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 - 32768*a^4*b^
```

```
4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)
/((b*x^2 + a)^9*a^4*b^5)
```

Mupad [B]
time = 4.77, size = 205, normalized size = 1.02

$$\frac{35\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{9/2}\,b^{11/2}}-\frac{\frac{x^9}{18\,b}-\frac{1289\,x^{11}}{32768\,a}+\frac{1289\,a\,x^7}{32768\,b^2}+\frac{35\,a^4\,x}{65536\,b^5}-\frac{581\,b\,x^{13}}{32768\,a^2}+\frac{581\,a^2\,x^5}{32768\,b^3}+\frac{455\,a^3\,x^3}{98304\,b^4}-\frac{455\,b^2\,x^{15}}{98304\,a^3}-\frac{35\,b^3\,x^{17}}{65536\,a^4}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^10,x)
```

```
[Out] (35*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(9/2)*b^(11/2)) - (x^9/(18*b) - (12
89*x^11)/(32768*a) + (1289*a*x^7)/(32768*b^2) + (35*a^4*x)/(65536*b^5) - (5
81*b*x^13)/(32768*a^2) + (581*a^2*x^5)/(32768*b^3) + (455*a^3*x^3)/(98304*b
^4) - (455*b^2*x^15)/(98304*a^3) - (35*b^3*x^17)/(65536*a^4))/(a^9 + b^9*x^
18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5
*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

3.217 $\int \frac{x^8}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=202

$$-\frac{x^7}{18b\,(a+bx^2)^9}-\frac{7x^5}{288b^2\,(a+bx^2)^8}-\frac{5x^3}{576b^3\,(a+bx^2)^7}-\frac{5x}{2304b^4\,(a+bx^2)^6}+\frac{x}{4608ab^4\,(a+bx^2)^5}+\frac{x}{4096a^2b^4\,(a$$

```
[Out] -1/18*x^7/b/(b*x^2+a)^9-7/288*x^5/b^2/(b*x^2+a)^8-5/576*x^3/b^3/(b*x^2+a)^7
-5/2304*x/b^4/(b*x^2+a)^6+1/4608*x/a/b^4/(b*x^2+a)^5+1/4096*x/a^2/b^4/(b*x^
2+a)^4+7/24576*x/a^3/b^4/(b*x^2+a)^3+35/98304*x/a^4/b^4/(b*x^2+a)^2+35/6553
6*x/a^5/b^4/(b*x^2+a)+35/65536*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)/b^(9/2)
```

Rubi [A]
time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 205, 211}

$$\frac{35\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}}+\frac{35x}{65536a^5b^4\,(a+bx^2)}+\frac{35x}{98304a^4b^4\,(a+bx^2)^2}+\frac{7x}{24576a^3b^4\,(a+bx^2)^3}+\frac{x}{4096a^2b^4\,(a+bx^2)^4}+\frac{x}{4608ab^4\,(a+bx^2)^5}-\frac{5x}{2304b^4\,(a+bx^2)^6}-\frac{5x^3}{576b^3\,(a+bx^2)^7}-\frac{7x^5}{288b^2\,(a+bx^2)^8}-\frac{x^7}{18b\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^10,x]
```

```
[Out] -1/18*x^7/(b*(a + b*x^2)^9) - (7*x^5)/(288*b^2*(a + b*x^2)^8) - (5*x^3)/(57
6*b^3*(a + b*x^2)^7) - (5*x)/(2304*b^4*(a + b*x^2)^6) + x/(4608*a*b^4*(a +
b*x^2)^5) + x/(4096*a^2*b^4*(a + b*x^2)^4) + (7*x)/(24576*a^3*b^4*(a + b*x^
2)^3) + (35*x)/(98304*a^4*b^4*(a + b*x^2)^2) + (35*x)/(65536*a^5*b^4*(a + b
*x^2)) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(11/2)*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^2)^{10}}\,dx = -\frac{x^7}{18b\,(a+bx^2)^9} + \frac{7\int \frac{x^6}{(a+bx^2)^9}\,dx}{18b}$$

$$= -\frac{x^7}{18b\,(a+bx^2)^9} - \frac{7x^5}{288b^2\,(a+bx^2)^8} + \frac{35\int \frac{x^4}{(a+bx^2)^8}\,dx}{288b^2}$$

$$= -\frac{x^7}{18b\,(a+bx^2)^9} - \frac{7x^5}{288b^2\,(a+bx^2)^8} - \frac{5x^3}{576b^3\,(a+bx^2)^7} + \frac{5\int \frac{x^2}{(a+bx^2)^7}\,dx}{192b^3}$$

$$= -\frac{x^7}{18b\,(a+bx^2)^9} - \frac{7x^5}{288b^2\,(a+bx^2)^8} - \frac{5x^3}{576b^3\,(a+bx^2)^7} - \frac{5x}{2304b^4\,(a+bx^2)^6} + \frac{5\int \frac{1}{(a+bx^2}}{2304b}$$

$$= -\frac{x^7}{18b\,(a+bx^2)^9} - \frac{7x^5}{288b^2\,(a+bx^2)^8} - \frac{5x^3}{576b^3\,(a+bx^2)^7} - \frac{5x}{2304b^4\,(a+bx^2)^6} + \frac{3}{4608ab^4\,(}$$

$$= -\frac{x^7}{18b\,(a+bx^2)^9} - \frac{7x^5}{288b^2\,(a+bx^2)^8} - \frac{5x^3}{576b^3\,(a+bx^2)^7} - \frac{5x}{2304b^4\,(a+bx^2)^6} + \frac{3}{4608ab^4\,(}$$

$$= -\frac{x^7}{18b\,(a+bx^2)^9} - \frac{7x^5}{288b^2\,(a+bx^2)^8} - \frac{5x^3}{576b^3\,(a+bx^2)^7} - \frac{5x}{2304b^4\,(a+bx^2)^6} + \frac{3}{4608ab^4\,(}$$

$$= -\frac{x^7}{18b\,(a+bx^2)^9} - \frac{7x^5}{288b^2\,(a+bx^2)^8} - \frac{5x^3}{576b^3\,(a+bx^2)^7} - \frac{5x}{2304b^4\,(a+bx^2)^6} + \frac{3}{4608ab^4\,(}$$

$$= -\frac{x^7}{18b\,(a+bx^2)^9} - \frac{7x^5}{288b^2\,(a+bx^2)^8} - \frac{5x^3}{576b^3\,(a+bx^2)^7} - \frac{5x}{2304b^4\,(a+bx^2)^6} + \frac{3}{4608ab^4\,(}$$

$$= -\frac{x^7}{18b\,(a+bx^2)^9} - \frac{7x^5}{288b^2\,(a+bx^2)^8} - \frac{5x^3}{576b^3\,(a+bx^2)^7} - \frac{5x}{2304b^4\,(a+bx^2)^6} + \frac{3}{4608ab^4\,(}$$

Mathematica [A]
time = 0.04, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x\left(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6+32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16}\right)}{(a+bx^2)^9}+315\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{589824a^{11/2}b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202*
a^5*b^3*x^6 + 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12 +
 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(589824*a^(11/2)*b^(9/2))
```

Maple [A]
time = 0.10, size = 122, normalized size = 0.60

method	result
default	$\frac{-\frac{35a^3x}{65536b^4}-\frac{455a^2x^3}{98304b^3}-\frac{581a\,x^5}{32768b^2}-\frac{1289x^7}{32768b}+\frac{x^9}{18a}+\frac{1289b\,x^{11}}{32768a^2}+\frac{581b^2x^{13}}{32768a^3}+\frac{455b^3x^{15}}{98304a^4}+\frac{35b^4x^{17}}{65536a^5}}{\left(b\,x^2+a\right)^9}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^5b^4\sqrt{ab}}$
risch	$\frac{-\frac{35a^3x}{65536b^4}-\frac{455a^2x^3}{98304b^3}-\frac{581a\,x^5}{32768b^2}-\frac{1289x^7}{32768b}+\frac{x^9}{18a}+\frac{1289b\,x^{11}}{32768a^2}+\frac{581b^2x^{13}}{32768a^3}+\frac{455b^3x^{15}}{98304a^4}+\frac{35b^4x^{17}}{65536a^5}}{\left(b\,x^2+a\right)^9}-\frac{35\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^4a^5}+\frac{35\ln\left(-bx+\sqrt{\right.}}{131072\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] (-35/65536*a^3*x/b^4-455/98304*a^2*x^3/b^3-581/32768*a*x^5/b^2-1289/32768*x
^7/b+1/18/a*x^9+1289/32768*b/a^2*x^11+581/32768*b^2/a^3*x^13+455/98304*b^3/
a^4*x^15+35/65536*b^4/a^5*x^17)/(b*x^2+a)^9+35/65536/a^5/b^4/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.53, size = 221, normalized size = 1.09

$$\frac{315\,b^8x^{17}+2730\,ab^7x^{15}+10458\,a^2b^6x^{13}+23202\,a^3b^5x^{11}+32768\,a^4b^4x^9-23202\,a^5b^3x^7-10458\,a^6b^2x^5-2730\,a^7bx^3-315\,a^8x}{589824\left(a^5b^{13}x^{18}+9\,a^6b^{12}x^{16}+36\,a^7b^{11}x^{14}+84\,a^8b^{10}x^{12}+126\,a^9b^9x^{10}+126\,a^{10}b^8x^8+84\,a^{11}b^7x^6+36\,a^{12}b^6x^4+9\,a^{13}b^5x^2+a^{14}b^4\right)}+\frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/589824*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b
^5*x^11 + 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*
a^7*b*x^3 - 315*a^8*x)/(a^5*b^13*x^18 + 9*a^6*b^12*x^16 + 36*a^7*b^11*x^14
+ 84*a^8*b^10*x^12 + 126*a^9*b^9*x^10 + 126*a^10*b^8*x^8 + 84*a^11*b^7*x^6
+ 36*a^12*b^6*x^4 + 9*a^13*b^5*x^2 + a^14*b^4) + 35/65536*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^5*b^4)
```

Fricas [A]
time = 1.05, size = 654, normalized size = 3.24

$$\left[\frac{630\,ab^9x^{17}+5460\,a^2b^8x^{15}+20916\,a^3b^7x^{13}+46404\,a^4b^6x^{11}+65536\,a^5b^5x^9-46404\,a^6b^4x^7-20916\,a^7b^3x^5-5460\,a^8b^2x^3-630\,a^9bx-315\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{1179648\left(a^6b^{14}x^{18}+9\,a^7b^{13}x^{16}+36\,a^8b^{12}x^{14}+84\,a^9b^{11}x^{12}+126\,a^{10}b^{10}x^{10}+126\,a^{11}b^9x^8+84\,a^{12}b^8x^6+36\,a^{13}b^7x^4+9\,a^{14}b^6x^2+a^{15}b^5\right)},\ \frac{315\,ab^9x^{17}+2730\,a^2b^8x^{15}+10458\,a^3b^7x^{13}+23202\,a^4b^6x^{11}+32768\,a^5b^5x^9-23202\,a^6b^4x^7-10458\,a^7b^3x^5-2730\,a^8b^2x^3-315\,a^9bx+315\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{589824\left(a^6b^{14}x^{18}+9\,a^7b^{13}x^{16}+36\,a^8b^{12}x^{14}+84\,a^9b^{11}x^{12}+126\,a^{10}b^{10}x^{10}+126\,a^{11}b^9x^8+84\,a^{12}b^8x^6+36\,a^{13}b^7x^4+9\,a^{14}b^6x^2+a^{15}b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="fricas")

[Out] [1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 46404
*a^4*b^6*x^11 + 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5 -
5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7
*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x
^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^6*b^14*x^18 + 9*a^7*b^13*x^16 + 36*a^8*b^12*x^14
+ 84*a^9*b^11*x^12 + 126*a^10*b^10*x^10 + 126*a^11*b^9*x^8 + 84*a^12*b^8*x
^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^2 + a^15*b^5), 1/589824*(315*a*b^9*x^17
+ 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 + 32768*a^5*
b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^3 - 315*a^
9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 +
126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^
8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^6*b^14*x^18 + 9*a^7*b^13
*x^16 + 36*a^8*b^12*x^14 + 84*a^9*b^11*x^12 + 126*a^10*b^10*x^10 + 126*a^11
*b^9*x^8 + 84*a^12*b^8*x^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^2 + a^15*b^5)]

Sympy [A]
time = 0.51, size = 291, normalized size = 1.44

$$-\frac{35\sqrt{-\frac{1}{a^{11}b^{9}}}\log\left(-a^{6}b^{4}\sqrt{-\frac{1}{a^{11}b^{9}}}+x\right)}{131072}+\frac{35\sqrt{-\frac{1}{a^{11}b^{9}}}\log\left(a^{6}b^{4}\sqrt{-\frac{1}{a^{11}b^{9}}}+x\right)}{131072}+\frac{-315a^{8}x-2730a^{7}bx^{3}-10458a^{6}b^{2}x^{5}-23202a^{5}b^{3}x^{7}+32768a^{4}b^{4}x^{9}+23202a^{3}b^{5}x^{11}+10458a^{2}b^{6}x^{13}+2730ab^{7}x^{15}+315b^{8}x^{17}}{589824a^{14}b^{4}+5308416a^{13}b^{5}x^{2}+21233664a^{12}b^{6}x^{4}+49545216a^{11}b^{7}x^{6}+74317824a^{10}b^{8}x^{8}+74317824a^{9}b^{9}x^{10}+49545216a^{8}b^{10}x^{12}+21233664a^{7}b^{11}x^{14}+5308416a^{6}b^{12}x^{16}+589824a^{5}b^{13}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**2+a)**10,x)

[Out] -35*sqrt(-1/(a**11*b**9))*log(-a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/131072
+ 35*sqrt(-1/(a**11*b**9))*log(a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/131072
+ (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b**3*
x**7 + 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x**13
+ 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**14*b**4 + 5308416*a**13*b
5*x2 + 21233664*a**12*b**6*x**4 + 49545216*a**11*b**7*x**6 + 74317824*a
10*b8*x**8 + 74317824*a**9*b**9*x**10 + 49545216*a**8*b**10*x**12 + 212
33664*a**7*b**11*x**14 + 5308416*a**6*b**12*x**16 + 589824*a**5*b**13*x**18
)

Giac [A]
time = 1.25, size = 128, normalized size = 0.63

$$\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^{5}b^{4}}+\frac{315\,b^{8}x^{17}+2730\,ab^{7}x^{15}+10458\,a^{2}b^{6}x^{13}+23202\,a^{3}b^{5}x^{11}+32768\,a^{4}b^{4}x^{9}-23202\,a^{5}b^{3}x^{7}-10458\,a^{6}b^{2}x^{5}-2730\,a^{7}bx^{3}-315\,a^{8}x}{589824\,(bx^{2}+a)^{9}a^{5}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="giac")

[Out] 35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5*b^4) + 1/589824*(315*b^8*x^17
+ 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 + 32768*a^4*b^

```
4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)
/((b*x^2 + a)^9*a^5*b^4)
```

Mupad [B]
time = 4.74, size = 204, normalized size = 1.01

$$\frac{\frac{x^9}{18\,a}-\frac{1289\,x^7}{32768\,b}-\frac{581\,a\,x^5}{32768\,b^2}-\frac{35\,a^3\,x}{65536\,b^4}+\frac{1289\,b\,x^{11}}{32768\,a^2}-\frac{455\,a^2\,x^3}{98304\,b^3}+\frac{581\,b^2\,x^{13}}{32768\,a^3}+\frac{455\,b^3\,x^{15}}{98304\,a^4}+\frac{35\,b^4\,x^{17}}{65536\,a^5}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{35\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{11/2}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^10,x)
```

```
[Out] (x^9/(18*a) - (1289*x^7)/(32768*b) - (581*a*x^5)/(32768*b^2) - (35*a^3*x)/(
65536*b^4) + (1289*b*x^11)/(32768*a^2) - (455*a^2*x^3)/(98304*b^3) + (581*b
^2*x^13)/(32768*a^3) + (455*b^3*x^15)/(98304*a^4) + (35*b^4*x^17)/(65536*a^
5))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*
b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7
*x^14) + (35*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(11/2)*b^(9/2))
```

3.218 $\int \frac{x^6}{(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=203

$$-\frac{x^5}{18b\,(a+bx^2)^9}-\frac{5x^3}{288b^2\,(a+bx^2)^8}-\frac{5x}{1344b^3\,(a+bx^2)^7}+\frac{5x}{16128ab^3\,(a+bx^2)^6}+\frac{11x}{32256a^2b^3\,(a+bx^2)^5}+\frac{}{28672a}$$

```
[Out] -1/18*x^5/b/(b*x^2+a)^9-5/288*x^3/b^2/(b*x^2+a)^8-5/1344*x/b^3/(b*x^2+a)^7+
5/16128*x/a/b^3/(b*x^2+a)^6+11/32256*x/a^2/b^3/(b*x^2+a)^5+11/28672*x/a^3/b
^3/(b*x^2+a)^4+11/24576*x/a^4/b^3/(b*x^2+a)^3+55/98304*x/a^5/b^3/(b*x^2+a)^
2+55/65536*x/a^6/b^3/(b*x^2+a)+55/65536*arctan(x*b^(1/2)/a^(1/2))/a^(13/2)/
b^(7/2)
```

Rubi [A]
time = 0.07, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 205, 211}

$$\frac{55\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{13/2}b^{7/2}}+\frac{55x}{65536a^6b^3\,(a+bx^2)}+\frac{55x}{98304a^5b^3\,(a+bx^2)^2}+\frac{11x}{24576a^4b^3\,(a+bx^2)^3}+\frac{11x}{28672a^3b^3\,(a+bx^2)^4}+\frac{11x}{32256a^2b^3\,(a+bx^2)^5}+\frac{5x}{16128ab^3\,(a+bx^2)^6}-\frac{5x}{1344b^3\,(a+bx^2)^7}-\frac{5x^3}{288b^2\,(a+bx^2)^8}-\frac{x^5}{18b\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^10,x]
```

```
[Out] -1/18*x^5/(b*(a + b*x^2)^9) - (5*x^3)/(288*b^2*(a + b*x^2)^8) - (5*x)/(1344
*b^3*(a + b*x^2)^7) + (5*x)/(16128*a*b^3*(a + b*x^2)^6) + (11*x)/(32256*a^2
*b^3*(a + b*x^2)^5) + (11*x)/(28672*a^3*b^3*(a + b*x^2)^4) + (11*x)/(24576*
a^4*b^3*(a + b*x^2)^3) + (55*x)/(98304*a^5*b^3*(a + b*x^2)^2) + (55*x)/(655
36*a^6*b^3*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(13/2)*
b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx = -\frac{x^5}{18b(a+bx^2)^9} + \frac{5\int \frac{x^4}{(a+bx^2)^9}\,dx}{18b}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} + \frac{5\int \frac{x^2}{(a+bx^2)^8}\,dx}{96b^2}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5\int \frac{1}{(a+bx^2)^7}\,dx}{1344b^3}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{55\int \cdots}{1\cdots}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{\cdots}{3225\cdots}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{\cdots}{3225\cdots}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{\cdots}{3225\cdots}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{\cdots}{3225\cdots}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{\cdots}{3225\cdots}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{\cdots}{3225\cdots}$$

Mathematica [A]
time = 0.04, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}\,x(-3465a^8-30030a^7bx^2-115038a^6b^2x^4+334602a^5b^3x^6+360448a^4b^4x^8+255222a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16})}{(a+bx^2)^9}+3465\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{4128768a^{13/2}b^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b*x^2)^10,x]

[Out] ((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 + 334602*a^5*b^3*x^6 + 360448*a^4*b^4*x^8 + 255222*a^3*b^5*x^10 + 115038*a^2*b^6*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(13/2)*b^(7/2))

Maple [A]
time = 0.11, size = 122, normalized size = 0.60

method	result
default	$\frac{-\frac{55a^2x}{65536b^3}-\frac{715a\,x^3}{98304b^2}-\frac{913x^5}{32768b}+\frac{18589x^7}{229376a}+\frac{11b\,x^9}{126a^2}+\frac{14179b^2x^{11}}{229376a^3}+\frac{913b^3x^{13}}{32768a^4}+\frac{715b^4x^{15}}{98304a^5}+\frac{55b^5x^{17}}{65536a^6}}{(b\,x^2+a)^9}+\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^6b^3\sqrt{ab}}$
risch	$\frac{-\frac{55a^2x}{65536b^3}-\frac{715a\,x^3}{98304b^2}-\frac{913x^5}{32768b}+\frac{18589x^7}{229376a}+\frac{11b\,x^9}{126a^2}+\frac{14179b^2x^{11}}{229376a^3}+\frac{913b^3x^{13}}{32768a^4}+\frac{715b^4x^{15}}{98304a^5}+\frac{55b^5x^{17}}{65536a^6}}{(b\,x^2+a)^9}-\frac{55\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^3a^6}+\frac{55\ln\left(-bx+\right.}{131072\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b*x^2+a)^10,x,method=_RETURNVERBOSE)

[Out] (-55/65536*a^2*x/b^3-715/98304*a*x^3/b^2-913/32768*x^5/b+18589/229376/a*x^7+11/126*b/a^2*x^9+14179/229376*b^2/a^3*x^11+913/32768*b^3/a^4*x^13+715/98304*b^4/a^5*x^15+55/65536*b^5/a^6*x^17)/(b*x^2+a)^9+55/65536/a^6/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Maxima [A]
time = 0.54, size = 221, normalized size = 1.09

$$\frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}+255222\,a^3b^5x^{11}+360448\,a^4b^4x^9+334602\,a^5b^3x^7-115038\,a^6b^2x^5-30030\,a^7bx^3-3465\,a^8x}{4128768\,(a^6b^{12}x^{18}+9\,a^7b^{11}x^{16}+36\,a^8b^{10}x^{14}+84\,a^9b^9x^{12}+126\,a^{10}b^8x^{10}+126\,a^{11}b^7x^8+84\,a^{12}b^6x^6+36\,a^{13}b^5x^4+9\,a^{14}b^4x^2+a^{15}b^3)}+\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="maxima")

[Out] 1/4128768*(3465*b^8*x^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 + 255222*a^3*b^5*x^11 + 360448*a^4*b^4*x^9 + 334602*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3 - 3465*a^8*x)/(a^6*b^12*x^18 + 9*a^7*b^11*x^16 + 36*a^8*b^10*x^14 + 84*a^9*b^9*x^12 + 126*a^10*b^8*x^10 + 126*a^11*b^7*x^8 + 84*a^12*b^6*x^6 + 36*a^13*b^5*x^4 + 9*a^14*b^4*x^2 + a^15*b^3) + 55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6*b^3)

Fricas [A]
time = 1.08, size = 654, normalized size = 3.22

$$\left[\frac{6930\,ab^9x^{17}+60060\,a^2b^8x^{15}+230076\,a^3b^7x^{13}+510444\,a^4b^6x^{11}+720896\,a^5b^5x^9+669204\,a^6b^4x^7-230076\,a^7b^3x^5-60060\,a^8b^2x^3-6930\,a^9bx-3465\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{8257536\,(a^7b^{13}x^{18}+9\,a^8b^{12}x^{16}+36\,a^9b^{11}x^{14}+84\,a^{10}b^{10}x^{12}+126\,a^{11}b^9x^{10}+126\,a^{12}b^8x^8+84\,a^{13}b^7x^6+36\,a^{14}b^6x^4+9\,a^{15}b^5x^2+a^{16}b^4)},\frac{3465\,ab^9x^{17}+30030\,a^2b^8x^{15}+115038\,a^3b^7x^{13}+255222\,a^4b^6x^{11}+360448\,a^5b^5x^9+334602\,a^6b^4x^7-115038\,a^7b^3x^5-30030\,a^8b^2x^3-3465\,a^9bx+3465\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{4128768\,(a^7b^{13}x^{18}+9\,a^8b^{12}x^{16}+36\,a^9b^{11}x^{14}+84\,a^{10}b^{10}x^{12}+126\,a^{11}b^9x^{10}+126\,a^{12}b^8x^8+84\,a^{13}b^7x^6+36\,a^{14}b^6x^4+9\,a^{15}b^5x^2+a^{16}b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 + 51
0444*a^4*b^6*x^11 + 720896*a^5*b^5*x^9 + 669204*a^6*b^4*x^7 - 230076*a^7*b^
3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16 +
36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84
*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^7*b^13*x^18 + 9*a^8*b^12*x^16 + 36*a^9
*b^11*x^14 + 84*a^10*b^10*x^12 + 126*a^11*b^9*x^10 + 126*a^12*b^8*x^8 + 84*
a^13*b^7*x^6 + 36*a^14*b^6*x^4 + 9*a^15*b^5*x^2 + a^16*b^4), 1/4128768*(346
5*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 + 255222*a^4*b^6*x^
11 + 360448*a^5*b^5*x^9 + 334602*a^6*b^4*x^7 - 115038*a^7*b^3*x^5 - 30030*a
^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14
 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 +
36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^7*b
^13*x^18 + 9*a^8*b^12*x^16 + 36*a^9*b^11*x^14 + 84*a^10*b^10*x^12 + 126*a^1
1*b^9*x^10 + 126*a^12*b^8*x^8 + 84*a^13*b^7*x^6 + 36*a^14*b^6*x^4 + 9*a^15*
b^5*x^2 + a^16*b^4)]
```

Sympy [A]
time = 0.49, size = 291, normalized size = 1.43

$$-\frac{55\sqrt{-\frac{1}{a^{13}b^{7}}}\log\left(-a^{7}b^{3}\sqrt{-\frac{1}{a^{13}b^{7}}}+x\right)}{131072}+\frac{55\sqrt{-\frac{1}{a^{13}b^{7}}}\log\left(a^{7}b^{3}\sqrt{-\frac{1}{a^{13}b^{7}}}+x\right)}{131072}+\frac{-3465a^{8}x-30030a^{7}bx^{3}-115038a^{6}b^{2}x^{5}+334602a^{5}b^{3}x^{7}+360448a^{4}b^{4}x^{9}+255222a^{3}b^{5}x^{11}+115038a^{2}b^{6}x^{13}+30030ab^{7}x^{15}+3465b^{8}x^{17}}{4128768a^{15}b^{3}+37158912a^{14}b^{4}x^{2}+148635648a^{13}b^{5}x^{4}+346816512a^{12}b^{6}x^{6}+520224768a^{11}b^{7}x^{8}+520224768a^{10}b^{8}x^{10}+346816512a^{9}b^{9}x^{12}+148635648a^{8}b^{10}x^{14}+37158912a^{7}b^{11}x^{16}+4128768a^{6}b^{12}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**10,x)
```

```
[Out] -55*sqrt(-1/(a**13*b**7))*log(-a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/131072
+ 55*sqrt(-1/(a**13*b**7))*log(a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/131072
+ (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 + 334602*a**5*b
**3*x**7 + 360448*a**4*b**4*x**9 + 255222*a**3*b**5*x**11 + 115038*a**2*b**
6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**15*b**3 + 37158
912*a**14*b**4*x**2 + 148635648*a**13*b**5*x**4 + 346816512*a**12*b**6*x**6
 + 520224768*a**11*b**7*x**8 + 520224768*a**10*b**8*x**10 + 346816512*a**9*
b**9*x**12 + 148635648*a**8*b**10*x**14 + 37158912*a**7*b**11*x**16 + 41287
68*a**6*b**12*x**18)
```

Giac [A]
time = 1.40, size = 128, normalized size = 0.63

$$\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^{6}b^{3}}+\frac{3465\,b^{8}x^{17}+30030\,ab^{7}x^{15}+115038\,a^{2}b^{6}x^{13}+255222\,a^{3}b^{5}x^{11}+360448\,a^{4}b^{4}x^{9}+334602\,a^{5}b^{3}x^{7}-115038\,a^{6}b^{2}x^{5}-30030\,a^{7}bx^{3}-3465\,a^{8}x}{4128768\,(bx^{2}+a)^{9}a^{6}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6*b^3) + 1/4128768*(3465*b^8*x^
17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 + 255222*a^3*b^5*x^11 + 360448*
a^4*b^4*x^9 + 334602*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3 - 3
465*a^8*x)/((b*x^2 + a)^9*a^6*b^3)
```

Mupad [B]
time = 4.79, size = 204, normalized size = 1.00

$$\frac{\frac{18589\,x^7}{229376\,a}-\frac{913\,x^5}{32768\,b}-\frac{715\,a\,x^3}{98304\,b^2}-\frac{55\,a^2\,x}{65536\,b^3}+\frac{11\,b\,x^9}{126\,a^2}+\frac{14179\,b^2\,x^{11}}{229376\,a^3}+\frac{913\,b^3\,x^{13}}{32768\,a^4}+\frac{715\,b^4\,x^{15}}{98304\,a^5}+\frac{55\,b^5\,x^{17}}{65536\,a^6}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{55\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{13/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^10,x)
```

```
[Out] ((18589*x^7)/(229376*a) - (913*x^5)/(32768*b) - (715*a*x^3)/(98304*b^2) - (
55*a^2*x)/(65536*b^3) + (11*b*x^9)/(126*a^2) + (14179*b^2*x^11)/(229376*a^3
) + (913*b^3*x^13)/(32768*a^4) + (715*b^4*x^15)/(98304*a^5) + (55*b^5*x^17)
/(65536*a^6))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4
 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 +
36*a^2*b^7*x^14) + (55*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(13/2)*b^(7/2))
```

3.219 $\int \frac{x^4}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=204

$$-\frac{x^3}{18b\left(a+bx^2\right)^9}-\frac{x}{96b^2\left(a+bx^2\right)^8}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{143x}{161280a^3b^2\left(a+bx^2\right)^5}+\frac{143}{}$$

```
[Out] -1/18*x^3/b/(b*x^2+a)^9-1/96*x/b^2/(b*x^2+a)^8+1/1344*x/a/b^2/(b*x^2+a)^7+1
3/16128*x/a^2/b^2/(b*x^2+a)^6+143/161280*x/a^3/b^2/(b*x^2+a)^5+143/143360*x
/a^4/b^2/(b*x^2+a)^4+143/122880*x/a^5/b^2/(b*x^2+a)^3+143/98304*x/a^6/b^2/(
b*x^2+a)^2+143/65536*x/a^7/b^2/(b*x^2+a)+143/65536*arctan(x*b^(1/2)/a^(1/2)
)/a^(15/2)/b^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 205, 211}

$$\frac{143\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{15/2}b^{5/2}}+\frac{143x}{65536a^7b^2\left(a+bx^2\right)}+\frac{143x}{98304a^6b^2\left(a+bx^2\right)^2}+\frac{143x}{122880a^5b^2\left(a+bx^2\right)^3}+\frac{143x}{143360a^4b^2\left(a+bx^2\right)^4}+\frac{143x}{161280a^3b^2\left(a+bx^2\right)^5}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}-\frac{x}{96b^2\left(a+bx^2\right)^8}-\frac{x^3}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^10,x]
```

```
[Out] -1/18*x^3/(b*(a + b*x^2)^9) - x/(96*b^2*(a + b*x^2)^8) + x/(1344*a*b^2*(a +
 b*x^2)^7) + (13*x)/(16128*a^2*b^2*(a + b*x^2)^6) + (143*x)/(161280*a^3*b^2
*(a + b*x^2)^5) + (143*x)/(143360*a^4*b^2*(a + b*x^2)^4) + (143*x)/(122880*
a^5*b^2*(a + b*x^2)^3) + (143*x)/(98304*a^6*b^2*(a + b*x^2)^2) + (143*x)/(6
5536*a^7*b^2*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(15/
2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx = -\frac{x^3}{18b\,(a+bx^2)^9} + \frac{\int \frac{x^2}{(a+bx^2)^9}\,dx}{6b}$$

$$= -\frac{x^3}{18b\,(a+bx^2)^9} - \frac{x}{96b^2\,(a+bx^2)^8} + \frac{\int \frac{1}{(a+bx^2)^8}\,dx}{96b^2}$$

$$= -\frac{x^3}{18b\,(a+bx^2)^9} - \frac{x}{96b^2\,(a+bx^2)^8} + \frac{x}{1344ab^2\,(a+bx^2)^7} + \frac{13\int \frac{1}{(a+bx^2)^7}\,dx}{1344ab^2}$$

$$= -\frac{x^3}{18b\,(a+bx^2)^9} - \frac{x}{96b^2\,(a+bx^2)^8} + \frac{x}{1344ab^2\,(a+bx^2)^7} + \frac{13x}{16128a^2b^2\,(a+bx^2)^6} + \frac{143\int\cdots}{1\cdots}$$

$$= -\frac{x^3}{18b\,(a+bx^2)^9} - \frac{x}{96b^2\,(a+bx^2)^8} + \frac{x}{1344ab^2\,(a+bx^2)^7} + \frac{13x}{16128a^2b^2\,(a+bx^2)^6} + \frac{\cdots}{1612\cdots}$$

$$= -\frac{x^3}{18b\,(a+bx^2)^9} - \frac{x}{96b^2\,(a+bx^2)^8} + \frac{x}{1344ab^2\,(a+bx^2)^7} + \frac{13x}{16128a^2b^2\,(a+bx^2)^6} + \frac{\cdots}{1612\cdots}$$

$$= -\frac{x^3}{18b\,(a+bx^2)^9} - \frac{x}{96b^2\,(a+bx^2)^8} + \frac{x}{1344ab^2\,(a+bx^2)^7} + \frac{13x}{16128a^2b^2\,(a+bx^2)^6} + \frac{\cdots}{1612\cdots}$$

$$= -\frac{x^3}{18b\,(a+bx^2)^9} - \frac{x}{96b^2\,(a+bx^2)^8} + \frac{x}{1344ab^2\,(a+bx^2)^7} + \frac{13x}{16128a^2b^2\,(a+bx^2)^6} + \frac{\cdots}{1612\cdots}$$

$$= -\frac{x^3}{18b\,(a+bx^2)^9} - \frac{x}{96b^2\,(a+bx^2)^8} + \frac{x}{1344ab^2\,(a+bx^2)^7} + \frac{13x}{16128a^2b^2\,(a+bx^2)^6} + \frac{\cdots}{1612\cdots}$$

$$= -\frac{x^3}{18b\,(a+bx^2)^9} - \frac{x}{96b^2\,(a+bx^2)^8} + \frac{x}{1344ab^2\,(a+bx^2)^7} + \frac{13x}{16128a^2b^2\,(a+bx^2)^6} + \frac{\cdots}{1612\cdots}$$

Mathematica [A]
time = 0.04, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x\left(-45045a^8-390390a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16}\right)}{(a+bx^2)^9}+45045\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{20643840a^{15/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 + 2633274*a^6*b^2*x^4 +
4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 1495494*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(15/2)*b^(5/2))
```

Maple [A]
time = 0.10, size = 122, normalized size = 0.60

method	result
default	$\frac{-\frac{143ax}{65536b^2}-\frac{1859x^3}{98304b}+\frac{20899x^5}{163840a}+\frac{241657b\,x^7}{1146880a^2}+\frac{143b^2x^9}{630a^3}+\frac{184327b^3x^{11}}{1146880a^4}+\frac{11869b^4x^{13}}{163840a^5}+\frac{1859b^5x^{15}}{98304a^6}+\frac{143b^6x^{17}}{65536a^7}}{(b\,x^2+a)^9}+\frac{143\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^7b^2\sqrt{ab}}$
risch	$\frac{-\frac{143ax}{65536b^2}-\frac{1859x^3}{98304b}+\frac{20899x^5}{163840a}+\frac{241657b\,x^7}{1146880a^2}+\frac{143b^2x^9}{630a^3}+\frac{184327b^3x^{11}}{1146880a^4}+\frac{11869b^4x^{13}}{163840a^5}+\frac{1859b^5x^{15}}{98304a^6}+\frac{143b^6x^{17}}{65536a^7}}{(b\,x^2+a)^9}-\frac{143\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^2a^7}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] (-143/65536*a*x/b^2-1859/98304*x^3/b+20899/163840/a*x^5+241657/1146880*b/a^
2*x^7+143/630*b^2/a^3*x^9+184327/1146880*b^3/a^4*x^11+11869/163840*b^4/a^5*
x^13+1859/98304*b^5/a^6*x^15+143/65536*b^6/a^7*x^17)/(b*x^2+a)^9+143/65536/
a^7/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.55, size = 221, normalized size = 1.08

$$\frac{45045\,b^8x^{17}+390390\,ab^7x^{15}+1495494\,a^2b^6x^{13}+3317886\,a^3b^5x^{11}+4685824\,a^4b^4x^9+4349826\,a^5b^3x^7+2633274\,a^6b^2x^5-390390\,a^7bx^3-45045\,a^8x}{20643840\,(a^7b^{11}x^{18}+9\,a^8b^{10}x^{16}+36\,a^9b^9x^{14}+84\,a^{10}b^8x^{12}+126\,a^{11}b^7x^{10}+126\,a^{12}b^6x^8+84\,a^{13}b^5x^6+36\,a^{14}b^4x^4+9\,a^{15}b^3x^2+a^{16}b^2)}+\frac{143\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20643840*(45045*b^8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 331
7886*a^3*b^5*x^11 + 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6
*b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a^7*b^11*x^18 + 9*a^8*b^10*x^16
 + 36*a^9*b^9*x^14 + 84*a^10*b^8*x^12 + 126*a^11*b^7*x^10 + 126*a^12*b^6*x^
8 + 84*a^13*b^5*x^6 + 36*a^14*b^4*x^4 + 9*a^15*b^3*x^2 + a^16*b^2) + 143/65
536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7*b^2)
```

Fricas [A]
time = 1.24, size = 654, normalized size = 3.21

$$\left[\frac{90090\,ab^9x^{17}+780780\,a^2b^8x^{15}+2990988\,a^3b^7x^{13}+6635772\,a^4b^6x^{11}+9371648\,a^5b^5x^9+8699652\,a^6b^4x^7+5266548\,a^7b^3x^5-780780\,a^8b^2x^3-90090\,a^9bx-45045\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{41287680\,(a^8b^{12}x^{18}+9\,a^9b^{11}x^{16}+36\,a^{10}b^{10}x^{14}+84\,a^{11}b^9x^{12}+126\,a^{12}b^8x^{10}+126\,a^{13}b^7x^8+84\,a^{14}b^6x^6+36\,a^{15}b^5x^4+9\,a^{16}b^4x^2+a^{17}b^3)},\ \frac{45045\,ab^9x^{17}+390390\,a^2b^8x^{15}+1495494\,a^3b^7x^{13}+3317886\,a^4b^6x^{11}+4685824\,a^5b^5x^9+4349826\,a^6b^4x^7+2633274\,a^7b^3x^5-390390\,a^8b^2x^3-45045\,a^9bx+45045\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{20643840\,(a^8b^{12}x^{18}+9\,a^9b^{11}x^{16}+36\,a^{10}b^{10}x^{14}+84\,a^{11}b^9x^{12}+126\,a^{12}b^8x^{10}+126\,a^{13}b^7x^8+84\,a^{14}b^6x^6+36\,a^{15}b^5x^4+9\,a^{16}b^4x^2+a^{17}b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="fricas")

[Out] [1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13 + 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 526654 8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a* b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b ^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^8*b^12*x^18 + 9*a^9*b^11*x^ 16 + 36*a^10*b^10*x^14 + 84*a^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*a^13*b^ 7*x^8 + 84*a^14*b^6*x^6 + 36*a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b^3), 1/2 0643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*x^13 + 33 17886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 + 2633274*a^ 7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8* x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x ^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^8*b^12*x^18 + 9*a^9*b^11*x^16 + 36*a^10*b^10*x^14 + 84*a ^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*a^13*b^7*x^8 + 84*a^14*b^6*x^6 + 36* a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b^3)]

Sympy [A]
time = 0.48, size = 291, normalized size = 1.43

$$-\frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(-a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072}+\frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072}+\frac{-45045a^8x-390390a^7bx^3+2633274a^6b^2x^5+4349826a^5b^3x^7+4685824a^4b^4x^9+3317886a^3b^5x^{11}+1495494a^2b^6x^{13}+390390ab^7x^{15}+45045b^8x^{17}}{20643840a^{16}b^2+185794560a^{15}b^3x^2+743178240a^{14}b^4x^4+1734082560a^{13}b^5x^6+2601123840a^{12}b^6x^8+2601123840a^{11}b^7x^{10}+1734082560a^{10}b^8x^{12}+743178240a^9b^9x^{14}+185794560a^8b^{10}x^{16}+20643840a^7b^{11}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**2+a)**10,x)

[Out] -143*sqrt(-1/(a**15*b**5))*log(-a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/131072 + 143*sqrt(-1/(a**15*b**5))*log(a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/13107 2 + (-45045*a**8*x - 390390*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 4349826* a**5*b**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 1495494 *a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a**16* b**2 + 185794560*a**15*b**3*x**2 + 743178240*a**14*b**4*x**4 + 1734082560*a **13*b**5*x**6 + 2601123840*a**12*b**6*x**8 + 2601123840*a**11*b**7*x**10 + 1734082560*a**10*b**8*x**12 + 743178240*a**9*b**9*x**14 + 185794560*a**8*b **10*x**16 + 20643840*a**7*b**11*x**18)

Giac [A]
time = 0.96, size = 128, normalized size = 0.63

$$\frac{143\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^7b^2}+\frac{45045\,b^8x^{17}+390390\,ab^7x^{15}+1495494\,a^2b^6x^{13}+3317886\,a^3b^5x^{11}+4685824\,a^4b^4x^9+4349826\,a^5b^3x^7+2633274\,a^6b^2x^5-390390\,a^7bx^3-45045\,a^8x}{20643840\,(bx^2+a)^9a^7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="giac")

```
[Out] 143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7*b^2) + 1/20643840*(45045*b^8
*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 4
685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 - 390390*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^7*b^2)
```

Mupad [B]
time = 4.74, size = 204, normalized size = 1.00

$$\frac{\frac{20899\,x^5}{163840\,a}-\frac{1859\,x^3}{98304\,b}+\frac{241657\,b\,x^7}{1146880\,a^2}+\frac{143\,b^2\,x^9}{630\,a^3}+\frac{184327\,b^3\,x^{11}}{1146880\,a^4}+\frac{11869\,b^4\,x^{13}}{163840\,a^5}+\frac{1859\,b^5\,x^{15}}{98304\,a^6}+\frac{143\,b^6\,x^{17}}{65536\,a^7}-\frac{143\,a\,x}{65536\,b^2}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{143\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{15/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^10,x)
```

```
[Out] ((20899*x^5)/(163840*a) - (1859*x^3)/(98304*b) + (241657*b*x^7)/(1146880*a^
2) + (143*b^2*x^9)/(630*a^3) + (184327*b^3*x^11)/(1146880*a^4) + (11869*b^4
*x^13)/(163840*a^5) + (1859*b^5*x^15)/(98304*a^6) + (143*b^6*x^17)/(65536*a
^7) - (143*a*x)/(65536*b^2))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 +
 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*
a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (143*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^
(15/2)*b^(5/2))
```

3.220 $\int \frac{x^2}{(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=205

$$-\frac{x}{18b\,(a+bx^2)^9}+\frac{x}{288ab\,(a+bx^2)^8}+\frac{5x}{1344a^2b\,(a+bx^2)^7}+\frac{65x}{16128a^3b\,(a+bx^2)^6}+\frac{143x}{32256a^4b\,(a+bx^2)^5}+\frac{}{28672a}$$

```
[Out] -1/18*x/b/(b*x^2+a)^9+1/288*x/a/b/(b*x^2+a)^8+5/1344*x/a^2/b/(b*x^2+a)^7+65
/16128*x/a^3/b/(b*x^2+a)^6+143/32256*x/a^4/b/(b*x^2+a)^5+143/28672*x/a^5/b/
(b*x^2+a)^4+143/24576*x/a^6/b/(b*x^2+a)^3+715/98304*x/a^7/b/(b*x^2+a)^2+715
/65536*x/a^8/b/(b*x^2+a)+715/65536*arctan(x*b^(1/2)/a^(1/2))/a^(17/2)/b^(3/
2)
```

Rubi [A]
time = 0.07, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {294, 205, 211}

$$\frac{715\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}}+\frac{715x}{65536a^8b\,(a+bx^2)}+\frac{715x}{98304a^7b\,(a+bx^2)^2}+\frac{143x}{24576a^6b\,(a+bx^2)^3}+\frac{143x}{28672a^5b\,(a+bx^2)^4}+\frac{143x}{32256a^4b\,(a+bx^2)^5}+\frac{65x}{16128a^3b\,(a+bx^2)^6}+\frac{5x}{1344a^2b\,(a+bx^2)^7}+\frac{x}{288ab\,(a+bx^2)^8}-\frac{x}{18b\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^10,x]
```

```
[Out] -1/18*x/(b*(a + b*x^2)^9) + x/(288*a*b*(a + b*x^2)^8) + (5*x)/(1344*a^2*b*(
a + b*x^2)^7) + (65*x)/(16128*a^3*b*(a + b*x^2)^6) + (143*x)/(32256*a^4*b*(
a + b*x^2)^5) + (143*x)/(28672*a^5*b*(a + b*x^2)^4) + (143*x)/(24576*a^6*b*
(a + b*x^2)^3) + (715*x)/(98304*a^7*b*(a + b*x^2)^2) + (715*x)/(65536*a^8*b
*(a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(17/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^2)^{10}}\,dx &= -\frac{x}{18b\,(a+bx^2)^9} + \frac{\int \frac{1}{(a+bx^2)^9}\,dx}{18b} \\
&= -\frac{x}{18b\,(a+bx^2)^9} + \frac{x}{288ab\,(a+bx^2)^8} + \frac{5\int \frac{1}{(a+bx^2)^8}\,dx}{96ab} \\
&= -\frac{x}{18b\,(a+bx^2)^9} + \frac{x}{288ab\,(a+bx^2)^8} + \frac{5x}{1344a^2b\,(a+bx^2)^7} + \frac{65\int \frac{1}{(a+bx^2)^7}\,dx}{1344a^2b} \\
&= -\frac{x}{18b\,(a+bx^2)^9} + \frac{x}{288ab\,(a+bx^2)^8} + \frac{5x}{1344a^2b\,(a+bx^2)^7} + \frac{65x}{16128a^3b\,(a+bx^2)^6} + \cdots \\
&= -\frac{x}{18b\,(a+bx^2)^9} + \frac{x}{288ab\,(a+bx^2)^8} + \frac{5x}{1344a^2b\,(a+bx^2)^7} + \frac{65x}{16128a^3b\,(a+bx^2)^6} + \cdots \\
&= -\frac{x}{18b\,(a+bx^2)^9} + \frac{x}{288ab\,(a+bx^2)^8} + \frac{5x}{1344a^2b\,(a+bx^2)^7} + \frac{65x}{16128a^3b\,(a+bx^2)^6} + \cdots \\
&= -\frac{x}{18b\,(a+bx^2)^9} + \frac{x}{288ab\,(a+bx^2)^8} + \frac{5x}{1344a^2b\,(a+bx^2)^7} + \frac{65x}{16128a^3b\,(a+bx^2)^6} + \cdots \\
&= -\frac{x}{18b\,(a+bx^2)^9} + \frac{x}{288ab\,(a+bx^2)^8} + \frac{5x}{1344a^2b\,(a+bx^2)^7} + \frac{65x}{16128a^3b\,(a+bx^2)^6} + \cdots \\
&= -\frac{x}{18b\,(a+bx^2)^9} + \frac{x}{288ab\,(a+bx^2)^8} + \frac{5x}{1344a^2b\,(a+bx^2)^7} + \frac{65x}{16128a^3b\,(a+bx^2)^6} + \cdots \\
&= -\frac{x}{18b\,(a+bx^2)^9} + \frac{x}{288ab\,(a+bx^2)^8} + \frac{5x}{1344a^2b\,(a+bx^2)^7} + \frac{65x}{16128a^3b\,(a+bx^2)^6} + \cdots
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 138, normalized size = 0.67

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x\left(-45045a^8+985866a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16}\right)}{(a+bx^2)^9}+45045\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{4128768a^{17/2}b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*x^2)^10,x]

[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 + 985866*a^7*b*x^2 + 2633274*a^6*b^2*x^4 + 4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 1495494*a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(17/2)*b^(3/2))

Maple [A]
time = 0.10, size = 124, normalized size = 0.60

method	result
default	$\frac{-\frac{715x}{65536b}+\frac{23473x^3}{98304a}+\frac{20899b\,x^5}{32768a^2}+\frac{241657b^2x^7}{229376a^3}+\frac{143b^3x^9}{126a^4}+\frac{184327b^4x^{11}}{229376a^5}+\frac{11869b^5x^{13}}{32768a^6}+\frac{9295b^6x^{15}}{98304a^7}+\frac{715b^7x^{17}}{65536a^8}}{(b\,x^2+a)^9}+\frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^8b\sqrt{ab}}$
risch	$\frac{-\frac{715x}{65536b}+\frac{23473x^3}{98304a}+\frac{20899b\,x^5}{32768a^2}+\frac{241657b^2x^7}{229376a^3}+\frac{143b^3x^9}{126a^4}+\frac{184327b^4x^{11}}{229376a^5}+\frac{11869b^5x^{13}}{32768a^6}+\frac{9295b^6x^{15}}{98304a^7}+\frac{715b^7x^{17}}{65536a^8}}{(b\,x^2+a)^9}-\frac{715\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b\,a^8}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^2+a)^10,x,method=_RETURNVERBOSE)

[Out] (-715/65536*x/b+23473/98304/a*x^3+20899/32768*b/a^2*x^5+241657/229376*b^2/a^3*x^7+143/126*b^3/a^4*x^9+184327/229376*b^4/a^5*x^11+11869/32768*b^5/a^6*x^13+9295/98304*b^6/a^7*x^15+715/65536*b^7/a^8*x^17)/(b*x^2+a)^9+715/65536/a^8/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Maxima [A]
time = 0.51, size = 219, normalized size = 1.07

$$\frac{45045\,b^8x^{17}+390390\,ab^7x^{15}+1495494\,a^2b^6x^{13}+3317886\,a^3b^5x^{11}+4685824\,a^4b^4x^9+4349826\,a^5b^3x^7+2633274\,a^6b^2x^5+985866\,a^7bx^3-45045\,a^8x}{4128768\,(a^8b^{10}x^{18}+9\,a^9b^9x^{16}+36\,a^{10}b^8x^{14}+84\,a^{11}b^7x^{12}+126\,a^{12}b^6x^{10}+126\,a^{13}b^5x^8+84\,a^{14}b^4x^6+36\,a^{15}b^3x^4+9\,a^{16}b^2x^2+a^{17}b)}+\frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="maxima")

[Out] 1/4128768*(45045*b^8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 + 985866*a^7*b*x^3 - 45045*a^8*x)/(a^8*b^10*x^18 + 9*a^9*b^9*x^16 + 36*a^10*b^8*x^14 + 84*a^11*b^7*x^12 + 126*a^12*b^6*x^10 + 126*a^13*b^5*x^8 + 84*a^14*b^4*x^6 + 36*a^15*b^3*x^4 + 9*a^16*b^2*x^2 + a^17*b) + 715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8*b)

Fricas [A]
time = 1.36, size = 654, normalized size = 3.19

$$\left[\frac{90090\,ab^9x^{17}+780780\,a^2b^8x^{15}+2990988\,a^3b^7x^{13}+6635772\,a^4b^6x^{11}+9371648\,a^5b^5x^9+8699652\,a^6b^4x^7+5266548\,a^7b^3x^5+1971732\,a^8b^2x^3-90090\,a^9bx-45045\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{8257536\,(a^9b^{11}x^{18}+9\,a^{10}b^{10}x^{16}+36\,a^{11}b^9x^{14}+84\,a^{12}b^8x^{12}+126\,a^{13}b^7x^{10}+126\,a^{14}b^6x^8+84\,a^{15}b^5x^6+36\,a^{16}b^4x^4+9\,a^{17}b^3x^2+a^{18}b^2)},\ \frac{45045\,ab^9x^{17}+390390\,a^2b^8x^{15}+1495494\,a^3b^7x^{13}+3317886\,a^4b^6x^{11}+4685824\,a^5b^5x^9+4349826\,a^6b^4x^7+2633274\,a^7b^3x^5+985866\,a^8b^2x^3-45045\,a^9bx+45045\,(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{4128768\,(a^9b^{11}x^{18}+9\,a^{10}b^{10}x^{16}+36\,a^{11}b^9x^{14}+84\,a^{12}b^8x^{12}+126\,a^{13}b^7x^{10}+126\,a^{14}b^6x^8+84\,a^{15}b^5x^6+36\,a^{16}b^4x^4+9\,a^{17}b^3x^2+a^{18}b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13 +
 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 5266548
*a^7*b^3*x^5 + 1971732*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^9*b^11*x^18 + 9*a^10*b^10*x
^16 + 36*a^11*b^9*x^14 + 84*a^12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*a^14*b^
6*x^8 + 84*a^15*b^5*x^6 + 36*a^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b^2), 1/4
128768*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*x^13 + 331
7886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 + 2633274*a^7
*b^3*x^5 + 985866*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*x
^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^
8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a))/(a^9*b^11*x^18 + 9*a^10*b^10*x^16 + 36*a^11*b^9*x^14 + 84*a^
12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*a^14*b^6*x^8 + 84*a^15*b^5*x^6 + 36*a
^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b^2)]
```

Sympy [A]
time = 0.47, size = 286, normalized size = 1.40

$$-\frac{715\sqrt{-\frac{1}{a^{17}b^{3}}}\log\left(-a^{9}b\sqrt{-\frac{1}{a^{17}b^{3}}}+x\right)}{131072}+\frac{715\sqrt{-\frac{1}{a^{17}b^{3}}}\log\left(a^{9}b\sqrt{-\frac{1}{a^{17}b^{3}}}+x\right)}{131072}+\frac{-45045a^{8}x+985866a^{7}bx^{3}+2633274a^{6}b^{2}x^{5}+4349826a^{5}b^{3}x^{7}+4685824a^{4}b^{4}x^{9}+3317886a^{3}b^{5}x^{11}+1495494a^{2}b^{6}x^{13}+390390ab^{7}x^{15}+45045b^{8}x^{17}}{4128768a^{17}b+37158912a^{16}b^{2}x^{2}+148635648a^{15}b^{3}x^{4}+346816512a^{14}b^{4}x^{6}+520224768a^{13}b^{5}x^{8}+520224768a^{12}b^{6}x^{10}+346816512a^{11}b^{7}x^{12}+148635648a^{10}b^{8}x^{14}+37158912a^{9}b^{9}x^{16}+4128768a^{8}b^{10}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**10,x)
```

```
[Out] -715*sqrt(-1/(a**17*b**3))*log(-a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 +
715*sqrt(-1/(a**17*b**3))*log(a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 + (-
45045*a**8*x + 985866*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 4349826*a**5*b
**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 1495494*a**2*
b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a**17*b + 371
58912*a**16*b**2*x**2 + 148635648*a**15*b**3*x**4 + 346816512*a**14*b**4*x*
*6 + 520224768*a**13*b**5*x**8 + 520224768*a**12*b**6*x**10 + 346816512*a**
11*b**7*x**12 + 148635648*a**10*b**8*x**14 + 37158912*a**9*b**9*x**16 + 412
8768*a**8*b**10*x**18)
```

Giac [A]
time = 0.83, size = 128, normalized size = 0.62

$$\frac{715\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{8}b}+\frac{45045\,b^{8}x^{17}+390390\,ab^{7}x^{15}+1495494\,a^{2}b^{6}x^{13}+3317886\,a^{3}b^{5}x^{11}+4685824\,a^{4}b^{4}x^{9}+4349826\,a^{5}b^{3}x^{7}+2633274\,a^{6}b^{2}x^{5}+985866\,a^{7}bx^{3}-45045\,a^{8}x}{4128768\,(bx^{2}+a)^{9}a^{8}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="giac")
```

[Out] 715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8*b) + 1/4128768*(45045*b^8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 + 985866*a^7*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^8*b)

Mupad [B]

time = 0.17, size = 206, normalized size = 1.00

$$\frac{\frac{23473\,x^3}{98304\,a}-\frac{715\,x}{65536\,b}+\frac{20899\,b\,x^5}{32768\,a^2}+\frac{241657\,b^2\,x^7}{229376\,a^3}+\frac{143\,b^3\,x^9}{126\,a^4}+\frac{184327\,b^4\,x^{11}}{229376\,a^5}+\frac{11869\,b^5\,x^{13}}{32768\,a^6}+\frac{9295\,b^6\,x^{15}}{98304\,a^7}+\frac{715\,b^7\,x^{17}}{65536\,a^8}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{715\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{17/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*x^2)^10,x)

[Out] ((23473*x^3)/(98304*a) - (715*x)/(65536*b) + (20899*b*x^5)/(32768*a^2) + (241657*b^2*x^7)/(229376*a^3) + (143*b^3*x^9)/(126*a^4) + (184327*b^4*x^11)/(229376*a^5) + (11869*b^5*x^13)/(32768*a^6) + (9295*b^6*x^15)/(98304*a^7) + (715*b^7*x^17)/(65536*a^8))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (715*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(17/2)*b^(3/2))

3.221 $\int \frac{1}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=181

$$\frac{x}{18a\left(a+bx^2\right)^9}+\frac{17x}{288a^2\left(a+bx^2\right)^8}+\frac{85x}{1344a^3\left(a+bx^2\right)^7}+\frac{1105x}{16128a^4\left(a+bx^2\right)^6}+\frac{2431x}{32256a^5\left(a+bx^2\right)^5}+\frac{243}{28672a^6\,(a}$$

```
[Out] 1/18*x/a/(b*x^2+a)^9+17/288*x/a^2/(b*x^2+a)^8+85/1344*x/a^3/(b*x^2+a)^7+110
5/16128*x/a^4/(b*x^2+a)^6+2431/32256*x/a^5/(b*x^2+a)^5+2431/28672*x/a^6/(b*
x^2+a)^4+2431/24576*x/a^7/(b*x^2+a)^3+12155/98304*x/a^8/(b*x^2+a)^2+12155/6
5536*x/a^9/(b*x^2+a)+12155/65536*arctan(x*b^(1/2)/a^(1/2))/a^(19/2)/b^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {205, 211}

$$\frac{12155\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{19/2}\sqrt{b}}+\frac{12155x}{65536a^9\left(a+bx^2\right)}+\frac{12155x}{98304a^8\left(a+bx^2\right)^2}+\frac{2431x}{24576a^7\left(a+bx^2\right)^3}+\frac{2431x}{28672a^6\left(a+bx^2\right)^4}+\frac{2431x}{32256a^5\left(a+bx^2\right)^5}+\frac{1105x}{16128a^4\left(a+bx^2\right)^6}+\frac{85x}{1344a^3\left(a+bx^2\right)^7}+\frac{17x}{288a^2\left(a+bx^2\right)^8}+\frac{x}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-10),x]
```

```
[Out] x/(18*a*(a + b*x^2)^9) + (17*x)/(288*a^2*(a + b*x^2)^8) + (85*x)/(1344*a^3*
(a + b*x^2)^7) + (1105*x)/(16128*a^4*(a + b*x^2)^6) + (2431*x)/(32256*a^5*(
a + b*x^2)^5) + (2431*x)/(28672*a^6*(a + b*x^2)^4) + (2431*x)/(24576*a^7*(a
 + b*x^2)^3) + (12155*x)/(98304*a^8*(a + b*x^2)^2) + (12155*x)/(65536*a^9*(
a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(19/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{10}}\,dx = \frac{x}{18a\,(a+bx^2)^9} + \frac{17\int \frac{1}{(a+bx^2)^9}\,dx}{18a}$$

$$= \frac{x}{18a\,(a+bx^2)^9} + \frac{17x}{288a^2\,(a+bx^2)^8} + \frac{85\int \frac{1}{(a+bx^2)^8}\,dx}{96a^2}$$

$$= \frac{x}{18a\,(a+bx^2)^9} + \frac{17x}{288a^2\,(a+bx^2)^8} + \frac{85x}{1344a^3\,(a+bx^2)^7} + \frac{1105\int \frac{1}{(a+bx^2)^7}\,dx}{1344a^3}$$

$$= \frac{x}{18a\,(a+bx^2)^9} + \frac{17x}{288a^2\,(a+bx^2)^8} + \frac{85x}{1344a^3\,(a+bx^2)^7} + \frac{1105x}{16128a^4\,(a+bx^2)^6} + \frac{12155\int}{16}$$

$$= \frac{x}{18a\,(a+bx^2)^9} + \frac{17x}{288a^2\,(a+bx^2)^8} + \frac{85x}{1344a^3\,(a+bx^2)^7} + \frac{1105x}{16128a^4\,(a+bx^2)^6} + \frac{2}{32256a^5}$$

$$= \frac{x}{18a\,(a+bx^2)^9} + \frac{17x}{288a^2\,(a+bx^2)^8} + \frac{85x}{1344a^3\,(a+bx^2)^7} + \frac{1105x}{16128a^4\,(a+bx^2)^6} + \frac{2}{32256a^5}$$

$$= \frac{x}{18a\,(a+bx^2)^9} + \frac{17x}{288a^2\,(a+bx^2)^8} + \frac{85x}{1344a^3\,(a+bx^2)^7} + \frac{1105x}{16128a^4\,(a+bx^2)^6} + \frac{2}{32256a^5}$$

$$= \frac{x}{18a\,(a+bx^2)^9} + \frac{17x}{288a^2\,(a+bx^2)^8} + \frac{85x}{1344a^3\,(a+bx^2)^7} + \frac{1105x}{16128a^4\,(a+bx^2)^6} + \frac{2}{32256a^5}$$

$$= \frac{x}{18a\,(a+bx^2)^9} + \frac{17x}{288a^2\,(a+bx^2)^8} + \frac{85x}{1344a^3\,(a+bx^2)^7} + \frac{1105x}{16128a^4\,(a+bx^2)^6} + \frac{2}{32256a^5}$$

$$= \frac{x}{18a\,(a+bx^2)^9} + \frac{17x}{288a^2\,(a+bx^2)^8} + \frac{85x}{1344a^3\,(a+bx^2)^7} + \frac{1105x}{16128a^4\,(a+bx^2)^6} + \frac{2}{32256a^5}$$

Mathematica [A]
time = 0.06, size = 131, normalized size = 0.72

$$\frac{\frac{3363003a^8x+16759722a^7bx^3+44765658a^6b^2x^5+73947042a^5b^3x^7+79659008a^4b^4x^9+56404062a^3b^5x^{11}+25423398a^2b^6x^{13}+6636630ab^7x^{15}+765765b^8x^{17}}{a^9(a+bx^2)^9}+\frac{765765\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{19/2}\sqrt{b}}}{4128768}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-10),x]
```

```
[Out] ((3363003*a^8*x + 16759722*a^7*b*x^3 + 44765658*a^6*b^2*x^5 + 73947042*a^5*
b^3*x^7 + 79659008*a^4*b^4*x^9 + 56404062*a^3*b^5*x^11 + 25423398*a^2*b^6*x
^13 + 6636630*a*b^7*x^15 + 765765*b^8*x^17)/(a^9*(a + b*x^2)^9) + (765765*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(19/2)*Sqrt[b]))/4128768
```

Maple [A]

time = 0.09, size = 204, normalized size = 1.13

method	result
risch	$\frac{\frac{53381x}{65536a}+\frac{399041b\,x^{3}}{98304a^{2}}+\frac{355283b^{2}x^{5}}{32768a^{3}}+\frac{4108169b^{3}x^{7}}{229376a^{4}}+\frac{2431b^{4}x^{9}}{126a^{5}}+\frac{3133559b^{5}x^{11}}{229376a^{6}}+\frac{201773b^{6}x^{13}}{32768a^{7}}+\frac{158015b^{7}x^{15}}{98304a^{8}}+\frac{12155b^{8}x^{17}}{65536a^{9}}}{(b\,x^{2}+a)^{9}}-\frac{12155\ln\left(bx+\right.}{131072\sqrt{-}}$

7

$9\left(\frac{7x}{48a\left(bx^2+a\right)^3}+\right.$

$11\left(\frac{9x}{80a\left(bx^2+a\right)^4}+\right.$

$13\left(\frac{11x}{120a\left(bx^2+a\right)^5}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*x/a/(b*x^2+a)^9+17/18/a*(1/16*x/a/(b*x^2+a)^8+15/16/a*(1/14*x/a/(b*x^2
+a)^7+13/14/a*(1/12*x/a/(b*x^2+a)^6+11/12/a*(1/10*x/a/(b*x^2+a)^5+9/10/a*(1
/8*x/a/(b*x^2+a)^4+7/8/a*(1/6*x/a/(b*x^2+a)^3+5/6/a*(1/4*x/a/(b*x^2+a)^2+3/
4/a*(1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))))))))
```

Maxima [A]
time = 0.52, size = 212, normalized size = 1.17

$$\frac{765765\,b^{8}x^{17}+6636630\,ab^{7}x^{15}+25423398\,a^{2}b^{6}x^{13}+56404062\,a^{3}b^{5}x^{11}+79659008\,a^{4}b^{4}x^{9}+73947042\,a^{5}b^{3}x^{7}+44765658\,a^{6}b^{2}x^{5}+16759722\,a^{7}bx^{3}+3363003\,a^{8}x}{4128768\,(a^{9}b^{9}x^{18}+9\,a^{10}b^{8}x^{16}+36\,a^{11}b^{7}x^{14}+84\,a^{12}b^{6}x^{12}+126\,a^{13}b^{5}x^{10}+126\,a^{14}b^{4}x^{8}+84\,a^{15}b^{3}x^{6}+36\,a^{16}b^{2}x^{4}+9\,a^{17}bx^{2}+a^{18})}+\frac{12155\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(765765*b^8*x^17 + 6636630*a*b^7*x^15 + 25423398*a^2*b^6*x^13 + 5
6404062*a^3*b^5*x^11 + 79659008*a^4*b^4*x^9 + 73947042*a^5*b^3*x^7 + 447656
58*a^6*b^2*x^5 + 16759722*a^7*b*x^3 + 3363003*a^8*x)/(a^9*b^9*x^18 + 9*a^10
*b^8*x^16 + 36*a^11*b^7*x^14 + 84*a^12*b^6*x^12 + 126*a^13*b^5*x^10 + 126*a
^14*b^4*x^8 + 84*a^15*b^3*x^6 + 36*a^16*b^2*x^4 + 9*a^17*b*x^2 + a^18) + 12
155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(153) = 306.
time = 0.68, size = 650, normalized size = 3.59

$$\left[\frac{1531530\,ab^{9}x^{17}+13273260\,a^{2}b^{8}x^{15}+50846796\,a^{3}b^{7}x^{13}+112808124\,a^{4}b^{6}x^{11}+159318016\,a^{5}b^{5}x^{9}+147894084\,a^{6}b^{4}x^{7}+89531316\,a^{7}b^{3}x^{5}+33519444\,a^{8}b^{2}x^{3}+6726006\,a^{9}bx-765765\,(b^{9}x^{18}+9\,ab^{8}x^{16}+36\,a^{2}b^{7}x^{14}+84\,a^{3}b^{6}x^{12}+126\,a^{4}b^{5}x^{10}+126\,a^{5}b^{4}x^{8}+84\,a^{6}b^{3}x^{6}+36\,a^{7}b^{2}x^{4}+9\,a^{8}bx^{2}+a^{9})\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{8257536\,(a^{10}b^{10}x^{18}+9\,a^{11}b^{9}x^{16}+36\,a^{12}b^{8}x^{14}+84\,a^{13}b^{7}x^{12}+126\,a^{14}b^{6}x^{10}+126\,a^{15}b^{5}x^{8}+84\,a^{16}b^{4}x^{6}+36\,a^{17}b^{3}x^{4}+9\,a^{18}b^{2}x^{2}+a^{19}b)},\ \frac{765765\,ab^{9}x^{17}+6636630\,a^{2}b^{8}x^{15}+25423398\,a^{3}b^{7}x^{13}+56404062\,a^{4}b^{6}x^{11}+79659008\,a^{5}b^{5}x^{9}+73947042\,a^{6}b^{4}x^{7}+44765658\,a^{7}b^{3}x^{5}+16759722\,a^{8}b^{2}x^{3}+3363003\,a^{9}bx+765765\,(b^{9}x^{18}+9\,ab^{8}x^{16}+36\,a^{2}b^{7}x^{14}+84\,a^{3}b^{6}x^{12}+126\,a^{4}b^{5}x^{10}+126\,a^{5}b^{4}x^{8}+84\,a^{6}b^{3}x^{6}+36\,a^{7}b^{2}x^{4}+9\,a^{8}bx^{2}+a^{9})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{4128768\,(a^{10}b^{10}x^{18}+9\,a^{11}b^{9}x^{16}+36\,a^{12}b^{8}x^{14}+84\,a^{13}b^{7}x^{12}+126\,a^{14}b^{6}x^{10}+126\,a^{15}b^{5}x^{8}+84\,a^{16}b^{4}x^{6}+36\,a^{17}b^{3}x^{4}+9\,a^{18}b^{2}x^{2}+a^{19}b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(1531530*a*b^9*x^17 + 13273260*a^2*b^8*x^15 + 50846796*a^3*b^7*x
^13 + 112808124*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 147894084*a^6*b^4*x^
7 + 89531316*a^7*b^3*x^5 + 33519444*a^8*b^2*x^3 + 6726006*a^9*b*x - 765765*
(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*
x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^
9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^10*b^10*x^1
8 + 9*a^11*b^9*x^16 + 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14*b^6*x^
10 + 126*a^15*b^5*x^8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*b^2*x^2
+ a^19*b), 1/4128768*(765765*a*b^9*x^17 + 6636630*a^2*b^8*x^15 + 25423398*a
^3*b^7*x^13 + 56404062*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 + 73947042*a^6*b
^4*x^7 + 44765658*a^7*b^3*x^5 + 16759722*a^8*b^2*x^3 + 3363003*a^9*b*x + 76
```

```
5765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4
*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2
 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^10*b^10*x^18 + 9*a^11*b^9*x^16
+ 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14*b^6*x^10 + 126*a^15*b^5*x^
8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*b^2*x^2 + a^19*b))]
```

Sympy [A]
time = 0.48, size = 272, normalized size = 1.50

$$-\frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(-a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072}+\frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072}+\frac{3363003a^8x+16759722a^7bx^3+44765658a^6b^2x^5+73947042a^5b^3x^7+79659008a^4b^4x^9+56404062a^3b^5x^{11}+25423398a^2b^6x^{13}+6636630ab^7x^{15}+765765b^8x^{17}}{4128768a^{18}+37158912a^{17}bx^2+148635648a^{16}b^2x^4+346816512a^{15}b^3x^6+520224768a^{14}b^4x^8+520224768a^{13}b^5x^{10}+346816512a^{12}b^6x^{12}+148635648a^{11}b^7x^{14}+37158912a^{10}b^8x^{16}+4128768a^9b^9x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**10,x)
```

```
[Out] -12155*sqrt(-1/(a**19*b))*log(-a**10*sqrt(-1/(a**19*b)) + x)/131072 + 12155
*sqrt(-1/(a**19*b))*log(a**10*sqrt(-1/(a**19*b)) + x)/131072 + (3363003*a**
8*x + 16759722*a**7*b*x**3 + 44765658*a**6*b**2*x**5 + 73947042*a**5*b**3*x
**7 + 79659008*a**4*b**4*x**9 + 56404062*a**3*b**5*x**11 + 25423398*a**2*b*
*6*x**13 + 6636630*a*b**7*x**15 + 765765*b**8*x**17)/(4128768*a**18 + 37158
912*a**17*b*x**2 + 148635648*a**16*b**2*x**4 + 346816512*a**15*b**3*x**6 +
520224768*a**14*b**4*x**8 + 520224768*a**13*b**5*x**10 + 346816512*a**12*b*
*6*x**12 + 148635648*a**11*b**7*x**14 + 37158912*a**10*b**8*x**16 + 4128768
*a**9*b**9*x**18)
```

Giac [A]
time = 1.39, size = 122, normalized size = 0.67

$$\frac{12155\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^9}+\frac{765765\,b^8x^{17}+6636630\,ab^7x^{15}+25423398\,a^2b^6x^{13}+56404062\,a^3b^5x^{11}+79659008\,a^4b^4x^9+73947042\,a^5b^3x^7+44765658\,a^6b^2x^5+16759722\,a^7bx^3+3363003\,a^8x}{4128768\,(bx^2+a)^9a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^9) + 1/4128768*(765765*b^8*x
^17 + 6636630*a*b^7*x^15 + 25423398*a^2*b^6*x^13 + 56404062*a^3*b^5*x^11 +
79659008*a^4*b^4*x^9 + 73947042*a^5*b^3*x^7 + 44765658*a^6*b^2*x^5 + 167597
22*a^7*b*x^3 + 3363003*a^8*x)/((b*x^2 + a)^9*a^9)
```

Mupad [B]
time = 4.74, size = 209, normalized size = 1.15

$$\frac{\frac{53381\,x}{65536\,a}+\frac{399041\,b\,x^3}{98304\,a^2}+\frac{355283\,b^2\,x^5}{32768\,a^3}+\frac{4108169\,b^3\,x^7}{229376\,a^4}+\frac{2431\,b^4\,x^9}{126\,a^5}+\frac{3133559\,b^5\,x^{11}}{229376\,a^6}+\frac{201773\,b^6\,x^{13}}{32768\,a^7}+\frac{158015\,b^7\,x^{15}}{98304\,a^8}+\frac{12155\,b^8\,x^{17}}{65536\,a^9}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{12155\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{19/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^10,x)
```

```
[Out] ((53381*x)/(65536*a) + (399041*b*x^3)/(98304*a^2) + (355283*b^2*x^5)/(32768
*a^3) + (4108169*b^3*x^7)/(229376*a^4) + (2431*b^4*x^9)/(126*a^5) + (313355
9*b^5*x^11)/(229376*a^6) + (201773*b^6*x^13)/(32768*a^7) + (158015*b^7*x^15
)/(98304*a^8) + (12155*b^8*x^17)/(65536*a^9))/(a^9 + b^9*x^18 + 9*a^8*b*x^2
 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a
^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (12155*atan((b^(1/2)*x)/
a^(1/2)))/(65536*a^(19/2)*b^(1/2))
```

3.222 $\int \frac{1}{x^2(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=209

$$-\frac{230945}{65536a^{10}x}+\frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{1615}{16128a^4x\left(a+bx^2\right)^6}+\frac{4199}{32256a^5x\left(a\right.}$$

```
[Out] -230945/65536/a^10/x+1/18/a/x/(b*x^2+a)^9+19/288/a^2/x/(b*x^2+a)^8+323/4032
/a^3/x/(b*x^2+a)^7+1615/16128/a^4/x/(b*x^2+a)^6+4199/32256/a^5/x/(b*x^2+a)^
5+46189/258048/a^6/x/(b*x^2+a)^4+46189/172032/a^7/x/(b*x^2+a)^3+46189/98304
/a^8/x/(b*x^2+a)^2+230945/196608/a^9/x/(b*x^2+a)-230945/65536*arctan(x*b^(1
/2)/a^(1/2))*b^(1/2)/a^(21/2)
```

Rubi [A]
time = 0.09, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {296, 331, 211}

$$-\frac{230945\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{21/2}}-\frac{230945}{65536a^{10}x}+\frac{230945}{196608a^9x\left(a+bx^2\right)}+\frac{46189}{98304a^8x\left(a+bx^2\right)^2}+\frac{46189}{172032a^7x\left(a+bx^2\right)^3}+\frac{46189}{258048a^6x\left(a+bx^2\right)^4}+\frac{4199}{32256a^5x\left(a+bx^2\right)^5}+\frac{1615}{16128a^4x\left(a+bx^2\right)^6}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{1}{18ax\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^10),x]
```

```
[Out] -230945/(65536*a^10*x) + 1/(18*a*x*(a + b*x^2)^9) + 19/(288*a^2*x*(a + b*x^
2)^8) + 323/(4032*a^3*x*(a + b*x^2)^7) + 1615/(16128*a^4*x*(a + b*x^2)^6) +
 4199/(32256*a^5*x*(a + b*x^2)^5) + 46189/(258048*a^6*x*(a + b*x^2)^4) + 46
189/(172032*a^7*x*(a + b*x^2)^3) + 46189/(98304*a^8*x*(a + b*x^2)^2) + 2309
45/(196608*a^9*x*(a + b*x^2)) - (230945*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(65536*a^(21/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^2)^{10}}\,dx = \frac{1}{18ax(a+bx^2)^9} + \frac{19\int \frac{1}{x^2(a+bx^2)^9}\,dx}{18a}$$

$$= \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323\int \frac{1}{x^2(a+bx^2)^8}\,dx}{288a^2}$$

$$= \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{1615\int \frac{1}{x^2(a+bx^2)^7}\,dx}{1344a^3}$$

$$= \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{1615}{16128a^4x(a+bx^2)^6} +$$

$$= \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{1615}{16128a^4x(a+bx^2)^6} +$$

$$= \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{1615}{16128a^4x(a+bx^2)^6} +$$

$$= \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{1615}{16128a^4x(a+bx^2)^6} +$$

$$= \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{1615}{16128a^4x(a+bx^2)^6} +$$

$$= \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{1615}{16128a^4x(a+bx^2)^6} +$$

$$= -\frac{230945}{65536a^{10}x} + \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{}{16128}$$

$$= -\frac{230945}{65536a^{10}x} + \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{}{16128}$$

Mathematica [A]

time = 0.07, size = 147, normalized size = 0.70

$$\frac{-\frac{\sqrt{a}\left(4128768a^9+63897057a^8bx^2+318434718a^7b^2x^4+850547502a^6b^3x^6+1404993798a^5b^4x^8+1513521152a^4b^5x^{10}+1071677178a^3b^6x^{12}+483044562a^2b^7x^{14}+126095970ab^8x^{16}+14549535b^9x^{18}\right)}{x(a+bx^2)^9}-14549535\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{4128768a^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^10),x]
```

```
[Out] (-((Sqrt[a]*(4128768*a^9 + 63897057*a^8*b*x^2 + 318434718*a^7*b^2*x^4 + 850
547502*a^6*b^3*x^6 + 1404993798*a^5*b^4*x^8 + 1513521152*a^4*b^5*x^10 + 107
1677178*a^3*b^6*x^12 + 483044562*a^2*b^7*x^14 + 126095970*a*b^8*x^16 + 1454
9535*b^9*x^18))/(x*(a + b*x^2)^9)) - 14549535*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(4128768*a^(21/2))
```

Maple [A]
time = 0.10, size = 131, normalized size = 0.63

method	result
default	$-\frac{b\left(\frac{\frac{424415}{65536}a^{8}x+\frac{4042835}{98304}a^{7}b\,x^{3}+\frac{3997865}{32768}a^{6}b^{2}x^{5}+\frac{49153835}{229376}a^{5}b^{3}x^{7}+\frac{30313}{126}a^{4}b^{4}x^{9}+\frac{40270037}{229376}a^{3}b^{5}x^{11}+\frac{2654039}{32768}a^{2}b^{6}x^{13}+\frac{2117549}{98304}a\,b^{7}x^{15}+\frac{165}{65}}{\left(b\,x^{2}+a\right)^{9}}\right.}{a^{10}}$
risch	$\frac{-\frac{1}{a}-\frac{1014239b\,x^{2}}{65536a^{2}}-\frac{7581779b^{2}x^{4}}{98304a^{3}}-\frac{6750377b^{3}x^{6}}{32768a^{4}}-\frac{78055211b^{4}x^{8}}{229376a^{5}}-\frac{46189b^{5}x^{10}}{126a^{6}}-\frac{595537621b^{6}x^{12}}{229376a^{7}}-\frac{3833687b^{7}x^{14}}{32768a^{8}}-\frac{3002285b^{8}x^{16}}{98304a^{9}}-\frac{230945b^{9}x^{18}}{65536a^{10}}}{x(b\,x^{2}+a)^{9}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^10*((424415/65536*a^8*x+4042835/98304*a^7*b*x^3+3997865/32768*a^6*b^2*
x^5+49153835/229376*a^5*b^3*x^7+30313/126*a^4*b^4*x^9+40270037/229376*a^3*b
^5*x^11+2654039/32768*a^2*b^6*x^13+2117549/98304*a*b^7*x^15+165409/65536*b^
8*x^17)/(b*x^2+a)^9+230945/65536/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/a^1
0/x
```

Maxima [A]
time = 0.54, size = 225, normalized size = 1.08

$$-\frac{14549535\,b^{9}x^{18}+126095970\,ab^{8}x^{16}+483044562\,a^{2}b^{7}x^{14}+1071677178\,a^{3}b^{6}x^{12}+1513521152\,a^{4}b^{5}x^{10}+1404993798\,a^{5}b^{4}x^{8}+850547502\,a^{6}b^{3}x^{6}+318434718\,a^{7}b^{2}x^{4}+63897057\,a^{8}bx^{2}+4128768\,a^{9}}{4128768\,(a^{10}b^{9}x^{19}+9\,a^{11}b^{8}x^{17}+36\,a^{12}b^{7}x^{15}+84\,a^{13}b^{6}x^{13}+126\,a^{14}b^{5}x^{11}+126\,a^{15}b^{4}x^{9}+84\,a^{16}b^{3}x^{7}+36\,a^{17}b^{2}x^{5}+9\,a^{18}bx^{3}+a^{19}x)}-\frac{230945\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/4128768*(14549535*b^9*x^18 + 126095970*a*b^8*x^16 + 483044562*a^2*b^7*x^
14 + 1071677178*a^3*b^6*x^12 + 1513521152*a^4*b^5*x^10 + 1404993798*a^5*b^4
*x^8 + 850547502*a^6*b^3*x^6 + 318434718*a^7*b^2*x^4 + 63897057*a^8*b*x^2 +
 4128768*a^9)/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15 + 84*a^13
*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^7 + 36*a^1
```

7*b^2*x^5 + 9*a^18*b*x^3 + a^19*x) - 230945/65536*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^10)

Fricas [A]
time = 1.09, size = 664, normalized size = 3.18

$$\left[-\frac{29099070\,b^{9}x^{18} + 252191940\,ab^{8}x^{16} + 966089124\,a^{2}b^{7}x^{14} + 2143354356\,a^{3}b^{6}x^{12} + 3027042304\,a^{4}b^{5}x^{10} + 2809987596\,a^{5}b^{4}x^{8} + 1701095004\,a^{6}b^{3}x^{6} + 636869436\,a^{7}b^{2}x^{4} + 127794114\,a^{8}bx^{2} + 8257536\,a^{9} - 14549535\,(b^{9}x^{19} + 9\,ab^{8}x^{17} + 36\,a^{2}b^{7}x^{15} + 84\,a^{3}b^{6}x^{13} + 126\,a^{4}b^{5}x^{11} + 126\,a^{5}b^{4}x^{9} + 84\,a^{6}b^{3}x^{7} + 36\,a^{7}b^{2}x^{5} + 9\,a^{8}bx^{3} + a^{9}x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2} - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^{2} + a}\right)}{8257536\,(a^{10}b^{9}x^{19} + 9\,a^{11}b^{8}x^{17} + 36\,a^{12}b^{7}x^{15} + 84\,a^{13}b^{6}x^{13} + 126\,a^{14}b^{5}x^{11} + 126\,a^{15}b^{4}x^{9} + 84\,a^{16}b^{3}x^{7} + 36\,a^{17}b^{2}x^{5} + 9\,a^{18}bx^{3} + a^{19}x)}, -\frac{14549535\,b^{9}x^{18} + 126095970\,ab^{8}x^{16} + 483044562\,a^{2}b^{7}x^{14} + 1071677178\,a^{3}b^{6}x^{12} + 1513521152\,a^{4}b^{5}x^{10} + 1404993798\,a^{5}b^{4}x^{8} + 850547502\,a^{6}b^{3}x^{6} + 318434718\,a^{7}b^{2}x^{4} + 63897057\,a^{8}bx^{2} + 4128768\,a^{9} + 14549535\,(b^{9}x^{19} + 9\,ab^{8}x^{17} + 36\,a^{2}b^{7}x^{15} + 84\,a^{3}b^{6}x^{13} + 126\,a^{4}b^{5}x^{11} + 126\,a^{5}b^{4}x^{9} + 84\,a^{6}b^{3}x^{7} + 36\,a^{7}b^{2}x^{5} + 9\,a^{8}bx^{3} + a^{9}x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{4128768\,(a^{10}b^{9}x^{19} + 9\,a^{11}b^{8}x^{17} + 36\,a^{12}b^{7}x^{15} + 84\,a^{13}b^{6}x^{13} + 126\,a^{14}b^{5}x^{11} + 126\,a^{15}b^{4}x^{9} + 84\,a^{16}b^{3}x^{7} + 36\,a^{17}b^{2}x^{5} + 9\,a^{18}bx^{3} + a^{19}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="fricas")

[Out] [-1/8257536*(29099070*b^9*x^18 + 252191940*a*b^8*x^16 + 966089124*a^2*b^7*x^14 + 2143354356*a^3*b^6*x^12 + 3027042304*a^4*b^5*x^10 + 2809987596*a^5*b^4*x^8 + 1701095004*a^6*b^3*x^6 + 636869436*a^7*b^2*x^4 + 127794114*a^8*b*x^2 + 8257536*a^9 - 14549535*(b^9*x^19 + 9*a*b^8*x^17 + 36*a^2*b^7*x^15 + 84*a^3*b^6*x^13 + 126*a^4*b^5*x^11 + 126*a^5*b^4*x^9 + 84*a^6*b^3*x^7 + 36*a^7*b^2*x^5 + 9*a^8*b*x^3 + a^9*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15 + 84*a^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^7 + 36*a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x), -1/4128768*(14549535*b^9*x^18 + 126095970*a*b^8*x^16 + 483044562*a^2*b^7*x^14 + 1071677178*a^3*b^6*x^12 + 1513521152*a^4*b^5*x^10 + 1404993798*a^5*b^4*x^8 + 850547502*a^6*b^3*x^6 + 318434718*a^7*b^2*x^4 + 63897057*a^8*b*x^2 + 4128768*a^9 + 14549535*(b^9*x^19 + 9*a*b^8*x^17 + 36*a^2*b^7*x^15 + 84*a^3*b^6*x^13 + 126*a^4*b^5*x^11 + 126*a^5*b^4*x^9 + 84*a^6*b^3*x^7 + 36*a^7*b^2*x^5 + 9*a^8*b*x^3 + a^9*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15 + 84*a^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^7 + 36*a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x)]

Sympy [A]
time = 0.62, size = 282, normalized size = 1.35

$$\frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(-\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b} + x\right)}{131072} - \frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b} + x\right)}{131072} + \frac{-4128768a^{9} - 63897057a^{8}bx^{2} - 318434718a^{7}b^{2}x^{4} - 850547502a^{6}b^{3}x^{6} - 1404993798a^{5}b^{4}x^{8} - 1513521152a^{4}b^{5}x^{10} - 1071677178a^{3}b^{6}x^{12} - 483044562a^{2}b^{7}x^{14} - 126095970ab^{8}x^{16} - 14549535b^{9}x^{18}}{4128768a^{19}x + 37158912a^{18}bx^{3} + 148635648a^{17}b^{2}x^{5} + 346816512a^{16}b^{3}x^{7} + 520224768a^{15}b^{4}x^{9} + 520224768a^{14}b^{5}x^{11} + 346816512a^{13}b^{6}x^{13} + 148635648a^{12}b^{7}x^{15} + 37158912a^{11}b^{8}x^{17} + 4128768a^{10}b^{9}x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**2+a)**10,x)

[Out] 230945*sqrt(-b/a**21)*log(-a**11*sqrt(-b/a**21)/b + x)/131072 - 230945*sqrt(-b/a**21)*log(a**11*sqrt(-b/a**21)/b + x)/131072 + (-4128768*a**9 - 63897057*a**8*b*x**2 - 318434718*a**7*b**2*x**4 - 850547502*a**6*b**3*x**6 - 1404993798*a**5*b**4*x**8 - 1513521152*a**4*b**5*x**10 - 1071677178*a**3*b**6*x**12 - 483044562*a**2*b**7*x**14 - 126095970*a*b**8*x**16 - 14549535*b**9*x**18)/(4128768*a**19*x + 37158912*a**18*b*x**3 + 148635648*a**17*b**2*x**5 + 346816512*a**16*b**3*x**7 + 520224768*a**15*b**4*x**9 + 520224768*a**14*b

```
**5*x**11 + 346816512*a**13*b**6*x**13 + 148635648*a**12*b**7*x**15 + 37158
912*a**11*b**8*x**17 + 4128768*a**10*b**9*x**19)
```

Giac [A]
time = 1.60, size = 134, normalized size = 0.64

$$-\frac{230945\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{10}}-\frac{1}{a^{10}x}-\frac{10420767\,b^{9}x^{17}+88937058\,ab^{8}x^{15}+334408914\,a^{2}b^{7}x^{13}+724860666\,a^{3}b^{6}x^{11}+993296384\,a^{4}b^{5}x^{9}+884769030\,a^{5}b^{4}x^{7}+503730990\,a^{6}b^{3}x^{5}+169799070\,a^{7}b^{2}x^{3}+26738145\,a^{8}bx}{4128768\,(bx^{2}+a)^{9}a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -230945/65536*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^10) - 1/(a^10*x) - 1/412
8768*(10420767*b^9*x^17 + 88937058*a*b^8*x^15 + 334408914*a^2*b^7*x^13 + 72
4860666*a^3*b^6*x^11 + 993296384*a^4*b^5*x^9 + 884769030*a^5*b^4*x^7 + 5037
30990*a^6*b^3*x^5 + 169799070*a^7*b^2*x^3 + 26738145*a^8*b*x)/((b*x^2 + a)^
9*a^10)
```

Mupad [B]
time = 5.09, size = 220, normalized size = 1.05

$$-\frac{\frac{1}{a}+\frac{1014239\,b\,x^{2}}{65536\,a^{2}}+\frac{7581779\,b^{2}\,x^{4}}{98304\,a^{3}}+\frac{6750377\,b^{3}\,x^{6}}{32768\,a^{4}}+\frac{78055211\,b^{4}\,x^{8}}{229376\,a^{5}}+\frac{46189\,b^{5}\,x^{10}}{126\,a^{6}}+\frac{59537621\,b^{6}\,x^{12}}{229376\,a^{7}}+\frac{3833687\,b^{7}\,x^{14}}{32768\,a^{8}}+\frac{3002285\,b^{8}\,x^{16}}{98304\,a^{9}}+\frac{230945\,b^{9}\,x^{18}}{65536\,a^{10}}}{a^{9}\,x+9\,a^{8}\,b\,x^{3}+36\,a^{7}\,b^{2}\,x^{5}+84\,a^{6}\,b^{3}\,x^{7}+126\,a^{5}\,b^{4}\,x^{9}+126\,a^{4}\,b^{5}\,x^{11}+84\,a^{3}\,b^{6}\,x^{13}+36\,a^{2}\,b^{7}\,x^{15}+9\,a\,b^{8}\,x^{17}+b^{9}\,x^{19}}-\frac{230945\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{21/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^10),x)
```

```
[Out] - (1/a + (1014239*b*x^2)/(65536*a^2) + (7581779*b^2*x^4)/(98304*a^3) + (675
0377*b^3*x^6)/(32768*a^4) + (78055211*b^4*x^8)/(229376*a^5) + (46189*b^5*x^
10)/(126*a^6) + (59537621*b^6*x^12)/(229376*a^7) + (3833687*b^7*x^14)/(3276
8*a^8) + (3002285*b^8*x^16)/(98304*a^9) + (230945*b^9*x^18)/(65536*a^10))/(
a^9*x + b^9*x^19 + 9*a^8*b*x^3 + 9*a*b^8*x^17 + 36*a^7*b^2*x^5 + 84*a^6*b^3
*x^7 + 126*a^5*b^4*x^9 + 126*a^4*b^5*x^11 + 84*a^3*b^6*x^13 + 36*a^2*b^7*x^
15) - (230945*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(21/2))
```

3.223 $\int \frac{1}{x^4(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=220

$$-\frac{1616615}{196608a^{10}x^3}+\frac{1616615b}{65536a^{11}x}+\frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right.}$$

```
[Out] -1616615/196608/a^10/x^3+1616615/65536*b/a^11/x+1/18/a/x^3/(b*x^2+a)^9+7/96
/a^2/x^3/(b*x^2+a)^8+19/192/a^3/x^3/(b*x^2+a)^7+323/2304/a^4/x^3/(b*x^2+a)^
6+323/1536/a^5/x^3/(b*x^2+a)^5+4199/12288/a^6/x^3/(b*x^2+a)^4+46189/73728/a
^7/x^3/(b*x^2+a)^3+46189/32768/a^8/x^3/(b*x^2+a)^2+323323/65536/a^9/x^3/(b*
x^2+a)+1616615/65536*b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(23/2)
```

Rubi [A]
time = 0.10, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {296, 331, 211}

$$\frac{1616615b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{23/2}}+\frac{1616615b}{65536a^{11}x}-\frac{1616615}{196608a^{10}x^3}+\frac{323323}{65536a^9x^3\left(a+bx^2\right)}+\frac{46189}{32768a^8x^3\left(a+bx^2\right)^2}+\frac{46189}{73728a^7x^3\left(a+bx^2\right)^3}+\frac{4199}{12288a^6x^3\left(a+bx^2\right)^4}+\frac{323}{1536a^5x^3\left(a+bx^2\right)^5}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{1}{18ax^3\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^10),x]
```

```
[Out] -1616615/(196608*a^10*x^3) + (1616615*b)/(65536*a^11*x) + 1/(18*a*x^3*(a +
b*x^2)^9) + 7/(96*a^2*x^3*(a + b*x^2)^8) + 19/(192*a^3*x^3*(a + b*x^2)^7) +
 323/(2304*a^4*x^3*(a + b*x^2)^6) + 323/(1536*a^5*x^3*(a + b*x^2)^5) + 4199
/(12288*a^6*x^3*(a + b*x^2)^4) + 46189/(73728*a^7*x^3*(a + b*x^2)^3) + 4618
9/(32768*a^8*x^3*(a + b*x^2)^2) + 323323/(65536*a^9*x^3*(a + b*x^2)) + (161
6615*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(23/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{10}}\,dx = \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7\int \frac{1}{x^4(a+bx^2)^9}\,dx}{6a}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{133\int \frac{1}{x^4(a+bx^2)^8}\,dx}{96a^2}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323\int \frac{1}{x^4(a+bx^2)^7}\,dx}{192a^3}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+$$

$$= -\frac{1616615}{196608a^{10}x^3}+\frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{}{2304}$$

$$= -\frac{1616615}{196608a^{10}x^3}+\frac{1616615b}{65536a^{11}x}+\frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+\right.}$$

$$= -\frac{1616615}{196608a^{10}x^3}+\frac{1616615b}{65536a^{11}x}+\frac{1}{18ax^3\left(a+bx^2\right)^9}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{19}{192a^3x^3\left(a+\right.}$$

Mathematica [A]

time = 0.06, size = 157, normalized size = 0.71

$$\frac{\frac{\sqrt{a}\,(-196608a^{10}+4128768a^9bx^2+63897057a^8b^2x^4+318434718a^7b^3x^6+850547502a^6b^4x^8+1404993798a^5b^5x^{10}+1513521152a^4b^6x^{12}+1071677178a^3b^7x^{14}+483044562a^2b^8x^{16}+126095970ab^9x^{18}+14549535b^{10}x^{20})}{x^3(a+bx^2)^9}+14549535b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{589824a^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^10),x]
```

```
[Out] ((Sqrt[a]*(-196608*a^10 + 4128768*a^9*b*x^2 + 63897057*a^8*b^2*x^4 + 318434
718*a^7*b^3*x^6 + 850547502*a^6*b^4*x^8 + 1404993798*a^5*b^5*x^10 + 1513521
152*a^4*b^6*x^12 + 1071677178*a^3*b^7*x^14 + 483044562*a^2*b^8*x^16 + 12609
5970*a*b^9*x^18 + 14549535*b^10*x^20))/(x^3*(a + b*x^2)^9) + 14549535*b^(3/
2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*a^(23/2))
```

Maple [A]
time = 0.13, size = 141, normalized size = 0.64

method	result
default	$\frac{b^2\left(\frac{\frac{1987865}{65536}a^8x+\frac{20435525}{98304}a^7bx^3+\frac{21103775}{32768}a^6b^2x^5+\frac{38143787}{32768}a^5b^3x^7+\frac{24013}{18}a^4b^4x^9+\frac{32405717}{32768}a^3b^5x^{11}+\frac{15137633}{32768}a^2b^6x^{13}+\frac{12201403}{98304}ab^7x^{1}}{(bx^2+a)^9}\right.}{a^{11}}$
risch	$\frac{-\frac{1}{3a}+\frac{7bx^2}{a^2}+\frac{7099673b^2x^4}{65536a^3}+\frac{530724531b^3x^6}{98304a^4}+\frac{47252639b^4x^8}{32768a^5}+\frac{78055211b^5x^{10}}{32768a^6}+\frac{46189b^6x^{12}}{18a^7}+\frac{595376621b^7x^{14}}{32768a^8}+\frac{268358809b^8x^{16}}{32768a^9}+\frac{21015995b^9x^{18}}{98304a^{10}}}{x^3(bx^2+a)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^11*((1987865/65536*a^8*x+20435525/98304*a^7*b*x^3+21103775/32768*a^6*
b^2*x^5+38143787/32768*a^5*b^3*x^7+24013/18*a^4*b^4*x^9+32405717/32768*a^3*
b^5*x^11+15137633/32768*a^2*b^6*x^13+12201403/98304*a*b^7*x^15+961255/65536
*b^8*x^17)/(b*x^2+a)^9+1616615/65536/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1
/3/a^10/x^3+10*b/a^11/x
```

Maxima [A]
time = 0.54, size = 240, normalized size = 1.09

$$\frac{14549535\,b^{10}x^{20}+126095970\,ab^9x^{18}+483044562\,a^2b^8x^{16}+1071677178\,a^3b^7x^{14}+1513521152\,a^4b^6x^{12}+1404993798\,a^5b^5x^{10}+850547502\,a^6b^4x^8+318434718\,a^7b^3x^6+63897057\,a^8b^2x^4+4128768\,a^9bx^2-196608\,a^{10}}{589824\,(a^{11}b^9x^{21}+9\,a^{12}b^8x^{19}+36\,a^{13}b^7x^{17}+84\,a^{14}b^6x^{15}+126\,a^{15}b^5x^{13}+126\,a^{16}b^4x^{11}+84\,a^{17}b^3x^9+36\,a^{18}b^2x^7+9\,a^{19}bx^5+a^{20}x^3)}+\frac{1616615\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/589824*(14549535*b^10*x^20 + 126095970*a*b^9*x^18 + 483044562*a^2*b^8*x^1
6 + 1071677178*a^3*b^7*x^14 + 1513521152*a^4*b^6*x^12 + 1404993798*a^5*b^5*
x^10 + 850547502*a^6*b^4*x^8 + 318434718*a^7*b^3*x^6 + 63897057*a^8*b^2*x^4
```

```
 + 4128768*a^9*b*x^2 - 196608*a^10)/(a^11*b^9*x^21 + 9*a^12*b^8*x^19 + 36*a
^13*b^7*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*b^4*x^11 + 8
4*a^17*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3) + 1616615/65536
*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^11)
```

Fricas [A]
time = 0.97, size = 700, normalized size = 3.18

$$\left[\frac{29099070\,b^{10}x^{20}+252191940\,ab^{9}x^{18}+966089124\,a^{2}b^{8}x^{16}+2143354356\,a^{3}b^{7}x^{14}+3027042304\,a^{4}b^{6}x^{12}+2809987596\,a^{5}b^{5}x^{10}+1701095004\,a^{6}b^{4}x^{8}+636869436\,a^{7}b^{3}x^{6}+127794114\,a^{8}b^{2}x^{4}+8257536\,a^{9}bx^{2}-393216\,a^{10}+14549535\,(b^{10}x^{21}+9\,ab^{9}x^{19}+36\,a^{2}b^{8}x^{17}+84\,a^{3}b^{7}x^{15}+126\,a^{4}b^{6}x^{13}+126\,a^{5}b^{5}x^{11}+84\,a^{6}b^{4}x^{9}+36\,a^{7}b^{3}x^{7}+9\,a^{8}b^{2}x^{5}+a^{9}bx^{3})\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{1179648\,(a^{11}b^{9}x^{21}+9\,a^{12}b^{8}x^{19}+36\,a^{13}b^{7}x^{17}+84\,a^{14}b^{6}x^{15}+126\,a^{15}b^{5}x^{13}+126\,a^{16}b^{4}x^{11}+84\,a^{17}b^{3}x^{9}+36\,a^{18}b^{2}x^{7}+9\,a^{19}bx^{5}+a^{20}x^{3})},\ \frac{14549535\,b^{10}x^{20}+126095970\,ab^{9}x^{18}+483044562\,a^{2}b^{8}x^{16}+1071677178\,a^{3}b^{7}x^{14}+1513521152\,a^{4}b^{6}x^{12}+1404993798\,a^{5}b^{5}x^{10}+850547502\,a^{6}b^{4}x^{8}+318434718\,a^{7}b^{3}x^{6}+63897057\,a^{8}b^{2}x^{4}+4128768\,a^{9}bx^{2}-196608\,a^{10}+14549535\,(b^{10}x^{21}+9\,ab^{9}x^{19}+36\,a^{2}b^{8}x^{17}+84\,a^{3}b^{7}x^{15}+126\,a^{4}b^{6}x^{13}+126\,a^{5}b^{5}x^{11}+84\,a^{6}b^{4}x^{9}+36\,a^{7}b^{3}x^{7}+9\,a^{8}b^{2}x^{5}+a^{9}bx^{3})\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{589824\,(a^{11}b^{9}x^{21}+9\,a^{12}b^{8}x^{19}+36\,a^{13}b^{7}x^{17}+84\,a^{14}b^{6}x^{15}+126\,a^{15}b^{5}x^{13}+126\,a^{16}b^{4}x^{11}+84\,a^{17}b^{3}x^{9}+36\,a^{18}b^{2}x^{7}+9\,a^{19}bx^{5}+a^{20}x^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(29099070*b^10*x^20 + 252191940*a*b^9*x^18 + 966089124*a^2*b^8*x
^16 + 2143354356*a^3*b^7*x^14 + 3027042304*a^4*b^6*x^12 + 2809987596*a^5*b^
5*x^10 + 1701095004*a^6*b^4*x^8 + 636869436*a^7*b^3*x^6 + 127794114*a^8*b^2
*x^4 + 8257536*a^9*b*x^2 - 393216*a^10 + 14549535*(b^10*x^21 + 9*a*b^9*x^19
 + 36*a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b^5*x^11
+ 84*a^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^11*b^9*x^21 + 9*a^12*b^8
*x^19 + 36*a^13*b^7*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*
b^4*x^11 + 84*a^17*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3), 1/
589824*(14549535*b^10*x^20 + 126095970*a*b^9*x^18 + 483044562*a^2*b^8*x^16
+ 1071677178*a^3*b^7*x^14 + 1513521152*a^4*b^6*x^12 + 1404993798*a^5*b^5*x^
10 + 850547502*a^6*b^4*x^8 + 318434718*a^7*b^3*x^6 + 63897057*a^8*b^2*x^4 +
 4128768*a^9*b*x^2 - 196608*a^10 + 14549535*(b^10*x^21 + 9*a*b^9*x^19 + 36*
a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b^5*x^11 + 84*a
^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqrt(b/a)*arctan(x
*sqrt(b/a)))/(a^11*b^9*x^21 + 9*a^12*b^8*x^19 + 36*a^13*b^7*x^17 + 84*a^14*
b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*b^4*x^11 + 84*a^17*b^3*x^9 + 36*a^1
8*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3)]
```

Sympy [A]
time = 0.65, size = 304, normalized size = 1.38

$$-\frac{1616615\sqrt{-\frac{b^{3}}{a^{23}}}\log\left(-\frac{a^{12}\sqrt{-\frac{b^{3}}{a^{23}}}}{b^{2}}+x\right)}{131072}+\frac{1616615\sqrt{-\frac{b^{3}}{a^{23}}}\log\left(\frac{a^{12}\sqrt{-\frac{b^{3}}{a^{23}}}}{b^{2}}+x\right)}{131072}+\frac{-196608a^{10}+4128768a^{9}bx^{2}+63897057a^{8}b^{2}x^{4}+318434718a^{7}b^{3}x^{6}+850547502a^{6}b^{4}x^{8}+1404993798a^{5}b^{5}x^{10}+1513521152a^{4}b^{6}x^{12}+1071677178a^{3}b^{7}x^{14}+483044562a^{2}b^{8}x^{16}+126095970ab^{9}x^{18}+14549535b^{10}x^{20}}{589824a^{20}x^{3}+5308416a^{19}bx^{5}+21233664a^{18}b^{2}x^{7}+49545216a^{17}b^{3}x^{9}+74317824a^{16}b^{4}x^{11}+74317824a^{15}b^{5}x^{13}+49545216a^{14}b^{6}x^{15}+21233664a^{13}b^{7}x^{17}+5308416a^{12}b^{8}x^{19}+589824a^{11}b^{9}x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**10,x)
```

```
[Out] -1616615*sqrt(-b**3/a**23)*log(-a**12*sqrt(-b**3/a**23)/b**2 + x)/131072 +
1616615*sqrt(-b**3/a**23)*log(a**12*sqrt(-b**3/a**23)/b**2 + x)/131072 + (-
196608*a**10 + 4128768*a**9*b*x**2 + 63897057*a**8*b**2*x**4 + 318434718*a*
*7*b**3*x**6 + 850547502*a**6*b**4*x**8 + 1404993798*a**5*b**5*x**10 + 1513
521152*a**4*b**6*x**12 + 1071677178*a**3*b**7*x**14 + 483044562*a**2*b**8*x
```

```
**16 + 126095970*a*b**9*x**18 + 14549535*b**10*x**20)/(589824*a**20*x**3 +
5308416*a**19*b*x**5 + 21233664*a**18*b**2*x**7 + 49545216*a**17*b**3*x**9
+ 74317824*a**16*b**4*x**11 + 74317824*a**15*b**5*x**13 + 49545216*a**14*b*
*6*x**15 + 21233664*a**13*b**7*x**17 + 5308416*a**12*b**8*x**19 + 589824*a*
*11*b**9*x**21)
```

Giac [A]
time = 1.31, size = 148, normalized size = 0.67

$$\frac{1616615\, b^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\, \sqrt{ab}\, a^{11}} + \frac{30\, bx^{2} - a}{3\, a^{11} x^{3}} + \frac{8651295\, b^{10} x^{17} + 73208418\, ab^{9} x^{15} + 272477394\, a^{2} b^{8} x^{13} + 583302906\, a^{3} b^{7} x^{11} + 786857984\, a^{4} b^{6} x^{9} + 686588166\, a^{5} b^{5} x^{7} + 379867950\, a^{6} b^{4} x^{5} + 122613150\, a^{7} b^{3} x^{3} + 17890785\, a^{8} b^{2} x}{589824\, (bx^{2} + a)^{9} a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1616615/65536*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^11) + 1/3*(30*b*x^2 -
a)/(a^11*x^3) + 1/589824*(8651295*b^10*x^17 + 73208418*a*b^9*x^15 + 2724773
94*a^2*b^8*x^13 + 583302906*a^3*b^7*x^11 + 786857984*a^4*b^6*x^9 + 68658816
6*a^5*b^5*x^7 + 379867950*a^6*b^4*x^5 + 122613150*a^7*b^3*x^3 + 17890785*a^
8*b^2*x)/((b*x^2 + a)^9*a^11)
```

Mupad [B]
time = 5.11, size = 234, normalized size = 1.06

$$\frac{\frac{7\, b\, x^{2}}{a^{2}} - \frac{1}{3\, a} + \frac{7099673\, b^{2}\, x^{4}}{65536\, a^{3}} + \frac{53072453\, b^{3}\, x^{6}}{98304\, a^{4}} + \frac{47252639\, b^{4}\, x^{8}}{32768\, a^{5}} + \frac{78055211\, b^{5}\, x^{10}}{32768\, a^{6}} + \frac{46189\, b^{6}\, x^{12}}{18\, a^{7}} + \frac{59537621\, b^{7}\, x^{14}}{32768\, a^{8}} + \frac{26835809\, b^{8}\, x^{16}}{32768\, a^{9}} + \frac{21015995\, b^{9}\, x^{18}}{98304\, a^{10}} + \frac{1616615\, b^{10}\, x^{20}}{65536\, a^{11}}}{a^{9}\, x^{3} + 9\, a^{8}\, b\, x^{5} + 36\, a^{7}\, b^{2}\, x^{7} + 84\, a^{6}\, b^{3}\, x^{9} + 126\, a^{5}\, b^{4}\, x^{11} + 126\, a^{4}\, b^{5}\, x^{13} + 84\, a^{3}\, b^{6}\, x^{15} + 36\, a^{2}\, b^{7}\, x^{17} + 9\, a\, b^{8}\, x^{19} + b^{9}\, x^{21}} + \frac{1616615\, b^{3/2}\, \mathrm{atan}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{65536\, a^{23/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^10),x)
```

```
[Out] ((7*b*x^2)/a^2 - 1/(3*a) + (7099673*b^2*x^4)/(65536*a^3) + (53072453*b^3*x^
6)/(98304*a^4) + (47252639*b^4*x^8)/(32768*a^5) + (78055211*b^5*x^10)/(3276
8*a^6) + (46189*b^6*x^12)/(18*a^7) + (59537621*b^7*x^14)/(32768*a^8) + (268
35809*b^8*x^16)/(32768*a^9) + (21015995*b^9*x^18)/(98304*a^10) + (1616615*b
^10*x^20)/(65536*a^11))/(a^9*x^3 + b^9*x^21 + 9*a^8*b*x^5 + 9*a*b^8*x^19 +
36*a^7*b^2*x^7 + 84*a^6*b^3*x^9 + 126*a^5*b^4*x^11 + 126*a^4*b^5*x^13 + 84*
a^3*b^6*x^15 + 36*a^2*b^7*x^17) + (1616615*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)
))/(65536*a^(23/2))
```

3.224 $\int \frac{1}{x^6(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=233

$$-\frac{7436429}{327680a^{10}x^5}+\frac{7436429b}{196608a^{11}x^3}-\frac{7436429b^2}{65536a^{12}x}+\frac{1}{18ax^5\left(a+bx^2\right)^9}+\frac{23}{288a^2x^5\left(a+bx^2\right)^8}+\frac{23}{192a^3x^5\left(a+bx^2\right)^7}+\frac{}{230}$$

```
[Out] -7436429/327680/a^10/x^5+7436429/196608*b/a^11/x^3-7436429/65536*b^2/a^12/x
+1/18/a/x^5/(b*x^2+a)^9+23/288/a^2/x^5/(b*x^2+a)^8+23/192/a^3/x^5/(b*x^2+a)
^7+437/2304/a^4/x^5/(b*x^2+a)^6+7429/23040/a^5/x^5/(b*x^2+a)^5+7429/12288/a
^6/x^5/(b*x^2+a)^4+96577/73728/a^7/x^5/(b*x^2+a)^3+1062347/294912/a^8/x^5/(
b*x^2+a)^2+1062347/65536/a^9/x^5/(b*x^2+a)-7436429/65536*b^(5/2)*arctan(x*b
^(1/2)/a^(1/2))/a^(25/2)
```

Rubi [A]
time = 0.11, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {296, 331, 211}

$$-\frac{7436429b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{25/2}}-\frac{7436429b^2}{65536a^{12}x}+\frac{7436429b}{196608a^{11}x^3}-\frac{7436429}{327680a^{10}x^5}+\frac{1062347}{65536a^9x^5\left(a+bx^2\right)}+\frac{1062347}{294912a^8x^5\left(a+bx^2\right)^2}+\frac{96577}{73728a^7x^5\left(a+bx^2\right)^3}+\frac{7429}{12288a^6x^5\left(a+bx^2\right)^4}+\frac{7429}{23040a^5x^5\left(a+bx^2\right)^5}+\frac{437}{2304a^4x^5\left(a+bx^2\right)^6}+\frac{23}{192a^3x^5\left(a+bx^2\right)^7}+\frac{23}{288a^2x^5\left(a+bx^2\right)^8}+\frac{1}{18ax^5\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^10),x]
```

```
[Out] -7436429/(327680*a^10*x^5) + (7436429*b)/(196608*a^11*x^3) - (7436429*b^2)/
(65536*a^12*x) + 1/(18*a*x^5*(a + b*x^2)^9) + 23/(288*a^2*x^5*(a + b*x^2)^8
) + 23/(192*a^3*x^5*(a + b*x^2)^7) + 437/(2304*a^4*x^5*(a + b*x^2)^6) + 742
9/(23040*a^5*x^5*(a + b*x^2)^5) + 7429/(12288*a^6*x^5*(a + b*x^2)^4) + 9657
7/(73728*a^7*x^5*(a + b*x^2)^3) + 1062347/(294912*a^8*x^5*(a + b*x^2)^2) +
1062347/(65536*a^9*x^5*(a + b*x^2)) - (7436429*b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(65536*a^(25/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6 (a+bx^2)^{10}}\,dx = \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23\int \frac{1}{x^6(a+bx^2)^9}\,dx}{18a}$$

$$= \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{161\int \frac{1}{x^6(a+bx^2)^8}\,dx}{96a^2}$$

$$= \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{23}{192a^3x^5 (a+bx^2)^7} + \frac{437\int \frac{1}{x^6(a+bx^2)^7}\,dx}{192a^3}$$

$$= \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{23}{192a^3x^5 (a+bx^2)^7} + \frac{437}{2304a^4x^5 (a+bx^2)^6} -$$

$$= \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{23}{192a^3x^5 (a+bx^2)^7} + \frac{437}{2304a^4x^5 (a+bx^2)^6} -$$

$$= \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{23}{192a^3x^5 (a+bx^2)^7} + \frac{437}{2304a^4x^5 (a+bx^2)^6} -$$

$$= \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{23}{192a^3x^5 (a+bx^2)^7} + \frac{437}{2304a^4x^5 (a+bx^2)^6} -$$

$$= \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{23}{192a^3x^5 (a+bx^2)^7} + \frac{437}{2304a^4x^5 (a+bx^2)^6} -$$

$$= \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{23}{192a^3x^5 (a+bx^2)^7} + \frac{437}{2304a^4x^5 (a+bx^2)^6} -$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{23}{192a^3x^5 (a+bx^2)^7} + \frac{}{230}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} + \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8} + \frac{23}{192a^3x^5 (a}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429b^2}{65536a^{12}x} + \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429b^2}{65536a^{12}x} + \frac{1}{18ax^5 (a+bx^2)^9} + \frac{23}{288a^2x^5 (a+bx^2)^8}$$

<u>Mathematica</u> [A]
time = 0.06, size = 169, normalized size = 0.73

$$\frac{-\frac{\sqrt{a}\left(589824a^{11}-4521984a^{10}bx^2+94961664a^9b^2x^4+1469632311a^8b^3x^6+7323998514a^7b^4x^8+19562592546a^6b^5x^{10}+32314857354a^5b^6x^{12}+34810986496a^4b^7x^{14}+24648575094a^3b^8x^{16}+11110024926a^2b^9x^{18}+2900207310ab^{10}x^{20}+334639305b^{11}x^{22}\right)}{x^5(a+bx^2)^9} - 334639305b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2949120a^{25/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^10),x]
```

```
[Out] (-((Sqrt[a]*(589824*a^11 - 4521984*a^10*b*x^2 + 94961664*a^9*b^2*x^4 + 1469
632311*a^8*b^3*x^6 + 7323998514*a^7*b^4*x^8 + 19562592546*a^6*b^5*x^10 + 32
314857354*a^5*b^6*x^12 + 34810986496*a^4*b^7*x^14 + 24648575094*a^3*b^8*x^1
6 + 11110024926*a^2*b^9*x^18 + 2900207310*a*b^10*x^20 + 334639305*b^11*x^22
))/(x^5*(a + b*x^2)^9)) - 334639305*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
949120*a^(25/2))
```

Maple [A]
time = 0.12, size = 153, normalized size = 0.66

method	result
default	$-\frac{b^{3}\left(\frac{\frac{6981491}{65536} a^{8} x+\frac{74539223}{98304} a^{7} b\,x^{3}+\frac{394553929}{163840} a^{6} b^{2} x^{5}+\frac{725918941}{163840} a^{5} b^{3} x^{7}+\frac{463199}{90} a^{4} b^{4} x^{9}+\frac{631790371}{163840} a^{3} b^{5} x^{11}+\frac{297702839}{163840} a^{2} b^{6} x^{13}+\frac{483407}{98304}}{\left(b\,x^{2}+a\right)^{9}}\right.}{a^{12}}$
risch	$\frac{-\frac{1}{5a}+\frac{23b\,x^{2}}{15a^{2}}-\frac{161b^{2}x^{4}}{5a^{3}}-\frac{163292479b^{3}x^{6}}{327680a^{4}}-\frac{1220666419b^{4}x^{8}}{491520a^{5}}-\frac{1086810697b^{5}x^{10}}{163840a^{6}}-\frac{1795269853b^{6}x^{12}}{163840a^{7}}-\frac{1062347b^{7}x^{14}}{90a^{8}}-\frac{1369365283b^{8}x^{16}}{163840a^{9}}-\frac{61722}{16}}{x^{5}\left(b\,x^{2}+a\right)^{9}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/a^12*((6981491/65536*a^8*x+74539223/98304*a^7*b*x^3+394553929/163840*a
^6*b^2*x^5+725918941/163840*a^5*b^3*x^7+463199/90*a^4*b^4*x^9+631790371/163
840*a^3*b^5*x^11+297702839/163840*a^2*b^6*x^13+48340777/98304*a*b^7*x^15+38
31949/65536*b^8*x^17)/(b*x^2+a)^9+7436429/65536/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2)))-1/5/a^10/x^5+10/3*b/a^11/x^3-55*b^2/a^12/x
```

Maxima [A]
time = 0.54, size = 251, normalized size = 1.08

$$-\frac{334639305\,b^{11}x^{22}+2900207310\,ab^{10}x^{20}+11110024926\,a^{2}b^{9}x^{18}+24648575094\,a^{3}b^{8}x^{16}+34810986496\,a^{4}b^{7}x^{14}+32314857354\,a^{5}b^{6}x^{12}+19562592546\,a^{6}b^{5}x^{10}+7323998514\,a^{7}b^{4}x^{8}+1469632311\,a^{8}b^{3}x^{6}+94961664\,a^{9}b^{2}x^{4}-4521984\,a^{10}bx^{2}+589824\,a^{11}}{2949120\,\left(a^{12}b^{9}x^{23}+9\,a^{13}b^{8}x^{21}+36\,a^{14}b^{7}x^{19}+84\,a^{15}b^{6}x^{17}+126\,a^{16}b^{5}x^{15}+126\,a^{17}b^{4}x^{13}+84\,a^{18}b^{3}x^{11}+36\,a^{19}b^{2}x^{9}+9\,a^{20}bx^{7}+a^{21}x^{5}\right)}-\frac{7436429\,b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/2949120*(334639305*b^11*x^22 + 2900207310*a*b^10*x^20 + 11110024926*a^2*
b^9*x^18 + 24648575094*a^3*b^8*x^16 + 34810986496*a^4*b^7*x^14 + 3231485735
4*a^5*b^6*x^12 + 19562592546*a^6*b^5*x^10 + 7323998514*a^7*b^4*x^8 + 146963
2311*a^8*b^3*x^6 + 94961664*a^9*b^2*x^4 - 4521984*a^10*b*x^2 + 589824*a^11)
/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7*x^19 + 84*a^15*b^6*x^17 + 1
26*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*b^3*x^11 + 36*a^19*b^2*x^9 +
 9*a^20*b*x^7 + a^21*x^5) - 7436429/65536*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a^12)
```

Fricas [A]
time = 0.77, size = 726, normalized size = 3.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/5898240*(669278610*b^11*x^22 + 5800414620*a*b^10*x^20 + 22220049852*a^2
*b^9*x^18 + 49297150188*a^3*b^8*x^16 + 69621972992*a^4*b^7*x^14 + 646297147
08*a^5*b^6*x^12 + 39125185092*a^6*b^5*x^10 + 14647997028*a^7*b^4*x^8 + 2939
264622*a^8*b^3*x^6 + 189923328*a^9*b^2*x^4 - 9043968*a^10*b*x^2 + 1179648*a
^11 - 334639305*(b^11*x^23 + 9*a*b^10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*b^8*x
^17 + 126*a^4*b^7*x^15 + 126*a^5*b^6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*b^4*x^
9 + 9*a^8*b^3*x^7 + a^9*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7*x^19 + 84*
a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*b^3*x^11 +
36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5), -1/2949120*(334639305*b^11*x^22
 + 2900207310*a*b^10*x^20 + 11110024926*a^2*b^9*x^18 + 24648575094*a^3*b^8*
x^16 + 34810986496*a^4*b^7*x^14 + 32314857354*a^5*b^6*x^12 + 19562592546*a^
6*b^5*x^10 + 7323998514*a^7*b^4*x^8 + 1469632311*a^8*b^3*x^6 + 94961664*a^9
*b^2*x^4 - 4521984*a^10*b*x^2 + 589824*a^11 + 334639305*(b^11*x^23 + 9*a*b^
10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*b^8*x^17 + 126*a^4*b^7*x^15 + 126*a^5*b^
6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*b^4*x^9 + 9*a^8*b^3*x^7 + a^9*b^2*x^5)*sq
rt(b/a)*arctan(x*sqrt(b/a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7
*x^19 + 84*a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*
b^3*x^11 + 36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5)]
```

Sympy [A]
time = 0.69, size = 316, normalized size = 1.36

$$\frac{7436429\sqrt{-\frac{b^5}{a^{25}}}\log\left(-\frac{a^{13}\sqrt{-\frac{b^5}{a^{25}}}}{b^3}+x\right)}{131072} - \frac{7436429\sqrt{-\frac{b^5}{a^{25}}}\log\left(\frac{a^{13}\sqrt{-\frac{b^5}{a^{25}}}}{b^3}+x\right)}{131072} + \frac{-589824a^{11}+4521984a^{10}bx^2-94961664a^9b^2x^4-1469632311a^8b^3x^6-7323998514a^7b^4x^8-19562592546a^6b^5x^{10}-32314857354a^5b^6x^{12}-34810986496a^4b^7x^{14}-24648575094a^3b^8x^{16}-11110024926a^2b^9x^{18}-2900207310ab^{10}x^{20}-334639305b^{11}x^{22}}{2949120a^{21}x^5+26542080a^{20}bx^7+106168320a^{19}b^2x^9+247726080a^{18}b^3x^{11}+371589120a^{17}b^4x^{13}+371589120a^{16}b^5x^{15}+247726080a^{15}b^6x^{17}+106168320a^{14}b^7x^{19}+26542080a^{13}b^8x^{21}+2949120a^{12}b^9x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**10,x)
```

```
[Out] 7436429*sqrt(-b**5/a**25)*log(-a**13*sqrt(-b**5/a**25)/b**3 + x)/131072 - 7
436429*sqrt(-b**5/a**25)*log(a**13*sqrt(-b**5/a**25)/b**3 + x)/131072 + (-5
89824*a**11 + 4521984*a**10*b*x**2 - 94961664*a**9*b**2*x**4 - 1469632311*a
**8*b**3*x**6 - 7323998514*a**7*b**4*x**8 - 19562592546*a**6*b**5*x**10 - 3
2314857354*a**5*b**6*x**12 - 34810986496*a**4*b**7*x**14 - 24648575094*a**3
*b**8*x**16 - 11110024926*a**2*b**9*x**18 - 2900207310*a*b**10*x**20 - 3346
39305*b**11*x**22)/(2949120*a**21*x**5 + 26542080*a**20*b*x**7 + 106168320*
a**19*b**2*x**9 + 247726080*a**18*b**3*x**11 + 371589120*a**17*b**4*x**13 +
```

```
 371589120*a**16*b**5*x**15 + 247726080*a**15*b**6*x**17 + 106168320*a**14*
b**7*x**19 + 26542080*a**13*b**8*x**21 + 2949120*a**12*b**9*x**23)
```

Giac [A]
time = 1.20, size = 159, normalized size = 0.68

$$-\frac{7436429\,b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{12}}-\frac{825\,b^{2}x^{4}-50\,abx^{2}+3\,a^{2}}{15\,a^{12}x^{5}}-\frac{172437705\,b^{11}x^{17}+1450223310\,ab^{10}x^{15}+5358651102\,a^{2}b^{9}x^{13}+11372226678\,a^{3}b^{8}x^{11}+15178104832\,a^{4}b^{7}x^{9}+13066540938\,a^{5}b^{6}x^{7}+7101970722\,a^{6}b^{5}x^{5}+2236176690\,a^{7}b^{4}x^{3}+314167095\,a^{8}b^{3}x}{2949120\,(bx^{2}+a)^{9}a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -7436429/65536*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^12) - 1/15*(825*b^2*x
^4 - 50*a*b*x^2 + 3*a^2)/(a^12*x^5) - 1/2949120*(172437705*b^11*x^17 + 1450
223310*a*b^10*x^15 + 5358651102*a^2*b^9*x^13 + 11372226678*a^3*b^8*x^11 + 1
5178104832*a^4*b^7*x^9 + 13066540938*a^5*b^6*x^7 + 7101970722*a^6*b^5*x^5 +
 2236176690*a^7*b^4*x^3 + 314167095*a^8*b^3*x)/((b*x^2 + a)^9*a^12)
```

Mupad [B]
time = 5.89, size = 246, normalized size = 1.06

$$-\frac{\frac{1}{5\,a}-\frac{23\,b\,x^{2}}{15\,a^{2}}+\frac{161\,b^{2}\,x^{4}}{5\,a^{3}}+\frac{163292479\,b^{3}\,x^{6}}{327680\,a^{4}}+\frac{1220666419\,b^{4}\,x^{8}}{491520\,a^{5}}+\frac{1086810697\,b^{5}\,x^{10}}{163840\,a^{6}}+\frac{1795269853\,b^{6}\,x^{12}}{163840\,a^{7}}+\frac{1062347\,b^{7}\,x^{14}}{90\,a^{8}}+\frac{1369365283\,b^{8}\,x^{16}}{163840\,a^{9}}+\frac{617223607\,b^{9}\,x^{18}}{163840\,a^{10}}+\frac{96673577\,b^{10}\,x^{20}}{98304\,a^{11}}+\frac{7436429\,b^{11}\,x^{22}}{65536\,a^{12}}}{a^{9}\,x^{5}+9\,a^{8}\,b\,x^{7}+36\,a^{7}\,b^{2}\,x^{9}+84\,a^{6}\,b^{3}\,x^{11}+126\,a^{5}\,b^{4}\,x^{13}+126\,a^{4}\,b^{5}\,x^{15}+84\,a^{3}\,b^{6}\,x^{17}+36\,a^{2}\,b^{7}\,x^{19}+9\,a\,b^{8}\,x^{21}+b^{9}\,x^{23}}-\frac{7436429\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{25/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^10),x)
```

```
[Out] - (1/(5*a) - (23*b*x^2)/(15*a^2) + (161*b^2*x^4)/(5*a^3) + (163292479*b^3*x
^6)/(327680*a^4) + (1220666419*b^4*x^8)/(491520*a^5) + (1086810697*b^5*x^10
)/(163840*a^6) + (1795269853*b^6*x^12)/(163840*a^7) + (1062347*b^7*x^14)/(9
0*a^8) + (1369365283*b^8*x^16)/(163840*a^9) + (617223607*b^9*x^18)/(163840*
a^10) + (96673577*b^10*x^20)/(98304*a^11) + (7436429*b^11*x^22)/(65536*a^12
))/(a^9*x^5 + b^9*x^23 + 9*a^8*b*x^7 + 9*a*b^8*x^21 + 36*a^7*b^2*x^9 + 84*a
^6*b^3*x^11 + 126*a^5*b^4*x^13 + 126*a^4*b^5*x^15 + 84*a^3*b^6*x^17 + 36*a^
2*b^7*x^19) - (7436429*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(25/2))
```

3.225 $\int \frac{x^3}{a-bx^2} dx$

Optimal. Leaf size=28

$$-\frac{x^2}{2b} - \frac{a\log(a-bx^2)}{2b^2}$$

[Out] -1/2*x^2/b-1/2*a*ln(-b*x^2+a)/b^2

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {272, 45}

$$-\frac{a\log(a-bx^2)}{2b^2} - \frac{x^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a - b*x^2),x]

[Out] -1/2*x^2/b - (a*Log[a - b*x^2])/(2*b^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a-bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a-bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{b} - \frac{a}{b(-a+bx)}\right)\,dx, x, x^2\right)\\ &= -\frac{x^2}{2b} - \frac{a\log(a-bx^2)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{x^2}{2b}-\frac{a\log(a-bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2),x]
[Out] -1/2*x^2/b - (a*Log[a - b*x^2])/(2*b^2)
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.89

method	result	size
default	$-\frac{x^2}{2b}-\frac{a\ln(-b\,x^2+a)}{2b^2}$	25
norman	$-\frac{x^2}{2b}-\frac{a\ln(-b\,x^2+a)}{2b^2}$	25
risch	$-\frac{x^2}{2b}-\frac{a\ln(-b\,x^2+a)}{2b^2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2/b-1/2*a*ln(-b*x^2+a)/b^2
```

Maxima [A]
time = 0.29, size = 25, normalized size = 0.89

$$-\frac{x^2}{2\,b}-\frac{a\log(bx^2-a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*x^2/b - 1/2*a*log(b*x^2 - a)/b^2
```

Fricas [A]
time = 0.71, size = 23, normalized size = 0.82

$$-\frac{bx^2+a\log(bx^2-a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(b*x^2 + a*log(b*x^2 - a))/b^2
```

Sympy [**A**]
time = 0.05, size = 22, normalized size = 0.79

$$-\frac{a\log\left(-a+bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a),x)
```

```
[Out] -a*log(-a + b*x**2)/(2*b**2) - x**2/(2*b)
```

Giac [**A**]
time = 1.24, size = 26, normalized size = 0.93

$$-\frac{x^2}{2\,b}-\frac{a\log\left(|bx^2-a|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*x^2/b - 1/2*a*log(abs(b*x^2 - a))/b^2
```

Mupad [**B**]
time = 0.05, size = 23, normalized size = 0.82

$$-\frac{b\,x^2+a\,\ln\left(b\,x^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2),x)
```

```
[Out] -(b*x^2 + a*log(b*x^2 - a))/(2*b^2)
```

3.226 $\int \frac{x^2}{a-bx^2}\,dx$

Optimal. Leaf size=31

$$-\frac{x}{b}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] -x/b+arctanh(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {327, 214}

$$\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2),x]
```

```
[Out] -(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a-bx^2}\,dx &= -\frac{x}{b}+\frac{a\int \frac{1}{a-bx^2}\,dx}{b}\\ &= -\frac{x}{b}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$-\frac{x}{b}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2),x]
[Out] -(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Maple [A]
time = 0.03, size = 27, normalized size = 0.87

method	result	size
default	$-\frac{x}{b}+\frac{a\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	27
risch	$-\frac{x}{b}-\frac{\sqrt{ab}\,\ln\left(\sqrt{ab}\,x-a\right)}{2b^2}+\frac{\sqrt{ab}\,\ln\left(-\sqrt{ab}\,x-a\right)}{2b^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -x/b+a/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 42, normalized size = 1.35

$$-\frac{a\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*a*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*b) - x/b
```

Fricas [A]

time = 0.90, size = 80, normalized size = 2.58

$$\left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{\frac{a}{b}}+a}{bx^2-a}\right)-2\,x}{2\,b}, -\frac{\sqrt{-\frac{a}{b}}\arctan\left(\frac{bx\sqrt{-\frac{a}{b}}}{a}\right)+x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a/b)*log((b*x^2 + 2*b*x*sqrt(a/b) + a)/(b*x^2 - a)) - 2*x)/b, -(
sqrt(-a/b)*arctan(b*x*sqrt(-a/b)/a) + x)/b]
```

Sympy [A]
time = 0.05, size = 49, normalized size = 1.58

$$-\frac{\sqrt{\frac{a}{b^3}}\log\left(-b\sqrt{\frac{a}{b^3}}+x\right)}{2}+\frac{\sqrt{\frac{a}{b^3}}\log\left(b\sqrt{\frac{a}{b^3}}+x\right)}{2}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a),x)
```

```
[Out] -sqrt(a/b**3)*log(-b*sqrt(a/b**3) + x)/2 + sqrt(a/b**3)*log(b*sqrt(a/b**3)
+ x)/2 - x/b
```

Giac [A]
time = 1.50, size = 29, normalized size = 0.94

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - x/b
```

Mupad [B]
time = 4.57, size = 23, normalized size = 0.74

$$\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2),x)
[Out] (a^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/b^(3/2) - x/b
```

3.227 $\int \frac{x}{a-bx^2} \, dx$

Optimal. Leaf size=16

$$-\frac{\log(a-bx^2)}{2b}$$

[Out] -1/2*ln(-b*x^2+a)/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {266}

$$-\frac{\log(a-bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b*x^2),x]

[Out] -1/2*Log[a - b*x^2]/b

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{a-bx^2} \, dx = -\frac{\log(a-bx^2)}{2b}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\log(a-bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b*x^2),x]

[Out] -1/2*Log[a - b*x^2]/b

Maple [A]
time = 0.02, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$-\frac{\ln(-b\,x^2+a)}{2b}$	15
default	$-\frac{\ln(-b\,x^2+a)}{2b}$	15
norman	$-\frac{\ln(-b\,x^2+a)}{2b}$	15
risch	$-\frac{\ln(-b\,x^2+a)}{2b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(-b*x^2+a)/b
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 - a)/b
```

Fricas [A]
time = 0.93, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*log(b*x^2 - a)/b
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.75

$$-\frac{\log\left(-a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a),x)
[Out] -log(-a + b*x**2)/(2*b)
```

Giac [A]
time = 0.93, size = 16, normalized size = 1.00

$$-\frac{\log\left(|bx^2 - a|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(b*x^2 - a))/b
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.94

$$-\frac{\ln\left(b\,x^2 - a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2),x)
```

```
[Out] -log(b*x^2 - a)/(2*b)
```

3.228 $\int \frac{1}{a-bx^2} dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{214\}$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-bx^2}\,dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-1),x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.04, size = 16, normalized size = 0.67

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$\frac{\ln\left(bx+\sqrt{ab}\right)}{2\sqrt{ab}}-\frac{\ln\left(-bx+\sqrt{ab}\right)}{2\sqrt{ab}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.52, size = 31, normalized size = 1.29

$$-\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/sqrt(a*b)
```

Fricas [A]
time = 0.90, size = 68, normalized size = 2.83

$$\left[\frac{\sqrt{ab}\,\log\left(\frac{bx^2+2\sqrt{ab}\,x+a}{bx^2-a}\right)}{2\,ab},\,-\frac{\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="fricas")
[Out] [1/2*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a))/(a*b), -sqrt(-a
*b)*arctan(sqrt(-a*b)*x/a)/(a*b)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.
time = 0.05, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{ab}}\log\left(-a\sqrt{\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{\frac{1}{ab}}\log\left(a\sqrt{\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a),x)
```

```
[Out] -sqrt(1/(a*b))*log(-a*sqrt(1/(a*b)) + x)/2 + sqrt(1/(a*b))*log(a*sqrt(1/(a*
b)) + x)/2
```

Giac [**A**]
time = 1.16, size = 18, normalized size = 0.75

$$-\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -arctan(b*x/sqrt(-a*b))/sqrt(-a*b)
```

Mupad [**B**]
time = 0.18, size = 16, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2),x)
```

```
[Out] atanh((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.229 $\int \frac{1}{x(a-bx^2)}\, dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^2\right)}{2a}$$

```
[Out] ln(x)/a-1/2*ln(-b*x^2+a)/a
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {272, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)),x]
```

```
[Out] Log[x]/a - Log[a - b*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a-bx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} + \frac{b\text{Subst}\left(\int \frac{1}{a-bx}\,dx, x, x^2\right)}{2a}$$
$$= \frac{\log(x)}{a} - \frac{\log\left(a-bx^2\right)}{2a}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log\left(a-bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)),x]
[Out] Log[x]/a - Log[a - b*x^2]/(2*a)
```

Maple [A]
time = 0.03, size = 22, normalized size = 0.96

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln\left(-b\,x^2+a\right)}{2a}$	22
norman	$\frac{\ln(x)}{a}-\frac{\ln\left(-b\,x^2+a\right)}{2a}$	22
risch	$\frac{\ln(x)}{a}-\frac{\ln\left(-b\,x^2+a\right)}{2a}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a),x,method=_RETURNVERBOSE)
[Out] ln(x)/a-1/2*ln(-b*x^2+a)/a
```

Maxima [A]
time = 0.29, size = 25, normalized size = 1.09

$$-\frac{\log\left(bx^2-a\right)}{2\,a} + \frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 - a)/a + 1/2*log(x^2)/a
```

Fricas [A]
time = 1.38, size = 20, normalized size = 0.87

$$-\frac{\log\left(bx^2 - a\right) - 2\,\log\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(log(b*x^2 - a) - 2*log(x))/a
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.65

$$\frac{\log\left(x\right)}{a} - \frac{\log\left(-\frac{a}{b} + x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a),x)
[Out] log(x)/a - log(-a/b + x**2)/(2*a)
```

Giac [A]
time = 1.10, size = 26, normalized size = 1.13

$$\frac{\log\left(x^2\right)}{2\,a} - \frac{\log\left(|bx^2 - a|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="giac")
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 - a))/a
```

Mupad [B]
time = 4.54, size = 21, normalized size = 0.91

$$\frac{\ln\left(x\right)}{a} - \frac{\ln\left(a - b\,x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)),x)
[Out] log(x)/a - log(a - b*x^2)/(2*a)
```

3.230 $\int \frac{1}{x^2(a-bx^2)} \, dx$

Optimal. Leaf size=33

$$-\frac{1}{ax}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -1/a/x+arctanh(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {331, 214}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a-bx^2\right)} \, dx &= -\frac{1}{ax}+\frac{b\int \frac{1}{a-bx^2} \, dx}{a}\\
&= -\frac{1}{ax}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{ax}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)),x]
[Out] -(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.03, size = 29, normalized size = 0.88

method	result	size
default	$\frac{b\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}$	29
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{3}\_Z^{2}-b\right)}\_R\ln\left(\left(3\_R^{2}a^{3}-2b\right)x+a^{2}\_R\right)\right)}{2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a),x,method=_RETURNVERBOSE)
[Out] b/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))-1/a/x
```

Maxima [A]
time = 0.57, size = 44, normalized size = 1.33

$$-\frac{b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a) - 1/(a*x)
```

Fricas [A]

time = 1.20, size = 82, normalized size = 2.48

$$\left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)-2}{2\,ax}, -\frac{x\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) - 2)/(a*x)
, -(x*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 1)/(a*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.
time = 0.07, size = 58, normalized size = 1.76

$$-\frac{\sqrt{\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{\frac{b}{a^3}}}{b}+x\right)}{2}+\frac{\sqrt{\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a),x)
```

```
[Out] -sqrt(b/a**3)*log(-a**2*sqrt(b/a**3)/b + x)/2 + sqrt(b/a**3)*log(a**2*sqrt(
b/a**3)/b + x)/2 - 1/(a*x)
```

Giac [A]
time = 1.29, size = 31, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/(a*x)
```

Mupad [B]
time = 4.61, size = 25, normalized size = 0.76

$$\frac{\sqrt{b}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)),x)
[Out] (b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/a^(3/2) - 1/(a*x)
```

3.231 $\int \frac{1}{x^3(a-bx^2)}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2\right)}{2a^2}$$

```
[Out] -1/2/a/x^2+b*ln(x)/a^2-1/2*b*ln(-b*x^2+a)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {272, 46}

$$-\frac{b\log\left(a-bx^2\right)}{2a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)),x]
```

```
[Out] -1/2*1/(a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a-bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{b}{a^2x}+\frac{b^2}{a^2(a-bx)}\right)\,dx,x,x^2\right)\\
&= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$-\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)),x]
[Out] -1/2*1/(a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

Maple [A]
time = 0.03, size = 32, normalized size = 0.91

method	result	size
default	$-\frac{1}{2a\,x^{2}}+\frac{b\ln(x)}{a^{2}}-\frac{b\ln\left(-b\,x^{2}+a\right)}{2a^{2}}$	32
norman	$-\frac{1}{2a\,x^{2}}+\frac{b\ln(x)}{a^{2}}-\frac{b\ln\left(-b\,x^{2}+a\right)}{2a^{2}}$	32
risch	$-\frac{1}{2a\,x^{2}}+\frac{b\ln(x)}{a^{2}}-\frac{b\ln\left(-b\,x^{2}+a\right)}{2a^{2}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2+b*ln(x)/a^2-1/2*b*ln(-b*x^2+a)/a^2
```

Maxima [A]
time = 0.31, size = 35, normalized size = 1.00

$$-\frac{b\log\left(bx^2-a\right)}{2\,a^2}+\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*b*log(b*x^2 - a)/a^2 + 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

Fricas [A]
time = 1.50, size = 33, normalized size = 0.94

$$-\frac{bx^2\log\left(bx^2-a\right)-2\,bx^2\log\left(x\right)+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(b*x^2*log(b*x^2 - a) - 2*b*x^2*log(x) + a)/(a^2*x^2)
```

Sympy [A]
time = 0.11, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(-\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a),x)
```

```
[Out] -1/(2*a*x**2) + b*log(x)/a**2 - b*log(-a/b + x**2)/(2*a**2)
```

Giac [A]
time = 1.19, size = 43, normalized size = 1.23

$$\frac{b\log(x^2)}{2\,a^2}-\frac{b\log(|bx^2-a|)}{2\,a^2}-\frac{bx^2+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*log(x^2)/a^2 - 1/2*b*log(abs(b*x^2 - a))/a^2 - 1/2*(b*x^2 + a)/(a^2*x
^2)
```

Mupad [B]
time = 0.07, size = 31, normalized size = 0.89

$$\frac{b\,\ln(x)}{a^2}-\frac{b\,\ln(a-b\,x^2)}{2\,a^2}-\frac{1}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)),x)
```

```
[Out] (b*log(x))/a^2 - (b*log(a - b*x^2))/(2*a^2) - 1/(2*a*x^2)
```

3.232 $\int \frac{x^3}{(a-bx^2)^2} dx$

Optimal. Leaf size=35

$$\frac{a}{2b^2(a-bx^2)}+\frac{\log(a-bx^2)}{2b^2}$$

[Out] 1/2*a/b^2/(-b*x^2+a)+1/2*ln(-b*x^2+a)/b^2

Rubi [A]

time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {272, 45}

$$\frac{a}{2b^2(a-bx^2)}+\frac{\log(a-bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a - b*x^2)^2,x]

[Out] a/(2*b^2*(a - b*x^2)) + Log[a - b*x^2]/(2*b^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a-bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a-bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a}{b(-a+bx)^2}+\frac{1}{b(-a+bx)}\right)dx,x,x^2\right)\\ &= \frac{a}{2b^2(a-bx^2)}+\frac{\log(a-bx^2)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.83

$$\frac{\frac{a}{a-bx^2} + \log(a - bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^2,x]
[Out] (a/(a - b*x^2) + Log[a - b*x^2])/(2*b^2)
```

Maple [A]
time = 0.03, size = 32, normalized size = 0.91

method	result	size
default	$\frac{a}{2b^2(-b\,x^2+a)} + \frac{\ln(-b\,x^2+a)}{2b^2}$	32
norman	$\frac{a}{2b^2(-b\,x^2+a)} + \frac{\ln(-b\,x^2+a)}{2b^2}$	32
risch	$\frac{a}{2b^2(-b\,x^2+a)} + \frac{\ln(-b\,x^2+a)}{2b^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*a/b^2/(-b*x^2+a)+1/2*ln(-b*x^2+a)/b^2
```

Maxima [A]
time = 0.28, size = 35, normalized size = 1.00

$$-\frac{a}{2(b^3x^2 - ab^2)} + \frac{\log(bx^2 - a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*a/(b^3*x^2 - a*b^2) + 1/2*log(b*x^2 - a)/b^2
```

Fricas [A]
time = 1.33, size = 42, normalized size = 1.20

$$\frac{(bx^2 - a)\log(bx^2 - a) - a}{2(b^3x^2 - ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x^2 - a)*log(b*x^2 - a) - a)/(b^3*x^2 - a*b^2)
```

Sympy [A]
time = 0.08, size = 29, normalized size = 0.83

$$-\frac{a}{-2ab^2+2b^3x^2}+\frac{\log(-a+bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**2,x)
[Out] -a/(-2*a*b**2 + 2*b**3*x**2) + log(-a + b*x**2)/(2*b**2)
```

Giac [A]
time = 1.24, size = 53, normalized size = 1.51

$$-\frac{\frac{\log\left(\frac{|bx^2-a|}{(bx^2-a)^2|b|}\right)}{b}+\frac{a}{(bx^2-a)b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="giac")
[Out] -1/2*(log(abs(b*x^2 - a)/((b*x^2 - a)^2*abs(b)))/b + a/((b*x^2 - a)*b))/b
```

Mupad [B]
time = 0.04, size = 32, normalized size = 0.91

$$\frac{\ln(b\,x^2-a)}{2\,b^2}+\frac{a}{2\,b^2\,(a-b\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2)^2,x)
[Out] log(b*x^2 - a)/(2*b^2) + a/(2*b^2*(a - b*x^2))
```

3.233 $\int \frac{x^2}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=46

$$\frac{x}{2b\,(a-bx^2)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

```
[Out] 1/2*x/b/(-b*x^2+a)-1/2*arctanh(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {294, 214}

$$\frac{x}{2b\,(a-bx^2)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^2,x]
```

```
[Out] x/(2*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a-bx^2)^2}\,dx &= \frac{x}{2b\,(a-bx^2)}-\frac{\int \frac{1}{a-bx^2}\,dx}{2b}\\ &= \frac{x}{2b\,(a-bx^2)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.02

$$-\frac{x}{2b\left(-a+bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^2,x]
[Out] -1/2*x/(b*(-a + b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Maple [A]
time = 0.04, size = 37, normalized size = 0.80

method	result	size
default	$\frac{x}{2b\left(-b\,x^{2}+a\right)}-\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}$	37
risch	$\frac{x}{2b\left(-b\,x^{2}+a\right)}+\frac{\ln\left(bx-\sqrt{ab}\right)}{4\sqrt{ab}\,b}-\frac{\ln\left(-bx-\sqrt{ab}\right)}{4\sqrt{ab}\,b}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x/b/(-b*x^2+a)-1/2/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 52, normalized size = 1.13

$$-\frac{x}{2\left(b^2x^2-ab\right)}+\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*x/(b^2*x^2 - a*b) + 1/4*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt
(a*b)*b)
```

Fricas [A]
time = 0.94, size = 127, normalized size = 2.76

$$\left[-\frac{2\,abx-\left(bx^2-a\right)\sqrt{ab}\,\log\left(\frac{bx^2-2\sqrt{ab}\,x+a}{bx^2-a}\right)}{4\left(ab^3x^2-a^2b^2\right)},-\frac{abx-\left(bx^2-a\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{2\left(ab^3x^2-a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 - 2*sqrt(a*b)*x + a)/(b*x
^2 - a)))/(a*b^3*x^2 - a^2*b^2), -1/2*(a*b*x - (b*x^2 - a)*sqrt(-a*b)*arcta
n(sqrt(-a*b)*x/a))/(a*b^3*x^2 - a^2*b^2)]
```

Sympy [A]
time = 0.09, size = 71, normalized size = 1.54

$$-\frac{x}{-2ab+2b^2x^2}+\frac{\sqrt{\frac{1}{ab^3}}\log\left(-ab\sqrt{\frac{1}{ab^3}}+x\right)}{4}-\frac{\sqrt{\frac{1}{ab^3}}\log\left(ab\sqrt{\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-2*a*b + 2*b**2*x**2) + sqrt(1/(a*b**3))*log(-a*b*sqrt(1/(a*b**3)) + x)
/4 - sqrt(1/(a*b**3))*log(a*b*sqrt(1/(a*b**3)) + x)/4
```

Giac [A]
time = 2.52, size = 39, normalized size = 0.85

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,b}-\frac{x}{2\,(bx^2-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - 1/2*x/((b*x^2 - a)*b)
```

Mupad [B]
time = 4.68, size = 34, normalized size = 0.74

$$\frac{x}{2\,b\,\left(a-b\,x^2\right)}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^2,x)
```

```
[Out] x/(2*b*(a - b*x^2)) - atanh((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2))
```

3.234 $\int \frac{x}{(a-bx^2)^2} dx$

Optimal. Leaf size=17

$$\frac{1}{2b(a-bx^2)}$$

```
[Out] 1/2/b/(-b*x^2+a)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {267}

$$\frac{1}{2b(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2)^2,x]
[Out] 1/(2*b*(a - b*x^2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a-bx^2)^2} dx = \frac{1}{2b(a-bx^2)}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2b(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2)^2,x]
[Out] 1/(2*b*(a - b*x^2))
```

Maple [A]
time = 0.02, size = 16, normalized size = 0.94

method	result	size
gosper	$\frac{1}{2b(-b\,x^{2}+a)}$	16
derivativedivides	$\frac{1}{2b(-b\,x^{2}+a)}$	16
default	$\frac{1}{2b(-b\,x^{2}+a)}$	16
norman	$\frac{1}{2b(-b\,x^{2}+a)}$	16
risch	$\frac{1}{2b(-b\,x^{2}+a)}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/(-b*x^2+a)
```

Maxima [A]
time = 0.30, size = 16, normalized size = 0.94

$$-\frac{1}{2\,(bx^{2}-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x^2 - a)*b)
```

Fricas [A]
time = 0.66, size = 16, normalized size = 0.94

$$-\frac{1}{2\,(b^{2}x^{2}-ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 - a*b)
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.88

$$-\frac{1}{-2ab+2b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**2,x)
```

```
[Out] -1/(-2*a*b + 2*b**2*x**2)
```

Giac [**A**]
time = 4.86, size = 16, normalized size = 0.94

$$-\frac{1}{2\,(bx^2-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 - a)*b)
```

Mupad [**B**]
time = 0.02, size = 15, normalized size = 0.88

$$\frac{1}{2\,b\,\left(a-b\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2)^2,x)
```

```
[Out] 1/(2*b*(a - b*x^2))
```

3.235 $\int \frac{1}{(a-bx^2)^2} dx$

Optimal. Leaf size=46

$$\frac{x}{2a(a-bx^2)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

```
[Out] 1/2*x/a/(-b*x^2+a)+1/2*arctanh(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {205, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-bx^2)^2}\,dx &= \frac{x}{2a(a-bx^2)}+\frac{\int \frac{1}{a-bx^2}\,dx}{2a}\\ &= \frac{x}{2a(a-bx^2)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.02

$$-\frac{x}{2a\left(-a+bx^2\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-2),x]
[Out] -1/2*x/(a*(-a + b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Maple [A]
time = 0.04, size = 37, normalized size = 0.80

method	result	size
default	$\frac{x}{2a\left(-b\,x^2+a\right)}+\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	37
risch	$\frac{x}{2a\left(-b\,x^2+a\right)}+\frac{\ln\left(bx+\sqrt{ab}\right)}{4\sqrt{ab}\,a}-\frac{\ln\left(-bx+\sqrt{ab}\right)}{4\sqrt{ab}\,a}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x/a/(-b*x^2+a)+1/2/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 52, normalized size = 1.13

$$-\frac{x}{2\left(abx^2-a^2\right)}-\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*x/(a*b*x^2 - a^2) - 1/4*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt
(a*b)*a)
```

Fricas [A]
time = 0.91, size = 126, normalized size = 2.74

$$\left[-\frac{2\,abx-\left(bx^2-a\right)\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{4\left(a^2b^2x^2-a^3b\right)},-\frac{abx+\left(bx^2-a\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{2\left(a^2b^2x^2-a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x
^2 - a)))/(a^2*b^2*x^2 - a^3*b), -1/2*(a*b*x + (b*x^2 - a)*sqrt(-a*b)*arcta
n(sqrt(-a*b)*x/a))/(a^2*b^2*x^2 - a^3*b)]
```

Sympy [A]
time = 0.09, size = 71, normalized size = 1.54

$$-\frac{x}{-2a^2+2abx^2}-\frac{\sqrt{\frac{1}{a^3b}}\log\left(-a^2\sqrt{\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{\frac{1}{a^3b}}\log\left(a^2\sqrt{\frac{1}{a^3b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-2*a**2 + 2*a*b*x**2) - sqrt(1/(a**3*b))*log(-a**2*sqrt(1/(a**3*b)) + x
)/4 + sqrt(1/(a**3*b))*log(a**2*sqrt(1/(a**3*b)) + x)/4
```

Giac [A]
time = 2.19, size = 39, normalized size = 0.85

$$-\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,a}-\frac{x}{2\,(bx^2-a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2*x/((b*x^2 - a)*a)
```

Mupad [B]
time = 4.51, size = 34, normalized size = 0.74

$$\frac{x}{2\,a\,\left(a-b\,x^2\right)}+\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^2,x)
```

```
[Out] x/(2*a*(a - b*x^2)) + atanh((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

3.236 $\int \frac{1}{x(a-bx^2)^2} dx$

Optimal. Leaf size=40

$$\frac{1}{2a(a-bx^2)}+\frac{\log(x)}{a^2}-\frac{\log(a-bx^2)}{2a^2}$$

```
[Out] 1/2/a/(-b*x^2+a)+ln(x)/a^2-1/2*ln(-b*x^2+a)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {272, 46}

$$-\frac{\log(a-bx^2)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^2),x]
```

```
[Out] 1/(2*a*(a - b*x^2)) + Log[x]/a^2 - Log[a - b*x^2]/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a-bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x}+\frac{b}{a(a-bx)^2}+\frac{b}{a^2(a-bx)}\right)\,dx,x,x^2\right)\\ &= \frac{1}{2a(a-bx^2)}+\frac{\log(x)}{a^2}-\frac{\log(a-bx^2)}{2a^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.88

$$\frac{\frac{a}{a-bx^2}+2\log(x)-\log(a-bx^2)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^2),x]
[Out] (a/(a - b*x^2) + 2*Log[x] - Log[a - b*x^2])/(2*a^2)
```

Maple [A]
time = 0.04, size = 44, normalized size = 1.10

method	result	size
risch	$\frac{1}{2a(-b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(-b\,x^2+a)}{2a^2}$	37
norman	$\frac{b\,x^2}{2a^2(-b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(-b\,x^2+a)}{2a^2}$	41
default	$\frac{b\left(\frac{a}{b\left(-b\,x^2+a\right)}-\frac{\ln\left(-b\,x^2+a\right)}{b}\right)}{2a^2}+\frac{\ln(x)}{a^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*b/a^2*(a/b/(-b*x^2+a)-ln(-b*x^2+a)/b)+ln(x)/a^2
```

Maxima [A]
time = 0.28, size = 41, normalized size = 1.02

$$-\frac{1}{2\,(abx^2-a^2)}-\frac{\log(bx^2-a)}{2\,a^2}+\frac{\log(x^2)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2/(a*b*x^2 - a^2) - 1/2*log(b*x^2 - a)/a^2 + 1/2*log(x^2)/a^2
```

Fricas [A]
time = 0.55, size = 53, normalized size = 1.32

$$-\frac{(bx^2-a)\log(bx^2-a)-2\,(bx^2-a)\log(x)+a}{2\,(a^2bx^2-a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 - a)*log(b*x^2 - a) - 2*(b*x^2 - a)*log(x) + a)/(a^2*b*x^2 - a
^3)
```

Sympy [A]
time = 0.13, size = 34, normalized size = 0.85

$$-\frac{1}{-2a^2+2abx^2}+\frac{\log{(x)}}{a^2}-\frac{\log{\left(-\frac{a}{b}+x^2\right)}}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**2,x)
```

```
[Out] -1/(-2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(-a/b + x**2)/(2*a**2)
```

Giac [A]
time = 1.21, size = 51, normalized size = 1.28

$$\frac{\log{(x^2)}}{2\,a^2}-\frac{\log{(|bx^2-a|)}}{2\,a^2}+\frac{bx^2-2\,a}{2\,(bx^2-a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 - a))/a^2 + 1/2*(b*x^2 - 2*a)/((b*x^2
- a)*a^2)
```

Mupad [B]
time = 0.06, size = 36, normalized size = 0.90

$$\frac{\ln{(x)}}{a^2}+\frac{1}{2\,a\,\left(a-b\,x^2\right)}-\frac{\ln{\left(a-b\,x^2\right)}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)^2),x)
```

```
[Out] log(x)/a^2 + 1/(2*a*(a - b*x^2)) - log(a - b*x^2)/(2*a^2)
```

3.237 $\int \frac{1}{x^2(a-bx^2)^2} \, dx$

Optimal. Leaf size=58

$$-\frac{3}{2a^2x}+\frac{1}{2ax\left(a-bx^2\right)}+\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] -3/2/a^2/x+1/2/a/x/(-b*x^2+a)+3/2*arctanh(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {296, 331, 214}

$$\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a - b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\int \frac{1}{x^2 (a-bx^2)^2} dx = \frac{1}{2ax(a-bx^2)} + \frac{3\int \frac{1}{x^2(a-bx^2)} dx}{2a}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax(a-bx^2)} + \frac{(3b)\int \frac{1}{a-bx^2} dx}{2a^2}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax(a-bx^2)} + \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.97

$$-\frac{1}{a^2x} - \frac{bx}{2a^2(-a+bx^2)} + \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^2),x]
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(-a + b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(5/2))
```

Maple [A]
time = 0.04, size = 45, normalized size = 0.78

method	result	size
default	$\frac{b\left(\frac{x}{-2b\,x^2+2a}+\frac{3\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2x}$	45
risch	$\frac{\frac{3b\,x^2}{2a^2}-\frac{1}{a}}{x(-b\,x^2+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2-b\right)}\_R\ln\left(\left(3\_R^2a^5-2b\right)x+a^3\_R\right)\right)}{4}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b/a^2*(1/2*x/(-b*x^2+a)+3/2/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2)))-1/a^2/x
```

Maxima [A]
time = 0.48, size = 65, normalized size = 1.12

$$-\frac{3\,bx^2-2\,a}{2\,(a^2bx^3-a^3x)}-\frac{3\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 - 2*a)/(a^2*b*x^3 - a^3*x) - 3/4*b*log((b*x - sqrt(a*b))/(b*x
 + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.70, size = 140, normalized size = 2.41

$$\left[-\frac{6\,bx^2-3\,(bx^3-ax)\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)-4\,a}{4\,(a^2bx^3-a^3x)},-\frac{3\,bx^2+3\,(bx^3-ax)\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)-2\,a}{2\,(a^2bx^3-a^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 - a*x)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a
)/(b*x^2 - a)) - 4*a)/(a^2*b*x^3 - a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 - a*x)*
sqrt(-b/a)*arctan(x*sqrt(-b/a)) - 2*a)/(a^2*b*x^3 - a^3*x)]
```

Sympy [A]
time = 0.13, size = 83, normalized size = 1.43

$$-\frac{3\sqrt{\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{3\sqrt{\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{2a-3bx^2}{-2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**2,x)
```

```
[Out] -3*sqrt(b/a**5)*log(-a**3*sqrt(b/a**5)/b + x)/4 + 3*sqrt(b/a**5)*log(a**3*s
qrt(b/a**5)/b + x)/4 + (2*a - 3*b*x**2)/(-2*a**3*x + 2*a**2*b*x**3)
```

Giac [A]
time = 4.51, size = 50, normalized size = 0.86

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}\,a^2}-\frac{3\,bx^2-2\,a}{2\,(bx^3-ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/2*(3*b*x^2 - 2*a)/((b*x^
3 - a*x)*a^2)
```

Mupad [B]
time = 4.63, size = 45, normalized size = 0.78

$$\frac{3\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{\frac{1}{a}-\frac{3\,b\,x^2}{2\,a^2}}{a\,x-b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^2),x)
```

```
[Out] (3*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) - (1/a - (3*b*x^2)/(2*a^
2))/(a*x - b*x^3)
```

3.238 $\int \frac{1}{x^3(a-bx^2)^2}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{2a^2x^2}+\frac{b}{2a^2\left(a-bx^2\right)}+\frac{2b\log(x)}{a^3}-\frac{b\log\left(a-bx^2\right)}{a^3}$$

```
[Out] -1/2/a^2/x^2+1/2*b/a^2/(-b*x^2+a)+2*b*ln(x)/a^3-b*ln(-b*x^2+a)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {272, 46}

$$-\frac{b\log\left(a-bx^2\right)}{a^3}+\frac{2b\log(x)}{a^3}+\frac{b}{2a^2\left(a-bx^2\right)}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^2),x]
```

```
[Out] -1/2*1/(a^2*x^2) + b/(2*a^2*(a - b*x^2)) + (2*b*Log[x])/a^3 - (b*Log[a - b*
x^2])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a-bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^2(a-bx)^2}+\frac{2b^2}{a^3(a-bx)}\right)\,dx,x,x^2\right)\\ &= -\frac{1}{2a^2x^2}+\frac{b}{2a^2\left(a-bx^2\right)}+\frac{2b\log(x)}{a^3}-\frac{b\log\left(a-bx^2\right)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.85

$$\frac{-\frac{a}{x^2}+\frac{ab}{a-bx^2}+4b\log(x)-2b\log\left(a-bx^2\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^2),x]
[Out] (-(a/x^2) + (a*b)/(a - b*x^2) + 4*b*Log[x] - 2*b*Log[a - b*x^2])/(2*a^3)
```

Maple [A]
time = 0.04, size = 56, normalized size = 1.08

method	result	size
risch	$\frac{\frac{b\,x^{2}}{a^{2}}-\frac{1}{2a}}{x^{2}\left(-b\,x^{2}+a\right)}+\frac{2b\ln\left(x\right)}{a^{3}}-\frac{b\ln\left(-b\,x^{2}+a\right)}{a^{3}}$	53
norman	$\frac{\frac{b^{2}x^{4}}{a^{3}}-\frac{1}{2a}}{x^{2}\left(-b\,x^{2}+a\right)}+\frac{2b\ln\left(x\right)}{a^{3}}-\frac{b\ln\left(-b\,x^{2}+a\right)}{a^{3}}$	55
default	$\frac{b^{2}\left(-\frac{2\ln\left(-b\,x^{2}+a\right)}{b}+\frac{a}{b\left(-b\,x^{2}+a\right)}\right)}{2a^{3}}-\frac{1}{2a^{2}x^{2}}+\frac{2b\ln\left(x\right)}{a^{3}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*b^2/a^3*(-2*ln(-b*x^2+a)/b+a/b/(-b*x^2+a))-1/2/a^2/x^2+2*b*ln(x)/a^3
```

Maxima [A]
time = 0.28, size = 57, normalized size = 1.10

$$-\frac{2\,bx^2-a}{2\left(a^2bx^4-a^3x^2\right)}-\frac{b\log\left(bx^2-a\right)}{a^3}+\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*(2*b*x^2 - a)/(a^2*b*x^4 - a^3*x^2) - b*log(b*x^2 - a)/a^3 + b*log(x^2
)/a^3
```

Fricas [A]
time = 0.61, size = 80, normalized size = 1.54

$$-\frac{2\,abx^2-a^2+2\left(b^2x^4-abx^2\right)\log\left(bx^2-a\right)-4\left(b^2x^4-abx^2\right)\log\left(x\right)}{2\left(a^3bx^4-a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 - a^2 + 2*(b^2*x^4 - a*b*x^2)*log(b*x^2 - a) - 4*(b^2*x^4 -
 a*b*x^2)*log(x))/(a^3*b*x^4 - a^4*x^2)
```

Sympy [A]
time = 0.16, size = 49, normalized size = 0.94

$$\frac{a - 2bx^2}{-2a^3x^2 + 2a^2bx^4} + \frac{2b\log\left(x\right)}{a^3} - \frac{b\log\left(-\frac{a}{b} + x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**2,x)
```

```
[Out] (a - 2*b*x**2)/(-2*a**3*x**2 + 2*a**2*b*x**4) + 2*b*log(x)/a**3 - b*log(-a/
b + x**2)/a**3
```

Giac [A]
time = 1.21, size = 56, normalized size = 1.08

$$\frac{b\log\left(x^2\right)}{a^3} - \frac{b\log\left(|bx^2 - a|\right)}{a^3} - \frac{2\,bx^2 - a}{2\,(bx^4 - ax^2)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] b*log(x^2)/a^3 - b*log(abs(b*x^2 - a))/a^3 - 1/2*(2*b*x^2 - a)/((b*x^4 - a*
x^2)*a^2)
```

Mupad [B]
time = 4.59, size = 55, normalized size = 1.06

$$\frac{2\,b\,\ln\left(x\right)}{a^3} - \frac{b\,\ln\left(a - b\,x^2\right)}{a^3} - \frac{\frac{1}{2\,a} - \frac{b\,x^2}{a^2}}{a\,x^2 - b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)^2),x)
```

```
[Out] (2*b*log(x))/a^3 - (b*log(a - b*x^2))/a^3 - (1/(2*a) - (b*x^2)/a^2)/(a*x^2
- b*x^4)
```

3.239 $\int \frac{x^3}{(a-bx^2)^3} dx$

Optimal. Leaf size=20

$$\frac{x^4}{4a(a-bx^2)^2}$$

```
[Out] 1/4*x^4/a/(-b*x^2+a)^2
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {270}

$$\frac{x^4}{4a(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^3,x]
```

```
[Out] x^4/(4*a*(a - b*x^2)^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(a-bx^2)^3} dx = \frac{x^4}{4a(a-bx^2)^2}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.25

$$-\frac{a-2bx^2}{4b^2(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^3,x]
```

```
[Out] -1/4*(a - 2*b*x^2)/(b^2*(a - b*x^2)^2)
```

Maple [A]
time = 0.03, size = 33, normalized size = 1.65

method	result	size
gosper	$-\frac{-2b\,x^2+a}{4(-b\,x^2+a)^2b^2}$	24
norman	$\frac{\frac{x^2}{2b}-\frac{a}{4b^2}}{(-b\,x^2+a)^2}$	27
risch	$\frac{\frac{x^2}{2b}-\frac{a}{4b^2}}{(-b\,x^2+a)^2}$	27
default	$\frac{a}{4b^2(-b\,x^2+a)^2}-\frac{1}{2b^2(-b\,x^2+a)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a/b^2/(-b*x^2+a)^2-1/2/b^2/(-b*x^2+a)
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.90

$$\frac{2\,bx^2-a}{4\,(b^4x^4-2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [A]
time = 0.90, size = 38, normalized size = 1.90

$$\frac{2\,bx^2-a}{4\,(b^4x^4-2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.11, size = 36, normalized size = 1.80

$$-\frac{a-2bx^2}{4a^2b^2-8ab^3x^2+4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**3,x)
```

```
[Out] -(a - 2*b*x**2)/(4*a**2*b**2 - 8*a*b**3*x**2 + 4*b**4*x**4)
```

Giac [A]
time = 1.77, size = 26, normalized size = 1.30

$$\frac{2\,bx^2-a}{4\,(bx^2-a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x^2 - a)/((b*x^2 - a)^2*b^2)
```

Mupad [B]
time = 0.04, size = 37, normalized size = 1.85

$$-\frac{\frac{a}{4\,b^2}-\frac{x^2}{2\,b}}{a^2-2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2)^3,x)
```

```
[Out] -(a/(4*b^2) - x^2/(2*b))/(a^2 + b^2*x^4 - 2*a*b*x^2)
```

3.240 $\int \frac{x^2}{(a-bx^2)^3} \, dx$

Optimal. Leaf size=67

$$\frac{x}{4b(a-bx^2)^2}-\frac{x}{8ab(a-bx^2)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

```
[Out] 1/4*x/b/(-b*x^2+a)^2-1/8*x/a/b/(-b*x^2+a)-1/8*arctanh(x*b^(1/2)/a^(1/2))/a^
(3/2)/b^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {294, 205, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}-\frac{x}{8ab(a-bx^2)}+\frac{x}{4b(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^3,x]
```

```
[Out] x/(4*b*(a - b*x^2)^2) - x/(8*a*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]
]/(8*a^(3/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a-bx^2)^3}\,dx &= \frac{x}{4b(a-bx^2)^2} - \frac{\int \frac{1}{(a-bx^2)^2}\,dx}{4b}\\ &= \frac{x}{4b(a-bx^2)^2} - \frac{x}{8ab(a-bx^2)} - \frac{\int \frac{1}{a-bx^2}\,dx}{8ab}\\ &= \frac{x}{4b(a-bx^2)^2} - \frac{x}{8ab(a-bx^2)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.84

$$\frac{x(a+bx^2)}{8ab(a-bx^2)^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^3,x]
[Out] (x*(a + b*x^2))/(8*a*b*(a - b*x^2)^2) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(8*a^(
3/2)*b^(3/2))
```

Maple [A]
time = 0.04, size = 50, normalized size = 0.75

method	result	size
default	$\frac{\frac{x^3}{8a}+\frac{x}{8b}}{(-b\,x^2+a)^2} - \frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8ba\sqrt{ab}}$	50
risch	$\frac{\frac{x^3}{8a}+\frac{x}{8b}}{(-b\,x^2+a)^2} + \frac{\ln\left(bx-\sqrt{ab}\right)}{16\sqrt{ab}\,ba} - \frac{\ln\left(-bx-\sqrt{ab}\right)}{16\sqrt{ab}\,ba}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] (1/8/a*x^3+1/8*x/b)/(-b*x^2+a)^2-1/8/b/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2
))
```

Maxima [A]
time = 0.53, size = 76, normalized size = 1.13

$$\frac{bx^3+ax}{8\,(ab^3x^4-2\,a^2b^2x^2+a^3b)}+\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 + a*x)/(a*b^3*x^4 - 2*a^2*b^2*x^2 + a^3*b) + 1/16*log((b*x - sqr
t(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.93, size = 188, normalized size = 2.81

$$\left[\frac{2\,ab^2x^3+2\,a^2bx+(b^2x^4-2\,abx^2+a^2)\sqrt{ab}\,\log\left(\frac{bx^2-2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{16\,(a^2b^4x^4-2\,a^3b^3x^2+a^4b^2)},\ \frac{ab^2x^3+a^2bx+(b^2x^4-2\,abx^2+a^2)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{8\,(a^2b^4x^4-2\,a^3b^3x^2+a^4b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 + 2*a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)*log(
(b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 + a^
4*b^2), 1/8*(a*b^2*x^3 + a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-a*b)*a
rctan(sqrt(-a*b)*x/a))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 + a^4*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(51) = 102.
time = 0.13, size = 105, normalized size = 1.57

$$\frac{\sqrt{\frac{1}{a^3b^3}}\,\log\left(-a^2b\sqrt{\frac{1}{a^3b^3}}+x\right)}{16}-\frac{\sqrt{\frac{1}{a^3b^3}}\,\log\left(a^2b\sqrt{\frac{1}{a^3b^3}}+x\right)}{16}-\frac{-ax-bx^3}{8a^3b-16a^2b^2x^2+8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**3,x)
```

```
[Out] sqrt(1/(a**3*b**3))*log(-a**2*b*sqrt(1/(a**3*b**3)) + x)/16 - sqrt(1/(a**3*
b**3))*log(a**2*b*sqrt(1/(a**3*b**3)) + x)/16 - (-a*x - b*x**3)/(8*a**3*b -
 16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

Giac [A]
time = 1.16, size = 53, normalized size = 0.79

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\,\sqrt{-ab}\,ab}+\frac{bx^3+ax}{8\,(bx^2-a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a*b) + 1/8*(b*x^3 + a*x)/((b*x^2 - a
)^2*a*b)
```

Mupad [B]
time = 4.62, size = 54, normalized size = 0.81

$$\frac{\frac{x}{8\,b}+\frac{x^3}{8\,a}}{a^2-2\,a\,b\,x^2+b^2\,x^4}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^3,x)
```

```
[Out] (x/(8*b) + x^3/(8*a))/(a^2 + b^2*x^4 - 2*a*b*x^2) - atanh((b^(1/2)*x)/a^(1/
2))/(8*a^(3/2)*b^(3/2))
```

3.241 $\int \frac{x}{(a-bx^2)^3} \, dx$

Optimal. Leaf size=17

$$\frac{1}{4b(a-bx^2)^2}$$

[Out] 1/4/b/(-b*x^2+a)^2

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {267}

$$\frac{1}{4b(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b*x^2)^3,x]

[Out] 1/(4*b*(a - b*x^2)^2)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a-bx^2)^3} \, dx = \frac{1}{4b(a-bx^2)^2}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{4b(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b*x^2)^3,x]

[Out] 1/(4*b*(a - b*x^2)^2)

Maple [A]
time = 0.02, size = 16, normalized size = 0.94

method	result	size
gosper	$\frac{1}{4b(-b\,x^2+a)^2}$	16
derivativedivides	$\frac{1}{4b(-b\,x^2+a)^2}$	16
default	$\frac{1}{4b(-b\,x^2+a)^2}$	16
norman	$\frac{1}{4b(-b\,x^2+a)^2}$	16
risch	$\frac{1}{4b(-b\,x^2+a)^2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/4/b/(-b*x^2+a)^2
```

Maxima [A]
time = 0.30, size = 16, normalized size = 0.94

$$\frac{1}{4\,(bx^2-a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/4/((b*x^2 - a)^2*b)
```

Fricas [A]
time = 0.93, size = 26, normalized size = 1.53

$$\frac{1}{4\,(b^3x^4-2\,ab^2x^2+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/4/(b^3*x^4 - 2*a*b^2*x^2 + a^2*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.10, size = 26, normalized size = 1.53

$$\frac{1}{4a^2b-8ab^2x^2+4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**3,x)
```

```
[Out] 1/(4*a**2*b - 8*a*b**2*x**2 + 4*b**3*x**4)
```

Giac [A]
time = 1.21, size = 16, normalized size = 0.94

$$\frac{1}{4\left(bx^2-a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/4/((b*x^2 - a)^2*b)
```

Mupad [B]
time = 0.03, size = 26, normalized size = 1.53

$$\frac{1}{4\,a^2\,b-8\,a\,b^2\,x^2+4\,b^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2)^3,x)
```

```
[Out] 1/(4*a^2*b + 4*b^3*x^4 - 8*a*b^2*x^2)
```

3.242 $\int \frac{1}{(a-bx^2)^3}\,dx$

Optimal. Leaf size=64

$$\frac{x}{4a\left(a-bx^2\right)^2}+\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

[Out] 1/4*x/a/(-b*x^2+a)^2+3/8*x/a^2/(-b*x^2+a)+3/8*arctanh(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {205, 214}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{x}{4a\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(a - b*x^2)^(-3),x]

[Out] x/(4*a*(a - b*x^2)^2) + (3*x)/(8*a^2*(a - b*x^2)) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b])

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^3}\,dx = \frac{x}{4a(a-bx^2)^2} + \frac{3\int \frac{1}{(a-bx^2)^2}\,dx}{4a}$$
$$= \frac{x}{4a(a-bx^2)^2} + \frac{3x}{8a^2(a-bx^2)} + \frac{3\int \frac{1}{a-bx^2}\,dx}{8a^2}$$
$$= \frac{x}{4a(a-bx^2)^2} + \frac{3x}{8a^2(a-bx^2)} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.88

$$\frac{5ax-3bx^3}{8a^2(a-bx^2)^2} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-3),x]
[Out] (5*a*x - 3*b*x^3)/(8*a^2*(a - b*x^2)^2) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(8*a^(5/2)*Sqrt[b])
```

Maple [A]
time = 0.04, size = 59, normalized size = 0.92

method	result	size
default	$\frac{x}{4a(-b\,x^2+a)^2}+\frac{\frac{3x}{8a(-b\,x^2+a)}+\frac{3\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{a}$	59
risch	$\frac{-\frac{3b\,x^3}{8a^2}+\frac{5x}{8a}}{(-b\,x^2+a)^2}+\frac{3\ln\left(bx+\sqrt{ab}\right)}{16\sqrt{ab}\,a^2}-\frac{3\ln\left(-bx+\sqrt{ab}\right)}{16\sqrt{ab}\,a^2}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*x/a/(-b*x^2+a)^2+3/4/a*(1/2*x/a/(-b*x^2+a)+1/2/a/(a*b)^(1/2)*arctanh(b*
x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.61, size = 73, normalized size = 1.14

$$-\frac{3\,bx^3-5\,ax}{8\,(a^2b^2x^4-2\,a^3bx^2+a^4)} - \frac{3\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*x^3 - 5*a*x)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 3/16*log((b*x -
sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 1.36, size = 188, normalized size = 2.94

$$\left[-\frac{6\,ab^2x^3-10\,a^2bx-3\,(b^2x^4-2\,abx^2+a^2)\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{16\,(a^3b^3x^4-2\,a^4b^2x^2+a^5b)},-\frac{3\,ab^2x^3-5\,a^2bx+3\,(b^2x^4-2\,abx^2+a^2)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{8\,(a^3b^3x^4-2\,a^4b^2x^2+a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(6*a*b^2*x^3 - 10*a^2*b*x - 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)*
log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2
+ a^5*b), -1/8*(3*a*b^2*x^3 - 5*a^2*b*x + 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqr
t(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [A]
time = 0.14, size = 99, normalized size = 1.55

$$-\frac{3\sqrt{\frac{1}{a^5b}}\log\left(-a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16}+\frac{3\sqrt{\frac{1}{a^5b}}\log\left(a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16}-\frac{-5ax+3bx^3}{8a^4-16a^3bx^2+8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(1/(a**5*b))*log(-a**3*sqrt(1/(a**5*b)) + x)/16 + 3*sqrt(1/(a**5*b))
*log(a**3*sqrt(1/(a**5*b)) + x)/16 - (-5*a*x + 3*b*x**3)/(8*a**4 - 16*a**3*
b*x**2 + 8*a**2*b**2*x**4)
```

Giac [A]
time = 1.88, size = 49, normalized size = 0.77

$$-\frac{3\,\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\,\sqrt{-ab}\,a^2}-\frac{3\,bx^3-5\,ax}{8\,(bx^2-a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/8*(3*b*x^3 - 5*a*x)/((b*x^
2 - a)^2*a^2)
```

Mupad [B]
time = 4.60, size = 55, normalized size = 0.86

$$\frac{\frac{5x}{8a}-\frac{3bx^3}{8a^2}}{a^2-2abx^2+b^2x^4}+\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^3,x)
```

```
[Out] ((5*x)/(8*a) - (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 - 2*a*b*x^2) + (3*atanh((b
^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

3.243 $\int \frac{1}{x(a-bx^2)^3} dx$

Optimal. Leaf size=57

$$\frac{1}{4a(a-bx^2)^2}+\frac{1}{2a^2(a-bx^2)}+\frac{\log(x)}{a^3}-\frac{\log(a-bx^2)}{2a^3}$$

```
[Out] 1/4/a/(-b*x^2+a)^2+1/2/a^2/(-b*x^2+a)+ln(x)/a^3-1/2*ln(-b*x^2+a)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {272, 46}

$$-\frac{\log(a-bx^2)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a^2(a-bx^2)}+\frac{1}{4a(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^3),x]
```

```
[Out] 1/(4*a*(a - b*x^2)^2) + 1/(2*a^2*(a - b*x^2)) + Log[x]/a^3 - Log[a - b*x^2]
/(2*a^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a-bx^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)^3} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x}+\frac{b}{a(a-bx)^3}+\frac{b}{a^2(a-bx)^2}+\frac{b}{a^3(a-bx)}\right) dx, x, x^2\right)\\ &= \frac{1}{4a(a-bx^2)^2}+\frac{1}{2a^2(a-bx^2)}+\frac{\log(x)}{a^3}-\frac{\log(a-bx^2)}{2a^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.79

$$\frac{\frac{a(3a-2bx^2)}{(a-bx^2)^2}+4\log(x)-2\log(a-bx^2)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^3),x]
[Out] ((a*(3*a - 2*b*x^2))/(a - b*x^2)^2 + 4*Log[x] - 2*Log[a - b*x^2])/(4*a^3)
```

Maple [A]
time = 0.05, size = 62, normalized size = 1.09

method	result	size
risch	$\frac{-\frac{b\,x^2}{2a^2}+\frac{3}{4a}}{(-b\,x^2+a)^2}+\frac{\ln(x)}{a^3}-\frac{\ln(-b\,x^2+a)}{2a^3}$	48
norman	$\frac{\frac{b\,x^2}{a^2}-\frac{3b^2x^4}{4a^3}}{(-b\,x^2+a)^2}+\frac{\ln(x)}{a^3}-\frac{\ln(-b\,x^2+a)}{2a^3}$	53
default	$\frac{b\left(\frac{a}{b(-b\,x^2+a)}-\frac{\ln(-b\,x^2+a)}{b}+\frac{a^2}{2b(-b\,x^2+a)^2}\right)}{2a^3}+\frac{\ln(x)}{a^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/2/a^3*b*(a/b/(-b*x^2+a)-ln(-b*x^2+a)/b+1/2/b*a^2/(-b*x^2+a)^2)+ln(x)/a^3
```

Maxima [A]
time = 0.28, size = 62, normalized size = 1.09

$$-\frac{2\,bx^2-3\,a}{4\,(a^2b^2x^4-2\,a^3bx^2+a^4)}-\frac{\log(bx^2-a)}{2\,a^3}+\frac{\log(x^2)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 - 3*a)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 - a)
/a^3 + 1/2*log(x^2)/a^3
```

Fricas [A]
time = 1.44, size = 92, normalized size = 1.61

$$-\frac{2\,abx^2-3\,a^2+2\,(b^2x^4-2\,abx^2+a^2)\log(bx^2-a)-4\,(b^2x^4-2\,abx^2+a^2)\log(x)}{4\,(a^3b^2x^4-2\,a^4bx^2+a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*x^2 - 3*a^2 + 2*(b^2*x^4 - 2*a*b*x^2 + a^2)*log(b*x^2 - a) - 4*
(b^2*x^4 - 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 - 2*a^4*b*x^2 + a^5)
```

Sympy [A]
time = 0.18, size = 56, normalized size = 0.98

$$-\frac{-3a+2bx^2}{4a^4-8a^3bx^2+4a^2b^2x^4}+\frac{\log(x)}{a^3}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**3,x)
```

```
[Out] -(-3*a + 2*b*x**2)/(4*a**4 - 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a**
3 - log(-a/b + x**2)/(2*a**3)
```

Giac [A]
time = 1.70, size = 63, normalized size = 1.11

$$\frac{\log(x^2)}{2a^3}-\frac{\log(|bx^2-a|)}{2a^3}+\frac{3b^2x^4-8abx^2+6a^2}{4(bx^2-a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 - a))/a^3 + 1/4*(3*b^2*x^4 - 8*a*b*x^2
 + 6*a^2)/((b*x^2 - a)^2*a^3)
```

Mupad [B]
time = 0.06, size = 57, normalized size = 1.00

$$\frac{\ln(x)}{a^3}+\frac{\frac{3}{4a}-\frac{bx^2}{2a^2}}{a^2-2abx^2+b^2x^4}-\frac{\ln(a-bx^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)^3),x)
```

```
[Out] log(x)/a^3 + (3/(4*a) - (b*x^2)/(2*a^2))/(a^2 + b^2*x^4 - 2*a*b*x^2) - log(
a - b*x^2)/(2*a^3)
```

3.244 $\int \frac{1}{x^2(a-bx^2)^3}\,dx$

Optimal. Leaf size=78

$$-\frac{15}{8a^3x}+\frac{1}{4ax\left(a-bx^2\right)^2}+\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}$$

```
[Out] -15/8/a^3/x+1/4/a/x/(-b*x^2+a)^2+5/8/a^2/x/(-b*x^2+a)+15/8*arctanh(x*b^(1/2
)/a^(1/2))*b^(1/2)/a^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {296, 331, 214}

$$\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{1}{4ax\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^3),x]
```

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a - b*x^2)^2) + 5/(8*a^2*x*(a - b*x^2)) + (15*Sqr
t[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 (a-bx^2)^3}\,dx &= \frac{1}{4ax(a-bx^2)^2}+\frac{5\int \frac{1}{x^2(a-bx^2)^2}\,dx}{4a}\\ &= \frac{1}{4ax(a-bx^2)^2}+\frac{5}{8a^2x(a-bx^2)}+\frac{15\int \frac{1}{x^2(a-bx^2)}\,dx}{8a^2}\\ &= -\frac{15}{8a^3x}+\frac{1}{4ax(a-bx^2)^2}+\frac{5}{8a^2x(a-bx^2)}+\frac{(15b)\int \frac{1}{a-bx^2}\,dx}{8a^3}\\ &= -\frac{15}{8a^3x}+\frac{1}{4ax(a-bx^2)^2}+\frac{5}{8a^2x(a-bx^2)}+\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.88

$$\frac{-8a^2+25abx^2-15b^2x^4}{8a^3x(a-bx^2)^2}+\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^3),x]
[Out] (-8*a^2 + 25*a*b*x^2 - 15*b^2*x^4)/(8*a^3*x*(a - b*x^2)^2) + (15*Sqrt[b]*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Maple [A]
time = 0.05, size = 54, normalized size = 0.69

method	result	size
default	$\frac{b\left(\frac{-\frac{7}{8}b\,x^3+\frac{9}{8}ax}{\left(-b\,x^2+a\right)^2}+\frac{15\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3x}$	54
risch	$\frac{-\frac{15b^2x^4}{8a^3}+\frac{25b\,x^2}{8a^2}-\frac{1}{a}}{x(-b\,x^2+a)^2}+\frac{15\left(\sum_{\_R=\mathrm{RootOf}\left(a^7\_Z^2-b\right)}\_R\ln\left(\left(3\_R^2a^7-2b\right)x+a^4\_R\right)\right)}{16}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*b*((-7/8*b*x^3+9/8*a*x)/(-b*x^2+a)^2+15/8/(a*b)^(1/2)*arctanh(b*x/(a*
b)^(1/2)))-1/a^3/x
```

Maxima [A]
time = 0.50, size = 86, normalized size = 1.10

$$-\frac{15\,b^2x^4-25\,abx^2+8\,a^2}{8\,(a^3b^2x^5-2\,a^4bx^3+a^5x)}-\frac{15\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b^2*x^4 - 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 - 2*a^4*b*x^3 + a^5*x)
- 15/16*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 1.22, size = 202, normalized size = 2.59

$$\left[-\frac{30\,b^2x^4-50\,abx^2-15\,(b^2x^5-2\,abx^3+a^2x)\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)+16\,a^2}{16\,(a^3b^2x^5-2\,a^4bx^3+a^5x)},-\frac{15\,b^2x^4-25\,abx^2+15\,(b^2x^5-2\,abx^3+a^2x)\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)+8\,a^2}{8\,(a^3b^2x^5-2\,a^4bx^3+a^5x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 - 50*a*b*x^2 - 15*(b^2*x^5 - 2*a*b*x^3 + a^2*x)*sqrt(b/a
)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) + 16*a^2)/(a^3*b^2*x^5 - 2
*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 - 25*a*b*x^2 + 15*(b^2*x^5 - 2*a*b*x^
3 + a^2*x)*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 8*a^2)/(a^3*b^2*x^5 - 2*a^4*b*
x^3 + a^5*x)]
```

Sympy [A]
time = 0.19, size = 107, normalized size = 1.37

$$-\frac{15\sqrt{\frac{b}{a^7}}\log\left(-\frac{a^4\sqrt{\frac{b}{a^7}}}{b}+x\right)}{16}+\frac{15\sqrt{\frac{b}{a^7}}\log\left(\frac{a^4\sqrt{\frac{b}{a^7}}}{b}+x\right)}{16}-\frac{8a^2-25abx^2+15b^2x^4}{8a^5x-16a^4bx^3+8a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**3,x)
```

```
[Out] -15*sqrt(b/a**7)*log(-a**4*sqrt(b/a**7)/b + x)/16 + 15*sqrt(b/a**7)*log(a**
4*sqrt(b/a**7)/b + x)/16 - (8*a**2 - 25*a*b*x**2 + 15*b**2*x**4)/(8*a**5*x
- 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

Giac [A]
time = 2.50, size = 61, normalized size = 0.78

$$-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\,\sqrt{-ab}\,a^3}-\frac{7\,b^2x^3-9\,abx}{8\,(bx^2-a)^2a^3}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3) - 1/8*(7*b^2*x^3 - 9*a*b*x)
/((b*x^2 - a)^2*a^3) - 1/(a^3*x)
```

Mupad [B]
time = 4.60, size = 66, normalized size = 0.85

$$\frac{15\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{\frac{1}{a}-\frac{25\,b\,x^2}{8\,a^2}+\frac{15\,b^2\,x^4}{8\,a^3}}{a^2\,x-2\,a\,b\,x^3+b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^3),x)
```

```
[Out] (15*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) - (1/a - (25*b*x^2)/(8*
a^2) + (15*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 - 2*a*b*x^3)
```

3.245 $\int \frac{1}{x^3(a-bx^2)^3}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{2a^3x^2}+\frac{b}{4a^2\left(a-bx^2\right)^2}+\frac{b}{a^3\left(a-bx^2\right)}+\frac{3b\log(x)}{a^4}-\frac{3b\log\left(a-bx^2\right)}{2a^4}$$

```
[Out] -1/2/a^3/x^2+1/4*b/a^2/(-b*x^2+a)^2+b/a^3/(-b*x^2+a)+3*b*ln(x)/a^4-3/2*b*ln
(-b*x^2+a)/a^4
```

Rubi [A]

time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{272, 46\}$

$$-\frac{3b\log\left(a-bx^2\right)}{2a^4}+\frac{3b\log(x)}{a^4}+\frac{b}{a^3\left(a-bx^2\right)}-\frac{1}{2a^3x^2}+\frac{b}{4a^2\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^3),x]
```

```
[Out] -1/2*1/(a^3*x^2) + b/(4*a^2*(a - b*x^2)^2) + b/(a^3*(a - b*x^2)) + (3*b*Log
[x])/a^4 - (3*b*Log[a - b*x^2])/(2*a^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a-bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{a^2(a-bx)^3}+\frac{2b^2}{a^3(a-bx)^2}+\frac{3b^2}{a^4(a-bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{2a^3x^2}+\frac{b}{4a^2\left(a-bx^2\right)^2}+\frac{b}{a^3\left(a-bx^2\right)}+\frac{3b\log(x)}{a^4}-\frac{3b\log\left(a-bx^2\right)}{2a^4}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.87

$$\frac{\frac{a(-2a^2+9abx^2-6b^2x^4)}{(ax-bx^3)^2}+12b\log(x)-6b\log\left(a-bx^2\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^3),x]
```

```
[Out] ((a*(-2*a^2 + 9*a*b*x^2 - 6*b^2*x^4))/(a*x - b*x^3)^2 + 12*b*Log[x] - 6*b*L
og[a - b*x^2])/(4*a^4)
```

Maple [A]
time = 0.05, size = 75, normalized size = 1.09

method	result	size
risch	$\frac{-\frac{3b^2x^4}{2a^3}+\frac{9b\,x^2}{4a^2}-\frac{1}{2a}}{x^2(-b\,x^2+a)^2}+\frac{3b\ln(x)}{a^4}-\frac{3b\ln\left(-b\,x^2+a\right)}{2a^4}$	65
norman	$\frac{-\frac{1}{2a}+\frac{3b^2x^4}{a^3}-\frac{9b^3x^6}{4a^4}}{x^2(-b\,x^2+a)^2}+\frac{3b\ln(x)}{a^4}-\frac{3b\ln\left(-b\,x^2+a\right)}{2a^4}$	67
default	$\frac{b^2\left(-\frac{3\ln\left(-b\,x^2+a\right)}{b}+\frac{2a}{b\left(-b\,x^2+a\right)}+\frac{a^2}{2b\left(-b\,x^2+a\right)^2}\right)}{2a^4}-\frac{1}{2a^3x^2}+\frac{3b\ln(x)}{a^4}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^4*b^2*(-3*ln(-b*x^2+a)/b+2*a/b/(-b*x^2+a)+1/2/b*a^2/(-b*x^2+a)^2)-1/2
/a^3/x^2+3*b*ln(x)/a^4
```

Maxima [A]
time = 0.38, size = 79, normalized size = 1.14

$$-\frac{6\,b^2x^4-9\,abx^2+2\,a^2}{4\,(a^3b^2x^6-2\,a^4bx^4+a^5x^2)}-\frac{3\,b\log\left(bx^2-a\right)}{2\,a^4}+\frac{3\,b\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*x^4 - 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 - 2*a^4*b*x^4 + a^5*x^2)
- 3/2*b*log(b*x^2 - a)/a^4 + 3/2*b*log(x^2)/a^4
```

Fricas [A]
time = 1.45, size = 121, normalized size = 1.75

$$-\frac{6\,ab^2x^4-9\,a^2bx^2+2\,a^3+6\,(b^3x^6-2\,ab^2x^4+a^2bx^2)\log\left(bx^2-a\right)-12\,(b^3x^6-2\,ab^2x^4+a^2bx^2)\log\left(x\right)}{4\,(a^4b^2x^6-2\,a^5bx^4+a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3 + 6*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 - a) - 12*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 - 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [A]
time = 0.23, size = 78, normalized size = 1.13

$$-\frac{2a^2 - 9abx^2 + 6b^2x^4}{4a^5x^2 - 8a^4bx^4 + 4a^3b^2x^6} + \frac{3b\log(x)}{a^4} - \frac{3b\log\left(-\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**3,x)
```

```
[Out] -(2*a**2 - 9*a*b*x**2 + 6*b**2*x**4)/(4*a**5*x**2 - 8*a**4*b*x**4 + 4*a**3*
b**2*x**6) + 3*b*log(x)/a**4 - 3*b*log(-a/b + x**2)/(2*a**4)
```

Giac [A]
time = 1.72, size = 84, normalized size = 1.22

$$\frac{3\,b\log(x^2)}{2\,a^4} - \frac{3\,b\log(|bx^2 - a|)}{2\,a^4} + \frac{9\,b^3x^4 - 22\,ab^2x^2 + 14\,a^2b}{4\,(bx^2 - a)^2a^4} - \frac{3\,bx^2 + a}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*b*log(x^2)/a^4 - 3/2*b*log(abs(b*x^2 - a))/a^4 + 1/4*(9*b^3*x^4 - 22*a*
b^2*x^2 + 14*a^2*b)/((b*x^2 - a)^2*a^4) - 1/2*(3*b*x^2 + a)/(a^4*x^2)
```

Mupad [B]
time = 4.61, size = 76, normalized size = 1.10

$$\frac{3\,b\,\ln(x)}{a^4} - \frac{3\,b\,\ln(a - b\,x^2)}{2\,a^4} - \frac{\frac{1}{2\,a} - \frac{9\,b\,x^2}{4\,a^2} + \frac{3\,b^2\,x^4}{2\,a^3}}{a^2\,x^2 - 2\,a\,b\,x^4 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)^3),x)
```

```
[Out] (3*b*log(x))/a^4 - (3*b*log(a - b*x^2))/(2*a^4) - (1/(2*a) - (9*b*x^2)/(4*a
^2) + (3*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 - 2*a*b*x^4)
```

3.246 $\int \frac{x^3}{(a-bx^2)^5} \, dx$

Optimal. Leaf size=36

$$\frac{a}{8b^2(a-bx^2)^4}-\frac{1}{6b^2(a-bx^2)^3}$$

[Out] 1/8*a/b^2/(-b*x^2+a)^4-1/6/b^2/(-b*x^2+a)^3

Rubi [A]

time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {272, 45}

$$\frac{a}{8b^2(a-bx^2)^4}-\frac{1}{6b^2(a-bx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a - b*x^2)^5,x]

[Out] a/(8*b^2*(a - b*x^2)^4) - 1/(6*b^2*(a - b*x^2)^3)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a-bx^2)^5} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a-bx)^5} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a}{b(a-bx)^5}-\frac{1}{b(a-bx)^4}\right) \, dx, x, x^2\right)\\
&= \frac{a}{8b^2(a-bx^2)^4}-\frac{1}{6b^2(a-bx^2)^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.69

$$-\frac{a-4bx^2}{24b^2\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^5,x]
```

```
[Out] -1/24*(a - 4*b*x^2)/(b^2*(a - b*x^2)^4)
```

Maple [A]
time = 0.04, size = 33, normalized size = 0.92

method	result	size
gosper	$-\frac{-4b\,x^2+a}{24\left(-b\,x^2+a\right)^4 b^2}$	24
norman	$\frac{\frac{x^2}{6b}-\frac{a}{24b^2}}{\left(-b\,x^2+a\right)^4}$	27
risch	$\frac{\frac{x^2}{6b}-\frac{a}{24b^2}}{\left(-b\,x^2+a\right)^4}$	27
default	$\frac{a}{8b^2\left(-b\,x^2+a\right)^4}-\frac{1}{6b^2\left(-b\,x^2+a\right)^3}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a/b^2/(-b*x^2+a)^4-1/6/b^2/(-b*x^2+a)^3
```

Maxima [A]
time = 0.33, size = 60, normalized size = 1.67

$$\frac{4\,bx^2-a}{24\left(b^6x^8-4\,ab^5x^6+6\,a^2b^4x^4-4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2 +
 a^4*b^2)
```

Fricas [A]
time = 1.11, size = 60, normalized size = 1.67

$$\frac{4\,bx^2-a}{24\left(b^6x^8-4\,ab^5x^6+6\,a^2b^4x^4-4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2 +
 a^4*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.
time = 0.19, size = 60, normalized size = 1.67

$$-\frac{a-4bx^2}{24a^4b^2-96a^3b^3x^2+144a^2b^4x^4-96ab^5x^6+24b^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**5,x)
```

```
[Out] -(a - 4*b*x**2)/(24*a**4*b**2 - 96*a**3*b**3*x**2 + 144*a**2*b**4*x**4 - 96
*a*b**5*x**6 + 24*b**6*x**8)
```

Giac [A]
time = 2.01, size = 39, normalized size = 1.08

$$\frac{\frac{4}{(bx^2-a)^3b}+\frac{3\,a}{(bx^2-a)^4b}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*(4/((b*x^2 - a)^3*b) + 3*a/((b*x^2 - a)^4*b))/b
```

Mupad [B]
time = 4.60, size = 59, normalized size = 1.64

$$-\frac{\frac{a}{24\,b^2}-\frac{x^2}{6\,b}}{a^4-4\,a^3\,b\,x^2+6\,a^2\,b^2\,x^4-4\,a\,b^3\,x^6+b^4\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2)^5,x)
```

```
[Out] -(a/(24*b^2) - x^2/(6*b))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^
2*b^2*x^4)
```

3.247 $\int \frac{x^2}{(a-bx^2)^5} \, dx$

Optimal. Leaf size=109

$$\frac{x}{8b(a-bx^2)^4}-\frac{x}{48ab(a-bx^2)^3}-\frac{5x}{192a^2b(a-bx^2)^2}-\frac{5x}{128a^3b(a-bx^2)}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

```
[Out] 1/8*x/b/(-b*x^2+a)^4-1/48*x/a/b/(-b*x^2+a)^3-5/192*x/a^2/b/(-b*x^2+a)^2-5/1
28*x/a^3/b/(-b*x^2+a)-5/128*arctanh(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {294, 205, 214}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}-\frac{5x}{128a^3b(a-bx^2)}-\frac{5x}{192a^2b(a-bx^2)^2}-\frac{x}{48ab(a-bx^2)^3}+\frac{x}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^5,x]
```

```
[Out] x/(8*b*(a - b*x^2)^4) - x/(48*a*b*(a - b*x^2)^3) - (5*x)/(192*a^2*b*(a - b*
x^2)^2) - (5*x)/(128*a^3*b*(a - b*x^2)) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(128*a^(7/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a-bx^2)^5}\,dx &= \frac{x}{8b\,(a-bx^2)^4}-\frac{\int \frac{1}{(a-bx^2)^4}\,dx}{8b}\\ &= \frac{x}{8b\,(a-bx^2)^4}-\frac{x}{48ab\,(a-bx^2)^3}-\frac{5\int \frac{1}{(a-bx^2)^3}\,dx}{48ab}\\ &= \frac{x}{8b\,(a-bx^2)^4}-\frac{x}{48ab\,(a-bx^2)^3}-\frac{5x}{192a^2b\,(a-bx^2)^2}-\frac{5\int \frac{1}{(a-bx^2)^2}\,dx}{64a^2b}\\ &= \frac{x}{8b\,(a-bx^2)^4}-\frac{x}{48ab\,(a-bx^2)^3}-\frac{5x}{192a^2b\,(a-bx^2)^2}-\frac{5x}{128a^3b\,(a-bx^2)}-\frac{5\int \frac{1}{a-bx^2}\,dx}{128a^3b}\\ &= \frac{x}{8b\,(a-bx^2)^4}-\frac{x}{48ab\,(a-bx^2)^3}-\frac{5x}{192a^2b\,(a-bx^2)^2}-\frac{5x}{128a^3b\,(a-bx^2)}-\frac{5\tanh^{-1}\left(\cdots\right)}{128a^{7/2}b\cdots}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 0.74

$$\frac{15a^3x+73a^2bx^3-55ab^2x^5+15b^3x^7}{384a^3b\,(a-bx^2)^4}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^5,x]
```

```
[Out] (15*a^3*x + 73*a^2*b*x^3 - 55*a*b^2*x^5 + 15*b^3*x^7)/(384*a^3*b*(a - b*x^2
)^4) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2))
```

Maple [A]
time = 0.06, size = 70, normalized size = 0.64

method	result	size
default	$\frac{\frac{5b^2x^7}{128a^3}-\frac{55b\,x^5}{384a^2}+\frac{73x^3}{384a}+\frac{5x}{128b}}{(-b\,x^2+a)^4}-\frac{5\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{128a^3b\sqrt{ab}}$	70

risch	$\frac{\frac{5b^2x^7}{128a^3}-\frac{55b\,x^5}{384a^2}+\frac{73x^3}{384a}+\frac{5x}{128b}}{(-b\,x^2+a)^4}+\frac{5\ln\left(bx-\sqrt{ab}\right)}{256\sqrt{ab}\,b\,a^3}-\frac{5\ln\left(-bx-\sqrt{ab}\right)}{256\sqrt{ab}\,b\,a^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (5/128*b^2/a^3*x^7-55/384*b/a^2*x^5+73/384/a*x^3+5/128*x/b)/(-b*x^2+a)^4-5/
128/a^3/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 124, normalized size = 1.14

$$\frac{15\,b^3x^7-55\,ab^2x^5+73\,a^2bx^3+15\,a^3x}{384\,(a^3b^5x^8-4\,a^4b^4x^6+6\,a^5b^3x^4-4\,a^6b^2x^2+a^7b)}+\frac{5\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^3*x^7 - 55*a*b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/(a^3*b^5*x^8 -
4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 - 4*a^6*b^2*x^2 + a^7*b) + 5/256*log((b*x - s
qrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^3*b)
```

Fricas [A]
time = 1.32, size = 324, normalized size = 2.97

$$\left[\frac{30\,ab^4x^7-110\,a^2b^3x^5+146\,a^3b^2x^3+30\,a^4bx+15\,(b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+a^4)\sqrt{ab}\,\log\left(\frac{bx^2-2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{768\,(a^4b^6x^8-4\,a^5b^5x^6+6\,a^6b^4x^4-4\,a^7b^3x^2+a^8b^2)},\ \frac{15\,ab^4x^7-55\,a^2b^3x^5+73\,a^3b^2x^3+15\,a^4bx+15\,(b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+a^4)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{384\,(a^4b^6x^8-4\,a^5b^5x^6+6\,a^6b^4x^4-4\,a^7b^3x^2+a^8b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 - 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 + 30*a^4*b*x + 15*
(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b)*log((
b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^4*b^6*x^8 - 4*a^5*b^5*x^6 + 6*a
^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 - 55*a^2*b^3*x^5
 + 73*a^3*b^2*x^3 + 15*a^4*b*x + 15*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4
- 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^4*b^6*x^8 - 4*a^
5*b^5*x^6 + 6*a^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2)]
```

Sympy [A]
time = 0.22, size = 160, normalized size = 1.47

$$\frac{5\sqrt{\frac{1}{a^7b^3}}\,\log\left(-a^4b\sqrt{\frac{1}{a^7b^3}}+x\right)}{256}-\frac{5\sqrt{\frac{1}{a^7b^3}}\,\log\left(a^4b\sqrt{\frac{1}{a^7b^3}}+x\right)}{256}-\frac{-15a^3x-73a^2bx^3+55ab^2x^5-15b^3x^7}{384a^7b-1536a^6b^2x^2+2304a^5b^3x^4-1536a^4b^4x^6+384a^3b^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**5,x)
```

```
[Out] 5*sqrt(1/(a**7*b**3))*log(-a**4*b*sqrt(1/(a**7*b**3)) + x)/256 - 5*sqrt(1/(
a**7*b**3))*log(a**4*b*sqrt(1/(a**7*b**3)) + x)/256 - (-15*a**3*x - 73*a**2
*b*x**3 + 55*a*b**2*x**5 - 15*b**3*x**7)/(384*a**7*b - 1536*a**6*b**2*x**2
+ 2304*a**5*b**3*x**4 - 1536*a**4*b**4*x**6 + 384*a**3*b**5*x**8)
```

Giac [A]
time = 1.40, size = 77, normalized size = 0.71

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128 \sqrt{-ab}\, a^3 b} + \frac{15 b^3 x^7 - 55 ab^2 x^5 + 73 a^2 b x^3 + 15 a^3 x}{384 (bx^2 - a)^4 a^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 5/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3*b) + 1/384*(15*b^3*x^7 - 55*a*
b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/((b*x^2 - a)^4*a^3*b)
```

Mupad [B]
time = 4.76, size = 96, normalized size = 0.88

$$\frac{\frac{5\,x}{128\,b} + \frac{73\,x^3}{384\,a} - \frac{55\,b\,x^5}{384\,a^2} + \frac{5\,b^2\,x^7}{128\,a^3}}{a^4 - 4\,a^3\,b\,x^2 + 6\,a^2\,b^2\,x^4 - 4\,a\,b^3\,x^6 + b^4\,x^8} - \frac{5\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{7/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^5,x)
```

```
[Out] ((5*x)/(128*b) + (73*x^3)/(384*a) - (55*b*x^5)/(384*a^2) + (5*b^2*x^7)/(128
*a^3))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4) - (5*ata
nh((b^(1/2)*x)/a^(1/2)))/(128*a^(7/2)*b^(3/2))
```

3.248 $\int \frac{x}{(a-bx^2)^5} dx$

Optimal. Leaf size=17

$$\frac{1}{8b(a-bx^2)^4}$$

[Out] 1/8/b/(-b*x^2+a)^4

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {267}

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b*x^2)^5,x]

[Out] 1/(8*b*(a - b*x^2)^4)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a-bx^2)^5} dx = \frac{1}{8b(a-bx^2)^4}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b*x^2)^5,x]

[Out] 1/(8*b*(a - b*x^2)^4)

Maple [A]
time = 0.04, size = 16, normalized size = 0.94

method	result	size
gosper	$\frac{1}{8b(-b\,x^2+a)^4}$	16
derivativedivides	$\frac{1}{8b(-b\,x^2+a)^4}$	16
default	$\frac{1}{8b(-b\,x^2+a)^4}$	16
norman	$\frac{1}{8b(-b\,x^2+a)^4}$	16
risch	$\frac{1}{8b(-b\,x^2+a)^4}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/8/b/(-b*x^2+a)^4
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.94

$$\frac{1}{8\left(bx^2-a\right)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="maxima")
[Out] 1/8/((b*x^2 - a)^4*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(16) = 32$.
time = 0.92, size = 48, normalized size = 2.82

$$\frac{1}{8\left(b^5x^8-4\,ab^4x^6+6\,a^2b^3x^4-4\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="fricas")
[Out] 1/8/(b^5*x^8 - 4*a*b^4*x^6 + 6*a^2*b^3*x^4 - 4*a^3*b^2*x^2 + a^4*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.
time = 0.18, size = 49, normalized size = 2.88

$$\frac{1}{8a^4b-32a^3b^2x^2+48a^2b^3x^4-32ab^4x^6+8b^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**5,x)
```

```
[Out] 1/(8*a**4*b - 32*a**3*b**2*x**2 + 48*a**2*b**3*x**4 - 32*a*b**4*x**6 + 8*b*
*5*x**8)
```

Giac [A]
time = 2.56, size = 16, normalized size = 0.94

$$\frac{1}{8\left(bx^2-a\right)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/8/((b*x^2 - a)^4*b)
```

Mupad [B]
time = 4.69, size = 48, normalized size = 2.82

$$\frac{1}{8\,a^4\,b-32\,a^3\,b^2\,x^2+48\,a^2\,b^3\,x^4-32\,a\,b^4\,x^6+8\,b^5\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2)^5,x)
```

```
[Out] 1/(8*a^4*b + 8*b^5*x^8 - 32*a*b^4*x^6 - 32*a^3*b^2*x^2 + 48*a^2*b^3*x^4)
```

3.249 $\int \frac{1}{(a-bx^2)^5} dx$

Optimal. Leaf size=100

$$\frac{x}{8a(a-bx^2)^4}+\frac{7x}{48a^2(a-bx^2)^3}+\frac{35x}{192a^3(a-bx^2)^2}+\frac{35x}{128a^4(a-bx^2)}+\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}$$

```
[Out] 1/8*x/a/(-b*x^2+a)^4+7/48*x/a^2/(-b*x^2+a)^3+35/192*x/a^3/(-b*x^2+a)^2+35/1
28*x/a^4/(-b*x^2+a)+35/128*arctanh(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {205, 214}

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}+\frac{35x}{128a^4(a-bx^2)}+\frac{35x}{192a^3(a-bx^2)^2}+\frac{7x}{48a^2(a-bx^2)^3}+\frac{x}{8a(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-5),x]
```

```
[Out] x/(8*a*(a - b*x^2)^4) + (7*x)/(48*a^2*(a - b*x^2)^3) + (35*x)/(192*a^3*(a -
 b*x^2)^2) + (35*x)/(128*a^4*(a - b*x^2)) + (35*ArcTanh[(Sqrt[b]*x)/Sqrt[a]
])/(128*a^(9/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^5}\,dx = \frac{x}{8a(a-bx^2)^4} + \frac{7\int \frac{1}{(a-bx^2)^4}\,dx}{8a}$$
$$= \frac{x}{8a(a-bx^2)^4} + \frac{7x}{48a^2(a-bx^2)^3} + \frac{35\int \frac{1}{(a-bx^2)^3}\,dx}{48a^2}$$
$$= \frac{x}{8a(a-bx^2)^4} + \frac{7x}{48a^2(a-bx^2)^3} + \frac{35x}{192a^3(a-bx^2)^2} + \frac{35\int \frac{1}{(a-bx^2)^2}\,dx}{64a^3}$$
$$= \frac{x}{8a(a-bx^2)^4} + \frac{7x}{48a^2(a-bx^2)^3} + \frac{35x}{192a^3(a-bx^2)^2} + \frac{35x}{128a^4(a-bx^2)} + \frac{35\int \frac{1}{a-bx^2}\,dx}{128a^4}$$
$$= \frac{x}{8a(a-bx^2)^4} + \frac{7x}{48a^2(a-bx^2)^3} + \frac{35x}{192a^3(a-bx^2)^2} + \frac{35x}{128a^4(a-bx^2)} + \frac{35\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{128a^{9/2}\sqrt{b}}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.79

$$\frac{\frac{\sqrt{a}\,x(279a^3-511a^2bx^2+385ab^2x^4-105b^3x^6)}{(a-bx^2)^4} + \frac{105\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}}{384a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-5),x]
```

```
[Out] ((Sqrt[a]*x*(279*a^3 - 511*a^2*b*x^2 + 385*a*b^2*x^4 - 105*b^3*x^6))/(a - b
*x^2)^4 + (105*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(384*a^(9/2))
```

Maple [A]
time = 0.05, size = 103, normalized size = 1.03

method	result	size
risch	$\frac{-\frac{35b^3x^7}{128a^4}+\frac{385b^2x^5}{384a^3}-\frac{511b\,x^3}{384a^2}+\frac{93x}{128a}}{(-b\,x^2+a)^4} + \frac{35\ln\left(bx+\sqrt{ab}\right)}{256\sqrt{ab}\;a^4} - \frac{35\ln\left(-bx+\sqrt{ab}\right)}{256\sqrt{ab}\;a^4}$	92

default	$\frac{x}{8a(-b\,x^{2}+a)^{4}}+\frac{\frac{7x}{48a\left(-b\,x^{2}+a\right)^{3}}+\frac{7\left(\frac{5x}{24a\left(-b\,x^{2}+a\right)^{2}}+\frac{5\left(\frac{3x}{8a\left(-b\,x^{2}+a\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{6a}\right)}{8a}}{a}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x/a/(-b*x^2+a)^4+7/8/a*(1/6*x/a/(-b*x^2+a)^3+5/6/a*(1/4*x/a/(-b*x^2+a)^
2+3/4/a*(1/2*x/a/(-b*x^2+a)+1/2/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2)))))
```

Maxima [A]
time = 0.48, size = 117, normalized size = 1.17

$$-\frac{105\,b^{3}x^{7}-385\,ab^{2}x^{5}+511\,a^{2}bx^{3}-279\,a^{3}x}{384\,(a^{4}b^{4}x^{8}-4\,a^{5}b^{3}x^{6}+6\,a^{6}b^{2}x^{4}-4\,a^{7}bx^{2}+a^{8})}-\frac{35\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/384*(105*b^3*x^7 - 385*a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/(a^4*b^4*x
^8 - 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 35/256*log((b*x -
 sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 1.08, size = 320, normalized size = 3.20

$$\left[-\frac{210\,ab^{4}x^{7}-770\,a^{2}b^{3}x^{5}+1022\,a^{3}b^{2}x^{3}-558\,a^{4}bx-105\,(b^{4}x^{8}-4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}-4\,a^{3}bx^{2}+a^{4})\sqrt{ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{ab}\,x+a}{bx^{2}-a}\right)}{768\,(a^{5}b^{5}x^{8}-4\,a^{6}b^{4}x^{6}+6\,a^{7}b^{3}x^{4}-4\,a^{8}b^{2}x^{2}+a^{9}b)},\,-\frac{105\,ab^{4}x^{7}-385\,a^{2}b^{3}x^{5}+511\,a^{3}b^{2}x^{3}-279\,a^{4}bx+105\,(b^{4}x^{8}-4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}-4\,a^{3}bx^{2}+a^{4})\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{384\,(a^{5}b^{5}x^{8}-4\,a^{6}b^{4}x^{6}+6\,a^{7}b^{3}x^{4}-4\,a^{8}b^{2}x^{2}+a^{9}b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(210*a*b^4*x^7 - 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 - 558*a^4*b*x -
 105*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b)*
log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^5*b^5*x^8 - 4*a^6*b^4*x^6
+ 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b), -1/384*(105*a*b^4*x^7 - 385*a^2*b
^3*x^5 + 511*a^3*b^2*x^3 - 279*a^4*b*x + 105*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2
*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^5*b^5*x
^8 - 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b)]
```

Sympy [A]
time = 0.23, size = 146, normalized size = 1.46

$$-\frac{35\sqrt{\frac{1}{a^9b}}\log\left(-a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256}+\frac{35\sqrt{\frac{1}{a^9b}}\log\left(a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256}-\frac{-279a^3x+511a^2bx^3-385ab^2x^5+105b^3x^7}{384a^8-1536a^7bx^2+2304a^6b^2x^4-1536a^5b^3x^6+384a^4b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**5,x)
```

```
[Out] -35*sqrt(1/(a**9*b))*log(-a**5*sqrt(1/(a**9*b)) + x)/256 + 35*sqrt(1/(a**9*
b))*log(a**5*sqrt(1/(a**9*b)) + x)/256 - (-279*a**3*x + 511*a**2*b*x**3 - 3
85*a*b**2*x**5 + 105*b**3*x**7)/(384*a**8 - 1536*a**7*b*x**2 + 2304*a**6*b*
*2*x**4 - 1536*a**5*b**3*x**6 + 384*a**4*b**4*x**8)
```

Giac [A]
time = 1.56, size = 71, normalized size = 0.71

$$-\frac{35\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\sqrt{-ab}\,a^4}-\frac{105\,b^3x^7-385\,ab^2x^5+511\,a^2bx^3-279\,a^3x}{384\,(bx^2-a)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] -35/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^4) - 1/384*(105*b^3*x^7 - 385*
a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/((b*x^2 - a)^4*a^4)
```

Mupad [B]
time = 4.79, size = 99, normalized size = 0.99

$$\frac{\frac{93\,x}{128\,a}-\frac{511\,b\,x^3}{384\,a^2}+\frac{385\,b^2\,x^5}{384\,a^3}-\frac{35\,b^3\,x^7}{128\,a^4}}{a^4-4\,a^3\,b\,x^2+6\,a^2\,b^2\,x^4-4\,a\,b^3\,x^6+b^4\,x^8}+\frac{35\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{9/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^5,x)
```

```
[Out] ((93*x)/(128*a) - (511*b*x^3)/(384*a^2) + (385*b^2*x^5)/(384*a^3) - (35*b^3
*x^7)/(128*a^4))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4
) + (35*atanh((b^(1/2)*x)/a^(1/2)))/(128*a^(9/2)*b^(1/2))
```

3.250 $\int \frac{1}{x(a-bx^2)^5} \, dx$

Optimal. Leaf size=91

$$\frac{1}{8a\left(a-bx^2\right)^4}+\frac{1}{6a^2\left(a-bx^2\right)^3}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{2a^4\left(a-bx^2\right)}+\frac{\log(x)}{a^5}-\frac{\log\left(a-bx^2\right)}{2a^5}$$

```
[Out] 1/8/a/(-b*x^2+a)^4+1/6/a^2/(-b*x^2+a)^3+1/4/a^3/(-b*x^2+a)^2+1/2/a^4/(-b*x^
2+a)+ln(x)/a^5-1/2*ln(-b*x^2+a)/a^5
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {272, 46}

$$-\frac{\log\left(a-bx^2\right)}{2a^5}+\frac{\log(x)}{a^5}+\frac{1}{2a^4\left(a-bx^2\right)}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{6a^2\left(a-bx^2\right)^3}+\frac{1}{8a\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^5),x]
```

```
[Out] 1/(8*a*(a - b*x^2)^4) + 1/(6*a^2*(a - b*x^2)^3) + 1/(4*a^3*(a - b*x^2)^2) +
 1/(2*a^4*(a - b*x^2)) + Log[x]/a^5 - Log[a - b*x^2]/(2*a^5)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a-bx^2\right)^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)^5}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^5x}+\frac{b}{a(a-bx)^5}+\frac{b}{a^2(a-bx)^4}+\frac{b}{a^3(a-bx)^3}+\frac{b}{a^4(a-bx)^2}+\frac{b}{a^5(a-\right.\right.\\ &= \frac{1}{8a\left(a-bx^2\right)^4}+\frac{1}{6a^2\left(a-bx^2\right)^3}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{2a^4\left(a-bx^2\right)}+\frac{\log(x)}{a^5}-\frac{\log\left(a-\right.}{2a^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 0.74

$$\frac{\frac{a(25a^3-52a^2bx^2+42ab^2x^4-12b^3x^6)}{(a-bx^2)^4}+24\log(x)-12\log\left(a-bx^2\right)}{24a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^5),x]
```

```
[Out] ((a*(25*a^3 - 52*a^2*b*x^2 + 42*a*b^2*x^4 - 12*b^3*x^6))/(a - b*x^2)^4 + 24
*Log[x] - 12*Log[a - b*x^2])/(24*a^5)
```

Maple [A]
time = 0.06, size = 98, normalized size = 1.08

method	result	size
risch	$\frac{-\frac{b^3x^6}{2a^4}+\frac{7b^2x^4}{4a^3}-\frac{13b\,x^2}{6a^2}+\frac{25}{24a}}{\left(-b\,x^2+a\right)^4}+\frac{\ln(x)}{a^5}-\frac{\ln\left(-b\,x^2+a\right)}{2a^5}$	70
norman	$\frac{\frac{2b\,x^2}{a^2}-\frac{9b^2x^4}{2a^3}+\frac{11b^3x^6}{3a^4}-\frac{25b^4x^8}{24a^5}}{\left(-b\,x^2+a\right)^4}+\frac{\ln(x)}{a^5}-\frac{\ln\left(-b\,x^2+a\right)}{2a^5}$	76
default	$\frac{b\left(\frac{a^3}{3b\left(-b\,x^2+a\right)^3}+\frac{a}{b\left(-b\,x^2+a\right)}+\frac{a^4}{4b\left(-b\,x^2+a\right)^4}+\frac{a^2}{2b\left(-b\,x^2+a\right)^2}-\frac{\ln\left(-b\,x^2+a\right)}{b}\right)}{2a^5}+\frac{\ln(x)}{a^5}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/a^5*(1/3/b*a^3/(-b*x^2+a)^3+a/b/(-b*x^2+a)+1/4/b*a^4/(-b*x^2+a)^4+1/2
/b*a^2/(-b*x^2+a)^2-ln(-b*x^2+a)/b)+ln(x)/a^5
```

Maxima [A]
time = 0.30, size = 106, normalized size = 1.16

$$-\frac{12\,b^3x^6-42\,ab^2x^4+52\,a^2bx^2-25\,a^3}{24\left(a^4b^4x^8-4\,a^5b^3x^6+6\,a^6b^2x^4-4\,a^7bx^2+a^8\right)}-\frac{\log\left(bx^2-a\right)}{2\,a^5}+\frac{\log\left(x^2\right)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*b^3*x^6 - 42*a*b^2*x^4 + 52*a^2*b*x^2 - 25*a^3)/(a^4*b^4*x^8 - 4*
a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 1/2*log(b*x^2 - a)/a^5 +
 1/2*log(x^2)/a^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(85) = 170.
time = 1.69, size = 180, normalized size = 1.98

$$-\frac{12\,ab^3x^6-42\,a^2b^2x^4+52\,a^3bx^2-25\,a^4+12\left(b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+a^4\right)\log\left(bx^2-a\right)-24\left(b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+a^4\right)\log\left(x\right)}{24\left(a^5b^4x^8-4\,a^6b^3x^6+6\,a^7b^2x^4-4\,a^8bx^2+a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a*b^3*x^6 - 42*a^2*b^2*x^4 + 52*a^3*b*x^2 - 25*a^4 + 12*(b^4*x^8
- 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(b*x^2 - a) - 24*(b^4
*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4*x^
8 - 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 - 4*a^8*b*x^2 + a^9)
```

Sympy [A]
time = 0.29, size = 104, normalized size = 1.14

$$-\frac{-25a^3+52a^2bx^2-42ab^2x^4+12b^3x^6}{24a^8-96a^7bx^2+144a^6b^2x^4-96a^5b^3x^6+24a^4b^4x^8}+\frac{\log(x)}{a^5}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**5,x)
```

```
[Out] -(-25*a**3 + 52*a**2*b*x**2 - 42*a*b**2*x**4 + 12*b**3*x**6)/(24*a**8 - 96*
a**7*b*x**2 + 144*a**6*b**2*x**4 - 96*a**5*b**3*x**6 + 24*a**4*b**4*x**8) +
 log(x)/a**5 - log(-a/b + x**2)/(2*a**5)
```

Giac [A]
time = 2.14, size = 85, normalized size = 0.93

$$\frac{\log(x^2)}{2\,a^5}-\frac{\log(|bx^2-a|)}{2\,a^5}+\frac{25\,b^4x^8-112\,ab^3x^6+192\,a^2b^2x^4-152\,a^3bx^2+50\,a^4}{24\,(bx^2-a)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^5 - 1/2*log(abs(b*x^2 - a))/a^5 + 1/24*(25*b^4*x^8 - 112*a*b
^3*x^6 + 192*a^2*b^2*x^4 - 152*a^3*b*x^2 + 50*a^4)/((b*x^2 - a)^4*a^5)
```

Mupad [B]
time = 5.24, size = 101, normalized size = 1.11

$$\frac{\ln(x)}{a^5}+\frac{\frac{25}{24\,a}-\frac{13\,b\,x^2}{6\,a^2}+\frac{7\,b^2\,x^4}{4\,a^3}-\frac{b^3\,x^6}{2\,a^4}}{a^4-4\,a^3\,b\,x^2+6\,a^2\,b^2\,x^4-4\,a\,b^3\,x^6+b^4\,x^8}-\frac{\ln(a-b\,x^2)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)^5),x)
```

```
[Out] log(x)/a^5 + (25/(24*a) - (13*b*x^2)/(6*a^2) + (7*b^2*x^4)/(4*a^3) - (b^3*x
^6)/(2*a^4))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4) -
log(a - b*x^2)/(2*a^5)
```

3.251 $\int \frac{1}{x^2 (a-bx^2)^5} \, dx$

Optimal. Leaf size=118

$$-\frac{315}{128a^5x}+\frac{1}{8ax\left(a-bx^2\right)^4}+\frac{3}{16a^2x\left(a-bx^2\right)^3}+\frac{21}{64a^3x\left(a-bx^2\right)^2}+\frac{105}{128a^4x\left(a-bx^2\right)}+\frac{315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}}$$

```
[Out] -315/128/a^5/x+1/8/a/x/(-b*x^2+a)^4+3/16/a^2/x/(-b*x^2+a)^3+21/64/a^3/x/(-b
*x^2+a)^2+105/128/a^4/x/(-b*x^2+a)+315/128*arctanh(x*b^(1/2)/a^(1/2))*b^(1/
2)/a^(11/2)
```

Rubi [A]
time = 0.03, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {296, 331, 214}

$$\frac{315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}}-\frac{315}{128a^5x}+\frac{105}{128a^4x\left(a-bx^2\right)}+\frac{21}{64a^3x\left(a-bx^2\right)^2}+\frac{3}{16a^2x\left(a-bx^2\right)^3}+\frac{1}{8ax\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^5),x]
```

```
[Out] -315/(128*a^5*x) + 1/(8*a*x*(a - b*x^2)^4) + 3/(16*a^2*x*(a - b*x^2)^3) + 2
1/(64*a^3*x*(a - b*x^2)^2) + 105/(128*a^4*x*(a - b*x^2)) + (315*Sqrt[b]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rubi steps

$$\int \frac{1}{x^2 (a-bx^2)^5} dx = \frac{1}{8ax (a-bx^2)^4} + \frac{9 \int \frac{1}{x^2 (a-bx^2)^4} dx}{8a}$$

$$= \frac{1}{8ax (a-bx^2)^4} + \frac{3}{16a^2 x (a-bx^2)^3} + \frac{21 \int \frac{1}{x^2 (a-bx^2)^3} dx}{16a^2}$$

$$= \frac{1}{8ax (a-bx^2)^4} + \frac{3}{16a^2 x (a-bx^2)^3} + \frac{21}{64a^3 x (a-bx^2)^2} + \frac{105 \int \frac{1}{x^2 (a-bx^2)^2} dx}{64a^3}$$

$$= \frac{1}{8ax (a-bx^2)^4} + \frac{3}{16a^2 x (a-bx^2)^3} + \frac{21}{64a^3 x (a-bx^2)^2} + \frac{105}{128a^4 x (a-bx^2)} + \frac{315 \int}{1}$$

$$= -\frac{315}{128a^5 x} + \frac{1}{8ax (a-bx^2)^4} + \frac{3}{16a^2 x (a-bx^2)^3} + \frac{21}{64a^3 x (a-bx^2)^2} + \frac{105}{128a^4 x (a-bx}$$

$$= -\frac{315}{128a^5 x} + \frac{1}{8ax (a-bx^2)^4} + \frac{3}{16a^2 x (a-bx^2)^3} + \frac{21}{64a^3 x (a-bx^2)^2} + \frac{105}{128a^4 x (a-bx}$$

Mathematica [A]
time = 0.04, size = 92, normalized size = 0.78

$$\frac{\frac{\sqrt{a} \left(-128a^4+837a^3bx^2-1533a^2b^2x^4+1155ab^3x^6-315b^4x^8\right)}{x(a-bx^2)^4} + 315\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{128a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^5),x]
```

```
[Out] ((Sqrt[a]*(-128*a^4 + 837*a^3*b*x^2 - 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 - 3
15*b^4*x^8))/(x*(a - b*x^2)^4) + 315*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(128*a^(11/2))
```

Maple [A]
time = 0.06, size = 76, normalized size = 0.64

method	result	size

default	$\dfrac{b\left(\dfrac{-\frac{187}{128}b^3x^7+\frac{643}{128}a\,b^2x^5-\frac{765}{128}a^2b\,x^3+\frac{325}{128}a^3x}{\left(-b\,x^2+a\right)^4}+\dfrac{315\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}}\right)}{a^5}-\dfrac{1}{a^5x}$	76
risch	$\dfrac{-\frac{315b^4x^8}{128a^5}+\frac{1155b^3x^6}{128a^4}-\frac{1533b^2x^4}{128a^3}+\frac{837b\,x^2}{128a^2}-\frac{1}{a}}{x(-b\,x^2+a)^4}+\dfrac{315\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^2-b\right)}\_R\ln\left(\left(3\_R^2a^{11}-2b\right)x+a^6\_R\right)\right)}{256}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*b*((-187/128*b^3*x^7+643/128*a*b^2*x^5-765/128*a^2*b*x^3+325/128*a^3*
x)/(-b*x^2+a)^4+315/128/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2)))-1/a^5/x
```

Maxima [A]
time = 0.51, size = 130, normalized size = 1.10

$$-\frac{315\,b^4x^8-1155\,ab^3x^6+1533\,a^2b^2x^4-837\,a^3bx^2+128\,a^4}{128\,(a^5b^4x^9-4\,a^6b^3x^7+6\,a^7b^2x^5-4\,a^8bx^3+a^9x)}-\frac{315\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/128*(315*b^4*x^8 - 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 - 837*a^3*b*x^2 + 1
28*a^4)/(a^5*b^4*x^9 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^9*x)
 - 315/256*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^5)
```

Fricas [A]
time = 1.20, size = 334, normalized size = 2.83

$$\left[-\frac{630\,b^4x^8-2310\,ab^3x^6+3066\,a^2b^2x^4-1674\,a^3bx^2+256\,a^4-315\,(b^4x^9-4\,ab^3x^7+6\,a^2b^2x^5-4\,a^3bx^3+a^4x)\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)}{256\,(a^5b^4x^9-4\,a^6b^3x^7+6\,a^7b^2x^5-4\,a^8bx^3+a^9x)},\ -\frac{315\,b^4x^8-1155\,ab^3x^6+1533\,a^2b^2x^4-837\,a^3bx^2+128\,a^4+315\,(b^4x^9-4\,ab^3x^7+6\,a^2b^2x^5-4\,a^3bx^3+a^4x)\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)}{128\,(a^5b^4x^9-4\,a^6b^3x^7+6\,a^7b^2x^5-4\,a^8bx^3+a^9x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/256*(630*b^4*x^8 - 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 - 1674*a^3*b*x^2 +
 256*a^4 - 315*(b^4*x^9 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4*x
)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)))/(a^5*b^4*x^9 -
4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x^8 -
 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 - 837*a^3*b*x^2 + 128*a^4 + 315*(b^4*x^9
 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4*x)*sqrt(-b/a)*arctan(x*s
```

```
qrt(-b/a)))/(a^5*b^4*x^9 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^
9*x)]
```

Sympy [A]
time = 0.31, size = 155, normalized size = 1.31

$$-\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(-\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256}+\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256}-\frac{128a^4-837a^3bx^2+1533a^2b^2x^4-1155ab^3x^6+315b^4x^8}{128a^9x-512a^8bx^3+768a^7b^2x^5-512a^6b^3x^7+128a^5b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**5,x)
[Out] -315*sqrt(b/a**11)*log(-a**6*sqrt(b/a**11)/b + x)/256 + 315*sqrt(b/a**11)*l
og(a**6*sqrt(b/a**11)/b + x)/256 - (128*a**4 - 837*a**3*b*x**2 + 1533*a**2*
b**2*x**4 - 1155*a*b**3*x**6 + 315*b**4*x**8)/(128*a**9*x - 512*a**8*b*x**3
 + 768*a**7*b**2*x**5 - 512*a**6*b**3*x**7 + 128*a**5*b**4*x**9)
```

Giac [A]
time = 1.63, size = 83, normalized size = 0.70

$$-\frac{315\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\,\sqrt{-ab}\,a^5}-\frac{1}{a^5x}-\frac{187\,b^4x^7-643\,ab^3x^5+765\,a^2b^2x^3-325\,a^3bx}{128\,(bx^2-a)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="giac")
[Out] -315/128*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^5) - 1/(a^5*x) - 1/128*(187
*b^4*x^7 - 643*a*b^3*x^5 + 765*a^2*b^2*x^3 - 325*a^3*b*x)/((b*x^2 - a)^4*a^
5)
```

Mupad [B]
time = 5.17, size = 110, normalized size = 0.93

$$\frac{315\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{11/2}}-\frac{\frac{1}{a}-\frac{837\,b\,x^2}{128\,a^2}+\frac{1533\,b^2\,x^4}{128\,a^3}-\frac{1155\,b^3\,x^6}{128\,a^4}+\frac{315\,b^4\,x^8}{128\,a^5}}{a^4\,x-4\,a^3\,b\,x^3+6\,a^2\,b^2\,x^5-4\,a\,b^3\,x^7+b^4\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^5),x)
[Out] (315*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(128*a^(11/2)) - (1/a - (837*b*x^2
)/(128*a^2) + (1533*b^2*x^4)/(128*a^3) - (1155*b^3*x^6)/(128*a^4) + (315*b^
4*x^8)/(128*a^5))/(a^4*x + b^4*x^9 - 4*a^3*b*x^3 - 4*a*b^3*x^7 + 6*a^2*b^2*
x^5)
```

3.252 $\int \frac{1}{x^3(a-bx^2)^5}\,dx$

Optimal. Leaf size=106

$$-\frac{1}{2a^5x^2}+\frac{b}{8a^2(a-bx^2)^4}+\frac{b}{3a^3(a-bx^2)^3}+\frac{3b}{4a^4(a-bx^2)^2}+\frac{2b}{a^5(a-bx^2)}+\frac{5b\log(x)}{a^6}-\frac{5b\log(a-bx^2)}{2a^6}$$

```
[Out] -1/2/a^5/x^2+1/8*b/a^2/(-b*x^2+a)^4+1/3*b/a^3/(-b*x^2+a)^3+3/4*b/a^4/(-b*x^
2+a)^2+2*b/a^5/(-b*x^2+a)+5*b*ln(x)/a^6-5/2*b*ln(-b*x^2+a)/a^6
```

Rubi [A]

time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {272, 46}

$$-\frac{5b\log(a-bx^2)}{2a^6}+\frac{5b\log(x)}{a^6}+\frac{2b}{a^5(a-bx^2)}-\frac{1}{2a^5x^2}+\frac{3b}{4a^4(a-bx^2)^2}+\frac{b}{3a^3(a-bx^2)^3}+\frac{b}{8a^2(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^5),x]
```

```
[Out] -1/2*1/(a^5*x^2) + b/(8*a^2*(a - b*x^2)^4) + b/(3*a^3*(a - b*x^2)^3) + (3*b
)/(4*a^4*(a - b*x^2)^2) + (2*b)/(a^5*(a - b*x^2)) + (5*b*Log[x])/a^6 - (5*b
*Log[a - b*x^2])/(2*a^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a-bx^2)^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)^5}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^5x^2}+\frac{5b}{a^6x}+\frac{b^2}{a^2(a-bx)^5}+\frac{2b^2}{a^3(a-bx)^4}+\frac{3b^2}{a^4(a-bx)^3}+\frac{4b^2}{a^5(a-bx)^2}\right.\right.\\ &= -\frac{1}{2a^5x^2}+\frac{b}{8a^2(a-bx^2)^4}+\frac{b}{3a^3(a-bx^2)^3}+\frac{3b}{4a^4(a-bx^2)^2}+\frac{2b}{a^5(a-bx^2)}+\frac{5b\log(x}{a^6}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 0.78

$$\frac{\frac{a(-12a^4+125a^3bx^2-260a^2b^2x^4+210ab^3x^6-60b^4x^8)}{x^2(a-bx^2)^4}+120b\log(x)-60b\log(a-bx^2)}{24a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^5),x]
[Out] ((a*(-12*a^4 + 125*a^3*b*x^2 - 260*a^2*b^2*x^4 + 210*a*b^3*x^6 - 60*b^4*x^8
))/(x^2*(a - b*x^2)^4) + 120*b*Log[x] - 60*b*Log[a - b*x^2])/(24*a^6)
```

Maple [A]
time = 0.06, size = 111, normalized size = 1.05

method	result	size
risch	$\frac{-\frac{5b^4x^8}{2a^5}+\frac{35b^3x^6}{4a^4}-\frac{65b^2x^4}{6a^3}+\frac{125b\,x^2}{24a^2}-\frac{1}{2a}}{x^2(-b\,x^2+a)^4}+\frac{5b\ln(x)}{a^6}-\frac{5b\ln(-b\,x^2+a)}{2a^6}$	87
norman	$\frac{-\frac{1}{2a}+\frac{10b^2x^4}{a^3}-\frac{45b^3x^6}{2a^4}+\frac{55b^4x^8}{3a^5}-\frac{125b^5x^{10}}{24a^6}}{x^2(-b\,x^2+a)^4}+\frac{5b\ln(x)}{a^6}-\frac{5b\ln(-b\,x^2+a)}{2a^6}$	89
default	$\frac{b^2\left(\frac{4a}{b(-b\,x^2+a)}+\frac{2a^3}{3b(-b\,x^2+a)^3}+\frac{a^4}{4b(-b\,x^2+a)^4}-\frac{5\ln(-b\,x^2+a)}{b}+\frac{3a^2}{2b(-b\,x^2+a)^2}\right)}{2a^6}-\frac{1}{2a^5x^2}+\frac{5b\ln(x)}{a^6}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/2*b^2/a^6*(4*a/b/(-b*x^2+a)+2/3/b*a^3/(-b*x^2+a)^3+1/4/b*a^4/(-b*x^2+a)^4
-5*ln(-b*x^2+a)/b+3/2/b*a^2/(-b*x^2+a)^2)-1/2/a^5/x^2+5*b*ln(x)/a^6
```

Maxima [A]
time = 0.27, size = 123, normalized size = 1.16

$$-\frac{60\,b^4x^8-210\,ab^3x^6+260\,a^2b^2x^4-125\,a^3bx^2+12\,a^4}{24\,(a^5b^4x^{10}-4\,a^6b^3x^8+6\,a^7b^2x^6-4\,a^8bx^4+a^9x^2)}-\frac{5\,b\log(bx^2-a)}{2\,a^6}+\frac{5\,b\log(x^2)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="maxima")
[Out] -1/24*(60*b^4*x^8 - 210*a*b^3*x^6 + 260*a^2*b^2*x^4 - 125*a^3*b*x^2 + 12*a^
4)/(a^5*b^4*x^10 - 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 - 4*a^8*b*x^4 + a^9*x^2) -
 5/2*b*log(b*x^2 - a)/a^6 + 5/2*b*log(x^2)/a^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(100) = 200.
time = 0.91, size = 209, normalized size = 1.97

$$-\frac{60\,ab^4x^8-210\,a^2b^3x^6+260\,a^3b^2x^4-125\,a^4bx^2+12\,a^5+60\,(b^5x^{10}-4\,ab^4x^8+6\,a^2b^3x^6-4\,a^3b^2x^4+a^4bx^2)\log(bx^2-a)-120\,(b^5x^{10}-4\,ab^4x^8+6\,a^2b^3x^6-4\,a^3b^2x^4+a^4bx^2)\log(x)}{24\,(a^6b^4x^{10}-4\,a^7b^3x^8+6\,a^8b^2x^6-4\,a^9bx^4+a^{10}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(60*a*b^4*x^8 - 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 - 125*a^4*b*x^2 + 1
2*a^5 + 60*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b^2*x^4 + a^4*b*
x^2)*log(b*x^2 - a) - 120*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b
^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 - 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 -
 4*a^9*b*x^4 + a^10*x^2)
```

Sympy [A]
time = 0.35, size = 126, normalized size = 1.19

$$-\frac{12a^4 - 125a^3bx^2 + 260a^2b^2x^4 - 210ab^3x^6 + 60b^4x^8}{24a^9x^2 - 96a^8bx^4 + 144a^7b^2x^6 - 96a^6b^3x^8 + 24a^5b^4x^{10}} + \frac{5b\log(x)}{a^6} - \frac{5b\log\left(-\frac{a}{b} + x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**5,x)
```

```
[Out] -(12*a**4 - 125*a**3*b*x**2 + 260*a**2*b**2*x**4 - 210*a*b**3*x**6 + 60*b**
4*x**8)/(24*a**9*x**2 - 96*a**8*b*x**4 + 144*a**7*b**2*x**6 - 96*a**6*b**3*
x**8 + 24*a**5*b**4*x**10) + 5*b*log(x)/a**6 - 5*b*log(-a/b + x**2)/(2*a**6
)
```

Giac [A]
time = 1.87, size = 106, normalized size = 1.00

$$\frac{5\,b\log(x^2)}{2\,a^6} - \frac{5\,b\log(|bx^2 - a|)}{2\,a^6} - \frac{5\,bx^2 + a}{2\,a^6x^2} + \frac{125\,b^5x^8 - 548\,ab^4x^6 + 912\,a^2b^3x^4 - 688\,a^3b^2x^2 + 202\,a^4b}{24\,(bx^2 - a)^4a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 5/2*b*log(x^2)/a^6 - 5/2*b*log(abs(b*x^2 - a))/a^6 - 1/2*(5*b*x^2 + a)/(a^6
*x^2) + 1/24*(125*b^5*x^8 - 548*a*b^4*x^6 + 912*a^2*b^3*x^4 - 688*a^3*b^2*x
^2 + 202*a^4*b)/((b*x^2 - a)^4*a^6)
```

Mupad [B]
time = 5.35, size = 120, normalized size = 1.13

$$\frac{5\,b\,\ln(x)}{a^6} - \frac{5\,b\,\ln(a - b\,x^2)}{2\,a^6} - \frac{\frac{1}{2\,a} - \frac{125\,b\,x^2}{24\,a^2} + \frac{65\,b^2\,x^4}{6\,a^3} - \frac{35\,b^3\,x^6}{4\,a^4} + \frac{5\,b^4\,x^8}{2\,a^5}}{a^4\,x^2 - 4\,a^3\,b\,x^4 + 6\,a^2\,b^2\,x^6 - 4\,a\,b^3\,x^8 + b^4\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)^5),x)
```

```
[Out] (5*b*log(x))/a^6 - (5*b*log(a - b*x^2))/(2*a^6) - (1/(2*a) - (125*b*x^2)/(2
4*a^2) + (65*b^2*x^4)/(6*a^3) - (35*b^3*x^6)/(4*a^4) + (5*b^4*x^8)/(2*a^5))
/(a^4*x^2 + b^4*x^10 - 4*a^3*b*x^4 - 4*a*b^3*x^8 + 6*a^2*b^2*x^6)
```

3.253 $\int \frac{1}{x(1+bx^2)}\,dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{2}\log\left(1 + bx^2\right)$$

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 36, 29, 31}

$$\log(x) - \frac{1}{2}\log\left(bx^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x^2)),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\,(1+bx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+bx)}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right) - \frac{1}{2}b\text{Subst}\left(\int \frac{1}{1+bx}\,dx,x,x^2\right)$$
$$= \log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x^2)),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Maple [A]
time = 0.06, size = 14, normalized size = 0.93

method	result	size
default	$\ln(x) - \frac{\ln(b\,x^2+1)}{2}$	14
norman	$\ln(x) - \frac{\ln(b\,x^2+1)}{2}$	14
risch	$\ln(x) - \frac{\ln(b\,x^2+1)}{2}$	14
meijerg	$-\frac{\ln(b\,x^2+1)}{2} + \ln(x) + \frac{\ln(b)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 1.13

$$-\frac{1}{2}\log\left(bx^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

Fricas [A]
time = 0.83, size = 13, normalized size = 0.87

$$-\frac{1}{2}\log\left(bx^2+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.80

$$\log\left(x\right)-\frac{\log\left(x^2+\frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+1),x)
```

```
[Out] log(x) - log(x**2 + 1/b)/2
```

Giac [A]
time = 1.48, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

Mupad [B]
time = 5.11, size = 14, normalized size = 0.93

$$\ln\left(x\right)-\frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + 1)),x)
```

```
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2
```

3.254 $\int \frac{1}{x(-1+bx^2)}\,dx$

Optimal. Leaf size=18

$$-\log(x)+\frac{1}{2}\log\left(1-bx^2\right)$$

```
[Out] -ln(x)+1/2*ln(-b*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 36, 29, 31}

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x^2)),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(-1+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-1+bx)}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)\right)+\frac{1}{2}b\text{Subst}\left(\int \frac{1}{-1+bx}\,dx,x,x^2\right)\\
&= -\log(x)+\frac{1}{2}\log\left(1-bx^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$
-\log(x)+\frac{1}{2}\log\left(1-bx^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x^2)),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.89

method	result	size
default	$\frac{\ln(b\,x^2-1)}{2}-\ln\left(x\right)$	16
norman	$\frac{\ln(b\,x^2-1)}{2}-\ln\left(x\right)$	16
risch	$-\ln\left(x\right)+\frac{\ln(-b\,x^2+1)}{2}$	17
meijerg	$\frac{\ln(-b\,x^2+1)}{2}-\ln\left(x\right)-\frac{\ln(-b)}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(b*x^2-1)-ln(x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.94

$$
\frac{1}{2}\log\left(bx^2-1\right)-\frac{1}{2}\log\left(x^2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 - 1) - 1/2*log(x^2)
```

Fricas [A]
time = 0.71, size = 15, normalized size = 0.83

$$\frac{1}{2}\log\left(bx^2-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.67

$$-\log\left(x\right)+\frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2-1),x)
```

```
[Out] -log(x) + log(x**2 - 1/b)/2
```

Giac [A]
time = 1.88, size = 18, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.89

$$\frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^2}{2}\right)}{2}-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 - 1)),x)
```

```
[Out] log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

3.255 $\int \frac{1}{x^3(1+bx^2)}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)$$

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 46}

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + b*x^2)),x]
```

```
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(1+bx^2\right)}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+bx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right)\,dx,x,x^2\right)\\&=-\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$-\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + b*x^2)),x]
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Maple [A]
time = 0.04, size = 23, normalized size = 0.88

method	result	size
default	$-\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(b\,x^2+1)}{2}$	23
norman	$-\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(b\,x^2+1)}{2}$	23
risch	$-\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(-b\,x^2-1)}{2}$	24
meijerg	$\frac{b\left(\ln(b\,x^2+1)-2\ln(x)-\ln(b)-\frac{1}{x^2 b}\right)}{2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.92

$$\frac{1}{2}\,b\log\left(bx^2+1\right)-\frac{1}{2}\,b\log\left(x^2\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 + 1) - 1/2*b*log(x^2) - 1/2/x^2
```

Fricas [A]
time = 1.81, size = 28, normalized size = 1.08

$$\frac{bx^2\log\left(bx^2+1\right)-2\,bx^2\log\left(x\right)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

Sympy [A]
time = 0.08, size = 22, normalized size = 0.85

$$-b\log\left(x\right)+\frac{b\log\left(x^2+\frac{1}{b}\right)}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+1),x)
[Out] -b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

Giac [A]
time = 1.67, size = 32, normalized size = 1.23

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2+1\right|\right)+\frac{bx^2-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

Mupad [B]
time = 0.06, size = 22, normalized size = 0.85

$$\frac{b\,\ln\left(b\,x^2+1\right)}{2}-b\,\ln\left(x\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + 1)),x)
[Out] (b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

3.256 $\int \frac{1}{x^3(-1+bx^2)}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)$$

```
[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(-b*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-1 + b*x^2)),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(-1+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-1+bx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right)\,dx, x, x^2\right)\\ &= \frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1 - bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-1 + b*x^2)),x]
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Maple [A]
time = 0.04, size = 23, normalized size = 0.85

method	result	size
default	$\frac{b\ln(b\,x^2-1)}{2}+\frac{1}{2x^2}-b\ln\left(x\right)$	23
norman	$\frac{b\ln(b\,x^2-1)}{2}+\frac{1}{2x^2}-b\ln\left(x\right)$	23
risch	$\frac{1}{2x^2}-b\ln\left(x\right)+\frac{b\ln(-b\,x^2+1)}{2}$	24
meijerg	$\frac{b\left(\ln(-b\,x^2+1)-2\ln(x)-\ln(-b)+\frac{1}{x^2 b}\right)}{2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2-1),x,method=_RETURNVERBOSE)
[Out] 1/2*b*ln(b*x^2-1)+1/2/x^2-b*ln(x)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.89

$$\frac{1}{2}\,b\log\left(bx^2 - 1\right) - \frac{1}{2}\,b\log\left(x^2\right) + \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 - 1) - 1/2*b*log(x^2) + 1/2/x^2
```

Fricas [A]
time = 1.25, size = 28, normalized size = 1.04

$$\frac{bx^2\log\left(bx^2 - 1\right) - 2\,bx^2\log\left(x\right) + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

Sympy [A]
time = 0.08, size = 22, normalized size = 0.81

$$-b\log(x) + \frac{b\log\left(x^2 - \frac{1}{b}\right)}{2} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2-1),x)
[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

Giac [A]
time = 1.14, size = 32, normalized size = 1.19

$$-\frac{1}{2}\,b\log\left(x^2\right) + \frac{1}{2}\,b\log\left(\left|bx^2 - 1\right|\right) + \frac{bx^2 + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

Mupad [B]
time = 0.06, size = 22, normalized size = 0.81

$$\frac{b\,\ln\left(b\,x^2 - 1\right)}{2} - b\,\ln\left(x\right) + \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 - 1)),x)
[Out] (b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

3.257 $\int \frac{1}{-1+a+ax^2}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

```
[Out] -arctanh(x*a^(1/2)/(1-a)^(1/2))/((1-a)*a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a + a*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*x)/Sqrt[1 - a]]/Sqrt[(1 - a)*a])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{-1+a+ax^2}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{-1+a}}\right)}{\sqrt{-1+a}\,\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a + a*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[-1 + a]]/(Sqrt[-1 + a]*Sqrt[a])
```

Maple [A]
time = 0.05, size = 20, normalized size = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{a x}{\sqrt{(-1+a) a}}\right)}{\sqrt{(-1+a) a}}$	20
risch	$-\frac{\ln\left(a x+\sqrt{-(-1+a) a}\right)}{2 \sqrt{-(-1+a) a}}+\frac{\ln\left(-a x+\sqrt{-(-1+a) a}\right)}{2 \sqrt{-(-1+a) a}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+a-1),x,method=_RETURNVERBOSE)
[Out] 1/((-1+a)*a)^(1/2)*arctan(a*x/((-1+a)*a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.83, size = 82, normalized size = 2.73

$$\left[-\frac{\sqrt{-a^{2}+a} \log\left(\frac{a x^{2}-2 \sqrt{-a^{2}+a}\, x-a+1}{a x^{2}+a-1}\right)}{2 \left(a^{2}-a\right)}, \frac{\arctan\left(\frac{\sqrt{a^{2}-a}\, x}{a-1}\right)}{\sqrt{a^{2}-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a^2 + a)*log((a*x^2 - 2*sqrt(-a^2 + a)*x - a + 1)/(a*x^2 + a -
1))/(a^2 - a), arctan(sqrt(a^2 - a)*x/(a - 1))/sqrt(a^2 - a)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(24) = 48$.

time = 0.06, size = 83, normalized size = 2.77

$$-\frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(-a\sqrt{-\frac{1}{a(a-1)}}+x+\sqrt{-\frac{1}{a(a-1)}}\right)}{2}+\frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(a\sqrt{-\frac{1}{a(a-1)}}+x-\sqrt{-\frac{1}{a(a-1)}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+a-1),x)
```

```
[Out] -sqrt(-1/(a*(a - 1)))*log(-a*sqrt(-1/(a*(a - 1))) + x + sqrt(-1/(a*(a - 1))
))/2 + sqrt(-1/(a*(a - 1)))*log(a*sqrt(-1/(a*(a - 1))) + x - sqrt(-1/(a*(a
- 1))))/2
```

Giac [A]

time = 1.44, size = 23, normalized size = 0.77

$$\frac{\arctan\left(\frac{ax}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="giac")
```

```
[Out] arctan(a*x/sqrt(a^2 - a))/sqrt(a^2 - a)
```

Mupad [B]

time = 0.17, size = 23, normalized size = 0.77

$$\frac{\mathrm{atan}\left(\frac{a\,x}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*x^2 - 1),x)
```

```
[Out] atan((a*x)/(a^2 - a)^(1/2))/(a^2 - a)^(1/2)
```

3.258 $\int \frac{1}{-c-d+(c-d)x^2} dx$

Optimal. Leaf size=37

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-d}\,x}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}$$

```
[Out] -arctanh(x*(c-d)^(1/2)/(c+d)^(1/2))/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{214\}$

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(-c - d + (c - d)*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(Sqrt[c - d]*x)/Sqrt[c + d]]/(Sqrt[c - d]*Sqrt[c + d]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{-c-d+(c-d)x^2}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{c-d}\,x}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.19

$$\frac{\tan^{-1}\left(\frac{\sqrt{c-d}\,x}{\sqrt{-c-d}}\right)}{\sqrt{-c-d}\,\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-c - d + (c - d)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c - d]*x)/Sqrt[-c - d]]/(Sqrt[-c - d]*Sqrt[c - d])
```

Maple [A]
time = 0.06, size = 33, normalized size = 0.89

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{(c-d)x}{\sqrt{(c+d)(c-d)}}\right)}{\sqrt{(c+d)(c-d)}}$	33
risch	$\frac{\ln\left((-c+d)x+\sqrt{c^2-d^2}\right)}{2\sqrt{c^2-d^2}}-\frac{\ln\left((c-d)x+\sqrt{c^2-d^2}\right)}{2\sqrt{c^2-d^2}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c-d+(c-d)*x^2),x,method=_RETURNVERBOSE)
[Out] -1/((c+d)*(c-d))^(1/2)*arctanh((c-d)*x/((c+d)*(c-d))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.98, size = 102, normalized size = 2.76

$$\left[\frac{\log\left(\frac{(c-d)x^2-2\sqrt{c^2-d^2}\,x+c+d}{(c-d)x^2-c-d}\right)}{2\sqrt{c^2-d^2}},\ \frac{\sqrt{-c^2+d^2}\arctan\left(\frac{\sqrt{-c^2+d^2}\,x}{c+d}\right)}{c^2-d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="fricas")
[Out] [1/2*log(((c - d)*x^2 - 2*sqrt(c^2 - d^2)*x + c + d)/((c - d)*x^2 - c - d))
/sqrt(c^2 - d^2), sqrt(-c^2 + d^2)*arctan(sqrt(-c^2 + d^2)*x/(c + d))/(c^2
- d^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.
time = 0.09, size = 87, normalized size = 2.35

$$\frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(-c\sqrt{\frac{1}{(c-d)(c+d)}}-d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2}-\frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(c\sqrt{\frac{1}{(c-d)(c+d)}}+d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x**2),x)
```

```
[Out] sqrt(1/((c - d)*(c + d)))*log(-c*sqrt(1/((c - d)*(c + d))) - d*sqrt(1/((c -
 d)*(c + d))) + x)/2 - sqrt(1/((c - d)*(c + d)))*log(c*sqrt(1/((c - d)*(c +
 d))) + d*sqrt(1/((c - d)*(c + d))) + x)/2
```

Giac [A]
time = 1.17, size = 33, normalized size = 0.89

$$\frac{\arctan\left(\frac{cx-dx}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="giac")
```

```
[Out] arctan((c*x - d*x)/sqrt(-c^2 + d^2))/sqrt(-c^2 + d^2)
```

Mupad [B]
time = 0.29, size = 29, normalized size = 0.78

$$-\frac{\operatorname{atanh}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c+d}\,\sqrt{c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(c + d - x^2*(c - d)),x)
```

```
[Out] -atanh((x*(c - d)^(1/2))/(c + d)^(1/2))/((c + d)^(1/2)*(c - d)^(1/2))
```

3.259 $\int \frac{1}{x(1+bx^2)^2}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2(1+bx^2)}+\log(x)-\frac{1}{2}\log\left(1+bx^2\right)$$

```
[Out] 1/2/(b*x^2+1)+ln(x)-1/2*ln(b*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 46}

$$\frac{1}{2(bx^2+1)}-\frac{1}{2}\log\left(bx^2+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x^2)^2),x]
```

```
[Out] 1/(2*(1 + b*x^2)) + Log[x] - Log[1 + b*x^2]/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{b}{(1+bx)^2}-\frac{b}{1+bx}\right)dx, x, x^2\right)\\ &= \frac{1}{2(1+bx^2)}+\log(x)-\frac{1}{2}\log\left(1+bx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.89

$$\frac{1}{2+2bx^2}+\log(x)-\frac{1}{2}\log\left(1+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x^2)^2),x]
[Out] (2 + 2*b*x^2)^(-1) + Log[x] - Log[1 + b*x^2]/2
```

Maple [A]
time = 0.07, size = 34, normalized size = 1.21

method	result	size
risch	$\frac{1}{2b\,x^2+2}+\ln\left(x\right)-\frac{\ln\left(b\,x^2+1\right)}{2}$	25
norman	$-\frac{b\,x^2}{2\left(b\,x^2+1\right)}+\ln\left(x\right)-\frac{\ln\left(b\,x^2+1\right)}{2}$	29
default	$\ln\left(x\right)-\frac{b\left(\frac{\ln\left(b\,x^2+1\right)}{b}-\frac{1}{b\left(b\,x^2+1\right)}\right)}{2}$	34
meijerg	$-\frac{b\,x^2}{2b\,x^2+2}-\frac{\ln\left(b\,x^2+1\right)}{2}+\frac{1}{2}+\ln\left(x\right)+\frac{\ln\left(b\right)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] ln(x)-1/2*b*(1/b*ln(b*x^2+1)-1/b/(b*x^2+1))
```

Maxima [A]
time = 0.27, size = 28, normalized size = 1.00

$$\frac{1}{2\left(bx^2+1\right)}-\frac{1}{2}\log\left(bx^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="maxima")
[Out] 1/2/(b*x^2 + 1) - 1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

Fricas [A]
time = 0.79, size = 40, normalized size = 1.43

$$-\frac{\left(bx^2+1\right)\log\left(bx^2+1\right)-2\left(bx^2+1\right)\log\left(x\right)-1}{2\left(bx^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*((b*x^2 + 1)*log(b*x^2 + 1) - 2*(b*x^2 + 1)*log(x) - 1)/(b*x^2 + 1)
```

Sympy [A]
time = 0.08, size = 22, normalized size = 0.79

$$\log(x) - \frac{\log\left(x^2 + \frac{1}{b}\right)}{2} + \frac{1}{2bx^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+1)**2,x)
[Out] log(x) - log(x**2 + 1/b)/2 + 1/(2*b*x**2 + 2)
```

Giac [A]
time = 1.38, size = 36, normalized size = 1.29

$$\frac{bx^2 + 2}{2\,(bx^2 + 1)} + \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|bx^2 + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="giac")
[Out] 1/2*(b*x^2 + 2)/(b*x^2 + 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.86

$$\ln(x) - \frac{\ln\left(\frac{3\,b\,x^2}{2} + \frac{3}{2}\right)}{2} + \frac{1}{2\,\left(b\,x^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + 1)^2),x)
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2 + 1/(2*(b*x^2 + 1))
```

3.260 $\int \frac{1}{x(-1+bx^2)^2} dx$

Optimal. Leaf size=30

$$\frac{1}{2(1-bx^2)} + \log(x) - \frac{1}{2}\log\left(1-bx^2\right)$$

```
[Out] 1/2/(-b*x^2+1)+ln(x)-1/2*ln(-b*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\frac{1}{2(1-bx^2)} - \frac{1}{2}\log\left(1-bx^2\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x^2)^2),x]
```

```
[Out] 1/(2*(1 - b*x^2)) + Log[x] - Log[1 - b*x^2]/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(-1+bx^2)^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-1+bx)^2} dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x} + \frac{b}{(-1+bx)^2} - \frac{b}{-1+bx}\right) dx, x, x^2\right) \\ &= \frac{1}{2(1-bx^2)} + \log(x) - \frac{1}{2}\log\left(1-bx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{1}{2-2bx^2}+\log(x)-\frac{1}{2}\log\left(1-bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x^2)^2),x]
```

```
[Out] (2 - 2*b*x^2)^(-1) + Log[x] - Log[1 - b*x^2]/2
```

Maple [A]
time = 0.05, size = 33, normalized size = 1.10

method	result	size
risch	$-\frac{1}{2(b\,x^2-1)}-\frac{\ln\left(b\,x^2-1\right)}{2}+\ln\left(x\right)$	25
norman	$-\frac{b\,x^2}{2(b\,x^2-1)}-\frac{\ln\left(b\,x^2-1\right)}{2}+\ln\left(x\right)$	29
default	$-\frac{b\left(\frac{\ln\left(b\,x^2-1\right)}{b}+\frac{1}{b\left(b\,x^2-1\right)}\right)}{2}+\ln\left(x\right)$	33
meijerg	$\frac{b\,x^2}{-2b\,x^2+2}-\frac{\ln\left(-b\,x^2+1\right)}{2}+\frac{1}{2}+\ln\left(x\right)+\frac{\ln\left(-b\right)}{2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2-1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*(1/b*ln(b*x^2-1)+1/b/(b*x^2-1))+ln(x)
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.93

$$-\frac{1}{2\left(bx^2-1\right)}-\frac{1}{2}\,\log\left(bx^2-1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(b*x^2 - 1) - 1/2*log(b*x^2 - 1) + 1/2*log(x^2)
```

Fricas [A]
time = 1.19, size = 40, normalized size = 1.33

$$-\frac{\left(bx^2-1\right)\log\left(bx^2-1\right)-2\left(bx^2-1\right)\log\left(x\right)+1}{2\left(bx^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="fricas")
[Out] -1/2*((b*x^2 - 1)*log(b*x^2 - 1) - 2*(b*x^2 - 1)*log(x) + 1)/(b*x^2 - 1)
```

Sympy [A]
time = 0.08, size = 22, normalized size = 0.73

$$\log(x) - \frac{\log\left(x^2 - \frac{1}{b}\right)}{2} - \frac{1}{2bx^2 - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2-1)**2,x)
[Out] log(x) - log(x**2 - 1/b)/2 - 1/(2*b*x**2 - 2)
```

Giac [A]
time = 1.33, size = 36, normalized size = 1.20

$$\frac{bx^2 - 2}{2(bx^2 - 1)} + \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|bx^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="giac")
[Out] 1/2*(b*x^2 - 2)/(b*x^2 - 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 - 1))
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.87

$$\ln(x) - \frac{\ln\left(\frac{3\,b\,x^2}{2} - \frac{3}{2}\right)}{2} - \frac{1}{2\,\left(b\,x^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 - 1)^2),x)
[Out] log(x) - log((3*b*x^2)/2 - 3/2)/2 - 1/(2*(b*x^2 - 1))
```

3.261 $\int \frac{1}{a+(b-ac)x^2}\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b-ac}}$$

```
[Out] arctan(x*(-a*c+b)^(1/2)/a^(1/2))/a^(1/2)/(-a*c+b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {211}

$$\frac{\text{ArcTan}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+(b-ac)x^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b-ac}}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{\sqrt{-b+ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{-b+ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])
```

Maple [A]
time = 0.06, size = 34, normalized size = 1.00

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{(ac-b)x}{\sqrt{a\,(ac-b)}}\right)}{\sqrt{a\,(ac-b)}}$	34
risch	$\frac{\ln\left((-ac+b)x-\sqrt{a\,(ac-b)}\right)}{2\sqrt{a\,(ac-b)}}-\frac{\ln\left((ac-b)x-\sqrt{a\,(ac-b)}\right)}{2\sqrt{a\,(ac-b)}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(-a*c+b)*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*(a*c-b))^(1/2)*arctanh((a*c-b)*x/(a*(a*c-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*c-b>0)', see `assume?` for more d
etails)
```

Fricas [A]
time = 1.45, size = 106, normalized size = 3.12

$$\left[\frac{\log\left(\frac{(ac-b)x^2+2\sqrt{a^2c-ab}\,x+a}{(ac-b)x^2-a}\right)}{2\sqrt{a^2c-ab}},-\frac{\sqrt{-a^2c+ab}\arctan\left(\frac{\sqrt{-a^2c+ab}\,x}{a}\right)}{a^2c-ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((a*c - b)*x^2 + 2*sqrt(a^2*c - a*b)*x + a)/((a*c - b)*x^2 - a))/s
qrt(a^2*c - a*b), -sqrt(-a^2*c + a*b)*arctan(sqrt(-a^2*c + a*b)*x/a)/(a^2*c
 - a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.
time = 0.11, size = 60, normalized size = 1.76

$$-\frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(-a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2}+\frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x**2),x)
```

```
[Out] -sqrt(1/(a*(a*c - b)))*log(-a*sqrt(1/(a*(a*c - b))) + x)/2 + sqrt(1/(a*(a*c
 - b)))*log(a*sqrt(1/(a*(a*c - b))) + x)/2
```

Giac [A]
time = 1.43, size = 37, normalized size = 1.09

$$-\frac{\arctan\left(\frac{acx-bx}{\sqrt{-a^2c+ab}}\right)}{\sqrt{-a^2c+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="giac")
```

```
[Out] -arctan((a*c*x - b*x)/sqrt(-a^2*c + a*b))/sqrt(-a^2*c + a*b)
```

Mupad [B]
time = 5.14, size = 38, normalized size = 1.12

$$-\frac{\operatorname{atanh}\left(\frac{x\,(2\,b-2\,a\,c)}{2\,\sqrt{a}\,\sqrt{a\,c-b}}\right)}{\sqrt{a}\,\sqrt{a\,c-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + x^2*(b - a*c)),x)
```

```
[Out] -atanh((x*(2*b - 2*a*c))/(2*a^(1/2)*(a*c - b)^(1/2)))/(a^(1/2)*(a*c - b)^(1
/2))
```

3.262 $\int \frac{1}{a-(b-ac)x^2} dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b-ac}}$$

```
[Out] arctanh(x*(-a*c+b)^(1/2)/a^(1/2))/a^(1/2)/(-a*c+b)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {214}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-(b-ac)x^2}\,dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b-ac}}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{\sqrt{-b+ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{-b+ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])
```

Maple [A]
time = 0.03, size = 34, normalized size = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{(ac-b)x}{\sqrt{a(ac-b)}}\right)}{\sqrt{a(ac-b)}}$	34
risch	$-\frac{\ln\left((ac-b)x+\sqrt{-a(ac-b)}\right)}{2\sqrt{-a(ac-b)}}+\frac{\ln\left((-ac+b)x+\sqrt{-a(ac-b)}\right)}{2\sqrt{-a(ac-b)}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-(-a*c+b)*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*(a*c-b))^(1/2)*arctan((a*c-b)*x/(a*(a*c-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*c-b>0)', see `assume?` for more d
etails)
```

Fricas [A]
time = 1.46, size = 105, normalized size = 3.09

$$\left[-\frac{\sqrt{-a^2c+ab}\,\log\left(\frac{(ac-b)x^2-2\sqrt{-a^2c+ab}\,x-a}{(ac-b)x^2+a}\right)}{2\,(a^2c-ab)},\ \frac{\arctan\left(\frac{\sqrt{a^2c-ab}\,x}{a}\right)}{\sqrt{a^2c-ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2*c + a*b)*log(((a*c - b)*x^2 - 2*sqrt(-a^2*c + a*b)*x - a)/(
(a*c - b)*x^2 + a))/(a^2*c - a*b), arctan(sqrt(a^2*c - a*b)*x/a)/sqrt(a^2*c
 - a*b)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.
time = 0.10, size = 66, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(-a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x**2),x)
```

```
[Out] -sqrt(-1/(a*(a*c - b)))*log(-a*sqrt(-1/(a*(a*c - b))) + x)/2 + sqrt(-1/(a*(
a*c - b)))*log(a*sqrt(-1/(a*(a*c - b))) + x)/2
```

Giac [**A**]
time = 1.47, size = 36, normalized size = 1.06

$$\frac{\arctan\left(\frac{acx-bx}{\sqrt{a^2c-ab}}\right)}{\sqrt{a^2c-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="giac")
```

```
[Out] arctan((a*c*x - b*x)/sqrt(a^2*c - a*b))/sqrt(a^2*c - a*b)
```

Mupad [**B**]
time = 4.64, size = 38, normalized size = 1.12

$$-\frac{\mathrm{atan}\left(\frac{x\,(2\,b-2\,a\,c)}{2\,\sqrt{a^2\,c-a\,b}}\right)}{\sqrt{a^2\,c-a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - x^2*(b - a*c)),x)
```

```
[Out] -atan((x*(2*b - 2*a*c))/(2*(a^2*c - a*b)^(1/2)))/(a^2*c - a*b)^(1/2)
```

3.263 $\int \frac{1}{c(a-d)-(b-c)x^2} \, dx$

Optimal. Leaf size=50

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b-c}\,x}{\sqrt{c}\,\sqrt{a-d}}\right)}{\sqrt{b-c}\,\sqrt{c}\,\sqrt{a-d}}$$

```
[Out] arctanh(x*(b-c)^(1/2)/c^(1/2)/(a-d)^(1/2))/(b-c)^(1/2)/c^(1/2)/(a-d)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{214\}$

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\,\sqrt{a-d}}\right)}{\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b - c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[b - c]*Sqrt[c]*Sqrt[a
- d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{c(a-d)-(b-c)x^2} \, dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b-c}\,x}{\sqrt{c}\,\sqrt{a-d}}\right)}{\sqrt{b-c}\,\sqrt{c}\,\sqrt{a-d}}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{-b+c}\,x}{\sqrt{c}\,\sqrt{a-d}}\right)}{\sqrt{c}\,\sqrt{-b+c}\,\sqrt{a-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[-b + c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[c]*Sqrt[-b + c]*Sqrt[a
 - d])
```

Maple [A]
time = 0.06, size = 39, normalized size = 0.78

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{(-b+c)x}{\sqrt{c(a-d)(b-c)}}\right)}{\sqrt{c(a-d)(b-c)}}$	39
risch	$\frac{\ln\left((b-c)x+\sqrt{c(a-d)(b-c)}\right)}{2\sqrt{c(a-d)(b-c)}}-\frac{\ln\left((-b+c)x+\sqrt{c(a-d)(b-c)}\right)}{2\sqrt{c(a-d)(b-c)}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a-d)-(b-c)*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(c*(a-d)*(b-c))^(1/2)*arctanh((-b+c)*x/(c*(a-d)*(b-c))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((c-b)*(d-a)>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(38) = 76.
time = 1.39, size = 182, normalized size = 3.64

$$\left[\frac{\log\left(\frac{(b-c)x^2+ac-cd+2\sqrt{abc-ac^2-(bc-c^2)d}\,x}{(b-c)x^2-ac+cd}\right)}{2\sqrt{abc-ac^2-(bc-c^2)d}},\ \frac{\sqrt{-abc+ac^2+(bc-c^2)d}\,\arctan\left(-\frac{\sqrt{-abc+ac^2+(bc-c^2)d}\,x}{ac-cd}\right)}{abc-ac^2-(bc-c^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((b - c)*x^2 + a*c - c*d + 2*sqrt(a*b*c - a*c^2 - (b*c - c^2)*d)*x
)/((b - c)*x^2 - a*c + c*d))/sqrt(a*b*c - a*c^2 - (b*c - c^2)*d), sqrt(-a*b
*c + a*c^2 + (b*c - c^2)*d)*arctan(-sqrt(-a*b*c + a*c^2 + (b*c - c^2)*d)*x/
(a*c - c*d))/(a*b*c - a*c^2 - (b*c - c^2)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(39) = 78$.
time = 0.14, size = 104, normalized size = 2.08

$$-\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(-ac\sqrt{\frac{1}{c(a-d)(b-c)}}+cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}+\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(ac\sqrt{\frac{1}{c(a-d)(b-c)}}-cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x**2),x)
```

```
[Out] -sqrt(1/(c*(a - d)*(b - c)))*log(-a*c*sqrt(1/(c*(a - d)*(b - c))) + c*d*sqr
t(1/(c*(a - d)*(b - c))) + x)/2 + sqrt(1/(c*(a - d)*(b - c)))*log(a*c*sqrt(
1/(c*(a - d)*(b - c))) - c*d*sqrt(1/(c*(a - d)*(b - c))) + x)/2
```

Giac [A]
time = 1.62, size = 58, normalized size = 1.16

$$-\frac{\arctan\left(\frac{bx-cx}{\sqrt{-abc+ac^2+bcd-c^2d}}\right)}{\sqrt{-abc+ac^2+bcd-c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="giac")
```

```
[Out] -arctan((b*x - c*x)/sqrt(-a*b*c + a*c^2 + b*c*d - c^2*d))/sqrt(-a*b*c + a*c
^2 + b*c*d - c^2*d)
```

Mupad [B]
time = 4.88, size = 46, normalized size = 0.92

$$\frac{\operatorname{atanh}\left(\frac{x\,(2\,b-2\,c)}{2\,\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}\right)}{\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a - d) - x^2*(b - c)),x)
```

```
[Out] atanh((x*(2*b - 2*c))/(2*c^(1/2)*(a - d)^(1/2)*(b - c)^(1/2)))/(c^(1/2)*(a
- d)^(1/2)*(b - c)^(1/2))
```

3.264 $\int x^{7/2}(a+bx^2)\,dx$

Optimal. Leaf size=21

$$\frac{2}{9}ax^{9/2}+\frac{2}{13}bx^{13/2}$$

```
[Out] 2/9*a*x^(9/2)+2/13*b*x^(13/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{9}ax^{9/2}+\frac{2}{13}bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2),x]
[Out] (2*a*x^(9/2))/9 + (2*b*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)\,dx &= \int \left(ax^{7/2}+bx^{11/2}\right)\,dx\\ &= \frac{2}{9}ax^{9/2}+\frac{2}{13}bx^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{117}\left(13ax^{9/2}+9bx^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2),x]
[Out] (2*(13*a*x^(9/2) + 9*b*x^(13/2)))/117
```

Maple [A]
time = 0.08, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{9}{2}}}{9}+\frac{2b\,x^{\frac{13}{2}}}{13}$	14
default	$\frac{2a\,x^{\frac{9}{2}}}{9}+\frac{2b\,x^{\frac{13}{2}}}{13}$	14
gosper	$\frac{2x^{\frac{9}{2}}\left(9b\,x^{2}+13a\right)}{117}$	16
trager	$\frac{2x^{\frac{9}{2}}\left(9b\,x^{2}+13a\right)}{117}$	16
risch	$\frac{2x^{\frac{9}{2}}\left(9b\,x^{2}+13a\right)}{117}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*a*x^(9/2)+2/13*b*x^(13/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.62

$$\frac{2}{13}\,bx^{\frac{13}{2}}+\frac{2}{9}\,ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

Fricas [A]
time = 1.63, size = 18, normalized size = 0.86

$$\frac{2}{117}\left(9\,bx^{6}+13\,ax^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/117*(9*b*x^6 + 13*a*x^4)*sqrt(x)
```

Sympy [A]
time = 0.45, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{9}{2}}}{9}+\frac{2bx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a),x)
```

```
[Out] 2*a*x**(9/2)/9 + 2*b*x**(13/2)/13
```

Giac [A]
time = 1.50, size = 13, normalized size = 0.62

$$\frac{2}{13}bx^{\frac{13}{2}}+\frac{2}{9}ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

Mupad [B]
time = 4.50, size = 15, normalized size = 0.71

$$\frac{2\,x^{9/2}\,\left(9\,b\,x^2+13\,a\right)}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2),x)
```

```
[Out] (2*x^(9/2)*(13*a + 9*b*x^2))/117
```

3.265 $\int x^{5/2}(a+bx^2)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

```
[Out] 2/7*a*x^(7/2)+2/11*b*x^(11/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{14\}$

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)\,dx &= \int \left(ax^{5/2}+bx^{9/2}\right)\,dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$\frac{2}{77}x^{7/2}\left(11a+7bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2),x]
```

```
[Out] (2*x^(7/2)*(11*a + 7*b*x^2))/77
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{11}{2}}}{11}$	14
default	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{11}{2}}}{11}$	14
gosper	$\frac{2x^{\frac{7}{2}}\left(7b\,x^{2}+11a\right)}{77}$	16
trager	$\frac{2x^{\frac{7}{2}}\left(7b\,x^{2}+11a\right)}{77}$	16
risch	$\frac{2x^{\frac{7}{2}}\left(7b\,x^{2}+11a\right)}{77}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*a*x^(7/2)+2/11*b*x^(11/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.62

$$\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Fricas [A]
time = 1.19, size = 18, normalized size = 0.86

$$\frac{2}{77}\left(7\,bx^{5}+11\,ax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b*x^5 + 11*a*x^3)*sqrt(x)
```

Sympy [A]
time = 0.27, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11
```

Giac [A]
time = 1.45, size = 13, normalized size = 0.62

$$\frac{2}{11}bx^{\frac{11}{2}}+\frac{2}{7}ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{7/2}\,\left(7\,b\,x^2+11\,a\right)}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2),x)
```

```
[Out] (2*x^(7/2)*(11*a + 7*b*x^2))/77
```

3.266 $\int x^{3/2}(a+bx^2)\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

```
[Out] 2/5*a*x^(5/2)+2/9*b*x^(9/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2),x]
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx^2)\,dx&=\int\left(ax^{3/2}+bx^{7/2}\right)dx\\&=\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{45}\left(9ax^{5/2}+5bx^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2),x]
[Out] (2*(9*a*x^(5/2) + 5*b*x^(9/2)))/45
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
default	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
gosper	$\frac{2x^{\frac{5}{2}}\left(5b\,x^{2}+9a\right)}{45}$	16
trager	$\frac{2x^{\frac{5}{2}}\left(5b\,x^{2}+9a\right)}{45}$	16
risch	$\frac{2x^{\frac{5}{2}}\left(5b\,x^{2}+9a\right)}{45}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a*x^(5/2)+2/9*b*x^(9/2)
```

Maxima [A]
time = 0.30, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Fricas [A]
time = 1.28, size = 18, normalized size = 0.86

$$\frac{2}{45}\left(5\,bx^{4}+9\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b*x^4 + 9*a*x^2)*sqrt(x)
```

Sympy [A]
time = 0.14, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a),x)
[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9
```

Giac [A]
time = 1.07, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="giac")
[Out] 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Mupad [B]
time = 0.02, size = 15, normalized size = 0.71

$$\frac{2\,x^{5/2}\,\left(5\,b\,x^2+9\,a\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2),x)
[Out] (2*x^(5/2)*(9*a + 5*b*x^2))/45
```

3.267 $\int \sqrt{x} \left(a+bx^2\right) \, dx$

Optimal. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

```
[Out] 2/3*a*x^(3/2)+2/7*b*x^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{14\}$

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2),x]
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x} \left(a+bx^2\right) \, dx &= \int \left(a\sqrt{x}+bx^{5/2}\right) \, dx \\ &= \frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{21}\left(7ax^{3/2}+3bx^{7/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2),x]
[Out] (2*(7*a*x^(3/2) + 3*b*x^(7/2)))/21
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
default	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
gosper	$\frac{2x^{\frac{3}{2}}\left(3b\,x^{2}+7a\right)}{21}$	16
trager	$\frac{2x^{\frac{3}{2}}\left(3b\,x^{2}+7a\right)}{21}$	16
risch	$\frac{2x^{\frac{3}{2}}\left(3b\,x^{2}+7a\right)}{21}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*x^(3/2)+2/7*b*x^(7/2)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Fricas [A]
time = 1.92, size = 16, normalized size = 0.76

$$\frac{2}{21}\,\left(3\,bx^{3}+7\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b*x^3 + 7*a*x)*sqrt(x)
```

Sympy [A]
time = 0.57, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*x**(1/2),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7
```

Giac [A]
time = 1.32, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{3/2}\,\left(3\,b\,x^2+7\,a\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2),x)
```

```
[Out] (2*x^(3/2)*(7*a + 3*b*x^2))/21
```

3.268 $\int \frac{a+bx^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=19

$$2a\sqrt{x} + \frac{2}{5}bx^{5/2}$$

```
[Out] 2/5*b*x^(5/2)+2*a*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$2a\sqrt{x} + \frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\sqrt{x}} \, dx &= \int \left(\frac{a}{\sqrt{x}} + bx^{3/2}\right) dx \\ &= 2a\sqrt{x} + \frac{2}{5}bx^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.05

$$\frac{2}{5}\left(5a\sqrt{x} + bx^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/Sqrt[x],x]
```

```
[Out] (2*(5*a*Sqrt[x] + b*x^(5/2)))/5
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{5}{2}}}{5}+2a\sqrt{x}$	14
default	$\frac{2b\,x^{\frac{5}{2}}}{5}+2a\sqrt{x}$	14
gosper	$\frac{2\sqrt{x}\,\left(b\,x^{2}+5a\right)}{5}$	15
trager	$\left(\frac{2b\,x^{2}}{5}+2a\right)\sqrt{x}$	15
risch	$\frac{2\sqrt{x}\,\left(b\,x^{2}+5a\right)}{5}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*b*x^(5/2)+2*a*x^(1/2)
```

Maxima [A]
time = 0.32, size = 13, normalized size = 0.68

$$\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Fricas [A]
time = 1.47, size = 14, normalized size = 0.74

$$\frac{2}{5}\left(bx^{2}+5\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b*x^2 + 5*a)*sqrt(x)
```

Sympy [A]
time = 0.06, size = 17, normalized size = 0.89

$$2a\sqrt{x}+\frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5
```

Giac [A]
time = 1.57, size = 13, normalized size = 0.68

$$\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.74

$$\frac{2\,\sqrt{x}\,\left(b\,x^2+5\,a\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(5*a + b*x^2))/5
```

3.269 $\int \frac{a+bx^2}{x^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}$$

```
[Out] 2/3*b*x^(3/2)-2*a/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{14\}$

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+b\sqrt{x}\right)\,dx\\ &= -\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-\frac{2(3a-bx^2)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(3/2),x]
```

```
[Out] (-2*(3*a - b*x^2))/(3*Sqrt[x])
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.74

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{3}{2}}}{3}-\frac{2a}{\sqrt{x}}$	14
default	$\frac{2b\,x^{\frac{3}{2}}}{3}-\frac{2a}{\sqrt{x}}$	14
gosper	$-\frac{2\left(-b\,x^{2}+3a\right)}{3\sqrt{x}}$	16
trager	$-\frac{2\left(-b\,x^{2}+3a\right)}{3\sqrt{x}}$	16
risch	$-\frac{2\left(-b\,x^{2}+3a\right)}{3\sqrt{x}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*b*x^(3/2)-2*a/x^(1/2)
```

Maxima [A]
time = 0.31, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="maxima")
[Out] 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Fricas [A]
time = 1.08, size = 14, normalized size = 0.74

$$\frac{2\left(bx^{2}-3\,a\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="fricas")
[Out] 2/3*(b*x^2 - 3*a)/sqrt(x)
```

Sympy [A]
time = 0.14, size = 17, normalized size = 0.89

$$-\frac{2a}{\sqrt{x}}+\frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3
```

Giac [A]
time = 1.08, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.79

$$-\frac{6\,a-2\,b\,x^2}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(3/2),x)
```

```
[Out] -(6*a - 2*b*x^2)/(3*x^(1/2))
```

3.270 $\int \frac{a+bx^2}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}$$

```
[Out] -2/3*a/x^(3/2)+2*b*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{14\}$

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}\right)dx\\ &= -\frac{2a}{3x^{3/2}}+2b\sqrt{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.89

$$-\frac{2(a-3bx^2)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(5/2),x]
```

```
[Out] (-2*(a - 3*b*x^2))/(3*x^(3/2))
```

Maple [A]
time = 0.04, size = 14, normalized size = 0.74

method	result	size
gosper	$-\frac{2(-3b\,x^2+a)}{3x^{\frac{3}{2}}}$	14
derivativedivides	$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}$	14
default	$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}$	14
trager	$-\frac{2(-3b\,x^2+a)}{3x^{\frac{3}{2}}}$	14
risch	$-\frac{2(-3b\,x^2+a)}{3x^{\frac{3}{2}}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a/x^(3/2)+2*b*x^(1/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Fricas [A]
time = 1.34, size = 15, normalized size = 0.79

$$\frac{2\,(3\,bx^2-a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b*x^2 - a)/x^(3/2)
```

Sympy [A]
time = 0.17, size = 17, normalized size = 0.89

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x)
```

Giac [A]
time = 1.61, size = 13, normalized size = 0.68

$$2b\sqrt{x} - \frac{2a}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.79

$$-\frac{2a - 6bx^2}{3x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(5/2),x)
```

```
[Out] -(2*a - 6*b*x^2)/(3*x^(3/2))
```

3.271 $\int \frac{a+bx^2}{x^{7/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

```
[Out] -2/5*a/x^(5/2)-2*b/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{7/2}}\,dx &= \int \left(\frac{a}{x^{7/2}}+\frac{b}{x^{3/2}}\right)dx\\ &= -\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.89

$$-\frac{2(a+5bx^2)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(7/2),x]
```

```
[Out] (-2*(a + 5*b*x^2))/(5*x^(5/2))
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.74

method	result	size
gosper	$-\frac{2(5b\,x^2+a)}{5x^{\frac{5}{2}}}$	14
derivativedivides	$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}$	14
default	$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}$	14
trager	$-\frac{2(5b\,x^2+a)}{5x^{\frac{5}{2}}}$	14
risch	$-\frac{2(5b\,x^2+a)}{5x^{\frac{5}{2}}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*a/x^(5/2)-2*b/x^(1/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$-\frac{2\,(5\,bx^2+a)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="maxima")
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

Fricas [A]
time = 1.58, size = 13, normalized size = 0.68

$$-\frac{2\,(5\,bx^2+a)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="fricas")
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

Sympy [A]
time = 0.29, size = 19, normalized size = 1.00

$$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(7/2),x)
[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x)
```

Giac [A]
time = 1.01, size = 13, normalized size = 0.68

$$-\frac{2\,(5\,bx^2+a)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="giac")
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.79

$$-\frac{10\,b\,x^2+2\,a}{5\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(7/2),x)
[Out] -(2*a + 10*b*x^2)/(5*x^(5/2))
```

3.272 $\int x^{7/2}(a+bx^2)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}$$

```
[Out] 2/9*a^2*x^(9/2)+4/13*a*b*x^(13/2)+2/17*b^2*x^(17/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {276}

$$\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(9/2))/9 + (4*a*b*x^(13/2))/13 + (2*b^2*x^(17/2))/17
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\,dx&=\int\left(a^2x^{7/2}+2abx^{11/2}+b^2x^{15/2}\right)\,dx\\&=\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.83

$$\frac{2x^{9/2}(221a^2+306abx^2+117b^2x^4)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(9/2)*(221*a^2 + 306*a*b*x^2 + 117*b^2*x^4))/1989
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2a^{2}x^{\frac{9}{2}}}{9}+\frac{4ab\,x^{\frac{13}{2}}}{13}+\frac{2b^{2}x^{\frac{17}{2}}}{17}$	25
default	$\frac{2a^{2}x^{\frac{9}{2}}}{9}+\frac{4ab\,x^{\frac{13}{2}}}{13}+\frac{2b^{2}x^{\frac{17}{2}}}{17}$	25
gosper	$\frac{2x^{\frac{9}{2}}\left(117b^{2}x^{4}+306ab\,x^{2}+221a^{2}\right)}{1989}$	27
trager	$\frac{2x^{\frac{9}{2}}\left(117b^{2}x^{4}+306ab\,x^{2}+221a^{2}\right)}{1989}$	27
risch	$\frac{2x^{\frac{9}{2}}\left(117b^{2}x^{4}+306ab\,x^{2}+221a^{2}\right)}{1989}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 2/9*a^2*x^(9/2)+4/13*a*b*x^(13/2)+2/17*b^2*x^(17/2)
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.67

$$\frac{2}{17}\,b^{2}x^{\frac{17}{2}}+\frac{4}{13}\,abx^{\frac{13}{2}}+\frac{2}{9}\,a^{2}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="maxima")
[Out] 2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

Fricas [A]
time = 1.59, size = 29, normalized size = 0.81

$$\frac{2}{1989}\left(117\,b^{2}x^{8}+306\,abx^{6}+221\,a^{2}x^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 2/1989*(117*b^2*x^8 + 306*a*b*x^6 + 221*a^2*x^4)*sqrt(x)
```

Sympy [A]
time = 0.70, size = 34, normalized size = 0.94

$$\frac{2a^{2}x^{\frac{9}{2}}}{9}+\frac{4abx^{\frac{13}{2}}}{13}+\frac{2b^{2}x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2,x)
[Out] 2*a**2*x**(9/2)/9 + 4*a*b*x**(13/2)/13 + 2*b**2*x**(17/2)/17
```

Giac [A]
time = 0.96, size = 24, normalized size = 0.67

$$\frac{2}{17}\,b^2x^{\frac{17}{2}}+\frac{4}{13}\,abx^{\frac{13}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="giac")
[Out] 2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.69

$$x^{9/2}\,\left(\frac{2\,a^2}{9}+\frac{4\,a\,b\,x^2}{13}+\frac{2\,b^2\,x^4}{17}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^2,x)
[Out] x^(9/2)*((2*a^2)/9 + (2*b^2*x^4)/17 + (4*a*b*x^2)/13)
```

3.273 $\int x^{5/2}(a+bx^2)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

```
[Out] 2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*b^2*x^(15/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {276}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*b^2*x^(15/2))/15
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\,dx&=\int\left(a^2x^{5/2}+2abx^{9/2}+b^2x^{13/2}\right)\,dx\\&=\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}(165a^2+210abx^2+77b^2x^4)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(7/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

Maple [A]
time = 0.06, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{2b^{2}x^{\frac{15}{2}}}{15}$	25
default	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{2b^{2}x^{\frac{15}{2}}}{15}$	25
gosper	$\frac{2x^{\frac{7}{2}}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)}{1155}$	27
trager	$\frac{2x^{\frac{7}{2}}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)}{1155}$	27
risch	$\frac{2x^{\frac{7}{2}}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)}{1155}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*b^2*x^(15/2)
```

Maxima [A]
time = 0.30, size = 24, normalized size = 0.67

$$\frac{2}{15}\,b^{2}x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^{2}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Fricas [A]
time = 1.57, size = 29, normalized size = 0.81

$$\frac{2}{1155}\left(77\,b^{2}x^{7}+210\,abx^{5}+165\,a^{2}x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*x^7 + 210*a*b*x^5 + 165*a^2*x^3)*sqrt(x)
```

Sympy [A]
time = 0.46, size = 34, normalized size = 0.94

$$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{2b^{2}x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 2*b**2*x**(15/2)/15
```

Giac [A]
time = 1.21, size = 24, normalized size = 0.67

$$\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{7/2}\,\left(165\,a^2+210\,a\,b\,x^2+77\,b^2\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^2,x)
```

```
[Out] (2*x^(7/2)*(165*a^2 + 77*b^2*x^4 + 210*a*b*x^2))/1155
```

3.274 $\int x^{3/2}(a+bx^2)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}b^2x^{13/2}$$

```
[Out] 2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*b^2*x^(13/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {276}

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}b^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(13/2))/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\,dx &= \int \left(a^2x^{3/2}+2abx^{7/2}+b^2x^{11/2}\right)\,dx\\ &= \frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}b^2x^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2}\left(117a^2+130abx^2+45b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(5/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

Maple [A]
time = 0.06, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2a^{2}x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2b^{2}x^{\frac{13}{2}}}{13}$	25
default	$\frac{2a^{2}x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2b^{2}x^{\frac{13}{2}}}{13}$	25
gosper	$\frac{2x^{\frac{5}{2}}\left(45b^{2}x^{4}+130ab\,x^{2}+117a^{2}\right)}{585}$	27
trager	$\frac{2x^{\frac{5}{2}}\left(45b^{2}x^{4}+130ab\,x^{2}+117a^{2}\right)}{585}$	27
risch	$\frac{2x^{\frac{5}{2}}\left(45b^{2}x^{4}+130ab\,x^{2}+117a^{2}\right)}{585}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*b^2*x^(13/2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{2}{13}\,b^{2}x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^{2}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="maxima")
[Out] 2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Fricas [A]
time = 1.58, size = 29, normalized size = 0.81

$$\frac{2}{585}\left(45\,b^{2}x^{6}+130\,abx^{4}+117\,a^{2}x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 2/585*(45*b^2*x^6 + 130*a*b*x^4 + 117*a^2*x^2)*sqrt(x)
```

Sympy [A]
time = 0.27, size = 34, normalized size = 0.94

$$\frac{2a^{2}x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^{2}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13
```

Giac [A]
time = 1.14, size = 24, normalized size = 0.67

$$\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{5/2}\,\left(117\,a^2+130\,a\,b\,x^2+45\,b^2\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^2,x)
```

```
[Out] (2*x^(5/2)*(117*a^2 + 45*b^2*x^4 + 130*a*b*x^2))/585
```

3.275 $\int \sqrt{x} \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}$$

```
[Out] 2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*b^2*x^(11/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {276}

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2,x]
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(11/2))/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x} \left(a+bx^2\right)^2 \, dx &= \int \left(a^2\sqrt{x}+2abx^{5/2}+b^2x^{9/2}\right) \, dx \\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2}\left(77a^2+66abx^2+21b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2,x]
[Out] (2*x^(3/2)*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

Maple [A]
time = 0.06, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2a^{2}x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^{2}x^{\frac{11}{2}}}{11}$	25
default	$\frac{2a^{2}x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^{2}x^{\frac{11}{2}}}{11}$	25
gosper	$\frac{2x^{\frac{3}{2}}\left(21b^{2}x^{4}+66ab\,x^{2}+77a^{2}\right)}{231}$	27
trager	$\frac{2x^{\frac{3}{2}}\left(21b^{2}x^{4}+66ab\,x^{2}+77a^{2}\right)}{231}$	27
risch	$\frac{2x^{\frac{3}{2}}\left(21b^{2}x^{4}+66ab\,x^{2}+77a^{2}\right)}{231}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*b^2*x^(11/2)
```

Maxima [A]
time = 0.32, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^{2}x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^{2}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Fricas [A]
time = 0.97, size = 27, normalized size = 0.75

$$\frac{2}{231}\left(21\,b^{2}x^{5}+66\,abx^{3}+77\,a^{2}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(x)
```

Sympy [A]
time = 0.75, size = 34, normalized size = 0.94

$$\frac{2a^{2}x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{2b^{2}x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2),x)
```

```
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(11/2)/11
```

Giac [A]
time = 1.44, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{3/2}\,\left(77\,a^2+66\,a\,b\,x^2+21\,b^2\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^2,x)
```

```
[Out] (2*x^(3/2)*(77*a^2 + 21*b^2*x^4 + 66*a*b*x^2))/231
```

3.276 $\int \frac{(a+bx^2)^2}{\sqrt{x}} dx$

Optimal. Leaf size=34

$$2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}b^2x^{9/2}$$

```
[Out] 4/5*a*b*x^(5/2)+2/9*b^2*x^(9/2)+2*a^2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{276\}$

$$2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*b^2*x^(9/2))/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}} + 2abx^{3/2} + b^2x^{7/2}\right) dx\\ &= 2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}b^2x^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.88

$$\frac{2}{45}\sqrt{x}\left(45a^2 + 18abx^2 + 5b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(45*a^2 + 18*a*b*x^2 + 5*b^2*x^4))/45
```

Maple [A]
time = 0.03, size = 25, normalized size = 0.74

method	result	size
derivativedivides	$\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2b^{2}x^{\frac{9}{2}}}{9}+2a^{2}\sqrt{x}$	25
default	$\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2b^{2}x^{\frac{9}{2}}}{9}+2a^{2}\sqrt{x}$	25
trager	$\left(\frac{2}{9}b^{2}x^{4}+\frac{4}{5}ab\,x^{2}+2a^{2}\right)\sqrt{x}$	26
gosper	$\frac{2\sqrt{x}\,\left(5b^{2}x^{4}+18ab\,x^{2}+45a^{2}\right)}{45}$	27
risch	$\frac{2\sqrt{x}\,\left(5b^{2}x^{4}+18ab\,x^{2}+45a^{2}\right)}{45}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/5*a*b*x^(5/2)+2/9*b^2*x^(9/2)+2*a^2*x^(1/2)
```

Maxima [A]
time = 0.41, size = 24, normalized size = 0.71

$$\frac{2}{9}\,b^{2}x^{\frac{9}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+2\,a^{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
[Out] 2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

Fricas [A]
time = 1.52, size = 26, normalized size = 0.76

$$\frac{2}{45}\left(5\,b^{2}x^{4}+18\,abx^{2}+45\,a^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(x)
```

Sympy [A]
time = 0.14, size = 32, normalized size = 0.94

$$2a^{2}\sqrt{x}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{2b^{2}x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9
```

Giac [A]
time = 1.06, size = 24, normalized size = 0.71

$$\frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.76

$$\frac{2\,\sqrt{x}\,\left(45\,a^2+18\,a\,b\,x^2+5\,b^2\,x^4\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(45*a^2 + 5*b^2*x^4 + 18*a*b*x^2))/45
```

3.277 $\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}$$

```
[Out] 4/3*a*b*x^(3/2)+2/7*b^2*x^(7/2)-2*a^2/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {276}

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(7/2))/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+2ab\sqrt{x}+b^2x^{5/2}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.88

$$-\frac{2(21a^2-14abx^2-3b^2x^4)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(3/2),x]
```

[Out] (-2*(21*a^2 - 14*a*b*x^2 - 3*b^2*x^4))/(21*Sqrt[x])

Maple [A]
time = 0.05, size = 25, normalized size = 0.74

method	result	size
derivativedivides	$\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^{2}x^{\frac{7}{2}}}{7}-\frac{2a^{2}}{\sqrt{x}}$	25
default	$\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^{2}x^{\frac{7}{2}}}{7}-\frac{2a^{2}}{\sqrt{x}}$	25
gosper	$-\frac{2\left(-3b^{2}x^{4}-14ab\,x^{2}+21a^{2}\right)}{21\sqrt{x}}$	27
trager	$-\frac{2\left(-3b^{2}x^{4}-14ab\,x^{2}+21a^{2}\right)}{21\sqrt{x}}$	27
risch	$-\frac{2\left(-3b^{2}x^{4}-14ab\,x^{2}+21a^{2}\right)}{21\sqrt{x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2/x^(3/2),x,method=_RETURNVERBOSE)

[Out] 4/3*a*b*x^(3/2)+2/7*b^2*x^(7/2)-2*a^2/x^(1/2)

Maxima [A]
time = 0.28, size = 24, normalized size = 0.71

$$\frac{2}{7}\,b^{2}x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="maxima")

[Out] 2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)

Fricas [A]
time = 1.04, size = 26, normalized size = 0.76

$$\frac{2\,\left(3\,b^{2}x^{4}+14\,abx^{2}-21\,a^{2}\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="fricas")

[Out] 2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)/sqrt(x)

Sympy [A]
time = 0.23, size = 32, normalized size = 0.94

$$-\frac{2a^{2}}{\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{2b^{2}x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(7/2)/7
```

Giac [A]
time = 1.29, size = 24, normalized size = 0.71

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.76

$$\frac{-42\,a^2+28\,a\,b\,x^2+6\,b^2\,x^4}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(3/2),x)
```

```
[Out] (6*b^2*x^4 - 42*a^2 + 28*a*b*x^2)/(21*x^(1/2))
```

3.278 $\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}$$

```
[Out] -2/3*a^2/x^(3/2)+2/5*b^2*x^(5/2)+4*a*b*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {276}

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*b^2*x^(5/2))/5
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{\sqrt{x}}+b^2x^{3/2}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.88

$$-\frac{2(5a^2-30abx^2-3b^2x^4)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(5/2),x]
```

```
[Out] (-2*(5*a^2 - 30*a*b*x^2 - 3*b^2*x^4))/(15*x^(3/2))
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.74

method	result	size
derivativedivides	$-\frac{2a^2}{3x^{\frac{3}{2}}}+\frac{2b^2x^{\frac{5}{2}}}{5}+4ab\sqrt{x}$	25
default	$-\frac{2a^2}{3x^{\frac{3}{2}}}+\frac{2b^2x^{\frac{5}{2}}}{5}+4ab\sqrt{x}$	25
gosper	$-\frac{2(-3b^2x^4-30ab\,x^2+5a^2)}{15x^{\frac{3}{2}}}$	27
trager	$-\frac{2(-3b^2x^4-30ab\,x^2+5a^2)}{15x^{\frac{3}{2}}}$	27
risch	$-\frac{2(-3b^2x^4-30ab\,x^2+5a^2)}{15x^{\frac{3}{2}}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*a^2/x^(3/2)+2/5*b^2*x^(5/2)+4*a*b*x^(1/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
[Out] 2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

Fricas [A]
time = 1.54, size = 26, normalized size = 0.76

$$\frac{2\,(3\,b^2x^4+30\,abx^2-5\,a^2)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)/x^(3/2)
```

Sympy [A]
time = 0.25, size = 32, normalized size = 0.94

$$-\frac{2a^2}{3x^{\frac{3}{2}}}+4ab\sqrt{x}+\frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5
```

Giac [A]
time = 2.11, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.76

$$\frac{-10\,a^2+60\,a\,b\,x^2+6\,b^2\,x^4}{15\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(5/2),x)
```

```
[Out] (6*b^2*x^4 - 10*a^2 + 60*a*b*x^2)/(15*x^(3/2))
```

3.279 $\int \frac{(a+bx^2)^2}{x^{7/2}} dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{5x^{5/2}} - \frac{4ab}{\sqrt{x}} + \frac{2}{3}b^2x^{3/2}$$

```
[Out] -2/5*a^2/x^(5/2)+2/3*b^2*x^(3/2)-4*a*b/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{276\}$

$$-\frac{2a^2}{5x^{5/2}} - \frac{4ab}{\sqrt{x}} + \frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx &= \int \left(\frac{a^2}{x^{7/2}} + \frac{2ab}{x^{3/2}} + b^2\sqrt{x}\right)dx\\ &= -\frac{2a^2}{5x^{5/2}} - \frac{4ab}{\sqrt{x}} + \frac{2}{3}b^2x^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.88

$$\frac{2(-3a^2 - 30abx^2 + 5b^2x^4)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(7/2),x]
```

```
[Out] (2*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*x^(5/2))
```

Maple [A]
time = 0.05, size = 25, normalized size = 0.74

method	result	size
derivativedivides	$-\frac{2a^{2}}{5x^{\frac{5}{2}}}+\frac{2b^{2}x^{\frac{3}{2}}}{3}-\frac{4ab}{\sqrt{x}}$	25
default	$-\frac{2a^{2}}{5x^{\frac{5}{2}}}+\frac{2b^{2}x^{\frac{3}{2}}}{3}-\frac{4ab}{\sqrt{x}}$	25
gosper	$-\frac{2\left(-5b^{2}x^{4}+30ab\,x^{2}+3a^{2}\right)}{15x^{\frac{5}{2}}}$	27
trager	$-\frac{2\left(-5b^{2}x^{4}+30ab\,x^{2}+3a^{2}\right)}{15x^{\frac{5}{2}}}$	27
risch	$-\frac{2\left(-5b^{2}x^{4}+30ab\,x^{2}+3a^{2}\right)}{15x^{\frac{5}{2}}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*a^2/x^(5/2)+2/3*b^2*x^(3/2)-4*a*b/x^(1/2)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.74

$$\frac{2}{3}\,b^{2}x^{\frac{3}{2}}-\frac{2\,\left(10\,abx^{2}+a^{2}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
[Out] 2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

Fricas [A]
time = 1.14, size = 26, normalized size = 0.76

$$\frac{2\,\left(5\,b^{2}x^{4}-30\,abx^{2}-3\,a^{2}\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
[Out] 2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)/x^(5/2)
```

Sympy [A]
time = 0.32, size = 32, normalized size = 0.94

$$-\frac{2a^{2}}{5x^{\frac{5}{2}}}-\frac{4ab}{\sqrt{x}}+\frac{2b^{2}x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2),x)
```

```
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 2*b**2*x**(3/2)/3
```

Giac [A]
time = 1.20, size = 25, normalized size = 0.74

$$\frac{2}{3}\,b^2x^{\frac{3}{2}}-\frac{2\,(10\,abx^2+a^2)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.76

$$-\frac{6\,a^2+60\,a\,b\,x^2-10\,b^2\,x^4}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(7/2),x)
```

```
[Out] -(6*a^2 - 10*b^2*x^4 + 60*a*b*x^2)/(15*x^(5/2))
```

3.280 $\int x^{7/2}(a+bx^2)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{9}a^3x^{9/2}+\frac{6}{13}a^2bx^{13/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}$$

```
[Out] 2/9*a^3*x^(9/2)+6/13*a^2*b*x^(13/2)+6/17*a*b^2*x^(17/2)+2/21*b^3*x^(21/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {276}

$$\frac{2}{9}a^3x^{9/2}+\frac{6}{13}a^2bx^{13/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(9/2))/9 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2*x^(17/2))/17 + (2*b^3*
x^(21/2))/21
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^3\,dx&=\int\left(a^3x^{7/2}+3a^2bx^{11/2}+3ab^2x^{15/2}+b^3x^{19/2}\right)\,dx\\&=\frac{2}{9}a^3x^{9/2}+\frac{6}{13}a^2bx^{13/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.80

$$\frac{2x^{9/2}(1547a^3+3213a^2bx^2+2457ab^2x^4+663b^3x^6)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^3,x]
```

[Out] (2*x^(9/2)*(1547*a^3 + 3213*a^2*b*x^2 + 2457*a*b^2*x^4 + 663*b^3*x^6))/1392
3

Maple [A]
time = 0.06, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2a^{3}x^{\frac{9}{2}}}{9}+\frac{6a^{2}b\,x^{\frac{13}{2}}}{13}+\frac{6a\,b^{2}x^{\frac{17}{2}}}{17}+\frac{2b^{3}x^{\frac{21}{2}}}{21}$	36
default	$\frac{2a^{3}x^{\frac{9}{2}}}{9}+\frac{6a^{2}b\,x^{\frac{13}{2}}}{13}+\frac{6a\,b^{2}x^{\frac{17}{2}}}{17}+\frac{2b^{3}x^{\frac{21}{2}}}{21}$	36
gosper	$\frac{2x^{\frac{9}{2}}\left(663b^{3}x^{6}+2457a\,b^{2}x^{4}+3213a^{2}b\,x^{2}+1547a^{3}\right)}{13923}$	38
trager	$\frac{2x^{\frac{9}{2}}\left(663b^{3}x^{6}+2457a\,b^{2}x^{4}+3213a^{2}b\,x^{2}+1547a^{3}\right)}{13923}$	38
risch	$\frac{2x^{\frac{9}{2}}\left(663b^{3}x^{6}+2457a\,b^{2}x^{4}+3213a^{2}b\,x^{2}+1547a^{3}\right)}{13923}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] 2/9*a^3*x^(9/2)+6/13*a^2*b*x^(13/2)+6/17*a*b^2*x^(17/2)+2/21*b^3*x^(21/2)

Maxima [A]
time = 0.27, size = 35, normalized size = 0.69

$$\frac{2}{21}\,b^{3}x^{\frac{21}{2}}+\frac{6}{17}\,ab^{2}x^{\frac{17}{2}}+\frac{6}{13}\,a^{2}bx^{\frac{13}{2}}+\frac{2}{9}\,a^{3}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="maxima")

[Out] 2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^(
9/2)

Fricas [A]
time = 1.16, size = 40, normalized size = 0.78

$$\frac{2}{13923}\left(663\,b^{3}x^{10}+2457\,ab^{2}x^{8}+3213\,a^{2}bx^{6}+1547\,a^{3}x^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="fricas")

[Out] 2/13923*(663*b^3*x^10 + 2457*a*b^2*x^8 + 3213*a^2*b*x^6 + 1547*a^3*x^4)*sqr
t(x)

Sympy [A]
time = 1.04, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{9}{2}}}{9}+\frac{6a^2bx^{\frac{13}{2}}}{13}+\frac{6ab^2x^{\frac{17}{2}}}{17}+\frac{2b^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(9/2)/9 + 6*a**2*b*x**(13/2)/13 + 6*a*b**2*x**(17/2)/17 + 2*b**3*
x**(21/2)/21
```

Giac [A]
time = 1.04, size = 35, normalized size = 0.69

$$\frac{2}{21}\,b^3x^{\frac{21}{2}}+\frac{6}{17}\,ab^2x^{\frac{17}{2}}+\frac{6}{13}\,a^2bx^{\frac{13}{2}}+\frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^(
9/2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{9/2}}{9}+\frac{2\,b^3\,x^{21/2}}{21}+\frac{6\,a^2\,b\,x^{13/2}}{13}+\frac{6\,a\,b^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(9/2))/9 + (2*b^3*x^(21/2))/21 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2*
x^(17/2))/17
```

3.281 $\int x^{5/2}(a+bx^2)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}$$

```
[Out] 2/7*a^3*x^(7/2)+6/11*a^2*b*x^(11/2)+2/5*a*b^2*x^(15/2)+2/19*b^3*x^(19/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {276}

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2*x^(15/2))/5 + (2*b^3*x
^(19/2))/19
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^3\,dx&=\int\left(a^3x^{5/2}+3a^2bx^{9/2}+3ab^2x^{13/2}+b^3x^{17/2}\right)\,dx\\&=\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.80

$$\frac{2x^{7/2}(1045a^3+1995a^2bx^2+1463ab^2x^4+385b^3x^6)}{7315}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(7/2)*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^2*x^4 + 385*b^3*x^6))/7315
```

Maple [A]
time = 0.06, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{6a^{2}b\,x^{\frac{11}{2}}}{11}+\frac{2a\,b^{2}x^{\frac{15}{2}}}{5}+\frac{2b^{3}x^{\frac{19}{2}}}{19}$	36
default	$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{6a^{2}b\,x^{\frac{11}{2}}}{11}+\frac{2a\,b^{2}x^{\frac{15}{2}}}{5}+\frac{2b^{3}x^{\frac{19}{2}}}{19}$	36
gosper	$\frac{2x^{\frac{7}{2}}\left(385b^{3}x^{6}+1463a\,b^{2}x^{4}+1995a^{2}b\,x^{2}+1045a^{3}\right)}{7315}$	38
trager	$\frac{2x^{\frac{7}{2}}\left(385b^{3}x^{6}+1463a\,b^{2}x^{4}+1995a^{2}b\,x^{2}+1045a^{3}\right)}{7315}$	38
risch	$\frac{2x^{\frac{7}{2}}\left(385b^{3}x^{6}+1463a\,b^{2}x^{4}+1995a^{2}b\,x^{2}+1045a^{3}\right)}{7315}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 2/7*a^3*x^(7/2)+6/11*a^2*b*x^(11/2)+2/5*a*b^2*x^(15/2)+2/19*b^3*x^(19/2)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.69

$$\frac{2}{19}\,b^{3}x^{\frac{19}{2}}+\frac{2}{5}\,ab^{2}x^{\frac{15}{2}}+\frac{6}{11}\,a^{2}bx^{\frac{11}{2}}+\frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="maxima")
[Out] 2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7
/2)
```

Fricas [A]
time = 1.27, size = 40, normalized size = 0.78

$$\frac{2}{7315}\left(385\,b^{3}x^{9}+1463\,ab^{2}x^{7}+1995\,a^{2}bx^{5}+1045\,a^{3}x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 2/7315*(385*b^3*x^9 + 1463*a*b^2*x^7 + 1995*a^2*b*x^5 + 1045*a^3*x^3)*sqrt(
x)
```

Sympy [A]
time = 0.70, size = 49, normalized size = 0.96

$$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{6a^{2}bx^{\frac{11}{2}}}{11}+\frac{2ab^{2}x^{\frac{15}{2}}}{5}+\frac{2b^{3}x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a*b**2*x**(15/2)/5 + 2*b**3*x
**(19/2)/19
```

Giac [A]
time = 2.05, size = 35, normalized size = 0.69

$$\frac{2}{19}\,b^3x^{\frac{19}{2}}+\frac{2}{5}\,ab^2x^{\frac{15}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7
/2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{7/2}}{7}+\frac{2\,b^3\,x^{19/2}}{19}+\frac{6\,a^2\,b\,x^{11/2}}{11}+\frac{2\,a\,b^2\,x^{15/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*b^3*x^(19/2))/19 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2*
x^(15/2))/5
```

3.282 $\int x^{3/2}(a+bx^2)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}$$

```
[Out] 2/5*a^3*x^(5/2)+2/3*a^2*b*x^(9/2)+6/13*a*b^2*x^(13/2)+2/17*b^3*x^(17/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {276}

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(13/2))/13 + (2*b^3*x^
(17/2))/17
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^3\,dx &= \int \left(a^3x^{3/2}+3a^2bx^{7/2}+3ab^2x^{11/2}+b^3x^{15/2}\right)\,dx\\ &= \frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.80

$$\frac{2x^{5/2}(663a^3+1105a^2bx^2+765ab^2x^4+195b^3x^6)}{3315}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^3,x]
```

[Out] (2*x^(5/2)*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*x^4 + 195*b^3*x^6))/3315

Maple [A]
time = 0.06, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{17}{2}}}{17}$	36
default	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{17}{2}}}{17}$	36
gosper	$\frac{2x^{\frac{5}{2}}\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)}{3315}$	38
trager	$\frac{2x^{\frac{5}{2}}\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)}{3315}$	38
risch	$\frac{2x^{\frac{5}{2}}\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)}{3315}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] 2/5*a^3*x^(5/2)+2/3*a^2*b*x^(9/2)+6/13*a*b^2*x^(13/2)+2/17*b^3*x^(17/2)

Maxima [A]
time = 0.27, size = 35, normalized size = 0.69

$$\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="maxima")

[Out] 2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/2)

Fricas [A]
time = 1.46, size = 40, normalized size = 0.78

$$\frac{2}{3315}\left(195\,b^3x^8+765\,ab^2x^6+1105\,a^2bx^4+663\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="fricas")

[Out] 2/3315*(195*b^3*x^8 + 765*a*b^2*x^6 + 1105*a^2*b*x^4 + 663*a^3*x^2)*sqrt(x)

Sympy [A]
time = 0.46, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6ab^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(13/2)/13 + 2*b**3*x*
*(17/2)/17
```

Giac [A]
time = 1.15, size = 35, normalized size = 0.69

$$\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/
2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,b^3\,x^{17/2}}{17}+\frac{2\,a^2\,b\,x^{9/2}}{3}+\frac{6\,a\,b^2\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*b^3*x^(17/2))/17 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^
(13/2))/13
```

3.283 $\int \sqrt{x}\,(a+bx^2)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

```
[Out] 2/3*a^3*x^(3/2)+6/7*a^2*b*x^(7/2)+6/11*a*b^2*x^(11/2)+2/15*b^3*x^(15/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {276}

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(15/2))/15
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx^2)^3\,dx &= \int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3ab^2x^{9/2}+b^3x^{13/2}\right)dx\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.80

$$\frac{2x^{3/2}(385a^3+495a^2bx^2+315ab^2x^4+77b^3x^6)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^3,x]
```

[Out] (2*x^(3/2)*(385*a^3 + 495*a^2*b*x^2 + 315*a*b^2*x^4 + 77*b^3*x^6))/1155

Maple [A]
time = 0.06, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2b\,x^{\frac{7}{2}}}{7}+\frac{6a\,b^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{15}{2}}}{15}$	36
default	$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2b\,x^{\frac{7}{2}}}{7}+\frac{6a\,b^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{15}{2}}}{15}$	36
gosper	$\frac{2x^{\frac{3}{2}}\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)}{1155}$	38
trager	$\frac{2x^{\frac{3}{2}}\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)}{1155}$	38
risch	$\frac{2x^{\frac{3}{2}}\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)}{1155}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^3*x^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/3*a^3*x^(3/2)+6/7*a^2*b*x^(7/2)+6/11*a*b^2*x^(11/2)+2/15*b^3*x^(15/2)

Maxima [A]
time = 0.34, size = 35, normalized size = 0.69

$$\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="maxima")

[Out] 2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/2)

Fricas [A]
time = 1.05, size = 38, normalized size = 0.75

$$\frac{2}{1155}\left(77\,b^3x^7+315\,ab^2x^5+495\,a^2bx^3+385\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="fricas")

[Out] 2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(x)

Sympy [A]
time = 0.98, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{6ab^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*x**(1/2),x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x*
*(15/2)/15
```

Giac [A]
time = 1.04, size = 35, normalized size = 0.69

$$\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/
2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,b^3\,x^{15/2}}{15}+\frac{6\,a^2\,b\,x^{7/2}}{7}+\frac{6\,a\,b^2\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(3/2))/3 + (2*b^3*x^(15/2))/15 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x^
(11/2))/11
```

3.284 $\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}$$

```
[Out] 6/5*a^2*b*x^(5/2)+2/3*a*b^2*x^(9/2)+2/13*b^3*x^(13/2)+2*a^3*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {276}

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(13/2)
)/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3ab^2x^{7/2}+b^3x^{11/2}\right)dx\\ &= 2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.84

$$\frac{2}{195}\sqrt{x}\left(195a^3+117a^2bx^2+65ab^2x^4+15b^3x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/Sqrt[x],x]
[Out] (2*Sqrt[x]*(195*a^3 + 117*a^2*b*x^2 + 65*a*b^2*x^4 + 15*b^3*x^6))/195
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.73

method	result	size
derivativedivides	$\frac{6a^2 b\,x^{\frac{5}{2}}}{5}+\frac{2a\,b^2 x^{\frac{9}{2}}}{3}+\frac{2b^3 x^{\frac{13}{2}}}{13}+2a^3\sqrt{x}$	36
default	$\frac{6a^2 b\,x^{\frac{5}{2}}}{5}+\frac{2a\,b^2 x^{\frac{9}{2}}}{3}+\frac{2b^3 x^{\frac{13}{2}}}{13}+2a^3\sqrt{x}$	36
trager	$\left(\frac{2}{13}b^3x^6+\frac{2}{3}a\,b^2x^4+\frac{6}{5}a^2b\,x^2+2a^3\right)\sqrt{x}$	37
gosper	$\frac{2\sqrt{x}\,\left(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3\right)}{195}$	38
risch	$\frac{2\sqrt{x}\,\left(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3\right)}{195}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 6/5*a^2*b*x^(5/2)+2/3*a*b^2*x^(9/2)+2/13*b^3*x^(13/2)+2*a^3*x^(1/2)
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.71

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
[Out] 2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

Fricas [A]
time = 1.16, size = 37, normalized size = 0.76

$$\frac{2}{195}\left(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
[Out] 2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(x)
```

Sympy [A]
time = 0.25, size = 48, normalized size = 0.98

$$2a^3\sqrt{x}+\frac{6a^2bx^{\frac{5}{2}}}{5}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(13/
2)/13
```

Giac [A]
time = 1.93, size = 35, normalized size = 0.71

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$2\,a^3\,\sqrt{x}+\frac{2\,b^3\,x^{13/2}}{13}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{2\,a\,b^2\,x^{9/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(1/2),x)
```

```
[Out] 2*a^3*x^(1/2) + (2*b^3*x^(13/2))/13 + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/2
))/3
```

3.285 $\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}$$

```
[Out] 2*a^2*b*x^(3/2)+6/7*a*b^2*x^(7/2)+2/11*b^3*x^(11/2)-2*a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {276}

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(11/2))
/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3ab^2x^{5/2}+b^3x^{9/2}\right)dx\\ &= -\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.87

$$-\frac{2(77a^3-77a^2bx^2-33ab^2x^4-7b^3x^6)}{77\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(3/2),x]
[Out] (-2*(77*a^3 - 77*a^2*b*x^2 - 33*a*b^2*x^4 - 7*b^3*x^6))/(77*Sqrt[x])
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.77

method	result	size
derivativedivides	$2a^{2}b\,x^{\frac{3}{2}}+\frac{6a\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{11}{2}}}{11}-\frac{2a^{3}}{\sqrt{x}}$	36
default	$2a^{2}b\,x^{\frac{3}{2}}+\frac{6a\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{11}{2}}}{11}-\frac{2a^{3}}{\sqrt{x}}$	36
gosper	$-\frac{2\left(-7b^{3}x^{6}-33a\,b^{2}x^{4}-77a^{2}b\,x^{2}+77a^{3}\right)}{77\sqrt{x}}$	38
trager	$-\frac{2\left(-7b^{3}x^{6}-33a\,b^{2}x^{4}-77a^{2}b\,x^{2}+77a^{3}\right)}{77\sqrt{x}}$	38
risch	$-\frac{2\left(-7b^{3}x^{6}-33a\,b^{2}x^{4}-77a^{2}b\,x^{2}+77a^{3}\right)}{77\sqrt{x}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*a^2*b*x^(3/2)+6/7*a*b^2*x^(7/2)+2/11*b^3*x^(11/2)-2*a^3/x^(1/2)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.74

$$\frac{2}{11}\,b^{3}x^{\frac{11}{2}}+\frac{6}{7}\,ab^{2}x^{\frac{7}{2}}+2\,a^{2}bx^{\frac{3}{2}}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
[Out] 2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

Fricas [A]
time = 1.14, size = 37, normalized size = 0.79

$$\frac{2\,(7\,b^{3}x^{6}+33\,ab^{2}x^{4}+77\,a^{2}bx^{2}-77\,a^{3})}{77\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
[Out] 2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)/sqrt(x)
```

Sympy [A]
time = 0.36, size = 46, normalized size = 0.98

$$-\frac{2a^3}{\sqrt{x}} + 2a^2bx^{\frac{3}{2}} + \frac{6ab^2x^{\frac{7}{2}}}{7} + \frac{2b^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(7/2)/7 + 2*b**3*x**(11/2
)/11
```

Giac [A]
time = 1.11, size = 35, normalized size = 0.74

$$\frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + 2\,a^2bx^{\frac{3}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.74

$$\frac{2\,b^3\,x^{11/2}}{11} - \frac{2\,a^3}{\sqrt{x}} + 2\,a^2\,b\,x^{3/2} + \frac{6\,a\,b^2\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(3/2),x)
```

```
[Out] (2*b^3*x^(11/2))/11 - (2*a^3)/x^(1/2) + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2))
/7
```

3.286 $\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}$$

```
[Out] -2/3*a^3/x^(3/2)+6/5*a*b^2*x^(5/2)+2/9*b^3*x^(9/2)+6*a^2*b*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {276}

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(9/
2))/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{\sqrt{x}}+3ab^2x^{3/2}+b^3x^{7/2}\right)dx\\ &= -\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.84

$$-\frac{2(15a^3-135a^2bx^2-27ab^2x^4-5b^3x^6)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(5/2),x]
[Out] (-2*(15*a^3 - 135*a^2*b*x^2 - 27*a*b^2*x^4 - 5*b^3*x^6))/(45*x^(3/2))
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.73

method	result	size
derivativedivides	$-\frac{2a^{3}}{3x^{\frac{3}{2}}}+\frac{6a\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2b^{3}x^{\frac{9}{2}}}{9}+6a^{2}b\sqrt{x}$	36
default	$-\frac{2a^{3}}{3x^{\frac{3}{2}}}+\frac{6a\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2b^{3}x^{\frac{9}{2}}}{9}+6a^{2}b\sqrt{x}$	36
gosper	$-\frac{2\left(-5b^{3}x^{6}-27a\,b^{2}x^{4}-135a^{2}b\,x^{2}+15a^{3}\right)}{45x^{\frac{3}{2}}}$	38
trager	$-\frac{2\left(-5b^{3}x^{6}-27a\,b^{2}x^{4}-135a^{2}b\,x^{2}+15a^{3}\right)}{45x^{\frac{3}{2}}}$	38
risch	$-\frac{2\left(-5b^{3}x^{6}-27a\,b^{2}x^{4}-135a^{2}b\,x^{2}+15a^{3}\right)}{45x^{\frac{3}{2}}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*a^3/x^(3/2)+6/5*a*b^2*x^(5/2)+2/9*b^3*x^(9/2)+6*a^2*b*x^(1/2)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.71

$$\frac{2}{9}\,b^{3}x^{\frac{9}{2}}+\frac{6}{5}\,ab^{2}x^{\frac{5}{2}}+6\,a^{2}b\sqrt{x}-\frac{2\,a^{3}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
[Out] 2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

Fricas [A]
time = 1.29, size = 37, normalized size = 0.76

$$\frac{2\,(5\,b^{3}x^{6}+27\,ab^{2}x^{4}+135\,a^{2}bx^{2}-15\,a^{3})}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
[Out] 2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)/x^(3/2)
```

Sympy [A]
time = 0.42, size = 48, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}} + 6a^2b\sqrt{x} + \frac{6ab^2x^{\frac{5}{2}}}{5} + \frac{2b^3x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(
9/2)/9
```

Giac [A]
time = 1.20, size = 35, normalized size = 0.71

$$\frac{2}{9}b^3x^{\frac{9}{2}} + \frac{6}{5}ab^2x^{\frac{5}{2}} + 6a^2b\sqrt{x} - \frac{2a^3}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.71

$$\frac{2b^3x^{9/2}}{9} - \frac{2a^3}{3x^{3/2}} + 6a^2b\sqrt{x} + \frac{6ab^2x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(5/2),x)
```

```
[Out] (2*b^3*x^(9/2))/9 - (2*a^3)/(3*x^(3/2)) + 6*a^2*b*x^(1/2) + (6*a*b^2*x^(5/2
))/5
```

3.287 $\int \frac{(a+bx^2)^3}{x^{7/2}} dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}$$

```
[Out] -2/5*a^3/x^(5/2)+2*a*b^2*x^(3/2)+2/7*b^3*x^(7/2)-6*a^2*b/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {276}

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(7/2)
)/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx &= \int \left(\frac{a^3}{x^{7/2}}+\frac{3a^2b}{x^{3/2}}+3ab^2\sqrt{x}+b^3x^{5/2}\right)dx\\ &= -\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.87

$$-\frac{2(7a^3+105a^2bx^2-35ab^2x^4-5b^3x^6)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(7/2),x]
```

```
[Out] (-2*(7*a^3 + 105*a^2*b*x^2 - 35*a*b^2*x^4 - 5*b^3*x^6))/(35*x^(5/2))
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.77

method	result	size
derivativedivides	$-\frac{2a^3}{5x^{\frac{5}{2}}}+2a\,b^2x^{\frac{3}{2}}+\frac{2b^3x^{\frac{7}{2}}}{7}-\frac{6a^2b}{\sqrt{x}}$	36
default	$-\frac{2a^3}{5x^{\frac{5}{2}}}+2a\,b^2x^{\frac{3}{2}}+\frac{2b^3x^{\frac{7}{2}}}{7}-\frac{6a^2b}{\sqrt{x}}$	36
gosper	$-\frac{2(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)}{35x^{\frac{5}{2}}}$	38
trager	$-\frac{2(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)}{35x^{\frac{5}{2}}}$	38
risch	$-\frac{2(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)}{35x^{\frac{5}{2}}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*a^3/x^(5/2)+2*a*b^2*x^(3/2)+2/7*b^3*x^(7/2)-6*a^2*b/x^(1/2)
```

Maxima [A]
time = 0.31, size = 36, normalized size = 0.77

$$\frac{2}{7}\,b^3x^{\frac{7}{2}}+2\,ab^2x^{\frac{3}{2}}-\frac{2\,(15\,a^2bx^2+a^3)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

Fricas [A]
time = 1.10, size = 37, normalized size = 0.79

$$\frac{2\,(5\,b^3x^6+35\,ab^2x^4-105\,a^2bx^2-7\,a^3)}{35\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)/x^(5/2)
```

Sympy [A]
time = 0.53, size = 46, normalized size = 0.98

$$-\frac{2a^3}{5x^{\frac{5}{2}}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{\frac{3}{2}} + \frac{2b^3x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(7/2),x)
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(7/
2)/7
```

Giac [A]
time = 1.36, size = 36, normalized size = 0.77

$$\frac{2}{7}\,b^3x^{\frac{7}{2}} + 2\,ab^2x^{\frac{3}{2}} - \frac{2\,(15\,a^2bx^2 + a^3)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="giac")
[Out] 2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.79

$$-\frac{14\,a^3 + 210\,a^2\,b\,x^2 - 70\,a\,b^2\,x^4 - 10\,b^3\,x^6}{35\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(7/2),x)
[Out] -(14*a^3 - 10*b^3*x^6 + 210*a^2*b*x^2 - 70*a*b^2*x^4)/(35*x^(5/2))
```

3.288 $\int \frac{x^{7/2}}{a+bx^2}\,dx$

Optimal. Leaf size=215

$$-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{5/2}}{5b}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}-\frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\,b^{9}}$$

```
[Out] 2/5*x^(5/2)/b-1/2*a^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(9/4)
*2^(1/2)+1/2*a^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(9/4)*2^(1
/2)-1/4*a^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(9/
4)*2^(1/2)+1/4*a^(5/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2)
)/b^(9/4)*2^(1/2)-2*a*x^(1/2)/b^2
```

Rubi [A]
time = 0.15, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{a^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{a^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{9/4}}-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(5/2))/(5*b) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4)) + (a^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{a+bx^2}\,dx = \frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{a+bx^2}\,dx}{b}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{b^2}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{a^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2} + \frac{a^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x\right.}{b^2}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{a^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} + \frac{a^{3/2}\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}} + \sqrt{}}\right.}{2b}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} - \frac{a^{5/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}} + \frac{a^{5/4}\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\,b^{9/}}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} - \frac{a^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}} + \frac{a^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}} - a^{5/}$$

Mathematica [A]
time = 0.16, size = 128, normalized size = 0.60

$$\frac{4\sqrt[4]{b}\sqrt{x}\,(-5a+bx^2) - 5\sqrt{2}\,a^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 5\sqrt{2}\,a^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{10b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2),x]
```

```
[Out] (4*b^(1/4)*Sqrt[x]*(-5*a + b*x^2) - 5*Sqrt[2]*a^(5/4)*ArcTan[(Sqrt[a] - Sqr
t[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*a^(5/4)*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(10*b^(9/4))
```

Maple [A]
time = 0.06, size = 125, normalized size = 0.58

method	result

derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{5}{2}}}{5}+a\sqrt{x}\right)}{b^{2}}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4b^{2}}$
default	$-\frac{2\left(-\frac{b\,x^{\frac{5}{2}}}{5}+a\sqrt{x}\right)}{b^{2}}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4b^{2}}$
risch	$-\frac{2\left(-b\,x^{2}+5a\right)\sqrt{x}}{5b^{2}}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^2*(-1/5*b*x^(5/2)+a*x^(1/2))+1/4*a/b^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b
)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)-1))
```

Maxima [A]
time = 0.53, size = 194, normalized size = 0.90

$$\frac{2\left(bx^{\frac{5}{2}}-5\,a\sqrt{x}\right)}{5\,b^{2}}+\frac{\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x^(5/2) - 5*a*sqrt(x))/b^2 + 1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqr
t(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b))
+ sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a)
)/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/b^(1/4))/b^2
```

Fricas [A]
time = 2.18, size = 170, normalized size = 0.79

$$\frac{20\,b^{2}\left(-\frac{a^{5}}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab^{7}\sqrt{x}\left(-\frac{a^{5}}{b^{9}}\right)^{\frac{3}{4}}-\sqrt{b^{4}\sqrt{-\frac{a^{5}}{b^{9}}}+a^{2}x}\,b^{7}\left(-\frac{a^{5}}{b^{9}}\right)^{\frac{3}{4}}}{a^{5}}\right)+5\,b^{2}\left(-\frac{a^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(b^{2}\left(-\frac{a^{5}}{b^{9}}\right)^{\frac{1}{4}}+a\sqrt{x}\right)-5\,b^{2}\left(-\frac{a^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(-b^{2}\left(-\frac{a^{5}}{b^{9}}\right)^{\frac{1}{4}}+a\sqrt{x}\right)+4\left(bx^{2}-5\,a\right)\sqrt{x}}{10\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*(20*b^2*(-a^5/b^9)^(1/4)*arctan(-(a*b^7*sqrt(x)*(-a^5/b^9)^(3/4) - sqr
t(b^4*sqrt(-a^5/b^9) + a^2*x)*b^7*(-a^5/b^9)^(3/4))/a^5) + 5*b^2*(-a^5/b^9)
^(1/4)*log(b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) - 5*b^2*(-a^5/b^9)^(1/4)*log(-
b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) + 4*(b*x^2 - 5*a)*sqrt(x))/b^2
```

Sympy [A]
time = 15.16, size = 136, normalized size = 0.63

$$\begin{cases} \tilde{\infty} x^{\frac{5}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{5}{2}}}{5 b} & \text{for}\: a = 0 \\ \frac{2 x^{\frac{9}{2}}}{9 a} & \text{for}\: b = 0 \\ - \frac{2 a \sqrt{x}}{b^{2}} - \frac{a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{2}} + \frac{a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{2}} + \frac{a \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{2}} + \frac{2 x^{\frac{5}{2}}}{5 b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*b), Eq(a, 0))
, (2*x**(9/2)/(9*a), Eq(b, 0)), (-2*a*sqrt(x)/b**2 - a*(-a/b)**(1/4)*log(sq
rt(x) - (-a/b)**(1/4))/(2*b**2) + a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/
4))/(2*b**2) + a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 + 2*x**(5/2
)/(5*b), True))
```

Giac [A]
time = 1.19, size = 196, normalized size = 0.91

$$\frac{\sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 b^{3}} + \frac{\sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} a \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 b^{3}} + \frac{\sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} a \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 b^{3}} - \frac{\sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} a \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 b^{3}} + \frac{2 \left(b^{4} x^{\frac{5}{2}} - 5 a b^{3} \sqrt{x}\right)}{5 b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/b^3 + 1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*(a*b^3)^(1/4
)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*(a*b
^3)^(1/4)*a*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 2/5*(b^
4*x^(5/2) - 5*a*b^3*sqrt(x))/b^5
```

Mupad [B]
time = 4.49, size = 67, normalized size = 0.31

$$\frac{2\,x^{5/2}}{5\,b}-\frac{2\,a\,\sqrt{x}}{b^2}-\frac{{\left(-a\right)}^{5/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{9/4}}+\frac{{\left(-a\right)}^{5/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}}\right)\,1{}\mathrm{i}}{b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(5/2))/(5*b) - (2*a*x^(1/2))/b^2 - ((-a)^(5/4)*atan((b^(1/4)*x^(1/2))/
(-a)^(1/4)))/b^(9/4) + ((-a)^(5/4)*atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*1i
)/b^(9/4)
```

3.289 $\int \frac{x^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=204

$$\frac{2x^{3/2}}{3b}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{2\sqrt{2}\,b^{7/4}}$$

```
[Out] 2/3*x^(3/2)/b+1/2*a^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(7/4)
*2^(1/2)-1/2*a^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(7/4)*2^(1
/2)-1/4*a^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(7/
4)*2^(1/2)+1/4*a^(3/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2)
)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{a^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{a^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{7/4}}-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2))/(3*b) + (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(7/4)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*b^(7/4)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{a+bx^2}\,dx}{b}$$
$$= \frac{2x^{3/2}}{3b} - \frac{(2a)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2x^{3/2}}{3b} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} - \frac{a\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}$$
$$= \frac{2x^{3/2}}{3b} - \frac{a\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x,\right.}{2b^2}$$
$$= \frac{2x^{3/2}}{3b} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{2\sqrt{2}\,b^{7/4}}$$
$$= \frac{2x^{3/2}}{3b} + \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{a^{3/4}\log\left(\sqrt{a}\right.}{}$$

Mathematica [A]
time = 0.13, size = 119, normalized size = 0.58

$$\frac{4b^{3/4}x^{3/2} + 3\sqrt{2}\,a^{3/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 3\sqrt{2}\,a^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{6b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (4*b^(3/4)*x^(3/2) + 3*Sqrt[2]*a^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*a^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(6*b^(7/4))
```

Maple [A]
time = 0.05, size = 116, normalized size = 0.57

method	result

derivativedivides	$\frac{2x^{\frac{3}{2}}}{3b}-\frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{2x^{\frac{3}{2}}}{3b}-\frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$\frac{2x^{\frac{3}{2}}}{3b}-\frac{a\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{a\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{a\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{2b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)/b-1/4*a/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.49, size = 186, normalized size = 0.91

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4b}+\frac{2x^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b +
 2/3*x^(3/2)/b
```

Fricas [A]
time = 1.45, size = 165, normalized size = 0.81

$$\frac{12\,b\left(-\frac{a^{3}}{b^{7}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^{2}b^{2}\sqrt{x}\left(-\frac{a^{3}}{b^{7}}\right)^{\frac{1}{4}}-\sqrt{-a^{3}b^{3}\sqrt{-\frac{a^{3}}{b^{7}}}+a^{4}x}\,b^{2}\left(-\frac{a^{3}}{b^{7}}\right)^{\frac{1}{4}}}{a^{3}}\right)-3\,b\left(-\frac{a^{3}}{b^{7}}\right)^{\frac{1}{4}}\log\left(b^{5}\left(-\frac{a^{3}}{b^{7}}\right)^{\frac{3}{4}}+a^{2}\sqrt{x}\right)+3\,b\left(-\frac{a^{3}}{b^{7}}\right)^{\frac{1}{4}}\log\left(-b^{5}\left(-\frac{a^{3}}{b^{7}}\right)^{\frac{3}{4}}+a^{2}\sqrt{x}\right)+4\,x^{\frac{3}{2}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*b*(-a^3/b^7)^(1/4)*arctan(-(a^2*b^2*sqrt(x)*(-a^3/b^7)^(1/4) - sqrt
(-a^3*b^3*sqrt(-a^3/b^7) + a^4*x)*b^2*(-a^3/b^7)^(1/4))/a^3) - 3*b*(-a^3/b^
7)^(1/4)*log(b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 3*b*(-a^3/b^7)^(1/4)*log
(-b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 4*x^(3/2))/b
```

Sympy [A]
time = 4.50, size = 124, normalized size = 0.61

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a = 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b = 0 \\ -\frac{a \log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b^2 \sqrt[4]{-\frac{a}{b}}} + \frac{a \log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b^2 \sqrt[4]{-\frac{a}{b}}} - \frac{a \operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2 \sqrt[4]{-\frac{a}{b}}} + \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(a, 0))
, (2*x**(7/2)/(7*a), Eq(b, 0)), (-a*log(sqrt(x) - (-a/b)**(1/4))/(2*b**2*(-
a/b)**(1/4)) + a*log(sqrt(x) + (-a/b)**(1/4))/(2*b**2*(-a/b)**(1/4)) - a*at
an(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b)**(1/4)) + 2*x**(3/2)/(3*b), True))
```

Giac [A]
time = 1.34, size = 178, normalized size = 0.87

$$\frac{2\,x^{\frac{3}{2}}}{3\,b} - \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4} - \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4} + \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}} \log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,b^4} - \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}} \log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/b - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*
(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/4*sq
rt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4
```

Mupad [B]
time = 0.09, size = 54, normalized size = 0.26

$$\frac{2\,x^{3/2}}{3\,b} + \frac{{(-a)}^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{b^{7/4}} - \frac{{(-a)}^{3/4}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(3/2))/(3*b) + ((-a)^(3/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4)
 - ((-a)^(3/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4)
```

3.290 $\int \frac{x^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=202

$$\frac{2\sqrt{x}}{b}+\frac{\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{a}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,b^{5/4}}$$

```
[Out] 1/2*a^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)*2^(1/2)-1/2*a
^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)*2^(1/2)+1/4*a^(1/4
)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)*2^(1/2)-1/4
*a^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)*2^(1
/2)+2*x^(1/2)/b
```

Rubi [A]
time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{a}\,\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\,\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{2\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x])/b + (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*b^(5/4)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*b^(5/4)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{a+bx^2}\,dx = \frac{2\sqrt{x}}{b} - \frac{a\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\right.}{2b^{3/2}}$$
$$= \frac{2\sqrt{x}}{b} + \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{2\sqrt{2}\,b^{5/4}}$$
$$= \frac{2\sqrt{x}}{b} + \frac{\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{a}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}} + \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}\right.}{}$$

Mathematica [A]
time = 0.13, size = 118, normalized size = 0.58

$$\frac{4\sqrt[4]{b}\sqrt{x}+\sqrt{2}\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\sqrt{2}\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{2b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (4*b^(1/4)*Sqrt[x] + Sqrt[2]*a^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])] - Sqrt[2]*a^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(2*b^(5/4))
```

Maple [A]
time = 0.05, size = 115, normalized size = 0.57

method	result
derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$

default	$\frac{2\sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$
risch	$\frac{2\sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 2*x^(1/2)/b-1/4/b*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))

Maxima [A]
time = 0.56, size = 185, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="maxima")

[Out] -1/4*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/b + 2*sqrt(x)/b

Fricas [A]
time = 1.02, size = 124, normalized size = 0.61

$$-\frac{4\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2\sqrt{-\frac{a}{b^5}}+x}\,b^4\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}-b^4\sqrt{x}\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}}{a}\right)+b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\sqrt{x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="fricas")

[Out] -1/2*(4*b*(-a/b^5)^(1/4)*arctan((sqrt(b^2*sqrt(-a/b^5) + x)*b^4*(-a/b^5)^(3/4) - b^4*sqrt(x)*(-a/b^5)^(3/4))/a) + b*(-a/b^5)^(1/4)*log(b*(-a/b^5)^(1/4

```
) + sqrt(x)) - b*(-a/b^5)^(1/4)*log(-b*(-a/b^5)^(1/4) + sqrt(x)) - 4*sqrt(x
))/b
```

Sympy [A]
time = 1.73, size = 110, normalized size = 0.54

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b} + \frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (2*x
**(5/2)/(5*a), Eq(b, 0)), (2*sqrt(x)/b + (-a/b)**(1/4)*log(sqrt(x) - (-a/b)
**(1/4))/(2*b) - (-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - (-a/b)*
*(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b, True))
```

Giac [A]
time = 1.38, size = 178, normalized size = 0.88

$$-\frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2} - \frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2} - \frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^2} + \frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^2} + \frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/b^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^2 - 1/4*sqrt(2)*(a*b^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 2*sqrt(x)/b
```

Mupad [B]
time = 0.09, size = 55, normalized size = 0.27

$$\frac{2\,\sqrt{x}}{b} - \frac{{(-a)}^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{b^{5/4}} - \frac{{(-a)}^{1/4}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(1/2))/b - ((-a)^(1/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4) - (
(-a)^(1/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4)
```

3.291 $\int \frac{\sqrt{x}}{a+bx^2} \, dx$

Optimal. Leaf size=192

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{\right.}$$

```
[Out] -1/2*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b^(3/4)*2^(1/2)+1/2*
arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b^(3/4)*2^(1/2)+1/4*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(3/4)*2^(1/2)-1/
4*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(3/4)*2^(
1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) +
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1
/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = 2\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} +$$

$$= \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}$$

Mathematica [A]
time = 0.11, size = 91, normalized size = 0.47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] -((ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(1/4
)*b^(3/4)))
```

Maple [A]
time = 0.03, size = 106, normalized size = 0.55

method	result	size
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	106

default	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac14}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac14}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac14}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(
x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.53, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{4\,a^{\frac14}b^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{4\,a^{\frac14}b^{\frac34}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + 1/4*sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
```

Fricas [A]
time = 0.98, size = 126, normalized size = 0.66

$$-2\left(-\frac{1}{ab^3}\right)^{\frac14}\arctan\left(\sqrt{-ab\sqrt{-\frac{1}{ab^3}}+x}\,b\left(-\frac{1}{ab^3}\right)^{\frac14}-b\sqrt{x}\left(-\frac{1}{ab^3}\right)^{\frac14}\right)+\frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac14}\log\left(ab^2\left(-\frac{1}{ab^3}\right)^{\frac34}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac14}\log\left(-ab^2\left(-\frac{1}{ab^3}\right)^{\frac34}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*(-1/(a*b^3))^(1/4)*arctan(sqrt(-a*b*sqrt(-1/(a*b^3)) + x)*b*(-1/(a*b^3))
^(1/4) - b*sqrt(x)*(-1/(a*b^3))^(1/4)) + 1/2*(-1/(a*b^3))^(1/4)*log(a*b^2*(
-1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*(-1/(a*b^3))^(1/4)*log(-a*b^2*(-1/(a*b^3
))^(3/4) + sqrt(x))
```

Sympy [A]

time = 0.97, size = 104, normalized size = 0.54

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b = 0 \\ \frac{\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} - \frac{\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b\sqrt[4]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
2*x**(3/2)/(3*a), Eq(b, 0)), (log(sqrt(x) - (-a/b)**(1/4))/(2*b*(-a/b)**(1/
4)) - log(sqrt(x) + (-a/b)**(1/4))/(2*b*(-a/b)**(1/4)) + atan(sqrt(x)/(-a/b
)**(1/4))/(b*(-a/b)**(1/4)), True))
```

Giac [A]
time = 1.10, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} + \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} - \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^3} + \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(a*b^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/4*sqrt(2
)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3)
```

Mupad [B]
time = 0.07, size = 38, normalized size = 0.20

$$\frac{\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right) - \operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{1/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2),x)
```

```
[Out] (atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/
((-a)^(1/4)*b^(3/4))
```

3.292 $\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx$

Optimal. Leaf size=192

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{a}\right.}{}$$

```
[Out] -1/2*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)+1/2*
arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)-1/4*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)+1/
4*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(1/4)*2^(
1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)) -
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(3
/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = 2\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\sqrt{b}}$$
$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} +$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{}}$$

Mathematica [A]
time = 0.10, size = 92, normalized size = 0.48

$$\frac{-\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (-ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(3/4)
*b^(1/4))
```

Maple [A]
time = 0.03, size = 106, normalized size = 0.55

method	result	size
derivativedivides	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a}$	106

default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{4\,a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{4\,a^{\frac{3}{4}}b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/4*sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 1/4*sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4))
```

Fricas [A]
time = 1.24, size = 126, normalized size = 0.66

$$2\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2\sqrt{-\frac{1}{a^3b}}+x}\,a^2b\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}-a^2b\sqrt{x}\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(a^3*b))^(1/4)*arctan(sqrt(a^2*sqrt(-1/(a^3*b)) + x)*a^2*b*(-1/(a^3*b
))^(3/4) - a^2*b*sqrt(x)*(-1/(a^3*b))^(3/4)) + 1/2*(-1/(a^3*b))^(1/4)*log(a
*(-1/(a^3*b))^(1/4) + sqrt(x)) - 1/2*(-1/(a^3*b))^(1/4)*log(-a*(-1/(a^3*b))
^(1/4) + sqrt(x))
```

Sympy [A]

time = 1.58, size = 104, normalized size = 0.54

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a = 0 \\ \frac{2\sqrt{x}}{a} & \text{for } b = 0 \\ -\frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0)
), (2*sqrt(x)/a, Eq(b, 0)), (-(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2
*a) + (-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a) + (-a/b)**(1/4)*atan
(sqrt(x)/(-a/b)**(1/4))/a, True))
```

Giac [A]
time = 1.57, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab} + \frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab} + \frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab} - \frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b) + 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b) + 1/4*sqrt(2)*(a*b^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b) - 1/4*sqrt(2)*(a*b
^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b)
```

Mupad [B]
time = 0.08, size = 37, normalized size = 0.19

$$-\frac{\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{3/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)),x)
```

```
[Out] -(atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))
/((-a)^(3/4)*b^(1/4))
```

3.293 $\int \frac{1}{x^{3/2}(a+bx^2)}\,dx$

Optimal. Leaf size=202

$$-\frac{2}{a\sqrt{x}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,a^{5/4}}$$

```
[Out] 1/2*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)*2^(1/2)-1/2*b
^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)*2^(1/2)-1/4*b^(1/4
)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)*2^(1/2)+1/4
*b^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)*2^(1
/2)-2/a/x^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] -2/(a*Sqrt[x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)) - (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx^2)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{b\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{(2b)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \right.}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\right.}{2a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{2\sqrt{2}\,a^{5/4}}$$
$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{b}\,\mathrm{l}}{}$$

Mathematica [A]
time = 0.14, size = 117, normalized size = 0.58

$$\frac{-\frac{4\sqrt[4]{a}}{\sqrt{x}}+\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)),x]
[Out] ((-4*a^(1/4))/Sqrt[x] + Sqrt[2]*b^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(2*a^(5/4))
```

Maple [A]
time = 0.05, size = 115, normalized size = 0.57

method	result

derivativedivides	$-\frac{2}{a\sqrt{x}}-\frac{\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$-\frac{2}{a\sqrt{x}}-\frac{\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2}{a\sqrt{x}}-\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4a\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{2a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/x^(1/2)-1/4/a/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.48, size = 186, normalized size = 0.92

$$-\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a -
 2/(a*sqrt(x))
```

Fricas [A]
time = 2.23, size = 142, normalized size = 0.70

$$\frac{4\,ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab\sqrt{x}\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}-\sqrt{-a^3b\sqrt{-\frac{b}{a^5}}+b^2x}\;a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}}{b}\right)-ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)+ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(-a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)-4\sqrt{x}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*(-b/a^5)^(1/4)*arctan(-(a*b*sqrt(x)*(-b/a^5)^(1/4) - sqrt(-a^3*b
*sqrt(-b/a^5) + b^2*x)*a*(-b/a^5)^(1/4))/b) - a*x*(-b/a^5)^(1/4)*log(a^4*(-
b/a^5)^(3/4) + b*sqrt(x)) + a*x*(-b/a^5)^(1/4)*log(-a^4*(-b/a^5)^(3/4) + b*
sqrt(x)) - 4*sqrt(x))/(a*x)
```

Sympy [A]
time = 3.47, size = 114, normalized size = 0.56

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} + \frac{\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}} - \frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) - (-a/b)**(1/4))/(2*a*(-a/b)**
(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) - atan(sqrt(x)/(-
a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*sqrt(x)), True))
```

Giac [A]
time = 0.82, size = 190, normalized size = 0.94

$$-\frac{2}{a\sqrt{x}} - \frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{2}b^{2}} - \frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{2}b^{2}} + \frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{2}b^{2}} - \frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2/(a*sqrt(x)) - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) - 1/2*sqrt(2)*(a*b^3)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) +
 1/4*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^2*b^2) - 1/4*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^2)
```

Mupad [B]
time = 4.50, size = 54, normalized size = 0.27

$$\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{5/4}}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{5/4}}-\frac{2}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(5/4) - ((-b)^(1/4)*atan
(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(5/4) - 2/(a*x^(1/2))
```

3.294 $\int \frac{1}{x^{5/2}(a+bx^2)}\,dx$

Optimal. Leaf size=204

$$-\frac{2}{3ax^{3/2}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}\right.}{2\sqrt{2}\,a^{7/4}}$$

```
[Out] -2/3/a/x^(3/2)+1/2*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4
)*2^(1/2)-1/2*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)*2^(
1/2)+1/4*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7
/4)*2^(1/2)-1/4*b^(3/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2
))/a^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}}+\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*a^(7/4)) + (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)) - (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx^2)}\,dx = -\frac{2}{3ax^{3/2}} - \frac{b\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{(2b)\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{b\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}} - \frac{b\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{b^{3/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}} - \frac{b^{3/4}\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{b^{3/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}} - \frac{b^{3/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}} + \frac{b^{3/4}\log\ldots}{\ldots}$$

Mathematica [A]
time = 0.14, size = 119, normalized size = 0.58

$$\frac{-\frac{4a^{3/4}}{x^{3/2}} + 3\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 3\sqrt{2}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{6a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] ((-4*a^(3/4))/x^(3/2) + 3*Sqrt[2]*b^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(6*a^(7/4))
```

Maple [A]
time = 0.05, size = 116, normalized size = 0.57

method	result
derivativedivides	$-\frac{2}{3a\,x^{\frac{3}{2}}} - \frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\ldots\right.\right.}{4a^2}$

default	$-\frac{2}{3a\,x^{\frac{3}{2}}}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.\right.}{4a^{2}}$
risch	$-\frac{2}{3a\,x^{\frac{3}{2}}}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4a^{2}}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^{2}}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)-1/4*b/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 187, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\,a}-\frac{2}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*
b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)
*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/a - 2
/3/(a*x^(3/2))
```

Fricas [A]
time = 1.14, size = 167, normalized size = 0.82

$$-\frac{12\,ax^{2}\left(-\frac{b^{3}}{a^{7}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^{5}b\sqrt{x}\left(-\frac{b^{3}}{a^{7}}\right)^{\frac{3}{4}}-\sqrt{a^{4}\sqrt{-\frac{b^{3}}{a^{7}}}+b^{2}x}\,a^{5}\left(-\frac{b^{3}}{a^{7}}\right)^{\frac{3}{4}}}{b^{3}}\right)+3\,ax^{2}\left(-\frac{b^{3}}{a^{7}}\right)^{\frac{1}{4}}\log\left(a^{2}\left(-\frac{b^{3}}{a^{7}}\right)^{\frac{1}{4}}+b\sqrt{x}\right)-3\,ax^{2}\left(-\frac{b^{3}}{a^{7}}\right)^{\frac{1}{4}}\log\left(-a^{2}\left(-\frac{b^{3}}{a^{7}}\right)^{\frac{1}{4}}+b\sqrt{x}\right)+4\sqrt{x}}{6\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*x^2*(-b^3/a^7)^(1/4)*arctan(-(a^5*b*sqrt(x)*(-b^3/a^7)^(3/4) - s
qrt(a^4*sqrt(-b^3/a^7) + b^2*x)*a^5*(-b^3/a^7)^(3/4))/b^3) + 3*a*x^2*(-b^3/
```

```
a^7)^(1/4)*log(a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) - 3*a*x^2*(-b^3/a^7)^(1/4)
*log(-a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) + 4*sqrt(x))/(a*x^2)
```

Sympy [A]
time = 8.35, size = 128, normalized size = 0.63

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for } b = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} + \frac{b\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a^2} - \frac{b\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a^2} - \frac{b\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (-2/(3*a*x**(3/2)), Eq(b, 0)), (-2/(3*a*x**(3/2)) + b*(-a/b)**(1/4)*log(
sqrt(x) - (-a/b)**(1/4))/(2*a**2) - b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(
1/4))/(2*a**2) - b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a**2, True))
```

Giac [A]
time = 0.76, size = 178, normalized size = 0.87

$$-\frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2} - \frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2} - \frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2} + \frac{\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2} - \frac{2}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/a^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^2 - 1/4*sqrt(2)*(a*b^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 - 2/3/(a*x^(3/2)
)
```

Mupad [B]
time = 0.09, size = 53, normalized size = 0.26

$$\frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{a^{7/4}} - \frac{2}{3\,a\,x^{3/2}} + \frac{(-b)^{3/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(7/4) - 2/(3*a*x^(3/2)) +
 ((-b)^(3/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(7/4)
```

3.295 $\int \frac{1}{x^{7/2}(a+bx^2)}\,dx$

Optimal. Leaf size=215

$$-\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\,a^{9}}$$

```
[Out] -2/5/a/x^(5/2)-1/2*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4
)*2^(1/2)+1/2*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)*2^(
1/2)+1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9
/4)*2^(1/2)-1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2
))/a^(9/4)*2^(1/2)+2*b/a^2/x^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+\frac{b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{9/4}}+\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)) - (b^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}(a+bx^2)}\,dx = -\frac{2}{5ax^{5/2}} - \frac{b\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{a}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b^2\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a^2}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{b^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{b^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,}{a+bx^4}\right.}{a^2}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a^2} + \frac{b\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}} + \cdots}\right.}{}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b^{5/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}} - \frac{b^{5/4}\log\left(\sqrt{a} + \cdots\right.}{2\cdots}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{b^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}} + \frac{b^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}$$

Mathematica [A]
time = 0.16, size = 127, normalized size = 0.59

$$\frac{-\frac{4\sqrt[4]{a}\,(a-5bx^2)}{x^{5/2}} - 5\sqrt{2}\,b^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 5\sqrt{2}\,b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{10a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] ((-4*a^(1/4)*(a - 5*b*x^2))/x^(5/2) - 5*Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[a] - S
qrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 5*Sqrt[2]*b^(5/4)*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(10*a^(9/4))
```

Maple [A]
time = 0.06, size = 125, normalized size = 0.58

method	result
derivativedivides	$\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2}{5a\,x^{\frac{5}{2}}}+$
default	$\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2}{5a\,x^{\frac{5}{2}}}+$
risch	$-\frac{2(-5b\,x^2+a)}{5a^2x^{\frac{5}{2}}}+\frac{b\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4a^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\mathrm{arctan}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b/a^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/5/a/x^(5/2)+2*b/a^2
/x^(1/2)
```

Maxima [A]

time = 0.49, size = 198, normalized size = 0.92

$$\frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,a^2}+\frac{2\,(5\,bx^2-a)}{5\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^
2 + 2/5*(5*b*x^2 - a)/(a^2*x^(5/2))
```

Fricas [A]

time = 2.20, size = 193, normalized size = 0.90

$$-\frac{20\,a^2x^3\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2b^4\sqrt{x}\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}-\sqrt{-a^5b^5\sqrt{-\frac{b^5}{a^9}}+b^8x}\,a^2\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}}{b^5}\right)-5\,a^2x^3\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}\log\left(a^7\left(-\frac{b^5}{a^9}\right)^{\frac{3}{4}}+b^4\sqrt{x}\right)+5\,a^2x^3\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}\log\left(-a^7\left(-\frac{b^5}{a^9}\right)^{\frac{3}{4}}+b^4\sqrt{x}\right)-4\,(5\,bx^2-a)\sqrt{x}}{10\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*x^3*(-b^5/a^9)^(1/4)*arctan(-(a^2*b^4*sqrt(x)*(-b^5/a^9)^(1/4
) - sqrt(-a^5*b^5*sqrt(-b^5/a^9) + b^8*x)*a^2*(-b^5/a^9)^(1/4))/b^5) - 5*a^
2*x^3*(-b^5/a^9)^(1/4)*log(a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) + 5*a^2*x^3*
(-b^5/a^9)^(1/4)*log(-a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) - 4*(5*b*x^2 - a)
*sqrt(x))/(a^2*x^3)
```

Sympy [A]
time = 28.94, size = 139, normalized size = 0.65

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{9bx^{\frac{9}{2}}} & \text{for } a = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} + \frac{b\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}} - \frac{b\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}} + \frac{b\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2\sqrt[4]{-\frac{a}{b}}} + \frac{2b}{a^2\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(a, 0)
), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + b*log(sqrt(x) - (-a/
b)**(1/4))/(2*a**2*(-a/b)**(1/4)) - b*log(sqrt(x) + (-a/b)**(1/4))/(2*a**2*
(-a/b)**(1/4)) + b*atan(sqrt(x)/(-a/b)**(1/4))/(a**2*(-a/b)**(1/4)) + 2*b/(
a**2*sqrt(x)), True))
```

Giac [A]
time = 0.93, size = 200, normalized size = 0.93

$$\frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b} + \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b} - \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b} + \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b} + \frac{2\,(5\,bx^2-a)}{5\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a^3*b) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) - 1/4*sqrt(2)*(a*b^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/4*sqrt(2
)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) +
 2/5*(5*b*x^2 - a)/(a^2*x^(5/2))
```

Mupad [B]
time = 4.48, size = 66, normalized size = 0.31

$$\frac{(-b)^{5/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{9/4}}-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{9/4}}-\frac{\frac{2}{5\,a}-\frac{2\,b\,x^2}{a^2}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(5/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(9/4) - ((-b)^(5/4)*atan
(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(9/4) - (2/(5*a) - (2*b*x^2)/a^2)/x^(5/2)
```

3.296 $\int \frac{x^{7/2}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=230

$$\frac{5\sqrt{x}}{2b^2}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{a}\,\log\left(\sqrt{a}\right.-$$

```
[Out] -1/2*x^(5/2)/b/(b*x^2+a)+5/8*a^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/
4))/b^(9/4)*2^(1/2)-5/8*a^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b
^(9/4)*2^(1/2)+5/16*a^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^
(1/2))/b^(9/4)*2^(1/2)-5/16*a^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^
(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+5/2*x^(1/2)/b^2
```

Rubi [A]

time = 0.12, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{5\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{a}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt{x}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2)^2,x]
```

```
[Out] (5*Sqrt[x])/(2*b^2) - x^(5/2)/(2*b*(a + b*x^2)) + (5*a^(1/4)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*a^(1/4)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(9/4))
 - (5*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx = -\frac{x^{5/2}}{2b(a+bx^2)} + \frac{5\int \frac{x^{3/2}}{a+bx^2}\,dx}{4b}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b(a+bx^2)} - \frac{(5a)\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{4b^2}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b(a+bx^2)} - \frac{(5a)\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b(a+bx^2)} - \frac{\left(5\sqrt{a}\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^2} - \frac{\left(5\sqrt{a}\right)\mathrm{Subst}\left(\int \frac{\sqrt{}}{}\right.}{}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b(a+bx^2)} - \frac{\left(5\sqrt{a}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{5/2}} - \frac{\left(5\sqrt{a}\right)\mathrm{Sub}}{}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b(a+bx^2)} + \frac{5\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{9/4}} - \frac{5\sqrt[4]{a}\log\left(\sqrt{a}+\right.}{}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b(a+bx^2)} + \frac{5\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{9/4}} - \frac{5\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,b^{9/4}}$$

Mathematica [A]
time = 0.27, size = 138, normalized size = 0.60

$$\frac{\frac{4\sqrt[4]{b}\sqrt{x}(5a+4bx^2)}{a+bx^2} + 5\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 5\sqrt{2}\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{8b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2)^2,x]
```

```
[Out] ((4*b^(1/4)*Sqrt[x]*(5*a + 4*b*x^2))/(a + b*x^2) + 5*Sqrt[2]*a^(1/4)*ArcTan
[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 5*Sqrt[2]*a^(1/
4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(8*b^(
9/4))
```

Maple [A]
time = 0.09, size = 136, normalized size = 0.59

method	result
derivativedivides	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{4(b x^2+a)}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\sqrt{\cdots}\right.\right.}{32a}\right)}{b^2}$
default	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{4(b x^2+a)}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\sqrt{\cdots}\right.\right.}{32a}\right)}{b^2}$
risch	$\frac{2\sqrt{x}}{b^2}+\frac{a\sqrt{x}}{2b^2(b x^2+a)}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{16b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\cdots\right.}{8b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^2-2*a/b^2*(-1/4*x^(1/2)/(b*x^2+a)+5/32*(a/b)^(1/4)/a*2^(1/2)*(l
n((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.53, size = 206, normalized size = 0.90

$$\frac{a\sqrt{x}}{2\,(b^3x^2+ab^2)}-\frac{5\left(\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}\right)}{16\,b^2}+\frac{2\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*sqrt(x)/(b^3*x^2 + a*b^2) - 5/16*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2
)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt
(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*
b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) +
 sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))
/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/b^(1/4))/b^2 + 2*sqrt(x)/b^2
```

Fricas [A]
time = 1.57, size = 192, normalized size = 0.83

$$-\frac{20\,(b^3x^2+ab^2)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4\sqrt{-\frac{a}{b^9}}+x}\,b^7\left(-\frac{a}{b^9}\right)^{\frac{3}{4}}-b^7\sqrt{x}\left(-\frac{a}{b^9}\right)^{\frac{3}{4}}}{a}\right)+5\,(b^3x^2+ab^2)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}\log\left(5\,b^2\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}+5\,\sqrt{x}\right)-5\,(b^3x^2+ab^2)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}\log\left(-5\,b^2\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}+5\,\sqrt{x}\right)-4\,(4\,bx^2+5\,a)\sqrt{x}}{8\,(b^3x^2+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*(b^3*x^2 + a*b^2)*(-a/b^9)^(1/4)*arctan((sqrt(b^4*sqrt(-a/b^9) + x
)*b^7*(-a/b^9)^(3/4) - b^7*sqrt(x)*(-a/b^9)^(3/4))/a) + 5*(b^3*x^2 + a*b^2)
*(-a/b^9)^(1/4)*log(5*b^2*(-a/b^9)^(1/4) + 5*sqrt(x)) - 5*(b^3*x^2 + a*b^2)
*(-a/b^9)^(1/4)*log(-5*b^2*(-a/b^9)^(1/4) + 5*sqrt(x)) - 4*(4*b*x^2 + 5*a)*
sqrt(x))/(b^3*x^2 + a*b^2)
```

Sympy [A]
time = 75.55, size = 338, normalized size = 1.47

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\ a=0\wedge b=0 \\ \frac{2\sqrt{x}}{b^2} & \text{for}\ a=0 \\ \frac{2x^{\frac{9}{2}}}{9a^2} & \text{for}\ b=0 \\ \frac{20a\sqrt{x}}{8ab^2+8b^3x^2}+\frac{5a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2+8b^3x^2}-\frac{5a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2+8b^3x^2}-\frac{10a\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8ab^2+8b^3x^2}+\frac{16bx^{\frac{5}{2}}}{8ab^2+8b^3x^2}+\frac{5bx^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2+8b^3x^2}-\frac{5bx^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2+8b^3x^2}-\frac{10bx^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8ab^2+8b^3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**2, Eq(a, 0)), (
2*x**(9/2)/(9*a**2), Eq(b, 0)), (20*a*sqrt(x)/(8*a*b**2 + 8*b**3*x**2) + 5*
a*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) - 5*a
*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) - 10*a
*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) + 16*b*
x**(5/2)/(8*a*b**2 + 8*b**3*x**2) + 5*b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-
a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) - 5*b*x**2*(-a/b)**(1/4)*log(sqrt(x)
+ (-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) - 10*b*x**2*(-a/b)**(1/4)*atan(sq
rt(x)/(-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2), True))
```

Giac [A]
time = 1.00, size = 196, normalized size = 0.85

$$-\frac{5\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^3}-\frac{5\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^3}-\frac{5\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^3}+\frac{5\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^3}+\frac{a\sqrt{x}}{2\,(bx^2+a)b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/b^3 - 5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
```

```
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 - 5/16*sqrt(2)*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 5/16*sqrt(2)*(a*b^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 1/2*a*sqrt(x
)/((b*x^2 + a)*b^2) + 2*sqrt(x)/b^2
```

Mupad [B]
time = 4.51, size = 80, normalized size = 0.35

$$\frac{2\sqrt{x}}{b^2}-\frac{5\,{(-a)}^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{4\,b^{9/4}}+\frac{a\,\sqrt{x}}{2\,\left(b^3\,x^2+a\,b^2\right)}+\frac{{(-a)}^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{(-a)}^{1/4}}\right)\,5{}\mathrm{i}}{4\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 - (5*(-a)^(1/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*b^(9
/4)) + ((-a)^(1/4)*atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*5i)/(4*b^(9/4)) +
(a*x^(1/2))/(2*(a*b^2 + b^3*x^2))
```

3.297 $\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{x^{3/2}}{2b(a+bx^2)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}$$

```
[Out] -1/2*x^(3/2)/b/(b*x^2+a)-3/8*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1
/4)/b^(7/4)*2^(1/2)+3/8*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b
^(7/4)*2^(1/2)+3/16*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(1/4)/b^(7/4)*2^(1/2)-3/16*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^
(1/2))/a^(1/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{x^{3/2}}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*x^(3/2)/(b*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = -\frac{x^{3/2}}{2b(a+bx^2)} + \frac{3\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4b}$$
$$= -\frac{x^{3/2}}{2b(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$
$$= -\frac{x^{3/2}}{2b(a+bx^2)} - \frac{3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}}$$
$$= -\frac{x^{3/2}}{2b(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^2} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}\right.}{8b^2}$$
$$= -\frac{x^{3/2}}{2b(a+bx^2)} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}} - \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\right.}}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}$$
$$= -\frac{x^{3/2}}{2b(a+bx^2)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}} + \frac{3\log\left(\sqrt{\right.}}{}$$

Mathematica [A]
time = 0.25, size = 128, normalized size = 0.59

$$\frac{-\frac{4b^{3/4}x^{3/2}}{a+bx^2} - \frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}} - \frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{a}}}{8b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] ((-4*b^(3/4)*x^(3/2))/(a + b*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)
/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(1/4) - (3*Sqrt[2]*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4))/(8*b^(7/4))
```

Maple [A]
time = 0.05, size = 124, normalized size = 0.57

method	result

derivativedivides	$-\frac{x^{\frac{3}{2}}}{2b(b\,x^{2}+a)}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.}{16b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$-\frac{x^{\frac{3}{2}}}{2b(b\,x^{2}+a)}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.}{16b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(3/2)/b/(b*x^2+a)+3/16/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1
))
```

Maxima [A]
time = 0.52, size = 195, normalized size = 0.89

$$-\frac{x^{\frac{3}{2}}}{2\,(b^{2}x^{2}+ab)}+\frac{3\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x^(3/2)/(b^2*x^2 + a*b) + 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
- sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/
4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/(a^(1/4)*b^(3/4)))/b
```

Fricas [A]
time = 1.27, size = 185, normalized size = 0.85

$$-\frac{12\,(b^{2}x^{2}+ab)\left(-\frac{1}{ab^{7}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-ab^{3}\sqrt{-\frac{1}{ab^{7}}}+x}\,b^{2}\left(-\frac{1}{ab^{7}}\right)^{\frac{1}{4}}-b^{2}\sqrt{x}\,\left(-\frac{1}{ab^{7}}\right)^{\frac{1}{4}}\right)-3\,(b^{2}x^{2}+ab)\left(-\frac{1}{ab^{7}}\right)^{\frac{1}{4}}\log\left(ab^{5}\left(-\frac{1}{ab^{7}}\right)^{\frac{3}{4}}+\sqrt{x}\right)+3\,(b^{2}x^{2}+ab)\left(-\frac{1}{ab^{7}}\right)^{\frac{1}{4}}\log\left(-ab^{5}\left(-\frac{1}{ab^{7}}\right)^{\frac{3}{4}}+\sqrt{x}\right)+4\,x^{\frac{3}{2}}}{8\,(b^{2}x^{2}+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*arctan(sqrt(-a*b^3*sqrt(-1/(a*b
^7)) + x)*b^2*(-1/(a*b^7))^(1/4) - b^2*sqrt(x)*(-1/(a*b^7))^(1/4)) - 3*(b^2
*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(a*b^5*(-1/(a*b^7))^(3/4) + sqrt(x)) + 3*
(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(-a*b^5*(-1/(a*b^7))^(3/4) + sqrt(x))
 + 4*x^(3/2))/(b^2*x^2 + a*b)
```

Sympy [A]
time = 49.48, size = 393, normalized size = 1.80

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{b^{2} \sqrt{x}} & \text{for}\: a = 0 \\ \frac{2 x^{\frac{7}{2}}}{7 a^{2}} & \text{for}\: b = 0 \\ \frac{3 a \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a b^{2} \sqrt[4]{- \frac{a}{b}} + 8 b^{3} x^{2} \sqrt[4]{- \frac{a}{b}}} - \frac{3 a \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a b^{2} \sqrt[4]{- \frac{a}{b}} + 8 b^{3} x^{2} \sqrt[4]{- \frac{a}{b}}} + \frac{6 a \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a b^{2} \sqrt[4]{- \frac{a}{b}} + 8 b^{3} x^{2} \sqrt[4]{- \frac{a}{b}}} - \frac{4 b x^{\frac{3}{2}} \sqrt[4]{- \frac{a}{b}}}{8 a b^{2} \sqrt[4]{- \frac{a}{b}} + 8 b^{3} x^{2} \sqrt[4]{- \frac{a}{b}}} + \frac{3 b x^{2} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a b^{2} \sqrt[4]{- \frac{a}{b}} + 8 b^{3} x^{2} \sqrt[4]{- \frac{a}{b}}} - \frac{3 b x^{2} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a b^{2} \sqrt[4]{- \frac{a}{b}} + 8 b^{3} x^{2} \sqrt[4]{- \frac{a}{b}}} + \frac{6 b x^{2} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a b^{2} \sqrt[4]{- \frac{a}{b}} + 8 b^{3} x^{2} \sqrt[4]{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**2*sqrt(x)), Eq(a, 0))
, (2*x**(7/2)/(7*a**2), Eq(b, 0)), (3*a*log(sqrt(x) - (-a/b)**(1/4))/(8*a*b
**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) - 3*a*log(sqrt(x) + (-a/b)**
(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) + 6*a*atan(sqrt
(x)/(-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) - 4
*b*x**(3/2)*(-a/b)**(1/4)/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/
4)) + 3*b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**
3*x**2*(-a/b)**(1/4)) - 3*b*x**2*log(sqrt(x) + (-a/b)**(1/4))/(8*a*b**2*(-a
/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) + 6*b*x**2*atan(sqrt(x)/(-a/b)**(1/
4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)), True))
```

Giac [A]
time = 1.09, size = 199, normalized size = 0.91

$$-\frac{x^{\frac{3}{2}}}{2 \, {\left(b x^{2} + a\right)} b} + \frac{3 \, \sqrt{2} \left(a b^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \, \sqrt{x}\right)}}{2 \, \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 \, a b^{4}} + \frac{3 \, \sqrt{2} \left(a b^{3}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \, \sqrt{x}\right)}}{2 \, \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 \, a b^{4}} - \frac{3 \, \sqrt{2} \left(a b^{3}\right)^{\frac{3}{4}} \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16 \, a b^{4}} + \frac{3 \, \sqrt{2} \left(a b^{3}\right)^{\frac{3}{4}} \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16 \, a b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((b*x^2 + a)*b) + 3/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 3/8*sqrt(2)*(a*b^
3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(a*b^4) - 3/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a*b^4) + 3/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/(a*b^4)
```

Mupad [B]

time = 0.08, size = 64, normalized size = 0.29

$$\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}-\frac{x^{3/2}}{2\,b\,\left(b\,x^2+a\right)}-\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2)^2,x)
```

```
[Out] (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^(7/4)) - x^(3/2)/(2*
b*(a + b*x^2)) - (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^(7
/4))
```

3.298 $\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{\sqrt{x}}{2b(a+bx^2)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\right.}{8\sqrt{2}\,a^{3/4}b^{5/4}}$$

```
[Out] -1/8*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2^(1/2)+1/8*
arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2^(1/2)-1/16*ln(a
^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1
/16*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(5/4)*2
^(1/2)-1/2*x^(1/2)/b/(b*x^2+a)
```

Rubi [A]
time = 0.10, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{\sqrt{x}}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*Sqrt[x]/(b*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
]/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)]/(4*Sqrt[2]*a^(3/4)*b^(5/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = -\frac{\sqrt{x}}{2b(a+bx^2)} + \frac{\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{4b}$$
$$= -\frac{\sqrt{x}}{2b(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$
$$= -\frac{\sqrt{x}}{2b(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{a}\,b} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{a}\,b}$$
$$= -\frac{\sqrt{x}}{2b(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{a}\,b^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+\right.}{8\sqrt{a}\,b^{3/2}}$$
$$= -\frac{\sqrt{x}}{2b(a+bx^2)} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{8\sqrt{2}\,a^{3/4}b^{5/4}}$$
$$= -\frac{\sqrt{x}}{2b(a+bx^2)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{\log\left(\sqrt{a}-\right.}{}$$

Mathematica [A]
time = 0.24, size = 127, normalized size = 0.58

$$\frac{-\frac{4\sqrt[4]{b}\sqrt{x}}{a+bx^2} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}}}{8b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((-4*b^(1/4)*Sqrt[x])/(a + b*x^2) - (Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4))/(8*b^(5/4))
```

Maple [A]
time = 0.05, size = 127, normalized size = 0.58

method	result

derivativedivides	$-\frac{\sqrt{x}}{2b(b\,x^2+a)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{16ba}$
default	$-\frac{\sqrt{x}}{2b(b\,x^2+a)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{16ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(1/2)/b/(b*x^2+a)+1/16/b*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1
))
```

Maxima [A]
time = 0.62, size = 195, normalized size = 0.89

$$\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,b}-\frac{\sqrt{x}}{2\,(b^2x^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b - 1
/2*sqrt(x)/(b^2*x^2 + a*b)
```

Fricas [A]
time = 1.24, size = 187, normalized size = 0.86

$$\frac{4\,(b^2x^2+ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x}\;a^2b^4\left(-\frac{1}{a^3b^5}\right)^{\frac{3}{4}}-a^2b^4\sqrt{x}\left(-\frac{1}{a^3b^5}\right)^{\frac{3}{4}}\right)+(b^2x^2+ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-(b^2x^2+ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\log\left(-ab\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\sqrt{x}}{8\,(b^2x^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*arctan(sqrt(a^2*b^2*sqrt(-1/(a^
3*b^5)) + x)*a^2*b^4*(-1/(a^3*b^5))^(3/4) - a^2*b^4*sqrt(x)*(-1/(a^3*b^5))^
```

```
(3/4)) + (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(a*b*(-1/(a^3*b^5))^(1/4)
+ sqrt(x)) - (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(-a*b*(-1/(a^3*b^5))^(
1/4) + sqrt(x)) - 4*sqrt(x))/(b^2*x^2 + a*b)
```

Sympy [A]
time = 27.07, size = 323, normalized size = 1.48

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{3 b^{2} x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ \frac{2 x^{\frac{5}{2}}}{5 a^{2}} & \text{for}\: b = 0 \\ - \frac{4 a \sqrt{x}}{8 a^{2} b + 8 a b^{2} x^{2}} - \frac{a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{2} b + 8 a b^{2} x^{2}} + \frac{a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{2} b + 8 a b^{2} x^{2}} + \frac{2 a \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a^{2} b + 8 a b^{2} x^{2}} - \frac{b x^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{2} b + 8 a b^{2} x^{2}} + \frac{b x^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{2} b + 8 a b^{2} x^{2}} + \frac{2 b x^{2} \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a^{2} b + 8 a b^{2} x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**2*x**(3/2)), Eq(a,
 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0)), (-4*a*sqrt(x)/(8*a**2*b + 8*a*b**2*x
**2) - a*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b + 8*a*b**2*x*
*2) + a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b + 8*a*b**2*x**
2) + 2*a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b + 8*a*b**2*x**
2) - b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b + 8*a*b**2
*x**2) + b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b + 8*a*
b**2*x**2) + 2*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b +
 8*a*b**2*x**2), True))
```

Giac [A]
time = 0.94, size = 199, normalized size = 0.91

$$\frac{\sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \, \sqrt{x}\right)}{2 \, \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 \, a b^{2}} + \frac{\sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \, \sqrt{x}\right)}{2 \, \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 \, a b^{2}} + \frac{\sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16 \, a b^{2}} - \frac{\sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16 \, a b^{2}} - \frac{\sqrt{x}}{2 \, {\left(b x^{2} + a\right)} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) + 1/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2) - 1/16*sqrt
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
 - 1/2*sqrt(x)/((b*x^2 + a)*b)
```

Mupad [B]
time = 4.65, size = 64, normalized size = 0.29

$$-\frac{\sqrt{x}}{2\,b\,\left(b\,x^2+a\right)}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}-\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2)^2,x)
```

```
[Out] - x^(1/2)/(2*b*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(3
/4)*b^(5/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(3/4)*b^(5/4))
```

3.299 $\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=218

$$\frac{x^{3/2}}{2a\,(a+bx^2)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right.}{8\sqrt{2}\,a^{5/4}b^{3/4}}$$

```
[Out] 1/2*x^(3/2)/a/(b*x^2+a)-1/8*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/
4)/b^(3/4)*2^(1/2)+1/8*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^
(3/4)*2^(1/2)+1/16*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^
(5/4)/b^(3/4)*2^(1/2)-1/16*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(
1/2))/a^(5/4)/b^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{x^{3/2}}{2a\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(
4*Sqrt[2]*a^(5/4)*b^(3/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(5/4)*b^(3/4)) + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{x^{3/2}}{2a(a+bx^2)} + \frac{\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a}$$

$$= \frac{x^{3/2}}{2a(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a}$$

$$= \frac{x^{3/2}}{2a(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a\sqrt{b}}$$

$$= \frac{x^{3/2}}{2a(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{8ab} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\right.}{8ab}$$

$$= \frac{x^{3/2}}{2a(a+bx^2)} + \frac{\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}} - \frac{\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \right.}{8\sqrt{2}a^{5/4}b^{3/4}}$$

$$= \frac{x^{3/2}}{2a(a+bx^2)} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{\log\left(\sqrt{a} - \sqrt{\right.}{8}$$

Mathematica [A]
time = 0.23, size = 128, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{a}x^{3/2}}{a+bx^2} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{b^{3/4}}}{8a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] ((4*a^(1/4)*x^(3/2))/(a + b*x^2) - (Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4))/(8*a^(5/4))
```

Maple [A]
time = 0.05, size = 127, normalized size = 0.58

method	result

derivativedivides	$\frac{x^{\frac{3}{2}}}{2a(b\,x^{2}+a)}+\frac{\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{x^{\frac{3}{2}}}{2a(b\,x^{2}+a)}+\frac{\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)/a/(b*x^2+a)+1/16/a/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
)
```

Maxima [A]
time = 0.54, size = 194, normalized size = 0.89

$$\frac{x^{\frac{3}{2}}}{2\,(abx^2+a^2)}+\frac{\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^(3/2)/(a*b*x^2 + a^2) + 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sq
rt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) -
 sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4
)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/(a^(1/4)*b^(3/4)))/a
```

Fricas [A]
time = 0.87, size = 182, normalized size = 0.83

$$-\frac{4\,(abx^2+a^2)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^3b\sqrt{-\frac{1}{a^5b^3}}+x}\;ab\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}-ab\sqrt{x}\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\right)-(abx^2+a^2)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)+(abx^2+a^2)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(-a^4b^2\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4\,x^{\frac{3}{2}}}{8\,(abx^2+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*arctan(sqrt(-a^3*b*sqrt(-1/(a^
5*b^3)) + x)*a*b*(-1/(a^5*b^3))^(1/4) - a*b*sqrt(x)*(-1/(a^5*b^3))^(1/4)) -
```

```
(a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(a^4*b^2*(-1/(a^5*b^3))^(3/4) + sq
rt(x)) + (a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-1/(a^5*b^3))^(
3/4) + sqrt(x)) - 4*x^(3/2))/(a*b*x^2 + a^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(197) = 394$.
time = 16.59, size = 400, normalized size = 1.83

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5b^2x^{\frac{5}{2}}} & \text{for } a = 0 \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for } b = 0 \\ \frac{a\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}} + 8ab^2x^2\sqrt[4]{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}} + 8ab^2x^2\sqrt[4]{-\frac{a}{b}}} + \frac{2a\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}} + 8ab^2x^2\sqrt[4]{-\frac{a}{b}}} + \frac{4bx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}}{8a^2b\sqrt[4]{-\frac{a}{b}} + 8ab^2x^2\sqrt[4]{-\frac{a}{b}}} + \frac{bx^2\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}} + 8ab^2x^2\sqrt[4]{-\frac{a}{b}}} - \frac{bx^2\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}} + 8ab^2x^2\sqrt[4]{-\frac{a}{b}}} + \frac{2bx^2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}} + 8ab^2x^2\sqrt[4]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a,
 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0)), (a*log(sqrt(x) - (-a/b)**(1/4))/(8*a
**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) - a*log(sqrt(x) + (-a/b)
**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) + 2*a*atan(
sqrt(x)/(-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4
)) + 4*b*x**(3/2)*(-a/b)**(1/4)/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a
/b)**(1/4)) + b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) +
 8*a*b**2*x**2*(-a/b)**(1/4)) - b*x**2*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2
*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) + 2*b*x**2*atan(sqrt(x)/(-a
/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)), True))
```

Giac [A]
time = 0.87, size = 199, normalized size = 0.91

$$\frac{x^{\frac{3}{2}}}{2\,(bx^2+a)a} + \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3} + \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3} - \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((b*x^2 + a)*a) + 1/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/8*sqrt(2)*(a*b
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a^2*b^3) - 1/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(a^2*b^3) + 1/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

Mupad [B]
time = 0.09, size = 64, normalized size = 0.29

$$\frac{x^{3/2}}{2\,a\,\left(b\,x^2+a\right)}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{5/4}\,b^{3/4}}+\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{5/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2)^2,x)
```

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/4
)*b^(3/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/4)*b^(3/4))
```

3.300 $\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx$

Optimal. Leaf size=218

$$\frac{\sqrt{x}}{2a\,(a+bx^2)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\right.}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}$$

```
[Out] -3/8*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(1/4)*2^(1/2)+3/8*
arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(1/4)*2^(1/2)-3/16*ln(a
^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(1/4)*2^(1/2)+3
/16*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(1/4)*2
^(1/2)+1/2*x^(1/2)/a/(b*x^2+a)
```

Rubi [A]

time = 0.10, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{\sqrt{x}}{2a\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{\sqrt{x}}{2a\,(a+bx^2)} + \frac{3\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{4a}$$
$$= \frac{\sqrt{x}}{2a\,(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a}$$
$$= \frac{\sqrt{x}}{2a\,(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{3/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{\right.}}{4a^{3/2}}$$
$$= \frac{\sqrt{x}}{2a\,(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{8a^{3/2}\sqrt{b}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}}\right.}{8a^{3/2}}$$
$$= \frac{\sqrt{x}}{2a\,(a+bx^2)} - \frac{3\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{3\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{8\sqrt{2}\,a^{7/4}\sqrt[4]{}}$$
$$= \frac{\sqrt{x}}{2a\,(a+bx^2)} - \frac{3\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{3\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}} - \frac{3\log\left(\right.}{}$$

Mathematica [A]
time = 0.24, size = 128, normalized size = 0.59

$$\frac{\frac{4a^{3/4}\sqrt{x}}{a+bx^2} - \frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{b}}}{8a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((4*a^(3/4)*Sqrt[x])/(a + b*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (3*Sqrt[2]*ArcTanh[(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4))/(8*a^(7/4))
```

Maple [A]
time = 0.05, size = 124, normalized size = 0.57

method	result

derivativedivides	$\frac{\sqrt{x}}{2a(bx^2+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{16a^2}$
default	$\frac{\sqrt{x}}{2a(bx^2+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{16a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)/a/(b*x^2+a)+3/16/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
)
```

Maxima [A]
time = 0.54, size = 194, normalized size = 0.89

$$\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{16a}+\frac{\sqrt{x}}{2(abx^2+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a + 1
/2*sqrt(x)/(a*b*x^2 + a^2)
```

Fricas [A]
time = 1.36, size = 179, normalized size = 0.82

$$\frac{12(abx^2+a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4\sqrt{-\frac{1}{a^7b}}+x}\,a^5b\left(-\frac{1}{a^7b}\right)^{\frac{3}{4}}-a^5b\sqrt{x}\left(-\frac{1}{a^7b}\right)^{\frac{3}{4}}\right)+3(abx^2+a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3(abx^2+a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(-a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\sqrt{x}}{8(abx^2+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(12*(a*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*arctan(sqrt(a^4*sqrt(-1/(a^7*b))
 + x)*a^5*b*(-1/(a^7*b))^(3/4) - a^5*b*sqrt(x)*(-1/(a^7*b))^(3/4)) + 3*(a*b
```

```
*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(a^2*(-1/(a^7*b))^(1/4) + sqrt(x)) - 3*(a
*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(-a^2*(-1/(a^7*b))^(1/4) + sqrt(x)) + 4
*sqrt(x))/(a*b*x^2 + a^2)
```

Sympy [A]
time = 23.73, size = 316, normalized size = 1.45

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{7 b^{2} x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ \frac{2 \sqrt{x}}{a^{2}} & \text{for}\: b = 0 \\ \frac{4 a \sqrt{x}}{8 a^{3} + 8 a^{2} b x^{2}} - \frac{3 a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} + \frac{3 a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} + \frac{6 a \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} - \frac{3 b x^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} + \frac{3 b x^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} + \frac{6 b x^{2} \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (2*sqrt(x)/a**2, Eq(b, 0)), (4*a*sqrt(x)/(8*a**3 + 8*a**2*b*x**2) - 3
*a*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) + 3*
a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) + 6*a
*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) - 3*b*x
**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) + 3
*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2)
 + 6*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x*
*2), True))
```

Giac [A]
time = 1.73, size = 199, normalized size = 0.91

$$\frac{3 \sqrt{2} \left(ab^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 a^{2} b} + \frac{3 \sqrt{2} \left(ab^{3}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 a^{2} b} + \frac{3 \sqrt{2} \left(ab^{3}\right)^{\frac{1}{4}} \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16 a^{2} b} - \frac{3 \sqrt{2} \left(ab^{3}\right)^{\frac{1}{4}} \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16 a^{2} b} + \frac{\sqrt{x}}{2 \left(bx^{2} + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a^2*b) + 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 3/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 3/16*sqrt
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b)
 + 1/2*sqrt(x)/((b*x^2 + a)*a)
```

Mupad [B]
time = 0.09, size = 64, normalized size = 0.29

$$\frac{\sqrt{x}}{2\,a\,\left(b\,x^2+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{7/4}\,b^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{7/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^2),x)
```

```
[Out] x^(1/2)/(2*a*(a + b*x^2)) + (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^
(7/4)*b^(1/4)) + (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(7/4)*b^(1
/4))
```

3.301 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx^2)}+\frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}}-\frac{5\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}}-\frac{5\sqrt[4]{b}\,\log}{}$$

```
[Out] 5/8*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)*2^(1/2)-5/8*b
^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)*2^(1/2)-5/16*b^(1/
4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)*2^(1/2)+5/
16*b^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)*2^
(1/2)-5/2/a^2/x^(1/2)+1/2/a/(b*x^2+a)/x^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{5\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}}-\frac{5\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{9/4}}-\frac{5\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}}-\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -5/(2*a^2*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x^2)) + (5*b^(1/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)
) + (5*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{1}{2a\sqrt{x}\,\left(a+bx^2\right)}+\frac{5\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{4a}$$

$$= -\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\,\left(a+bx^2\right)}-\frac{(5b)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^2}$$

$$= -\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\,\left(a+bx^2\right)}-\frac{(5b)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a^2}$$

$$= -\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\,\left(a+bx^2\right)}+\frac{\left(5\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^2}-\frac{\left(5\sqrt{b}\right.}{}$$

$$= -\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\,\left(a+bx^2\right)}-\frac{5\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^2}-\frac{5\mathrm{Subs}}{}$$

$$= -\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\,\left(a+bx^2\right)}-\frac{5\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{b}}{}$$

$$= -\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\,\left(a+bx^2\right)}+\frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}}-\frac{5\sqrt[4]{b}\,\tan^{-1}\left(1+\right.}{4\sqrt{2}}$$

Mathematica [A]
time = 0.28, size = 138, normalized size = 0.60

$$\frac{-\frac{4\sqrt[4]{a}\,(4a+5bx^2)}{\sqrt{x}\,(a+bx^2)}+5\sqrt{2}\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+5\sqrt{2}\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{8a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2),x]
[Out] ((-4*a^(1/4)*(4*a + 5*b*x^2))/(Sqrt[x]*(a + b*x^2)) + 5*Sqrt[2]*b^(1/4)*Arc
Tan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^
```

```
(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(8*
a^(9/4))
```

Maple [A]
time = 0.08, size = 136, normalized size = 0.59

method	result
derivativedivides	$-\frac{2b\left(\frac{x^{\frac{3}{2}}}{4b\,x^{2}+4a}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}}$
default	$-\frac{2b\left(\frac{x^{\frac{3}{2}}}{4b\,x^{2}+4a}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}}$
risch	$-\frac{2}{a^{2}\sqrt{x}}-\frac{b\,x^{\frac{3}{2}}}{2a^{2}\left(b\,x^{2}+a\right)}-\frac{5\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{16a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/a^2*(1/4*x^(3/2)/(b*x^2+a)+5/32/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1)))-2/a^2/x^(1/2)
```

Maxima [A]
time = 0.68, size = 208, normalized size = 0.90

$$-\frac{5\,bx^{2}+4\,a}{2\left(a^{2}bx^{\frac{5}{2}}+a^{3}\sqrt{x}\right)}-\frac{5\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(5*b*x^2 + 4*a)/(a^2*b*x^(5/2) + a^3*sqrt(x)) - 5/16*b*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
```

```
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

Fricas [A]
time = 1.10, size = 208, normalized size = 0.90

$$\frac{20\left(a^{2}bx^{3}+a^{3}x\right)\left(-\frac{b}{a^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,a^{2}b\sqrt{x}\left(-\frac{b}{a^{9}}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^{5}b\sqrt{-\frac{b}{a^{9}}}+15625\,b^{2}x}\,a^{2}\left(-\frac{b}{a^{9}}\right)^{\frac{1}{4}}}{125\,b}\right)-5\left(a^{2}bx^{3}+a^{3}x\right)\left(-\frac{b}{a^{9}}\right)^{\frac{1}{4}}\log\left(125\,a^{7}\left(-\frac{b}{a^{9}}\right)^{\frac{3}{4}}+125\,b\sqrt{x}\right)+5\left(a^{2}bx^{3}+a^{3}x\right)\left(-\frac{b}{a^{9}}\right)^{\frac{1}{4}}\log\left(-125\,a^{7}\left(-\frac{b}{a^{9}}\right)^{\frac{3}{4}}+125\,b\sqrt{x}\right)-4\left(5\,bx^{2}+4\,a\right)\sqrt{x}}{8\left(a^{2}bx^{3}+a^{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(20*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*arctan(-1/125*(125*a^2*b*sqrt(x)
*(-b/a^9)^(1/4) - sqrt(-15625*a^5*b*sqrt(-b/a^9) + 15625*b^2*x)*a^2*(-b/a^9
)^(1/4))/b) - 5*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*log(125*a^7*(-b/a^9)^(3/
4) + 125*b*sqrt(x)) + 5*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*log(-125*a^7*(-b
/a^9)^(3/4) + 125*b*sqrt(x)) - 4*(5*b*x^2 + 4*a)*sqrt(x))/(a^2*b*x^3 + a^3*
x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(218) = 436.
time = 49.02, size = 512, normalized size = 2.23

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{9 b^{2} x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{a^{2} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{5 a \sqrt{x} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} + \frac{5 a \sqrt{x} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{10 a \sqrt{x} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{16 a \sqrt[4]{- \frac{a}{b}}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{5 b x^{\frac{5}{2}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} + \frac{5 b x^{\frac{5}{2}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{10 b x^{\frac{5}{2}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{20 b x^{2} \sqrt[4]{- \frac{a}{b}}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b**2*x**(9/2)), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-5*a*sqrt(x)*log(sqrt(x) - (-a/b)**(1
/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) + 5*a
*sqrt(x)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**
2*b*x**(5/2)*(-a/b)**(1/4)) - 10*a*sqrt(x)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a
**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 16*a*(-a/b)*
*(1/4)/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 5
*b*x**(5/2)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*
a**2*b*x**(5/2)*(-a/b)**(1/4)) + 5*b*x**(5/2)*log(sqrt(x) + (-a/b)**(1/4))/
(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 10*b*x**
(5/2)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*
x**(5/2)*(-a/b)**(1/4)) - 20*b*x**2*(-a/b)**(1/4)/(8*a**3*sqrt(x)*(-a/b)**(
1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)), True))
```

Giac [A]
time = 1.69, size = 210, normalized size = 0.91

$$-\frac{5\,bx^2+4\,a}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^2}-\frac{5\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^2}-\frac{5\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^2}+\frac{5\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^2}-\frac{5\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b*x^2 + 4*a)/((b*x^(5/2) + a*sqrt(x))*a^2) - 5/8*sqrt(2)*(a*b^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*
b^2) - 5/8*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 5/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 5/16*sqrt(2)*(a*b^3)^(3/4
)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2)
```

Mupad [B]
time = 0.08, size = 77, normalized size = 0.33

$$\frac{5\,{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{9/4}}-\frac{5\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{9/4}}-\frac{\frac{2}{a}+\frac{5\,b\,x^2}{2\,a^2}}{a\,\sqrt{x}+b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^2),x)
```

```
[Out] (5*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(9/4)) - (5*(-b)^(1
/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(9/4)) - (2/a + (5*b*x^2)/(2*a
^2))/(a*x^(1/2) + b*x^(5/2))
```

3.302 $\int \frac{1}{x^{5/2}(a+bx^2)^2} dx$

Optimal. Leaf size=230

$$-\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx^2)}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}+\frac{7b^{3/4}\log\left(\cdots\right.}{}$$

```
[Out] -7/6/a^2/x^(3/2)+1/2/a/x^(3/2)/(b*x^2+a)+7/8*b^(3/4)*arctan(1-b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/a^(11/4)*2^(1/2)-7/8*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x
^(1/2)/a^(1/4))/a^(11/4)*2^(1/2)+7/16*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)*2^(1/2)-7/16*b^(3/4)*ln(a^(1/2)+x*b^(1/2)
+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{7b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}}+\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}-\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -7/(6*a^2*x^(3/2)) + 1/(2*a*x^(3/2)*(a + b*x^2)) + (7*b^(3/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) - (7*b^(3/4)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) + (7*b^(3/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11
/4)) - (7*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx^2)^2}\,dx = \frac{1}{2ax^{3/2}(a+bx^2)} + \frac{7\int \frac{1}{x^{5/2}(a+bx^2)}\,dx}{4a}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}(a+bx^2)} - \frac{(7b)\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{4a^2}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}(a+bx^2)} - \frac{(7b)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^2}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}(a+bx^2)} - \frac{(7b)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{5/2}} - \frac{(7b)\text{Subst}\left(\right.}{}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}(a+bx^2)} - \frac{\left(7\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^{5/2}} -$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}(a+bx^2)} + \frac{7b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{11/4}} - \frac{7b^{3/4}}{}$$

$$= -\frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}(a+bx^2)} + \frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}} - \frac{7b^{3/4}\tan^{-1}\left(1+\right.}{4\sqrt{2}\,a}$$

Mathematica [A]
time = 0.28, size = 138, normalized size = 0.60

$$\frac{-\frac{4a^{3/4}(4a+7bx^2)}{x^{3/2}(a+bx^2)} + 21\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 21\sqrt{2}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{24a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-4*a^(3/4)*(4*a + 7*b*x^2))/(x^(3/2)*(a + b*x^2)) + 21*Sqrt[2]*b^(3/4)*Ar
cTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 21*Sqrt[2]*
b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(
24*a^(11/4))
```

Maple [A]
time = 0.08, size = 136, normalized size = 0.59

method	result
derivativedivides	$-\frac{2b\left(\frac{\sqrt{x}}{4b\,x^{2}+4a}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.\right.}{32a}\right)}{a^{2}}$
default	$-\frac{2b\left(\frac{\sqrt{x}}{4b\,x^{2}+4a}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.\right.}{32a}\right)}{a^{2}}$
risch	$-\frac{2}{3a^{2}x^{\frac{3}{2}}}-\frac{b\sqrt{x}}{2a^{2}\left(b\,x^{2}+a\right)}-\frac{7b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{16a^{3}}-\frac{7b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{8a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/a^2*(1/4*x^(1/2)/(b*x^2+a)+7/32*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1)))-2/3/a^2/x^(3/2)
```

Maxima [A]
time = 0.52, size = 209, normalized size = 0.91

$$-\frac{7\,bx^2+4\,a}{6\left(a^2bx^{\frac{7}{2}}+a^3x^{\frac{3}{2}}\right)}-\frac{7\left(\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(7*b*x^2 + 4*a)/(a^2*b*x^(7/2) + a^3*x^(3/2)) - 7/16*(2*sqrt(2)*b*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/a^2
```

Fricas [A]
time = 1.46, size = 228, normalized size = 0.99

$$-\frac{84\,(a^2bx^4+a^3x^2)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^8b\sqrt{x}\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{4}}-\sqrt{a^6\sqrt{-\frac{b^3}{a^{11}}}+b^2x}\,a^8\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{4}}}{b^3}\right)+21\,(a^2bx^4+a^3x^2)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(7\,a^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}+7\,b\sqrt{x}\right)-21\,(a^2bx^4+a^3x^2)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(-7\,a^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}+7\,b\sqrt{x}\right)+4\,(7\,bx^2+4\,a)\sqrt{x}}{24\,(a^2bx^4+a^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(84*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*arctan(-(a^8*b*sqrt(x)*(-
b^3/a^11)^(3/4) - sqrt(a^6*sqrt(-b^3/a^11) + b^2*x)*a^8*(-b^3/a^11)^(3/4))/
b^3) + 21*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(7*a^3*(-b^3/a^11)^(1/
4) + 7*b*sqrt(x)) - 21*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(-7*a^3*(
-b^3/a^11)^(1/4) + 7*b*sqrt(x)) + 4*(7*b*x^2 + 4*a)*sqrt(x))/(a^2*b*x^4 + a
^3*x^2)
```

Sympy [A]
time = 108.93, size = 425, normalized size = 1.85

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{11b^2x^{\frac{11}{2}}} & \text{for}\: a = 0 \\ -\frac{2}{3a^2x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{16a^2}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}+\frac{21abx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}-\frac{21abx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}-\frac{42abx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}-\frac{28abx^2}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}+\frac{21b^2x^{\frac{7}{2}}\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}-\frac{21b^2x^{\frac{7}{2}}\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}-\frac{42b^2x^{\frac{7}{2}}\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(11/2), Eq(a, 0) & Eq(b, 0)), (-2/(11*b**2*x**(11/2)), Eq
(a, 0)), (-2/(3*a**2*x**(3/2)), Eq(b, 0)), (-16*a**2/(24*a**4*x**(3/2) + 24
*a**3*b*x**(7/2)) + 21*a*b*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/
4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 21*a*b*x**(3/2)*(-a/b)**(1/4)
*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 42*
a*b*x**(3/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*x**(3/2) +
24*a**3*b*x**(7/2)) - 28*a*b*x**2/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) +
 21*b**2*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24*a**4*x**(3
/2) + 24*a**3*b*x**(7/2)) - 21*b**2*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) + (-
a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 42*b**2*x**(7/2)*(-a
/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/
2)), True))
```

Giac [A]
time = 1.17, size = 196, normalized size = 0.85

$$-\frac{7\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3}-\frac{7\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3}-\frac{7\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3}+\frac{7\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3}-\frac{b\sqrt{x}}{2\,(bx^2+a)a^2}-\frac{2}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/a^3 - 7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^3 - 7/16*sqrt(2)*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 + 7/16*sqrt(2)*(a*b^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 - 1/2*b*sqrt(x
)/((b*x^2 + a)*a^2) - 2/3/(a^2*x^(3/2))
```

Mupad [B]
time = 4.68, size = 77, normalized size = 0.33

$$\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{11/4}}-\frac{\frac{2}{3\,a}+\frac{7\,b\,x^2}{6\,a^2}}{a\,x^{3/2}+b\,x^{7/2}}+\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^2),x)
```

```
[Out] (7*(-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(11/4)) - (2/(3*a) +
 (7*b*x^2)/(6*a^2))/(a*x^(3/2) + b*x^(7/2)) + (7*(-b)^(3/4)*atanh(((-b)^(1/
4)*x^(1/2))/a^(1/4)))/(4*a^(11/4))
```

3.303 $\int \frac{1}{x^{7/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=243

$$-\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}(a+bx^2)}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}+\frac{9b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}+$$

```
[Out] -9/10/a^2/x^(5/2)+1/2/a/x^(5/2)/(b*x^2+a)-9/8*b^(5/4)*arctan(1-b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)+9/8*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*
x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)+9/16*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)
*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)-9/16*b^(5/4)*ln(a^(1/2)+x*b^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)+9/2*b/a^3/x^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{9b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}+\frac{9b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{13/4}}+\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}+\frac{9b}{2a^3\sqrt{x}}-\frac{9}{10a^2x^{5/2}}+\frac{1}{2ax^{5/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -9/(10*a^2*x^(5/2)) + (9*b)/(2*a^3*Sqrt[x]) + 1/(2*a*x^(5/2)*(a + b*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]
*a^(13/4)) + (9*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(8*Sqrt[2]*a^(13/4)) - (9*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx &= \frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{9\int \frac{1}{x^{7/2}(a+bx^2)}\,dx}{4a}\\
&= -\frac{9}{10a^2x^{5/2}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}-\frac{(9b)\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{4a^2}\\
&= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{\left(9b^2\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^3}\\
&= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{\left(9b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a^3}\\
&= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}-\frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^3}\\
&= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{(9b)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^3}\\
&= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{9b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\right.}{8\sqrt{2}\,a^{13/4}}\\
&= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}+\frac{9b^{5/4}}{}
\end{aligned}$$

Mathematica [A]
time = 0.31, size = 149, normalized size = 0.61

$$\frac{\frac{4\sqrt[4]{a}\left(-4a^2+36abx^2+45b^2x^4\right)}{x^{5/2}(a+bx^2)}-45\sqrt{2}\,b^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-45\sqrt{2}\,b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{40a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2),x]
```

[Out] ((4*a^(1/4)*(-4*a^2 + 36*a*b*x^2 + 45*b^2*x^4))/(x^(5/2)*(a + b*x^2)) - 45*
Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x])] - 45*Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a
] + Sqrt[b]*x)])/(40*a^(13/4))

Maple [A]
time = 0.09, size = 147, normalized size = 0.60

method	result
derivativedivides	$\frac{2b^{2}\left(\frac{x^{\frac{3}{2}}}{4b\,x^{2}+4a}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}}$
default	$\frac{2b^{2}\left(\frac{x^{\frac{3}{2}}}{4b\,x^{2}+4a}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}}$
risch	$-\frac{2\left(-10b\,x^{2}+a\right)}{5a^{3}x^{\frac{5}{2}}}+\frac{b^{2}x^{\frac{3}{2}}}{2a^{3}\left(b\,x^{2}+a\right)}+\frac{9b\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{16a^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{9b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] 2*b^2/a^3*(1/4*x^(3/2)/(b*x^2+a)+9/32/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)-1)))-2/5/a^2/x^(5/2)+4*b/a^3/x^(1/2)

Maxima [A]
time = 0.57, size = 221, normalized size = 0.91

$$\frac{45\,b^{2}x^{4}+36\,abx^{2}-4\,a^{2}}{10\left(a^{3}bx^{\frac{9}{2}}+a^{4}x^{\frac{5}{2}}\right)}+\frac{9\,b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/10*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)/(a^3*b*x^(9/2) + a^4*x^(5/2)) + 9/16
b^2(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr

```
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3
```

Fricas [A]
time = 1.72, size = 251, normalized size = 1.03

$$-\frac{180\,(a^3bx^5+a^4x^3)\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\,a^3b^4\sqrt{x}\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}-\sqrt{-531441\,a^7b^5\sqrt{-\frac{b^5}{a^{13}}}+531441\,b^8x}\,a^3\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}}{729\,b^5}\right)-45\,(a^3bx^5+a^4x^3)\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}\log\left(729\,a^{10}\left(-\frac{b^5}{a^{13}}\right)^{\frac{3}{4}}+729\,b^4\sqrt{x}\right)+45\,(a^3bx^5+a^4x^3)\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}\log\left(-729\,a^{10}\left(-\frac{b^5}{a^{13}}\right)^{\frac{3}{4}}+729\,b^4\sqrt{x}\right)-4\,(45\,b^2x^4+36\,abx^2-4\,a^2)\sqrt{x}}{40\,(a^3bx^5+a^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(180*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)^(1/4)*arctan(-1/729*(729*a^3*b
^4*sqrt(x)*(-b^5/a^13)^(1/4) - sqrt(-531441*a^7*b^5*sqrt(-b^5/a^13) + 53144
1*b^8*x)*a^3*(-b^5/a^13)^(1/4))/b^5) - 45*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)
^(1/4)*log(729*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sqrt(x)) + 45*(a^3*b*x^5 +
a^4*x^3)*(-b^5/a^13)^(1/4)*log(-729*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sqrt(x
)) - 4*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)*sqrt(x))/(a^3*b*x^5 + a^4*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.05, size = 220, normalized size = 0.91

$$\frac{b^2x^{\frac{3}{2}}}{2\,(bx^2+a)a^3}+\frac{9\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b}+\frac{9\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b}-\frac{9\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b}+\frac{9\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b}+\frac{2\,(10\,bx^2-a)}{5\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^(3/2)/((b*x^2 + a)*a^3) + 9/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 9/8*sqrt(2)*
(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(
1/4))/(a^4*b) - 9/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(a^4*b) + 9/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 2/5*(10*b*x^2 - a)/(a^3*x^(5/2))
```

Mupad [B]
time = 4.69, size = 87, normalized size = 0.36

$$\frac{\frac{18\,b\,x^2}{5\,a^2}-\frac{2}{5\,a}+\frac{9\,b^2\,x^4}{2\,a^3}}{a\,x^{5/2}+b\,x^{9/2}}-\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{13/4}}+\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^2),x)
[Out] ((18*b*x^2)/(5*a^2) - 2/(5*a) + (9*b^2*x^4)/(2*a^3))/(a*x^(5/2) + b*x^(9/2)
) - (9*(-b)^(5/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(13/4)) + (9*(-b
)^(5/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(13/4))
```

3.304 $\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}-\frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{5\log\left(\sqrt{a}-\cdots\right.}{64\cdots}$$

```
[Out] -1/4*x^(5/2)/b/(b*x^2+a)^2-5/64*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a
^(3/4)/b^(9/4)*2^(1/2)+5/64*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/
4)/b^(9/4)*2^(1/2)-5/128*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/
2))/a^(3/4)/b^(9/4)*2^(1/2)+5/128*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1
/2)*x^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)-5/16*x^(1/2)/b^2/(b*x^2+a)
```

Rubi [A]
time = 0.12, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{5\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)}-\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2)^3,x]
```

```
[Out] -1/4*x^(5/2)/(b*(a + b*x^2)^2) - (5*Sqrt[x])/(16*b^2*(a + b*x^2)) - (5*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)) + (
5*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4
)) - (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqr
t[2]*a^(3/4)*b^(9/4)) + (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx = -\frac{x^{5/2}}{4b(a+bx^2)^2} + \frac{5\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx}{8b}$$
$$= -\frac{x^{5/2}}{4b(a+bx^2)^2} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} + \frac{5\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{32b^2}$$
$$= -\frac{x^{5/2}}{4b(a+bx^2)^2} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} + \frac{5\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$
$$= -\frac{x^{5/2}}{4b(a+bx^2)^2} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} + \frac{5\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32\sqrt{a}\,b^2} + \frac{5\text{Subst}\left(\int \frac{\sqrt{a}}{a}\right.}{32}$$
$$= -\frac{x^{5/2}}{4b(a+bx^2)^2} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} + \frac{5\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{a}\,b^{5/2}} + \frac{5\text{Subst}}{}$$
$$= -\frac{x^{5/2}}{4b(a+bx^2)^2} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} - \frac{5\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}} + \frac{5\log\left(\sqrt{a}\right.}{}$$
$$= -\frac{x^{5/2}}{4b(a+bx^2)^2} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} - \frac{5\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}} + \frac{5\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{32\sqrt{2}\,a^{3/4}b^{9/4}}$$

Mathematica [A]
time = 0.34, size = 138, normalized size = 0.58

$$\frac{-\frac{4\sqrt[4]{b}\sqrt{x}(5a+9bx^2)}{(a+bx^2)^2} - \frac{5\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}} + \frac{5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}}}{64b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2)^3,x]
```

```
[Out] ((-4*b^(1/4)*Sqrt[x]*(5*a + 9*b*x^2))/(a + b*x^2)^2 - (5*Sqrt[2]*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (5*Sqrt[2]
*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4))
/(64*b^(9/4))
```

Maple [A]
time = 0.06, size = 139, normalized size = 0.58

method	result
derivativedivides	$\frac{-\frac{9x^{\frac{5}{2}}}{16b}-\frac{5a\sqrt{x}}{16b^2}}{(b\,x^2+a)^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right.}\right.\right.}{128b^2a}$
default	$\frac{-\frac{9x^{\frac{5}{2}}}{16b}-\frac{5a\sqrt{x}}{16b^2}}{(b\,x^2+a)^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right.}\right.\right.}{128b^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-9/32*x^(5/2)/b-5/32*a*x^(1/2)/b^2)/(b*x^2+a)^2+5/128/b^2*(a/b)^(1/4)/a*
2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 218, normalized size = 0.91

$$-\frac{9\,bx^{\frac{5}{2}}+5\,a\sqrt{x}}{16\,(b^4x^4+2\,ab^3x^2+a^2b^2)}+\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*b*x^(5/2) + 5*a*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 5/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

Fricas [A]
time = 1.66, size = 254, normalized size = 1.06

$$\frac{20\,(b^4x^4+2\,ab^3x^2+a^2b^2)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x}\;a^2b^7\left(-\frac{1}{a^3b^9}\right)^{\frac{3}{4}}-a^2b^7\sqrt{x}\left(-\frac{1}{a^3b^9}\right)^{\frac{3}{4}}\right)+5\,(b^4x^4+2\,ab^3x^2+a^2b^2)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}+\sqrt{x}\right)-5\,(b^4x^4+2\,ab^3x^2+a^2b^2)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\,(9\,bx^2+5\,a)\sqrt{x}}{64\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b^9))^(1/4)*arctan(sqrt
(a^2*b^4*sqrt(-1/(a^3*b^9)) + x)*a^2*b^7*(-1/(a^3*b^9))^(3/4) - a^2*b^7*sqr
t(x)*(-1/(a^3*b^9))^(3/4)) + 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b
^9))^(1/4)*log(a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt(x)) - 5*(b^4*x^4 + 2*a*b^3
*x^2 + a^2*b^2)*(-1/(a^3*b^9))^(1/4)*log(-a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt
(x)) - 4*(9*b*x^2 + 5*a)*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.50, size = 209, normalized size = 0.87

$$\frac{5\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3}+\frac{5\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3}+\frac{5\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^3}-\frac{5\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^3}-\frac{9\,bx^{\frac{5}{2}}+5\,a\sqrt{x}}{16\,(bx^2+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a*b^3) + 5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) + 5/128*sqrt(2)*(a*b
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) - 5/128*
sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*
b^3) - 1/16*(9*b*x^(5/2) + 5*a*sqrt(x))/((b*x^2 + a)^2*b^2)
```

Mupad [**B**]
time = 4.69, size = 87, normalized size = 0.36

$$-\frac{\frac{9\,x^{5/2}}{16\,b}+\frac{5\,a\,\sqrt{x}}{16\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{32\,{(-a)}^{3/4}\,b^{9/4}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{32\,{(-a)}^{3/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2)^3,x)
```

```
[Out] - ((9*x^(5/2))/(16*b) + (5*a*x^(1/2))/(16*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
 - (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(3/4)*b^(9/4)) - (5*atan
h((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(3/4)*b^(9/4))
```

3.305 $\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{3\log\left(\sqrt{a}-\right.}{}$$

```
[Out] -1/4*x^(3/2)/b/(b*x^2+a)^2+3/16*x^(3/2)/a/b/(b*x^2+a)-3/64*arctan(1-b^(1/4)
*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(7/4)*2^(1/2)+3/64*arctan(1+b^(1/4)*2^(
1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(7/4)*2^(1/2)+3/128*ln(a^(1/2)+x*b^(1/2)-a^
(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)-3/128*ln(a^(1/2)+x*b
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}-\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] -1/4*x^(3/2)/(b*(a + b*x^2)^2) + (3*x^(3/2))/(16*a*b*(a + b*x^2)) - (3*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4)) + (
3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4
)) + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqr
t[2]*a^(5/4)*b^(7/4)) - (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = -\frac{x^{3/2}}{4b(a+bx^2)^2} + \frac{3\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx}{8b}$$

$$= -\frac{x^{3/2}}{4b(a+bx^2)^2} + \frac{3x^{3/2}}{16ab(a+bx^2)} + \frac{3\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32ab}$$

$$= -\frac{x^{3/2}}{4b(a+bx^2)^2} + \frac{3x^{3/2}}{16ab(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16ab}$$

$$= -\frac{x^{3/2}}{4b(a+bx^2)^2} + \frac{3x^{3/2}}{16ab(a+bx^2)} - \frac{3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32ab^{3/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{}}{}\right.}{}$$

$$= -\frac{x^{3/2}}{4b(a+bx^2)^2} + \frac{3x^{3/2}}{16ab(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{64ab^2} + \frac{3\text{Subs}}{}$$

$$= -\frac{x^{3/2}}{4b(a+bx^2)^2} + \frac{3x^{3/2}}{16ab(a+bx^2)} + \frac{3\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{7/4}} - \frac{3\log\left(\sqrt{}\right.}{}$$

$$= -\frac{x^{3/2}}{4b(a+bx^2)^2} + \frac{3x^{3/2}}{16ab(a+bx^2)} - \frac{3\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}} + \frac{3\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{32\sqrt{2}\,a^{5/4}b^{7/}}$$

Mathematica [A]
time = 0.34, size = 136, normalized size = 0.56

$$\frac{-\frac{4\sqrt[4]{a}\,b^{3/4}x^{3/2}(a-3bx^2)}{(a+bx^2)^2} - 3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{64a^{5/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] ((-4*a^(1/4)*b^(3/4)*x^(3/2)*(a - 3*b*x^2))/(a + b*x^2)^2 - 3*Sqrt[2]*ArcTa
n[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*ArcT
```

```
anh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(5/4)*b
^(7/4))
```

Maple [A]
time = 0.06, size = 138, normalized size = 0.57

method	result
derivativedivides	$\frac{\frac{3x^{\frac{7}{2}}}{16a}-\frac{x^{\frac{3}{2}}}{16b}}{(bx^2+a)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128b^2a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{3x^{\frac{7}{2}}}{16a}-\frac{x^{\frac{3}{2}}}{16b}}{(bx^2+a)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128b^2a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/32/a*x^(7/2)-1/32*x^(3/2)/b)/(b*x^2+a)^2+3/128/b^2/a/(a/b)^(1/4)*2^(1/
2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2
^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.53, size = 222, normalized size = 0.92

$$\frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*b*x^(7/2) - a*x^(3/2))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b)
```

Fricas [A]
time = 1.97, size = 260, normalized size = 1.07

$$-\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x}\,ab^2\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}-ab^2\sqrt{x}\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\right)-3\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\log\left(a^4b^5\left(-\frac{1}{a^5b^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)+3\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\log\left(-a^4b^5\left(-\frac{1}{a^5b^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4\left(3\,bx^3-ax\right)\sqrt{x}}{64\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*arctan(s
qrt(-a^3*b^3*sqrt(-1/(a^5*b^7)) + x)*a*b^2*(-1/(a^5*b^7))^(1/4) - a*b^2*sqr
t(x)*(-1/(a^5*b^7))^(1/4)) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5
*b^7))^(1/4)*log(a^4*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) + 3*(a*b^3*x^4 + 2
*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-1/(a^5*b^7))^(3/4
) + sqrt(x)) - 4*(3*b*x^3 - a*x)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*
b)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.24, size = 212, normalized size = 0.88

$$\frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\,(bx^2+a)^2ab}+\frac{3\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}+\frac{3\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}-\frac{3\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^4}+\frac{3\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*b*x^(7/2) - a*x^(3/2))/((b*x^2 + a)^2*a*b) + 3/64*sqrt(2)*(a*b^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2
*b^4) + 3/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) - 3/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 3/128*sqrt(2)*(a*b^3)^
(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4)
```

Mupad [**B**]
time = 0.08, size = 85, normalized size = 0.35

$$\frac{\frac{3\,x^{7/2}}{16\,a}-\frac{x^{3/2}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{5/4}\,b^{7/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{5/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2)^3,x)
```

```
[Out] ((3*x^(7/2))/(16*a) - x^(3/2)/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (3*atan
((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4)) + (3*atanh((b^(1/4)
*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4))
```

3.306 $\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{3\log\left(\sqrt{a}-\cdots\right.}{6\cdots}$$

```
[Out] -3/64*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)+3/6
4*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)-3/128*l
n(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(5/4)*2^(1/2
)+3/128*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(5/
4)*2^(1/2)-1/4*x^(1/2)/b/(b*x^2+a)^2+1/16*x^(1/2)/a/b/(b*x^2+a)
```

Rubi [A]

time = 0.11, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] -1/4*Sqrt[x]/(b*(a + b*x^2)^2) + Sqrt[x]/(16*a*b*(a + b*x^2)) - (3*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) + (3*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) -
 (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(7/4)*b^(5/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\int \frac{1}{\sqrt{x}(a+bx^2)^2}\,dx}{8b}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} + \frac{3\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{32ab}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16ab}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^{3/2}b} + \frac{3\text{Subst}\left(\int \frac{\sqrt{a}}{a}\right.}{3}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{64a^{3/2}b^{3/2}} + \frac{3\text{Subst}}{}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} - \frac{3\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}} + \frac{3\log\left(\sqrt{a}\right.}{}$$

$$= -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} - \frac{3\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}} + \frac{3\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{32\sqrt{2}a^{7/4}b^{5/4}}$$

Mathematica [**A**]
time = 0.32, size = 137, normalized size = 0.57

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(-3a+bx^2)}{(a+bx^2)^2} - 3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{64a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-3*a + b*x^2))/(a + b*x^2)^2 - 3*Sqrt[2]*ArcTa
n[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*ArcT
```

```
anh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(7/4)*b
^(5/4))
```

Maple [A]
time = 0.06, size = 138, normalized size = 0.57

method	result
derivativedivides	$\frac{\frac{x^{\frac{5}{2}}}{16a}-\frac{3\sqrt{x}}{16b}}{\left(b\,x^{2}+a\right)^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128b\,a^{2}}$
default	$\frac{\frac{x^{\frac{5}{2}}}{16a}-\frac{3\sqrt{x}}{16b}}{\left(b\,x^{2}+a\right)^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128b\,a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32/a*x^(5/2)-3/32*x^(1/2)/b)/(b*x^2+a)^2+3/128/b/a^2*(a/b)^(1/4)*2^(1/
2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2
^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.49, size = 221, normalized size = 0.91

$$\frac{bx^{\frac{5}{2}}-3\,a\sqrt{x}}{16\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b*x^(5/2) - 3*a*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

Fricas [A]
time = 1.34, size = 257, normalized size = 1.06

$$\frac{12\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)\left(-\frac{1}{a^{7}b^{5}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{4}b^{2}\sqrt{-\frac{1}{a^{7}b^{5}}}+x}\,a^{5}b^{4}\left(-\frac{1}{a^{7}b^{5}}\right)^{\frac{3}{4}}-a^{5}b^{4}\sqrt{x}\left(-\frac{1}{a^{7}b^{5}}\right)^{\frac{3}{4}}\right)+3\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)\left(-\frac{1}{a^{7}b^{5}}\right)^{\frac{1}{4}}\log\left(a^{2}b\left(-\frac{1}{a^{7}b^{5}}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)\left(-\frac{1}{a^{7}b^{5}}\right)^{\frac{1}{4}}\log\left(-a^{2}b\left(-\frac{1}{a^{7}b^{5}}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\left(bx^{2}-3\,a\right)\sqrt{x}}{64\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*arctan(sq
rt(a^4*b^2*sqrt(-1/(a^7*b^5)) + x)*a^5*b^4*(-1/(a^7*b^5))^(3/4) - a^5*b^4*s
qrt(x)*(-1/(a^7*b^5))^(3/4)) + 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a
^7*b^5))^(1/4)*log(a^2*b*(-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3*(a*b^3*x^4 + 2
*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*log(-a^2*b*(-1/(a^7*b^5))^(1/4)
+ sqrt(x)) + 4*(b*x^2 - 3*a)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(223) = 446$.
time = 155.12, size = 666, normalized size = 2.75

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{7 b^{3} x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ \frac{2 x^{\frac{5}{2}}}{5 a^{3}} & \text{for}\: b = 0 \\ - \frac{12 a^{2} \sqrt{x}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} - \frac{3 a^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} + \frac{3 a^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} + \frac{6 a^{2} \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} + \frac{4 a b x^{\frac{5}{2}}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} - \frac{6 a b x^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} + \frac{6 a b x^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} + \frac{12 a b x^{2} \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} - \frac{3 b^{2} x^{4} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} + \frac{3 b^{2} x^{4} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} + \frac{6 b^{2} x^{4} \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{64 a^{4} b + 128 a^{3} b^{2} x^{2} + 64 a^{2} b^{3} x^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**3*x**(7/2)), Eq(a,
 0)), (2*x**(5/2)/(5*a**3), Eq(b, 0)), (-12*a**2*sqrt(x)/(64*a**4*b + 128*a
**3*b**2*x**2 + 64*a**2*b**3*x**4) - 3*a**2*(-a/b)**(1/4)*log(sqrt(x) - (-a
/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 3*a**2*(
-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 +
 64*a**2*b**3*x**4) + 6*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*
a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 4*a*b*x**(5/2)/(64*a**4*
b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) - 6*a*b*x**2*(-a/b)**(1/4)*log(
sqrt(x) - (-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**
4) + 6*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b + 128
*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 12*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(
x)/(-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) - 3*
b**2*x**4*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**4*b + 128*a**3*
b**2*x**2 + 64*a**2*b**3*x**4) + 3*b**2*x**4*(-a/b)**(1/4)*log(sqrt(x) + (-
a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 6*b**2*
x**4*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x
**2 + 64*a**2*b**3*x**4), True))
```

Giac [A]
time = 0.73, size = 211, normalized size = 0.87

$$\frac{3 \sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^{2} b^{2}} + \frac{3 \sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^{2} b^{2}} + \frac{3 \sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^{2} b^{2}} - \frac{3 \sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^{2} b^{2}} + \frac{b x^{\frac{5}{2}} - 3 a \sqrt{x}}{16 \left(b x^{2} + a\right)^{2} a b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

[Out] 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 3/128*sqrt(2)*(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) - 3/128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) + 1/16*(b*x^(5/2) - 3*a*sqrt(x))/((b*x^2 + a)^2*a*b)

Mupad [B]
time = 4.67, size = 85, normalized size = 0.35

$$\frac{\frac{x^{5/2}}{16\,a}-\frac{3\,\sqrt{x}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{7/4}\,b^{5/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{7/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(a + b*x^2)^3,x)

[Out] (x^(5/2)/(16*a) - (3*x^(1/2))/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4)) + (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4))

3.307 $\int \frac{\sqrt{x}}{(a+bx^2)^3} dx$

Optimal. Leaf size=239

$$\frac{x^{3/2}}{4a(a+bx^2)^2}+\frac{5x^{3/2}}{16a^2(a+bx^2)}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{5\log\left(\sqrt{a}-\sqrt{\right.}}{64}$$

```
[Out] 1/4*x^(3/2)/a/(b*x^2+a)^2+5/16*x^(3/2)/a^2/(b*x^2+a)-5/64*arctan(1-b^(1/4)*
2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(3/4)*2^(1/2)+5/64*arctan(1+b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(3/4)*2^(1/2)+5/128*ln(a^(1/2)+x*b^(1/2)-a^(
1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(3/4)*2^(1/2)-5/128*ln(a^(1/2)+x*b^
(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{5\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{5x^{3/2}}{16a^2(a+bx^2)}+\frac{x^{3/2}}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] x^(3/2)/(4*a*(a + b*x^2)^2) + (5*x^(3/2))/(16*a^2*(a + b*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4))
+ (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2
]*a^(9/4)*b^(3/4)) - (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{x^{3/2}}{4a\,(a+bx^2)^2} + \frac{5\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx}{8a}$$

$$= \frac{x^{3/2}}{4a\,(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2\,(a+bx^2)} + \frac{5\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^2}$$

$$= \frac{x^{3/2}}{4a\,(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2\,(a+bx^2)} + \frac{5\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2}$$

$$= \frac{x^{3/2}}{4a\,(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2\,(a+bx^2)} - \frac{5\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^2\sqrt{b}} + \frac{5\text{Subst}\left(\int \frac{\sqrt{a}+ \cdots}{a+\cdots}\right.}{32a\cdots}$$

$$= \frac{x^{3/2}}{4a\,(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2\,(a+bx^2)} + \frac{5\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{64a^2b} + \frac{5\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{x^{3/2}}{4a\,(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2\,(a+bx^2)} + \frac{5\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{3/4}} - \frac{5\log\left(\sqrt{a}\cdots\right.}{\cdots}$$

$$= \frac{x^{3/2}}{4a\,(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2\,(a+bx^2)} - \frac{5\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}} + \frac{5\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}}\right.}{32\sqrt{2}\,a^{9/4}b^{3/4}}$$

Mathematica [A]
time = 0.23, size = 138, normalized size = 0.58

$$\frac{\frac{4\sqrt[4]{a}\,x^{3/2}(9a+5bx^2)}{(a+bx^2)^2} - \frac{5\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}} - \frac{5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{3/4}}}{64a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] ((4*a^(1/4)*x^(3/2)*(9*a + 5*b*x^2))/(a + b*x^2)^2 - (5*Sqrt[2]*ArcTan[(Sqr
t[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (5*Sqrt[2]*
ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4))/
(64*a^(9/4))
```

Maple [A]
time = 0.06, size = 150, normalized size = 0.63

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{4a(b\,x^2+a)^2}+\frac{\frac{5x^{\frac{3}{2}}}{16a\left(b\,x^2+a\right)}+\frac{5\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)}\right.\right.}{128ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a}$
default	$\frac{x^{\frac{3}{2}}}{4a(b\,x^2+a)^2}+\frac{\frac{5x^{\frac{3}{2}}}{16a\left(b\,x^2+a\right)}+\frac{5\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)}\right.\right.}{128ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)/a/(b*x^2+a)^2+5/4/a*(1/4*x^(3/2)/a/(b*x^2+a)+1/32/a/b/(a/b)^(1/
4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.49, size = 217, normalized size = 0.91

$$\frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\,(a^2b^2x^4+2\,a^3bx^2+a^4)}+\frac{5\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 5/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

Fricas [A]

time = 1.85, size = 250, normalized size = 1.05

$$-\frac{20\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^5b\sqrt{-\frac{1}{a^9b^3}}+x}\,a^2b\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}-a^2b\sqrt{x}\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\right)-5\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\log\left(a^7b^2\left(-\frac{1}{a^9b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)+5\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\log\left(-a^7b^2\left(-\frac{1}{a^9b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4\,(5\,bx^3+9\,ax)\sqrt{x}}{64\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(20*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*arctan(sqr
t(-a^5*b*sqrt(-1/(a^9*b^3)) + x)*a^2*b*(-1/(a^9*b^3))^(1/4) - a^2*b*sqrt(x)
*(-1/(a^9*b^3))^(1/4)) - 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))
^(1/4)*log(a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt(x)) + 5*(a^2*b^2*x^4 + 2*a^3
*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*log(-a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt
(x)) - 4*(5*b*x^3 + 9*a*x)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(224) = 448$.

time = 99.92, size = 887, normalized size = 3.71

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{9 b^{3} x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 a^{3}} & \text{for}\: b = 0 \\ \frac{5 a^{2} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} - \frac{5 a^{2} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} + \frac{10 a^{2} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} + \frac{36 a b x^{\frac{3}{2}} \sqrt[4]{- \frac{a}{b}}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} + \frac{10 a b x^{2} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} - \frac{10 a b x^{2} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} + \frac{20 a b x^{2} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} + \frac{20 b^{2} x^{\frac{7}{2}} \sqrt[4]{- \frac{a}{b}}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} + \frac{5 b^{2} x^{4} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} - \frac{5 b^{2} x^{4} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} + \frac{10 b^{2} x^{4} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{64 a^{4} b \sqrt[4]{- \frac{a}{b}} + 128 a^{3} b^{2} x^{2} \sqrt[4]{- \frac{a}{b}} + 64 a^{2} b^{3} x^{4} \sqrt[4]{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b**3*x**(9/2)), Eq(a,
 0)), (2*x**(3/2)/(3*a**3), Eq(b, 0)), (5*a**2*log(sqrt(x) - (-a/b)**(1/4))
/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3
*x**4*(-a/b)**(1/4)) - 5*a**2*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b*(-a/b
)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/
4)) + 10*a**2*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a*
*3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 36*a*b*x**(
3/2)*(-a/b)**(1/4)/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1
/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 10*a*b*x**2*log(sqrt(x) - (-a/b)**
(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**
2*b**3*x**4*(-a/b)**(1/4)) - 10*a*b*x**2*log(sqrt(x) + (-a/b)**(1/4))/(64*a
**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*
(-a/b)**(1/4)) + 20*a*b*x**2*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b*(-a/b)*
*(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)
) + 20*b**2*x**(7/2)*(-a/b)**(1/4)/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2
*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 5*b**2*x**4*log(sq
rt(x) - (-a/b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)
**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) - 5*b**2*x**4*log(sqrt(x) + (-a/
b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64
*a**2*b**3*x**4*(-a/b)**(1/4)) + 10*b**2*x**4*atan(sqrt(x)/(-a/b)**(1/4))/(
64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x
**4*(-a/b)**(1/4)), True))
```

Giac [**A**]
time = 1.21, size = 209, normalized size = 0.87

$$\frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\,(bx^2+a)^2a^2}+\frac{5\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3}+\frac{5\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3}-\frac{5\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^3}+\frac{5\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/((b*x^2 + a)^2*a^2) + 5/64*sqrt(2)*(a*b^3)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a
^3*b^3) + 5/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 5/128*sqrt(2)*(a*b^3)^(3/4)*log(sq
rt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 5/128*sqrt(2)*(a*b^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

Mupad [B]
time = 0.08, size = 86, normalized size = 0.36

$$\frac{\frac{9\,x^{3/2}}{16\,a}+\frac{5\,b\,x^{7/2}}{16\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{9/4}\,b^{3/4}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{9/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2)^3,x)
```

```
[Out] ((9*x^(3/2))/(16*a) + (5*b*x^(7/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) +
 (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4)) - (5*atanh(
(b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4))
```

3.308 $\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx$

Optimal. Leaf size=239

$$\frac{\sqrt{x}}{4a\,(a+bx^2)^2}+\frac{7\sqrt{x}}{16a^2\,(a+bx^2)}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{21\log\left(\sqrt{a}\right. -}{6}$$

```
[Out] -21/64*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)+2
1/64*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)-21/
128*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/b^(1/4)*
2^(1/2)+21/128*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/
4)/b^(1/4)*2^(1/2)+1/4*x^(1/2)/a/(b*x^2+a)^2+7/16*x^(1/2)/a^2/(b*x^2+a)
```

Rubi [A]
time = 0.11, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{21\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{21\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{7\sqrt{x}}{16a^2\,(a+bx^2)}+\frac{\sqrt{x}}{4a\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*a*(a + b*x^2)^2) + (7*Sqrt[x])/(16*a^2*(a + b*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/
4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*S
qrt[2]*a^(11/4)*b^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{\sqrt{x}}{4a\,(a+bx^2)^2} + \frac{7\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx}{8a}$$

$$= \frac{\sqrt{x}}{4a\,(a+bx^2)^2} + \frac{7\sqrt{x}}{16a^2\,(a+bx^2)} + \frac{21\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{32a^2}$$

$$= \frac{\sqrt{x}}{4a\,(a+bx^2)^2} + \frac{7\sqrt{x}}{16a^2\,(a+bx^2)} + \frac{21\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2}$$

$$= \frac{\sqrt{x}}{4a\,(a+bx^2)^2} + \frac{7\sqrt{x}}{16a^2\,(a+bx^2)} + \frac{21\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^{5/2}} + \frac{21\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{x}}{4a\,(a+bx^2)^2} + \frac{7\sqrt{x}}{16a^2\,(a+bx^2)} + \frac{21\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{64a^{5/2}\sqrt{b}} + \frac{21\text{S}}{}$$

$$= \frac{\sqrt{x}}{4a\,(a+bx^2)^2} + \frac{7\sqrt{x}}{16a^2\,(a+bx^2)} - \frac{21\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}} + \frac{21\log}{}$$

$$= \frac{\sqrt{x}}{4a\,(a+bx^2)^2} + \frac{7\sqrt{x}}{16a^2\,(a+bx^2)} - \frac{21\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}} + \frac{21\tan^{-1}\left(1 + \sqrt{2}\right.}{32\sqrt{2}\,a^{11/4}}$$

Mathematica [A]
time = 0.22, size = 138, normalized size = 0.58

$$\frac{\frac{4a^{3/4}\sqrt{x}\,(11a+7bx^2)}{(a+bx^2)^2} - \frac{21\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{21\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{b}}}{64a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((4*a^(3/4)*Sqrt[x]*(11*a + 7*b*x^2))/(a + b*x^2)^2 - (21*Sqrt[2]*ArcTan[(S
qrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])]])/b^(1/4) + (21*Sqrt[
2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4
))/(64*a^(11/4))
```

Maple [A]
time = 0.06, size = 147, normalized size = 0.62

method	result
derivativedivides	$\frac{\sqrt{x}}{4a(bx^2+a)^2}+\frac{\frac{7\sqrt{x}}{16a(bx^2+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128a^2}}{a}$
default	$\frac{\sqrt{x}}{4a(bx^2+a)^2}+\frac{\frac{7\sqrt{x}}{16a(bx^2+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128a^2}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(1/2)/a/(b*x^2+a)^2+7/4/a*(1/4*x^(1/2)/a/(b*x^2+a)+3/32/a^2*(a/b)^(1/
4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.49, size = 217, normalized size = 0.91

$$\frac{7\,bx^{\frac{5}{2}}+11\,a\sqrt{x}}{16\,(a^2b^2x^4+2\,a^3bx^2+a^4)}+\frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 21/12
8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1
/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)
*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^2
```

Fricas [A]

time = 1.60, size = 241, normalized size = 1.01

$$\frac{84\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^6\sqrt{-\frac{1}{a^{11}b}}+x}\,a^8b\left(-\frac{1}{a^{11}b}\right)^{\frac{3}{4}}-a^8b\sqrt{x}\left(-\frac{1}{a^{11}b}\right)^{\frac{3}{4}}\right)+21\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-21\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\log\left(-a^3\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\,(7\,bx^2+11\,a)\sqrt{x}}{64\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(84*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*arctan(sqrt(
a^6*sqrt(-1/(a^11*b)) + x)*a^8*b*(-1/(a^11*b))^(3/4) - a^8*b*sqrt(x)*(-1/(a
^11*b))^(3/4)) + 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*l
og(a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) - 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4
)*(-1/(a^11*b))^(1/4)*log(-a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) + 4*(7*b*x^2
+ 11*a)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(224) = 448$.
time = 140.11, size = 627, normalized size = 2.62

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{11 b^{3} x^{\frac{11}{2}}} & \text{for}\: a = 0 \\ \frac{2 \sqrt{x}}{a^{3}} & \text{for}\: b = 0 \\ \frac{44 a^{2} \sqrt{x}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} - \frac{21 a^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} + \frac{21 a^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} + \frac{42 a^{2} \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} + \frac{28 a b x^{\frac{5}{2}}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} - \frac{42 a b x^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} + \frac{42 a b x^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} + \frac{84 a b x^{2} \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} - \frac{21 b^{2} x^{4} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} + \frac{21 b^{2} x^{4} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} + \frac{42 b^{2} x^{4} \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{64 a^{5} + 128 a^{4} b x^{2} + 64 a^{3} b^{2} x^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(11/2), Eq(a, 0) & Eq(b, 0)), (-2/(11*b**3*x**(11/2)), Eq
(a, 0)), (2*sqrt(x)/a**3, Eq(b, 0)), (44*a**2*sqrt(x)/(64*a**5 + 128*a**4*b
*x**2 + 64*a**3*b**2*x**4) - 21*a**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1
/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4) + 21*a**2*(-a/b)**(1/4
)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x*
*4) + 42*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**5 + 128*a**4
*b*x**2 + 64*a**3*b**2*x**4) + 28*a*b*x**(5/2)/(64*a**5 + 128*a**4*b*x**2 +
 64*a**3*b**2*x**4) - 42*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4)
)/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4) + 42*a*b*x**2*(-a/b)**(1/
4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x
**4) + 84*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**5 + 128
*a**4*b*x**2 + 64*a**3*b**2*x**4) - 21*b**2*x**4*(-a/b)**(1/4)*log(sqrt(x)
- (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4) + 21*b**2*
x**4*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2
+ 64*a**3*b**2*x**4) + 42*b**2*x**4*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4
))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4), True))
```

Giac [A]
time = 0.74, size = 209, normalized size = 0.87

$$\frac{21 \sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^{3} b} + \frac{21 \sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^{3} b} + \frac{21 \sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^{3} b} - \frac{21 \sqrt{2} \left(a b^{3}\right)^{\frac{1}{4}} \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^{3} b} + \frac{7 b x^{\frac{5}{2}} + 11 a \sqrt{x}}{16 \left(b x^{2} + a\right)^{2} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/(a^3*b) + 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 21/128*sqrt(2)*(
```

```
a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) - 21/
128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^3*b) + 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/((b*x^2 + a)^2*a^2)
```

Mupad [B]
time = 4.67, size = 86, normalized size = 0.36

$$\frac{\frac{11\,\sqrt{x}}{16\,a}+\frac{7\,b\,x^{5/2}}{16\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{21\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{11/4}\,b^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{11/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^3),x)
```

```
[Out] ((11*x^(1/2))/(16*a) + (7*b*x^(5/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
- (21*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4)) - (21*at
anh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4))
```

3.309 $\int \frac{1}{x^{3/2}(a+bx^2)^3}\,dx$

Optimal. Leaf size=251

$$-\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\,(a+bx^2)^2}+\frac{9}{16a^2\sqrt{x}\,(a+bx^2)}+\frac{45\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{13/4}}-\frac{45\sqrt[4]{b}\,\tan^{-1}\left(1+\cdots\right.}{32\sqrt{2}\,a^{1\cdots}}$$

```
[Out] 45/64*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-45
/64*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-45/1
28*b^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2
^(1/2)+45/128*b^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))
/a^(13/4)*2^(1/2)-45/16/a^3/x^(1/2)+1/4/a/(b*x^2+a)^2/x^(1/2)+9/16/a^2/(b*x
^2+a)/x^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{45\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{13/4}}-\frac{45\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{13/4}}-\frac{45\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{13/4}}-\frac{45}{16a^3\sqrt{x}}+\frac{9}{16a^2\sqrt{x}\,(a+bx^2)}+\frac{1}{4a\sqrt{x}\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] -45/(16*a^3*Sqrt[x]) + 1/(4*a*Sqrt[x]*(a + b*x^2)^2) + 9/(16*a^2*Sqrt[x]*(a
 + b*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32
*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)])/(32*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4)) + (45*b^(1/4)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx^2)^3}\,dx = \frac{1}{4a\sqrt{x}\,(a+bx^2)^2} + \frac{9\int \frac{1}{x^{3/2}(a+bx^2)^2}\,dx}{8a}$$

$$= \frac{1}{4a\sqrt{x}\,(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}\,(a+bx^2)} + \frac{45\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{32a^2}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}\,(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}\,(a+bx^2)} - \frac{(45b)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}\,(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}\,(a+bx^2)} - \frac{(45b)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{\cdots}\right.}{16a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}\,(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}\,(a+bx^2)} + \frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\cdots}{a+bx^4}\right.}{32a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}\,(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}\,(a+bx^2)} - \frac{45\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}\cdots}\right.}{64a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}\,(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}\,(a+bx^2)} - \frac{45\sqrt[4]{b}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\cdots\right.}{64\sqrt{2}\,a^{13/4}\cdots}$$

$$= -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}\,(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}\,(a+bx^2)} + \frac{45\sqrt[4]{b}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}}\right.}{32\sqrt{2}\,a^{13/4}}$$

Mathematica [A]
time = 0.37, size = 149, normalized size = 0.59

$$\frac{-\frac{4\sqrt[4]{a}\,(32a^2+81abx^2+45b^2x^4)}{\sqrt{x}\,(a+bx^2)^2} + 45\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 45\sqrt{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{64a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a^(1/4)*(32*a^2 + 81*a*b*x^2 + 45*b^2*x^4))/(Sqrt[x]*(a + b*x^2)^2) +
45*Sqrt[2]*b^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])] + 45*Sqrt[2]*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqr
t[a] + Sqrt[b]*x)])/(64*a^(13/4))
```

Maple [A]

time = 0.11, size = 145, normalized size = 0.58

method	result
derivativedivides	$-\frac{2b\left(\frac{\frac{13b\,x^{\frac{7}{2}}}{32}+\frac{17a\,x^{\frac{3}{2}}}{32}}{\left(b\,x^{2}+a\right)^{2}}+\frac{45\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{256b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^{3}}$
default	$-\frac{2b\left(\frac{\frac{13b\,x^{\frac{7}{2}}}{32}+\frac{17a\,x^{\frac{3}{2}}}{32}}{\left(b\,x^{2}+a\right)^{2}}+\frac{45\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{256b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^{3}}$
risch	$-\frac{2}{a^{3}\sqrt{x}}-\frac{13b^{2}x^{\frac{7}{2}}}{16a^{3}\left(b\,x^{2}+a\right)^{2}}-\frac{17b\,x^{\frac{3}{2}}}{16a^{2}\left(b\,x^{2}+a\right)^{2}}-\frac{45\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{128a^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{45\sqrt{2}\,\arctan}{64a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^3*b*((13/32*b*x^(7/2)+17/32*a*x^(3/2))/(b*x^2+a)^2+45/256/b/(a/b)^(1/4
)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/a^3/x^(1/2)
```

Maxima [A]

time = 0.50, size = 230, normalized size = 0.92

$$-\frac{45\,b^{2}x^{4}+81\,abx^{2}+32\,a^{2}}{16\,\left(a^{3}b^{2}x^{\frac{9}{2}}+2\,a^{4}bx^{\frac{5}{2}}+a^{5}\sqrt{x}\right)}-\frac{45\,b\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2)/(a^3*b^2*x^(9/2) + 2*a^4*b*x^(5/2)
 + a^5*sqrt(x)) - 45/128*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
```

```
^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(1/4)*b^(3/4)))/a^3
```

Fricas [A]
time = 1.33, size = 263, normalized size = 1.05

$$\frac{180\,(a^3b^2x^5+2\,a^4bx^3+a^5x)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,a^3b\sqrt{x}\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,a^7b\sqrt{-\frac{b}{a^{13}}}+8303765625\,b^2x}\,a^3\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}}{91125\,b}\right)-45\,(a^3b^2x^5+2\,a^4bx^3+a^5x)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}\log\left(91125\,a^{10}\left(-\frac{b}{a^{13}}\right)^{\frac{3}{4}}+91125\,b\sqrt{x}\right)+45\,(a^3b^2x^5+2\,a^4bx^3+a^5x)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}\log\left(-91125\,a^{10}\left(-\frac{b}{a^{13}}\right)^{\frac{3}{4}}+91125\,b\sqrt{x}\right)-4\,(45\,b^2x^4+81\,abx^2+32\,a^2)\sqrt{x}}{64\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(180*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-b/a^13)^(1/4)*arctan(-1/911
25*(91125*a^3*b*sqrt(x)*(-b/a^13)^(1/4) - sqrt(-8303765625*a^7*b*sqrt(-b/a^
13) + 8303765625*b^2*x)*a^3*(-b/a^13)^(1/4))/b) - 45*(a^3*b^2*x^5 + 2*a^4*b
*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(91125*a^10*(-b/a^13)^(3/4) + 91125*b*sqrt
(x)) + 45*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(-91125*a^
10*(-b/a^13)^(3/4) + 91125*b*sqrt(x)) - 4*(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2
)*sqrt(x))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.71, size = 220, normalized size = 0.88

$$-\frac{2}{a^3\sqrt{x}}-\frac{45\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^2}-\frac{45\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^2}+\frac{45\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^2}-\frac{45\,\sqrt{2}\,(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^2}-\frac{13\,b^2x^{\frac{7}{2}}+17\,abx^{\frac{3}{2}}}{16\,(bx^2+a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -2/(a^3*sqrt(x)) - 45/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 45/64*sqrt(2)*(a*b^3)^(3/
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*
b^2) + 45/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + s
```

```
qrt(a/b))/(a^4*b^2) - 45/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*(13*b^2*x^(7/2) + 17*a*b*x^(3/2)
)/((b*x^2 + a)^2*a^3)
```

Mupad [B]
time = 0.10, size = 99, normalized size = 0.39

$$\frac{45\,{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{13/4}}-\frac{45\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{13/4}}-\frac{\frac{2}{a}+\frac{81\,b\,x^2}{16\,a^2}+\frac{45\,b^2\,x^4}{16\,a^3}}{a^2\,\sqrt{x}+b^2\,x^{9/2}+2\,a\,b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^3),x)
```

```
[Out] (45*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(13/4)) - (45*(-b
)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(13/4)) - (2/a + (81*b*x^
2)/(16*a^2) + (45*b^2*x^4)/(16*a^3))/(a^2*x^(1/2) + b^2*x^(9/2) + 2*a*b*x^(
5/2))
```

3.310 $\int \frac{1}{x^{5/2}(a+bx^2)^3}\,dx$

Optimal. Leaf size=251

$$-\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{15/4}}-\frac{77b^{3/4}\tan^{-1}\left(1+\sqrt{\ldots}\right.}{32\sqrt{2}\,a^{15/\ldots}}$$

```
[Out] -77/48/a^3/x^(3/2)+1/4/a/x^(3/2)/(b*x^2+a)^2+11/16/a^2/x^(3/2)/(b*x^2+a)+77
/64*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4)*2^(1/2)-77/6
4*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4)*2^(1/2)+77/128
*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(15/4)*2^(
1/2)-77/128*b^(3/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(15/4)*2^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{77b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{15/4}}-\frac{77b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{15/4}}+\frac{77b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}\,a^{15/4}}-\frac{77b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}\,a^{15/4}}-\frac{77}{48a^3x^{3/2}}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] -77/(48*a^3*x^(3/2)) + 1/(4*a*x^(3/2)*(a + b*x^2)^2) + 11/(16*a^2*x^(3/2)*(
a + b*x^2)) + (77*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(3
2*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(32*Sqrt[2]*a^(15/4)) + (77*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx^2)^3}\,dx = \frac{1}{4ax^{3/2}(a+bx^2)^2} + \frac{11\int \frac{1}{x^{5/2}(a+bx^2)^2}\,dx}{8a}$$

$$= \frac{1}{4ax^{3/2}(a+bx^2)^2} + \frac{11}{16a^2x^{3/2}(a+bx^2)} + \frac{77\int \frac{1}{x^{5/2}(a+bx^2)}\,dx}{32a^2}$$

$$= -\frac{77}{48a^3x^{3/2}} + \frac{1}{4ax^{3/2}(a+bx^2)^2} + \frac{11}{16a^2x^{3/2}(a+bx^2)} - \frac{(77b)\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{32a^3}$$

$$= -\frac{77}{48a^3x^{3/2}} + \frac{1}{4ax^{3/2}(a+bx^2)^2} + \frac{11}{16a^2x^{3/2}(a+bx^2)} - \frac{(77b)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right.}{16a^3}$$

$$= -\frac{77}{48a^3x^{3/2}} + \frac{1}{4ax^{3/2}(a+bx^2)^2} + \frac{11}{16a^2x^{3/2}(a+bx^2)} - \frac{(77b)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx\right.}{32a^{7/2}}$$

$$= -\frac{77}{48a^3x^{3/2}} + \frac{1}{4ax^{3/2}(a+bx^2)^2} + \frac{11}{16a^2x^{3/2}(a+bx^2)} - \frac{\left(77\sqrt{b}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}}{}}\right.}{64a^{7/2}}$$

$$= -\frac{77}{48a^3x^{3/2}} + \frac{1}{4ax^{3/2}(a+bx^2)^2} + \frac{11}{16a^2x^{3/2}(a+bx^2)} + \frac{77b^{3/4}\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{\right.}{64\sqrt{2}\,a^{15/4}}$$

$$= -\frac{77}{48a^3x^{3/2}} + \frac{1}{4ax^{3/2}(a+bx^2)^2} + \frac{11}{16a^2x^{3/2}(a+bx^2)} + \frac{77b^{3/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{}}{\sqrt[4]{a}}\right.}{32\sqrt{2}\,a^{15/4}}$$

Mathematica [A]
time = 0.36, size = 149, normalized size = 0.59

$$\frac{-\frac{4a^{3/4}(32a^2+121abx^2+77b^2x^4)}{x^{3/2}(a+bx^2)^2} + 231\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right) - 231\sqrt{2}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{192a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^3),x]
```

[Out] ((-4*a^(3/4)*(32*a^2 + 121*a*b*x^2 + 77*b^2*x^4))/(x^(3/2)*(a + b*x^2)^2) + 231*Sqrt[2]*b^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 231*Sqrt[2]*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(192*a^(15/4))

Maple [A]
time = 0.11, size = 145, normalized size = 0.58

method	result
derivativedivides	$-\frac{2b\left(\frac{\frac{15 b x^{\frac{5}{2}}}{32}+\frac{19 a\sqrt{x}}{32}}{\left(b x^{2}+a\right)^{2}}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\sqrt{2}\cdots\right)\right)}{256a}\right)}{a^{3}}$
default	$-\frac{2b\left(\frac{\frac{15 b x^{\frac{5}{2}}}{32}+\frac{19 a\sqrt{x}}{32}}{\left(b x^{2}+a\right)^{2}}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\sqrt{2}\cdots\right)\right)}{256a}\right)}{a^{3}}$
risch	$-\frac{2}{3a^{3}x^{\frac{3}{2}}}-\frac{15b^{2}x^{\frac{5}{2}}}{16a^{3}\left(b x^{2}+a\right)^{2}}-\frac{19b\sqrt{x}}{16a^{2}\left(b x^{2}+a\right)^{2}}-\frac{77b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{128a^{4}}-\frac{77b\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] -2/a^3*b*((15/32*b*x^(5/2)+19/32*a*x^(1/2))/(b*x^2+a)^2+77/256*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/3/a^3/x^(3/2)

Maxima [A]
time = 0.49, size = 231, normalized size = 0.92

$$-\frac{77\,b^{2}x^{4}+121\,abx^{2}+32\,a^{2}}{48\left(a^{3}b^{2}x^{\frac{11}{2}}+2\,a^{4}bx^{\frac{7}{2}}+a^{5}x^{\frac{3}{2}}\right)}-\frac{77\left(\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)}{128\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")

[Out] -1/48*(77*b^2*x^4 + 121*a*b*x^2 + 32*a^2)/(a^3*b^2*x^(11/2) + 2*a^4*b*x^(7/2) + a^5*x^(3/2)) - 77/128*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)

```
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqr
t(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(
3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/a^(3/4))/a^3
```

Fricas [A]
time = 1.71, size = 283, normalized size = 1.13

$$-\frac{924\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)\left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^{11}b\sqrt{x}\left(-\frac{b^3}{a^{15}}\right)^{\frac{3}{4}}-\sqrt{a^8\sqrt{-\frac{b^3}{a^{15}}}+b^2x}\,a^{11}\left(-\frac{b^3}{a^{15}}\right)^{\frac{3}{4}}}{b^3}\right)+231\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)\left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}\log\left(77\,a^4\left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}+77\,b\sqrt{x}\right)-231\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)\left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}\log\left(-77\,a^4\left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}+77\,b\sqrt{x}\right)+4\,(77\,b^2x^4+121\,abx^2+32\,a^2)\sqrt{x}}{192\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(924*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*arctan(
-(a^11*b*sqrt(x)*(-b^3/a^15)^(3/4) - sqrt(a^8*sqrt(-b^3/a^15) + b^2*x)*a^11
*(-b^3/a^15)^(3/4))/b^3) + 231*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/
a^15)^(1/4)*log(77*a^4*(-b^3/a^15)^(1/4) + 77*b*sqrt(x)) - 231*(a^3*b^2*x^6
 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*log(-77*a^4*(-b^3/a^15)^(1/4) +
 77*b*sqrt(x)) + 4*(77*b^2*x^4 + 121*a*b*x^2 + 32*a^2)*sqrt(x))/(a^3*b^2*x^
6 + 2*a^4*b*x^4 + a^5*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.82, size = 208, normalized size = 0.83

$$-\frac{77\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4}-\frac{77\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4}-\frac{77\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4}+\frac{77\,\sqrt{2}\,(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4}-\frac{15\,b^2x^{\frac{5}{2}}+19\,ab\sqrt{x}}{16\,(bx^2+a)^2a^3}-\frac{2}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/a^4 - 77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^4 - 77/128*sqrt(2)*(a*b^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 + 77/128*sqrt(2)
```

```
*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 - 1/16
*(15*b^2*x^(5/2) + 19*a*b*sqrt(x))/((b*x^2 + a)^2*a^3) - 2/3/(a^3*x^(3/2))
```

Mupad [B]
time = 4.66, size = 99, normalized size = 0.39

$$\frac{77\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{15/4}}-\frac{\frac{2}{3\,a}+\frac{121\,b\,x^2}{48\,a^2}+\frac{77\,b^2\,x^4}{48\,a^3}}{a^2\,x^{3/2}+b^2\,x^{11/2}+2\,a\,b\,x^{7/2}}+\frac{77\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^3),x)
```

```
[Out] (77*(-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(15/4)) - (2/(3*a)
 + (121*b*x^2)/(48*a^2) + (77*b^2*x^4)/(48*a^3))/(a^2*x^(3/2) + b^2*x^(11/2
) + 2*a*b*x^(7/2)) + (77*(-b)^(3/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(3
2*a^(15/4))
```

3.311 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{117}{80a^3x^{5/2}}+\frac{117b}{16a^4\sqrt{x}}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}+\frac{13}{16a^2x^{5/2}\left(a+bx^2\right)}-\frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{17/4}}+\frac{117b^{5/4}\mathrm{t}}{}$$

```
[Out] -117/80/a^3/x^(5/2)+1/4/a/x^(5/2)/(b*x^2+a)^2+13/16/a^2/x^(5/2)/(b*x^2+a)-1
17/64*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+11
7/64*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+117
/128*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(17/4)
*2^(1/2)-117/128*b^(5/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/
2))/a^(17/4)*2^(1/2)+117/16*b/a^4/x^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{117b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{17/4}}+\frac{117b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{17/4}}+\frac{117b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-\frac{117b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}+\frac{117b}{16a^4\sqrt{x}}-\frac{117}{80a^3x^{5/2}}+\frac{13}{16a^2x^{5/2}\left(a+bx^2\right)}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] -117/(80*a^3*x^(5/2)) + (117*b)/(16*a^4*Sqrt[x]) + 1/(4*a*x^(5/2)*(a + b*x^
2)^2) + 13/(16*a^2*x^(5/2)*(a + b*x^2)) - (117*b^(5/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(17
/4)) - (117*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(64*Sqrt[2]*a^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}(a+bx^2)^3}\,dx = \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13\int \frac{1}{x^{7/2}(a+bx^2)^2}\,dx}{8a}$$

$$= \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13}{16a^2x^{5/2}(a+bx^2)} + \frac{117\int \frac{1}{x^{7/2}(a+bx^2)}\,dx}{32a^2}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13}{16a^2x^{5/2}(a+bx^2)} - \frac{(117b)\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{32a^3}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13}{16a^2x^{5/2}(a+bx^2)} + \frac{(117b^2)\int \frac{\sqrt{x}}{a+bx^2}\,d}{32a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13}{16a^2x^{5/2}(a+bx^2)} + \frac{(117b^2)\,\mathrm{Subst}\left(\int\right.}{16}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13}{16a^2x^{5/2}(a+bx^2)} - \frac{(117b^{3/2})\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13}{16a^2x^{5/2}(a+bx^2)} + \frac{(117b)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13}{16a^2x^{5/2}(a+bx^2)} + \frac{117b^{5/4}\log\left(\sqrt{a}\right.}{}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13}{16a^2x^{5/2}(a+bx^2)} - \frac{117b^{5/4}\tan^{-1}\left(1\right.}{32\sqrt{2}}$$

Mathematica [A]
time = 0.37, size = 160, normalized size = 0.61

$$\frac{\frac{4\sqrt[4]{a}\,(-32a^3+416a^2bx^2+1053ab^2x^4+585b^3x^6)}{x^{5/2}(a+bx^2)^2} - 585\sqrt{2}\,b^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 585\sqrt{2}\,b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{320a^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] ((4*a^(1/4)*(-32*a^3 + 416*a^2*b*x^2 + 1053*a*b^2*x^4 + 585*b^3*x^6))/(x^(5
/2)*(a + b*x^2)^2) - 585*Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 585*Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(320*a^(17/4))
```

Maple [A]
time = 0.13, size = 156, normalized size = 0.59

method	result
derivativedivides	$\frac{2b^2\left(\frac{\frac{21 b x^{\frac{7}{2}}}{32}+\frac{25 a x^{\frac{3}{2}}}{32}}{(b x^2+a)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{256 b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^4}$
default	$\frac{2b^2\left(\frac{\frac{21 b x^{\frac{7}{2}}}{32}+\frac{25 a x^{\frac{3}{2}}}{32}}{(b x^2+a)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{256 b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^4}$
risch	$-\frac{2(-15 b x^2+a)}{5a^4 x^{\frac{5}{2}}}+\frac{21 b^3 x^{\frac{7}{2}}}{16a^4(b x^2+a)^2}+\frac{25 b^2 x^{\frac{3}{2}}}{16a^3(b x^2+a)^2}+\frac{117 b\sqrt{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{128a^4\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{117 b\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^4*b^2*((21/32*b*x^(7/2)+25/32*a*x^(3/2))/(b*x^2+a)^2+117/256/b/(a/b)^(1
/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/5/a^3/x^(5/2)+6*b/a^4/x^(1/2)
```

Maxima [A]
time = 0.50, size = 243, normalized size = 0.92

$$\frac{585\,b^3x^6+1053\,ab^2x^4+416\,a^2bx^2-32\,a^3}{80\left(a^4b^2x^{\frac{13}{2}}+2\,a^5bx^{\frac{9}{2}}+a^6x^{\frac{5}{2}}\right)}+\frac{117\,b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(585*b^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^3)/(a^4*b^2*x^(13
/2) + 2*a^5*b*x^(9/2) + a^6*x^(5/2)) + 117/128*b^2*(2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^
(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqr
t(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
 sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4
```

Fricas [A]
time = 1.37, size = 306, normalized size = 1.16

$$-\frac{2340\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)\left(-\frac{b^5}{a^{17}}\right)^{\frac14}\arctan\left(-\frac{1601613\,a^4b^4\sqrt{x}\left(-\frac{b^5}{a^{17}}\right)^{\frac14}-\sqrt{-2565164201769\,a^9b^5\sqrt{-\frac{b^5}{a^{17}}}+2565164201769\,b^8x}\,a^4\left(-\frac{b^5}{a^{17}}\right)^{\frac14}}{1601613\,b^5}\right)-585\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)\left(-\frac{b^5}{a^{17}}\right)^{\frac14}\log\left(1601613\,a^{13}\left(-\frac{b^5}{a^{17}}\right)^{\frac34}+1601613\,b^4\sqrt{x}\right)+585\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)\left(-\frac{b^5}{a^{17}}\right)^{\frac14}\log\left(-1601613\,a^{13}\left(-\frac{b^5}{a^{17}}\right)^{\frac34}+1601613\,b^4\sqrt{x}\right)-4\,(585\,b^3x^6+1053\,ab^2x^4+416\,a^2bx^2-32\,a^3)\sqrt{x}}{320\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(2340*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*arctan
(-1/1601613*(1601613*a^4*b^4*sqrt(x)*(-b^5/a^17)^(1/4) - sqrt(-256516420176
9*a^9*b^5*sqrt(-b^5/a^17) + 2565164201769*b^8*x)*a^4*(-b^5/a^17)^(1/4))/b^5
) - 585*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*log(1601613
*a^13*(-b^5/a^17)^(3/4) + 1601613*b^4*sqrt(x)) + 585*(a^4*b^2*x^7 + 2*a^5*b
*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*log(-1601613*a^13*(-b^5/a^17)^(3/4) + 160
1613*b^4*sqrt(x)) - 4*(585*b^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^
3)*sqrt(x))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.75, size = 232, normalized size = 0.88

$$\frac{117\,\sqrt{2}\,(ab^3)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{64\,a^5b}+\frac{117\,\sqrt{2}\,(ab^3)^{\frac34}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{64\,a^5b}-\frac{117\,\sqrt{2}\,(ab^3)^{\frac34}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b}+\frac{117\,\sqrt{2}\,(ab^3)^{\frac34}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b}+\frac{21\,b^3x^{\frac72}+25\,ab^2x^{\frac32}}{16\,(bx^2+a)^2a^4}+\frac{2\,(15\,bx^2-a)}{5\,a^4x^{\frac52}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/(a^5*b) + 117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*b) - 117/128*sqrt(2
)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b) +
117/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a
/b))/(a^5*b) + 1/16*(21*b^3*x^(7/2) + 25*a*b^2*x^(3/2))/((b*x^2 + a)^2*a^4)
 + 2/5*(15*b*x^2 - a)/(a^4*x^(5/2))
```

Mupad [B]
time = 4.65, size = 109, normalized size = 0.41

$$\frac{\frac{26\,b\,x^2}{5\,a^2}-\frac{2}{5\,a}+\frac{1053\,b^2\,x^4}{80\,a^3}+\frac{117\,b^3\,x^6}{16\,a^4}}{a^2\,x^{5/2}+b^2\,x^{13/2}+2\,a\,b\,x^{9/2}}-\frac{117\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{17/4}}+\frac{117\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^3),x)
```

```
[Out] ((26*b*x^2)/(5*a^2) - 2/(5*a) + (1053*b^2*x^4)/(80*a^3) + (117*b^3*x^6)/(16
*a^4))/(a^2*x^(5/2) + b^2*x^(13/2) + 2*a*b*x^(9/2)) - (117*(-b)^(5/4)*atan(
((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(17/4)) + (117*(-b)^(5/4)*atanh(((-b)^
(1/4)*x^(1/2))/a^(1/4)))/(32*a^(17/4))
```

3.312 $\int \frac{\sqrt{x}}{a-bx^2} dx$

Optimal. Leaf size=58

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

```
[Out] -arctan(b^(1/4)*x^(1/2)/a^(1/4))/a^(1/4)/b^(3/4)+arctanh(b^(1/4)*x^(1/2)/a^
(1/4))/a^(1/4)/b^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {335, 304, 211, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x^2),x]
```

```
[Out] -(ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))) + ArcTanh[(b^(1/4)*S
qrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx = 2\text{Subst}\left(\int \frac{x^2}{a-bx^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}+\sqrt{b}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.83

$$\frac{-\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x^2),x]
```

```
[Out] (-ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)] + ArcTanh[(b^(1/4)*Sqrt[x])/a^(1/4)])/(
a^(1/4)*b^(3/4))
```

Maple [A]
time = 0.03, size = 58, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	58
default	$-\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/(a/b)^(1/4)*(2*arctan(x^(1/2)/(a/b)^(1/4))-ln((x^(1/2)+(a/b)^(1/4))/
(x^(1/2)-(a/b)^(1/4))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(38) = 76.
time = 0.50, size = 86, normalized size = 1.48

$$-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\log\left(\frac{\sqrt{b}\sqrt{x}-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sqrt{x}+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(b)*sqrt(x)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(
b)) - 1/2*log((sqrt(b)*sqrt(x) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sqrt(x) +
sqrt(sqrt(a)*sqrt(b))))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(38) = 76.
time = 1.54, size = 117, normalized size = 2.02

$$2\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{ab\sqrt{\frac{1}{ab^3}}+x}\,b\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}-b\sqrt{x}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2*(1/(a*b^3))^(1/4)*arctan(sqrt(a*b*sqrt(1/(a*b^3)) + x)*b*(1/(a*b^3))^(1/4
) - b*sqrt(x)*(1/(a*b^3))^(1/4)) + 1/2*(1/(a*b^3))^(1/4)*log(a*b^2*(1/(a*b^
3))^(3/4) + sqrt(x)) - 1/2*(1/(a*b^3))^(1/4)*log(-a*b^2*(1/(a*b^3))^(3/4) +
 sqrt(x))
```

Sympy [A]
time = 1.00, size = 92, normalized size = 1.59

$$\begin{cases}\frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\ a=0\wedge b=0\\ \frac{2}{b\sqrt{x}} & \text{for}\ a=0\\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for}\ b=0\\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{\frac{a}{b}}\right)}{2b\sqrt[4]{\frac{a}{b}}}+\frac{\log\left(\sqrt{x}+\sqrt[4]{\frac{a}{b}}\right)}{2b\sqrt[4]{\frac{a}{b}}}-\frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{\frac{a}{b}}}\right)}{b\sqrt[4]{\frac{a}{b}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x**2+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2/(b*sqrt(x)), Eq(a, 0)), (2
*x**(3/2)/(3*a), Eq(b, 0)), (-log(sqrt(x) - (a/b)**(1/4))/(2*b*(a/b)**(1/4)
) + log(sqrt(x) + (a/b)**(1/4))/(2*b*(a/b)**(1/4)) - atan(sqrt(x)/(a/b)**(1
/4))/(b*(a/b)**(1/4)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(38) = 76.
time = 0.96, size = 194, normalized size = 3.34

$$\frac{\sqrt{2}\,(-ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}+2\sqrt{x}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{2\,ab^3}+\frac{\sqrt{2}\,(-ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-\frac{a}{b})^{\frac{1}{4}}-2\sqrt{x}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{2\,ab^3}-\frac{\sqrt{2}\,(-ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,(-\frac{a}{b})^{\frac{1}{4}}+x+\sqrt{-\frac{a}{b}}\right)}{4\,ab^3}+\frac{\sqrt{2}\,(-ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,(-\frac{a}{b})^{\frac{1}{4}}+x+\sqrt{-\frac{a}{b}}\right)}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sqr
t(x))/(-a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a/b)^(1/4) - 2*sqrt(x))/(-a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(-
a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt(-a/b))/(a*b^3) + 1
/4*sqrt(2)*(-a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt(-a/b
))/(a*b^3)
```

Mupad [B]
time = 0.08, size = 33, normalized size = 0.57

$$-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{a^{1/4}}\right)-\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{a^{1/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x^2),x)
```

```
[Out] -(atan((b^(1/4)*x^(1/2))/a^(1/4)) - atanh((b^(1/4)*x^(1/2))/a^(1/4)))/(a^(1
/4)*b^(3/4))
```

3.313 $\int \frac{x^{7/2}}{1+x^2} dx$

Optimal. Leaf size=108

$$-2\sqrt{x}+\frac{2x^{5/2}}{5}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}$$

```
[Out] 2/5*x^(5/2)+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(
1/2))*2^(1/2)-1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)*x^(1/2
))*2^(1/2)-2*x^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}+\frac{2x^{5/2}}{5}-2\sqrt{x}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2),x]
```

```
[Out] -2*Sqrt[x] + (2*x^(5/2))/5 - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{1+x^2}\,dx &= \frac{2x^{5/2}}{5} - \int \frac{x^{3/2}}{1+x^2}\,dx \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + \int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + 2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,\right. \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x,\right.}{\sqrt{2}} \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 59, normalized size = 0.55

$$
\frac{2}{5}\sqrt{x}\left(-5+x^2\right) + \frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2),x]
[Out] (2*Sqrt[x]*(-5 + x^2))/5 + ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] + Arc
Tanh[(Sqrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

Maple [A]
time = 0.89, size = 67, normalized size = 0.62

method	result
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
default	$\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$

risch	$\frac{2(x^2-5)\sqrt{x}}{5}+\frac{\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)}{4}+\frac{\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}+\frac{\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{}}{2}$
meijerg	$-\frac{2\sqrt{x}\,(-9x^2+45)}{45}+\frac{\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\right.}{}\right.}{2}$
trager	$\left(\frac{2x^2}{5}-2\right)\sqrt{x}-\frac{\text{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{-\text{RootOf}\left(\_Z^4+1\right)^5x+\text{RootOf}\left(\_Z^4+1\right)^5-2\,\text{RootOf}\left(\_Z^4+1\right)^3+\text{R}}{\text{RootOf}\left(\_Z^4+1\right)^2x-\text{RootOf}\left(\_Z\right.}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)-2*x^(1/2)+1/4*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^
(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.51, size = 84, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x)
```

Fricas [A]
time = 1.72, size = 117, normalized size = 1.08

$$\frac{2}{5}(x^2-5)\sqrt{x}-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 - 5)*sqrt(x) - sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1
) - sqrt(2)*sqrt(x) - 1) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(
x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) +
4*x + 4) - 1/4*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A]
time = 0.57, size = 105, normalized size = 0.97

$$\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} + \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right)}{2} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1),x)
```

```
[Out] 2*x**(5/2)/5 - 2*sqrt(x) - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sq
rt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1
)/2 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/2
```

Giac [A]
time = 0.74, size = 84, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}} + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2\sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2\sqrt{x}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x} + x + 1\right) - \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x} + x + 1\right) - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x)
```

Mupad [B]
time = 0.08, size = 47, normalized size = 0.44

$$\frac{2\,x^{5/2}}{5} - 2\,\sqrt{x} + \sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2*x^(1/2) + (2*x^(5/2))/5
```

3.314 $\int \frac{x^{5/2}}{1+x^2} \, dx$

Optimal. Leaf size=101

$$\frac{2x^{3/2}}{3}+\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}$$

```
[Out] 2/3*x^(3/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^(
1/2))*2^(1/2)-1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)*x^(1/2
))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{\sqrt{2}}+\frac{2x^{3/2}}{3}-\frac{\log\left(x-\sqrt{2}\,\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\,\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2),x]
```

```
[Out] (2*x^(3/2))/3 + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sq
rt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]
*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{1+x^2}\,dx &= \frac{2x^{3/2}}{3} - \int \frac{\sqrt{x}}{1+x^2}\,dx \\
&= \frac{2x^{3/2}}{3} - 2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3} + \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\right.}{\sqrt{2}} \\
&= \frac{2x^{3/2}}{3} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.54

$$\frac{2x^{3/2}}{3} - \frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2),x]
[Out] (2*x^(3/2))/3 - ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] + ArcTanh[(Sqrt[
2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

Maple [A]
time = 0.78, size = 62, normalized size = 0.61

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3} - \frac{\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
default	$\frac{2x^{\frac{3}{2}}}{3} - \frac{\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$

risch	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}-\frac{\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}-\frac{\sqrt{2}\,\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right)}{4}$
meijerg	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}-\right.}{2}$
trager	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\operatorname{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^4+1\right)^5x-\operatorname{RootOf}\left(\_Z^4+1\right)^5-2\operatorname{RootOf}\left(\_Z^4+1\right)^3x+\operatorname{RootOf}\left(\_Z^4+1\right)x-}{\operatorname{RootOf}\left(\_Z^4+1\right)^2x-\operatorname{RootOf}\left(\_Z^4+1\right)^2+x+1}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)-1/4*2^(1/2)*(ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))+2*
arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.51, size = 79, normalized size = 0.78

$$\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Fricas [A]
time = 1.89, size = 110, normalized size = 1.09

$$\frac{2}{3}x^{\frac{3}{2}}+\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2) + sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2
)*sqrt(x) - 1) + sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x +
 4) - sqrt(2)*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) -
 1/4*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A]
time = 0.31, size = 99, normalized size = 0.98

$$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*
sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2
)*atan(sqrt(2)*sqrt(x) + 1)/2
```

Giac [A]
time = 0.78, size = 79, normalized size = 0.78

$$\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Mupad [B]
time = 0.04, size = 42, normalized size = 0.42

$$\frac{2\,x^{3/2}}{3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2 + 1),x)
```

```
[Out] (2*x^(3/2))/3 - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) - 2
^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2)
```

3.315 $\int \frac{x^{3/2}}{1+x^2}\,dx$

Optimal. Leaf size=99

$$2\sqrt{x}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/
2)+1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/4*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)+
2*x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}+2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2),x]
```

```
[Out] 2*Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[x
]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqr
t[x] + x]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{1+x^2}\,dx = 2\sqrt{x} - \int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= 2\sqrt{x} - 2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{x} - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{x} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) +$$
$$= 2\sqrt{x} + \frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,\surd\right.}{\sqrt{2}}$$
$$= 2\sqrt{x} + \frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\right.}{}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 0.55

$$2\sqrt{x} - \frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\,\sqrt{x}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2),x]
[Out] 2*Sqrt[x] - ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] - ArcTanh[(Sqrt[2]*S
qrt[x])/(1 + x)]/Sqrt[2]
```

Maple [A]
time = 0.80, size = 62, normalized size = 0.63

method	result
derivativedivides	$2\sqrt{x}-\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$
default	$2\sqrt{x}-\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$

risch	$2\sqrt{x}-\frac{\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}-\frac{\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}-\frac{\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)}{4}$
meijerg	$2\sqrt{x}-\frac{\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\right.}{2(x^2)^{\frac{1}{4}}}\right.}{2}$
trager	$2\sqrt{x}-\frac{\operatorname{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^4+1\right)^5x-\operatorname{RootOf}\left(\_Z^4+1\right)^5-2\operatorname{RootOf}\left(\_Z^4+1\right)^3x+\operatorname{RootOf}\left(\_Z^4+1\right.}{\operatorname{RootOf}\left(\_Z^4+1\right)^2x-\operatorname{RootOf}\left(\_Z^4+1\right)^2+x+1}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)-1/4*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))+2*ar
ctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.50, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

Fricas [A]
time = 2.19, size = 110, normalized size = 1.11

$$\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1)
 + sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*
sqrt(x) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 1/4*sqrt(2)*l
og(-4*sqrt(2)*sqrt(x) + 4*x + 4) + 2*sqrt(x)
```

Sympy [A]
time = 0.19, size = 97, normalized size = 0.98

$$2\sqrt{x}+\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1),x)
```

```
[Out] 2*sqrt(x) + sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqr
t(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*a
tan(sqrt(2)*sqrt(x) + 1)/2
```

Giac [A]
time = 1.13, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

Mupad [B]
time = 0.04, size = 42, normalized size = 0.42

$$2\sqrt{x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2 + 1),x)
```

```
[Out] 2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2^(1/
2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2)
```

3.316 $\int \frac{\sqrt{x}}{1+x^2} dx$

Optimal. Leaf size=92

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2
)+1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/4*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{1+x^2}\,dx &= 2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right)+\frac{\text{Subst}\Big(
\\
&= \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-
\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 0.45

$$\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2),x]
[Out] (ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] - ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/S
qrt[2]
```

Maple [A]
time = 0.58, size = 56, normalized size = 0.61

method	result
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
meijerg	$\frac{x^{\frac{3}{2}}\sqrt{2}\ln\left(1-\sqrt{2}(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{4(x^2)^{\frac{3}{4}}}+\frac{x^{\frac{3}{2}}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2)^{\frac{1}{4}}}{2-\sqrt{2}(x^2)^{\frac{1}{4}}}\right)}{2(x^2)^{\frac{3}{4}}}-\frac{x^{\frac{3}{2}}\sqrt{2}\ln\left(1+\sqrt{2}(x^2)^{\frac{1}{4}}+\sqrt{\ldots}\right.}{4(x^2)^{\frac{3}{4}}}$ …
trager	$\frac{\mathrm{RootOf}(\_Z^4+1)\ln\left(-\frac{\mathrm{RootOf}(\_Z^4+1)^5x-\mathrm{RootOf}(\_Z^4+1)^5+2\,\mathrm{RootOf}(\_Z^4+1)^3-\mathrm{RootOf}(\_Z^4+1)x+4\sqrt{x}-\ldots}{\mathrm{RootOf}(\_Z^4+1)^2x-\mathrm{RootOf}(\_Z^4+1)^2-x-1}\right.}{2}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/4*2^(1/2)*(ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))+2*arctan(1+2^(
1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.49, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Fricas [A]
time = 1.43, size = 107, normalized size = 1.16

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1
) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)
*sqrt(x) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 1/4*sqrt(2)*
log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A]
time = 0.15, size = 90, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2
```

Giac [A]
time = 0.76, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.40

$$\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2)
```

3.317 $\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$

Optimal. Leaf size=92

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2
)-1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\,\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\,\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx &= 2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)\\ &= \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) -\\ &= -\frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\right.}{\sqrt{2}}\\ &= -\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 0.42

$$\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)),x]
[Out] (ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] + ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/S
qrt[2]
```

Maple [A]
time = 0.33, size = 56, normalized size = 0.61

method	result
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
meijerg	$-\frac{\sqrt{x}\sqrt{2}\ln\left(1-\sqrt{2}(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{4(x^2)^{\frac{1}{4}}}+\frac{\sqrt{x}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2)^{\frac{1}{4}}}{2-\sqrt{2}(x^2)^{\frac{1}{4}}}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{x}\sqrt{2}\ln\left(1+\sqrt{2}(x\right.}{4(x^2)^{\frac{1}{4}}}$
trager	$-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^5x+\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x-\mathrm{RootOf}\left(\_Z^4+1\right)x+4\sqrt{x}}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2+x+1}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))+2*arctan(1+2^(
1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.49, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Fricas [A]
time = 1.73, size = 107, normalized size = 1.16

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1
) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)
*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 1/4*sqrt(2)*
log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A]
time = 0.19, size = 90, normalized size = 0.98

$$-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/x**(1/2),x)
```

```
[Out] -sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)
*sqrt(x) + 1)/2
```

Giac [A]
time = 0.96, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Mupad [B]
time = 0.03, size = 37, normalized size = 0.40

$$\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^2 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2)
```

3.318 $\int \frac{1}{x^{3/2}(1+x^2)}\,dx$

Optimal. Leaf size=99

$$-\frac{2}{\sqrt{x}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/
2)-1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)-
2/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{2}{\sqrt{x}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)),x]
```

```
[Out] -2/Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[
x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sq
rt[x] + x]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(1+x^2)}\,dx = -\frac{2}{\sqrt{x}} - \int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{2}{\sqrt{x}} - 2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{\sqrt{x}} + \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{\sqrt{x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x\right.$$
$$= -\frac{2}{\sqrt{x}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{\sqrt{2}}$$
$$= -\frac{2}{\sqrt{x}} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} +$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 0.54

$$-\frac{2}{\sqrt{x}} - \frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)),x]
```

```
[Out] -2/Sqrt[x] - ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] + ArcTanh[(Sqrt[2]*
Sqrt[x])/(1 + x)]/Sqrt[2]
```

Maple [A]
time = 0.44, size = 62, normalized size = 0.63

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}-\frac{2}{\sqrt{x}}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}-\frac{2}{\sqrt{x}}$

risch	$-\frac{2}{\sqrt{x}}-\frac{\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}-\frac{\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}-\frac{\sqrt{2}\,\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right)}{4}$
meijerg	$-\frac{2}{\sqrt{x}}-\frac{x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right.}{2(x^2)^{\frac{3}{4}}}\right.}{2}$
trager	$-\frac{2}{\sqrt{x}}+\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^5x+\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+\mathrm{RootOf}\left(\_Z^4+1\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-x-1}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*(ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))+2*arctan(1+2^
(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/x^(1/2)
```

Maxima [A]
time = 0.52, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

Fricas [A]
time = 1.26, size = 120, normalized size = 1.21

$$\frac{4\sqrt{2}\,x\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}\,x\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\sqrt{2}\,x\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\sqrt{2}\,x\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\sqrt{x}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqr
t(x) - 1) + 4*sqrt(2)*x*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x +
4) - sqrt(2)*sqrt(x) + 1) + sqrt(2)*x*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - sq
rt(2)*x*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*sqrt(x))/x
```

Sympy [A]
time = 0.34, size = 97, normalized size = 0.98

$$-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1),x)
```

```
[Out] -sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2)
*sqrt(x) + 1)/2 - 2/sqrt(x)
```

Giac [A]
time = 0.82, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

Mupad [B]
time = 0.04, size = 42, normalized size = 0.42

$$-\frac{2}{\sqrt{x}}+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^2 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) - 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) - 2/x^(1/2)
```

3.319 $\int \frac{1}{x^{5/2}(1+x^2)}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{3x^{3/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}$$

```
[Out] -2/3/x^(3/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^
(1/2))*2^(1/2)+1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/4*ln(1+x+2^(1/2)*x^(1/
2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{2}{3x^{3/2}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)),x]
```

```
[Out] -2/(3*x^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*S
qrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2
]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(1+x^2)}\,dx = -\frac{2}{3x^{3/2}} - \int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= -\frac{2}{3x^{3/2}} - 2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{3x^{3/2}} - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{3x^{3/2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x,\right.$$
$$= -\frac{2}{3x^{3/2}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{\sqrt{2}}$$
$$= -\frac{2}{3x^{3/2}} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} -$$

Mathematica [A]
time = 0.07, size = 56, normalized size = 0.55

$$-\frac{2}{3x^{3/2}} - \frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)),x]
[Out] -2/(3*x^(3/2)) - ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] - ArcTanh[(Sqrt
[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

Maple [A]
time = 0.63, size = 62, normalized size = 0.61

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}-\frac{2}{3x^{\frac{3}{2}}}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}-\frac{2}{3x^{\frac{3}{2}}}$

risch	$-\frac{2}{3x^{\frac{3}{2}}}-\frac{\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}-\frac{\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}-\frac{\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)}{4}$
meijerg	$-\frac{2}{3x^{\frac{3}{2}}}-\frac{\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^{2})^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2(x^{2})^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^{2})^{\frac{1}{4}}}{2-\sqrt{2}\,(x^{2})^{\frac{1}{4}}}\right)}{(x^{2})^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^{2})^{\frac{1}{4}}+\right.}{2(x^{2})^{\frac{1}{4}}}\right.}{2}$
trager	$-\frac{2}{3x^{\frac{3}{2}}}+\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}x+\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}+2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x-\mathrm{RootOf}\left(\_Z^{4}\right.}{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+x+1}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))+2*arctan(1+2^
(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/3/x^(3/2)
```

Maxima [A]
time = 0.52, size = 79, normalized size = 0.78

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

Fricas [A]
time = 0.95, size = 129, normalized size = 1.28

$$\frac{12\sqrt{2}\,x^{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\,x^{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-3\sqrt{2}\,x^{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+3\sqrt{2}\,x^{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\sqrt{x}}{12x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)
*sqrt(x) - 1) + 12*sqrt(2)*x^2*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) +
 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*x^2*log(4*sqrt(2)*sqrt(x) + 4*
x + 4) + 3*sqrt(2)*x^2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*sqrt(x))/x^2
```

Sympy [A]
time = 0.48, size = 99, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2 - 2/(3*x**(3/2))
```

Giac [A]
time = 0.61, size = 79, normalized size = 0.78

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

Mupad [B]
time = 0.04, size = 42, normalized size = 0.42

$$-\frac{2}{3\,x^{3/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^2 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) - 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2/(3*x^(3/2))
```

3.320 $\int \frac{1}{x^{7/2}(1+x^2)} \, dx$

Optimal. Leaf size=108

$$-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}\right.}{2\sqrt{2}}$$

```
[Out] -2/5/x^(5/2)+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^
(1/2))*2^(1/2)+1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/4*ln(1+x+2^(1/2)*x^(1/
2))*2^(1/2)+2/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)),x]
```

```
[Out] -2/(5*x^(5/2)) + 2/Sqrt[x] - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}(1+x^2)}\,dx = -\frac{2}{5x^{5/2}} - \int \frac{1}{x^{3/2}(1+x^2)}\,dx$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + \int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + 2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + \frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x}\right.}{}$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} - \frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,\sqrt{x}\right.}{2\sqrt{2}}$$

Mathematica [A]
time = 0.07, size = 62, normalized size = 0.57

$$\frac{2(-1+5x^2)}{5x^{5/2}} + \frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\,\sqrt{x}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)),x]
[Out] (2*(-1 + 5*x^2))/(5*x^(5/2)) + ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] -
 ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

Maple [A]
time = 0.78, size = 67, normalized size = 0.62

method	result
derivativedivides	$\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4} - \frac{2}{5x^{\frac{5}{2}}} + \frac{2}{\sqrt{x}}$

default	$\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}-\frac{2}{5x^{\frac{5}{2}}}+\frac{2}{\sqrt{x}}$
risch	$\frac{2x^{2}-\frac{2}{5}}{x^{\frac{5}{2}}}+\frac{\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}+\frac{\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{2}+\frac{\sqrt{2}\,\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right)}{4}$
meijerg	$\frac{2}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}+\frac{x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^{2})^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2(x^{2})^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^{2})^{\frac{1}{4}}}{2-\sqrt{2}\,(x^{2})^{\frac{1}{4}}}\right)}{(x^{2})^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^{2})^{\frac{1}{4}}+\right.}{2(x^{2})^{\frac{3}{4}}}\right.}{2}$
trager	$\frac{2x^{2}-\frac{2}{5}}{x^{\frac{5}{2}}}+\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{5}x-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{5}+2\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3}-\operatorname{RootOf}\left(\_Z^{4}+1\right)x}{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}x-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}-x-1}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*(ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))+2*arctan(1+2^(
1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/5/x^(5/2)+2/x^(1/2)
```

Maxima [A]
time = 0.49, size = 86, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{2\left(5x^{2}-1\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2)
```

Fricas [A]
time = 1.01, size = 136, normalized size = 1.26

$$-\frac{20\sqrt{2}\,x^{3}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\,x^{3}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+5\sqrt{2}\,x^{3}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-5\sqrt{2}\,x^{3}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(5x^{2}-1\right)\sqrt{x}}{20x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2
)*sqrt(x) - 1) + 20*sqrt(2)*x^3*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x)
```

```
+ 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*x^3*log(4*sqrt(2)*sqrt(x) + 4
*x + 4) - 5*sqrt(2)*x^3*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(5*x^2 - 1)*s
qrt(x))/x^3
```

Sympy [A]
time = 0.88, size = 105, normalized size = 0.97

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}+\frac{2}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2 + 2/sqrt(x) - 2/(5*x**(5/2))
```

Giac [A]
time = 0.67, size = 86, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{2\left(5x^{2}-1\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2)
```

Mupad [B]
time = 0.04, size = 48, normalized size = 0.44

$$\frac{2x^2-\frac{2}{5}}{x^{5/2}}+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^2 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) + (2*x^2 - 2/5)/x^(5/2)
```

3.321 $\int \frac{x^{7/2}}{(1+x^2)^2} dx$

Optimal. Leaf size=122

$$\frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2(1+x^2)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\log\left(1+\right.}{8}$$

[Out] -1/2*x^(5/2)/(x^2+1)-5/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-5/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+5/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-5/16*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)+5/2*x^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{5\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}-\frac{x^{5/2}}{2(x^2+1)}+\frac{5\sqrt{x}}{2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)/(1 + x^2)^2,x]

[Out] (5*Sqrt[x])/2 - x^(5/2)/(2*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{(1+x^2)^2}\,dx = -\frac{x^{5/2}}{2(1+x^2)}+\frac{5}{4}\int \frac{x^{3/2}}{1+x^2}\,dx$$
$$=\frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2(1+x^2)}-\frac{5}{4}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$=\frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2(1+x^2)}-\frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$=\frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2(1+x^2)}-\frac{5}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{5}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$=\frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2(1+x^2)}-\frac{5}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{5}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+}\right.$$
$$=\frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2(1+x^2)}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\text{Subst}\left(\int\right.}{}$$
$$=\frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2(1+x^2)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\right.}{8\sqrt{2}}$$

Mathematica [A]
time = 0.13, size = 72, normalized size = 0.59

$$\frac{1}{8}\left(\frac{4\sqrt{x}\,(5+4x^2)}{1+x^2}-5\sqrt{2}\,\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)-5\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2)^2,x]
```

```
[Out] ((4*Sqrt[x]*(5 + 4*x^2))/(1 + x^2) - 5*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqr
t[x])] - 5*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Maple [A]
time = 0.46, size = 74, normalized size = 0.61

method	result
derivativedivides	$2\sqrt{x}+\frac{\sqrt{x}}{2x^2+2}-\frac{5\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$
default	$2\sqrt{x}+\frac{\sqrt{x}}{2x^2+2}-\frac{5\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$

risch	$\frac{(4x^2+5)\sqrt{x}}{2x^2+2} - \frac{5\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{8} - \frac{5\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{8} - \frac{5\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}\,\cdots}{1+x-\sqrt{2}\,\cdots}\right.}{16}$
meijerg	$\frac{\sqrt{x}\,(36x^2+45)}{18x^2+18} - \frac{5\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}} + \frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}} + \frac{\sqrt{2}\,\ln\left(1+\sqrt{\cdots}\right.}{2\cdots}\right.}{8}$
trager	$\frac{(4x^2+5)\sqrt{x}}{2x^2+2} + \frac{5\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^5 x+\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+\mathrm{RootOf}\cdots}{\mathrm{RootOf}\left(\_Z^4+1\right)^2 x-\mathrm{RootOf}\left(\_Z^4+1\cdots\right.}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)+1/2*x^(1/2)/(x^2+1)-5/16*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+x-2
^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.53, size = 91, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}+\frac{\sqrt{x}}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqrt(x
)/(x^2 + 1)
```

Fricas [A]
time = 1.50, size = 148, normalized size = 1.21

$$\frac{20\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}(x^2+1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-5\sqrt{2}(x^2+1)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+5\sqrt{2}(x^2+1)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)+8(4x^2+5)\sqrt{x}}{16(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 5*sqrt(2)*(x^2 + 1)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4) + 5*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x
 + 4) + 8*(4*x^2 + 5)*sqrt(x))/(x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(110) = 220$.

time = 1.18, size = 277, normalized size = 2.27

$$\frac{32x^{\frac{5}{2}}}{16x^2+16}+\frac{40\sqrt{x}}{16x^2+16}+\frac{5\sqrt{2}\,x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{5\sqrt{2}\,x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{10\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}-\frac{10\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}+\frac{5\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{5\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}-\frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1)**2,x)
```

```
[Out] 32*x**(5/2)/(16*x**2 + 16) + 40*sqrt(x)/(16*x**2 + 16) + 5*sqrt(2)*x**2*log
(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*sqrt(2)*x**2*log(4*sqrt(2
)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x)
- 1)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 +
16) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*sqrt(2
)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*atan(sqrt(2)
*sqrt(x) - 1)/(16*x**2 + 16) - 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**
2 + 16)
```

Giac [A]

time = 0.56, size = 91, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqrt(x
)/(x^2 + 1)
```

Mupad [B]

time = 4.62, size = 55, normalized size = 0.45

$$\frac{\sqrt{x}}{2\,\left(x^2+1\right)}+2\,\sqrt{x}+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{8}-\frac{5}{8}\mathrm{i}\right)+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{8}+\frac{5}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2 + 1)^2,x)
```

```
[Out] x^(1/2)/(2*(x^2 + 1)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/8 - 5
i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/8 + 5i/8) + 2*x^(1/2)
```

3.322 $\int \frac{x^{5/2}}{(1+x^2)^2} \, dx$

Optimal. Leaf size=113

$$-\frac{x^{3/2}}{2(1+x^2)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}\right.}{8\sqrt{2}}$$

```
[Out] -1/2*x^(3/2)/(x^2+1)+3/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/8*arctan(1+2^
(1/2)*x^(1/2))*2^(1/2)+3/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-3/16*ln(1+x+2^(
1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}-\frac{x^{3/2}}{2(x^2+1)}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2)^2,x]
```

```
[Out] -1/2*x^(3/2)/(1 + x^2) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/
(8*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{(1+x^2)^2} dx = -\frac{x^{3/2}}{2(1+x^2)} + \frac{3}{4}\int \frac{\sqrt{x}}{1+x^2} dx$$
$$= -\frac{x^{3/2}}{2(1+x^2)} + \frac{3}{2}\text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{x}\right)$$
$$= -\frac{x^{3/2}}{2(1+x^2)} - \frac{3}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{x}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{x}\right)$$
$$= -\frac{x^{3/2}}{2(1+x^2)} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{x}\right) + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx,\right.$$
$$= -\frac{x^{3/2}}{2(1+x^2)} + \frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2} \right.}{4}$$
$$= -\frac{x^{3/2}}{2(1+x^2)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}\sqrt{x}\right.}{8\sqrt{2}}$$

Mathematica [A]
time = 0.13, size = 65, normalized size = 0.58

$$\frac{1}{8}\left(-\frac{4x^{3/2}}{1+x^2} + 3\sqrt{2}\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2)^2,x]
[Out] ((-4*x^(3/2))/(1 + x^2) + 3*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] - 3*
Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Maple [A]
time = 0.77, size = 69, normalized size = 0.61

method	result
derivativedivides	$-\frac{x^{\frac{3}{2}}}{2(x^2+1)} + \frac{3\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$
default	$-\frac{x^{\frac{3}{2}}}{2(x^2+1)} + \frac{3\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$

risch	$-\frac{x^{\frac{3}{2}}}{2(x^2+1)}+\frac{3\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{8}+\frac{3\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{8}+\frac{3\sqrt{2}\,\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right.}{16}$
meijerg	$-\frac{x^{\frac{3}{2}}}{2(x^2+1)}+\frac{3x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{\right.}}{2(x^2)^{\frac{3}{4}}}\right.}{8}$
trager	$-\frac{x^{\frac{3}{2}}}{2(x^2+1)}-\frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^5x+\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+\mathrm{RootOf}\left(\_Z\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-x-}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(3/2)/(x^2+1)+3/16*2^(1/2)*(ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^
(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.51, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1)
```

Fricas [A]
time = 1.32, size = 141, normalized size = 1.25

$$-\frac{12\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+3\sqrt{2}\left(x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-3\sqrt{2}\left(x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)+8x^{\frac{3}{2}}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt
(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 3*sqrt(2)*(x^2 + 1)*log(4*s
qrt(2)*sqrt(x) + 4*x + 4) - 3*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*
x + 4) + 8*x^(3/2))/(x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(102) = 204$.
time = 0.87, size = 264, normalized size = 2.34

$$-\frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{3\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{3\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{16x^2+16}+\frac{3\sqrt{2}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{3\sqrt{2}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1)**2,x)
```

```
[Out] -8*x**(3/2)/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x +
4)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**
2 + 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sqrt(
2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + 3*sqrt(2)*log(-4*sqrt(2)
*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x
+ 4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) +
6*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Giac [A]
time = 0.99, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1)
```

Mupad [B]
time = 4.62, size = 51, normalized size = 0.45

$$-\frac{x^{3/2}}{2\,\left(x^2+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}-\frac{3}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}+\frac{3}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2 + 1)^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/8 - 3i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(3/8 + 3i/8) - x^(3/2)/(2*(x^2 + 1))
```

3.323 $\int \frac{x^{3/2}}{(1+x^2)^2} \, dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{x}}{2(1+x^2)}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}$$

```
[Out] 1/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2
)-1/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/16*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
-1/2*x^(1/2)/(x^2+1)
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}-\frac{\sqrt{x}}{2(x^2+1)}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2)^2,x]
```

```
[Out] -1/2*Sqrt[x]/(1 + x^2) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) - Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
+ Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{(1+x^2)^2}\,dx = -\frac{\sqrt{x}}{2(1+x^2)} + \frac{1}{4}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= -\frac{\sqrt{x}}{2(1+x^2)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\sqrt{x}}{2(1+x^2)} + \frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\sqrt{x}}{2(1+x^2)} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x\right.$$
$$= -\frac{\sqrt{x}}{2(1+x^2)} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \right.}{4\sqrt{2}}$$
$$= -\frac{\sqrt{x}}{2(1+x^2)} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} -$$

Mathematica [A]
time = 0.13, size = 63, normalized size = 0.56

$$\frac{1}{8}\left(-\frac{4\sqrt{x}}{1+x^2} + \sqrt{2}\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) + \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2)^2,x]
[Out] ((-4*Sqrt[x])/(1 + x^2) + Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] + Sqrt
[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Maple [A]
time = 0.62, size = 69, normalized size = 0.61

method	result
derivativedivides	$-\frac{\sqrt{x}}{2(x^2+1)} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$
default	$-\frac{\sqrt{x}}{2(x^2+1)} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$

risch	$-\frac{\sqrt{x}}{2(x^2+1)}+\frac{\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{8}+\frac{\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{8}+\frac{\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)}{16}$
meijerg	$-\frac{\sqrt{x}}{2(x^2+1)}+\frac{\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)\cdots\right.}{2(x^2)^{\frac{1}{4}}}\cdots\right.}{8}$
trager	$-\frac{\sqrt{x}}{2(x^2+1)}-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^5x+\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+\mathrm{RootOf}\left(\cdots\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-\cdots}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(1/2)/(x^2+1)+1/16*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^
(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.50, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{\sqrt{x}}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
```

Fricas [A]
time = 1.40, size = 140, normalized size = 1.24

$$-\frac{4\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}(x^2+1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\sqrt{2}(x^2+1)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+\sqrt{2}(x^2+1)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)+8\sqrt{x}}{16(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 4*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2
)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - sqrt(2)*(x^2 + 1)*log(4*sqrt(
2)*sqrt(x) + 4*x + 4) + sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
 + 8*sqrt(x))/(x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(95) = 190$.
time = 0.63, size = 257, normalized size = 2.27

$$-\frac{8\sqrt{x}}{16x^2+16}-\frac{\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{16x^2+16}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{\sqrt{2}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1)**2,x)
```

```
[Out] -8*sqrt(x)/(16*x**2 + 16) - sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/
(16*x**2 + 16) + sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 1
6) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)*x*
*2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - sqrt(2)*log(-4*sqrt(2)*sqrt(x
) + 4*x + 4)/(16*x**2 + 16) + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Giac [A]
time = 0.81, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\,\sqrt{x}+x+1\right)-\frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\,\sqrt{x}+x+1\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
```

Mupad [B]
time = 4.68, size = 51, normalized size = 0.45

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)}+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2 + 1)^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/8 - 1i/8) - x^(1/2)/(2*(x^2 + 1))
```

3.324 $\int \frac{\sqrt{x}}{(1+x^2)^2} \, dx$

Optimal. Leaf size=113

$$\frac{x^{3/2}}{2(1+x^2)}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}$$

```
[Out] 1/2*x^(3/2)/(x^2+1)+1/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/8*arctan(1+2^(
1/2)*x^(1/2))*2^(1/2)+1/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/16*ln(1+x+2^(1
/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}+\frac{x^{3/2}}{2(x^2+1)}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2)^2,x]
```

```
[Out] x^(3/2)/(2*(1 + x^2)) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
+ Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2]) -
 Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx = \frac{x^{3/2}}{2(1+x^2)} + \frac{1}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= \frac{x^{3/2}}{2(1+x^2)} + \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{x^{3/2}}{2(1+x^2)} - \frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{x^{3/2}}{2(1+x^2)} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x$$
$$= \frac{x^{3/2}}{2(1+x^2)} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{4\sqrt{2}}$$
$$= \frac{x^{3/2}}{2(1+x^2)} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} -$$

Mathematica [A]
time = 0.11, size = 64, normalized size = 0.57

$$\frac{1}{8}\left(\frac{4x^{3/2}}{1+x^2} + \sqrt{2}\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2)^2,x]
[Out] ((4*x^(3/2))/(1 + x^2) + Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] - Sqrt[
2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Maple [A]
time = 0.44, size = 69, normalized size = 0.61

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{2x^2+2} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$
default	$\frac{x^{\frac{3}{2}}}{2x^2+2} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$
risch	$\frac{x^{\frac{3}{2}}}{2x^2+2} + \frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{8} + \frac{\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{8} + \frac{\sqrt{2}\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)}{16}$

meijerg	$\frac{2x^{\frac{3}{2}}}{4x^2+4}+\frac{x^{\frac{3}{2}}\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{16(x^2)^{\frac{3}{4}}}+\frac{x^{\frac{3}{2}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{8(x^2)^{\frac{3}{4}}}-\frac{x^{\frac{3}{2}}\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x\right.}{16(x^2)^{\frac{3}{4}}}$
trager	$\frac{x^{\frac{3}{2}}}{2x^2+2}-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^5x+\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+\mathrm{RootOf}\left(\_Z^4+1\right)}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-x-1}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)/(x^2+1)+1/16*2^(1/2)*(ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(
1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.49, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\,\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{16}\sqrt{2}\,\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{16}\sqrt{2}\,\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1)
```

Fricas [A]
time = 1.27, size = 140, normalized size = 1.24

$$-\frac{4\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\sqrt{2}\left(x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\sqrt{2}\left(x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8x^{\frac{3}{2}}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 4*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2
)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + sqrt(2)*(x^2 + 1)*log(4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
 - 8*x^(3/2))/(x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(95) = 190.
time = 0.51, size = 257, normalized size = 2.27

$$\frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{\sqrt{2}\,x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{\sqrt{2}\,x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}+\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)**2,x)
```

```
[Out] 8*x**(3/2)/(16*x**2 + 16) + sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/
(16*x**2 + 16) - sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 1
6) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)*x*
*2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + sqrt(2)*log(-4*sqrt(2)*sqrt(x
) + 4*x + 4)/(16*x**2 + 16) - sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Giac [A]
time = 0.73, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{x^{\frac{3}{2}}}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1)
```

Mupad [B]
time = 0.04, size = 50, normalized size = 0.44

$$\frac{x^{3/2}}{2\,(x^2+1)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 + 1)^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/8 - 1i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/8 + 1i/8) + x^(3/2)/(2*(x^2 + 1))
```

3.325 $\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{x}}{2(1+x^2)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt{x}\right.}{8\sqrt{2}}$$

```
[Out] 3/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2
)-3/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+3/16*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
+1/2*x^(1/2)/(x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}+\frac{\sqrt{x}}{2(x^2+1)}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Ar
cTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(
8*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
```

```
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx = \frac{\sqrt{x}}{2(1+x^2)} + \frac{3}{4}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= \frac{\sqrt{x}}{2(1+x^2)} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{x}}{2(1+x^2)} + \frac{3}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{x}}{2(1+x^2)} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx\right.$$
$$= \frac{\sqrt{x}}{2(1+x^2)} - \frac{3\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\right.}{}$$
$$= \frac{\sqrt{x}}{2(1+x^2)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{4\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{4\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\,\sqrt{x}\right.}{8\sqrt{2}}$$

Mathematica [A]
time = 0.11, size = 65, normalized size = 0.58

$$\frac{1}{8}\left(\frac{4\sqrt{x}}{1+x^2} + 3\sqrt{2}\,\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\,\sqrt{x}}\right) + 3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)^2),x]
[Out] ((4*Sqrt[x])/(1 + x^2) + 3*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] + 3*S
qrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Maple [A]
time = 0.61, size = 69, normalized size = 0.61

method	result
derivativedivides	$\frac{\sqrt{x}}{2x^2+2}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
default	$\frac{\sqrt{x}}{2x^2+2}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
risch	$\frac{\sqrt{x}}{2x^2+2}+\frac{3\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{8}+\frac{3\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{8}+\frac{3\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)}{16}$

meijerg	$\frac{2\sqrt{x}}{4x^2+4} - \frac{3\sqrt{x}\,\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{16(x^2)^{\frac{1}{4}}} + \frac{3\sqrt{x}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{8(x^2)^{\frac{1}{4}}} + \frac{3\sqrt{x}\,\sqrt{2}\,}{}$
trager	$\frac{\sqrt{x}}{2x^2+2} + \frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)^3 \ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5 x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-\mathrm{RootOf}\left(\_Z^4+\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2 x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-x-1}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)/(x^2+1)+3/16*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(
1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.55, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1)
```

Fricas [A]
time = 1.05, size = 141, normalized size = 1.25

$$-\frac{12\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-3\sqrt{2}\left(x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+3\sqrt{2}\left(x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\sqrt{x}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt
(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*(x^2 + 1)*log(4*s
qrt(2)*sqrt(x) + 4*x + 4) + 3*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*
x + 4) - 8*sqrt(x))/(x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(102) = 204$.
time = 0.60, size = 264, normalized size = 2.34

$$\frac{8\sqrt{x}}{16x^2+16}-\frac{3\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{3\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}-\frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/x**(1/2),x)
```

```
[Out] 8*sqrt(x)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2
+ 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sqrt(2)
*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - 3*sqrt(2)*log(-4*sqrt(2)*s
qrt(x) + 4*x + 4)/(16*x**2 + 16) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x +
4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*
sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Giac [A]
time = 3.20, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1)
```

Mupad [B]
time = 4.66, size = 50, normalized size = 0.44

$$\frac{\sqrt{x}}{2\left(x^2+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}+\frac{3}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}-\frac{3}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^2 + 1)^2),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/8 + 3i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(3/8 - 3i/8) + x^(1/2)/(2*(x^2 + 1))
```

3.326 $\int \frac{1}{x^{3/2}(1+x^2)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\,(1+x^2)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}$$

```
[Out] -5/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-5/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/
2)-5/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+5/16*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2
)-5/2/x^(1/2)+1/2/(x^2+1)/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{5\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}+\frac{1}{2\sqrt{x}\,(x^2+1)}-\frac{5}{2\sqrt{x}}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)^2),x]
```

```
[Out] -5/(2*Sqrt[x]) + 1/(2*Sqrt[x]*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (5*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(1+x^2)^2}\,dx = \frac{1}{2\sqrt{x}\,(1+x^2)} + \frac{5}{4}\int \frac{1}{x^{3/2}(1+x^2)}\,dx$$
$$= -\frac{5}{2\sqrt{x}} + \frac{1}{2\sqrt{x}\,(1+x^2)} - \frac{5}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{5}{2\sqrt{x}} + \frac{1}{2\sqrt{x}\,(1+x^2)} - \frac{5}{2}\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{5}{2\sqrt{x}} + \frac{1}{2\sqrt{x}\,(1+x^2)} + \frac{5}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \frac{5}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx\right.$$
$$= -\frac{5}{2\sqrt{x}} + \frac{1}{2\sqrt{x}\,(1+x^2)} - \frac{5}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) - \frac{5}{8}\text{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{5}{2\sqrt{x}} + \frac{1}{2\sqrt{x}\,(1+x^2)} - \frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} -$$
$$= -\frac{5}{2\sqrt{x}} + \frac{1}{2\sqrt{x}\,(1+x^2)} + \frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{5\,\mathrm{l}}{}$$

Mathematica [A]
time = 0.17, size = 72, normalized size = 0.59

$$\frac{1}{8}\left(-\frac{4(4+5x^2)}{\sqrt{x}\,(1+x^2)} - 5\sqrt{2}\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) + 5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)^2),x]
```

```
[Out] ((-4*(4 + 5*x^2))/(Sqrt[x]*(1 + x^2)) - 5*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*
Sqrt[x])] + 5*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Maple [A]
time = 0.44, size = 74, normalized size = 0.61

method	result
derivativedivides	$-\frac{x^{\frac{3}{2}}}{2(x^2+1)} - \frac{5\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16} - \frac{2}{\sqrt{x}}$
default	$-\frac{x^{\frac{3}{2}}}{2(x^2+1)} - \frac{5\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16} - \frac{2}{\sqrt{x}}$

risch	$-\frac{5x^2+4}{2\sqrt{x}\,(x^2+1)}-\frac{5\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{8}-\frac{5\sqrt{2}\,\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right)}{16}-\frac{5\arctan\left(1+\sqrt{2}\,\sqrt{x}\right.}{8}$
meijerg	$-\frac{2(5x^2+4)}{\sqrt{x}\,(4x^2+4)}-\frac{5x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2\right.}{2(x^2)^{\frac{3}{4}}}\right.}{8}$
trager	$-\frac{5x^2+4}{2\sqrt{x}\,(x^2+1)}+\frac{5\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^5x+\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+\mathrm{RootC}}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right.}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(3/2)/(x^2+1)-5/16*2^(1/2)*(ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^
(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/x^(1/2)
```

Maxima [A]

time = 0.50, size = 92, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{5x^2+4}{2\left(x^{\frac{5}{2}}+\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^(5/
2) + sqrt(x))
```

Fricas [A]

time = 1.22, size = 148, normalized size = 1.21

$$\frac{20\sqrt{2}(x^3+x)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}(x^3+x)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+5\sqrt{2}(x^3+x)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-5\sqrt{2}(x^3+x)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8(5x^2+4)\sqrt{x}}{16(x^3+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*sqrt(2)*(x^3 + x)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^3 + x)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*(x^3 + x)*log(4*sq
```

```
rt(2)*sqrt(x) + 4*x + 4) - 5*sqrt(2)*(x^3 + x)*log(-4*sqrt(2)*sqrt(x) + 4*x
 + 4) - 8*(5*x^2 + 4)*sqrt(x))/(x^3 + x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(112) = 224$.
time = 0.84, size = 366, normalized size = 3.00

$$-\frac{5\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}+\frac{5\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{5\sqrt{2}\sqrt{x}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}+\frac{5\sqrt{2}\sqrt{x}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{40x^{2}}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{32}{16x^{\frac{5}{2}}+16\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1)**2,x)
```

```
[Out] -5*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2) + 16*sqr
t(x)) + 5*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2) +
16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**(5/2) +
16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**(5/2) +
16*sqrt(x)) - 5*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5
/2) + 16*sqrt(x)) + 5*sqrt(2)*sqrt(x)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1)/(16*x
**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1)/(16*x*
*(5/2) + 16*sqrt(x)) - 40*x**2/(16*x**(5/2) + 16*sqrt(x)) - 32/(16*x**(5/2)
 + 16*sqrt(x))
```

Giac [A]
time = 2.04, size = 92, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{5x^2+4}{2\left(x^{\frac{5}{2}}+\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^(5/
2) + sqrt(x))
```

Mupad [B]
time = 4.68, size = 55, normalized size = 0.45

$$-\frac{\frac{5x^2}{2}+2}{\sqrt{x}+x^{5/2}}+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}+\frac{5}{8}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}-\frac{5}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^2 + 1)^2),x)
```

```
[Out] - ((5*x^2)/2 + 2)/(x^(1/2) + x^(5/2)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 -
 1i/2))*(5/8 - 5i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/8 + 5i
/8)
```

3.327 $\int \frac{1}{x^{5/2}(1+x^2)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}(1+x^2)}+\frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{7\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{7\log}{}$$

```
[Out] -7/6/x^(3/2)+1/2/x^(3/2)/(x^2+1)-7/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-7/8
*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+7/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-7/1
6*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{7\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}-\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}(x^2+1)}+\frac{7\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{7\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)^2),x]
```

```
[Out] -7/(6*x^(3/2)) + 1/(2*x^(3/2)*(1 + x^2)) + (7*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (7*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (7*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
```

```
b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(1+x^2)^2}\,dx = \frac{1}{2x^{3/2}(1+x^2)} + \frac{7}{4}\int \frac{1}{x^{5/2}(1+x^2)}\,dx$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}(1+x^2)} - \frac{7}{4}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}(1+x^2)} - \frac{7}{2}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}(1+x^2)} - \frac{7}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \frac{7}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x\right.$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}(1+x^2)} - \frac{7}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) - \frac{7}{8}\text{Subst}\left(\int \frac{}{1+}\right.$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}(1+x^2)} + \frac{7\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{7\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{7}{}$$
$$= -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}(1+x^2)} + \frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{7\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{7\log}{}$$

Mathematica [A]
time = 0.15, size = 72, normalized size = 0.59

$$\frac{1}{24}\left(-\frac{4(4+7x^2)}{x^{3/2}(1+x^2)} - 21\sqrt{2}\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 21\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)^2),x]
[Out] ((-4*(4 + 7*x^2))/(x^(3/2)*(1 + x^2)) - 21*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]
*Sqrt[x])] - 21*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/24
```

Maple [A]
time = 0.47, size = 74, normalized size = 0.61

method	result
derivativedivides	$-\frac{\sqrt{x}}{2(x^2+1)} - \frac{7\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16} - \frac{2}{3x^{\frac{3}{2}}}$
default	$-\frac{\sqrt{x}}{2(x^2+1)} - \frac{7\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16} - \frac{2}{3x^{\frac{3}{2}}}$

risch	$-\frac{7x^2+4}{6(x^2+1)x^{\frac{3}{2}}}-\frac{7\arctan(1+\sqrt{2}\,\sqrt{x})\sqrt{2}}{8}-\frac{7\arctan(-1+\sqrt{2}\,\sqrt{x})\sqrt{2}}{8}-\frac{7\sqrt{2}\,\ln\Big(\frac{1+x+\sqrt{2}\,\sqrt{}}{1+x-\sqrt{2}\,\sqrt{}}}{16}$
meijerg	$-\frac{2(7x^2+4)}{3x^{\frac{3}{2}}(4x^2+4)}-\frac{7\sqrt{x}\,\Big(-\frac{\sqrt{2}\,\ln(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2})}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln(1+\sqrt{2}}{2(x}}{8}$
trager	$-\frac{7x^2+4}{6(x^2+1)x^{\frac{3}{2}}}+\frac{7\,\mathrm{RootOf}(\_Z^4+1)^3\ln\Big(-\frac{-\mathrm{RootOf}(\_Z^4+1)^5x+\mathrm{RootOf}(\_Z^4+1)^5-2\,\mathrm{RootOf}(\_Z^4+1)^3+\mathrm{RootO}}{\mathrm{RootOf}(\_Z^4+1)^2x-\mathrm{RootOf}(\_Z^4+1}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(1/2)/(x^2+1)-7/16*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^
(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/3/x^(3/
2)
```

Maxima [A]
time = 0.62, size = 92, normalized size = 0.75

$$-\frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{7}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{7}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{7}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{7x^2+4}{6\left(x^{\frac{7}{2}}+x^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/6*(7*x^2 + 4)/(x^(7/
2) + x^(3/2))
```

Fricas [A]
time = 1.06, size = 158, normalized size = 1.30

$$\frac{84\sqrt{2}(x^4+x^2)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+84\sqrt{2}(x^4+x^2)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-21\sqrt{2}(x^4+x^2)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+21\sqrt{2}(x^4+x^2)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8(7x^2+4)\sqrt{x}}{48(x^4+x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(84*sqrt(2)*(x^4 + x^2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
 sqrt(2)*sqrt(x) - 1) + 84*sqrt(2)*(x^4 + x^2)*arctan(1/2*sqrt(2)*sqrt(-4*s
qrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 21*sqrt(2)*(x^4 + x^2)*l
```

```
og(4*sqrt(2)*sqrt(x) + 4*x + 4) + 21*sqrt(2)*(x^4 + x^2)*log(-4*sqrt(2)*sqr
t(x) + 4*x + 4) - 8*(7*x^2 + 4)*sqrt(x))/(x^4 + x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(112) = 224$.
time = 1.39, size = 366, normalized size = 3.00

$$\frac{21\sqrt{2}\,x^{\frac{7}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{21\sqrt{2}\,x^{\frac{7}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}\,x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}\,x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}+\frac{21\sqrt{2}\,x^{\frac{3}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{21\sqrt{2}\,x^{\frac{3}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}\,x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}\,x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{56x^2}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{32}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1)**2,x)
```

```
[Out] 21*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2) + 48*x**
(3/2)) - 21*sqrt(2)*x**(7/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2)
+ 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(48*x**(7/2)
 + 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) + 1)/(48*x**(7/2
) + 48*x**(3/2)) + 21*sqrt(2)*x**(3/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(4
8*x**(7/2) + 48*x**(3/2)) - 21*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x) + 4*x
 + 4)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(x
) - 1)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(
x) + 1)/(48*x**(7/2) + 48*x**(3/2)) - 56*x**2/(48*x**(7/2) + 48*x**(3/2)) -
 32/(48*x**(7/2) + 48*x**(3/2))
```

Giac [A]
time = 1.02, size = 91, normalized size = 0.75

$$-\frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{7}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{7}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{7}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
- 2/3/x^(3/2)
```

Mupad [B]
time = 0.08, size = 55, normalized size = 0.45

$$-\frac{\frac{7x^2}{6}+\frac{2}{3}}{x^{3/2}+x^{7/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{7}{8}-\frac{7}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{7}{8}+\frac{7}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^2 + 1)^2),x)
```

```
[Out] - ((7*x^2)/6 + 2/3)/(x^(3/2) + x^(7/2)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2
 - 1i/2))*(7/8 + 7i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(7/8 -
7i/8)
```

3.328 $\int \frac{1}{x^{7/2}(1+x^2)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}(1+x^2)}-\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\log\left(1-\sqrt{2}\sqrt{x}+\right.}{8\sqrt{2}}$$

```
[Out] -9/10/x^(5/2)+1/2/x^(5/2)/(x^2+1)+9/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+9/
8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+9/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-9/
16*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)+9/2/x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{9\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}-\frac{9}{10x^{5/2}}+\frac{1}{2x^{5/2}(x^2+1)}+\frac{9}{2\sqrt{x}}+\frac{9\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)^2),x]
```

```
[Out] -9/(10*x^(5/2)) + 9/(2*Sqrt[x]) + 1/(2*x^(5/2)*(1 + x^2)) - (9*ArcTan[1 - S
qrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2])
+ (9*Log[1 - Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (9*Log[1 + Sqrt[2]*Sqrt[x]
 + x])/(8*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}(1+x^2)^2}\,dx = \frac{1}{2x^{5/2}(1+x^2)} + \frac{9}{4}\int \frac{1}{x^{7/2}(1+x^2)}\,dx$$
$$= -\frac{9}{10x^{5/2}} + \frac{1}{2x^{5/2}(1+x^2)} - \frac{9}{4}\int \frac{1}{x^{3/2}(1+x^2)}\,dx$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}(1+x^2)} + \frac{9}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}(1+x^2)} + \frac{9}{2}\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}(1+x^2)} - \frac{9}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{9}{4}\text{Subst}\left(\int\right.$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}(1+x^2)} + \frac{9}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{9}{8}\text{Su}$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}(1+x^2)} + \frac{9\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{9\log\left(1+\sqrt{2}\sqrt{x}\right.}{8\sqrt{2}}$$
$$= -\frac{9}{10x^{5/2}} + \frac{9}{2\sqrt{x}} + \frac{1}{2x^{5/2}(1+x^2)} - \frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{9\tan^{-1}\left(1+\sqrt{2}\sqrt{\right.}}{4\sqrt{2}}$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 0.59

$$\frac{1}{40}\left(\frac{4(-4+36x^2+45x^4)}{x^{5/2}(1+x^2)} + 45\sqrt{2}\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 45\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)^2),x]
[Out] ((4*(-4 + 36*x^2 + 45*x^4))/(x^(5/2)*(1 + x^2)) + 45*Sqrt[2]*ArcTan[(-1 + x
)/(Sqrt[2]*Sqrt[x])] - 45*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/40
```

Maple [A]
time = 0.46, size = 79, normalized size = 0.60

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{2x^2+2} + \frac{9\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16} - \frac{2}{5x^{\frac{5}{2}}} +$

default	$\frac{x^{\frac{3}{2}}}{2x^{2}+2}+\frac{9\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}-\frac{2}{5x^{\frac{5}{2}}}+\frac{}{\sqrt{}}$
risch	$\frac{45x^{4}+36x^{2}-4}{10(x^{2}+1)x^{\frac{5}{2}}}+\frac{9\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{8}+\frac{9\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{8}+\frac{9\sqrt{2}\ln\left(\frac{1+x-\sqrt{2}\sqrt{}}{1+x+\sqrt{2}\sqrt{}}\right.}{16}$
meijerg	$-\frac{2(-45x^{4}-36x^{2}+4)}{5x^{\frac{5}{2}}(4x^{2}+4)}+\frac{9x^{\frac{3}{2}}\left(\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\ln\left(1+\sqrt{2}\right.}{2\left(x\right.}\right.}{8}$
trager	$\frac{45x^{4}+36x^{2}-4}{10(x^{2}+1)x^{\frac{5}{2}}}+\frac{9\,\mathrm{RootOf}\left(\_Z^{4}+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}+2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}-\mathrm{RootOf}\left(\_\right.}{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}-}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)/(x^2+1)+9/16*2^(1/2)*(ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(
1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/5/x^(5/2
)+4/x^(1/2)
```

Maxima [A]
time = 0.56, size = 97, normalized size = 0.74

$$\frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{9}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{45x^{4}+36x^{2}-4}{10\left(x^{\frac{9}{2}}+x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/10*(45*x^4 + 36*x^2 -
 4)/(x^(9/2) + x^(5/2))
```

Fricas [A]
time = 1.62, size = 163, normalized size = 1.24

$$-\frac{180\sqrt{2}\left(x^{5}+x^{3}\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+180\sqrt{2}\left(x^{5}+x^{3}\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+45\sqrt{2}\left(x^{5}+x^{3}\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-45\sqrt{2}\left(x^{5}+x^{3}\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(45x^{4}+36x^{2}-4\right)\sqrt{x}}{80\left(x^{5}+x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/80*(180*sqrt(2)*(x^5 + x^3)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1)
 - sqrt(2)*sqrt(x) - 1) + 180*sqrt(2)*(x^5 + x^3)*arctan(1/2*sqrt(2)*sqrt(-
4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 45*sqrt(2)*(x^5 + x^3
)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 45*sqrt(2)*(x^5 + x^3)*log(-4*sqrt(2)*
sqrt(x) + 4*x + 4) - 8*(45*x^4 + 36*x^2 - 4)*sqrt(x))/(x^5 + x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(121) = 242$.
time = 2.64, size = 384, normalized size = 2.93

$$\frac{45\sqrt{2}\,x^{\frac{9}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}-\frac{45\sqrt{2}\,x^{\frac{9}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}\,x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}\,x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{45\sqrt{2}\,x^{\frac{5}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}-\frac{45\sqrt{2}\,x^{\frac{5}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}\,x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}\,x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{360x^4}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{288x^2}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}-\frac{32}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1)**2,x)
```

```
[Out] 45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2) + 80*x**
(5/2)) - 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2)
+ 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(80*x**(9/2)
 + 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(80*x**(9/2
) + 80*x**(5/2)) + 45*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(8
0*x**(9/2) + 80*x**(5/2)) - 45*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x
 + 4)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x
) - 1)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(
x) + 1)/(80*x**(9/2) + 80*x**(5/2)) + 360*x**4/(80*x**(9/2) + 80*x**(5/2))
+ 288*x**2/(80*x**(9/2) + 80*x**(5/2)) - 32/(80*x**(9/2) + 80*x**(5/2))
```

Giac [A]
time = 0.98, size = 98, normalized size = 0.75

$$\frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{9}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}+\frac{2\left(10x^2-1\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1) +
 2/5*(10*x^2 - 1)/x^(5/2)
```

Mupad [B]
time = 0.07, size = 59, normalized size = 0.45

$$\frac{\frac{9x^4}{2}+\frac{18x^2}{5}-\frac{2}{5}}{x^{5/2}+x^{9/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{9}{8}-\frac{9}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{9}{8}+\frac{9}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^2 + 1)^2),x)
```

```
[Out] ((18*x^2)/5 + (9*x^4)/2 - 2/5)/(x^(5/2) + x^(9/2)) + 2^(1/2)*atan(2^(1/2)*x
^(1/2)*(1/2 - 1i/2))*(9/8 - 9i/8) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/
2))*(9/8 + 9i/8)
```

3.329 $\int \frac{x^{7/2}}{(1+x^2)^3} dx$

Optimal. Leaf size=129

$$-\frac{x^{5/2}}{4(1+x^2)^2}-\frac{5\sqrt{x}}{16(1+x^2)}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\cdots$$

```
[Out] -1/4*x^(5/2)/(x^2+1)^2+5/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+5/64*arctan(
1+2^(1/2)*x^(1/2))*2^(1/2)-5/128*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+5/128*ln(1
+x+2^(1/2)*x^(1/2))*2^(1/2)-5/16*x^(1/2)/(x^2+1)
```

Rubi [A]
time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{5\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}-\frac{5\sqrt{x}}{16(x^2+1)}-\frac{x^{5/2}}{4(x^2+1)^2}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2)^3,x]
```

```
[Out] -1/4*x^(5/2)/(1 + x^2)^2 - (5*Sqrt[x])/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
(5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{(1+x^2)^3}\,dx = -\frac{x^{5/2}}{4(1+x^2)^2} + \frac{5}{8}\int \frac{x^{3/2}}{(1+x^2)^2}\,dx$$
$$= -\frac{x^{5/2}}{4(1+x^2)^2} - \frac{5\sqrt{x}}{16(1+x^2)} + \frac{5}{32}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= -\frac{x^{5/2}}{4(1+x^2)^2} - \frac{5\sqrt{x}}{16(1+x^2)} + \frac{5}{16}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{x^{5/2}}{4(1+x^2)^2} - \frac{5\sqrt{x}}{16(1+x^2)} + \frac{5}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{5}{32}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx$$
$$= -\frac{x^{5/2}}{4(1+x^2)^2} - \frac{5\sqrt{x}}{16(1+x^2)} + \frac{5}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{5}{64}\text{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{x^{5/2}}{4(1+x^2)^2} - \frac{5\sqrt{x}}{16(1+x^2)} - \frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{5S}{}$$
$$= -\frac{x^{5/2}}{4(1+x^2)^2} - \frac{5\sqrt{x}}{16(1+x^2)} - \frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} - \frac{5\log}{}$$

Mathematica [A]
time = 0.19, size = 72, normalized size = 0.56

$$\frac{1}{64}\left(-\frac{4\sqrt{x}\,(5+9x^2)}{(1+x^2)^2} + 5\sqrt{2}\,\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) + 5\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2)^3,x]
[Out] ((-4*Sqrt[x]*(5 + 9*x^2))/(1 + x^2)^2 + 5*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*
Sqrt[x])] + 5*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Maple [A]
time = 0.46, size = 77, normalized size = 0.60

method	result
derivativedivides	$\frac{-\frac{9x^{\frac{5}{2}}}{16}-\frac{5\sqrt{x}}{16}}{(x^2+1)^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$

default	$\frac{-\frac{9x^{\frac{5}{2}}}{16}-\frac{5\sqrt{x}}{16}}{(x^2+1)^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
risch	$-\frac{(9x^2+5)\sqrt{x}}{16(x^2+1)^2}+\frac{5\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{64}+\frac{5\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{64}+\frac{5\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}\cdots}{1+x-\sqrt{2}\cdots}\right.}{128}$
meijerg	$-\frac{\sqrt{x}\,(81x^2+45)}{144(x^2+1)^2}+\frac{5\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\cdots\right.}{2\cdots}\right.}{64}$
trager	$-\frac{(9x^2+5)\sqrt{x}}{16(x^2+1)^2}+\frac{5\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x+\mathrm{RootOf}\cdots}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right.\cdots}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-9/32*x^(5/2)-5/32*x^(1/2))/(x^2+1)^2+5/128*2^(1/2)*(ln((1+x+2^(1/2)*x^(
1/2))/(1+x-2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2
)*x^(1/2)))
```

Maxima [A]
time = 0.51, size = 99, normalized size = 0.77

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{9x^{\frac{5}{2}}+5\sqrt{x}}{16\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2) + 5
*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Fricas [A]
time = 1.45, size = 173, normalized size = 1.34

$$-\frac{20\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-5\sqrt{2}\left(x^4+2x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+5\sqrt{2}\left(x^4+2x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)+8\left(9x^2+5\right)\sqrt{x}}{128\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 5*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 5*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) + 8*(9*x^2 + 5)*sqrt(x))/(x^4 + 2*x
^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(117) = 234$.
time = 3.01, size = 481, normalized size = 3.73

$$-\frac{72x^{\frac{5}{2}}}{128x^4+256x^2+128}-\frac{40\sqrt{x}}{128x^4+256x^2+128}-\frac{5\sqrt{2}\,x^4\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}\,x^4\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,x^4\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,x^4\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{10\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{20\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{20\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{5\sqrt{2}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1)**3,x)
```

```
[Out] -72*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 40*sqrt(x)/(128*x**4 + 256*x**2
+ 128) - 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x
**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2
 + 128) - 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 25
6*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x
**2 + 128) - 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) + 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128)
+ 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A]
time = 1.51, size = 94, normalized size = 0.73

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{9x^{\frac{5}{2}}+5\sqrt{x}}{16\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2) + 5
*sqrt(x))/(x^2 + 1)^2
```

Mupad [B]
time = 4.73, size = 62, normalized size = 0.48

$$-\frac{\frac{5\sqrt{x}}{16}+\frac{9x^{5/2}}{16}}{x^4+2x^2+1}+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}+\frac{5}{64}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}-\frac{5}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/64 + 5i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/64 - 5i/64) - ((5*x^(1/2))/16 + (9*x^(5/2))/
16)/(2*x^2 + x^4 + 1)
```

3.330 $\int \frac{x^{5/2}}{(1+x^2)^3} \, dx$

Optimal. Leaf size=129

$$-\frac{x^{3/2}}{4(1+x^2)^2}+\frac{3x^{3/2}}{16(1+x^2)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}$$

```
[Out] -1/4*x^(3/2)/(x^2+1)^2+3/16*x^(3/2)/(x^2+1)+3/64*arctan(-1+2^(1/2)*x^(1/2))
*2^(1/2)+3/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+3/128*ln(1+x-2^(1/2)*x^(1/2
))*2^(1/2)-3/128*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}+\frac{3x^{3/2}}{16(x^2+1)}-\frac{x^{3/2}}{4(x^2+1)^2}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2)^3,x]
```

```
[Out] -1/4*x^(3/2)/(1 + x^2)^2 + (3*x^(3/2))/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) +
(3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{(1+x^2)^3}\,dx = -\frac{x^{3/2}}{4(1+x^2)^2}+\frac{3}{8}\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2}+\frac{3x^{3/2}}{16(1+x^2)}+\frac{3}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2}+\frac{3x^{3/2}}{16(1+x^2)}+\frac{3}{16}\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2}+\frac{3x^{3/2}}{16(1+x^2)}-\frac{3}{32}\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{32}\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx\right.$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2}+\frac{3x^{3/2}}{16(1+x^2)}+\frac{3}{64}\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{64}\mathrm{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2}+\frac{3x^{3/2}}{16(1+x^2)}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3S}{}$$
$$= -\frac{x^{3/2}}{4(1+x^2)^2}+\frac{3x^{3/2}}{16(1+x^2)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\log}{}$$

Mathematica [A]
time = 0.17, size = 72, normalized size = 0.56

$$\frac{1}{64}\left(\frac{4x^{3/2}(-1+3x^2)}{(1+x^2)^2}+3\sqrt{2}\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)-3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2)^3,x]
[Out] ((4*x^(3/2)*(-1 + 3*x^2))/(1 + x^2)^2 + 3*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*
Sqrt[x])] - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Maple [A]
time = 0.45, size = 77, normalized size = 0.60

method	result
derivativedivides	$\frac{\frac{3x^{\frac{7}{2}}}{16}-\frac{x^{\frac{3}{2}}}{16}}{(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$

default	$\frac{\frac{3x^{\frac{7}{2}}}{16}-\frac{x^{\frac{3}{2}}}{16}}{(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
risch	$\frac{x^{\frac{3}{2}}(3x^2-1)}{16(x^2+1)^2}+\frac{3\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{64}+\frac{3\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{64}+\frac{3\sqrt{2}\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right.}{128}$
meijerg	$-\frac{x^{\frac{3}{2}}(-21x^2+7)}{112(x^2+1)^2}+\frac{3x^{\frac{3}{2}}\left(\frac{\sqrt{2}\ln\left(1-\sqrt{2}(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2)^{\frac{1}{4}}}{2-\sqrt{2}(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\ln\left(1+\sqrt{2}(x^2\right.}{2(x^2)^{\frac{3}{4}}}\right.}{64}$
trager	$\frac{x^{\frac{3}{2}}(3x^2-1)}{16(x^2+1)^2}-\frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^5x+\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+\mathrm{RootOf}\left(\_\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-x}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/32*x^(7/2)-1/32*x^(3/2))/(x^2+1)^2+3/128*2^(1/2)*(ln((1+x-2^(1/2)*x^(1
/2))/(1+x+2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)
*x^(1/2)))
```

Maxima [A]

time = 0.50, size = 99, normalized size = 0.77

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{3x^{\frac{7}{2}}-x^{\frac{3}{2}}}{16(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2) - x
^(3/2))/(x^4 + 2*x^2 + 1)
```

Fricas [A]

time = 1.29, size = 175, normalized size = 1.36

$$-\frac{12\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}(x^4+2x^2+1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+3\sqrt{2}(x^4+2x^2+1)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-3\sqrt{2}(x^4+2x^2+1)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8(3x^3-x)\sqrt{x}}{128(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 3*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 3*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(3*x^3 - x)*sqrt(x))/(x^4 + 2*x
^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(117) = 234$.

time = 2.45, size = 481, normalized size = 3.73

$$\frac{24x^{\frac{7}{2}}}{128x^4+256x^2+128}-\frac{8x^{\frac{3}{2}}}{128x^4+256x^2+128}+\frac{3\sqrt{2}\,x^4\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{3\sqrt{2}\,x^4\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\,x^4\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\,x^4\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{6\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{12\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{12\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}+\frac{3\sqrt{2}\,\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{3\sqrt{2}\,\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1)**3,x)
```

```
[Out] 24*x**(7/2)/(128*x**4 + 256*x**2 + 128) - 8*x**(3/2)/(128*x**4 + 256*x**2 +
 128) + 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) - 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**
2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128) + 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**
2 + 128) - 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**
2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
 128) + 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 12
8) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128) + 6*sq
rt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A]

time = 0.70, size = 94, normalized size = 0.73

$$\frac{3}{64}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+2\,\sqrt{x}\right)\right)+\frac{3}{64}\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-2\,\sqrt{x}\right)\right)-\frac{3}{128}\,\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt{x}+x+1\right)+\frac{3}{128}\,\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt{x}+x+1\right)+\frac{3\,x^{\frac{7}{2}}-x^{\frac{3}{2}}}{16\,(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2) - x
^(3/2))/(x^2 + 1)^2
```

Mupad [B]

time = 0.07, size = 62, normalized size = 0.48

$$-\frac{\frac{x^{3/2}}{16}-\frac{3\,x^{7/2}}{16}}{x^4+2\,x^2+1}+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/64 - 3i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(3/64 + 3i/64) - (x^(3/2)/16 - (3*x^(7/2))/16)/
(2*x^2 + x^4 + 1)
```

3.331 $\int \frac{x^{3/2}}{(1+x^2)^3}\,dx$

Optimal. Leaf size=129

$$-\frac{\sqrt{x}}{4(1+x^2)^2}+\frac{\sqrt{x}}{16(1+x^2)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+$$

```
[Out] 3/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/64*arctan(1+2^(1/2)*x^(1/2))*2^(1
/2)-3/128*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+3/128*ln(1+x+2^(1/2)*x^(1/2))*2^(
1/2)-1/4*x^(1/2)/(x^2+1)^2+1/16*x^(1/2)/(x^2+1)
```

Rubi [A]
time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}+\frac{\sqrt{x}}{16(x^2+1)}-\frac{\sqrt{x}}{4(x^2+1)^2}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2)^3,x]
```

```
[Out] -1/4*Sqrt[x]/(1 + x^2)^2 + Sqrt[x]/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*S
qrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (3*L
og[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x]
)/(64*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{(1+x^2)^3}\,dx = -\frac{\sqrt{x}}{4(1+x^2)^2}+\frac{1}{8}\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx$$
$$= -\frac{\sqrt{x}}{4(1+x^2)^2}+\frac{\sqrt{x}}{16(1+x^2)}+\frac{3}{32}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= -\frac{\sqrt{x}}{4(1+x^2)^2}+\frac{\sqrt{x}}{16(1+x^2)}+\frac{3}{16}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{\sqrt{x}}{4(1+x^2)^2}+\frac{\sqrt{x}}{16(1+x^2)}+\frac{3}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{32}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx$$
$$= -\frac{\sqrt{x}}{4(1+x^2)^2}+\frac{\sqrt{x}}{16(1+x^2)}+\frac{3}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{64}\text{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{\sqrt{x}}{4(1+x^2)^2}+\frac{\sqrt{x}}{16(1+x^2)}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3S}{}$$
$$= -\frac{\sqrt{x}}{4(1+x^2)^2}+\frac{\sqrt{x}}{16(1+x^2)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{3\log}{}$$

Mathematica [A]
time = 0.16, size = 70, normalized size = 0.54

$$\frac{1}{64}\left(\frac{4\sqrt{x}\,(-3+x^2)}{(1+x^2)^2}+3\sqrt{2}\,\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)+3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2)^3,x]
```

```
[Out] ((4*Sqrt[x]*(-3 + x^2))/(1 + x^2)^2 + 3*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sq
rt[x])] + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Maple [A]
time = 0.47, size = 77, normalized size = 0.60

method	result
derivativedivides	$\frac{\frac{x^{\frac{5}{2}}}{16}-\frac{3\sqrt{x}}{16}}{(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$

default	$\frac{\frac{x^{\frac{5}{2}}}{16}-\frac{3\sqrt{x}}{16}}{(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
risch	$\frac{(x^2-3)\sqrt{x}}{16(x^2+1)^2}+\frac{3\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{64}+\frac{3\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{64}+\frac{3\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right.}{128}$
meijerg	$-\frac{\sqrt{x}\,(-5x^2+15)}{80(x^2+1)^2}+\frac{3\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\right.}{}\right.}{64}$
trager	$\frac{(x^2-3)\sqrt{x}}{16(x^2+1)^2}+\frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-\mathrm{RootOf}\left(\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*x^(5/2)-3/32*x^(1/2))/(x^2+1)^2+3/128*2^(1/2)*(ln((1+x+2^(1/2)*x^(1
/2))/(1+x-2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)
*x^(1/2)))
```

Maxima [A]

time = 0.50, size = 97, normalized size = 0.75

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{x^{\frac{5}{2}}-3\sqrt{x}}{16\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) - 3*s
qrt(x))/(x^4 + 2*x^2 + 1)
```

Fricas [A]

time = 1.09, size = 171, normalized size = 1.33

$$-\frac{12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-3\sqrt{2}\left(x^4+2x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+3\sqrt{2}\left(x^4+2x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(x^2-3\right)\sqrt{x}}{128\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 3*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(x^2 - 3)*sqrt(x))/(x^4 + 2*x^2
 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(116) = 232$.
time = 1.66, size = 481, normalized size = 3.73

$$\frac{8x^{\frac{5}{2}}}{128x^4 + 256x^2 + 128} - \frac{24\sqrt{x}}{128x^4 + 256x^2 + 128} - \frac{3\sqrt{2}\,x^4\log\left(-4\sqrt{2}\,\sqrt{x} + 4x + 4\right)}{128x^4 + 256x^2 + 128} + \frac{3\sqrt{2}\,x^4\log\left(4\sqrt{2}\,\sqrt{x} + 4x + 4\right)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}\,x^4\operatorname{atan}\left(\sqrt{2}\,\sqrt{x} - 1\right)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}\,x^4\operatorname{atan}\left(\sqrt{2}\,\sqrt{x} + 1\right)}{128x^4 + 256x^2 + 128} - \frac{6\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x} + 4x + 4\right)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x} + 4x + 4\right)}{128x^4 + 256x^2 + 128} + \frac{12\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x} - 1\right)}{128x^4 + 256x^2 + 128} + \frac{12\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x} + 1\right)}{128x^4 + 256x^2 + 128} - \frac{3\sqrt{2}\log\left(-4\sqrt{2}\,\sqrt{x} + 4x + 4\right)}{128x^4 + 256x^2 + 128} + \frac{3\sqrt{2}\log\left(4\sqrt{2}\,\sqrt{x} + 4x + 4\right)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x} - 1\right)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x} + 1\right)}{128x^4 + 256x^2 + 128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1)**3,x)
```

```
[Out] 8*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 24*sqrt(x)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**
2 + 128) + 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2
 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 1
28) - 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2
 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 1
28) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 128
) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128) + 6*sqr
t(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A]
time = 0.94, size = 92, normalized size = 0.71

$$\frac{3}{64}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2} + 2\,\sqrt{x}\right)\right) + \frac{3}{64}\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2} - 2\,\sqrt{x}\right)\right) + \frac{3}{128}\,\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt{x} + x + 1\right) - \frac{3}{128}\,\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt{x} + x + 1\right) + \frac{x^{\frac{5}{2}} - 3\,\sqrt{x}}{16\,(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) - 3*s
qrt(x))/(x^2 + 1)^2
```

Mupad [B]
time = 4.69, size = 62, normalized size = 0.48

$$-\frac{\frac{3\,\sqrt{x}}{16} - \frac{x^{5/2}}{16}}{x^4 + 2\,x^2 + 1} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{64} + \frac{3}{64}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{64} - \frac{3}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/64 + 3i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(3/64 - 3i/64) - ((3*x^(1/2))/16 - x^(5/2)/16)/
(2*x^2 + x^4 + 1)
```

3.332 $\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx$

Optimal. Leaf size=129

$$\frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{5\,\mathrm{l}}{}$$

```
[Out] 1/4*x^(3/2)/(x^2+1)^2+5/16*x^(3/2)/(x^2+1)+5/64*arctan(-1+2^(1/2)*x^(1/2))*
2^(1/2)+5/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+5/128*ln(1+x-2^(1/2)*x^(1/2)
)*2^(1/2)-5/128*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{5\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}+\frac{5x^{3/2}}{16\left(x^2+1\right)}+\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2)^3,x]
```

```
[Out] x^(3/2)/(4*(1 + x^2)^2) + (5*x^(3/2))/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[2
]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (
5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] +
 x])/(64*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx = \frac{x^{3/2}}{4(1+x^2)^2} + \frac{5}{8}\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx$$
$$= \frac{x^{3/2}}{4(1+x^2)^2} + \frac{5x^{3/2}}{16(1+x^2)} + \frac{5}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= \frac{x^{3/2}}{4(1+x^2)^2} + \frac{5x^{3/2}}{16(1+x^2)} + \frac{5}{16}\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{x^{3/2}}{4(1+x^2)^2} + \frac{5x^{3/2}}{16(1+x^2)} - \frac{5}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{5}{32}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x$$
$$= \frac{x^{3/2}}{4(1+x^2)^2} + \frac{5x^{3/2}}{16(1+x^2)} + \frac{5}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{5}{64}\text{Subst}\left(\int \frac{}{1+}\right.$$
$$= \frac{x^{3/2}}{4(1+x^2)^2} + \frac{5x^{3/2}}{16(1+x^2)} + \frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{5\text{Su}}{}$$
$$= \frac{x^{3/2}}{4(1+x^2)^2} + \frac{5x^{3/2}}{16(1+x^2)} - \frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{5\log\left(1\right.}{}$$

Mathematica [A]
time = 0.10, size = 72, normalized size = 0.56

$$\frac{1}{64}\left(\frac{4x^{3/2}(9+5x^2)}{(1+x^2)^2} + 5\sqrt{2}\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2)^3,x]
```

```
[Out] ((4*x^(3/2)*(9 + 5*x^2))/(1 + x^2)^2 + 5*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*S
qrt[x])] - 5*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Maple [A]
time = 0.44, size = 81, normalized size = 0.63

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{4(x^2+1)^2} + \frac{5x^{\frac{3}{2}}}{16(x^2+1)} + \frac{5\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right.\right.}{128}$

default	$\frac{x^{\frac{3}{2}}}{4(x^2+1)^2}+\frac{5x^{\frac{3}{2}}}{16(x^2+1)}+\frac{5\sqrt{2}\,\left(\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right.}{128}$
risch	$\frac{x^{\frac{3}{2}}(5x^2+9)}{16(x^2+1)^2}+\frac{5\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{64}+\frac{5\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{64}+\frac{5\sqrt{2}\,\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right.}{128}$
meijerg	$\frac{x^{\frac{3}{2}}(15x^2+27)}{48(x^2+1)^2}+\frac{5x^{\frac{3}{2}}\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{128(x^2)^{\frac{3}{4}}}+\frac{5x^{\frac{3}{2}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{64(x^2)^{\frac{3}{4}}}-\frac{5x^{\frac{3}{2}}\sqrt{2}\,\ln\left(\right.}{}$
trager	$\frac{x^{\frac{3}{2}}(5x^2+9)}{16(x^2+1)^2}+\frac{5\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x+\mathrm{RootOf}\left(\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2+}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)/(x^2+1)^2+5/16*x^(3/2)/(x^2+1)+5/128*2^(1/2)*(ln((1+x-2^(1/2)*x
^(1/2))/(1+x+2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1
/2)*x^(1/2)))
```

Maxima [A]
time = 0.51, size = 99, normalized size = 0.77

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{5x^{\frac{7}{2}}+9x^{\frac{3}{2}}}{16(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2) + 9
*x^(3/2))/(x^4 + 2*x^2 + 1)
```

Fricas [A]
time = 0.95, size = 175, normalized size = 1.36

$$-\frac{20\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}(x^4+2x^2+1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+5\sqrt{2}(x^4+2x^2+1)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-5\sqrt{2}(x^4+2x^2+1)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8(5x^3+9x)\sqrt{x}}{128(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
```

```
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 5*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(5*x^3 + 9*x)*sqrt(x))/(x^4 + 2
*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(117) = 234$.
time = 1.23, size = 481, normalized size = 3.73

$$\frac{40x^{\frac{7}{2}}}{128x^4+256x^2+128}+\frac{72x^{\frac{3}{2}}}{128x^4+256x^2+128}+\frac{5\sqrt{2}\,x^4\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{5\sqrt{2}\,x^4\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,x^4\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,x^4\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{10\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{20\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{20\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{5\sqrt{2}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)**3,x)
```

```
[Out] 40*x**(7/2)/(128*x**4 + 256*x**2 + 128) + 72*x**(3/2)/(128*x**4 + 256*x**2
+ 128) + 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) - 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x
**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) - 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 25
6*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x
**2 + 128) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) - 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128)
+ 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A]
time = 0.93, size = 94, normalized size = 0.73

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{5x^{\frac{7}{2}}+9x^{\frac{3}{2}}}{16\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2) + 9
*x^(3/2))/(x^2 + 1)^2
```

Mupad [B]
time = 0.04, size = 61, normalized size = 0.47

$$\frac{\frac{9\,x^{3/2}}{16}+\frac{5\,x^{7/2}}{16}}{x^4+2\,x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{5}{64}-\frac{5}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{5}{64}+\frac{5}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/64 - 5i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/64 + 5i/64) + ((9*x^(3/2))/16 + (5*x^(7/2))/
16)/(2*x^2 + x^4 + 1)
```

3.333 $\int \frac{1}{\sqrt{x}\,(1+x^2)^3}\,dx$

Optimal. Leaf size=129

$$\frac{\sqrt{x}}{4(1+x^2)^2}+\frac{7\sqrt{x}}{16(1+x^2)}-\frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{21\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+$$

```
[Out] 21/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+21/64*arctan(1+2^(1/2)*x^(1/2))*2^
(1/2)-21/128*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+21/128*ln(1+x+2^(1/2)*x^(1/2))
*2^(1/2)+1/4*x^(1/2)/(x^2+1)^2+7/16*x^(1/2)/(x^2+1)
```

Rubi [A]

time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{21\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}+\frac{7\sqrt{x}}{16(x^2+1)}+\frac{\sqrt{x}}{4(x^2+1)^2}-\frac{21\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{21\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*(1 + x^2)^2) + (7*Sqrt[x])/(16*(1 + x^2)) - (21*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (21*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
 (21*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (21*Log[1 + Sqrt[2]*Sqrt[
x] + x])/(64*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^3}\,dx = \frac{\sqrt{x}}{4(1+x^2)^2} + \frac{7}{8}\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx$$
$$= \frac{\sqrt{x}}{4(1+x^2)^2} + \frac{7\sqrt{x}}{16(1+x^2)} + \frac{21}{32}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= \frac{\sqrt{x}}{4(1+x^2)^2} + \frac{7\sqrt{x}}{16(1+x^2)} + \frac{21}{16}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{x}}{4(1+x^2)^2} + \frac{7\sqrt{x}}{16(1+x^2)} + \frac{21}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{21}{32}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.$$
$$= \frac{\sqrt{x}}{4(1+x^2)^2} + \frac{7\sqrt{x}}{16(1+x^2)} + \frac{21}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{21}{64}\text{Subst}\left(\int\right.$$
$$= \frac{\sqrt{x}}{4(1+x^2)^2} + \frac{7\sqrt{x}}{16(1+x^2)} - \frac{21\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{21\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}$$
$$= \frac{\sqrt{x}}{4(1+x^2)^2} + \frac{7\sqrt{x}}{16(1+x^2)} - \frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{21\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} -$$

Mathematica [A]
time = 0.10, size = 72, normalized size = 0.56

$$\frac{1}{64}\left(\frac{4\sqrt{x}\,(11+7x^2)}{(1+x^2)^2} + 21\sqrt{2}\,\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) + 21\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)^3),x]
[Out] ((4*Sqrt[x]*(11 + 7*x^2))/(1 + x^2)^2 + 21*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]
*Sqrt[x])] + 21*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Maple [A]
time = 0.46, size = 81, normalized size = 0.63

method	result
derivativedivides	$\frac{\sqrt{x}}{4(x^2+1)^2} + \frac{7\sqrt{x}}{16(x^2+1)} + \frac{21\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right.\right.}{128}$

default	$\frac{\sqrt{x}}{4(x^2+1)^2}+\frac{7\sqrt{x}}{16(x^2+1)}+\frac{21\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\,\sqrt{x}}{1+x-\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right.}{128}$
risch	$\frac{(7x^2+11)\sqrt{x}}{16(x^2+1)^2}+\frac{21\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{64}+\frac{21\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}}{64}+\frac{21\sqrt{2}\,\ln\left(\frac{1+x+\sqrt{2}}{1+x-\sqrt{2}}\right.}{128}$
meijerg	$\frac{(7x^2+11)\sqrt{x}}{16(x^2+1)^2}-\frac{21\sqrt{x}\,\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{128(x^2)^{\frac{1}{4}}}+\frac{21\sqrt{x}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{64(x^2)^{\frac{1}{4}}}+\frac{21\sqrt{}}{}$
trager	$\frac{(7x^2+11)\sqrt{x}}{16(x^2+1)^2}-\frac{21\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^5x+\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x-\mathrm{RootOf}}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right.}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(1/2)/(x^2+1)^2+7/16*x^(1/2)/(x^2+1)+21/128*2^(1/2)*(ln((1+x+2^(1/2)*
x^(1/2))/(1+x-2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(
1/2)*x^(1/2)))
```

Maxima [A]
time = 0.50, size = 99, normalized size = 0.77

$$\frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{21}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{7x^{\frac{5}{2}}+11\sqrt{x}}{16\,(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt(x
) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5/2)
 + 11*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Fricas [A]
time = 1.21, size = 173, normalized size = 1.34

$$-\frac{84\sqrt{2}\,(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+84\sqrt{2}\,(x^4+2x^2+1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-21\sqrt{2}\,(x^4+2x^2+1)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+21\sqrt{2}\,(x^4+2x^2+1)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\,(7x^2+11)\sqrt{x}}{128\,(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(84*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 84*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
```

```
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 21*sqrt(2)
*(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 21*sqrt(2)*(x^4 + 2*x
^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(7*x^2 + 11)*sqrt(x))/(x^4 +
2*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(117) = 234$.
time = 1.55, size = 481, normalized size = 3.73

$$\frac{56x^{\frac{5}{2}}}{128x^4+256x^2+128}+\frac{88\sqrt{x}}{128x^4+256x^2+128}-\frac{21\sqrt{2}\,x^4\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{21\sqrt{2}\,x^4\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}\,x^4\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}\,x^4\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{42\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{84\sqrt{2}\,x^2\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{84\sqrt{2}\,x^2\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{21\sqrt{2}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{21\sqrt{2}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/x**(1/2),x)
```

```
[Out] 56*x**(5/2)/(128*x**4 + 256*x**2 + 128) + 88*sqrt(x)/(128*x**4 + 256*x**2 +
 128) - 21*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 21*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 2
56*x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*
x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**
2 + 128) - 42*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 42*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
+ 256*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 2
56*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*
x**2 + 128) - 21*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 21*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 12
8) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Giac [A]
time = 2.99, size = 94, normalized size = 0.73

$$\frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{21}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{7x^{\frac{5}{2}}+11\sqrt{x}}{16\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt(x
) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5/2)
 + 11*sqrt(x))/(x^2 + 1)^2
```

Mupad [B]
time = 4.75, size = 61, normalized size = 0.47

$$\frac{\frac{11\sqrt{x}}{16}+\frac{7x^{5/2}}{16}}{x^4+2x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{21}{64}+\frac{21}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{21}{64}-\frac{21}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^2 + 1)^3),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(21/64 + 21i/64) + 2^(1/2)*atan(
2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(21/64 - 21i/64) + ((11*x^(1/2))/16 + (7*x^(5
/2))/16)/(2*x^2 + x^4 + 1)
```

3.334 $\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\,\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\,\left(1+x^2\right)}+\frac{45\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{32\sqrt{2}}-\frac{45\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{32\sqrt{2}}-\frac{45\log\left(\right.}{}$$

[Out] -45/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-45/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-45/128*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+45/128*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)-45/16/x^(1/2)+1/4/(x^2+1)^2/x^(1/2)+9/16/(x^2+1)/x^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{45\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{x}\right)}{32\sqrt{2}}-\frac{45\text{ArcTan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{32\sqrt{2}}+\frac{9}{16\sqrt{x}\,\left(x^2+1\right)}+\frac{1}{4\sqrt{x}\,\left(x^2+1\right)^2}-\frac{45}{16\sqrt{x}}-\frac{45\log\left(x-\sqrt{2}\,\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\log\left(x+\sqrt{2}\,\sqrt{x}+1\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)^3),x]
```

```
[Out] -45/(16*Sqrt[x]) + 1/(4*Sqrt[x]*(1 + x^2)^2) + 9/(16*Sqrt[x]*(1 + x^2)) + (45*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (45*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (45*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (45*Log[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx &= \frac{1}{4\sqrt{x}\,\left(1+x^2\right)^2}+\frac{9}{8}\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx\\
&= \frac{1}{4\sqrt{x}\,\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\,\left(1+x^2\right)}+\frac{45}{32}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\,\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\,\left(1+x^2\right)}-\frac{45}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\,\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\,\left(1+x^2\right)}-\frac{45}{16}\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\,\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\,\left(1+x^2\right)}+\frac{45}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\,\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\,\left(1+x^2\right)}-\frac{45}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\right.\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\,\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\,\left(1+x^2\right)}-\frac{45\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{45\log}{}\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\,\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\,\left(1+x^2\right)}+\frac{45\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{32\sqrt{2}}-\frac{45\tan^{-}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 77, normalized size = 0.56

$$
\frac{1}{64}\left(-\frac{4(32+81x^2+45x^4)}{\sqrt{x}\,\left(1+x^2\right)^2}-45\sqrt{2}\,\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\,\sqrt{x}}\right)+45\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x}}{1+x}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)^3),x]
[Out] ((-4*(32 + 81*x^2 + 45*x^4))/(Sqrt[x]*(1 + x^2)^2) - 45*Sqrt[2]*ArcTan[(-1
+ x)/(Sqrt[2]*Sqrt[x])] + 45*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Maple [A]
time = 0.62, size = 82, normalized size = 0.59

method	result
derivativedivides	$-\frac{2\left(\frac{13x^{\frac{7}{2}}}{32}+\frac{17x^{\frac{3}{2}}}{32}\right)}{(x^2+1)^2}-\frac{45\sqrt{2}\,\left(\ln\left(\frac{1+x-\sqrt{2}\,\sqrt{x}}{1+x+\sqrt{2}\,\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$

default	$-\frac{2\left(\frac{13x^{\frac{7}{2}}}{32}+\frac{17x^{\frac{3}{2}}}{32}\right)}{(x^2+1)^2}-\frac{45\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
risch	$-\frac{45x^4+81x^2+32}{16\sqrt{x}\,(x^2+1)^2}-\frac{45\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{64}-\frac{45\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{64}-\frac{45\sqrt{2}\ln\left(\frac{1+x-}{1+x+}\right.}{128}$
meijerg	$-\frac{45x^4+81x^2+32}{16\sqrt{x}\,(x^2+1)^2}-\frac{45x^{\frac{3}{2}}\left(\frac{\sqrt{2}\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\ln\left(1+\sqrt{2}\right.}{2(x^2}\right.}{64}$
trager	$-\frac{45x^4+81x^2+32}{16\sqrt{x}\,(x^2+1)^2}-\frac{45\operatorname{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^4+1\right)^5x-\operatorname{RootOf}\left(\_Z^4+1\right)^5-2\operatorname{RootOf}\left(\_Z^4+1\right)^3x+\operatorname{R}}{\operatorname{RootOf}\left(\_Z^4+1\right)^2x-\operatorname{RootOf}\left(\_Z\right.}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(13/32*x^(7/2)+17/32*x^(3/2))/(x^2+1)^2-45/128*2^(1/2)*(ln((1+x-2^(1/2)*
x^(1/2))/(1+x+2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(
1/2)*x^(1/2)))-2/x^(1/2)
```

Maxima [A]
time = 0.51, size = 102, normalized size = 0.74

$$-\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{45}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{45x^4+81x^2+32}{16\left(x^{\frac{9}{2}}+2x^{\frac{5}{2}}+\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(45*x^4 +
 81*x^2 + 32)/(x^(9/2) + 2*x^(5/2) + sqrt(x))
```

Fricas [A]
time = 1.20, size = 178, normalized size = 1.29

$$\frac{180\sqrt{2}\,(x^5+2x^3+x)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+180\sqrt{2}\,(x^5+2x^3+x)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+45\sqrt{2}\,(x^5+2x^3+x)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-45\sqrt{2}\,(x^5+2x^3+x)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\,(45x^4+81x^2+32)\sqrt{x}}{128\,(x^5+2x^3+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(180*sqrt(2)*(x^5 + 2*x^3 + x)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 180*sqrt(2)*(x^5 + 2*x^3 + x)*arctan(1/2*sq
rt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 45*sqrt(2
)*(x^5 + 2*x^3 + x)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 45*sqrt(2)*(x^5 + 2*
x^3 + x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(45*x^4 + 81*x^2 + 32)*sqrt(
x))/(x^5 + 2*x^3 + x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(128) = 256$.
time = 2.20, size = 653, normalized size = 4.73

$$-\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}+\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}+\frac{90\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{180\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{180\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{45\sqrt{2}\sqrt{x}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}+\frac{45\sqrt{2}\sqrt{x}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{360x^4}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{648x^2}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{256}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1)**3,x)
```

```
[Out] -45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*
x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x +
 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(2)*x**(9/2)*atan(
sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(
2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sq
rt(x)) - 90*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2
) + 256*x**(5/2) + 128*sqrt(x)) + 90*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 180*sqrt(2)*x**(5
/2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) -
 180*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**(5/2
) + 128*sqrt(x)) - 45*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(12
8*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*sqrt(x)*log(4*sqrt(2)
*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(2
)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt
(x)) - 90*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**
(5/2) + 128*sqrt(x)) - 360*x**4/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x))
 - 648*x**2/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 256/(128*x**(9/2)
 + 256*x**(5/2) + 128*sqrt(x))
```

Giac [A]
time = 2.06, size = 99, normalized size = 0.72

$$-\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{45}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{\sqrt{x}}-\frac{13x^{\frac{7}{2}}+17x^{\frac{3}{2}}}{16\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqrt(
```

```
x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x) - 1/
16*(13*x^(7/2) + 17*x^(3/2))/(x^2 + 1)^2
```

Mupad [B]
time = 4.72, size = 65, normalized size = 0.47

$$-\frac{\frac{45\,x^4}{16}+\frac{81\,x^2}{16}+2}{\sqrt{x}+2\,x^{5/2}+x^{9/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{45}{64}+\frac{45}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{45}{64}-\frac{45}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^2 + 1)^3),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(45/64 - 45i/64) - 2^(1/2)*ata
n(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(45/64 + 45i/64) - ((81*x^2)/16 + (45*x^4)/
16 + 2)/(x^(1/2) + 2*x^(5/2) + x^(9/2))
```

3.335 $\int \frac{1}{x^{5/2}(1+x^2)^3} dx$

Optimal. Leaf size=138

$$-\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}(1+x^2)^2}+\frac{11}{16x^{3/2}(1+x^2)}+\frac{77\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{77\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{77\log\left(1\right.}{}$$

```
[Out] -77/48/x^(3/2)+1/4/x^(3/2)/(x^2+1)^2+11/16/x^(3/2)/(x^2+1)-77/64*arctan(-1+
2^(1/2)*x^(1/2))*2^(1/2)-77/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+77/128*ln(
1+x-2^(1/2)*x^(1/2))*2^(1/2)-77/128*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{77\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{77\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}-\frac{77}{48x^{3/2}}+\frac{11}{16x^{3/2}(x^2+1)}+\frac{1}{4x^{3/2}(x^2+1)^2}+\frac{77\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{77\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)^3),x]
```

```
[Out] -77/(48*x^(3/2)) + 1/(4*x^(3/2)*(1 + x^2)^2) + 11/(16*x^(3/2)*(1 + x^2)) +
(77*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (77*ArcTan[1 + Sqrt[2]*Sqrt
[x]])/(32*Sqrt[2]) + (77*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (77*L
og[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
```

```
b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(1+x^2)^3}\,dx = \frac{1}{4x^{3/2}(1+x^2)^2} + \frac{11}{8}\int \frac{1}{x^{5/2}(1+x^2)^2}\,dx$$

$$= \frac{1}{4x^{3/2}(1+x^2)^2} + \frac{11}{16x^{3/2}(1+x^2)} + \frac{77}{32}\int \frac{1}{x^{5/2}(1+x^2)}\,dx$$

$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}(1+x^2)^2} + \frac{11}{16x^{3/2}(1+x^2)} - \frac{77}{32}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$

$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}(1+x^2)^2} + \frac{11}{16x^{3/2}(1+x^2)} - \frac{77}{16}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}(1+x^2)^2} + \frac{11}{16x^{3/2}(1+x^2)} - \frac{77}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - $$

$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}(1+x^2)^2} + \frac{11}{16x^{3/2}(1+x^2)} - \frac{77}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \right.$$

$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}(1+x^2)^2} + \frac{11}{16x^{3/2}(1+x^2)} + \frac{77\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} - \frac{77\log}{}$$

$$= -\frac{77}{48x^{3/2}} + \frac{1}{4x^{3/2}(1+x^2)^2} + \frac{11}{16x^{3/2}(1+x^2)} + \frac{77\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} - \frac{77\tan^{-1}}{}$$

Mathematica [A]
time = 0.21, size = 77, normalized size = 0.56

$$\frac{1}{192}\left(-\frac{4(32+121x^2+77x^4)}{x^{3/2}(1+x^2)^2} - 231\sqrt{2}\,\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 231\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)^3),x]
[Out] ((-4*(32 + 121*x^2 + 77*x^4))/(x^(3/2)*(1 + x^2)^2) - 231*Sqrt[2]*ArcTan[(-
1 + x)/(Sqrt[2]*Sqrt[x])] - 231*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])
/192
```

Maple [A]
time = 0.63, size = 82, normalized size = 0.59

method	result
derivativedivides	$-\frac{2\left(\frac{15x^{\frac{5}{2}}}{32}+\frac{19\sqrt{x}}{32}\right)}{(x^2+1)^2} - \frac{77\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right.\right.}{128}$

default	$-\frac{2\left(\frac{15x^{\frac{5}{2}}}{32}+\frac{19\sqrt{x}}{32}\right)}{(x^2+1)^2}-\frac{77\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right.}{128}$
risch	$-\frac{77x^4+121x^2+32}{48(x^2+1)^2x^{\frac{3}{2}}}-\frac{77\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{64}-\frac{77\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{64}-\frac{77\sqrt{2}\ln\left(\frac{1+x+}{1+x-}\right.}{128}$
meijerg	$-\frac{77x^4+121x^2+32}{48(x^2+1)^2x^{\frac{3}{2}}}-\frac{77\sqrt{x}\left(-\frac{\sqrt{2}\ln\left(1-\sqrt{2}(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2)^{\frac{1}{4}}}{2-\sqrt{2}(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\ln\left(1+\sqrt{}\right.}{}\right.}{64}$
trager	$-\frac{77x^4+121x^2+32}{48(x^2+1)^2x^{\frac{3}{2}}}-\frac{77\operatorname{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^4+1\right)^5x-\operatorname{RootOf}\left(\_Z^4+1\right)^5+2\operatorname{RootOf}\left(\_Z^4+1\right)^3-\operatorname{Roo}}{\operatorname{RootOf}\left(\_Z^4+1\right)^2x-\operatorname{RootOf}\left(\_Z\right.}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(15/32*x^(5/2)+19/32*x^(1/2))/(x^2+1)^2-77/128*2^(1/2)*(ln((1+x+2^(1/2)*
x^(1/2))/(1+x-2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(
1/2)*x^(1/2)))-2/3/x^(3/2)
```

Maxima [A]
time = 0.52, size = 102, normalized size = 0.74

$$-\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{77}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{77x^4+121x^2+32}{48\left(x^{\frac{11}{2}}+2x^{\frac{7}{2}}+x^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/48*(77*x^4 +
 121*x^2 + 32)/(x^(11/2) + 2*x^(7/2) + x^(3/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(93) = 186.
time = 0.91, size = 188, normalized size = 1.36

$$\frac{924\sqrt{2}(x^6+2x^4+x^2)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+924\sqrt{2}(x^6+2x^4+x^2)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-231\sqrt{2}(x^6+2x^4+x^2)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+231\sqrt{2}(x^6+2x^4+x^2)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8(77x^4+121x^2+32)\sqrt{x}}{384(x^6+2x^4+x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(924*sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x)
+ x + 1) - sqrt(2)*sqrt(x) - 1) + 924*sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(1/
2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 231*s
qrt(2)*(x^6 + 2*x^4 + x^2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 231*sqrt(2)*(
x^6 + 2*x^4 + x^2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(77*x^4 + 121*x^2
+ 32)*sqrt(x))/(x^6 + 2*x^4 + x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(128) = 256$.
time = 3.60, size = 653, normalized size = 4.73

$$\frac{231\sqrt{2}\,x^{\frac{11}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{231\sqrt{2}\,x^{\frac{11}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}\,x^{\frac{11}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}\,x^{\frac{11}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}+\frac{462\sqrt{2}\,x^{\frac{7}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}\,x^{\frac{7}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{924\sqrt{2}\,x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{924\sqrt{2}\,x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}+\frac{231\sqrt{2}\,x^{\frac{3}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{231\sqrt{2}\,x^{\frac{3}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}\,x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}\,x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{616x^{4}}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{968x^{2}}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{256}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1)**3,x)
```

```
[Out] 231*sqrt(2)*x**(11/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 76
8*x**(7/2) + 384*x**(3/2)) - 231*sqrt(2)*x**(11/2)*log(4*sqrt(2)*sqrt(x) +
4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(11
/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2))
 - 462*sqrt(2)*x**(11/2)*atan(sqrt(2)*sqrt(x) + 1)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) + 462*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) + 4*x +
 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(7/2)*lo
g(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)
) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) + 1)/(384
*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) + 231*sqrt(2)*x**(3/2)*log(-4*sqr
t(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 231
*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(x) - 1)/(384
*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(3/2)*atan(sqrt(
2)*sqrt(x) + 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 616*x**4/(3
84*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 968*x**2/(384*x**(11/2) + 768
*x**(7/2) + 384*x**(3/2)) - 256/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2
))
```

Giac [A]
time = 1.31, size = 99, normalized size = 0.72

$$-\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{77}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{15x^{\frac{5}{2}}+19\sqrt{x}}{16\left(x^2+1\right)^2}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqrt(
```

```
x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(15*x^(5/
2) + 19*sqrt(x))/(x^2 + 1)^2 - 2/3/x^(3/2)
```

Mupad [B]
time = 4.72, size = 65, normalized size = 0.47

$$-\frac{\frac{77\,x^4}{48}+\frac{121\,x^2}{48}+\frac{2}{3}}{x^{3/2}+2\,x^{7/2}+x^{11/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{77}{64}-\frac{77}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{77}{64}+\frac{77}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^2 + 1)^3),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(77/64 + 77i/64) - 2^(1/2)*ata
n(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(77/64 - 77i/64) - ((121*x^2)/48 + (77*x^4)
/48 + 2/3)/(x^(3/2) + 2*x^(7/2) + x^(11/2))
```

3.336 $\int \frac{1}{x^{7/2}(1+x^2)^3}\,dx$

Optimal. Leaf size=147

$$-\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{16x^{5/2}(1+x^2)}-\frac{117\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{117\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}$$

```
[Out] -117/80/x^(5/2)+1/4/x^(5/2)/(x^2+1)^2+13/16/x^(5/2)/(x^2+1)+117/64*arctan(-
1+2^(1/2)*x^(1/2))*2^(1/2)+117/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+117/128
*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-117/128*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)+11
7/16/x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{117\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{117\text{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}-\frac{117}{80x^{5/2}}+\frac{13}{16x^{5/2}(x^2+1)}+\frac{1}{4x^{5/2}(x^2+1)^2}+\frac{117}{16\sqrt{x}}+\frac{117\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)^3),x]
```

```
[Out] -117/(80*x^(5/2)) + 117/(16*Sqrt[x]) + 1/(4*x^(5/2)*(1 + x^2)^2) + 13/(16*x
^(5/2)*(1 + x^2)) - (117*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*Log[1 - Sqrt[2]*Sqrt[x] + x
])/(64*Sqrt[2]) - (117*Log[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}(1+x^2)^3}\,dx &= \frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{8}\int \frac{1}{x^{7/2}(1+x^2)^2}\,dx\\
&= \frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{16x^{5/2}(1+x^2)}+\frac{117}{32}\int \frac{1}{x^{7/2}(1+x^2)}\,dx\\
&= -\frac{117}{80x^{5/2}}+\frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{16x^{5/2}(1+x^2)}-\frac{117}{32}\int \frac{1}{x^{3/2}(1+x^2)}\,dx\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{16x^{5/2}(1+x^2)}+\frac{117}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{16x^{5/2}(1+x^2)}+\frac{117}{16}\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x\right.\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{16x^{5/2}(1+x^2)}-\frac{117}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x\right.\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{16x^{5/2}(1+x^2)}+\frac{117}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+\right.\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{16x^{5/2}(1+x^2)}+\frac{117\log\left(1-\sqrt{2}\sqrt{x}+x\right.}{64\sqrt{2}}\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}(1+x^2)^2}+\frac{13}{16x^{5/2}(1+x^2)}-\frac{117\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 82, normalized size = 0.56

$$
\frac{1}{320}\left(\frac{4(-32+416x^2+1053x^4+585x^6)}{x^{5/2}(1+x^2)^2}+585\sqrt{2}\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)-585\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)^3),x]
```

```
[Out] ((4*(-32 + 416*x^2 + 1053*x^4 + 585*x^6))/(x^(5/2)*(1 + x^2)^2) + 585*Sqrt[
2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] - 585*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x
])/(1 + x)])/320
```

Maple [A]
time = 0.45, size = 87, normalized size = 0.59

method	result
derivativedivides	$\frac{\frac{21x^{\frac{7}{2}}}{16}+\frac{25x^{\frac{3}{2}}}{16}}{(x^2+1)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}-\frac{2}{5x\cdots}$
default	$\frac{\frac{21x^{\frac{7}{2}}}{16}+\frac{25x^{\frac{3}{2}}}{16}}{(x^2+1)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{1+x-\sqrt{2}\sqrt{x}}{1+x+\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}-\frac{2}{5x\cdots}$
risch	$\frac{585x^6+1053x^4+416x^2-32}{80(x^2+1)^2x^{\frac{5}{2}}}+\frac{117\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{64}+\frac{117\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\sqrt{2}}{64}+\frac{117\sqrt{2}\cdots}{\cdots}$
meijerg	$-\frac{-585x^6-1053x^4-416x^2+32}{80x^{\frac{5}{2}}(x^2+1)^2}+\frac{117x^{\frac{3}{2}}\left(\frac{\sqrt{2}\ln\left(1-\sqrt{2}(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2)^{\frac{1}{4}}}{2-\sqrt{2}(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\cdots}{\cdots}\right.}{64\cdots}$
trager	$\frac{585x^6+1053x^4+416x^2-32}{80(x^2+1)^2x^{\frac{5}{2}}}+\frac{117\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\cdots\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootO}\cdots}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(21/32*x^(7/2)+25/32*x^(3/2))/(x^2+1)^2+117/128*2^(1/2)*(ln((1+x-2^(1/2)*
x^(1/2))/(1+x+2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(
1/2)*x^(1/2)))-2/5/x^(5/2)+6/x^(1/2)
```

Maxima [A]

time = 0.51, size = 107, normalized size = 0.73

$$\frac{117}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{117}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{117}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{117}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{585x^6+1053x^4+416x^2-32}{80\left(x^{\frac{13}{2}}+2x^{\frac{9}{2}}+x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/80*(585*x
^6 + 1053*x^4 + 416*x^2 - 32)/(x^(13/2) + 2*x^(9/2) + x^(5/2))
```

Fricas [A]

time = 0.84, size = 193, normalized size = 1.31

$$-\frac{2340\sqrt{2}(x^7+2x^5+x^3)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+2340\sqrt{2}(x^7+2x^5+x^3)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+585\sqrt{2}(x^7+2x^5+x^3)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-585\sqrt{2}(x^7+2x^5+x^3)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8(585x^6+1053x^4+416x^2-32)\sqrt{x}}{640(x^7+2x^5+x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/640*(2340*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x
) + x + 1) - sqrt(2)*sqrt(x) - 1) + 2340*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan
(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 58
5*sqrt(2)*(x^7 + 2*x^5 + x^3)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 585*sqrt(2
)*(x^7 + 2*x^5 + x^3)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(585*x^6 + 1053
*x^4 + 416*x^2 - 32)*sqrt(x))/(x^7 + 2*x^5 + x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(136) = 272$.

time = 6.65, size = 678, normalized size = 4.61

$$\frac{585\sqrt{2}\,x^{\frac{13}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}-\frac{585\sqrt{2}\,x^{\frac{13}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}\,x^{\frac{13}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}\,x^{\frac{13}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}\,x^{\frac{9}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}-\frac{1170\sqrt{2}\,x^{\frac{9}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{2340\sqrt{2}\,x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{2340\sqrt{2}\,x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{585\sqrt{2}\,x^{\frac{5}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}-\frac{585\sqrt{2}\,x^{\frac{5}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}\,x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}\,x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{4680x^{6}}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{8424x^{4}}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{3328x^{2}}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}-\frac{256}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1)**3,x)
```

```
[Out] 585*sqrt(2)*x**(13/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 12
80*x**(9/2) + 640*x**(5/2)) - 585*sqrt(2)*x**(13/2)*log(4*sqrt(2)*sqrt(x) +
 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**
(13/2)*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5
/2)) + 1170*sqrt(2)*x**(13/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2) + 12
80*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x)
+ 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) - 1170*sqrt(2)*x*
*(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 64
0*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2
) + 1280*x**(9/2) + 640*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt
(x) + 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 585*sqrt(2)*x**(5
/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*
x**(5/2)) - 585*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(
13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(5/2)*atan(sqrt(2)*
sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*
x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**
(5/2)) + 4680*x**6/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 8424*x*
*4/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 3328*x**2/(640*x**(13/2
) + 1280*x**(9/2) + 640*x**(5/2)) - 256/(640*x**(13/2) + 1280*x**(9/2) + 64
0*x**(5/2))
```

Giac [A]

time = 0.90, size = 106, normalized size = 0.72

$$\frac{117}{64}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+2\,\sqrt{x}\right)\right)+\frac{117}{64}\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-2\,\sqrt{x}\right)\right)-\frac{117}{128}\,\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt{x}+x+1\right)+\frac{117}{128}\,\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt{x}+x+1\right)+\frac{21\,x^{\frac{7}{2}}+25\,x^{\frac{3}{2}}}{16\,{\left(x^{2}+1\right)}^{2}}+\frac{2\,{\left(15\,x^{2}-1\right)}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(21*x^
(7/2) + 25*x^(3/2))/(x^2 + 1)^2 + 2/5*(15*x^2 - 1)/x^(5/2)
```

Mupad [B]
time = 0.07, size = 69, normalized size = 0.47

$$\frac{\frac{117\,x^6}{16}+\frac{1053\,x^4}{80}+\frac{26\,x^2}{5}-\frac{2}{5}}{x^{5/2}+2\,x^{9/2}+x^{13/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{117}{64}-\frac{117}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{117}{64}+\frac{117}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^2 + 1)^3),x)
```

```
[Out] ((26*x^2)/5 + (1053*x^4)/80 + (117*x^6)/16 - 2/5)/(x^(5/2) + 2*x^(9/2) + x^
(13/2)) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(117/64 - 117i/64) + 2
^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(117/64 + 117i/64)
```

3.337 $\int \frac{\sqrt{x}}{1-x^2} \, dx$

Optimal. Leaf size=15

$$-\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 304, 209, 212}

$$\tanh^{-1}\left(\sqrt{x}\right)-\text{ArcTan}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 - x^2),x]
```

```
[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{1-x^2}\,dx = 2\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{x}\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= -\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A]
time = 0.02, size = 15, normalized size = 1.00

$$-\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 - x^2),x]
[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.
time = 0.19, size = 24, normalized size = 1.60

method	result	size
derivativedivides	$-\frac{\ln\left(\sqrt{x}-1\right)}{2}+\frac{\ln\left(\sqrt{x}+1\right)}{2}-\arctan\left(\sqrt{x}\right)$	24
default	$-\frac{\ln\left(\sqrt{x}-1\right)}{2}+\frac{\ln\left(\sqrt{x}+1\right)}{2}-\arctan\left(\sqrt{x}\right)$	24
meijerg	$-\frac{x^{\frac{3}{2}}\left(\ln\left(1-\left(x^2\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^2\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^2\right)^{\frac{1}{4}}\right)\right)}{2\left(x^2\right)^{\frac{3}{4}}}$	40
trager	$\frac{\ln\left(\frac{2\sqrt{x}+1+x}{x-1}\right)}{2}-\frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\text{RootOf}\left(\_Z^2+1\right)x+2\sqrt{x}+\text{RootOf}\left(\_Z^2+1\right)}{x+1}\right)}{2}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^(1/2)-1)+1/2*ln(x^(1/2)+1)-arctan(x^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.
time = 0.52, size = 23, normalized size = 1.53

$$-\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 1.06, size = 23, normalized size = 1.53

$$-\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.10, size = 26, normalized size = 1.73

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}-\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-x**2+1),x)
```

```
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 - atan(sqrt(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.
time = 1.11, size = 24, normalized size = 1.60

$$-\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.73

$$\operatorname{atanh}\left(\sqrt{x}\right)-\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(x^2 - 1),x)
```

```
[Out] atanh(x^(1/2)) - atan(x^(1/2))
```

3.338 $\int \frac{x^{2/3}}{1+x^2} \, dx$

Optimal. Leaf size=73

$$-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{x}\right)+\tan^{-1}\left(\sqrt[3]{x}\right)-\frac{1}{2}\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x}}{1+x^{2/3}}\right)$$

```
[Out] arctan(x^(1/3))+1/2*arctan(2*x^(1/3)-3^(1/2))+1/2*arctan(2*x^(1/3)+3^(1/2))
-1/2*arctanh(x^(1/3)*3^(1/2)/(1+x^(2/3)))*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 100, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {335, 301, 648, 632, 210, 642, 209}

$$-\frac{1}{2}\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\text{ArcTan}\left(2\sqrt[3]{x}+\sqrt{3}\right)+\text{ArcTan}\left(\sqrt[3]{x}\right)+\frac{1}{4}\sqrt{3}\log\left(x^{2/3}-\sqrt{3}\sqrt[3]{x}+1\right)-\frac{1}{4}\sqrt{3}\log\left(x^{2/3}+\sqrt{3}\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(1 + x^2),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - 2*x^(1/3)] + ArcTan[Sqrt[3] + 2*x^(1/3)]/2 + ArcTan[x
^(1/3)] + (Sqrt[3]*Log[1 - Sqrt[3]*x^(1/3) + x^(2/3)])/4 - (Sqrt[3]*Log[1 +
 Sqrt[3]*x^(1/3) + x^(2/3)])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
```

```
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{2/3}}{1+x^2}\,dx &= 3\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx,x,\sqrt[3]{x}\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt[3]{x}\right)+\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}\,x}{2}}{1-\sqrt{3}\,x+x^2}\,dx,x,\sqrt[3]{x}\right)+\text{Subst}\left(\int \frac{-\frac{1}{2}-\ldots}{1+\sqrt{\ldots}}\right.\\ &= \tan^{-1}\left(\sqrt[3]{x}\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx,x,\sqrt[3]{x}\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx,x,\ldots\right.\\ &= \tan^{-1}\left(\sqrt[3]{x}\right)+\frac{1}{4}\sqrt{3}\,\log\left(1-\sqrt{3}\,\sqrt[3]{x}+x^{2/3}\right)-\frac{1}{4}\sqrt{3}\,\log\left(1+\sqrt{3}\,\sqrt[3]{x}+x^{2/3}\right)-\frac{1}{2}\text{Subs}\ldots\\ &= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{x}\right)+\tan^{-1}\left(\sqrt[3]{x}\right)+\frac{1}{4}\sqrt{3}\,\log\left(1-\sqrt{3}\,\sqrt[3]{\ldots}\right.\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 79, normalized size = 1.08

$$\tan^{-1}\left(\sqrt[3]{x}\right)+\frac{1}{2}\left(1-i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt[3]{x}\right)+\frac{1}{2}\left(1+i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(1 + x^2),x]
```

```
[Out] ArcTan[x^(1/3)] + ((1 - I*Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^(1/3))/2])/2 +
 ((1 + I*Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^(1/3))/2])/2
```

Maple [A]
time = 2.54, size = 69, normalized size = 0.95

method	result
derivativedivides	$\arctan\left(x^{\frac{1}{3}}\right)+\frac{\arctan\left(2x^{\frac{1}{3}}-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x^{\frac{1}{3}}+\sqrt{3}\right)}{2}+\frac{\ln\left(1+x^{\frac{2}{3}}-x^{\frac{1}{3}}\sqrt{3}\right)\sqrt{3}}{4}-\frac{\ln\left(1+x^{\frac{2}{3}}+x^{\frac{1}{3}}\right.}{4}$
default	$\arctan\left(x^{\frac{1}{3}}\right)+\frac{\arctan\left(2x^{\frac{1}{3}}-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x^{\frac{1}{3}}+\sqrt{3}\right)}{2}+\frac{\ln\left(1+x^{\frac{2}{3}}-x^{\frac{1}{3}}\sqrt{3}\right)\sqrt{3}}{4}-\frac{\ln\left(1+x^{\frac{2}{3}}+x^{\frac{1}{3}}\right.}{4}$
meijerg	$\frac{x^{\frac{5}{3}}\sqrt{3}\,\ln\left(1-\sqrt{3}\,(x^2)^{\frac{1}{6}}+(x^2)^{\frac{1}{3}}\right)}{4(x^2)^{\frac{5}{6}}}+\frac{x^{\frac{5}{3}}\arctan\left(\frac{(x^2)^{\frac{1}{6}}}{2-\sqrt{3}\,(x^2)^{\frac{1}{6}}}\right)}{2(x^2)^{\frac{5}{6}}}+\frac{x^{\frac{5}{3}}\arctan\left((x^2)^{\frac{1}{6}}\right)}{(x^2)^{\frac{5}{6}}}-\frac{x^{\frac{5}{3}}\sqrt{3}\,\ln\left(1+\sqrt{\right.}}{4(}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x^(1/3))+1/2*arctan(2*x^(1/3)-3^(1/2))+1/2*arctan(2*x^(1/3)+3^(1/2))
+1/4*ln(1+x^(2/3)-x^(1/3)*3^(1/2))*3^(1/2)-1/4*ln(1+x^(2/3)+x^(1/3)*3^(1/2)
)*3^(1/2)
```

Maxima [A]
time = 0.52, size = 68, normalized size = 0.93

$$-\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}\,x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}\,x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2\,x^{\frac{1}{3}}\right)+\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)*
x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sqrt
(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(51) = 102$.

time = 1.16, size = 108, normalized size = 1.48

$$-\frac{1}{4}\sqrt{3}\log\left(16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16\right)+\frac{1}{4}\sqrt{3}\log\left(-16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16\right)-\arctan\left(\sqrt{3}+\frac{1}{2}\sqrt{-16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16}-2x^{\frac{1}{3}}\right)-\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1}-2x^{\frac{1}{3}}\right)+\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*log(16*sqrt(3)*x^(1/3) + 16*x^(2/3) + 16) + 1/4*sqrt(3)*log(-1
6*sqrt(3)*x^(1/3) + 16*x^(2/3) + 16) - arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3
)*x^(1/3) + 16*x^(2/3) + 16) - 2*x^(1/3)) - arctan(-sqrt(3) + 2*sqrt(sqrt(3
)*x^(1/3) + x^(2/3) + 1) - 2*x^(1/3)) + arctan(x^(1/3))
```

Sympy [A]

time = 0.37, size = 94, normalized size = 1.29

$$\frac{\sqrt{3}\log\left(4x^{\frac{2}{3}}-4\sqrt{3}\sqrt[3]{x}+4\right)}{4}-\frac{\sqrt{3}\log\left(4x^{\frac{2}{3}}+4\sqrt{3}\sqrt[3]{x}+4\right)}{4}+\operatorname{atan}\left(\sqrt[3]{x}\right)+\frac{\operatorname{atan}\left(2\sqrt[3]{x}-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2\sqrt[3]{x}+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(x**2+1),x)
```

```
[Out] sqrt(3)*log(4*x**(2/3) - 4*sqrt(3)*x**(1/3) + 4)/4 - sqrt(3)*log(4*x**(2/3)
 + 4*sqrt(3)*x**(1/3) + 4)/4 + atan(x**(1/3)) + atan(2*x**(1/3) - sqrt(3))/
2 + atan(2*x**(1/3) + sqrt(3))/2
```

Giac [A]

time = 0.79, size = 68, normalized size = 0.93

$$-\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)+\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)*
x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sqrt
(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

Mupad [B]

time = 4.77, size = 57, normalized size = 0.78

$$\operatorname{atan}\left(x^{1/3}\right)-\operatorname{atan}\left(\frac{486\,x^{1/3}}{-243+\sqrt{3}\,243\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\operatorname{atan}\left(\frac{486\,x^{1/3}}{243+\sqrt{3}\,243\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(x^2 + 1),x)
```

```
[Out] atan(x^(1/3)) - atan((486*x^(1/3))/(3^(1/2)*243i - 243))*((3^(1/2)*1i)/2 +
1/2) - atan((486*x^(1/3))/(3^(1/2)*243i + 243))*((3^(1/2)*1i)/2 - 1/2)
```

3.339 $\int x^m (a+bx^2)^5 \, dx$

Optimal. Leaf size=97

$$\frac{a^5x^{1+m}}{1+m}+\frac{5a^4bx^{3+m}}{3+m}+\frac{10a^3b^2x^{5+m}}{5+m}+\frac{10a^2b^3x^{7+m}}{7+m}+\frac{5ab^4x^{9+m}}{9+m}+\frac{b^5x^{11+m}}{11+m}$$

```
[Out] a^5*x^(1+m)/(1+m)+5*a^4*b*x^(3+m)/(3+m)+10*a^3*b^2*x^(5+m)/(5+m)+10*a^2*b^3
*x^(7+m)/(7+m)+5*a*b^4*x^(9+m)/(9+m)+b^5*x^(11+m)/(11+m)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$\frac{a^5x^{m+1}}{m+1}+\frac{5a^4bx^{m+3}}{m+3}+\frac{10a^3b^2x^{m+5}}{m+5}+\frac{10a^2b^3x^{m+7}}{m+7}+\frac{5ab^4x^{m+9}}{m+9}+\frac{b^5x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^(1 + m))/(1 + m) + (5*a^4*b*x^(3 + m))/(3 + m) + (10*a^3*b^2*x^(5 +
m))/(5 + m) + (10*a^2*b^3*x^(7 + m))/(7 + m) + (5*a*b^4*x^(9 + m))/(9 + m)
+ (b^5*x^(11 + m))/(11 + m)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^5\,dx &= \int \left(a^5x^m+5a^4bx^{2+m}+10a^3b^2x^{4+m}+10a^2b^3x^{6+m}+5ab^4x^{8+m}+b^5x^{10+m}\right)dx\\ &= \frac{a^5x^{1+m}}{1+m}+\frac{5a^4bx^{3+m}}{3+m}+\frac{10a^3b^2x^{5+m}}{5+m}+\frac{10a^2b^3x^{7+m}}{7+m}+\frac{5ab^4x^{9+m}}{9+m}+\frac{b^5x^{11+m}}{11+m}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 88, normalized size = 0.91

$$x^{1+m}\left(\frac{a^5}{1+m}+\frac{5a^4bx^2}{3+m}+\frac{10a^3b^2x^4}{5+m}+\frac{10a^2b^3x^6}{7+m}+\frac{5ab^4x^8}{9+m}+\frac{b^5x^{10}}{11+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^5,x]
```

```
[Out] x^(1 + m)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (10*a^3*b^2*x^4)/(5 + m) +
 (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m) + (b^5*x^10)/(11 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(97) = 194$.
time = 0.03, size = 431, normalized size = 4.44

method	result
risch	$\frac{x(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689m\,x^{10}b^5+290a}{}$
gosper	$\frac{x^{1+m}(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689m\,x^{10}b^5+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*a*b^4*
m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b^5*m*x
^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^5*x^4+
3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*b^3*m^
2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b^3*m*x
^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+2030*a
^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+20790*a^3*b
^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+9129*a^5*
m+10395*a^5)*x^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [A]
time = 0.30, size = 97, normalized size = 1.00

$$\frac{b^5x^{m+11}}{m+11}+\frac{5\,ab^4x^{m+9}}{m+9}+\frac{10\,a^2b^3x^{m+7}}{m+7}+\frac{10\,a^3b^2x^{m+5}}{m+5}+\frac{5\,a^4bx^{m+3}}{m+3}+\frac{a^5x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] b^5*x^(m + 11)/(m + 11) + 5*a*b^4*x^(m + 9)/(m + 9) + 10*a^2*b^3*x^(m + 7)/
(m + 7) + 10*a^3*b^2*x^(m + 5)/(m + 5) + 5*a^4*b*x^(m + 3)/(m + 3) + a^5*x^
(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(97) = 194$.
time = 1.21, size = 367, normalized size = 3.78

$$\frac{((b^5m^5+25\,b^5m^4+230\,b^5m^3+950\,b^5m^2+1689\,b^5m+945\,b^5)x^{11}+5\,(ab^4m^5+27\,ab^4m^4+262\,ab^4m^3+1122\,ab^4m^2+2041\,ab^4m+1155\,ab^4)x^9+10\,(a^2b^3m^5+29\,a^2b^3m^4+302\,a^2b^3m^3+1366\,a^2b^3m^2+2577\,a^2b^3m+1485\,a^2b^3)x^7+10\,(a^3b^2m^5+31\,a^3b^2m^4+350\,a^3b^2m^3+1730\,a^3b^2m^2+3489\,a^3b^2m+2079\,a^3b^2)x^5+5\,(a^4bm^5+33\,a^4bm^4+406\,a^4bm^3+2262\,a^4bm^2+5353\,a^4bm+3465\,a^4b)x^3+(a^5m^5+35\,a^5m^4+470\,a^5m^3+3010\,a^5m^2+9129\,a^5m+10395\,a^5)x)x^m}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="fricas")

[Out] ((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)*x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 2041*a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b^3*m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b^2*m^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^2*m + 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 2262*a^4*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 470*a^5*m^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1999 vs. $2(87) = 174$.
time = 0.67, size = 1999, normalized size = 20.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(b*x**2+a)**5,x)

[Out] Piecewise((-a**5/(10*x**10) - 5*a**4*b/(8*x**8) - 5*a**3*b**2/(3*x**6) - 5*a**2*b**3/(2*x**4) - 5*a*b**4/(2*x**2) + b**5*log(x), Eq(m, -11)), (-a**5/(8*x**8) - 5*a**4*b/(6*x**6) - 5*a**3*b**2/(2*x**4) - 5*a**2*b**3/x**2 + 5*a*b**4*log(x) + b**5*x**2/2, Eq(m, -9)), (-a**5/(6*x**6) - 5*a**4*b/(4*x**4) - 5*a**3*b**2/x**2 + 10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4, Eq(m, -7)), (-a**5/(4*x**4) - 5*a**4*b/(2*x**2) + 10*a**3*b**2*log(x) + 5*a**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6, Eq(m, -5)), (-a**5/(2*x**2) + 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 + 5*a*b**4*x**6/6 + b**5*x**8/8, Eq(m, -3)), (a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**3*b**2*x**4/2 + 5*a**2*b**3*x**6/3 + 5*a*b**4*x**8/8 + b**5*x**10/10, Eq(m, -1)), (a**5*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35*a**5*m**4*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**5*m**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3010*a**5*m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129*a**5*m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5*a**4*b*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 165*a**4*b*m**4*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2030*a**4*b*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 11310*a**4*b*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 26765*a**4*b*m*x**3*x**m

```
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1
7325*a**4*b*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 10*a**3*b**2*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 310*a**3*b**2*m**4*x**5*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35
00*a**3*b**2*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 17300*a**3*b**2*m**2*x**5*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 34890*a**3*b**2*m*x*
*5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 20790*a**3*b**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 10*a**2*b**3*m**5*x**7*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 290*a**2*b**3*m**4*
x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 3020*a**2*b**3*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 13660*a**2*b**3*m**2*x**7*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25770*a**2
*b**3*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 195
24*m + 10395) + 14850*a**2*b**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 5*a*b**4*m**5*x**9*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 135*a*b**4*m*
*4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 1310*a*b**4*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 5610*a*b**4*m**2*x**9*x**m/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10205*a*b**4*m
*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
10395) + 5775*a*b**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + b**5*m**5*x**11*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25*b**5*m**4*x**11*x**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 230*b
**5*m**3*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 950*b**5*m**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 1689*b**5*m*x**11*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 945*b**5*x**1
1*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(97) = 194$.
time = 0.91, size = 540, normalized size = 5.57

$$\frac{b^{5} m^{5} x^{11} x^{m} + 25 b^{5} m^{4} x^{11} x^{m} + 5 a b^{4} m^{5} x^{9} x^{m} + 230 b^{5} m^{3} x^{11} x^{m} + 135 a b^{4} m^{4} x^{9} x^{m} + 950 b^{5} m^{2} x^{11} x^{m} + 10 a^{2} b^{3} m^{5} x^{7} x^{m} + 1310 a b^{4} m^{3} x^{9} x^{m} + 1689 b^{5} m x^{11} x^{m} + 290 a^{2} b^{3} m^{4} x^{7} x^{m} + 5610 a b^{4} m^{2} x^{9} x^{m} + 945 b^{5} x^{11} x^{m} + 10 a^{3} b^{2} m^{5} x^{5} x^{m} + 3020 a^{2} b^{3} m^{3} x^{7} x^{m} + 10205 a b^{4} m x^{9} x^{m} + 310 a^{3} b^{2} m^{4} x^{5} x^{m} + 13660 a^{2} b^{3} m^{2} x^{7} x^{m} + 5775 a b^{4} x^{9} x^{m} + 5 a^{4} b m^{5} x^{3} x^{m} + 3500 a^{3} b^{2} m^{3} x^{5} x^{m} + 25770 a^{2} b^{3} m x^{7} x^{m} + 165 a^{4} b m^{4} x^{3} x^{m} + 17300 a^{3} b^{2} m^{2} x^{5} x^{m} + 14850 a^{2} b^{3} x^{7} x^{m} + a^{5} m^{5} x x^{m} + 2030 a^{4} b m^{3} x^{3} x^{m} + 34890 a^{3} b^{2} m x^{5} x^{m} + 35 a^{5} m^{4} x x^{m} + 11310 a^{4} b m^{2} x^{3} x^{m} + 20790 a^{3} b^{2} x^{5} x^{m} + 470 a^{5} m^{3} x x^{m} + 26765 a^{4} b m x^{3} x^{m} + 3010 a^{5} m^{2} x x^{m} + 17325 a^{4} b x^{3} x^{m} + 9129 a^{5} m x x^{m} + 10395 a^{5} x x^{m}}{m^{6} + 36 m^{5} + 505 m^{4} + 3480 m^{3} + 12139 m^{2} + 19524 m + 10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] (b^5*m^5*x^11*x^m + 25*b^5*m^4*x^11*x^m + 5*a*b^4*m^5*x^9*x^m + 230*b^5*m^3
*x^11*x^m + 135*a*b^4*m^4*x^9*x^m + 950*b^5*m^2*x^11*x^m + 10*a^2*b^3*m^5*x
^7*x^m + 1310*a*b^4*m^3*x^9*x^m + 1689*b^5*m*x^11*x^m + 290*a^2*b^3*m^4*x^7
*x^m + 5610*a*b^4*m^2*x^9*x^m + 945*b^5*x^11*x^m + 10*a^3*b^2*m^5*x^5*x^m +
 3020*a^2*b^3*m^3*x^7*x^m + 10205*a*b^4*m*x^9*x^m + 310*a^3*b^2*m^4*x^5*x^m
 + 13660*a^2*b^3*m^2*x^7*x^m + 5775*a*b^4*x^9*x^m + 5*a^4*b*m^5*x^3*x^m + 3
500*a^3*b^2*m^3*x^5*x^m + 25770*a^2*b^3*m*x^7*x^m + 165*a^4*b*m^4*x^3*x^m +
 17300*a^3*b^2*m^2*x^5*x^m + 14850*a^2*b^3*x^7*x^m + a^5*m^5*x*x^m + 2030*a
^4*b*m^3*x^3*x^m + 34890*a^3*b^2*m*x^5*x^m + 35*a^5*m^4*x*x^m + 11310*a^4*b
*m^2*x^3*x^m + 20790*a^3*b^2*x^5*x^m + 470*a^5*m^3*x*x^m + 26765*a^4*b*m*x^
3*x^m + 3010*a^5*m^2*x*x^m + 17325*a^4*b*x^3*x^m + 9129*a^5*m*x*x^m + 10395
*a^5*x*x^m)/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 1039
5)
```

Mupad [B]
time = 5.11, size = 389, normalized size = 4.01

$$\frac{a^5\,x\,x^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{b^5\,x^m\,x^{11}\,\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{5\,a\,b^4\,x^m\,x^9\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{5\,a^4\,b\,x^m\,x^3\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{10\,a^2\,b^3\,x^m\,x^7\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+1485\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{10\,a^3\,b^2\,x^m\,x^5\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+2079\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x*x^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395))/(19524*m +
 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (b^5*x^m*x^11*(16
89*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))/(19524*m + 12139*m^2 + 3480
*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (5*a*b^4*x^m*x^9*(2041*m + 1122*m^
2 + 262*m^3 + 27*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m
^4 + 36*m^5 + m^6 + 10395) + (5*a^4*b*x^m*x^3*(5353*m + 2262*m^2 + 406*m^3
+ 33*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5
+ m^6 + 10395) + (10*a^2*b^3*x^m*x^7*(2577*m + 1366*m^2 + 302*m^3 + 29*m^4
+ m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 1
0395) + (10*a^3*b^2*x^m*x^5*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4 + m^5 + 2
079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
```

3.340 $\int x^m (a+bx^2)^4 \, dx$

Optimal. Leaf size=79

$$\frac{a^4x^{1+m}}{1+m}+\frac{4a^3bx^{3+m}}{3+m}+\frac{6a^2b^2x^{5+m}}{5+m}+\frac{4ab^3x^{7+m}}{7+m}+\frac{b^4x^{9+m}}{9+m}$$

```
[Out] a^4*x^(1+m)/(1+m)+4*a^3*b*x^(3+m)/(3+m)+6*a^2*b^2*x^(5+m)/(5+m)+4*a*b^3*x^(
7+m)/(7+m)+b^4*x^(9+m)/(9+m)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$\frac{a^4x^{m+1}}{m+1}+\frac{4a^3bx^{m+3}}{m+3}+\frac{6a^2b^2x^{m+5}}{m+5}+\frac{4ab^3x^{m+7}}{m+7}+\frac{b^4x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^4,x]
```

```
[Out] (a^4*x^(1 + m))/(1 + m) + (4*a^3*b*x^(3 + m))/(3 + m) + (6*a^2*b^2*x^(5 + m
))/(5 + m) + (4*a*b^3*x^(7 + m))/(7 + m) + (b^4*x^(9 + m))/(9 + m)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^4\,dx &= \int \left(a^4x^m+4a^3bx^{2+m}+6a^2b^2x^{4+m}+4ab^3x^{6+m}+b^4x^{8+m}\right)\,dx\\ &= \frac{a^4x^{1+m}}{1+m}+\frac{4a^3bx^{3+m}}{3+m}+\frac{6a^2b^2x^{5+m}}{5+m}+\frac{4ab^3x^{7+m}}{7+m}+\frac{b^4x^{9+m}}{9+m}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 72, normalized size = 0.91

$$x^{1+m}\left(\frac{a^4}{1+m}+\frac{4a^3bx^2}{3+m}+\frac{6a^2b^2x^4}{5+m}+\frac{4ab^3x^6}{7+m}+\frac{b^4x^8}{9+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^4,x]
```

```
[Out] x^(1 + m)*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
(4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(79) = 158.
time = 0.02, size = 290, normalized size = 3.67

method	result
risch	$x\left(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2b^2m^3x^4+888m\right.$
gosper	$x^{1+m}\left(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2b^2m^3x^4+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^3*m^3*x
^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+120*a^2*b^
2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*a*b^3*x^6
+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a^2*b^2
*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m+945*a
^4)*x^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [A]
time = 0.30, size = 79, normalized size = 1.00

$$\frac{b^4x^{m+9}}{m+9}+\frac{4\,ab^3x^{m+7}}{m+7}+\frac{6\,a^2b^2x^{m+5}}{m+5}+\frac{4\,a^3bx^{m+3}}{m+3}+\frac{a^4x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] b^4*x^(m + 9)/(m + 9) + 4*a*b^3*x^(m + 7)/(m + 7) + 6*a^2*b^2*x^(m + 5)/(m
+ 5) + 4*a^3*b*x^(m + 3)/(m + 3) + a^4*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(79) = 158.
time = 1.12, size = 251, normalized size = 3.18

$$\frac{\left(\left(b^4m^4+16\,b^4m^3+86\,b^4m^2+176\,b^4m+105\,b^4\right)x^9+4\left(ab^3m^4+18\,ab^3m^3+104\,ab^3m^2+222\,ab^3m+135\,ab^3\right)x^7+6\left(a^2b^2m^4+20\,a^2b^2m^3+130\,a^2b^2m^2+300\,a^2b^2m+189\,a^2b^2\right)x^5+4\left(a^3bm^4+22\,a^3bm^3+164\,a^3bm^2+458\,a^3bm+315\,a^3b\right)x^3+\left(a^4m^4+24\,a^4m^3+206\,a^4m^2+744\,a^4m+945\,a^4\right)x\right)x^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

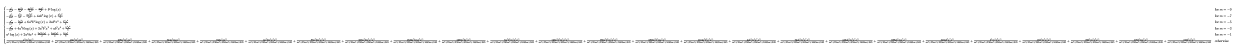
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] ((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*m
^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2*b
```

```
^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)*x^
5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3*b)*
x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*x^m/(m^
5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1221 vs. $2(70) = 140$.
time = 0.50, size = 1221, normalized size = 15.46



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**4,x)
```

```
[Out] Piecewise((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a*
b**3/x**2 + b**4*log(x), Eq(m, -9)), (-a**4/(6*x**6) - a**3*b/x**4 - 3*a**2
*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2, Eq(m, -7)), (-a**4/(4*x**4) - 2
*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4, Eq(m, -5))
, (-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 + b**4
*x**6/6, Eq(m, -3)), (a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*
a*b**3*x**6/3 + b**4*x**8/8, Eq(m, -1)), (a**4*m**4*x*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 24*a**4*m**3*x*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 206*a**4*m**2*x*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*a**4*m*x*x**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 945*a**4*x*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a**3*b*m**4*x**3*x**m/(m**5 + 25*m
**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*m**3*x**3*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*m**2*x**3*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1832*a**3*b*m*x**3
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1260*a**3*b*x
**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 6*a**2*b**
2*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 12
0*a**2*b**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 780*a**2*b**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
+ 1689*m + 945) + 1800*a**2*b**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 1134*a**2*b**2*x**5*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 4*a*b**3*m**4*x**7*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 72*a*b**3*m**3*x**7*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 416*a*b**3*m**2*x**7*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 888*a*b**3*m*x**7*x**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 540*a*b**3*x**7*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**4*m**4*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**4*m**3*x**9*
x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**4*m**2*x
**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4*m
```

```
*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*b**4
*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(79) = 158.
time = 1.23, size = 365, normalized size = 4.62

$$\frac{b^4m^4x^9x^m + 16\,b^4m^3x^9x^m + 4\,ab^3m^4x^7x^m + 86\,b^4m^2x^9x^m + 72\,ab^3m^3x^7x^m + 176\,b^4mx^9x^m + 6\,a^2b^2m^4x^5x^m + 416\,ab^3m^2x^7x^m + 105\,b^4x^9x^m + 120\,a^2b^2m^3x^5x^m + 888\,ab^3mx^7x^m + 4\,a^3bm^4x^3x^m + 780\,a^2b^2m^2x^5x^m + 540\,ab^3x^7x^m + 88\,a^3bm^3x^3x^m + 1800\,a^2b^2mx^5x^m + a^4m^4xx^m + 656\,a^3bm^2x^3x^m + 1134\,a^2b^2x^5x^m + 24\,a^4m^3xx^m + 1832\,a^3bmx^3x^m + 206\,a^4m^2xx^m + 1260\,a^3bx^3x^m + 744\,a^4mxx^m + 945\,a^4xx^m}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] (b^4*m^4*x^9*x^m + 16*b^4*m^3*x^9*x^m + 4*a*b^3*m^4*x^7*x^m + 86*b^4*m^2*x^
9*x^m + 72*a*b^3*m^3*x^7*x^m + 176*b^4*m*x^9*x^m + 6*a^2*b^2*m^4*x^5*x^m +
416*a*b^3*m^2*x^7*x^m + 105*b^4*x^9*x^m + 120*a^2*b^2*m^3*x^5*x^m + 888*a*b
^3*m*x^7*x^m + 4*a^3*b*m^4*x^3*x^m + 780*a^2*b^2*m^2*x^5*x^m + 540*a*b^3*x^
7*x^m + 88*a^3*b*m^3*x^3*x^m + 1800*a^2*b^2*m*x^5*x^m + a^4*m^4*x*x^m + 656
*a^3*b*m^2*x^3*x^m + 1134*a^2*b^2*x^5*x^m + 24*a^4*m^3*x*x^m + 1832*a^3*b*m
*x^3*x^m + 206*a^4*m^2*x*x^m + 1260*a^3*b*x^3*x^m + 744*a^4*m*x*x^m + 945*a
^4*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Mupad [B]
time = 4.99, size = 272, normalized size = 3.44

$$\frac{a^4\,x\,x^m\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{b^4\,x^m\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{6\,a^2\,b^2\,x^m\,x^5\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{4\,a\,b^3\,x^m\,x^7\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{4\,a^3\,b\,x^m\,x^3\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^4,x)
```

```
[Out] (a^4*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 230*
m^3 + 25*m^4 + m^5 + 945) + (b^4*x^m*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 1
05))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (6*a^2*b^2*x^m*x^5
*(300*m + 130*m^2 + 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m
^4 + m^5 + 945) + (4*a*b^3*x^m*x^7*(222*m + 104*m^2 + 18*m^3 + m^4 + 135))/
(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (4*a^3*b*x^m*x^3*(458*m
 + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^
5 + 945)
```

3.341 $\int x^m (a+bx^2)^3 \, dx$

Optimal. Leaf size=61

$$\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{3+m}}{3+m}+\frac{3ab^2x^{5+m}}{5+m}+\frac{b^3x^{7+m}}{7+m}$$

```
[Out] a^3*x^(1+m)/(1+m)+3*a^2*b*x^(3+m)/(3+m)+3*a*b^2*x^(5+m)/(5+m)+b^3*x^(7+m)/(
7+m)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+3}}{m+3}+\frac{3ab^2x^{m+5}}{m+5}+\frac{b^3x^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(3 + m))/(3 + m) + (3*a*b^2*x^(5 + m))
/(5 + m) + (b^3*x^(7 + m))/(7 + m)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^3\,dx &= \int\left(a^3x^m+3a^2bx^{2+m}+3ab^2x^{4+m}+b^3x^{6+m}\right)\,dx\\ &= \frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{3+m}}{3+m}+\frac{3ab^2x^{5+m}}{5+m}+\frac{b^3x^{7+m}}{7+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.92

$$x^{1+m}\left(\frac{a^3}{1+m}+\frac{3a^2bx^2}{3+m}+\frac{3ab^2x^4}{5+m}+\frac{b^3x^6}{7+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^2)/(3 + m) + (3*a*b^2*x^4)/(5 + m) + (b
^3*x^6)/(7 + m))
```

Maple [A]
time = 0.03, size = 72, normalized size = 1.18

method	result
norman	$\frac{a^{3} x \,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b^{3} x^{7} {\mathrm e}^{m \ln \left(x \right)}}{7+m}+\frac{3 a \,b^{2} x^{5} {\mathrm e}^{m \ln \left(x \right)}}{5+m}+\frac{3 a^{2} b \,x^{3} {\mathrm e}^{m \ln \left(x \right)}}{3+m}$
risch	$\frac{x \left(b^{3} m^{3} x^{6}+9 b^{3} m^{2} x^{6}+3 a \,b^{2} m^{3} x^{4}+23 m \,x^{6} b^{3}+33 a \,b^{2} m^{2} x^{4}+15 b^{3} x^{6}+3 a^{2} b \,m^{3} x^{2}+93 m \,x^{4} a \,b^{2}+39 a^{2} b \,m^{2} x^{2}+63 a \,b^{2} x^{4}+a^{3} m^{3}+141 m \,x}{\left(7+m \right) \left(5+m \right) \left(3+m \right) \left(1+m \right)}$
gosper	$\frac{x^{1+m} \left(b^{3} m^{3} x^{6}+9 b^{3} m^{2} x^{6}+3 a \,b^{2} m^{3} x^{4}+23 m \,x^{6} b^{3}+33 a \,b^{2} m^{2} x^{4}+15 b^{3} x^{6}+3 a^{2} b \,m^{3} x^{2}+93 m \,x^{4} a \,b^{2}+39 a^{2} b \,m^{2} x^{2}+63 a \,b^{2} x^{4}+a^{3} m^{3}+14}{\left(7+m \right) \left(5+m \right) \left(3+m \right) \left(1+m \right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/(1+m)*x*exp(m*ln(x))+b^3/(7+m)*x^7*exp(m*ln(x))+3*a*b^2/(5+m)*x^5*exp(m
*ln(x))+3*a^2*b/(3+m)*x^3*exp(m*ln(x))
```

Maxima [A]
time = 0.27, size = 61, normalized size = 1.00

$$\frac{b^{3} x^{m+7}}{m+7} + \frac{3 \, a b^{2} x^{m+5}}{m+5} + \frac{3 \, a^{2} b x^{m+3}}{m+3} + \frac{a^{3} x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 7)/(m + 7) + 3*a*b^2*x^(m + 5)/(m + 5) + 3*a^2*b*x^(m + 3)/(m +
3) + a^3*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(61) = 122.
time = 1.20, size = 157, normalized size = 2.57

$$\frac{{\left({\left(b^{3} m^{3} + 9 \, b^{3} m^{2} + 23 \, b^{3} m + 15 \, b^{3}\right)} x^{7} + 3 \, {\left(a b^{2} m^{3} + 11 \, a b^{2} m^{2} + 31 \, a b^{2} m + 21 \, a b^{2}\right)} x^{5} + 3 \, {\left(a^{2} b m^{3} + 13 \, a^{2} b m^{2} + 47 \, a^{2} b m + 35 \, a^{2} b\right)} x^{3} + {\left(a^{3} m^{3} + 15 \, a^{3} m^{2} + 71 \, a^{3} m + 105 \, a^{3}\right)} x\right)} x^{m}}{m^{4} + 16 \, m^{3} + 86 \, m^{2} + 176 \, m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m^
2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m +
 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*x^m/(m^4 +
16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(53) = 106$.
time = 0.35, size = 683, normalized size = 11.20

$$\begin{cases} -\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3\log(x) & \text{for } m = -7 \\ -\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2\log(x) + \frac{b^3x^2}{2} & \text{for } m = -5 \\ -\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4} & \text{for } m = -3 \\ a^3\log(x) + \frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6} & \text{for } m = -1 \\ \frac{a^3m^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{15a^3m^2xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{71a^3mxx^m}{m^4+16m^3+86m^2+176m+105} + \frac{105a^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{3a^2bm^3x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{39a^2bm^2x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{141a^2bmx^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{105a^2bx^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{3ab^2m^3x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{33ab^2m^2x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{93ab^2mx^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{63ab^2x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{b^3m^3x^7x^m}{m^4+16m^3+86m^2+176m+105} + \frac{9b^3m^2x^7x^m}{m^4+16m^3+86m^2+176m+105} + \frac{23b^3mx^7x^m}{m^4+16m^3+86m^2+176m+105} + \frac{15b^3x^7x^m}{m^4+16m^3+86m^2+176m+105} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a*b**2/(2*x**2) + b**3*lo
g(x), Eq(m, -7)), (-a**3/(4*x**4) - 3*a**2*b/(2*x**2) + 3*a*b**2*log(x) + b
**3*x**2/2, Eq(m, -5)), (-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2
 + b**3*x**4/4, Eq(m, -3)), (a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/
4 + b**3*x**6/6, Eq(m, -1)), (a**3*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 +
176*m + 105) + 15*a**3*m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + 71*a**3*m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*a**3*x*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 3*a**2*b*m**3*x**3*x**m/(m**
4 + 16*m**3 + 86*m**2 + 176*m + 105) + 39*a**2*b*m**2*x**3*x**m/(m**4 + 16*
m**3 + 86*m**2 + 176*m + 105) + 141*a**2*b*m*x**3*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 105*a**2*b*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 3*a*b**2*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
) + 33*a*b**2*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 93*
a*b**2*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 63*a*b**2*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**3*m**3*x**7*x**m/(m**4
 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*b**3*m**2*x**7*x**m/(m**4 + 16*m**3
 + 86*m**2 + 176*m + 105) + 23*b**3*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 +
 176*m + 105) + 15*b**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105),
 True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(61) = 122$.
time = 1.65, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^7x^m + 9\,b^3m^2x^7x^m + 3\,ab^2m^3x^5x^m + 23\,b^3mx^7x^m + 33\,ab^2m^2x^5x^m + 15\,b^3x^7x^m + 3\,a^2bm^3x^3x^m + 93\,ab^2mx^5x^m + 39\,a^2bm^2x^3x^m + 63\,ab^2x^5x^m + a^3m^3xx^m + 141\,a^2bmx^3x^m + 15\,a^3m^2xx^m + 105\,a^2bx^3x^m + 71\,a^3mxx^m + 105\,a^3xx^m}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*x^m + 9*b^3*m^2*x^7*x^m + 3*a*b^2*m^3*x^5*x^m + 23*b^3*m*x^7*x
^m + 33*a*b^2*m^2*x^5*x^m + 15*b^3*x^7*x^m + 3*a^2*b*m^3*x^3*x^m + 93*a*b^2
*m*x^5*x^m + 39*a^2*b*m^2*x^3*x^m + 63*a*b^2*x^5*x^m + a^3*m^3*x*x^m + 141*
a^2*b*m*x^3*x^m + 15*a^3*m^2*x*x^m + 105*a^2*b*x^3*x^m + 71*a^3*m*x*x^m + 1
05*a^3*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Mupad [B]
time = 4.79, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b^3\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{3\,a\,b^2\,x^5\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{3\,a^2\,b\,x^3\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^3,x)
```

```
[Out] x^m*((a^3*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 + m^4 + 1
05) + (b^3*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 +
105) + (3*a*b^2*x^5*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m^3 +
m^4 + 105) + (3*a^2*b*x^3*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*
m^3 + m^4 + 105))
```

3.342 $\int x^m (a+bx^2)^2 \, dx$

Optimal. Leaf size=43

$$\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{3+m}}{3+m}+\frac{b^2x^{5+m}}{5+m}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(3+m)/(3+m)+b^2*x^(5+m)/(5+m)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{276\}$

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+3}}{m+3}+\frac{b^2x^{m+5}}{m+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(3 + m))/(3 + m) + (b^2*x^(5 + m))/(5 +
m)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^2 \, dx &= \int \left(a^2x^m+2abx^{2+m}+b^2x^{4+m}\right) \, dx\\ &= \frac{a^2x^{1+m}}{1+m}+\frac{2abx^{3+m}}{3+m}+\frac{b^2x^{5+m}}{5+m}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.93

$$x^{1+m}\left(\frac{a^2}{1+m}+\frac{2abx^2}{3+m}+\frac{b^2x^4}{5+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

Maple [A]
time = 0.03, size = 51, normalized size = 1.19

method	result	size
norman	$\frac{a^{2} x\,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b^{2} x^{5} {\mathrm e}^{m \ln \left(x \right)}}{5+m}+\frac{2 a b\,x^{3} {\mathrm e}^{m \ln \left(x \right)}}{3+m}$	51
risch	$\frac{x \left(b^{2} m^{2} x^{4}+4 m\,x^{4} b^{2}+2 a b\,m^{2} x^{2}+3 b^{2} x^{4}+12 m\,x^{2} a b+a^{2} m^{2}+10 a b\,x^{2}+8 m\,a^{2}+15 a^{2}\right) x^{m}}{\left(5+m \right) \left(3+m \right) \left(1+m \right)}$	92
gosper	$\frac{x^{1+m} \left(b^{2} m^{2} x^{4}+4 m\,x^{4} b^{2}+2 a b\,m^{2} x^{2}+3 b^{2} x^{4}+12 m\,x^{2} a b+a^{2} m^{2}+10 a b\,x^{2}+8 m\,a^{2}+15 a^{2}\right)}{\left(5+m \right) \left(3+m \right) \left(1+m \right)}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(1+m)*x*exp(m*ln(x))+b^2/(5+m)*x^5*exp(m*ln(x))+2*a*b/(3+m)*x^3*exp(m*l
n(x))
```

Maxima [A]
time = 0.29, size = 43, normalized size = 1.00

$$\frac{b^{2} x^{m+5}}{m+5}+\frac{2\,a b x^{m+3}}{m+3}+\frac{a^{2} x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 5)/(m + 5) + 2*a*b*x^(m + 3)/(m + 3) + a^2*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 1.25, size = 85, normalized size = 1.98

$$\frac{((b^{2} m^{2}+4\,b^{2} m+3\,b^{2})x^{5}+2\,(a b m^{2}+6\,a b m+5\,a b)x^{3}+(a^{2} m^{2}+8\,a^{2} m+15\,a^{2})x)x^{m}}{m^{3}+9\,m^{2}+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^2
*m^2 + 8*a^2*m + 15*a^2)*x)*x^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(36) = 72.
time = 0.24, size = 306, normalized size = 7.12

$$\begin{cases} - \frac{a^{2}}{4 x^{4}} - \frac{a b}{x^{2}} + b^{2} \log{\left(x \right)} & \text{for}\: m = -5 \\ - \frac{a^{2}}{2 x^{2}} + 2 a b \log{\left(x \right)} + \frac{b^{2} x^{2}}{2} & \text{for}\: m = -3 \\ a^{2} \log{\left(x \right)} + a b x^{2} + \frac{b^{2} x^{4}}{4} & \text{for}\: m = -1 \\ \frac{a^{2} m^{2} x x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{8 a^{2} m x x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{15 a^{2} x x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{2 a b m^{2} x^{3} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{12 a b m x^{3} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{10 a b x^{3} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{b^{2} m^{2} x^{5} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{4 b^{2} m x^{5} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{3 b^{2} x^{5} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x), Eq(m, -5)), (-a**2/(2*x
**2) + 2*a*b*log(x) + b**2*x**2/2, Eq(m, -3)), (a**2*log(x) + a*b*x**2 + b*
*2*x**4/4, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a*
*2*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a**2*x*x**m/(m**3 + 9*m**2 + 2
3*m + 15) + 2*a*b*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 12*a*b*m*x**
3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 10*a*b*x**3*x**m/(m**3 + 9*m**2 + 23*m
 + 15) + b**2*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*m*x**5*x*
*m/(m**3 + 9*m**2 + 23*m + 15) + 3*b**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 1
5), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(43) = 86.
time = 1.20, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^5x^m + 4\,b^2mx^5x^m + 2\,abm^2x^3x^m + 3\,b^2x^5x^m + 12\,abmx^3x^m + a^2m^2xx^m + 10\,abx^3x^m + 8\,a^2mxx^m + 15\,a^2xx^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^5*x^m + 4*b^2*m*x^5*x^m + 2*a*b*m^2*x^3*x^m + 3*b^2*x^5*x^m + 12
*a*b*m*x^3*x^m + a^2*m^2*x*x^m + 10*a*b*x^3*x^m + 8*a^2*m*x*x^m + 15*a^2*x*
x^m)/(m^3 + 9*m^2 + 23*m + 15)
```

Mupad [B]
time = 4.75, size = 93, normalized size = 2.16

$$x^m\,\left(\frac{a^2\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}+\frac{b^2\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{2\,a\,b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^2,x)
```

```
[Out] x^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*m + m
^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*m + 9*
m^2 + m^3 + 15))
```

3.343 $\int x^m(a+bx^2)\,dx$

Optimal. Leaf size=25

$$\frac{ax^{1+m}}{1+m}+\frac{bx^{3+m}}{3+m}$$

```
[Out] a*x^(1+m)/(1+m)+b*x^(3+m)/(3+m)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{14\}$

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2),x]
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)\,dx&=\int\left(ax^m+bx^{2+m}\right)\,dx\\&=\frac{ax^{1+m}}{1+m}+\frac{bx^{3+m}}{3+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$\frac{ax^{1+m}}{1+m}+\frac{bx^{3+m}}{3+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2),x]
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

Maple [A]
time = 0.01, size = 30, normalized size = 1.20

method	result	size
norman	$\frac{a x \,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b \,x^{3} {\mathrm e}^{m \ln \left(x \right)}}{3+m}$	30
risch	$\frac{x \left(b m \,x^{2}+b \,x^{2}+a m +3 a \right) x^{m}}{\left(3+m \right) \left(1+m \right)}$	34
gosper	$\frac{x^{1+m} \left(b m \,x^{2}+b \,x^{2}+a m +3 a \right)}{\left(3+m \right) \left(1+m \right)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] a/(1+m)*x*exp(m*ln(x))+b/(3+m)*x^3*exp(m*ln(x))
```

Maxima [A]
time = 0.30, size = 25, normalized size = 1.00

$$\frac{bx^{m+3}}{m+3} + \frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 3)/(m + 3) + a*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 1.78, size = 33, normalized size = 1.32

$$\frac{((bm+b)x^3 + (am+3\,a)x)x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^3 + (a*m + 3*a)*x)*x^m/(m^2 + 4*m + 3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(19) = 38$.
time = 0.13, size = 94, normalized size = 3.76

$$\begin{cases} -\frac{a}{2x^2} + b\log{(x)} & \text{for}\: m = -3 \\ a\log{(x)} + \frac{bx^2}{2} & \text{for}\: m = -1 \\ \frac{amxx^m}{m^2+4m+3} + \frac{3axx^m}{m^2+4m+3} + \frac{bmx^3x^m}{m^2+4m+3} + \frac{bx^3x^m}{m^2+4m+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a),x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*log(x), Eq(m, -3)), (a*log(x) + b*x**2/2, Eq(m,
-1)), (a*m*x*x**m/(m**2 + 4*m + 3) + 3*a*x*x**m/(m**2 + 4*m + 3) + b*m*x**3
*x**m/(m**2 + 4*m + 3) + b*x**3*x**m/(m**2 + 4*m + 3), True))
```

Giac [A]
time = 0.90, size = 43, normalized size = 1.72

$$\frac{bmx^3x^m + bx^3x^m + amxx^m + 3\,axx^m}{m^2 + 4\,m + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b*m*x^3*x^m + b*x^3*x^m + a*m*x*x^m + 3*a*x*x^m)/(m^2 + 4*m + 3)
```

Mupad [B]
time = 4.80, size = 34, normalized size = 1.36

$$\frac{x^{m+1}\,\left(3\,a+a\,m+b\,x^2+b\,m\,x^2\right)}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2),x)
```

```
[Out] (x^(m + 1)*(3*a + a*m + b*x^2 + b*m*x^2))/(4*m + m^2 + 3)
```

3.344 $\int \frac{x^m}{a+bx^2} dx$

Optimal. Leaf size=39

$$\frac{x^{1+m} {}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

```
[Out] x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {371}

$$\frac{x^{m+1} {}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1
+ m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{a+bx^2} dx = \frac{x^{1+m} {}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.05

$$\frac{x^{1+m} {}_2F_1\left(1,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{b\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a),x)
```

```
[Out] int(x^m/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.54, size = 88, normalized size = 2.26

$$\frac{mxx^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{xx^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a),x)
```

```
[Out] m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(4*a*gamma(m/2 + 3/2)) + x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 +
 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a),x, algorithm="giac")
[Out] integrate(x^m/(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2),x)
[Out] int(x^m/(a + b*x^2), x)
```

3.345 $\int \frac{x^m}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=39

$$\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)}$$

```
[Out] x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {371}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^2*(
1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)^2}\,dx = \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.05

$$\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^2*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(b\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2,x)
```

```
[Out] int(x^m/(b*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.87, size = 374, normalized size = 9.59

$$-\frac{am^{2}xx^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^{3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^{2}bx^{2}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2amxx^{m}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^{3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^{2}bx^{2}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{axx^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^{3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^{2}bx^{2}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2axx^{m}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^{3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^{2}bx^{2}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}-\frac{bm^{2}x^{3}x^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^{3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^{2}bx^{2}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{bx^{3}x^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^{3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^{2}bx^{2}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2,x)
```

```
[Out] -a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*m*x*
x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 +
```

```
3/2)) + a*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*x*
x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 +
3/2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*g
amma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))
+ b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(x^m/(b*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^2,x)
[Out] int(x^m/(a + b*x^2)^2, x)
```

3.346 $\int \frac{x^m}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=39

$$\frac{x^{1+m}\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^3(1+m)}$$

```
[Out] x^(1+m)*hypergeom([3, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{371\}$

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^3*(
1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)^3}\,dx = \frac{x^{1+m}\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^3(1+m)}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.05

$$\frac{x^{1+m}\,{}_2F_1\left(3,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a^3(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^3*(1 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^3,x)
```

```
[Out] int(x^m/(b*x^2+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.05, size = 1556, normalized size = 39.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**3,x)
```

```
[Out] a**2*m**3*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a*
*3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*m**2*x*x**m*lerchphi(b*x**2*exp_pol
```

```
ar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a
**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a**2*
m**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gam
ma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - a**2*m*x*x**m*lerchph
i(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m
/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 +
 3/2)) + 8*a**2*m*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a*
*4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*a**2*x
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32
*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**
4*gamma(m/2 + 3/2)) + 10*a**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 +
3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)
) + 2*a*b*m**3*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*g
amma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2)
 + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 6*a*b*m**2*x**3*x**m*lerchphi(b*x*
*2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3
/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2))
 - 2*a*b*m**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**
4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a*b*m*x
**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*
x**4*gamma(m/2 + 3/2)) + 4*a*b*m*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(
m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2
+ 3/2)) + 6*a*b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2
) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 6*a*b*x**3*x**m*gamma(m/2 + 1/2)/
(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*
x**4*gamma(m/2 + 3/2)) + b**2*m**3*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x
**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*b**2*m**2*x*
*5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(
32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x
**4*gamma(m/2 + 3/2)) - b**2*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a,
 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*
gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*b**2*x**5*x**m*l
erchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*g
amma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma
(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(b\,x^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^3,x)
```

```
[Out] int(x^m/(a + b*x^2)^3, x)
```

3.347 $\int \frac{(cx)^{1+m}}{a+bx^2} \, dx$

Optimal. Leaf size=44

$$\frac{(cx)^{2+m} \, _2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac(2+m)}$$

```
[Out] (c*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-b*x^2/a)/a/c/(2+m)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {371}

$$\frac{(cx)^{m+2} \, _2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1 + m)/(a + b*x^2),x]
```

```
[Out] ((c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a
*c*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{1+m}}{a+bx^2} \, dx = \frac{(cx)^{2+m} \, _2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac(2+m)}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 1.02

$$\frac{cx^2(cx)^m \, _2F_1\left(1,\frac{2+m}{2};1+\frac{2+m}{2};-\frac{bx^2}{a}\right)}{a(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1 + m)/(a + b*x^2),x]
[Out] (c*x^2*(c*x)^m*Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, -((b*x^2)/a)]
)/(a*(2 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{1+m}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1+m)/(b*x^2+a),x)
[Out] int((c*x)^(1+m)/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((c*x)^(m + 1)/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="fricas")
[Out] integral((c*x)^(m + 1)/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.42, size = 92, normalized size = 2.09

$$\frac{cc^m m x^2 x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} + \frac{cc^m x^2 x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1+m)/(b*x**2+a),x)
[Out] c*c**m*m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2
 + 1)/(4*a*gamma(m/2 + 2)) + c*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((c*x)^(m + 1)/(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{m+1}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m + 1)/(a + b*x^2),x)
[Out] int((c*x)^(m + 1)/(a + b*x^2), x)
```

3.348 $\int \frac{(cx)^m}{a+bx^2} dx$

Optimal. Leaf size=44

$$\frac{(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}$$

```
[Out] (c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {371}

$$\frac{(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a + b*x^2),x]
```

```
[Out] ((c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
*c*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m}{a+bx^2}\,dx = \frac{(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.95

$$\frac{x(cx)^m\,{}_2F_1\left(1,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
*(1 + m))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m}}{b\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2+a),x)
```

```
[Out] int((c*x)^m/(b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(b*x^2 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(b*x^2 + a), x)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.55, size = 95, normalized size = 2.16

$$\frac{c^{m}mxx^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{c^{m}xx^{m}\Phi\left(\frac{bx^{2}e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(b*x**2+a),x)
```

```
[Out] c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^m}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a + b*x^2),x)
```

```
[Out] int((c*x)^m/(a + b*x^2), x)
```

3.349 $\int \frac{(cx)^{-1+m}}{a+bx^2} dx$

Optimal. Leaf size=38

$$\frac{(cx)^m {}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{acm}$$

```
[Out] (c*x)^m*hypergeom([1, 1/2*m],[1+1/2*m],-b*x^2/a)/a/c/m
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {371}

$$\frac{(cx)^m {}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{acm}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + m)/(a + b*x^2),x]
```

```
[Out] ((c*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, -((b*x^2)/a)])/(a*c*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-1+m}}{a+bx^2} dx = \frac{(cx)^m {}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{acm}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.00

$$\frac{x(cx)^{-1+m} {}_2F_1\left(1,\frac{m}{2};1+\frac{m}{2};-\frac{bx^2}{a}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + m)/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^(-1 + m)*Hypergeometric2F1[1, m/2, 1 + m/2, -((b*x^2)/a)])/(a*m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-1+m}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+m)/(b*x^2+a),x)
[Out] int((c*x)^(-1+m)/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((c*x)^(m - 1)/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="fricas")
[Out] integral((c*x)^(m - 1)/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.24, size = 39, normalized size = 1.03

$$\frac{c^m m x^m \Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}\right)\Gamma\left(\frac{m}{2}\right)}{4ac\Gamma\left(\frac{m}{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+m)/(b*x**2+a),x)
[Out] c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2)*gamma(m/2)/(4*a*c*ga
mma(m/2 + 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(m - 1)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,x)^{m-1}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m - 1)/(a + b*x^2),x)
```

```
[Out] int((c*x)^(m - 1)/(a + b*x^2), x)
```

3.350 $\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx$

Optimal. Leaf size=47

$$-\frac{(cx)^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{bx^2}{a}\right)}{ac(1-m)}$$

```
[Out] -(c*x)^(-1+m)*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-b*x^2/a)/a/c/(1-m)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{371\}$

$$-\frac{(cx)^{m-1}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{ac(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-2 + m)/(a + b*x^2),x]
```

```
[Out] -(((c*x)^(-1 + m)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((b*x^2)/a)]
)/(a*c*(1 - m)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx = -\frac{(cx)^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{bx^2}{a}\right)}{ac(1-m)}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.94

$$\frac{x(cx)^{-2+m}\,{}_2F_1\left(1,\frac{1}{2}(-1+m);1+\frac{1}{2}(-1+m);-\frac{bx^2}{a}\right)}{a(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-2 + m)/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^(-2 + m)*Hypergeometric2F1[1, (-1 + m)/2, 1 + (-1 + m)/2, -((b*x^2
)/a)])/(a*(-1 + m))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-2+m}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-2+m)/(b*x^2+a),x)
```

```
[Out] int((c*x)^(-2+m)/(b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(m - 2)/(b*x^2 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m - 2)/(b*x^2 + a), x)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 4.96, size = 102, normalized size = 2.17

$$\frac{c^m m x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}-\frac{1}{2}\right)\Gamma\left(\frac{m}{2}-\frac{1}{2}\right)}{4ac^2x\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)} - \frac{c^m x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}-\frac{1}{2}\right)\Gamma\left(\frac{m}{2}-\frac{1}{2}\right)}{4ac^2x\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-2+m)/(b*x**2+a),x)
```

[Out] c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 - 1/2)*gamma(m/2 - 1/2)/(4*a*c**2*x*gamma(m/2 + 1/2)) - c**m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 - 1/2)*gamma(m/2 - 1/2)/(4*a*c**2*x*gamma(m/2 + 1/2))

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="giac")

[Out] integrate((c*x)^(m - 2)/(b*x^2 + a), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{m-2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(m - 2)/(a + b*x^2),x)

[Out] int((c*x)^(m - 2)/(a + b*x^2), x)

3.351 $\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx$

Optimal. Leaf size=45

$$-\frac{(cx)^{-2+m}\,{}_2F_1\left(1,\frac{1}{2}(-2+m);\frac{m}{2};-\frac{bx^2}{a}\right)}{ac(2-m)}$$

```
[Out] -(c*x)^(-2+m)*hypergeom([1, -1+1/2*m],[1/2*m],-b*x^2/a)/a/c/(2-m)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {371}

$$-\frac{(cx)^{m-2}\,{}_2F_1\left(1,\frac{m-2}{2};\frac{m}{2};-\frac{bx^2}{a}\right)}{ac(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-3 + m)/(a + b*x^2),x]
```

```
[Out] -(((c*x)^(-2 + m)*Hypergeometric2F1[1, (-2 + m)/2, m/2, -((b*x^2)/a)])/(a*c
*(2 - m)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = -\frac{(cx)^{-2+m}\,{}_2F_1\left(1,\frac{1}{2}(-2+m);\frac{m}{2};-\frac{bx^2}{a}\right)}{ac(2-m)}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.98

$$\frac{x(cx)^{-3+m}\,{}_2F_1\left(1,\frac{1}{2}(-2+m);1+\frac{1}{2}(-2+m);-\frac{bx^2}{a}\right)}{a(-2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-3 + m)/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^(-3 + m)*Hypergeometric2F1[1, (-2 + m)/2, 1 + (-2 + m)/2, -((b*x^2
)/a)])/(a*(-2 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-3+m}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-3+m)/(b*x^2+a),x)
```

```
[Out] int((c*x)^(-3+m)/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(m - 3)/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m - 3)/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.69, size = 92, normalized size = 2.04

$$\frac{c^m m x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}-1\right)\Gamma\left(\frac{m}{2}-1\right)}{4ac^3x^2\Gamma\left(\frac{m}{2}\right)} - \frac{c^m x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}-1\right)\Gamma\left(\frac{m}{2}-1\right)}{2ac^3x^2\Gamma\left(\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-3+m)/(b*x**2+a),x)
```

```
[Out] c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 - 1)*gamma(m/2 - 1)/(
4*a*c**3*x**2*gamma(m/2)) - c**m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
 m/2 - 1)*gamma(m/2 - 1)/(2*a*c**3*x**2*gamma(m/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(m - 3)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{m-3}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m - 3)/(a + b*x^2),x)
```

```
[Out] int((c*x)^(m - 3)/(a + b*x^2), x)
```

3.352 $\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2} dx$

Optimal. Leaf size=36

$$\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{ax^2}{b}\right)}{1+m}$$

```
[Out] x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-a*x^2/b)/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {371}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{ax^2}{b}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + (a*x^2)/b)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((a*x^2)/b)])/(1 + m
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2} dx = \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{ax^2}{b}\right)}{1+m}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.06

$$\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{ax^2}{b}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(1 + (a*x^2)/b)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((a*x^2)/b)])/(1
 + m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.06, size = 92, normalized size = 2.56

method	result	size
meijerg	$\frac{\left(\frac{a}{b}\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}\left(\frac{a}{b}\right)^{\frac{1}{2}+\frac{m}{2}}}{2+\frac{2a\,x^2}{b}}+\frac{2x^{1+m}\left(\frac{a}{b}\right)^{\frac{1}{2}+\frac{m}{2}}\left(-\frac{m^2}{4}+\frac{1}{4}\right)\Phi\left(-\frac{a\,x^2}{b},1,\frac{1}{2}+\frac{m}{2}\right)}{1+m}\right)}{2}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+a*x^2/b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a/b)^(-1/2-1/2*m)*(2*x^(1+m)*(a/b)^(1/2+1/2*m)/(2+2*a*x^2/b)+2/(1+m)*x
^(1+m)*(a/b)^(1/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(-a*x^2/b,1,1/2+1/2*m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+a*x^2/b)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(a*x^2/b + 1)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+a*x^2/b)^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^m/(a^2*x^4 + 2*a*b*x^2 + b^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.01, size = 343, normalized size = 9.53

$$-\frac{am^2x^3x^m\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{ax^3x^m\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}-\frac{bm^2xx^m\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2bmxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{bxx^m\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2bxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+a*x**2/b)**2,x)
```

```
[Out] -a*m**2*x**3*x**m*lerchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/
2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) + a*x**3*x**m*l
erchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2*
gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) - b*m**2*x*x**m*lerchphi(a*x**2*ex
p_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2)
+ 8*b*gamma(m/2 + 3/2)) + 2*b*m*x*x**m*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2
 + 3/2) + 8*b*gamma(m/2 + 3/2)) + b*x*x**m*lerchphi(a*x**2*exp_polar(I*pi)/
b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/
2 + 3/2)) + 2*b*x*x**m*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*ga
mma(m/2 + 3/2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+a*x^2/b)^2,x, algorithm="giac")
[Out] integrate(x^m/(a*x^2/b + 1)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(\frac{a\,x^2}{b}+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x^2)/b + 1)^2,x)
[Out] int(x^m/((a*x^2)/b + 1)^2, x)
```

3.353 $\int x^7 \sqrt{a+bx^2}\, dx$

Optimal. Leaf size=80

$$-\frac{a^3(a+bx^2)^{3/2}}{3b^4}+\frac{3a^2(a+bx^2)^{5/2}}{5b^4}-\frac{3a(a+bx^2)^{7/2}}{7b^4}+\frac{(a+bx^2)^{9/2}}{9b^4}$$

```
[Out] -1/3*a^3*(b*x^2+a)^(3/2)/b^4+3/5*a^2*(b*x^2+a)^(5/2)/b^4-3/7*a*(b*x^2+a)^(7
/2)/b^4+1/9*(b*x^2+a)^(9/2)/b^4
```

Rubi [A]

time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$-\frac{a^3(a+bx^2)^{3/2}}{3b^4}+\frac{3a^2(a+bx^2)^{5/2}}{5b^4}+\frac{(a+bx^2)^{9/2}}{9b^4}-\frac{3a(a+bx^2)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + b*x^2],x]
```

```
[Out] -1/3*(a^3*(a + b*x^2)^(3/2))/b^4 + (3*a^2*(a + b*x^2)^(5/2))/(5*b^4) - (3*a
*(a + b*x^2)^(7/2))/(7*b^4) + (a + b*x^2)^(9/2)/(9*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\sqrt{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3\sqrt{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx,x,\right.\\ &= -\frac{a^3(a+bx^2)^{3/2}}{3b^4}+\frac{3a^2(a+bx^2)^{5/2}}{5b^4}-\frac{3a(a+bx^2)^{7/2}}{7b^4}+\frac{(a+bx^2)^{9/2}}{9b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.76

$$\frac{\sqrt{a+bx^2}\,(-16a^4+8a^3bx^2-6a^2b^2x^4+5ab^3x^6+35b^4x^8)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^4 + 8*a^3*b*x^2 - 6*a^2*b^2*x^4 + 5*a*b^3*x^6 + 35*
b^4*x^8))/(315*b^4)
```

Maple [A]
time = 0.05, size = 82, normalized size = 1.02

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}\,(-35b^3x^6+30a\,b^2x^4-24a^2b\,x^2+16a^3)}{315b^4}$	47
trager	$-\frac{(-35b^4x^8-5a\,b^3x^6+6a^2b^2x^4-8a^3b\,x^2+16a^4)\sqrt{b\,x^2+a}}{315b^4}$	58
risch	$-\frac{(-35b^4x^8-5a\,b^3x^6+6a^2b^2x^4-8a^3b\,x^2+16a^4)\sqrt{b\,x^2+a}}{315b^4}$	58
default	$\frac{x^6(b\,x^2+a)^{\frac{3}{2}}}{9b}-\frac{2a\left(\frac{x^4(b\,x^2+a)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)}{7b}\right)}{3b}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*x^6*(b*x^2+a)^(3/2)/b-2/3*a/b*(1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x
^2*(b*x^2+a)^(3/2)/b-2/15*a*(b*x^2+a)^(3/2)/b^2))
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.91

$$\frac{(bx^2+a)^{\frac{3}{2}}x^6}{9\,b}-\frac{2\,(bx^2+a)^{\frac{3}{2}}ax^4}{21\,b^2}+\frac{8\,(bx^2+a)^{\frac{3}{2}}a^2x^2}{105\,b^3}-\frac{16\,(bx^2+a)^{\frac{3}{2}}a^3}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(3/2)*x^6/b - 2/21*(b*x^2 + a)^(3/2)*a*x^4/b^2 + 8/105*(b*x
^2 + a)^(3/2)*a^2*x^2/b^3 - 16/315*(b*x^2 + a)^(3/2)*a^3/b^4
```

Fricas [A]
time = 0.56, size = 57, normalized size = 0.71

$$\frac{\left(35\,b^4x^8+5\,ab^3x^6-6\,a^2b^2x^4+8\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*x^8 + 5*a*b^3*x^6 - 6*a^2*b^2*x^4 + 8*a^3*b*x^2 - 16*a^4)*sqr
t(b*x^2 + a)/b^4
```

Sympy [A]
time = 0.25, size = 110, normalized size = 1.38

$$\begin{cases} -\frac{16a^4\sqrt{a+bx^2}}{315b^4}+\frac{8a^3x^2\sqrt{a+bx^2}}{315b^3}-\frac{2a^2x^4\sqrt{a+bx^2}}{105b^2}+\frac{ax^6\sqrt{a+bx^2}}{63b}+\frac{x^8\sqrt{a+bx^2}}{9} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\,x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x*
*2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a +
b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True)
)
```

Giac [A]
time = 1.36, size = 57, normalized size = 0.71

$$\frac{35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/
2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)/b^4
```

Mupad [B]
time = 4.59, size = 55, normalized size = 0.69

$$\sqrt{b\,x^2+a}\,\left(\frac{x^8}{9}-\frac{16\,a^4}{315\,b^4}+\frac{a\,x^6}{63\,b}-\frac{2\,a^2\,x^4}{105\,b^2}+\frac{8\,a^3\,x^2}{315\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*(x^8/9 - (16*a^4)/(315*b^4) + (a*x^6)/(63*b) - (2*a^2*x^4
)/(105*b^2) + (8*a^3*x^2)/(315*b^3))
```

3.354 $\int x^5\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=59

$$\frac{a^2(a+bx^2)^{3/2}}{3b^3}-\frac{2a(a+bx^2)^{5/2}}{5b^3}+\frac{(a+bx^2)^{7/2}}{7b^3}$$

```
[Out] 1/3*a^2*(b*x^2+a)^(3/2)/b^3-2/5*a*(b*x^2+a)^(5/2)/b^3+1/7*(b*x^2+a)^(7/2)/b
^3
```

Rubi [A]

time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{a^2(a+bx^2)^{3/2}}{3b^3}+\frac{(a+bx^2)^{7/2}}{7b^3}-\frac{2a(a+bx^2)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(a + b*x^2)^(3/2))/(3*b^3) - (2*a*(a + b*x^2)^(5/2))/(5*b^3) + (a + b*
x^2)^(7/2)/(7*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{a^2(a+bx^2)^{3/2}}{3b^3}-\frac{2a(a+bx^2)^{5/2}}{5b^3}+\frac{(a+bx^2)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A]

time = 0.02, size = 50, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\,(8a^3-4a^2bx^2+3ab^2x^4+15b^3x^6)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^3 - 4*a^2*b*x^2 + 3*a*b^2*x^4 + 15*b^3*x^6))/(105*b^3
)
```

Maple [A]

time = 0.05, size = 58, normalized size = 0.98

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{3}{2}}(15b^2x^4-12ab\,x^2+8a^2)}{105b^3}$	36
trager	$\frac{(15b^3x^6+3a\,b^2x^4-4a^2b\,x^2+8a^3)\sqrt{b\,x^2+a}}{105b^3}$	47
risch	$\frac{(15b^3x^6+3a\,b^2x^4-4a^2b\,x^2+8a^3)\sqrt{b\,x^2+a}}{105b^3}$	47
default	$\frac{x^4(b\,x^2+a)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)}{7b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a*(b*x^2+
a)^(3/2)/b^2)
```

Maxima [A]

time = 0.41, size = 53, normalized size = 0.90

$$\frac{(bx^2+a)^{\frac{3}{2}}x^4}{7\,b}-\frac{4\,(bx^2+a)^{\frac{3}{2}}ax^2}{35\,b^2}+\frac{8\,(bx^2+a)^{\frac{3}{2}}a^2}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(3/2)*x^4/b - 4/35*(b*x^2 + a)^(3/2)*a*x^2/b^2 + 8/105*(b*x
^2 + a)^(3/2)*a^2/b^3
```

Fricas [A]

time = 0.72, size = 46, normalized size = 0.78

$$\frac{(15\,b^3x^6+3\,ab^2x^4-4\,a^2bx^2+8\,a^3)\sqrt{bx^2+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*x^6 + 3*a*b^2*x^4 - 4*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a)/b^3
```

Sympy [A]
time = 0.15, size = 87, normalized size = 1.47

$$\begin{cases} \frac{8a^3\sqrt{a+bx^2}}{105b^3} - \frac{4a^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{ax^4\sqrt{a+bx^2}}{35b} + \frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2
)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne
(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [A]
time = 1.54, size = 43, normalized size = 0.73

$$\frac{15\,(bx^2+a)^{\frac{7}{2}} - 42\,(bx^2+a)^{\frac{5}{2}}a + 35\,(bx^2+a)^{\frac{3}{2}}a^2}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)
*a^2)/b^3
```

Mupad [B]
time = 4.66, size = 44, normalized size = 0.75

$$\sqrt{b\,x^2+a}\,\left(\frac{x^6}{7}+\frac{8\,a^3}{105\,b^3}+\frac{a\,x^4}{35\,b}-\frac{4\,a^2\,x^2}{105\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*(x^6/7 + (8*a^3)/(105*b^3) + (a*x^4)/(35*b) - (4*a^2*x^2)
/(105*b^2))
```

3.355 $\int x^3\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=38

$$-\frac{a(a+bx^2)^{3/2}}{3b^2}+\frac{(a+bx^2)^{5/2}}{5b^2}$$

```
[Out] -1/3*a*(b*x^2+a)^(3/2)/b^2+1/5*(b*x^2+a)^(5/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{(a+bx^2)^{5/2}}{5b^2}-\frac{a(a+bx^2)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2],x]
```

```
[Out] -1/3*(a*(a + b*x^2)^(3/2))/b^2 + (a + b*x^2)^(5/2)/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx,x,x^2\right)\\ &= -\frac{a(a+bx^2)^{3/2}}{3b^2}+\frac{(a+bx^2)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\,(-2a^2+abx^2+3b^2x^4)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(-2*a^2 + a*b*x^2 + 3*b^2*x^4))/(15*b^2)
```

Maple [A]
time = 0.03, size = 34, normalized size = 0.89

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}(-3b\,x^2+2a)}{15b^2}$	25
default	$\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}$	34
trager	$-\frac{(-3b^2x^4-ab\,x^2+2a^2)\sqrt{b\,x^2+a}}{15b^2}$	36
risch	$-\frac{(-3b^2x^4-ab\,x^2+2a^2)\sqrt{b\,x^2+a}}{15b^2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a*(b*x^2+a)^(3/2)/b^2
```

Maxima [A]
time = 0.36, size = 33, normalized size = 0.87

$$\frac{(bx^2+a)^{\frac{3}{2}}x^2}{5\,b}-\frac{2\,(bx^2+a)^{\frac{3}{2}}a}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/5*(b*x^2 + a)^(3/2)*x^2/b - 2/15*(b*x^2 + a)^(3/2)*a/b^2
```

Fricas [A]
time = 0.89, size = 34, normalized size = 0.89

$$\frac{(3\,b^2x^4+abx^2-2\,a^2)\sqrt{bx^2+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*x^4 + a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

time = 0.09, size = 63, normalized size = 1.66

$$\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2} + \frac{ax^2\sqrt{a+bx^2}}{15b} + \frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15
*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

Giac [A]

time = 1.10, size = 29, normalized size = 0.76

$$\frac{3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)/b^2
```

Mupad [B]

time = 4.65, size = 33, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{x^4}{5}-\frac{2\,a^2}{15\,b^2}+\frac{a\,x^2}{15\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*(x^4/5 - (2*a^2)/(15*b^2) + (a*x^2)/(15*b))
```

3.356 $\int x\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=18

$$\frac{(a+bx^2)^{3/2}}{3b}$$

```
[Out] 1/3*(b*x^2+a)^(3/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\frac{(a+bx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2],x]
```

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{a+bx^2}\,dx = \frac{(a+bx^2)^{3/2}}{3b}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{(a+bx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2],x]
```

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

Maple [A]
time = 0.04, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
derivativedivides	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
default	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
trager	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
risch	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(b*x^2+a)^(3/2)/b
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.78

$$\frac{(bx^2+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

Fricas [A]
time = 0.85, size = 14, normalized size = 0.78

$$\frac{(bx^2+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.
time = 0.06, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b \neq 0 \\ \frac{\sqrt{a}\,x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (
sqrt(a)*x**2/2, True))
```

Giac [A]
time = 1.16, size = 14, normalized size = 0.78

$$\frac{(bx^2+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

Mupad [B]
time = 4.63, size = 14, normalized size = 0.78

$$\frac{(b\,x^2+a)^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

3.357 $\int \frac{\sqrt{a+bx^2}}{x}\,dx$

Optimal. Leaf size=37

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 52, 65, 214}

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x,x]
```

```
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \sqrt{a+bx^2}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \sqrt{a+bx^2}+\frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= \sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.00

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x,x]
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Maple [A]
time = 0.03, size = 39, normalized size = 1.05

method	result	size
default	$\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [A]
time = 0.33, size = 27, normalized size = 0.73

$$-\sqrt{a}\ \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)
```

Fricas [A]
time = 1.02, size = 77, normalized size = 2.08

$$\left[\frac{1}{2}\sqrt{a}\ \log\left(-\frac{bx^2-2\sqrt{bx^2+a}\ \sqrt{a}\ +2\,a}{x^2}\right)+\sqrt{bx^2+a}\ ,\sqrt{-a}\ \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(b*x
^2 + a), sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)]
```

Sympy [A]
time = 0.70, size = 56, normalized size = 1.51

$$-\sqrt{a}\ \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{a}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) +
sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A]
time = 1.88, size = 33, normalized size = 0.89

$$\frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)
```

Mupad [B]
time = 4.66, size = 29, normalized size = 0.78

$$\sqrt{b\,x^2+a}-\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x,x)
[Out] (a + b*x^2)^(1/2) - a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2))
```

3.358 $\int \frac{\sqrt{a+bx^2}}{x^3} \, dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)-1/2*(b*x^2+a)^(1/2)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 43, 65, 214}

$$-\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*x^2]/x^2 - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{2x^2}+\frac{1}{4}b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= -\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 47, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^3,x]
[Out] -1/2*Sqrt[a + b*x^2]/x^2 - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Maple [A]
time = 0.04, size = 63, normalized size = 1.34

method	result	size
risch	$-\frac{\sqrt{b\,x^2+a}}{2x^2}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2\sqrt{a}}$	45
default	$-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.32, size = 51, normalized size = 1.09

$$-\frac{b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,\sqrt{a}}+\frac{\sqrt{bx^2+a}\,b}{2\,a}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*sqrt(b*x^2 + a)*b/a - 1/
2*(b*x^2 + a)^(3/2)/(a*x^2)
```

Fricas [A]
time = 1.10, size = 106, normalized size = 2.26

$$\left[\frac{\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,\sqrt{bx^2+a}\,a}{4\,ax^2},\ \frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,a}{2\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2
 + a)) - sqrt(b*x^2 + a)*a)/(a*x^2)]
```

Sympy [A]
time = 0.93, size = 42, normalized size = 0.89

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - b*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(
a))
```

Giac [A]
time = 1.06, size = 46, normalized size = 0.98

$$\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^2+a}\, b}{x^2}}{2\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^2 + a)*b/x^2)
/b
```

Mupad [B]
time = 4.80, size = 35, normalized size = 0.74

$$-\frac{\sqrt{b\, x^2+a}}{2\, x^2} - \frac{b\, \mathrm{atanh}\left(\frac{\sqrt{b\, x^2+a}}{\sqrt{a}}\right)}{2\, \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^3,x)
```

```
[Out] - (a + b*x^2)^(1/2)/(2*x^2) - (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/
2))
```

3.359 $\int \frac{\sqrt{a+bx^2}}{x^5}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{a+bx^2}}{4x^4}-\frac{b\sqrt{a+bx^2}}{8ax^2}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

```
[Out] 1/8*b^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/4*(b*x^2+a)^(1/2)/x^4-1/
8*b*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 44, 65, 214}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b\sqrt{a+bx^2}}{8ax^2}-\frac{\sqrt{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^5,x]
```

```
[Out] -1/4*Sqrt[a + b*x^2]/x^4 - (b*Sqrt[a + b*x^2])/(8*a*x^2) + (b^2*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} + \frac{1}{8}b\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} - \frac{b^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{16a}\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} - \frac{b\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{8a}\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 62, normalized size = 0.87

$$\frac{(-2a-bx^2)\sqrt{a+bx^2}}{8ax^4} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^5,x]
[Out] ((-2*a - b*x^2)*Sqrt[a + b*x^2])/(8*a*x^4) + (b^2*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(8*a^(3/2))
```

Maple [A]
time = 0.04, size = 87, normalized size = 1.23

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(b\,x^{2}+2 a\right)}{8 x^{4} a}+\frac{b^{2} \ln \left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{8 a^{\frac{3}{2}}}$	59
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4 a\,x^{4}}-\frac{b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{2 a\,x^{2}}+\frac{b \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln \left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2 a}\right)}{4 a}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*
x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Maxima [A]
time = 0.29, size = 73, normalized size = 1.03

$$\frac{b^{2} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^{2}+a}\,b^{2}}{8\,a^{2}}+\frac{(bx^{2}+a)^{\frac{3}{2}}b}{8\,a^{2}x^{2}}-\frac{(bx^{2}+a)^{\frac{3}{2}}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/8*sqrt(b*x^2 + a)*b^2/a^2
 + 1/8*(b*x^2 + a)^(3/2)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(3/2)/(a*x^4)
```

Fricas [A]
time = 1.24, size = 131, normalized size = 1.85

$$\left[\frac{\sqrt{a}\,b^{2}x^{4}\log\left(-\frac{bx^{2}+2\sqrt{bx^{2}+a}\,\sqrt{a}+2a}{x^{2}}\right)-2\,(abx^{2}+2\,a^{2})\sqrt{bx^{2}+a}}{16\,a^{2}x^{4}},-\frac{\sqrt{-a}\,b^{2}x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(abx^{2}+2\,a^{2})\sqrt{bx^{2}+a}}{8\,a^{2}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(a)*b^2*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
- 2*(a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a^2*x^4), -1/8*(sqrt(-a)*b^2*x^4*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) + (a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a^2*x^
4)]
```

Sympy [A]
time = 2.19, size = 92, normalized size = 1.30

$$-\frac{a}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{b^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}}+\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(8*x**3*sqrt(a/(b*x**2
) + 1)) - b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + b**2*asinh(sqrt(a)/(sqrt(
b)*x))/(8*a**(3/2))
```

Giac [A]
time = 2.37, size = 72, normalized size = 1.01

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{(bx^2+a)^{\frac{3}{2}}b^3+\sqrt{bx^2+a}\,ab^3}{ab^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*(b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x^2 + a)^(3/2
)*b^3 + sqrt(b*x^2 + a)*a*b^3)/(a*b^2*x^4))/b
```

Mupad [B]
time = 4.91, size = 54, normalized size = 0.76

$$\frac{b^2\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{3/2}}-\frac{\sqrt{b\,x^2+a}}{8\,x^4}-\frac{(b\,x^2+a)^{3/2}}{8\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^5,x)
```

```
[Out] (b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (a + b*x^2)^(1/2)/(8*x
^4) - (a + b*x^2)^(3/2)/(8*a*x^4)
```

3.360 $\int \frac{\sqrt{a+bx^2}}{x^7}\,dx$

Optimal. Leaf size=95

$$-\frac{\sqrt{a+bx^2}}{6x^6}-\frac{b\sqrt{a+bx^2}}{24ax^4}+\frac{b^2\sqrt{a+bx^2}}{16a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

```
[Out] -1/16*b^3*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*(b*x^2+a)^(1/2)/x^6-
1/24*b*(b*x^2+a)^(1/2)/a/x^4+1/16*b^2*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 44, 65, 214}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b^2\sqrt{a+bx^2}}{16a^2x^2}-\frac{\sqrt{a+bx^2}}{6x^6}-\frac{b\sqrt{a+bx^2}}{24ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^7,x]
```

```
[Out] -1/6*Sqrt[a + b*x^2]/x^6 - (b*Sqrt[a + b*x^2])/(24*a*x^4) + (b^2*Sqrt[a + b
*x^2])/(16*a^2*x^2) - (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{6x^6}+\frac{1}{12}b\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{6x^6}-\frac{b\sqrt{a+bx^2}}{24ax^4}-\frac{b^2\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{16a}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6}-\frac{b\sqrt{a+bx^2}}{24ax^4}+\frac{b^2\sqrt{a+bx^2}}{16a^2x^2}+\frac{b^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{32a^2}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6}-\frac{b\sqrt{a+bx^2}}{24ax^4}+\frac{b^2\sqrt{a+bx^2}}{16a^2x^2}+\frac{b^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{16a^2}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6}-\frac{b\sqrt{a+bx^2}}{24ax^4}+\frac{b^2\sqrt{a+bx^2}}{16a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\,(-8a^2-2abx^2+3b^2x^4)}{48a^2x^6}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2 - 2*a*b*x^2 + 3*b^2*x^4))/(48*a^2*x^6) - (b^3*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Maple [A]
time = 0.06, size = 111, normalized size = 1.17

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-3 b^{2} x^{4}+2 a b\,x^{2}+8 a^{2}\right)}{48 x^{6} a^{2}}-\frac{b^{3} \ln \left(\frac{2 a +2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{16 a^{\frac{5}{2}}}$	71
default	$-\frac{\left(b\,x^{2}+a \right)^{\frac{3}{2}}}{6 a\,x^{6}}-\frac{b \left(-\frac{\left(b\,x^{2}+a \right)^{\frac{3}{2}}}{4 a\,x^{4}}-\frac{b \left(-\frac{\left(b\,x^{2}+a \right)^{\frac{3}{2}}}{2 a\,x^{2}}+\frac{b \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln \left(\frac{2 a +2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2 a}\right)}{4 a}\right)}{2 a}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/
2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*
(b*x^2+a)^(1/2))/x))))
```

Maxima [A]
time = 0.29, size = 93, normalized size = 0.98

$$-\frac{b^{3} \operatorname{arsinh}\left(\frac{a}{\sqrt{a b} {\left| x \right|}}\right)}{16 \, a^{\frac{5}{2}}} + \frac{\sqrt{b x^{2} + a} b^{3}}{16 \, a^{3}} - \frac{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b^{2}}{16 \, a^{3} x^{2}} + \frac{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b}{8 \, a^{2} x^{4}} - \frac{{\left(b x^{2} + a\right)}^{\frac{3}{2}}}{6 \, a x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/16*sqrt(b*x^2 + a)*b^3/
a^3 - 1/16*(b*x^2 + a)^(3/2)*b^2/(a^3*x^2) + 1/8*(b*x^2 + a)^(3/2)*b/(a^2*x
^4) - 1/6*(b*x^2 + a)^(3/2)/(a*x^6)
```

Fricas [A]
time = 1.06, size = 157, normalized size = 1.65

$$\left[\frac{3 \, \sqrt{a} b^{3} x^{6} \log\left(-\frac{b x^{2} - 2 \, \sqrt{b x^{2} + a} \sqrt{a} + 2 \, a}{x^{2}}\right) + 2 \, {\left(3 \, a b^{2} x^{4} - 2 \, a^{2} b x^{2} - 8 \, a^{3}\right)} \sqrt{b x^{2} + a}}{96 \, a^{3} x^{6}}, \frac{3 \, \sqrt{-a} b^{3} x^{6} \arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^{2} + a}}\right) + {\left(3 \, a b^{2} x^{4} - 2 \, a^{2} b x^{2} - 8 \, a^{3}\right)} \sqrt{b x^{2} + a}}{48 \, a^{3} x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*a*b^2*x^4 - 2*a^2*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6), 1/48*
(3*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b^2*x^4 - 2*a^2
*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

Sympy [A]
time = 5.27, size = 117, normalized size = 1.23

$$-\frac{a}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{5\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*sqrt(b)/(24*x**5*sqrt(a/(b*x**
2) + 1)) + b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(16*a**2*x*
sqrt(a/(b*x**2) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2))
```

Giac [A]
time = 1.09, size = 92, normalized size = 0.97

$$\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,(bx^2+a)^{\frac{5}{2}}b^4-8\,(bx^2+a)^{\frac{3}{2}}ab^4-3\,\sqrt{bx^2+a}\,a^2b^4}{a^2b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a
)^(5/2)*b^4 - 8*(b*x^2 + a)^(3/2)*a*b^4 - 3*sqrt(b*x^2 + a)*a^2*b^4)/(a^2*b
^3*x^6))/b
```

Mupad [B]
time = 4.94, size = 74, normalized size = 0.78

$$\frac{\left(b\,x^2+a\right)^{5/2}}{16\,a^2\,x^6}-\frac{\left(b\,x^2+a\right)^{3/2}}{6\,a\,x^6}-\frac{\sqrt{b\,x^2+a}}{16\,x^6}+\frac{b^3\operatorname{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{16\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^7,x)
```

```
[Out] (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (a + b*x^2)^(1
/2)/(16*x^6) - (a + b*x^2)^(3/2)/(6*a*x^6) + (a + b*x^2)^(5/2)/(16*a^2*x^6)
```

3.361 $\int x^4\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=94

$$-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

```
[Out] 1/16*a^3*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-1/16*a^2*x*(b*x^2+a)^(1
/2)/b^2+1/24*a*x^3*(b*x^2+a)^(1/2)/b+1/6*x^5*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 212}

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2],x]
```

```
[Out] -1/16*(a^2*x*Sqrt[a + b*x^2])/b^2 + (a*x^3*Sqrt[a + b*x^2])/(24*b) + (x^5*S
qrt[a + b*x^2])/6 + (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^4\sqrt{a+bx^2}\,dx &= \frac{1}{6}x^5\sqrt{a+bx^2}+\frac{1}{6}a\int\frac{x^4}{\sqrt{a+bx^2}}\,dx\\
&= \frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}-\frac{a^2\int\frac{x^2}{\sqrt{a+bx^2}}\,dx}{8b}\\
&= -\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{a^3\int\frac{1}{\sqrt{a+bx^2}}\,dx}{16b^2}\\
&= -\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{a^3\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16b^2}\\
&= -\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 74, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\,(-3a^2x+2abx^3+8b^2x^5)}{48b^2}-\frac{a^3\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(-3*a^2*x + 2*a*b*x^3 + 8*b^2*x^5))/(48*b^2) - (a^3*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Maple [A]
time = 0.04, size = 82, normalized size = 0.87

method	result	size
risch	$-\frac{x\left(-8b^2x^4-2ab\,x^2+3a^2\right)\sqrt{b\,x^2+a}}{48b^2}+\frac{a^3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{16b^{\frac{5}{2}}}$	62

default	$\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(
b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.73

$$\frac{(bx^2+a)^{\frac{3}{2}}x^3}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2}+\frac{\sqrt{bx^2+a}\,a^2x}{16\,b^2}+\frac{a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*x^3/b - 1/8*(b*x^2 + a)^(3/2)*a*x/b^2 + 1/16*sqrt(b*x
^2 + a)*a^2*x/b^2 + 1/16*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 1.02, size = 146, normalized size = 1.55

$$\left[\frac{3\,a^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(8\,b^3x^5+2\,ab^2x^3-3\,a^2bx)\sqrt{bx^2+a}}{96\,b^3},\,-\frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(8\,b^3x^5+2\,ab^2x^3-3\,a^2bx)\sqrt{bx^2+a}}{48\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8
*b^3*x^5 + 2*a*b^2*x^3 - 3*a^2*b*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*a^3*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 2*a*b^2*x^3 - 3*a^2*
b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 5.10, size = 117, normalized size = 1.24

$$-\frac{a^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}\,x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{a^3\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{bx^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - a**(3/2)*x**3/(48*b*sqrt(1 + b*x
**2/a)) + 5*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*b**(5/2)) + b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.50, size = 64, normalized size = 0.68

$$\frac{1}{48}\left(2\left(4\,x^2+\frac{a}{b}\right)x^2-\frac{3\,a^2}{b^2}\right)\sqrt{bx^2+a}\;x-\frac{a^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2 + a/b)*x^2 - 3*a^2/b^2)*sqrt(b*x^2 + a)*x - 1/16*a^3*log(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(1/2), x)
```

3.362 $\int x^2\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=70

$$\frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

```
[Out] -1/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/8*a*x*(b*x^2+a)^(1/2)
/b+1/4*x^3*(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 212}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2],x]
```

```
[Out] (a*x*Sqrt[a + b*x^2])/(8*b) + (x^3*Sqrt[a + b*x^2])/4 - (a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx^2}\,dx &= \frac{1}{4}x^3\sqrt{a+bx^2}+\frac{1}{4}a\int\frac{x^2}{\sqrt{a+bx^2}}\,dx\\ &= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\ &= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b}\\ &= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.87

$$\frac{x\sqrt{a+bx^2}\,(a+2bx^2)}{8b}+\frac{a^2\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(a + 2*b*x^2))/(8*b) + (a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(8*b^(3/2))
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.83

method	result	size
risch	$\frac{x(2b\,x^2+a)\sqrt{b\,x^2+a}}{8b}-\frac{a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}$	49

default	$\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.29, size = 49, normalized size = 0.70

$$\frac{(bx^2+a)^{\frac{3}{2}}x}{4\,b}-\frac{\sqrt{bx^2+a}\,ax}{8\,b}-\frac{a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*x/b - 1/8*sqrt(b*x^2 + a)*a*x/b - 1/8*a^2*arcsinh(b*x
/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 1.22, size = 119, normalized size = 1.70

$$\left[\frac{a^2\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(2\,b^2x^3+abx)\sqrt{bx^2+a}}{16\,b^2},\frac{a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(2\,b^2x^3+abx)\sqrt{bx^2+a}}{8\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b
^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2, 1/8*(a^2*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (2*b^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A]
time = 2.02, size = 92, normalized size = 1.31

$$\frac{a^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{a^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{bx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*sqrt(a)*x**3/(8*sqrt(1 + b*x**2/a))
 - a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + b*x**5/(4*sqrt(a)*sqrt(1 +
b*x**2/a))
```

Giac [A]
time = 1.61, size = 50, normalized size = 0.71

$$\frac{1}{8}\sqrt{bx^2+a}\left(2x^2+\frac{a}{b}\right)x+\frac{a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*x^2 + a/b)*x + 1/8*a^2*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/2), x)
```

3.363 $\int \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+1/2*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 223, 212}

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}x\sqrt{a+bx^2} + \frac{1}{2}a\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{a+bx^2} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{1}{2}x\sqrt{a+bx^2} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 1.04

$$\frac{1}{2}x\sqrt{a+bx^2} - \frac{a\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Maple [A]
time = 0.03, size = 36, normalized size = 0.78

method	result	size
default	$\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$	36
risch	$\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.30, size = 28, normalized size = 0.61

$$\frac{1}{2}\sqrt{bx^2+a}\,x + \frac{a\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x + 1/2*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 1.25, size = 94, normalized size = 2.04

$$\left[\frac{2\sqrt{bx^2+a}\,bx+a\sqrt{b}\,\log\left(-2\,bx^2-2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b},\ \frac{\sqrt{bx^2+a}\,bx-a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)))/b]
```

Sympy [A]
time = 0.97, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Giac [A]
time = 1.08, size = 37, normalized size = 0.80

$$\frac{1}{2}\sqrt{bx^2+a}\,x-\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

Mupad [B]
time = 4.67, size = 35, normalized size = 0.76

$$\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2),x)
```

```
[Out] (x*(a + b*x^2)^(1/2))/2 + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2)
)
```

3.364 $\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{a+bx^2}}{x}+\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)-(b*x^2+a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {283, 223, 212}

$$\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^2}\,dx &= -\frac{\sqrt{a+bx^2}}{x} + b\int \frac{1}{\sqrt{a+bx^2}}\,dx \\ &= -\frac{\sqrt{a+bx^2}}{x} + b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\ &= -\frac{\sqrt{a+bx^2}}{x} + \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 1.07

$$-\frac{\sqrt{a+bx^2}}{x} - \sqrt{b}\,\log\left(-\sqrt{b}\,x + \sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2]/x) - Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [A]
time = 0.03, size = 60, normalized size = 1.43

method	result	size
risch	$-\frac{\sqrt{b\,x^2+a}}{x}+\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)$	36
default	$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{a}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.67

$$\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\sqrt{bx^2+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
[Out] sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)/x
```

Fricas [A]
time = 1.22, size = 88, normalized size = 2.10

$$\left[\frac{\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,\sqrt{bx^2+a}}{2\,x},-\frac{\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
[Out] [1/2*(sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sqrt(b*
x^2 + a))/x, -(sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + sqrt(b*x^2 +
 a))/x]
```

Sympy [A]
time = 0.74, size = 56, normalized size = 1.33

$$-\frac{\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+\sqrt{b}\ \operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{bx}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**2,x)
[Out] -sqrt(a)/(x*sqrt(1 + b*x**2/a)) + sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - b*x/(s
qrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.53, size = 57, normalized size = 1.36

$$-\frac{1}{2}\,\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
[Out] -1/2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a*sqrt(b)/((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B]
time = 4.82, size = 56, normalized size = 1.33

$$-\frac{\sqrt{b\,x^2+a}}{x}-\frac{\sqrt{b}\ \operatorname{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\ 1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^2,x)
```

```
[Out] - (a + b*x^2)^(1/2)/x - (b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(
1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

3.365 $\int \frac{\sqrt{a+bx^2}}{x^4} \, dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{3/2}}{3ax^3}$$

```
[Out] -1/3*(b*x^2+a)^(3/2)/a/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{(a+bx^2)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^4,x]
```

```
[Out] -1/3*(a + b*x^2)^(3/2)/(a*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^4} \, dx = -\frac{(a+bx^2)^{3/2}}{3ax^3}$$

Mathematica [A]
time = 0.04, size = 21, normalized size = 1.00

$$-\frac{(a+bx^2)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^4,x]
```

```
[Out] -1/3*(a + b*x^2)^(3/2)/(a*x^3)
```

Maple [A]
time = 0.04, size = 18, normalized size = 0.86

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}}{3a\,x^3}$	18
default	$-\frac{(b\,x^2+a)^{\frac{3}{2}}}{3a\,x^3}$	18
trager	$-\frac{(b\,x^2+a)^{\frac{3}{2}}}{3a\,x^3}$	18
risch	$-\frac{(b\,x^2+a)^{\frac{3}{2}}}{3a\,x^3}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^2+a)^(3/2)/a/x^3
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.81

$$-\frac{(bx^2+a)^{\frac{3}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(b*x^2 + a)^(3/2)/(a*x^3)
```

Fricas [A]
time = 1.24, size = 17, normalized size = 0.81

$$-\frac{(bx^2+a)^{\frac{3}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(b*x^2 + a)^(3/2)/(a*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.
time = 0.37, size = 42, normalized size = 2.00

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(17) = 34$.
time = 1.35, size = 59, normalized size = 2.81

$$\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4 b^{\frac{3}{2}}+a^2 b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) + a^2*b^(3/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [B]
time = 4.57, size = 17, normalized size = 0.81

$$-\frac{{\left(b\,x^2+a\right)}^{3/2}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^4,x)
```

```
[Out] -(a + b*x^2)^(3/2)/(3*a*x^3)
```

3.366 $\int \frac{\sqrt{a+bx^2}}{x^6} \, dx$

Optimal. Leaf size=44

$$-\frac{(a+bx^2)^{3/2}}{5ax^5}+\frac{2b(a+bx^2)^{3/2}}{15a^2x^3}$$

```
[Out] -1/5*(b*x^2+a)^(3/2)/a/x^5+2/15*b*(b*x^2+a)^(3/2)/a^2/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$\frac{2b(a+bx^2)^{3/2}}{15a^2x^3}-\frac{(a+bx^2)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^6,x]
```

```
[Out] -1/5*(a + b*x^2)^(3/2)/(a*x^5) + (2*b*(a + b*x^2)^(3/2))/(15*a^2*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^6} \, dx &= -\frac{(a+bx^2)^{3/2}}{5ax^5}-\frac{(2b)\int \frac{\sqrt{a+bx^2}}{x^4} \, dx}{5a}\\ &= -\frac{(a+bx^2)^{3/2}}{5ax^5}+\frac{2b(a+bx^2)^{3/2}}{15a^2x^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 0.95

$$\frac{\sqrt{a+bx^2}\,(-3a^2-abx^2+2b^2x^4)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2 - a*b*x^2 + 2*b^2*x^4))/(15*a^2*x^5)
```

Maple [A]
time = 0.04, size = 37, normalized size = 0.84

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}\,(-2b\,x^2+3a)}{15a^2x^5}$	28
default	$-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}$	37
trager	$-\frac{(-2b^2x^4+ab\,x^2+3a^2)\sqrt{b\,x^2+a}}{15a^2x^5}$	38
risch	$-\frac{(-2b^2x^4+ab\,x^2+3a^2)\sqrt{b\,x^2+a}}{15a^2x^5}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(b*x^2+a)^(3/2)/a/x^5+2/15*b*(b*x^2+a)^(3/2)/a^2/x^3
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.82

$$\frac{2\,(bx^2+a)^{\frac{3}{2}}b}{15\,a^2x^3}-\frac{(bx^2+a)^{\frac{3}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^2 + a)^(3/2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(3/2)/(a*x^5)
```

Fricas [A]
time = 0.72, size = 38, normalized size = 0.86

$$\frac{(2\,b^2x^4-abx^2-3\,a^2)\sqrt{bx^2+a}}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(2*b^2*x^4 - a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a)/(a^2*x^5)
```

Sympy [**A**]
time = 0.47, size = 68, normalized size = 1.55

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*
a*x**2) + 2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(36) = 72.
time = 1.05, size = 112, normalized size = 2.55

$$\frac{4\left(15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6b^{\frac{5}{2}}+5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4ab^{\frac{5}{2}}+5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{5}{2}}-a^3b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2) - a^3*b
^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [**B**]
time = 4.77, size = 37, normalized size = 0.84

$$-\frac{\sqrt{b\,x^2+a}\,\left(3\,a^2+a\,b\,x^2-2\,b^2\,x^4\right)}{15\,a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^6,x)
```

```
[Out] -((a + b*x^2)^(1/2)*(3*a^2 - 2*b^2*x^4 + a*b*x^2))/(15*a^2*x^5)
```

3.367 $\int \frac{\sqrt{a+bx^2}}{x^8}\,dx$

Optimal. Leaf size=68

$$-\frac{(a+bx^2)^{3/2}}{7ax^7}+\frac{4b(a+bx^2)^{3/2}}{35a^2x^5}-\frac{8b^2(a+bx^2)^{3/2}}{105a^3x^3}$$

```
[Out] -1/7*(b*x^2+a)^(3/2)/a/x^7+4/35*b*(b*x^2+a)^(3/2)/a^2/x^5-8/105*b^2*(b*x^2+
a)^(3/2)/a^3/x^3
```

Rubi [A]

time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$-\frac{8b^2(a+bx^2)^{3/2}}{105a^3x^3}+\frac{4b(a+bx^2)^{3/2}}{35a^2x^5}-\frac{(a+bx^2)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^8,x]
```

```
[Out] -1/7*(a + b*x^2)^(3/2)/(a*x^7) + (4*b*(a + b*x^2)^(3/2))/(35*a^2*x^5) - (8*
b^2*(a + b*x^2)^(3/2))/(105*a^3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^8}\,dx &= -\frac{(a+bx^2)^{3/2}}{7ax^7}-\frac{(4b)\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{7a}\\
&= -\frac{(a+bx^2)^{3/2}}{7ax^7}+\frac{4b(a+bx^2)^{3/2}}{35a^2x^5}+\frac{(8b^2)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{35a^2}\\
&= -\frac{(a+bx^2)^{3/2}}{7ax^7}+\frac{4b(a+bx^2)^{3/2}}{35a^2x^5}-\frac{8b^2(a+bx^2)^{3/2}}{105a^3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 0.78

$$\frac{\sqrt{a+bx^2}\,(-15a^3-3a^2bx^2+4ab^2x^4-8b^3x^6)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^8,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^3 - 3*a^2*b*x^2 + 4*a*b^2*x^4 - 8*b^3*x^6))/(105*a^
3*x^7)
```

Maple [A]
time = 0.05, size = 61, normalized size = 0.90

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}\,(8b^2x^4-12ab\,x^2+15a^2)}{105a^3x^7}$	39
trager	$-\frac{(8b^3x^6-4a\,b^2x^4+3a^2b\,x^2+15a^3)\sqrt{b\,x^2+a}}{105a^3x^7}$	50
risch	$-\frac{(8b^3x^6-4a\,b^2x^4+3a^2b\,x^2+15a^3)\sqrt{b\,x^2+a}}{105a^3x^7}$	50
default	$-\frac{(b\,x^2+a)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*(b*x^2+a)^(3/2)/a/x^7-4/7*b/a*(-1/5*(b*x^2+a)^(3/2)/a/x^5+2/15*b*(b*x^
2+a)^(3/2)/a^2/x^3)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.82

$$-\frac{8\,(bx^2+a)^{\frac{3}{2}}b^2}{105\,a^3x^3}+\frac{4\,(bx^2+a)^{\frac{3}{2}}b}{35\,a^2x^5}-\frac{(bx^2+a)^{\frac{3}{2}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -8/105*(b*x^2 + a)^(3/2)*b^2/(a^3*x^3) + 4/35*(b*x^2 + a)^(3/2)*b/(a^2*x^5)
 - 1/7*(b*x^2 + a)^(3/2)/(a*x^7)
```

Fricas [A]
time = 0.77, size = 49, normalized size = 0.72

$$-\frac{(8\,b^3x^6-4\,ab^2x^4+3\,a^2bx^2+15\,a^3)\sqrt{bx^2+a}}{105\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(8*b^3*x^6 - 4*a*b^2*x^4 + 3*a^2*b*x^2 + 15*a^3)*sqrt(b*x^2 + a)/(a^
3*x^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(61) = 122$.
time = 0.65, size = 359, normalized size = 5.28

$$-\frac{15a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33a^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17a^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3a^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12ab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8b^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**8,x)
```

```
[Out] -15*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*
x**8 + 105*a**3*b**6*x**10) - 33*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*a**3*b*
*(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
+ 105*a**3*b**6*x**10) - 3*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*a*b**(17/2)*x
**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**
3*b**6*x**10) - 8*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
+ 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(56) = 112$.
time = 1.38, size = 138, normalized size = 2.03

$$\frac{16\left(70\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8b^{\frac{7}{2}}+35\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6ab^{\frac{7}{2}}+21\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{7}{2}}-7\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{7}{2}}+a^4b^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(7/2) + 35*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(7/2) - 7
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(7/2) + a^4*b^(7/2))/((sqrt(b)*x - s
qrt(b*x^2 + a))^2 - a)^7
```

Mupad [B]
time = 4.71, size = 73, normalized size = 1.07

$$\frac{4\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3}-\frac{b\,\sqrt{b\,x^2+a}}{35\,a\,x^5}-\frac{\sqrt{b\,x^2+a}}{7\,x^7}-\frac{8\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^8,x)
```

```
[Out] (4*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^3) - (b*(a + b*x^2)^(1/2))/(35*a*x^5)
- (a + b*x^2)^(1/2)/(7*x^7) - (8*b^3*(a + b*x^2)^(1/2))/(105*a^3*x)
```

3.368 $\int \frac{\sqrt{a+bx^2}}{x^{10}} \, dx$

Optimal. Leaf size=92

$$-\frac{(a+bx^2)^{3/2}}{9ax^9}+\frac{2b(a+bx^2)^{3/2}}{21a^2x^7}-\frac{8b^2(a+bx^2)^{3/2}}{105a^3x^5}+\frac{16b^3(a+bx^2)^{3/2}}{315a^4x^3}$$

```
[Out] -1/9*(b*x^2+a)^(3/2)/a/x^9+2/21*b*(b*x^2+a)^(3/2)/a^2/x^7-8/105*b^2*(b*x^2+
a)^(3/2)/a^3/x^5+16/315*b^3*(b*x^2+a)^(3/2)/a^4/x^3
```

Rubi [A]

time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$\frac{16b^3(a+bx^2)^{3/2}}{315a^4x^3}-\frac{8b^2(a+bx^2)^{3/2}}{105a^3x^5}+\frac{2b(a+bx^2)^{3/2}}{21a^2x^7}-\frac{(a+bx^2)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^10,x]
```

```
[Out] -1/9*(a + b*x^2)^(3/2)/(a*x^9) + (2*b*(a + b*x^2)^(3/2))/(21*a^2*x^7) - (8*
b^2*(a + b*x^2)^(3/2))/(105*a^3*x^5) + (16*b^3*(a + b*x^2)^(3/2))/(315*a^4*
x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx = -\frac{(a+bx^2)^{3/2}}{9ax^9} - \frac{(2b)\int \frac{\sqrt{a+bx^2}}{x^8}\,dx}{3a}$$
$$= -\frac{(a+bx^2)^{3/2}}{9ax^9} + \frac{2b(a+bx^2)^{3/2}}{21a^2x^7} + \frac{(8b^2)\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{21a^2}$$
$$= -\frac{(a+bx^2)^{3/2}}{9ax^9} + \frac{2b(a+bx^2)^{3/2}}{21a^2x^7} - \frac{8b^2(a+bx^2)^{3/2}}{105a^3x^5} - \frac{(16b^3)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{105a^3}$$
$$= -\frac{(a+bx^2)^{3/2}}{9ax^9} + \frac{2b(a+bx^2)^{3/2}}{21a^2x^7} - \frac{8b^2(a+bx^2)^{3/2}}{105a^3x^5} + \frac{16b^3(a+bx^2)^{3/2}}{315a^4x^3}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\,(-35a^4-5a^3bx^2+6a^2b^2x^4-8ab^3x^6+16b^4x^8)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^10,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4 - 5*a^3*b*x^2 + 6*a^2*b^2*x^4 - 8*a*b^3*x^6 + 16*
b^4*x^8))/(315*a^4*x^9)
```

Maple [A]
time = 0.07, size = 85, normalized size = 0.92

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}(-16b^3x^6+24a\,b^2x^4-30a^2b\,x^2+35a^3)}{315x^9a^4}$	50
trager	$-\frac{(-16b^4x^8+8a\,b^3x^6-6a^2b^2x^4+5a^3b\,x^2+35a^4)\sqrt{b\,x^2+a}}{315x^9a^4}$	61
risch	$-\frac{(-16b^4x^8+8a\,b^3x^6-6a^2b^2x^4+5a^3b\,x^2+35a^4)\sqrt{b\,x^2+a}}{315x^9a^4}$	61
default	$-\frac{(b\,x^2+a)^{\frac{3}{2}}}{9a\,x^9}-\frac{2b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)}{3a}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(b*x^2+a)^(3/2)/a/x^9-2/3*b/a*(-1/7*(b*x^2+a)^(3/2)/a/x^7-4/7*b/a*(-1/
5*(b*x^2+a)^(3/2)/a/x^5+2/15*b*(b*x^2+a)^(3/2)/a^2/x^3))
```

Maxima [A]
time = 0.27, size = 76, normalized size = 0.83

$$\frac{16\,(bx^2+a)^{\frac{3}{2}}b^3}{315\,a^4x^3}-\frac{8\,(bx^2+a)^{\frac{3}{2}}b^2}{105\,a^3x^5}+\frac{2\,(bx^2+a)^{\frac{3}{2}}b}{21\,a^2x^7}-\frac{(bx^2+a)^{\frac{3}{2}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] 16/315*(b*x^2 + a)^(3/2)*b^3/(a^4*x^3) - 8/105*(b*x^2 + a)^(3/2)*b^2/(a^3*x
^5) + 2/21*(b*x^2 + a)^(3/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(3/2)/(a*x^9)
```

Fricas [A]
time = 0.78, size = 60, normalized size = 0.65

$$\frac{(16\,b^4x^8-8\,ab^3x^6+6\,a^2b^2x^4-5\,a^3bx^2-35\,a^4)\sqrt{bx^2+a}}{315\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(16*b^4*x^8 - 8*a*b^3*x^6 + 6*a^2*b^2*x^4 - 5*a^3*b*x^2 - 35*a^4)*sqr
t(b*x^2 + a)/(a^4*x^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(85) = 170$.
time = 0.87, size = 575, normalized size = 6.25

$$-\frac{35a^7b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{110a^6b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{114a^5b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{40a^4b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{5a^3b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{30a^2b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{40ab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{16b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**10,x)
```

```
[Out] -35*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**1
0*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*a**6*b**(21/2)
*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*a**5*b**(23/2)*x**4*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x
**12 + 315*a**4*b**12*x**14) - 40*a**4*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/
(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**
4*b**12*x**14) + 5*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*
x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14)
+ 30*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a*
```

```
*6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 40*a*b**(31
/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 16*b**(33/2)*x**14*sqrt(a/(
b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x*
*12 + 315*a**4*b**12*x**14)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(76) = 152$.
time = 1.09, size = 166, normalized size = 1.80

$$\frac{32\left(315\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}b^{\frac{9}{2}}+189\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{9}{2}}+84\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{9}{2}}-36\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^3b^{\frac{9}{2}}+9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{2}a^4b^{\frac{9}{2}}-a^5b^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(9/2) + 189*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a*b^(9/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(9/2)
- 36*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(9/2) + 9*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^4*b^(9/2) - a^5*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^
9
```

Mupad [B]
time = 4.95, size = 93, normalized size = 1.01

$$\frac{2\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^5}-\frac{b\,\sqrt{b\,x^2+a}}{63\,a\,x^7}-\frac{\sqrt{b\,x^2+a}}{9\,x^9}-\frac{8\,b^3\,\sqrt{b\,x^2+a}}{315\,a^3\,x^3}+\frac{16\,b^4\,\sqrt{b\,x^2+a}}{315\,a^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^10,x)
```

```
[Out] (2*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^5) - (b*(a + b*x^2)^(1/2))/(63*a*x^7)
- (a + b*x^2)^(1/2)/(9*x^9) - (8*b^3*(a + b*x^2)^(1/2))/(315*a^3*x^3) + (16
*b^4*(a + b*x^2)^(1/2))/(315*a^4*x)
```

3.369 $\int x^7(a+bx^2)^{3/2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3(a+bx^2)^{5/2}}{5b^4}+\frac{3a^2(a+bx^2)^{7/2}}{7b^4}-\frac{a(a+bx^2)^{9/2}}{3b^4}+\frac{(a+bx^2)^{11/2}}{11b^4}$$

```
[Out] -1/5*a^3*(b*x^2+a)^(5/2)/b^4+3/7*a^2*(b*x^2+a)^(7/2)/b^4-1/3*a*(b*x^2+a)^(9
/2)/b^4+1/11*(b*x^2+a)^(11/2)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {272, 45}

$$-\frac{a^3(a+bx^2)^{5/2}}{5b^4}+\frac{3a^2(a+bx^2)^{7/2}}{7b^4}+\frac{(a+bx^2)^{11/2}}{11b^4}-\frac{a(a+bx^2)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(3/2),x]
```

```
[Out] -1/5*(a^3*(a + b*x^2)^(5/2))/b^4 + (3*a^2*(a + b*x^2)^(7/2))/(7*b^4) - (a*(
a + b*x^2)^(9/2))/(3*b^4) + (a + b*x^2)^(11/2)/(11*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{3/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx\right.\\ &= -\frac{a^3(a+bx^2)^{5/2}}{5b^4}+\frac{3a^2(a+bx^2)^{7/2}}{7b^4}-\frac{a(a+bx^2)^{9/2}}{3b^4}+\frac{(a+bx^2)^{11/2}}{11b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.62

$$\frac{(a+bx^2)^{5/2}(-16a^3+40a^2bx^2-70ab^2x^4+105b^3x^6)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-16*a^3 + 40*a^2*b*x^2 - 70*a*b^2*x^4 + 105*b^3*x^6))/(
1155*b^4)
```

Maple [A]
time = 0.05, size = 82, normalized size = 1.02

method	result	size
gosper	$-\frac{(bx^2+a)^{\frac{5}{2}}(-105b^3x^6+70ab^2x^4-40a^2bx^2+16a^3)}{1155b^4}$	47
trager	$-\frac{(-105b^5x^{10}-140ab^4x^8-5a^2b^3x^6+6a^3b^2x^4-8a^4bx^2+16a^5)\sqrt{bx^2+a}}{1155b^4}$	69
risch	$-\frac{(-105b^5x^{10}-140ab^4x^8-5a^2b^3x^6+6a^3b^2x^4-8a^4bx^2+16a^5)\sqrt{bx^2+a}}{1155b^4}$	69
default	$\frac{x^6(bx^2+a)^{\frac{5}{2}}}{11b}-\frac{6a\left(\frac{x^4(bx^2+a)^{\frac{5}{2}}}{9b}-\frac{4a\left(\frac{x^2(bx^2+a)^{\frac{5}{2}}}{7b}-\frac{2a(bx^2+a)^{\frac{5}{2}}}{35b^2}\right)}{9b}\right)}{11b}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*x^6*(b*x^2+a)^(5/2)/b-6/11*a/b*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7
*x^2*(b*x^2+a)^(5/2)/b-2/35*a*(b*x^2+a)^(5/2)/b^2))
```

Maxima [A]
time = 0.30, size = 73, normalized size = 0.91

$$\frac{(bx^2+a)^{\frac{5}{2}}x^6}{11\,b}-\frac{2\,(bx^2+a)^{\frac{5}{2}}ax^4}{33\,b^2}+\frac{8\,(bx^2+a)^{\frac{5}{2}}a^2x^2}{231\,b^3}-\frac{16\,(bx^2+a)^{\frac{5}{2}}a^3}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(5/2)*x^6/b - 2/33*(b*x^2 + a)^(5/2)*a*x^4/b^2 + 8/231*(b*
x^2 + a)^(5/2)*a^2*x^2/b^3 - 16/1155*(b*x^2 + a)^(5/2)*a^3/b^4
```

Fricas [A]
time = 1.03, size = 68, normalized size = 0.85

$$\frac{\left(105\, b^{5} x^{10}+140\, a b^{4} x^{8}+5\, a^{2} b^{3} x^{6}-6\, a^{3} b^{2} x^{4}+8\, a^{4} b x^{2}-16\, a^{5}\right) \sqrt{b x^{2}+a}}{1155\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*b^5*x^10 + 140*a*b^4*x^8 + 5*a^2*b^3*x^6 - 6*a^3*b^2*x^4 + 8*a^
4*b*x^2 - 16*a^5)*sqrt(b*x^2 + a)/b^4
```

Sympy [A]
time = 0.37, size = 133, normalized size = 1.66

$$\begin{cases} - \frac{16 a^{5} \sqrt{a + b x^{2}}}{1155 b^{4}} + \frac{8 a^{4} x^{2} \sqrt{a + b x^{2}}}{1155 b^{3}} - \frac{2 a^{3} x^{4} \sqrt{a + b x^{2}}}{385 b^{2}} + \frac{a^{2} x^{6} \sqrt{a + b x^{2}}}{231 b} + \frac{4 a x^{8} \sqrt{a + b x^{2}}}{33} + \frac{b x^{10} \sqrt{a + b x^{2}}}{11} & \text{for}\: b \neq 0 \\ \frac{a^{\frac{3}{2}} x^{8}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-16*a**5*sqrt(a + b*x**2)/(1155*b**4) + 8*a**4*x**2*sqrt(a + b*x
**2)/(1155*b**3) - 2*a**3*x**4*sqrt(a + b*x**2)/(385*b**2) + a**2*x**6*sqrt
(a + b*x**2)/(231*b) + 4*a*x**8*sqrt(a + b*x**2)/33 + b*x**10*sqrt(a + b*x*
*2)/11, Ne(b, 0)), (a**(3/2)*x**8/8, True))
```

Giac [A]
time = 1.14, size = 57, normalized size = 0.71

$$\frac{105\, {\left(b x^{2}+a\right)}^{\frac{11}{2}}-385\, {\left(b x^{2}+a\right)}^{\frac{9}{2}} a+495\, {\left(b x^{2}+a\right)}^{\frac{7}{2}} a^{2}-231\, {\left(b x^{2}+a\right)}^{\frac{5}{2}} a^{3}}{1155\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1155*(105*(b*x^2 + a)^(11/2) - 385*(b*x^2 + a)^(9/2)*a + 495*(b*x^2 + a)^
(7/2)*a^2 - 231*(b*x^2 + a)^(5/2)*a^3)/b^4
```

Mupad [B]
time = 4.60, size = 64, normalized size = 0.80

$$\sqrt{b\,x^2+a}\,\left(\frac{4\,a\,x^8}{33}+\frac{b\,x^{10}}{11}-\frac{16\,a^5}{1155\,b^4}+\frac{a^2\,x^6}{231\,b}-\frac{2\,a^3\,x^4}{385\,b^2}+\frac{8\,a^4\,x^2}{1155\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((4*a*x^8)/33 + (b*x^10)/11 - (16*a^5)/(1155*b^4) + (a^2*
x^6)/(231*b) - (2*a^3*x^4)/(385*b^2) + (8*a^4*x^2)/(1155*b^3))
```

3.370 $\int x^5(a+bx^2)^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{a^2(a+bx^2)^{5/2}}{5b^3}-\frac{2a(a+bx^2)^{7/2}}{7b^3}+\frac{(a+bx^2)^{9/2}}{9b^3}$$

```
[Out] 1/5*a^2*(b*x^2+a)^(5/2)/b^3-2/7*a*(b*x^2+a)^(7/2)/b^3+1/9*(b*x^2+a)^(9/2)/b
^3
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{a^2(a+bx^2)^{5/2}}{5b^3}+\frac{(a+bx^2)^{9/2}}{9b^3}-\frac{2a(a+bx^2)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(3/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(5/2))/(5*b^3) - (2*a*(a + b*x^2)^(7/2))/(7*b^3) + (a + b*
x^2)^(9/2)/(9*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{3/2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{3/2}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx,x,x^2\right)\\&=\frac{a^2(a+bx^2)^{5/2}}{5b^3}-\frac{2a(a+bx^2)^{7/2}}{7b^3}+\frac{(a+bx^2)^{9/2}}{9b^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{(a+bx^2)^{5/2}(8a^2-20abx^2+35b^2x^4)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(3/2),x]
[Out] ((a + b*x^2)^(5/2)*(8*a^2 - 20*a*b*x^2 + 35*b^2*x^4))/(315*b^3)
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.98

method	result	size
gosper	$\frac{(bx^2+a)^{\frac{5}{2}}(35b^2x^4-20abx^2+8a^2)}{315b^3}$	36
default	$\frac{x^4(bx^2+a)^{\frac{5}{2}}}{9b}-\frac{4a\left(\frac{x^2(bx^2+a)^{\frac{5}{2}}}{7b}-\frac{2a(bx^2+a)^{\frac{5}{2}}}{35b^2}\right)}{9b}$	58
trager	$\frac{(35b^4x^8+50ab^3x^6+3a^2b^2x^4-4a^3bx^2+8a^4)\sqrt{bx^2+a}}{315b^3}$	58
risch	$\frac{(35b^4x^8+50ab^3x^6+3a^2b^2x^4-4a^3bx^2+8a^4)\sqrt{bx^2+a}}{315b^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a*(b*x^2+
a)^(5/2)/b^2)
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.90

$$\frac{(bx^2+a)^{\frac{5}{2}}x^4}{9\,b}-\frac{4\,(bx^2+a)^{\frac{5}{2}}ax^2}{63\,b^2}+\frac{8\,(bx^2+a)^{\frac{5}{2}}a^2}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] 1/9*(b*x^2 + a)^(5/2)*x^4/b - 4/63*(b*x^2 + a)^(5/2)*a*x^2/b^2 + 8/315*(b*x
^2 + a)^(5/2)*a^2/b^3
```

Fricas [A]
time = 0.86, size = 57, normalized size = 0.97

$$\frac{(35\,b^4x^8+50\,ab^3x^6+3\,a^2b^2x^4-4\,a^3bx^2+8\,a^4)\sqrt{bx^2+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*x^8 + 50*a*b^3*x^6 + 3*a^2*b^2*x^4 - 4*a^3*b*x^2 + 8*a^4)*sqr
t(b*x^2 + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(51) = 102$.
time = 0.27, size = 109, normalized size = 1.85

$$\begin{cases} \frac{8a^4\sqrt{a+bx^2}}{315b^3} - \frac{4a^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{a^2x^4\sqrt{a+bx^2}}{105b} + \frac{10ax^6\sqrt{a+bx^2}}{63} + \frac{bx^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((8*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*a**3*x**2*sqrt(a + b*x**2
)/(315*b**2) + a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*a*x**6*sqrt(a + b*x*
*2)/63 + b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*x**6/6, True))
```

Giac [A]
time = 0.92, size = 43, normalized size = 0.73

$$\frac{35\left(bx^2+a\right)^{\frac{9}{2}} - 90\left(bx^2+a\right)^{\frac{7}{2}}a + 63\left(bx^2+a\right)^{\frac{5}{2}}a^2}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2) - 90*(b*x^2 + a)^(7/2)*a + 63*(b*x^2 + a)^(5/2)
*a^2)/b^3
```

Mupad [B]
time = 4.66, size = 53, normalized size = 0.90

$$\sqrt{b\,x^2+a}\,\left(\frac{10\,a\,x^6}{63}+\frac{b\,x^8}{9}+\frac{8\,a^4}{315\,b^3}+\frac{a^2\,x^4}{105\,b}-\frac{4\,a^3\,x^2}{315\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((10*a*x^6)/63 + (b*x^8)/9 + (8*a^4)/(315*b^3) + (a^2*x^4
)/(105*b) - (4*a^3*x^2)/(315*b^2))
```

3.371 $\int x^3(a+bx^2)^{3/2}\,dx$

Optimal. Leaf size=38

$$-\frac{a(a+bx^2)^{5/2}}{5b^2}+\frac{(a+bx^2)^{7/2}}{7b^2}$$

```
[Out] -1/5*a*(b*x^2+a)^(5/2)/b^2+1/7*(b*x^2+a)^(7/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{(a+bx^2)^{7/2}}{7b^2}-\frac{a(a+bx^2)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(3/2),x]
```

```
[Out] -1/5*(a*(a + b*x^2)^(5/2))/b^2 + (a + b*x^2)^(7/2)/(7*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{3/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right)\,dx,x,x^2\right)\\ &= -\frac{a(a+bx^2)^{5/2}}{5b^2}+\frac{(a+bx^2)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.74

$$\frac{(a+bx^2)^{5/2}\,(-2a+5bx^2)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(3/2),x]
[Out] ((a + b*x^2)^(5/2)*(-2*a + 5*b*x^2))/(35*b^2)
```

Maple [A]
time = 0.04, size = 34, normalized size = 0.89

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{5}{2}}\,(-5b\,x^2+2a)}{35b^2}$	25
default	$\frac{x^2\,(b\,x^2+a)^{\frac{5}{2}}}{7b}-\frac{2a\,(b\,x^2+a)^{\frac{5}{2}}}{35b^2}$	34
trager	$-\frac{(-5b^3x^6-8a\,b^2x^4-a^2b\,x^2+2a^3)\sqrt{b\,x^2+a}}{35b^2}$	47
risch	$-\frac{(-5b^3x^6-8a\,b^2x^4-a^2b\,x^2+2a^3)\sqrt{b\,x^2+a}}{35b^2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a*(b*x^2+a)^(5/2)/b^2
```

Maxima [A]
time = 0.34, size = 33, normalized size = 0.87

$$\frac{(bx^2+a)^{\frac{5}{2}}x^2}{7\,b}-\frac{2\,(bx^2+a)^{\frac{5}{2}}a}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] 1/7*(b*x^2 + a)^(5/2)*x^2/b - 2/35*(b*x^2 + a)^(5/2)*a/b^2
```

Fricas [A]
time = 0.91, size = 45, normalized size = 1.18

$$\frac{(5\,b^3x^6+8\,ab^2x^4+a^2bx^2-2\,a^3)\sqrt{bx^2+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^3*x^6 + 8*a*b^2*x^4 + a^2*b*x^2 - 2*a^3)*sqrt(b*x^2 + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.
time = 0.18, size = 85, normalized size = 2.24

$$\begin{cases} -\frac{2a^3\sqrt{a+bx^2}}{35b^2} + \frac{a^2x^2\sqrt{a+bx^2}}{35b} + \frac{8ax^4\sqrt{a+bx^2}}{35} + \frac{bx^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*sqrt(a + b*x**2)/(35*b**2) + a**2*x**2*sqrt(a + b*x**2)/
(35*b) + 8*a*x**4*sqrt(a + b*x**2)/35 + b*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)
), (a**(3/2)*x**4/4, True))
```

Giac [A]
time = 0.65, size = 29, normalized size = 0.76

$$\frac{5\,(bx^2+a)^{\frac{7}{2}} - 7\,(bx^2+a)^{\frac{5}{2}}a}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35*(5*(b*x^2 + a)^(7/2) - 7*(b*x^2 + a)^(5/2)*a)/b^2
```

Mupad [B]
time = 4.64, size = 42, normalized size = 1.11

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a\,x^4}{35}+\frac{b\,x^6}{7}-\frac{2\,a^3}{35\,b^2}+\frac{a^2\,x^2}{35\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a*x^4)/35 + (b*x^6)/7 - (2*a^3)/(35*b^2) + (a^2*x^2)/
(35*b))
```

3.372 $\int x(a+bx^2)^{3/2}\,dx$

Optimal. Leaf size=18

$$\frac{(a+bx^2)^{5/2}}{5b}$$

[Out] 1/5*(b*x^2+a)^(5/2)/b

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\frac{(a+bx^2)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^(3/2),x]

[Out] (a + b*x^2)^(5/2)/(5*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{3/2}\,dx=\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{(a+bx^2)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^(3/2),x]

[Out] (a + b*x^2)^(5/2)/(5*b)

Maple [A]
time = 0.03, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{5}{2}}}{5b}$	15
derivativedivides	$\frac{(b\,x^2+a)^{\frac{5}{2}}}{5b}$	15
default	$\frac{(b\,x^2+a)^{\frac{5}{2}}}{5b}$	15
trager	$\frac{(b^2x^4+2ab\,x^2+a^2)\sqrt{b\,x^2+a}}{5b}$	33
risch	$\frac{(b^2x^4+2ab\,x^2+a^2)\sqrt{b\,x^2+a}}{5b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(b*x^2+a)^(5/2)/b
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.78

$$\frac{(bx^2+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(5/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.
time = 0.97, size = 32, normalized size = 1.78

$$\frac{(b^2x^4+2\,abx^2+a^2)\sqrt{bx^2+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.

time = 0.12, size = 61, normalized size = 3.39

$$\begin{cases} \frac{a^2\sqrt{a+bx^2}}{5b} + \frac{2ax^2\sqrt{a+bx^2}}{5} + \frac{bx^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((a**2*sqrt(a + b*x**2)/(5*b) + 2*a*x**2*sqrt(a + b*x**2)/5 + b*x*
*4*sqrt(a + b*x**2)/5, Ne(b, 0)), (a**(3/2)*x**2/2, True))
```

Giac [A]
time = 1.32, size = 14, normalized size = 0.78

$$\frac{(bx^2+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/5*(b*x^2 + a)^(5/2)/b
```

Mupad [B]
time = 4.57, size = 14, normalized size = 0.78

$$\frac{{\left(b\,x^2+a\right)}^{5/2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(5/2)/(5*b)
```

3.373 $\int \frac{(a+bx^2)^{3/2}}{x} dx$

Optimal. Leaf size=54

$$a\sqrt{a+bx^2}+\frac{1}{3}(a+bx^2)^{3/2}-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/3*(b*x^2+a)^(3/2)-a^(3/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+a*(b*x^2+a)^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 52, 65, 214}

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a\sqrt{a+bx^2}+\frac{1}{3}(a+bx^2)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x,x]
```

```
[Out] a*Sqrt[a + b*x^2] + (a + b*x^2)^(3/2)/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{3}\left(a+bx^2\right)^{3/2} + \frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= a\sqrt{a+bx^2} + \frac{1}{3}\left(a+bx^2\right)^{3/2} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= a\sqrt{a+bx^2} + \frac{1}{3}\left(a+bx^2\right)^{3/2} + \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= a\sqrt{a+bx^2} + \frac{1}{3}\left(a+bx^2\right)^{3/2} - a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.93

$$\frac{1}{3}\sqrt{a+bx^2}\left(4a+bx^2\right) - a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*a + b*x^2))/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

Maple [A]
time = 0.03, size = 53, normalized size = 0.98

method	result	size
default	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\;\ln\left(\frac{2a+2\sqrt{a}\;\sqrt{b\,x^2+a}}{x}\right)\right)$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x))
```

Maxima [A]
time = 0.33, size = 40, normalized size = 0.74

$$-a^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}+\sqrt{bx^{2}+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2) + sqrt(b*x^2
 + a)*a
```

Fricas [A]
time = 1.10, size = 100, normalized size = 1.85

$$\left[\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}\,+2\,a}{x^{2}}\right)+\frac{1}{3}\,(bx^{2}+4\,a)\,\sqrt{bx^{2}+a}\,,\,\sqrt{-a}\,a\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\frac{1}{3}\,(bx^{2}+4\,a)\,\sqrt{bx^{2}+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(3/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/3*(b*x
^2 + 4*a)*sqrt(b*x^2 + a), sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 1/
3*(b*x^2 + 4*a)*sqrt(b*x^2 + a)]
```

Sympy [A]
time = 1.09, size = 78, normalized size = 1.44

$$\frac{4a^{\frac{3}{2}}\sqrt{1+\frac{bx^{2}}{a}}}{3}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^{2}}{a}\right)}{2}-a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)+\frac{\sqrt{a}\,bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x,x)
```

```
[Out] 4*a**(3/2)*sqrt(1 + b*x**2/a)/3 + a**(3/2)*log(b*x**2/a)/2 - a**(3/2)*log(s
qrt(1 + b*x**2/a) + 1) + sqrt(a)*b*x**2*sqrt(1 + b*x**2/a)/3
```

Giac [A]
time = 1.03, size = 48, normalized size = 0.89

$$\frac{a^{2}\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}+\sqrt{bx^{2}+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*(b*x^2 + a)^(3/2) + sqr
t(b*x^2 + a)*a
```

Mupad [B]
time = 4.70, size = 42, normalized size = 0.78

$$a\,\sqrt{b\,x^2+a}\,-a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+\frac{{\left(b\,x^2+a\right)}^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x,x)
```

```
[Out] a*(a + b*x^2)^(1/2) - a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + (a + b*x^2
)^(3/2)/3
```

3.374 $\int \frac{(a+bx^2)^{3/2}}{x^3} dx$

Optimal. Leaf size=63

$$\frac{3}{2}b\sqrt{a+bx^2} - \frac{(a+bx^2)^{3/2}}{2x^2} - \frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -1/2*(b*x^2+a)^(3/2)/x^2-3/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+3/2
*b*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 52, 65, 214}

$$-\frac{(a+bx^2)^{3/2}}{2x^2} + \frac{3}{2}b\sqrt{a+bx^2} - \frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^3,x]
```

```
[Out] (3*b*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx,x,x^2\right)\\ &= -\frac{(a+bx^2)^{3/2}}{2x^2}+\frac{1}{4}(3b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\ &= \frac{3}{2}b\sqrt{a+bx^2}-\frac{(a+bx^2)^{3/2}}{2x^2}+\frac{1}{4}(3ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{3}{2}b\sqrt{a+bx^2}-\frac{(a+bx^2)^{3/2}}{2x^2}+\frac{1}{2}(3a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)\\ &= \frac{3}{2}b\sqrt{a+bx^2}-\frac{(a+bx^2)^{3/2}}{2x^2}-\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.90

$$\frac{\sqrt{a+bx^2}\,(-a+2bx^2)}{2x^2}-\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-a + 2*b*x^2))/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/2
```

Maple [A]
time = 0.05, size = 77, normalized size = 1.22

method	result	size
risch	$-\frac{a\sqrt{b\,x^{2}+a}}{2x^{2}}-\frac{3\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)b}{2}+b\sqrt{b\,x^{2}+a}$	57
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2a}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-
a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Maxima [A]
time = 0.33, size = 63, normalized size = 1.00

$$-\frac{3}{2}\sqrt{a}\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{3}{2}\sqrt{bx^2+a}\,b+\frac{(bx^2+a)^{\frac{3}{2}}b}{2\,a}-\frac{(bx^2+a)^{\frac{5}{2}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*b + 1/2*
(b*x^2 + a)^(3/2)*b/a - 1/2*(b*x^2 + a)^(5/2)/(a*x^2)
```

Fricas [A]
time = 1.67, size = 119, normalized size = 1.89

$$\left[\frac{3\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)+2\,(2\,bx^2-a)\sqrt{bx^2+a}}{4\,x^2},\ \frac{3\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,bx^2-a)\sqrt{bx^2+a}}{2\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2, 1/2*(3*sqrt(-a)*b*x^2*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) + (2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A]
time = 1.29, size = 88, normalized size = 1.40

$$-\frac{3\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{a^2}{2\sqrt{b}\,x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{a\sqrt{b}}{2x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - a**2/(2*sqrt(b)*x**3*sqrt(a/(b*
x**2) + 1)) + a*sqrt(b)/(2*x*sqrt(a/(b*x**2) + 1)) + b**(3/2)*x/sqrt(a/(b*x
**2) + 1)
```

Giac [A]
time = 1.49, size = 63, normalized size = 1.00

$$\frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx^2+a}\,b^2-\frac{\sqrt{bx^2+a}\,ab}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x^2 + a)*
b^2 - sqrt(b*x^2 + a)*a*b/x^2)/b
```

Mupad [B]
time = 4.82, size = 47, normalized size = 0.75

$$b\sqrt{b\,x^2+a}-\frac{a\sqrt{b\,x^2+a}}{2\,x^2}-\frac{3\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^3,x)
```

```
[Out] b*(a + b*x^2)^(1/2) - (a*(a + b*x^2)^(1/2))/(2*x^2) - (3*a^(1/2)*b*atanh((a
 + b*x^2)^(1/2)/a^(1/2)))/2
```

3.375 $\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=68

$$-\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{(a+bx^2)^{3/2}}{4x^4}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

```
[Out] -1/4*(b*x^2+a)^(3/2)/x^4-3/8*b^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)-3
/8*b*(b*x^2+a)^(1/2)/x^2
```

Rubi [A]

time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 43, 65, 214}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{(a+bx^2)^{3/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^5,x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2])/(8*x^2) - (a + b*x^2)^(3/2)/(4*x^4) - (3*b^2*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^2\right) \\
&= -\frac{(a+bx^2)^{3/2}}{4x^4} + \frac{1}{8}(3b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2} - \frac{(a+bx^2)^{3/2}}{4x^4} + \frac{1}{16}(3b^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2} - \frac{(a+bx^2)^{3/2}}{4x^4} + \frac{1}{8}(3b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right) \\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2} - \frac{(a+bx^2)^{3/2}}{4x^4} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 0.87

$$\frac{(-2a-5bx^2)\sqrt{a+bx^2}}{8x^4} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^5,x]
[Out] ((-2*a - 5*b*x^2)*Sqrt[a + b*x^2])/(8*x^4) - (3*b^2*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/(8*Sqrt[a])
```

Maple [A]
time = 0.04, size = 101, normalized size = 1.49

method	result	size
risch	$-\frac{\sqrt{b\,x^2+a}\,(5b\,x^2+2a)}{8x^4} - \frac{3b^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{8\sqrt{a}}$	57

default	$-\frac{(b x^2+a)^{\frac{5}{2}}}{4a x^4}+\frac{b\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{2a x^2}+\frac{3b\left(\frac{(b x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right)\right)\right)}{2a}\right)}{4a}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3
*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/
2))/x))))
```

Maxima [A]
time = 0.37, size = 90, normalized size = 1.32

$$-\frac{3\,b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,\sqrt{a}}+\frac{(bx^2+a)^{\frac{3}{2}}b^2}{8\,a^2}+\frac{3\,\sqrt{bx^2+a}\,b^2}{8\,a}-\frac{(bx^2+a)^{\frac{5}{2}}b}{8\,a^2x^2}-\frac{(bx^2+a)^{\frac{5}{2}}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -3/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(b*x^2 + a)^(3/2)*b^2/
a^2 + 3/8*sqrt(b*x^2 + a)*b^2/a - 1/8*(b*x^2 + a)^(5/2)*b/(a^2*x^2) - 1/4*(
b*x^2 + a)^(5/2)/(a*x^4)
```

Fricas [A]
time = 1.73, size = 136, normalized size = 2.00

$$\left[\frac{3\sqrt{a}\,b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)-2\,(5\,abx^2+2\,a^2)\sqrt{bx^2+a}}{16\,ax^4},\ \frac{3\sqrt{-a}\,b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(5\,abx^2+2\,a^2)\sqrt{bx^2+a}}{8\,ax^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 2*(5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a*x^4), 1/8*(3*sqrt(-a)*b^2*x^4
*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a
*x^4)]
```

Sympy [A]
time = 1.75, size = 71, normalized size = 1.04

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{4x^3}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{8x}-\frac{3b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**5,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(4*x**3) - 5*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(8*x) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a))
```

Giac [A]
time = 0.87, size = 70, normalized size = 1.03

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,(bx^2+a)^{\frac{3}{2}}b^3-3\,\sqrt{bx^2+a}\,ab^3}{b^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (5*(b*x^2 + a)^(3/2)
*b^3 - 3*sqrt(b*x^2 + a)*a*b^3)/(b^2*x^4))/b
```

Mupad [B]
time = 4.91, size = 52, normalized size = 0.76

$$\frac{3\,a\,\sqrt{b\,x^2+a}}{8\,x^4}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{5\,{\left(b\,x^2+a\right)}^{3/2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^5,x)
```

```
[Out] (3*a*(a + b*x^2)^(1/2))/(8*x^4) - (3*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/
(8*a^(1/2)) - (5*(a + b*x^2)^(3/2))/(8*x^4)
```

3.376 $\int \frac{(a+bx^2)^{3/2}}{x^7} dx$

Optimal. Leaf size=92

$$-\frac{b\sqrt{a+bx^2}}{8x^4}-\frac{b^2\sqrt{a+bx^2}}{16ax^2}-\frac{(a+bx^2)^{3/2}}{6x^6}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

```
[Out] -1/6*(b*x^2+a)^(3/2)/x^6+1/16*b^3*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-
1/8*b*(b*x^2+a)^(1/2)/x^4-1/16*b^2*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 44, 65, 214}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}-\frac{b^2\sqrt{a+bx^2}}{16ax^2}-\frac{(a+bx^2)^{3/2}}{6x^6}-\frac{b\sqrt{a+bx^2}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^7,x]
```

```
[Out] -1/8*(b*Sqrt[a + b*x^2])/x^4 - (b^2*Sqrt[a + b*x^2])/(16*a*x^2) - (a + b*x^
2)^(3/2)/(6*x^6) + (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^{3/2}}{6x^6} + \frac{1}{4}b\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4} - \frac{(a+bx^2)^{3/2}}{6x^6} + \frac{1}{16}b^2\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{(a+bx^2)^{3/2}}{6x^6} - \frac{b^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{32a}\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{(a+bx^2)^{3/2}}{6x^6} - \frac{b^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{16a}\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{(a+bx^2)^{3/2}}{6x^6} + \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\,(-8a^2-14abx^2-3b^2x^4)}{48ax^6} + \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2 - 14*a*b*x^2 - 3*b^2*x^4))/(48*a*x^6) + (b^3*ArcTa
nh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Maple [A]
time = 0.06, size = 125, normalized size = 1.36

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(3 b^{2} x^{4}+14 a b\,x^{2}+8 a^{2}\right)}{48 x^{6} a}+\frac{b^{3} \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{16 a^{\frac{3}{2}}}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6 a\,x^{6}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{4 a\,x^{4}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2 a\,x^{2}}+\frac{3 b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2 a}\right)}{4 a}\right)}{6 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/
2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(
1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

Maxima [A]
time = 0.32, size = 110, normalized size = 1.20

$$\frac{b^{3} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{3}{2}}}-\frac{(bx^{2}+a)^{\frac{3}{2}}b^{3}}{48\,a^{3}}-\frac{\sqrt{bx^{2}+a}\,b^{3}}{16\,a^{2}}+\frac{(bx^{2}+a)^{\frac{5}{2}}b^{2}}{48\,a^{3}x^{2}}+\frac{(bx^{2}+a)^{\frac{5}{2}}b}{24\,a^{2}x^{4}}-\frac{(bx^{2}+a)^{\frac{5}{2}}}{6\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/48*(b*x^2 + a)^(3/2)*b^3
/a^3 - 1/16*sqrt(b*x^2 + a)*b^3/a^2 + 1/48*(b*x^2 + a)^(5/2)*b^2/(a^3*x^2)
+ 1/24*(b*x^2 + a)^(5/2)*b/(a^2*x^4) - 1/6*(b*x^2 + a)^(5/2)/(a*x^6)
```

Fricas [A]
time = 1.32, size = 157, normalized size = 1.71

$$\left[\frac{3\,\sqrt{a}\,b^{3}x^{6}\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\,(3\,ab^{2}x^{4}+14\,a^{2}bx^{2}+8\,a^{3})\sqrt{bx^{2}+a}}{96\,a^{2}x^{6}},\ -\frac{3\,\sqrt{-a}\,b^{3}x^{6}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(3\,ab^{2}x^{4}+14\,a^{2}bx^{2}+8\,a^{3})\sqrt{bx^{2}+a}}{48\,a^{2}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 2*(3*a*b^2*x^4 + 14*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6), -1/4
8*(3*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b^2*x^4 + 14*
a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6)]
```

Sympy [A]
time = 3.87, size = 119, normalized size = 1.29

$$-\frac{a^2}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{11a\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{17b^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{b^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}}+\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**7,x)
```

```
[Out] -a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*a*sqrt(b)/(24*x**5*sqrt(a/
(b*x**2) + 1)) - 17*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - b**(5/2)/(16*
a*x*sqrt(a/(b*x**2) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2))
```

Giac [A]
time = 0.87, size = 92, normalized size = 1.00

$$-\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3\,(bx^2+a)^{\frac{5}{2}}b^4+8\,(bx^2+a)^{\frac{3}{2}}ab^4-3\,\sqrt{bx^2+a}\,a^2b^4}{ab^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x^2 + a)
^(5/2)*b^4 + 8*(b*x^2 + a)^(3/2)*a*b^4 - 3*sqrt(b*x^2 + a)*a^2*b^4)/(a*b^3*
x^6))/b
```

Mupad [B]
time = 4.94, size = 72, normalized size = 0.78

$$\frac{a\sqrt{b\,x^2+a}}{16\,x^6}-\frac{\left(b\,x^2+a\right)^{3/2}}{6\,x^6}-\frac{\left(b\,x^2+a\right)^{5/2}}{16\,a\,x^6}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{16\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^7,x)
```

```
[Out] (a*(a + b*x^2)^(1/2))/(16*x^6) - (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*
1i)/(16*a^(3/2)) - (a + b*x^2)^(3/2)/(6*x^6) - (a + b*x^2)^(5/2)/(16*a*x^6)
```

3.377 $\int \frac{(a+bx^2)^{3/2}}{x^9} \, dx$

Optimal. Leaf size=116

$$-\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{(a+bx^2)^{3/2}}{8x^8}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}$$

```
[Out] -1/8*(b*x^2+a)^(3/2)/x^8-3/128*b^4*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
-1/16*b*(b*x^2+a)^(1/2)/x^6-1/64*b^2*(b*x^2+a)^(1/2)/a/x^4+3/128*b^3*(b*x^2
+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {272, 43, 44, 65, 214}

$$-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}-\frac{(a+bx^2)^{3/2}}{8x^8}-\frac{b\sqrt{a+bx^2}}{16x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^9,x]
```

```
[Out] -1/16*(b*Sqrt[a + b*x^2])/x^6 - (b^2*Sqrt[a + b*x^2])/(64*a*x^4) + (3*b^3*S
qrt[a + b*x^2])/(128*a^2*x^2) - (a + b*x^2)^(3/2)/(8*x^8) - (3*b^4*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx,x,x^2\right)\\
&= -\frac{(a+bx^2)^{3/2}}{8x^8}+\frac{1}{16}(3b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{(a+bx^2)^{3/2}}{8x^8}+\frac{1}{32}b^2\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}-\frac{(a+bx^2)^{3/2}}{8x^8}-\frac{(3b^3)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{128a}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{(a+bx^2)^{3/2}}{8x^8}+\frac{(3b^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{256a^2}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{(a+bx^2)^{3/2}}{8x^8}+\frac{(3b^3)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{128a^2}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{(a+bx^2)^{3/2}}{8x^8}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 84, normalized size = 0.72

$$
\frac{\sqrt{a+bx^2}\,(-16a^3-24a^2bx^2-2ab^2x^4+3b^3x^6)}{128a^2x^8}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^3 - 24*a^2*b*x^2 - 2*a*b^2*x^4 + 3*b^3*x^6))/(128*a
^2*x^8) - (3*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Maple [A]
time = 0.07, size = 149, normalized size = 1.28

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-3b^3x^6+2a\,b^2x^4+24a^2b\,x^2+16a^3\right)}{128x^8a^2}-\frac{3b^4\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{128a^{\frac{5}{2}}}$
default	$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{8a\,x^8}-\frac{3b\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b \cdots}}{x}\right.\right.\right.}{2a}\right.}{4a}\right.}{6a}\right.}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(5/2)-3/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/
4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b
*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x))))))
```

Maxima [A]
time = 0.30, size = 130, normalized size = 1.12

$$-\frac{3\,b^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{5}{2}}}+\frac{(bx^2+a)^{\frac{3}{2}}b^4}{128\,a^4}+\frac{3\,\sqrt{bx^2+a}\,b^4}{128\,a^3}-\frac{(bx^2+a)^{\frac{5}{2}}b^3}{128\,a^4x^2}-\frac{(bx^2+a)^{\frac{5}{2}}b^2}{64\,a^3x^4}+\frac{(bx^2+a)^{\frac{5}{2}}b}{16\,a^2x^6}-\frac{(bx^2+a)^{\frac{5}{2}}}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -3/128*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/128*(b*x^2 + a)^(3/2)*
b^4/a^4 + 3/128*sqrt(b*x^2 + a)*b^4/a^3 - 1/128*(b*x^2 + a)^(5/2)*b^3/(a^4*
x^2) - 1/64*(b*x^2 + a)^(5/2)*b^2/(a^3*x^4) + 1/16*(b*x^2 + a)^(5/2)*b/(a^2
*x^6) - 1/8*(b*x^2 + a)^(5/2)/(a*x^8)
```

Fricas [A]
time = 1.66, size = 179, normalized size = 1.54

$$\left[\frac{3\,\sqrt{a}\,b^{4}x^{8}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(3\,ab^{3}x^{6}-2\,a^{2}b^{2}x^{4}-24\,a^{3}bx^{2}-16\,a^{4})\sqrt{bx^{2}+a}}{256\,a^{3}x^{8}},\ \frac{3\,\sqrt{-a}\,b^{4}x^{8}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(3\,ab^{3}x^{6}-2\,a^{2}b^{2}x^{4}-24\,a^{3}bx^{2}-16\,a^{4})\sqrt{bx^{2}+a}}{128\,a^{3}x^{8}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/256*(3*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a
))/(a^3*x^8), 1/128*(3*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
(3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a))/(a^3
*x^8)]
```

Sympy [A]
time = 11.62, size = 148, normalized size = 1.28

$$-\frac{a^{2}}{8\sqrt{b}\,x^{9}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5a\sqrt{b}}{16x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{13b^{\frac{3}{2}}}{64x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{\frac{5}{2}}}{128ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b^{\frac{7}{2}}}{128a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3b^{4}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**9,x)
```

```
[Out] -a**2/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 5*a*sqrt(b)/(16*x**7*sqrt(a/(
b*x**2) + 1)) - 13*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(128*
a*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(7/2)/(128*a**2*x*sqrt(a/(b*x**2) + 1))
 - 3*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(5/2))
```

Giac [A]
time = 0.70, size = 109, normalized size = 0.94

$$\frac{\frac{3\,b^{5}\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{3\,(bx^{2}+a)^{\frac{7}{2}}b^{5}-11\,(bx^{2}+a)^{\frac{5}{2}}ab^{5}-11\,(bx^{2}+a)^{\frac{3}{2}}a^{2}b^{5}+3\,\sqrt{bx^{2}+a}\,a^{3}b^{5}}{a^{2}b^{4}x^{8}}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/128*(3*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 +
a)^(7/2)*b^5 - 11*(b*x^2 + a)^(5/2)*a*b^5 - 11*(b*x^2 + a)^(3/2)*a^2*b^5 +
3*sqrt(b*x^2 + a)*a^3*b^5)/(a^2*b^4*x^8))/b
```

Mupad [B]
time = 5.21, size = 89, normalized size = 0.77

$$\frac{3\,a\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{11\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^8}-\frac{11\,{\left(b\,x^2+a\right)}^{5/2}}{128\,a\,x^8}+\frac{3\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^2\,x^8}+\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{128\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^9,x)
```

```
[Out] (b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (11*(a + b*x^
2)^(3/2))/(128*x^8) + (3*a*(a + b*x^2)^(1/2))/(128*x^8) - (11*(a + b*x^2)^(
5/2))/(128*a*x^8) + (3*(a + b*x^2)^(7/2))/(128*a^2*x^8)
```

3.378 $\int x^4(a+bx^2)^{3/2}\,dx$

Optimal. Leaf size=115

$$-\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

```
[Out] 1/8*x^5*(b*x^2+a)^(3/2)+3/128*a^4*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2
)-3/128*a^3*x*(b*x^2+a)^(1/2)/b^2+1/64*a^2*x^3*(b*x^2+a)^(1/2)/b+1/16*a*x^5
*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 212}

$$\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}-\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}ax^5\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/2),x]
```

```
[Out] (-3*a^3*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*x^3*Sqrt[a + b*x^2])/(64*b) + (
a*x^5*Sqrt[a + b*x^2])/16 + (x^5*(a + b*x^2)^(3/2))/8 + (3*a^4*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+bx^2\right)^{3/2}\,dx &= \frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{8}(3a)\int x^4\sqrt{a+bx^2}\,dx\\
&= \frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}a^2\int \frac{x^4}{\sqrt{a+bx^2}}\,dx\\
&= \frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}-\frac{\left(3a^3\right)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{64b}\\
&= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{\left(3a^4\right)}{}\\
&= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{\left(3a^4\right)}{}\\
&= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{3a^4\,\mathrm{ta}}{}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 85, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-3a^3x+2a^2bx^3+24ab^2x^5+16b^3x^7\right)}{128b^2}-\frac{3a^4\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{128b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^3*x + 2*a^2*b*x^3 + 24*a*b^2*x^5 + 16*b^3*x^7))/(128
*b^2) - (3*a^4*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Maple [A]
time = 0.04, size = 98, normalized size = 0.85

method	result	size
risch	$-\frac{x(-16b^3x^6-24a\,b^2x^4-2a^2b\,x^2+3a^3)\sqrt{b\,x^2+a}}{128b^2}+\frac{3a^4\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{128b^{\frac{5}{2}}}$	73
default	$\frac{x^3(b\,x^2+a)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(
b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x
^2+a)^(1/2)))))
```

Maxima [A]
time = 0.29, size = 87, normalized size = 0.76

$$\frac{(bx^2+a)^{\frac{5}{2}}x^3}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}ax}{16\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}a^2x}{64\,b^2}+\frac{3\,\sqrt{bx^2+a}\,a^3x}{128\,b^2}+\frac{3\,a^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*x^3/b - 1/16*(b*x^2 + a)^(5/2)*a*x/b^2 + 1/64*(b*x^2
+ a)^(3/2)*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3*x/b^2 + 3/128*a^4*arcsinh(
b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 1.25, size = 168, normalized size = 1.46

$$\left[\frac{3\,a^4\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(16\,b^4x^7+24\,ab^3x^5+2\,a^2b^2x^3-3\,a^3bx)\sqrt{bx^2+a}}{256\,b^3},-\frac{3\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(16\,b^4x^7+24\,ab^3x^5+2\,a^2b^2x^3-3\,a^3bx)\sqrt{bx^2+a}}{128\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
16*b^4*x^7 + 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b^3
, -1/128*(3*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (16*b^4*x^7 +
 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 11.51, size = 148, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13a^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}\,bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^4\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^2x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - a**(5/2)*x**3/(128*b*sqrt(1 +
 b*x**2/a)) + 13*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*sqrt(a)*b*x**7/(
16*sqrt(1 + b*x**2/a)) + 3*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + b
**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.54, size = 76, normalized size = 0.66

$$\frac{1}{128}\left(2\left(4\left(2\,bx^2+3\,a\right)x^2+\frac{a^2}{b}\right)x^2-\frac{3\,a^3}{b^2}\right)\sqrt{bx^2+a}\,x-\frac{3\,a^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*(2*(4*(2*b*x^2 + 3*a)*x^2 + a^2/b)*x^2 - 3*a^3/b^2)*sqrt(b*x^2 + a)*x
 - 3/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(3/2), x)
```

3.379 $\int x^2(a+bx^2)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

```
[Out] 1/6*x^3*(b*x^2+a)^(3/2)-1/16*a^3*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)
+1/16*a^2*x*(b*x^2+a)^(1/2)/b+1/8*a*x^3*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {285, 327, 223, 212}

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a + b*x^2])/(16*b) + (a*x^3*Sqrt[a + b*x^2])/8 + (x^3*(a + b*x^
2)^(3/2))/6 - (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^2\right)^{3/2}\,dx &= \frac{1}{6}x^3\left(a+bx^2\right)^{3/2}+\frac{1}{2}a\int x^2\sqrt{a+bx^2}\,dx\\
&= \frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}a^2\int \frac{x^2}{\sqrt{a+bx^2}}\,dx\\
&= \frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\int \frac{1}{\sqrt{a+bx^2}}\,dx}{16b}\\
&= \frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16b}\\
&= \frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 74, normalized size = 0.81

$$\frac{x\sqrt{a+bx^2}\left(3a^2+14abx^2+8b^2x^4\right)}{48b}+\frac{a^3\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/2),x]
[Out] (x*Sqrt[a + b*x^2]*(3*a^2 + 14*a*b*x^2 + 8*b^2*x^4))/(48*b) + (a^3*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Maple [A]
time = 0.04, size = 74, normalized size = 0.81

method	result	size
risch	$\frac{x\left(8b^2x^4+14ab\,x^2+3a^2\right)\sqrt{b\,x^2+a}}{48b}-\frac{a^3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{16b^{\frac{3}{2}}}$	62

default	$\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.27, size = 67, normalized size = 0.74

$$\frac{(bx^2+a)^{\frac{5}{2}}x}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}ax}{24\,b}-\frac{\sqrt{bx^2+a}\,a^2x}{16\,b}-\frac{a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*x/b - 1/24*(b*x^2 + a)^(3/2)*a*x/b - 1/16*sqrt(b*x^2
+ a)*a^2*x/b - 1/16*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 1.79, size = 145, normalized size = 1.59

$$\left[\frac{3\,a^3\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(8\,b^3x^5+14\,ab^2x^3+3\,a^2bx)\sqrt{bx^2+a}}{96\,b^2},\ \frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(8\,b^3x^5+14\,ab^2x^3+3\,a^2bx)\sqrt{bx^2+a}}{48\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8
*b^3*x^5 + 14*a*b^2*x^3 + 3*a^2*b*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*a^3*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*b^3*x^5 + 14*a*b^2*x^3 + 3*a^2
*b*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A]
time = 3.64, size = 119, normalized size = 1.31

$$\frac{a^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}\,bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{a^3\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{b^2x^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*x**3/(48*sqrt(1 + b*x**2
/a)) + 11*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*b**(3/2)) + b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.02, size = 63, normalized size = 0.69

$$\frac{1}{48}\left(2\left(4\,bx^2+7\,a\right)x^2+\frac{3\,a^2}{b}\right)\sqrt{bx^2+a}\,x+\frac{a^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2 + 7*a)*x^2 + 3*a^2/b)*sqrt(b*x^2 + a)*x + 1/16*a^3*log(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(3/2), x)
```

3.380 $\int (a+bx^2)^{3/2} \, dx$

Optimal. Leaf size=65

$$\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+3/
8*a*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2} dx = \frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+bx^2}\, dx$$
$$= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\, dx$$
$$= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\, dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

Mathematica [A]
time = 0.06, size = 60, normalized size = 0.92

$$\frac{1}{8}x\sqrt{a+bx^2}\left(5a+2bx^2\right)-\frac{3a^2\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(5*a + 2*b*x^2))/8 - (3*a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(8*Sqrt[b])
```

Maple [A]
time = 0.03, size = 52, normalized size = 0.80

method	result	size
risch	$\frac{x\left(2b\,x^2+5a\right)\sqrt{b\,x^2+a}}{8}+\frac{3a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}$	48
default	$\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/
2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.33, size = 43, normalized size = 0.66

$$\frac{1}{4}\left(bx^{2}+a\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{bx^{2}+a}\,ax+\frac{3\,a^{2}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*x + 3/8*sqrt(b*x^2 + a)*a*x + 3/8*a^2*arcsinh(b*x/sqr
t(a*b))/sqrt(b)
```

Fricas [A]
time = 1.28, size = 124, normalized size = 1.91

$$\left[\frac{3\,a^{2}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^{2}x^{3}+5\,abx\right)\sqrt{bx^{2}+a}}{16\,b},-\frac{3\,a^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\left(2\,b^{2}x^{3}+5\,abx\right)\sqrt{bx^{2}+a}}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 1.55, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^{2}}{a}}}{8}+\frac{\sqrt{a}\,bx^{3}\sqrt{1+\frac{bx^{2}}{a}}}{4}+\frac{3a^{2}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 + 3
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

Giac [A]
time = 1.93, size = 49, normalized size = 0.75

$$\frac{1}{8}\left(2\,bx^{2}+5\,a\right)\sqrt{bx^{2}+a}\,x-\frac{3\,a^{2}\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/sqrt(b)
```

Mupad [B]
time = 4.61, size = 37, normalized size = 0.57

$$\frac{x\,{\left(b\,x^2+a\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2),x)
```

```
[Out] (x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(3/2)
```

3.381 $\int \frac{(a+bx^2)^{3/2}}{x^2} dx$

Optimal. Leaf size=63

$$\frac{3}{2}bx\sqrt{a+bx^2} - \frac{(a+bx^2)^{3/2}}{x} + \frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -(b*x^2+a)^(3/2)/x+3/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)+3/2*b*x
*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 201, 223, 212}

$$-\frac{(a+bx^2)^{3/2}}{x} + \frac{3}{2}bx\sqrt{a+bx^2} + \frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^2,x]
```

```
[Out] (3*b*x*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/x + (3*a*Sqrt[b]*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx &= -\frac{(a+bx^2)^{3/2}}{x}+(3b)\int \sqrt{a+bx^2}\,dx\\
&= \frac{3}{2}bx\sqrt{a+bx^2}-\frac{(a+bx^2)^{3/2}}{x}+\frac{1}{2}(3ab)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{3}{2}bx\sqrt{a+bx^2}-\frac{(a+bx^2)^{3/2}}{x}+\frac{1}{2}(3ab)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{3}{2}bx\sqrt{a+bx^2}-\frac{(a+bx^2)^{3/2}}{x}+\frac{3}{2}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 0.94

$$
\frac{(-2a+bx^2)\sqrt{a+bx^2}}{2x}-\frac{3}{2}a\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^2,x]
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(2*x) - (3*a*Sqrt[b]*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/2
```

Maple [A]
time = 0.04, size = 76, normalized size = 1.21

method	result	size
risch	$-\frac{\sqrt{b\,x^2+a}\,(-b\,x^2+2a)}{2x}+\frac{3a\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2}$	48

default	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^
(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.68

$$\frac{3}{2}\sqrt{bx^2+a}\,bx+\frac{3}{2}\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{(bx^2+a)^{\frac{3}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*b*x + 3/2*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a
)^(3/2)/x
```

Fricas [A]
time = 1.29, size = 112, normalized size = 1.78

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\sqrt{bx^2+a}\,(bx^2-2\,a)}{4\,x},-\frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,(bx^2-2\,a)}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqr
t(b*x^2 + a)*(b*x^2 - 2*a))/x, -1/2*(3*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - sqrt(b*x^2 + a)*(b*x^2 - 2*a))/x]
```

Sympy [A]
time = 1.28, size = 88, normalized size = 1.40

$$-\frac{a^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,bx}{2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}+\frac{b^2x^3}{2\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -a**(3/2)/(x*sqrt(1 + b*x**2/a)) - sqrt(a)*b*x/(2*sqrt(1 + b*x**2/a)) + 3*a
*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2 + b**2*x**3/(2*sqrt(a)*sqrt(1 + b*x**2/
a))
```

Giac [A]
time = 1.35, size = 73, normalized size = 1.16

$$\frac{1}{2}\sqrt{bx^2+a}\,bx-\frac{3}{4}\,a\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b*x - 3/4*a*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2
) + 2*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B]
time = 5.15, size = 40, normalized size = 0.63

$$-\frac{\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^2,x)
```

```
[Out] -((a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a
 + 1)^(3/2))
```

3.382 $\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=61

$$-\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}+b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/3*(b*x^2+a)^(3/2)/x^3+b^(3/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))-b*(b*x^
2+a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {283, 223, 212}

$$b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^4,x]
```

```
[Out] -((b*Sqrt[a + b*x^2])/x) - (a + b*x^2)^(3/2)/(3*x^3) + b^(3/2)*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx = -\frac{(a+bx^2)^{3/2}}{3x^3} + b\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$$
$$= -\frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3} + b^2\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= -\frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3} + b^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= -\frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3} + b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.93

$$\frac{(-a-4bx^2)\sqrt{a+bx^2}}{3x^3} - b^{3/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^4,x]
```

```
[Out] ((-a - 4*b*x^2)*Sqrt[a + b*x^2])/(3*x^3) - b^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.
time = 0.04, size = 100, normalized size = 1.64

method	result	size
risch	$-\frac{\sqrt{b\,x^2+a}\,(4b\,x^2+a)}{3x^3}+b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)$	44
default	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{3a\,x^3}+\frac{2b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)}{3a}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(5/2)+2/3*b/a*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*
x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2
+a)^(1/2)))))
```

Maxima [A]
time = 0.33, size = 66, normalized size = 1.08

$$\frac{\sqrt{bx^2+a}\,b^2x}{a}+b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{2\,(bx^2+a)^{\frac{3}{2}}b}{3\,ax}-\frac{(bx^2+a)^{\frac{5}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*b^2*x/a + b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 2/3*(b*x^2 + a)^
(3/2)*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)/(a*x^3)
```

Fricas [A]
time = 0.93, size = 112, normalized size = 1.84

$$\left[\frac{3\,b^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(4\,bx^2+a)\sqrt{bx^2+a}}{6\,x^3},-\frac{3\,\sqrt{-b}\,bx^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(4\,bx^2+a)\sqrt{bx^2+a}}{3\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(4*
b*x^2 + a)*sqrt(b*x^2 + a))/x^3, -1/3*(3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (4*b*x^2 + a)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A]
time = 1.14, size = 78, normalized size = 1.28

$$-\frac{a\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}-\frac{b^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**4,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 4*b**(3/2)*sqrt(a/(b*x**2) + 1)/
3 - b**(3/2)*log(a/(b*x**2))/2 + b**(3/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(49) = 98$.
time = 0.97, size = 114, normalized size = 1.87

$$-\frac{1}{2}\,b^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{4\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4ab^{\frac{3}{2}}-3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{3}{2}}+2\,a^3b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 4/3*(3*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a*b^(3/2) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2) +
 2*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^4,x)
```

```
[Out] int((a + b*x^2)^(3/2)/x^4, x)
```

3.383 $\int \frac{(a+bx^2)^{3/2}}{x^6} dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

[Out] -1/5*(b*x^2+a)^(5/2)/a/x^5

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(3/2)/x^6,x]

[Out] -1/5*(a + b*x^2)^(5/2)/(a*x^5)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^6} dx = -\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Mathematica [A]
time = 0.06, size = 21, normalized size = 1.00

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(3/2)/x^6,x]

[Out] -1/5*(a + b*x^2)^(5/2)/(a*x^5)

Maple [A]
time = 0.04, size = 18, normalized size = 0.86

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5a\,x^{5}}$	18
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5a\,x^{5}}$	18
trager	$-\frac{\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\sqrt{b\,x^{2}+a}}{5a\,x^{5}}$	36
risch	$-\frac{\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\sqrt{b\,x^{2}+a}}{5a\,x^{5}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*(b*x^2+a)^(5/2)/a/x^5
```

Maxima [A]
time = 0.33, size = 17, normalized size = 0.81

$$-\frac{(bx^2+a)^{\frac{5}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="maxima")
[Out] -1/5*(b*x^2 + a)^(5/2)/(a*x^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 0.85, size = 35, normalized size = 1.67

$$-\frac{(b^2x^4+2\,abx^2+a^2)\sqrt{bx^2+a}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="fricas")
[Out] -1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/(a*x^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.
time = 0.45, size = 68, normalized size = 3.24

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2}-\frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**6,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(5*x**2) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(17) = 34$.
time = 0.71, size = 86, normalized size = 4.10

$$\frac{2\left(5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8 b^{\frac{5}{2}}+10\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4 a^2 b^{\frac{5}{2}}+a^4 b^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2) + 10*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(5/2) + a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [B]
time = 5.29, size = 17, normalized size = 0.81

$$-\frac{{\left(b\,x^2+a\right)}^{5/2}}{5\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^6,x)
```

```
[Out] -(a + b*x^2)^(5/2)/(5*a*x^5)
```

3.384 $\int \frac{(a+bx^2)^{3/2}}{x^8} dx$

Optimal. Leaf size=44

$$-\frac{(a+bx^2)^{5/2}}{7ax^7}+\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}$$

```
[Out] -1/7*(b*x^2+a)^(5/2)/a/x^7+2/35*b*(b*x^2+a)^(5/2)/a^2/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}-\frac{(a+bx^2)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^8,x]
```

```
[Out] -1/7*(a + b*x^2)^(5/2)/(a*x^7) + (2*b*(a + b*x^2)^(5/2))/(35*a^2*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx &= -\frac{(a+bx^2)^{5/2}}{7ax^7}-\frac{(2b)\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx}{7a}\\ &= -\frac{(a+bx^2)^{5/2}}{7ax^7}+\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 31, normalized size = 0.70

$$\frac{(a+bx^2)^{5/2}(-5a+2bx^2)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^8,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-5*a + 2*b*x^2))/(35*a^2*x^7)
```

Maple [A]
time = 0.06, size = 37, normalized size = 0.84

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{5}{2}}(-2b\,x^2+5a)}{35x^7a^2}$	28
default	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{7a\,x^7}+\frac{2b(b\,x^2+a)^{\frac{5}{2}}}{35a^2x^5}$	37
trager	$-\frac{(-2b^3x^6+a\,b^2x^4+8a^2b\,x^2+5a^3)\sqrt{b\,x^2+a}}{35x^7a^2}$	49
risch	$-\frac{(-2b^3x^6+a\,b^2x^4+8a^2b\,x^2+5a^3)\sqrt{b\,x^2+a}}{35x^7a^2}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*(b*x^2+a)^(5/2)/a/x^7+2/35*b*(b*x^2+a)^(5/2)/a^2/x^5
```

Maxima [A]
time = 0.33, size = 36, normalized size = 0.82

$$\frac{2\,(bx^2+a)^{\frac{5}{2}}b}{35\,a^2x^5}-\frac{(bx^2+a)^{\frac{5}{2}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] 2/35*(b*x^2 + a)^(5/2)*b/(a^2*x^5) - 1/7*(b*x^2 + a)^(5/2)/(a*x^7)
```

Fricas [A]
time = 1.11, size = 49, normalized size = 1.11

$$\frac{(2\,b^3x^6-ab^2x^4-8\,a^2bx^2-5\,a^3)\sqrt{bx^2+a}}{35\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(2*b^3*x^6 - a*b^2*x^4 - 8*a^2*b*x^2 - 5*a^3)*sqrt(b*x^2 + a)/(a^2*x^7
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(37) = 74$.
time = 0.58, size = 94, normalized size = 2.14

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6}-\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^4}-\frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{35ax^2}+\frac{2b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**8,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 8*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(35*x**4) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*a*x**2) + 2*b**(7/2)*sqrt(a/(
b*x**2) + 1)/(35*a**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(36) = 72$.
time = 1.30, size = 166, normalized size = 3.77

$$\frac{4\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}}+35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{7}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{7}{2}}+14\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^3b^{\frac{7}{2}}+7\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^4b^{\frac{7}{2}}-a^5b^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 4/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2) + 35*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*a*b^(7/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2) + 14
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2) + 7*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^4*b^(7/2) - a^5*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

Mupad [B]
time = 5.65, size = 71, normalized size = 1.61

$$\frac{2\,b^3\,\sqrt{b\,x^2+a}}{35\,a^2\,x}-\frac{8\,b\,\sqrt{b\,x^2+a}}{35\,x^5}-\frac{b^2\,\sqrt{b\,x^2+a}}{35\,a\,x^3}-\frac{a\,\sqrt{b\,x^2+a}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^8,x)
```

```
[Out] (2*b^3*(a + b*x^2)^(1/2))/(35*a^2*x) - (8*b*(a + b*x^2)^(1/2))/(35*x^5) - (
b^2*(a + b*x^2)^(1/2))/(35*a*x^3) - (a*(a + b*x^2)^(1/2))/(7*x^7)
```

3.385 $\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=68

$$-\frac{(a+bx^2)^{5/2}}{9ax^9}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}$$

```
[Out] -1/9*(b*x^2+a)^(5/2)/a/x^9+4/63*b*(b*x^2+a)^(5/2)/a^2/x^7-8/315*b^2*(b*x^2+
a)^(5/2)/a^3/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}-\frac{(a+bx^2)^{5/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^10,x]
```

```
[Out] -1/9*(a + b*x^2)^(5/2)/(a*x^9) + (4*b*(a + b*x^2)^(5/2))/(63*a^2*x^7) - (8*
b^2*(a + b*x^2)^(5/2))/(315*a^3*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx &= -\frac{(a+bx^2)^{5/2}}{9ax^9}-\frac{(4b)\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx}{9a}\\
&= -\frac{(a+bx^2)^{5/2}}{9ax^9}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}+\frac{(8b^2)\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx}{63a^2}\\
&= -\frac{(a+bx^2)^{5/2}}{9ax^9}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 42, normalized size = 0.62

$$\frac{(a+bx^2)^{5/2}\,(-35a^2+20abx^2-8b^2x^4)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^10,x]
[Out] ((a + b*x^2)^(5/2)*(-35*a^2 + 20*a*b*x^2 - 8*b^2*x^4))/(315*a^3*x^9)
```

Maple [A]
time = 0.08, size = 61, normalized size = 0.90

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{5}{2}}\,(8b^2x^4-20ab\,x^2+35a^2)}{315x^9a^3}$	39
default	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{9a\,x^9}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{7a\,x^7}+\frac{2b(b\,x^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}$	61
trager	$-\frac{(8b^4x^8-4a\,b^3x^6+3a^2b^2x^4+50a^3b\,x^2+35a^4)\sqrt{b\,x^2+a}}{315x^9a^3}$	61
risch	$-\frac{(8b^4x^8-4a\,b^3x^6+3a^2b^2x^4+50a^3b\,x^2+35a^4)\sqrt{b\,x^2+a}}{315x^9a^3}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^10,x,method=_RETURNVERBOSE)
[Out] -1/9*(b*x^2+a)^(5/2)/a/x^9-4/9*b/a*(-1/7*(b*x^2+a)^(5/2)/a/x^7+2/35*b*(b*x^
2+a)^(5/2)/a^2/x^5)
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.82

$$-\frac{8\,(bx^2+a)^{\frac{5}{2}}b^2}{315\,a^3x^5}+\frac{4\,(bx^2+a)^{\frac{5}{2}}b}{63\,a^2x^7}-\frac{(bx^2+a)^{\frac{5}{2}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="maxima")
[Out] -8/315*(b*x^2 + a)^(5/2)*b^2/(a^3*x^5) + 4/63*(b*x^2 + a)^(5/2)*b/(a^2*x^7)
 - 1/9*(b*x^2 + a)^(5/2)/(a*x^9)
```

Fricas [A]
time = 0.86, size = 60, normalized size = 0.88

$$-\frac{(8\,b^4x^8-4\,ab^3x^6+3\,a^2b^2x^4+50\,a^3bx^2+35\,a^4)\sqrt{bx^2+a}}{315\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(8*b^4*x^8 - 4*a*b^3*x^6 + 3*a^2*b^2*x^4 + 50*a^3*b*x^2 + 35*a^4)*sq
rt(b*x^2 + a)/(a^3*x^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(61) = 122$.
time = 0.79, size = 420, normalized size = 6.18

$$-\frac{35a^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{120a^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{138a^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{52a^3b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{3a^2b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{12ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{8b^{\frac{21}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**10,x)
```

```
[Out] -35*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*
x**10 + 315*a**3*b**6*x**12) - 120*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)
/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 138*a**
4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x
**10 + 315*a**3*b**6*x**12) - 52*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(
315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 3*a**2*b*
*(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10
 + 315*a**3*b**6*x**12) - 12*a*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a*
*5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 8*b**(21/2)*x**
12*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**
3*b**6*x**12)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(56) = 112$.
time = 0.70, size = 192, normalized size = 2.82

$$\frac{16\left(210\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}b^{\frac{9}{2}}+315\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}ab^{\frac{9}{2}}+441\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}a^2b^{\frac{9}{2}}+126\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{6}a^3b^{\frac{9}{2}}+36\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^4b^{\frac{9}{2}}-9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{2}a^5b^{\frac{9}{2}}+a^6b^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(9/2) + 315*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a*b^(9/2) + 441*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(9/2
) + 126*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(9/2) + 36*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a^4*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(9/2) +
 a^6*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

Mupad [B]
time = 5.71, size = 91, normalized size = 1.34

$$\frac{4\,b^3\,\sqrt{b\,x^2+a}}{315\,a^2\,x^3}-\frac{10\,b\,\sqrt{b\,x^2+a}}{63\,x^7}-\frac{b^2\,\sqrt{b\,x^2+a}}{105\,a\,x^5}-\frac{a\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{8\,b^4\,\sqrt{b\,x^2+a}}{315\,a^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^10,x)
```

```
[Out] (4*b^3*(a + b*x^2)^(1/2))/(315*a^2*x^3) - (10*b*(a + b*x^2)^(1/2))/(63*x^7)
 - (b^2*(a + b*x^2)^(1/2))/(105*a*x^5) - (a*(a + b*x^2)^(1/2))/(9*x^9) - (8
*b^4*(a + b*x^2)^(1/2))/(315*a^3*x)
```

3.386 $\int \frac{(a+bx^2)^{3/2}}{x^{12}} \, dx$

Optimal. Leaf size=92

$$-\frac{(a+bx^2)^{5/2}}{11ax^{11}}+\frac{2b(a+bx^2)^{5/2}}{33a^2x^9}-\frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7}+\frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}$$

```
[Out] -1/11*(b*x^2+a)^(5/2)/a/x^11+2/33*b*(b*x^2+a)^(5/2)/a^2/x^9-8/231*b^2*(b*x^
2+a)^(5/2)/a^3/x^7+16/1155*b^3*(b*x^2+a)^(5/2)/a^4/x^5
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$\frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}-\frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7}+\frac{2b(a+bx^2)^{5/2}}{33a^2x^9}-\frac{(a+bx^2)^{5/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^12,x]
```

```
[Out] -1/11*(a + b*x^2)^(5/2)/(a*x^11) + (2*b*(a + b*x^2)^(5/2))/(33*a^2*x^9) - (
8*b^2*(a + b*x^2)^(5/2))/(231*a^3*x^7) + (16*b^3*(a + b*x^2)^(5/2))/(1155*a
^4*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx = -\frac{(a+bx^2)^{5/2}}{11ax^{11}} - \frac{(6b)\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx}{11a}$$
$$= -\frac{(a+bx^2)^{5/2}}{11ax^{11}} + \frac{2b(a+bx^2)^{5/2}}{33a^2x^9} + \frac{(8b^2)\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx}{33a^2}$$
$$= -\frac{(a+bx^2)^{5/2}}{11ax^{11}} + \frac{2b(a+bx^2)^{5/2}}{33a^2x^9} - \frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7} - \frac{(16b^3)\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx}{231a^3}$$
$$= -\frac{(a+bx^2)^{5/2}}{11ax^{11}} + \frac{2b(a+bx^2)^{5/2}}{33a^2x^9} - \frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7} + \frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}$$

Mathematica [A]
time = 0.09, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{5/2}\,(-105a^3+70a^2bx^2-40ab^2x^4+16b^3x^6)}{1155a^4x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^12,x]
[Out] ((a + b*x^2)^(5/2)*(-105*a^3 + 70*a^2*b*x^2 - 40*a*b^2*x^4 + 16*b^3*x^6))/(
1155*a^4*x^11)
```

Maple [A]
time = 0.10, size = 85, normalized size = 0.92

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{5}{2}}\,(-16b^3x^6+40a\,b^2x^4-70a^2b\,x^2+105a^3)}{1155x^{11}a^4}$	50
trager	$-\frac{(-16b^5x^{10}+8a\,b^4x^8-6a^2b^3x^6+5a^3b^2x^4+140a^4b\,x^2+105a^5)\sqrt{b\,x^2+a}}{1155x^{11}a^4}$	72
risch	$-\frac{(-16b^5x^{10}+8a\,b^4x^8-6a^2b^3x^6+5a^3b^2x^4+140a^4b\,x^2+105a^5)\sqrt{b\,x^2+a}}{1155x^{11}a^4}$	72
default	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{11a\,x^{11}}-\frac{6b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{9a\,x^9}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{7a\,x^7}+\frac{2b(b\,x^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)}{11a}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

[Out] -1/11*(b*x^2+a)^(5/2)/a/x^11-6/11*b/a*(-1/9*(b*x^2+a)^(5/2)/a/x^9-4/9*b/a*(-1/7*(b*x^2+a)^(5/2)/a/x^7+2/35*b*(b*x^2+a)^(5/2)/a^2/x^5))

Maxima [A]
time = 0.30, size = 76, normalized size = 0.83

$$\frac{16\,(bx^2+a)^{\frac{5}{2}}b^3}{1155\,a^4x^5}-\frac{8\,(bx^2+a)^{\frac{5}{2}}b^2}{231\,a^3x^7}+\frac{2\,(bx^2+a)^{\frac{5}{2}}b}{33\,a^2x^9}-\frac{(bx^2+a)^{\frac{5}{2}}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="maxima")

[Out] 16/1155*(b*x^2 + a)^(5/2)*b^3/(a^4*x^5) - 8/231*(b*x^2 + a)^(5/2)*b^2/(a^3*x^7) + 2/33*(b*x^2 + a)^(5/2)*b/(a^2*x^9) - 1/11*(b*x^2 + a)^(5/2)/(a*x^11)

Fricas [A]
time = 1.37, size = 71, normalized size = 0.77

$$\frac{(16\,b^5x^{10}-8\,ab^4x^8+6\,a^2b^3x^6-5\,a^3b^2x^4-140\,a^4bx^2-105\,a^5)\sqrt{bx^2+a}}{1155\,a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="fricas")

[Out] 1/1155*(16*b^5*x^10 - 8*a*b^4*x^8 + 6*a^2*b^3*x^6 - 5*a^3*b^2*x^4 - 140*a^4*b*x^2 - 105*a^5)*sqrt(b*x^2 + a)/(a^4*x^11)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(85) = 170.
time = 1.05, size = 648, normalized size = 7.04

$$-\frac{105a^8b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{455a^7b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{740a^6b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{534a^5b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{145a^4b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}+\frac{5a^3b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}+\frac{30a^2b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}+\frac{40ab^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}+\frac{16b^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(3/2)/x**12,x)

[Out] -105*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 455*a**7*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 740*a**6*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 534*a**5*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 145*a**4*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) + 5*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/

```
(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 115
5*a**4*b**12*x**16) + 30*a**2*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(1155*a*
*7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b
**12*x**16) + 40*a*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**
10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16)
 + 16*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**
6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(76) = 152$.
time = 1.01, size = 220, normalized size = 2.39

$$\frac{32\left(1155\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{14}b^{\frac{11}{2}}+2079\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{11}{2}}+2541\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{11}{2}}+825\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}a^3b^{\frac{11}{2}}+165\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{6}a^4b^{\frac{11}{2}}-55\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^5b^{\frac{11}{2}}+11\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{2}a^6b^{\frac{11}{2}}-a^7b^{\frac{11}{2}}\right)}{1155\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] 32/1155*(1155*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(11/2) + 2079*(sqrt(b)*x -
 sqrt(b*x^2 + a))^12*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2
*b^(11/2) + 825*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(11/2) + 165*(sqrt(b)
*x - sqrt(b*x^2 + a))^6*a^4*b^(11/2) - 55*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a
^5*b^(11/2) + 11*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(11/2) - a^7*b^(11/2
))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^11
```

Mupad [B]
time = 5.78, size = 111, normalized size = 1.21

$$\frac{2\,b^3\,\sqrt{b\,x^2+a}}{385\,a^2\,x^5}-\frac{4\,b\,\sqrt{b\,x^2+a}}{33\,x^9}-\frac{b^2\,\sqrt{b\,x^2+a}}{231\,a\,x^7}-\frac{a\,\sqrt{b\,x^2+a}}{11\,x^{11}}-\frac{8\,b^4\,\sqrt{b\,x^2+a}}{1155\,a^3\,x^3}+\frac{16\,b^5\,\sqrt{b\,x^2+a}}{1155\,a^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^12,x)
```

```
[Out] (2*b^3*(a + b*x^2)^(1/2))/(385*a^2*x^5) - (4*b*(a + b*x^2)^(1/2))/(33*x^9)
- (b^2*(a + b*x^2)^(1/2))/(231*a*x^7) - (a*(a + b*x^2)^(1/2))/(11*x^11) - (
8*b^4*(a + b*x^2)^(1/2))/(1155*a^3*x^3) + (16*b^5*(a + b*x^2)^(1/2))/(1155*
a^4*x)
```

3.387 $\int x^7 (a + bx^2)^{5/2} \, dx$

Optimal. Leaf size=80

$$-\frac{a^3(a+bx^2)^{7/2}}{7b^4}+\frac{a^2(a+bx^2)^{9/2}}{3b^4}-\frac{3a(a+bx^2)^{11/2}}{11b^4}+\frac{(a+bx^2)^{13/2}}{13b^4}$$

```
[Out] -1/7*a^3*(b*x^2+a)^(7/2)/b^4+1/3*a^2*(b*x^2+a)^(9/2)/b^4-3/11*a*(b*x^2+a)^(
11/2)/b^4+1/13*(b*x^2+a)^(13/2)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$-\frac{a^3(a+bx^2)^{7/2}}{7b^4}+\frac{a^2(a+bx^2)^{9/2}}{3b^4}+\frac{(a+bx^2)^{13/2}}{13b^4}-\frac{3a(a+bx^2)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(5/2),x]
```

```
[Out] -1/7*(a^3*(a + b*x^2)^(7/2))/b^4 + (a^2*(a + b*x^2)^(9/2))/(3*b^4) - (3*a*(
a + b*x^2)^(11/2))/(11*b^4) + (a + b*x^2)^(13/2)/(13*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^{5/2} \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{5/2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right) dx,\right.\\
&= -\frac{a^3(a+bx^2)^{7/2}}{7b^4}+\frac{a^2(a+bx^2)^{9/2}}{3b^4}-\frac{3a(a+bx^2)^{11/2}}{11b^4}+\frac{(a+bx^2)^{13/2}}{13b^4}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.62

$$\frac{(a+bx^2)^{7/2}(-16a^3+56a^2bx^2-126ab^2x^4+231b^3x^6)}{3003b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(a + b*x^2)^(5/2),x]

[Out] ((a + b*x^2)^(7/2)*(-16*a^3 + 56*a^2*b*x^2 - 126*a*b^2*x^4 + 231*b^3*x^6))/(3003*b^4)

Maple [A]
time = 0.05, size = 82, normalized size = 1.02

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}(-231b^3x^6+126a\,b^2x^4-56a^2b\,x^2+16a^3)}{3003b^4}$	47
trager	$-\frac{(-231b^6x^{12}-567a\,b^5x^{10}-371a^2b^4x^8-5a^3x^6b^3+6a^4b^2x^4-8a^5b\,x^2+16a^6)\sqrt{b\,x^2+a}}{3003b^4}$	80
risch	$-\frac{(-231b^6x^{12}-567a\,b^5x^{10}-371a^2b^4x^8-5a^3x^6b^3+6a^4b^2x^4-8a^5b\,x^2+16a^6)\sqrt{b\,x^2+a}}{3003b^4}$	80
default	$\frac{x^6(b\,x^2+a)^{\frac{7}{2}}}{13b}-\frac{6a\left(\frac{x^4(b\,x^2+a)^{\frac{7}{2}}}{11b}-\frac{4a\left(\frac{x^2(b\,x^2+a)^{\frac{7}{2}}}{9b}-\frac{2a(b\,x^2+a)^{\frac{7}{2}}}{63b^2}\right)}{11b}\right)}{13b}$	82

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/13*x^6*(b*x^2+a)^(7/2)/b-6/13*a/b*(1/11*x^4*(b*x^2+a)^(7/2)/b-4/11*a/b*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a*(b*x^2+a)^(7/2)/b^2))

Maxima [A]
time = 0.29, size = 73, normalized size = 0.91

$$\frac{(bx^2+a)^{\frac{7}{2}}x^6}{13\,b}-\frac{6\,(bx^2+a)^{\frac{7}{2}}ax^4}{143\,b^2}+\frac{8\,(bx^2+a)^{\frac{7}{2}}a^2x^2}{429\,b^3}-\frac{16\,(bx^2+a)^{\frac{7}{2}}a^3}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="maxima")

[Out] 1/13*(b*x^2 + a)^(7/2)*x^6/b - 6/143*(b*x^2 + a)^(7/2)*a*x^4/b^2 + 8/429*(b*x^2 + a)^(7/2)*a^2*x^2/b^3 - 16/3003*(b*x^2 + a)^(7/2)*a^3/b^4

Fricas [A]
time = 1.31, size = 79, normalized size = 0.99

$$\frac{\left(231\, b^{6} x^{12} + 567\, a b^{5} x^{10} + 371\, a^{2} b^{4} x^{8} + 5\, a^{3} b^{3} x^{6} - 6\, a^{4} b^{2} x^{4} + 8\, a^{5} b x^{2} - 16\, a^{6}\right) \sqrt{b x^{2} + a}}{3003\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(231*b^6*x^12 + 567*a*b^5*x^10 + 371*a^2*b^4*x^8 + 5*a^3*b^3*x^6 - 6
*a^4*b^2*x^4 + 8*a^5*b*x^2 - 16*a^6)*sqrt(b*x^2 + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(70) = 140.
time = 0.63, size = 158, normalized size = 1.98

$$\begin{cases} - \frac{16 a^{6} \sqrt{a + b x^{2}}}{3003 b^{4}} + \frac{8 a^{5} x^{2} \sqrt{a + b x^{2}}}{3003 b^{3}} - \frac{2 a^{4} x^{4} \sqrt{a + b x^{2}}}{1001 b^{2}} + \frac{5 a^{3} x^{6} \sqrt{a + b x^{2}}}{3003 b} + \frac{53 a^{2} x^{8} \sqrt{a + b x^{2}}}{429} + \frac{27 a b x^{10} \sqrt{a + b x^{2}}}{143} + \frac{b^{2} x^{12} \sqrt{a + b x^{2}}}{13} & \text{for}\: b \neq 0 \\ \frac{a^{\frac{5}{2}} x^{8}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-16*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*a**5*x**2*sqrt(a + b*x
**2)/(3003*b**3) - 2*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5*a**3*x**6*s
qrt(a + b*x**2)/(3003*b) + 53*a**2*x**8*sqrt(a + b*x**2)/429 + 27*a*b*x**10
*sqrt(a + b*x**2)/143 + b**2*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(5/2
)*x**8/8, True))
```

Giac [A]
time = 1.17, size = 57, normalized size = 0.71

$$\frac{231\, {\left(b x^{2} + a\right)}^{\frac{13}{2}} - 819\, {\left(b x^{2} + a\right)}^{\frac{11}{2}} a + 1001\, {\left(b x^{2} + a\right)}^{\frac{9}{2}} a^{2} - 429\, {\left(b x^{2} + a\right)}^{\frac{7}{2}} a^{3}}{3003\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3003*(231*(b*x^2 + a)^(13/2) - 819*(b*x^2 + a)^(11/2)*a + 1001*(b*x^2 + a
)^(9/2)*a^2 - 429*(b*x^2 + a)^(7/2)*a^3)/b^4
```

Mupad [B]
time = 4.86, size = 75, normalized size = 0.94

$$\sqrt{b\,x^2+a}\,\left(\frac{53\,a^2\,x^8}{429}-\frac{16\,a^6}{3003\,b^4}+\frac{b^2\,x^{12}}{13}+\frac{5\,a^3\,x^6}{3003\,b}-\frac{2\,a^4\,x^4}{1001\,b^2}+\frac{8\,a^5\,x^2}{3003\,b^3}+\frac{27\,a\,b\,x^{10}}{143}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((53*a^2*x^8)/429 - (16*a^6)/(3003*b^4) + (b^2*x^12)/13 +
 (5*a^3*x^6)/(3003*b) - (2*a^4*x^4)/(1001*b^2) + (8*a^5*x^2)/(3003*b^3) + (
27*a*b*x^10)/143)
```

3.388 $\int x^5(a+bx^2)^{5/2}\,dx$

Optimal. Leaf size=59

$$\frac{a^2(a+bx^2)^{7/2}}{7b^3}-\frac{2a(a+bx^2)^{9/2}}{9b^3}+\frac{(a+bx^2)^{11/2}}{11b^3}$$

```
[Out] 1/7*a^2*(b*x^2+a)^(7/2)/b^3-2/9*a*(b*x^2+a)^(9/2)/b^3+1/11*(b*x^2+a)^(11/2)
/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{a^2(a+bx^2)^{7/2}}{7b^3}+\frac{(a+bx^2)^{11/2}}{11b^3}-\frac{2a(a+bx^2)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(7/2))/(7*b^3) - (2*a*(a + b*x^2)^(9/2))/(9*b^3) + (a + b*
x^2)^(11/2)/(11*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{5/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{5/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right)\,dx,x,x^2\right)\\ &= \frac{a^2(a+bx^2)^{7/2}}{7b^3}-\frac{2a(a+bx^2)^{9/2}}{9b^3}+\frac{(a+bx^2)^{11/2}}{11b^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{(a+bx^2)^{7/2}(8a^2-28abx^2+63b^2x^4)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(5/2),x]
[Out] ((a + b*x^2)^(7/2)*(8*a^2 - 28*a*b*x^2 + 63*b^2*x^4))/(693*b^3)
```

Maple [A]
time = 0.05, size = 58, normalized size = 0.98

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{7}{2}}(63b^2x^4-28ab\,x^2+8a^2)}{693b^3}$	36
default	$\frac{x^4(b\,x^2+a)^{\frac{7}{2}}}{11b}-\frac{4a\left(\frac{x^2(b\,x^2+a)^{\frac{7}{2}}}{9b}-\frac{2a(b\,x^2+a)^{\frac{7}{2}}}{63b^2}\right)}{11b}$	58
trager	$\frac{(63b^5x^{10}+161a\,b^4x^8+113a^2b^3x^6+3a^3b^2x^4-4a^4b\,x^2+8a^5)\sqrt{b\,x^2+a}}{693b^3}$	69
risch	$\frac{(63b^5x^{10}+161a\,b^4x^8+113a^2b^3x^6+3a^3b^2x^4-4a^4b\,x^2+8a^5)\sqrt{b\,x^2+a}}{693b^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/11*x^4*(b*x^2+a)^(7/2)/b-4/11*a/b*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a*(b*x^
2+a)^(7/2)/b^2)
```

Maxima [A]
time = 0.36, size = 53, normalized size = 0.90

$$\frac{(bx^2+a)^{\frac{7}{2}}x^4}{11\,b}-\frac{4\,(bx^2+a)^{\frac{7}{2}}ax^2}{99\,b^2}+\frac{8\,(bx^2+a)^{\frac{7}{2}}a^2}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] 1/11*(b*x^2 + a)^(7/2)*x^4/b - 4/99*(b*x^2 + a)^(7/2)*a*x^2/b^2 + 8/693*(b*
x^2 + a)^(7/2)*a^2/b^3
```

Fricas [A]
time = 1.50, size = 68, normalized size = 1.15

$$\frac{(63\,b^5x^{10}+161\,ab^4x^8+113\,a^2b^3x^6+3\,a^3b^2x^4-4\,a^4bx^2+8\,a^5)\sqrt{bx^2+a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/693*(63*b^5*x^10 + 161*a*b^4*x^8 + 113*a^2*b^3*x^6 + 3*a^3*b^2*x^4 - 4*a^
4*b*x^2 + 8*a^5)*sqrt(b*x^2 + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(51) = 102$.
time = 0.47, size = 133, normalized size = 2.25

$$\begin{cases} \frac{8a^5\sqrt{a+bx^2}}{693b^3} - \frac{4a^4x^2\sqrt{a+bx^2}}{693b^2} + \frac{a^3x^4\sqrt{a+bx^2}}{231b} + \frac{113a^2x^6\sqrt{a+bx^2}}{693} + \frac{23abx^8\sqrt{a+bx^2}}{99} + \frac{b^2x^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*a**4*x**2*sqrt(a + b*x**2
)/(693*b**2) + a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*a**2*x**6*sqrt(a +
b*x**2)/693 + 23*a*b*x**8*sqrt(a + b*x**2)/99 + b**2*x**10*sqrt(a + b*x**2)
/11, Ne(b, 0)), (a**(5/2)*x**6/6, True))
```

Giac [A]
time = 1.32, size = 43, normalized size = 0.73

$$\frac{63\,(bx^2+a)^{\frac{11}{2}} - 154\,(bx^2+a)^{\frac{9}{2}}a + 99\,(bx^2+a)^{\frac{7}{2}}a^2}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/693*(63*(b*x^2 + a)^(11/2) - 154*(b*x^2 + a)^(9/2)*a + 99*(b*x^2 + a)^(7/
2)*a^2)/b^3
```

Mupad [B]
time = 4.75, size = 64, normalized size = 1.08

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a^5}{693\,b^3}+\frac{113\,a^2\,x^6}{693}+\frac{b^2\,x^{10}}{11}+\frac{a^3\,x^4}{231\,b}-\frac{4\,a^4\,x^2}{693\,b^2}+\frac{23\,a\,b\,x^8}{99}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^5)/(693*b^3) + (113*a^2*x^6)/693 + (b^2*x^10)/11 +
(a^3*x^4)/(231*b) - (4*a^4*x^2)/(693*b^2) + (23*a*b*x^8)/99)
```

3.389 $\int x^3(a+bx^2)^{5/2}\,dx$

Optimal. Leaf size=38

$$-\frac{a(a+bx^2)^{7/2}}{7b^2}+\frac{(a+bx^2)^{9/2}}{9b^2}$$

```
[Out] -1/7*a*(b*x^2+a)^(7/2)/b^2+1/9*(b*x^2+a)^(9/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{(a+bx^2)^{9/2}}{9b^2}-\frac{a(a+bx^2)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(5/2),x]
```

```
[Out] -1/7*(a*(a + b*x^2)^(7/2))/b^2 + (a + b*x^2)^(9/2)/(9*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{5/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{5/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right)dx,x,x^2\right)\\ &= -\frac{a(a+bx^2)^{7/2}}{7b^2}+\frac{(a+bx^2)^{9/2}}{9b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.74

$$\frac{(a+bx^2)^{7/2}\,(-2a+7bx^2)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(5/2),x]
[Out] ((a + b*x^2)^(7/2)*(-2*a + 7*b*x^2))/(63*b^2)
```

Maple [A]
time = 0.04, size = 34, normalized size = 0.89

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}\,(-7b\,x^2+2a)}{63b^2}$	25
default	$\frac{x^2\,(b\,x^2+a)^{\frac{7}{2}}}{9b}-\frac{2a\,(b\,x^2+a)^{\frac{7}{2}}}{63b^2}$	34
trager	$-\frac{(-7b^4x^8-19a\,b^3x^6-15a^2b^2x^4-a^3b\,x^2+2a^4)\sqrt{b\,x^2+a}}{63b^2}$	58
risch	$-\frac{(-7b^4x^8-19a\,b^3x^6-15a^2b^2x^4-a^3b\,x^2+2a^4)\sqrt{b\,x^2+a}}{63b^2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a*(b*x^2+a)^(7/2)/b^2
```

Maxima [A]
time = 0.30, size = 33, normalized size = 0.87

$$\frac{(bx^2+a)^{\frac{7}{2}}x^2}{9\,b}-\frac{2\,(bx^2+a)^{\frac{7}{2}}a}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] 1/9*(b*x^2 + a)^(7/2)*x^2/b - 2/63*(b*x^2 + a)^(7/2)*a/b^2
```

Fricas [A]
time = 1.61, size = 56, normalized size = 1.47

$$\frac{(7\,b^4x^8+19\,ab^3x^6+15\,a^2b^2x^4+a^3bx^2-2\,a^4)\sqrt{bx^2+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^4*x^8 + 19*a*b^3*x^6 + 15*a^2*b^2*x^4 + a^3*b*x^2 - 2*a^4)*sqrt(b
*x^2 + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.
time = 0.35, size = 109, normalized size = 2.87

$$\begin{cases} -\frac{2a^4\sqrt{a+bx^2}}{63b^2} + \frac{a^3x^2\sqrt{a+bx^2}}{63b} + \frac{5a^2x^4\sqrt{a+bx^2}}{21} + \frac{19abx^6\sqrt{a+bx^2}}{63} + \frac{b^2x^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*a**4*sqrt(a + b*x**2)/(63*b**2) + a**3*x**2*sqrt(a + b*x**2)/
(63*b) + 5*a**2*x**4*sqrt(a + b*x**2)/21 + 19*a*b*x**6*sqrt(a + b*x**2)/63
+ b**2*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

Giac [A]
time = 0.98, size = 29, normalized size = 0.76

$$\frac{7(bx^2+a)^{\frac{9}{2}} - 9(bx^2+a)^{\frac{7}{2}}a}{63b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/63*(7*(b*x^2 + a)^(9/2) - 9*(b*x^2 + a)^(7/2)*a)/b^2
```

Mupad [B]
time = 4.74, size = 53, normalized size = 1.39

$$\sqrt{b\,x^2+a}\,\left(\frac{5\,a^2\,x^4}{21}-\frac{2\,a^4}{63\,b^2}+\frac{b^2\,x^8}{9}+\frac{a^3\,x^2}{63\,b}+\frac{19\,a\,b\,x^6}{63}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((5*a^2*x^4)/21 - (2*a^4)/(63*b^2) + (b^2*x^8)/9 + (a^3*x
^2)/(63*b) + (19*a*b*x^6)/63)
```

3.390 $\int x(a+bx^2)^{5/2}\,dx$

Optimal. Leaf size=18

$$\frac{(a+bx^2)^{7/2}}{7b}$$

```
[Out] 1/7*(b*x^2+a)^(7/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{(a+bx^2)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(5/2),x]
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{5/2}\,dx = \frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{(a+bx^2)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(5/2),x]
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

Maple [A]
time = 0.04, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{7}{2}}}{7b}$	15
derivativedivides	$\frac{(b\,x^2+a)^{\frac{7}{2}}}{7b}$	15
default	$\frac{(b\,x^2+a)^{\frac{7}{2}}}{7b}$	15
trager	$\frac{(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)\sqrt{b\,x^2+a}}{7b}$	44
risch	$\frac{(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)\sqrt{b\,x^2+a}}{7b}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(b*x^2+a)^(7/2)/b
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.78

$$\frac{(bx^2+a)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(7/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.
time = 1.19, size = 43, normalized size = 2.39

$$\frac{(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{bx^2+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(12) = 24$.

time = 0.22, size = 85, normalized size = 4.72

$$\begin{cases} \frac{a^3\sqrt{a+bx^2}}{7b} + \frac{3a^2x^2\sqrt{a+bx^2}}{7} + \frac{3abx^4\sqrt{a+bx^2}}{7} + \frac{b^2x^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((a**3*sqrt(a + b*x**2)/(7*b) + 3*a**2*x**2*sqrt(a + b*x**2)/7 + 3
*a*b*x**4*sqrt(a + b*x**2)/7 + b**2*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (a*
*(5/2)*x**2/2, True))
```

Giac [A]
time = 1.48, size = 14, normalized size = 0.78

$$\frac{(bx^2+a)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7*(b*x^2 + a)^(7/2)/b
```

Mupad [B]
time = 4.60, size = 14, normalized size = 0.78

$$\frac{(b\,x^2+a)^{7/2}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

3.391 $\int \frac{(a+bx^2)^{5/2}}{x} dx$

Optimal. Leaf size=72

$$a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}-a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/3*a*(b*x^2+a)^(3/2)+1/5*(b*x^2+a)^(5/2)-a^(5/2)*arctanh((b*x^2+a)^(1/2)/a
^(1/2))+a^2*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 52, 65, 214}

$$a^{5/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x,x]
```

```
[Out] a^2*Sqrt[a + b*x^2] + (a*(a + b*x^2)^(3/2))/3 + (a + b*x^2)^(5/2)/5 - a^(5/
2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx,x,x^2\right)\\ &= \frac{1}{5}(a+bx^2)^{5/2}+\frac{1}{2}a\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)\\ &= \frac{1}{3}a(a+bx^2)^{3/2}+\frac{1}{5}(a+bx^2)^{5/2}+\frac{1}{2}a^2\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\ &= a^2\sqrt{a+bx^2}+\frac{1}{3}a(a+bx^2)^{3/2}+\frac{1}{5}(a+bx^2)^{5/2}+\frac{1}{2}a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= a^2\sqrt{a+bx^2}+\frac{1}{3}a(a+bx^2)^{3/2}+\frac{1}{5}(a+bx^2)^{5/2}+\frac{a^3\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\ &= a^2\sqrt{a+bx^2}+\frac{1}{3}a(a+bx^2)^{3/2}+\frac{1}{5}(a+bx^2)^{5/2}-a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.86

$$\frac{1}{15}\sqrt{a+bx^2}\,(23a^2+11abx^2+3b^2x^4)-a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(23*a^2 + 11*a*b*x^2 + 3*b^2*x^4))/15 - a^(5/2)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]]
```

Maple [A]
time = 0.04, size = 67, normalized size = 0.93

method	result	size

default	$\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2
*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.75

$$-a^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a+\sqrt{bx^2+a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2) + 1/3*(b*x^2
 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

Fricas [A]
time = 1.22, size = 126, normalized size = 1.75

$$\left[\frac{1}{2}\,a^{\frac{5}{2}}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}\,+2\,a}{x^2}\right)+\frac{1}{15}\left(3\,b^2x^4+11\,abx^2+23\,a^2\right)\sqrt{bx^2+a}\,,\,\sqrt{-a}\,a^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\frac{1}{15}\left(3\,b^2x^4+11\,abx^2+23\,a^2\right)\sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(5/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/15*(3*
b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2 + a), sqrt(-a)*a^2*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) + 1/15*(3*b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2 + a)
]
```

Sympy [A]
time = 2.26, size = 105, normalized size = 1.46

$$\frac{23a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}}{15}+\frac{a^{\frac{5}{2}}\log\left(\frac{bx^2}{a}\right)}{2}-a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)+\frac{11a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}{15}+\frac{\sqrt{a}\,b^2x^4\sqrt{1+\frac{bx^2}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x,x)
```

```
[Out] 23*a**(5/2)*sqrt(1 + b*x**2/a)/15 + a**(5/2)*log(b*x**2/a)/2 - a**(5/2)*log
(sqrt(1 + b*x**2/a) + 1) + 11*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)/15 + sqrt(
a)*b**2*x**4*sqrt(1 + b*x**2/a)/5
```

Giac [A]
time = 1.03, size = 62, normalized size = 0.86

$$\frac{a^3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a + \sqrt{bx^2+a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/5*(b*x^2 + a)^(5/2) + 1/3
*(b*x^2 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

Mupad [B]
time = 4.68, size = 59, normalized size = 0.82

$$\frac{a\,{\left(b\,x^2+a\right)}^{3/2}}{3}+\frac{{\left(b\,x^2+a\right)}^{5/2}}{5}+a^2\,\sqrt{b\,x^2+a}+a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x,x)
```

```
[Out] a^(5/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (a*(a + b*x^2)^(3/2))/3 +
 (a + b*x^2)^(5/2)/5 + a^2*(a + b*x^2)^(1/2)
```

3.392 $\int \frac{(a+bx^2)^{5/2}}{x^3} dx$

Optimal. Leaf size=80

$$\frac{5}{2}ab\sqrt{a+bx^2} + \frac{5}{6}b(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{2x^2} - \frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 5/6*b*(b*x^2+a)^(3/2)-1/2*(b*x^2+a)^(5/2)/x^2-5/2*a^(3/2)*b*arctanh((b*x^2+
a)^(1/2)/a^(1/2))+5/2*a*b*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 52, 65, 214}

$$-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{(a+bx^2)^{5/2}}{2x^2} + \frac{5}{6}b(a+bx^2)^{3/2} + \frac{5}{2}ab\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (5*a*b*Sqrt[a + b*x^2])/2 + (5*b*(a + b*x^2)^(3/2))/6 - (a + b*x^2)^(5/2)/(
2*x^2) - (5*a^(3/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{(a+bx^2)^{5/2}}{2x^2} + \frac{1}{4}(5b)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right) \\
&= \frac{5}{6}b(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{2x^2} + \frac{1}{4}(5ab)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right) \\
&= \frac{5}{2}ab\sqrt{a+bx^2} + \frac{5}{6}b(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{2x^2} + \frac{1}{4}(5a^2b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x\right. \\
&= \frac{5}{2}ab\sqrt{a+bx^2} + \frac{5}{6}b(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{2x^2} + \frac{1}{2}(5a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right. \\
&= \frac{5}{2}ab\sqrt{a+bx^2} + \frac{5}{6}b(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{2x^2} - \frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\,(-3a^2+14abx^2+2b^2x^4)}{6x^2} - \frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2 + 14*a*b*x^2 + 2*b^2*x^4))/(6*x^2) - (5*a^(3/2)*b*
ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Maple [A]
time = 0.06, size = 91, normalized size = 1.14

method	result	size
risch	$-\frac{a^{2} \sqrt{b\,x^{2}+a}}{2 x^{2}}+\frac{b^{2} x^{2} \sqrt{b\,x^{2}+a}}{3}+\frac{7 a b \sqrt{b\,x^{2}+a}}{3}-\frac{5 b\,a^{\frac{3}{2}} \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b\,x^{2}+a}}{x}\right)}{2}$	78
default	$-\frac{\left(b\,x^{2}+a \right)^{\frac{7}{2}}}{2 a\,x^{2}}+\frac{5 b \left(\frac{\left(b\,x^{2}+a \right)^{\frac{5}{2}}}{5}+a \left(\frac{\left(b\,x^{2}+a \right)^{\frac{3}{2}}}{3}+a \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\, \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b\,x^{2}+a}}{x}\right)\right)\right)\right)}{2 a}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3
/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.95

$$-\frac{5}{2}\,a^{\frac{3}{2}}b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{5}{6}\,\left(bx^{2}+a\right)^{\frac{3}{2}}b+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}b}{2\,a}+\frac{5}{2}\,\sqrt{bx^{2}+a}\,ab-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -5/2*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 5/6*(b*x^2 + a)^(3/2)*b + 1/
2*(b*x^2 + a)^(5/2)*b/a + 5/2*sqrt(b*x^2 + a)*a*b - 1/2*(b*x^2 + a)^(7/2)/(
a*x^2)
```

Fricas [A]
time = 1.40, size = 142, normalized size = 1.78

$$\left[\frac{15\,a^{\frac{3}{2}}bx^{2}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(2\,b^{2}x^{4}+14\,abx^{2}-3\,a^{2}\right)\sqrt{bx^{2}+a}}{12\,x^{2}},\ \frac{15\,\sqrt{-a}\,abx^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(2\,b^{2}x^{4}+14\,abx^{2}-3\,a^{2}\right)\sqrt{bx^{2}+a}}{6\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(15*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(2*b^2*x^4 + 14*a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a))/x^2, 1/6*(15*sqrt(-a
)*a*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*b^2*x^4 + 14*a*b*x^2 - 3*a^
2)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A]
time = 2.10, size = 112, normalized size = 1.40

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}}{2x^2}+\frac{7a^{\frac{3}{2}}b\sqrt{1+\frac{bx^2}{a}}}{3}+\frac{5a^{\frac{3}{2}}b\log\left(\frac{bx^2}{a}\right)}{4}-\frac{5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2}+\frac{\sqrt{a}\,b^2x^2\sqrt{1+\frac{bx^2}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 7*a**(3/2)*b*sqrt(1 + b*x**2/a)/3 +
 5*a**(3/2)*b*log(b*x**2/a)/4 - 5*a**(3/2)*b*log(sqrt(1 + b*x**2/a) + 1)/2
+ sqrt(a)*b**2*x**2*sqrt(1 + b*x**2/a)/3
```

Giac [A]
time = 0.85, size = 82, normalized size = 1.02

$$\frac{\frac{15\,a^2b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,(bx^2+a)^{\frac{3}{2}}b^2+12\,\sqrt{bx^2+a}\,ab^2-\frac{3\,\sqrt{bx^2+a}\,a^2b}{x^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^2*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*(b*x^2 + a)^(
3/2)*b^2 + 12*sqrt(b*x^2 + a)*a*b^2 - 3*sqrt(b*x^2 + a)*a^2*b/x^2)/b
```

Mupad [B]
time = 4.95, size = 66, normalized size = 0.82

$$\frac{b\,{\left(b\,x^2+a\right)}^{3/2}}{3}-\frac{a^2\,\sqrt{b\,x^2+a}}{2\,x^2}+2\,a\,b\,\sqrt{b\,x^2+a}+\frac{a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^3,x)
```

```
[Out] (b*(a + b*x^2)^(3/2))/3 - (a^2*(a + b*x^2)^(1/2))/(2*x^2) + (a^(3/2)*b*atan
(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2 + 2*a*b*(a + b*x^2)^(1/2)
```

3.393 $\int \frac{(a+bx^2)^{5/2}}{x^5} dx$

Optimal. Leaf size=86

$$\frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b(a+bx^2)^{3/2}}{8x^2} - \frac{(a+bx^2)^{5/2}}{4x^4} - \frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -5/8*b*(b*x^2+a)^(3/2)/x^2-1/4*(b*x^2+a)^(5/2)/x^4-15/8*b^2*arctanh((b*x^2+
a)^(1/2)/a^(1/2))*a^(1/2)+15/8*b^2*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 52, 65, 214}

$$\frac{15}{8}b^2\sqrt{a+bx^2} - \frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{5b(a+bx^2)^{3/2}}{8x^2} - \frac{(a+bx^2)^{5/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^5,x]
```

```
[Out] (15*b^2*Sqrt[a + b*x^2])/8 - (5*b*(a + b*x^2)^(3/2))/(8*x^2) - (a + b*x^2)^
(5/2)/(4*x^4) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^{5/2}}{4x^4} + \frac{1}{8}(5b)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{5b(a+bx^2)^{3/2}}{8x^2} - \frac{(a+bx^2)^{5/2}}{4x^4} + \frac{1}{16}(15b^2)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b(a+bx^2)^{3/2}}{8x^2} - \frac{(a+bx^2)^{5/2}}{4x^4} + \frac{1}{16}(15ab^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b(a+bx^2)^{3/2}}{8x^2} - \frac{(a+bx^2)^{5/2}}{4x^4} + \frac{1}{8}(15ab)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.\\
&= \frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b(a+bx^2)^{3/2}}{8x^2} - \frac{(a+bx^2)^{5/2}}{4x^4} - \frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 70, normalized size = 0.81

$$
\frac{\sqrt{a+bx^2}\,(-2a^2-9abx^2+8b^2x^4)}{8x^4} - \frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a^2 - 9*a*b*x^2 + 8*b^2*x^4))/(8*x^4) - (15*Sqrt[a]*b^
2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Maple [A]
time = 0.06, size = 115, normalized size = 1.34

method	result
risch	$-\frac{a\sqrt{b\,x^{2}+a}\,(9b\,x^{2}+2a)}{8x^{4}}-\frac{15\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)b^{2}}{8}+b^{2}\sqrt{b\,x^{2}+a}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{4a\,x^{4}}+\frac{3b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2a\,x^{2}}+\frac{5b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)\right)}{2a}\right)}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5
*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+
2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

Maxima [A]
time = 0.27, size = 104, normalized size = 1.21

$$-\frac{15}{8}\sqrt{a}\,b^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{15}{8}\sqrt{bx^{2}+a}\,b^{2}+\frac{3\,(bx^{2}+a)^{\frac{5}{2}}b^{2}}{8\,a^{2}}+\frac{5\,(bx^{2}+a)^{\frac{3}{2}}b^{2}}{8\,a}-\frac{3\,(bx^{2}+a)^{\frac{7}{2}}b}{8\,a^{2}x^{2}}-\frac{(bx^{2}+a)^{\frac{7}{2}}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -15/8*sqrt(a)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) + 15/8*sqrt(b*x^2 + a)*b^2
+ 3/8*(b*x^2 + a)^(5/2)*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*b^2/a - 3/8*(b*x^2
+ a)^(7/2)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(7/2)/(a*x^4)
```

Fricas [A]
time = 1.41, size = 145, normalized size = 1.69

$$\left[\frac{15\,\sqrt{a}\,b^{2}x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(8\,b^{2}x^{4}-9\,abx^{2}-2\,a^{2})\sqrt{bx^{2}+a}}{16\,x^{4}},\ \frac{15\,\sqrt{-a}\,b^{2}x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(8\,b^{2}x^{4}-9\,abx^{2}-2\,a^{2})\sqrt{bx^{2}+a}}{8\,x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(15*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(8*b^2*x^4 - 9*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^4, 1/8*(15*sqrt(-
a)*b^2*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*b^2*x^4 - 9*a*b*x^2 - 2*a^
2)*sqrt(b*x^2 + a))/x^4]
```

Sympy [A]
time = 2.39, size = 117, normalized size = 1.36

$$-\frac{15\sqrt{a}\,b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8}-\frac{a^3}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{11a^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**5,x)
```

```
[Out] -15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**3/(4*sqrt(b)*x**5*sqrt(a
/(b*x**2) + 1)) - 11*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - a*b**(3/2
)/(8*x*sqrt(a/(b*x**2) + 1)) + b**(5/2)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A]
time = 0.66, size = 88, normalized size = 1.02

$$\frac{\frac{15\,ab^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\,\sqrt{bx^2+a}\,b^3-\frac{9\,(bx^2+a)^{\frac{3}{2}}ab^3-7\,\sqrt{bx^2+a}\,a^2b^3}{b^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(15*a*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x^2 + a)
*b^3 - (9*(b*x^2 + a)^(3/2)*a*b^3 - 7*sqrt(b*x^2 + a)*a^2*b^3)/(b^2*x^4))/b
```

Mupad [B]
time = 5.01, size = 71, normalized size = 0.83

$$b^2\,\sqrt{b\,x^2+a}-\frac{9\,a\,{\left(b\,x^2+a\right)}^{3/2}}{8\,x^4}+\frac{7\,a^2\,\sqrt{b\,x^2+a}}{8\,x^4}+\frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,15\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^5,x)
```

```
[Out] b^2*(a + b*x^2)^(1/2) + (a^(1/2)*b^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1
5i)/8 - (9*a*(a + b*x^2)^(3/2))/(8*x^4) + (7*a^2*(a + b*x^2)^(1/2))/(8*x^4)
```

3.394 $\int \frac{(a+bx^2)^{5/2}}{x^7} dx$

Optimal. Leaf size=89

$$-\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

```
[Out] -5/24*b*(b*x^2+a)^(3/2)/x^4-1/6*(b*x^2+a)^(5/2)/x^6-5/16*b^3*arctanh((b*x^2
+a)^(1/2)/a^(1/2))/a^(1/2)-5/16*b^2*(b*x^2+a)^(1/2)/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 43, 65, 214}

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{(a+bx^2)^{5/2}}{6x^6}-\frac{5b(a+bx^2)^{3/2}}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^7,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(16*x^2) - (5*b*(a + b*x^2)^(3/2))/(24*x^4) - (a +
 b*x^2)^(5/2)/(6*x^6) - (5*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{(a+bx^2)^{5/2}}{6x^6}+\frac{1}{12}(5b)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}+\frac{1}{16}\left(5b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}+\frac{1}{32}\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x\right.\\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}+\frac{1}{16}\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\right.\\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 70, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\left(-8a^2-26abx^2-33b^2x^4\right)}{48x^6}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2 - 26*a*b*x^2 - 33*b^2*x^4))/(48*x^6) - (5*b^3*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a])
```

Maple [A]
time = 0.06, size = 139, normalized size = 1.56

method	result

risch	$-\frac{\sqrt{b x^2+a}\,(33b^2x^4+26ab\,x^2+8a^2)}{48x^6}-\frac{5b^3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{16\sqrt{a}}$
default	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{6a\,x^6}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{4a\,x^4}+\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x}}{x}\right.\right.\right.\right.}{2a}\right.}{4a}\right.}{6a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(7/2)+1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/
2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)
+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))
```

Maxima [A]
time = 0.30, size = 127, normalized size = 1.43

$$-\frac{5\,b^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,\sqrt{a}}+\frac{(bx^2+a)^{\frac{5}{2}}b^3}{16\,a^3}+\frac{5\,(bx^2+a)^{\frac{3}{2}}b^3}{48\,a^2}+\frac{5\,\sqrt{bx^2+a}\,b^3}{16\,a}-\frac{(bx^2+a)^{\frac{7}{2}}b^2}{16\,a^3x^2}-\frac{(bx^2+a)^{\frac{7}{2}}b}{24\,a^2x^4}-\frac{(bx^2+a)^{\frac{7}{2}}}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] -5/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*(b*x^2 + a)^(5/2)*b^
3/a^3 + 5/48*(b*x^2 + a)^(3/2)*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*b^3/a - 1/16*
(b*x^2 + a)^(7/2)*b^2/(a^3*x^2) - 1/24*(b*x^2 + a)^(7/2)*b/(a^2*x^4) - 1/6*
(b*x^2 + a)^(7/2)/(a*x^6)
```

Fricas [A]
time = 1.27, size = 158, normalized size = 1.78

$$\left[\frac{15\,\sqrt{a}\,b^3x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(33\,ab^2x^4+26\,a^2bx^2+8\,a^3)\sqrt{bx^2+a}}{96\,ax^6},\ \frac{15\,\sqrt{-a}\,b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(33\,ab^2x^4+26\,a^2bx^2+8\,a^3)\sqrt{bx^2+a}}{48\,ax^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) - 2*(33*a*b^2*x^4 + 26*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6), 1/48
*(15*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (33*a*b^2*x^4 + 26
*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6)]
```

Sympy [A]
time = 2.84, size = 99, normalized size = 1.11

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{6x^5}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{24x^3}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{16x}-\frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**7,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(6*x**5) - 13*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(24*x**3) - 11*b**(5/2)*sqrt(a/(b*x**2) + 1)/(16*x) - 5*b**3*asinh(sq
rt(a)/(sqrt(b)*x))/(16*sqrt(a))
```

Giac [A]
time = 0.63, size = 87, normalized size = 0.98

$$\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{33\,(bx^2+a)^{\frac{5}{2}}b^4-40\,(bx^2+a)^{\frac{3}{2}}ab^4+15\,\sqrt{bx^2+a}\,a^2b^4}{b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(15*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (33*(b*x^2 + a)^(5
/2)*b^4 - 40*(b*x^2 + a)^(3/2)*a*b^4 + 15*sqrt(b*x^2 + a)*a^2*b^4)/(b^3*x^6
))/b
```

Mupad [B]
time = 5.20, size = 72, normalized size = 0.81

$$\frac{5\,a\,{\left(b\,x^2+a\right)}^{3/2}}{6\,x^6}-\frac{11\,{\left(b\,x^2+a\right)}^{5/2}}{16\,x^6}-\frac{5\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^7,x)
```

```
[Out] (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) - (11*(a + b*x^2
)^(5/2))/(16*x^6) + (5*a*(a + b*x^2)^(3/2))/(6*x^6) - (5*a^2*(a + b*x^2)^(1
/2))/(16*x^6)
```

3.395 $\int \frac{(a+bx^2)^{5/2}}{x^9} \, dx$

Optimal. Leaf size=113

$$-\frac{5b^2\sqrt{a+bx^2}}{64x^4}-\frac{5b^3\sqrt{a+bx^2}}{128ax^2}-\frac{5b(a+bx^2)^{3/2}}{48x^6}-\frac{(a+bx^2)^{5/2}}{8x^8}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

```
[Out] -5/48*b*(b*x^2+a)^(3/2)/x^6-1/8*(b*x^2+a)^(5/2)/x^8+5/128*b^4*arctanh((b*x^
2+a)^(1/2)/a^(1/2))/a^(3/2)-5/64*b^2*(b*x^2+a)^(1/2)/x^4-5/128*b^3*(b*x^2+a
)^(1/2)/a/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 44, 65, 214}

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}-\frac{5b^3\sqrt{a+bx^2}}{128ax^2}-\frac{5b^2\sqrt{a+bx^2}}{64x^4}-\frac{(a+bx^2)^{5/2}}{8x^8}-\frac{5b(a+bx^2)^{3/2}}{48x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^9,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(64*x^4) - (5*b^3*Sqrt[a + b*x^2])/(128*a*x^2) - (
5*b*(a + b*x^2)^(3/2))/(48*x^6) - (a + b*x^2)^(5/2)/(8*x^8) + (5*b^4*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx, x, x^2\right) \\
&= -\frac{(a+bx^2)^{5/2}}{8x^8} + \frac{1}{16}(5b)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx, x, x^2\right) \\
&= -\frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} + \frac{1}{32}\left(5b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right) \\
&= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} + \frac{1}{128}\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,\right. \\
&= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} - \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{25} \\
&= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}}\right.}{1} \\
&= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8} + \frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.}{128a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 84, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-48a^3-136a^2bx^2-118ab^2x^4-15b^3x^6\right)}{384ax^8}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3 - 136*a^2*b*x^2 - 118*a*b^2*x^4 - 15*b^3*x^6))/(3
84*a*x^8) + (5*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Maple [A]
time = 0.07, size = 163, normalized size = 1.44

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(15b^3x^6+118a\,b^2x^4+136a^2b\,x^2+48a^3)}{384x^8a}+\frac{5b^4\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{128a^{\frac{3}{2}}}$
default	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{8a\,x^8}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{6a\,x^6}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{4a\,x^4}+\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\cdots\right.\right.\right.\right)}{2a}\right)}{4a}\right)}{6a}\right)}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(7/2)-1/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(7/2)+1/6*b/a*(-1/
4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5*(b
*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a
^(1/2)*(b*x^2+a)^(1/2))/x)))))))
```

Maxima [A]
time = 0.29, size = 147, normalized size = 1.30

$$\frac{5\,b^4\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{3}{2}}}-\frac{(bx^2+a)^{\frac{5}{2}}b^4}{128\,a^4}-\frac{5\,(bx^2+a)^{\frac{3}{2}}b^4}{384\,a^3}-\frac{5\,\sqrt{bx^2+a}\,b^4}{128\,a^2}+\frac{(bx^2+a)^{\frac{7}{2}}b^3}{128\,a^4x^2}+\frac{(bx^2+a)^{\frac{7}{2}}b^2}{192\,a^3x^4}+\frac{(bx^2+a)^{\frac{7}{2}}b}{48\,a^2x^6}-\frac{(bx^2+a)^{\frac{7}{2}}}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 5/128*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/128*(b*x^2 + a)^(5/2)*b
^4/a^4 - 5/384*(b*x^2 + a)^(3/2)*b^4/a^3 - 5/128*sqrt(b*x^2 + a)*b^4/a^2 +
1/128*(b*x^2 + a)^(7/2)*b^3/(a^4*x^2) + 1/192*(b*x^2 + a)^(7/2)*b^2/(a^3*x^
4) + 1/48*(b*x^2 + a)^(7/2)*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(7/2)/(a*x^8)
```

Fricas [A]
time = 1.45, size = 179, normalized size = 1.58

$$\left[\frac{15\,\sqrt{a}\,b^4x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(15\,ab^3x^6+118\,a^2b^2x^4+136\,a^3bx^2+48\,a^4)\sqrt{bx^2+a}}{768\,a^2x^8},-\frac{15\,\sqrt{-a}\,b^4x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(15\,ab^3x^6+118\,a^2b^2x^4+136\,a^3bx^2+48\,a^4)\sqrt{bx^2+a}}{384\,a^2x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/768*(15*sqrt(a)*b^4*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 2*(15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt(b*x^
2 + a))/(a^2*x^8), -1/384*(15*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqrt(b*x^2 +
 a)) + (15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt(b*x^2
 + a))/(a^2*x^8)]
```

Sympy [A]
time = 7.63, size = 150, normalized size = 1.33

$$-\frac{a^3}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{23a^2\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{127ab^{\frac{3}{2}}}{192x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{133b^{\frac{5}{2}}}{384x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{7}{2}}}{128ax\sqrt{\frac{a}{bx^2}+1}}+\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**9,x)
```

```
[Out] -a**3/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 23*a**2*sqrt(b)/(48*x**7*sqrt
(a/(b*x**2) + 1)) - 127*a*b**(3/2)/(192*x**5*sqrt(a/(b*x**2) + 1)) - 133*b*
*(5/2)/(384*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(7/2)/(128*a*x*sqrt(a/(b*x**2
) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(3/2))
```

Giac [A]
time = 0.71, size = 109, normalized size = 0.96

$$-\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{15\,(bx^2+a)^{\frac{7}{2}}b^5+73\,(bx^2+a)^{\frac{5}{2}}ab^5-55\,(bx^2+a)^{\frac{3}{2}}a^2b^5+15\,\sqrt{bx^2+a}\,a^3b^5}{ab^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/384*(15*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x^2 +
 a)^(7/2)*b^5 + 73*(b*x^2 + a)^(5/2)*a*b^5 - 55*(b*x^2 + a)^(3/2)*a^2*b^5 +
 15*sqrt(b*x^2 + a)*a^3*b^5)/(a*b^4*x^8))/b
```

Mupad [B]
time = 5.43, size = 89, normalized size = 0.79

$$\frac{55\,a\,{\left(b\,x^2+a\right)}^{3/2}}{384\,x^8}-\frac{73\,{\left(b\,x^2+a\right)}^{5/2}}{384\,x^8}-\frac{5\,a^2\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{5\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^8}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^9,x)
```

```
[Out] (55*a*(a + b*x^2)^(3/2))/(384*x^8) - (b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/
2))*5i)/(128*a^(3/2)) - (73*(a + b*x^2)^(5/2))/(384*x^8) - (5*a^2*(a + b*x^
2)^(1/2))/(128*x^8) - (5*(a + b*x^2)^(7/2))/(128*a*x^8)
```

3.396 $\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=137

$$-\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b(a+bx^2)^{3/2}}{16x^8}-\frac{(a+bx^2)^{5/2}}{10x^{10}}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}$$

```
[Out] -1/16*b*(b*x^2+a)^(3/2)/x^8-1/10*(b*x^2+a)^(5/2)/x^10-3/256*b^5*arctanh((b*
x^2+a)^(1/2)/a^(1/2))/a^(5/2)-1/32*b^2*(b*x^2+a)^(1/2)/x^6-1/128*b^3*(b*x^2
+a)^(1/2)/a/x^4+3/256*b^4*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 44, 65, 214}

$$-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}-\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{(a+bx^2)^{5/2}}{10x^{10}}-\frac{b(a+bx^2)^{3/2}}{16x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^11,x]
```

```
[Out] -1/32*(b^2*Sqrt[a + b*x^2])/x^6 - (b^3*Sqrt[a + b*x^2])/(128*a*x^4) + (3*b^
4*Sqrt[a + b*x^2])/(256*a^2*x^2) - (b*(a + b*x^2)^(3/2))/(16*x^8) - (a + b*
x^2)^(5/2)/(10*x^10) - (3*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(5/2
))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{1}{4}b\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{1}{32}(3b^2)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{1}{64}b^3\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} - \frac{(3b^4)\,\text{Subst}\left(\int \frac{}{x^2\sqrt{a}}\right.}{256a}$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{(3b^5}{}$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{(3b^4}{}$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} - \frac{3b^5}{}$$

Mathematica [A]
time = 0.15, size = 95, normalized size = 0.69

$$\frac{\sqrt{a+bx^2}\left(-128a^4-336a^3bx^2-248a^2b^2x^4-10ab^3x^6+15b^4x^8\right)}{1280a^2x^{10}}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-128*a^4 - 336*a^3*b*x^2 - 248*a^2*b^2*x^4 - 10*a*b^3*x^6
 + 15*b^4*x^8))/(1280*a^2*x^10) - (3*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(256*a^(5/2))
```

Maple [A]
time = 0.10, size = 187, normalized size = 1.36

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-15b^4x^8+10a\,b^3x^6+248a^2b^2x^4+336a^3b\,x^2+128a^4\right)}{1280x^{10}a^2}-\frac{3b^5\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{256a^{\frac{5}{2}}}$

default	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{10a\,x^{10}}-\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{8a\,x^8}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{6a\,x^6}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{4a\,x^4}+\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2 \ldots}\right.\right.\right.}{\ldots}\right.}{4a}\right.}{6a}\right.}{8a}\right.}{10a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/a/x^10*(b*x^2+a)^(7/2)-3/10*b/a*(-1/8/a/x^8*(b*x^2+a)^(7/2)-1/8*b/a*(
-1/6/a/x^6*(b*x^2+a)^(7/2)+1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/
2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)
+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))))
```

Maxima [A]

time = 0.33, size = 167, normalized size = 1.22

$$-\frac{3\,b^5\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,a^{\frac{5}{2}}}+\frac{3\,(bx^2+a)^{\frac{5}{2}}b^5}{1280\,a^5}+\frac{(bx^2+a)^{\frac{3}{2}}b^5}{256\,a^4}+\frac{3\,\sqrt{bx^2+a}\,b^5}{256\,a^3}-\frac{3\,(bx^2+a)^{\frac{7}{2}}b^4}{1280\,a^5x^2}-\frac{(bx^2+a)^{\frac{7}{2}}b^3}{640\,a^4x^4}-\frac{(bx^2+a)^{\frac{7}{2}}b^2}{160\,a^3x^6}+\frac{3\,(bx^2+a)^{\frac{7}{2}}b}{80\,a^2x^8}-\frac{(bx^2+a)^{\frac{7}{2}}}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] -3/256*b^5*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/1280*(b*x^2 + a)^(5/2)
*b^5/a^5 + 1/256*(b*x^2 + a)^(3/2)*b^5/a^4 + 3/256*sqrt(b*x^2 + a)*b^5/a^3
- 3/1280*(b*x^2 + a)^(7/2)*b^4/(a^5*x^2) - 1/640*(b*x^2 + a)^(7/2)*b^3/(a^4
*x^4) - 1/160*(b*x^2 + a)^(7/2)*b^2/(a^3*x^6) + 3/80*(b*x^2 + a)^(7/2)*b/(a
^2*x^8) - 1/10*(b*x^2 + a)^(7/2)/(a*x^10)
```

Fricas [**A**]
time = 1.32, size = 201, normalized size = 1.47

$$\left[\frac{15\,\sqrt{a}\,b^5x^{10}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(15\,ab^4x^8-10\,a^2b^3x^6-248\,a^3b^2x^4-336\,a^4bx^2-128\,a^5)\sqrt{bx^2+a}}{2560\,a^3x^{10}},\ \frac{15\,\sqrt{-a}\,b^5x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(15\,ab^4x^8-10\,a^2b^3x^6-248\,a^3b^2x^4-336\,a^4bx^2-128\,a^5)\sqrt{bx^2+a}}{1280\,a^3x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)
/x^2) + 2*(15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3*b^2*x^4 - 336*a^4*b*x^2
- 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10), 1/1280*(15*sqrt(-a)*b^5*x^10*arctan
(sqrt(-a)/sqrt(b*x^2 + a)) + (15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3*b^2*x
^4 - 336*a^4*b*x^2 - 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10)]
```

Sympy [**A**]
time = 30.76, size = 175, normalized size = 1.28

$$-\frac{a^3}{10\sqrt{b}\,x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{29a^2\sqrt{b}}{80x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{73ab^{\frac{3}{2}}}{160x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{129b^{\frac{5}{2}}}{640x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{7}{2}}}{256ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{9}{2}}}{256a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3b^5\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{256a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**11,x)
```

```
[Out] -a**3/(10*sqrt(b)*x**11*sqrt(a/(b*x**2) + 1)) - 29*a**2*sqrt(b)/(80*x**9*sq
rt(a/(b*x**2) + 1)) - 73*a*b**(3/2)/(160*x**7*sqrt(a/(b*x**2) + 1)) - 129*b
**(5/2)/(640*x**5*sqrt(a/(b*x**2) + 1)) + b**(7/2)/(256*a*x**3*sqrt(a/(b*x*
*2) + 1)) + 3*b**(9/2)/(256*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**5*asinh(sqr
t(a)/(sqrt(b)*x))/(256*a**(5/2))
```

Giac [A]
time = 0.68, size = 126, normalized size = 0.92

$$\frac{\frac{15\,b^6\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{15\,(bx^2+a)^{\frac{9}{2}}b^6-70\,(bx^2+a)^{\frac{7}{2}}ab^6-128\,(bx^2+a)^{\frac{5}{2}}a^2b^6+70\,(bx^2+a)^{\frac{3}{2}}a^3b^6-15\,\sqrt{bx^2+a}\,a^4b^6}{a^2b^5x^{10}}}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(15*b^6*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (15*(b*x^2
 + a)^(9/2)*b^6 - 70*(b*x^2 + a)^(7/2)*a*b^6 - 128*(b*x^2 + a)^(5/2)*a^2*b^
6 + 70*(b*x^2 + a)^(3/2)*a^3*b^6 - 15*sqrt(b*x^2 + a)*a^4*b^6)/(a^2*b^5*x^1
0))/b
```

Mupad [B]
time = 5.68, size = 106, normalized size = 0.77

$$\frac{7\,a\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^{10}}-\frac{{\left(b\,x^2+a\right)}^{5/2}}{10\,x^{10}}-\frac{3\,a^2\,\sqrt{b\,x^2+a}}{256\,x^{10}}-\frac{7\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^{10}}+\frac{3\,{\left(b\,x^2+a\right)}^{9/2}}{256\,a^2\,x^{10}}+\frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{256\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^11,x)
```

```
[Out] (b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(256*a^(5/2)) - (a + b*x^2)^(
5/2)/(10*x^10) + (7*a*(a + b*x^2)^(3/2))/(128*x^10) - (3*a^2*(a + b*x^2)^(1
/2))/(256*x^10) - (7*(a + b*x^2)^(7/2))/(128*a*x^10) + (3*(a + b*x^2)^(9/2)
)/(256*a^2*x^10)
```

3.397 $\int x^4(a+bx^2)^{5/2}\,dx$

Optimal. Leaf size=136

$$-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{3a^5\tanh^{-1}\left(\right.}{256}$$

```
[Out] 1/16*a*x^5*(b*x^2+a)^(3/2)+1/10*x^5*(b*x^2+a)^(5/2)+3/256*a^5*arctanh(x*b^(
1/2)/(b*x^2+a)^(1/2))/b^(5/2)-3/256*a^4*x*(b*x^2+a)^(1/2)/b^2+1/128*a^3*x^3
*(b*x^2+a)^(1/2)/b+1/32*a^2*x^5*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 212}

$$\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a^4*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*x^3*Sqrt[a + b*x^2])/(128*b) +
(a^2*x^5*Sqrt[a + b*x^2])/32 + (a*x^5*(a + b*x^2)^(3/2))/16 + (x^5*(a + b*x
^2)^(5/2))/10 + (3*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{5/2}\,dx=\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{2}a\int x^4\left(a+bx^2\right)^{3/2}\,dx$$

$$=\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^4\sqrt{a+bx^2}\,dx$$

$$=\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{32}a^3\int\frac{x^4}{\sqrt{a+bx^2}}\,d$$

$$=\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}-\frac{(3}{}$$

$$=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}$$

$$=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}$$

$$=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}$$

Mathematica [A]
time = 0.11, size = 96, normalized size = 0.71

$$\frac{\sqrt{a+bx^2}\left(-15a^4x+10a^3bx^3+248a^2b^2x^5+336ab^3x^7+128b^4x^9\right)}{1280b^2}-\frac{3a^5\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{256b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^4*x + 10*a^3*b*x^3 + 248*a^2*b^2*x^5 + 336*a*b^3*x^
7 + 128*b^4*x^9))/(1280*b^2) - (3*a^5*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/
(256*b^(5/2))
```

Maple [A]
time = 0.05, size = 114, normalized size = 0.84

method	result
risch	$-\frac{x\left(-128 b^{4} x^{8}-336 a\,b^{3} x^{6}-248 a^{2} b^{2} x^{4}-10 a^{3} b\,x^{2}+15 a^{4}\right) \sqrt{b\,x^{2}+a}}{1280 b^{2}}+\frac{3 a^{5} \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{256 b^{\frac{5}{2}}}$
default	$\frac{x^{3} \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{10 b}-\frac{3 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8 b}-\frac{a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6}\right)}{8 b}\right)}{10 b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x
*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+
1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.28, size = 105, normalized size = 0.77

$$\frac{(bx^2+a)^{\frac{7}{2}}x^3}{10\,b}-\frac{3\,(bx^2+a)^{\frac{7}{2}}ax}{80\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}a^2x}{160\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}a^3x}{128\,b^2}+\frac{3\,\sqrt{bx^2+a}\,a^4x}{256\,b^2}+\frac{3\,a^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*x^3/b - 3/80*(b*x^2 + a)^(7/2)*a*x/b^2 + 1/160*(b*x^
2 + a)^(5/2)*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*a^3*x/b^2 + 3/256*sqrt(b*x
^2 + a)*a^4*x/b^2 + 3/256*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 1.31, size = 190, normalized size = 1.40

$$\left[\frac{15\,a^{5}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,(128\,b^{5}x^{9}+336\,ab^{4}x^{7}+248\,a^{2}b^{3}x^{5}+10\,a^{3}b^{2}x^{3}-15\,a^{4}bx)\sqrt{bx^{2}+a}}{2560\,b^{3}},\ -\frac{15\,a^{5}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(128\,b^{5}x^{9}+336\,ab^{4}x^{7}+248\,a^{2}b^{3}x^{5}+10\,a^{3}b^{2}x^{3}-15\,a^{4}bx)\sqrt{bx^{2}+a}}{1280\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(15*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^2*x^3 - 15*a^4*b
*x)*sqrt(b*x^2 + a))/b^3, -1/1280*(15*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^2*x^
3 - 15*a^4*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 30.41, size = 175, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}x}{256b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{a^{\frac{7}{2}}x^{3}}{256b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{129a^{\frac{5}{2}}x^{5}}{640\sqrt{1+\frac{bx^{2}}{a}}}+\frac{73a^{\frac{3}{2}}bx^{7}}{160\sqrt{1+\frac{bx^{2}}{a}}}+\frac{29\sqrt{a}\,b^{2}x^{9}}{80\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{5}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{b^{3}x^{11}}{10\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*x/(256*b**2*sqrt(1 + b*x**2/a)) - a**(7/2)*x**3/(256*b*sqrt(1 +
 b*x**2/a)) + 129*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*a**(3/2)*b*x*
*7/(160*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a))
+ 3*a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + b**3*x**11/(10*sqrt(a)*s
qrt(1 + b*x**2/a))
```

Giac [A]
time = 0.83, size = 91, normalized size = 0.67

$$-\frac{3\,a^{5}\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{256\,b^{\frac{5}{2}}}+\frac{1}{1280}\left(2\left(4\left(2\left(8\,b^{2}x^{2}+21\,ab\right)x^{2}+31\,a^{2}\right)x^{2}+\frac{5\,a^{3}}{b}\right)x^{2}-\frac{15\,a^{4}}{b^{2}}\right)\sqrt{bx^{2}+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -3/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/1280*(2*(4*(2
*(8*b^2*x^2 + 21*a*b)*x^2 + 31*a^2)*x^2 + 5*a^3/b)*x^2 - 15*a^4/b^2)*sqrt(b
*x^2 + a)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4}\,{\left(b\,x^{2}+a\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(5/2),x)
[Out] int(x^4*(a + b*x^2)^(5/2), x)
```

3.398 $\int x^2(a+bx^2)^{5/2}\,dx$

Optimal. Leaf size=112

$$\frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

```
[Out] 5/48*a*x^3*(b*x^2+a)^(3/2)+1/8*x^3*(b*x^2+a)^(5/2)-5/128*a^4*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))/b^(3/2)+5/128*a^3*x*(b*x^2+a)^(1/2)/b+5/64*a^2*x^3*(b*
x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {285, 327, 223, 212}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^3*x*Sqrt[a + b*x^2])/(128*b) + (5*a^2*x^3*Sqrt[a + b*x^2])/64 + (5*a*x
^3*(a + b*x^2)^(3/2))/48 + (x^3*(a + b*x^2)^(5/2))/8 - (5*a^4*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx^2)^{5/2}\,dx &= \frac{1}{8}x^3(a+bx^2)^{5/2}+\frac{1}{8}(5a)\int x^2(a+bx^2)^{3/2}\,dx\\ &= \frac{5}{48}ax^3(a+bx^2)^{3/2}+\frac{1}{8}x^3(a+bx^2)^{5/2}+\frac{1}{16}(5a^2)\int x^2\sqrt{a+bx^2}\,dx\\ &= \frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3(a+bx^2)^{3/2}+\frac{1}{8}x^3(a+bx^2)^{5/2}+\frac{1}{64}(5a^3)\int \frac{x^2}{\sqrt{a+bx^2}}\,d\\ &= \frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3(a+bx^2)^{3/2}+\frac{1}{8}x^3(a+bx^2)^{5/2}-\frac{(5a^4)}{}\\ &= \frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3(a+bx^2)^{3/2}+\frac{1}{8}x^3(a+bx^2)^{5/2}-\frac{(5a^4)}{}\\ &= \frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3(a+bx^2)^{3/2}+\frac{1}{8}x^3(a+bx^2)^{5/2}-\frac{5a^4\,\mathrm{t}}{}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 85, normalized size = 0.76

$$\frac{x\sqrt{a+bx^2}\,(15a^3+118a^2bx^2+136ab^2x^4+48b^3x^6)}{384b}+\frac{5a^4\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{128b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(15*a^3 + 118*a^2*b*x^2 + 136*a*b^2*x^4 + 48*b^3*x^6))/(
384*b) + (5*a^4*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Maple [A]
time = 0.04, size = 90, normalized size = 0.80

method	result	size

risch	$\frac{x\left(48b^{3}x^{6}+136a\,b^{2}x^{4}+118a^{2}b\,x^{2}+15a^{3}\right)\sqrt{b\,x^{2}+a}}{384b}-\frac{5a^{4}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{3}{2}}}$	73
default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+
a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2)))))
```

Maxima [A]
time = 0.28, size = 85, normalized size = 0.76

$$\frac{(bx^2+a)^{\frac{7}{2}}x}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}ax}{48\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}}a^2x}{192\,b}-\frac{5\,\sqrt{bx^2+a}\,a^3x}{128\,b}-\frac{5\,a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(7/2)*x/b - 1/48*(b*x^2 + a)^(5/2)*a*x/b - 5/192*(b*x^2 + a
)^(3/2)*a^2*x/b - 5/128*sqrt(b*x^2 + a)*a^3*x/b - 5/128*a^4*arcsinh(b*x/sqr
t(a*b))/b^(3/2)
```

Fricas [A]
time = 1.15, size = 167, normalized size = 1.49

$$\left[\frac{15\,a^4\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(48\,b^4x^7+136\,ab^3x^5+118\,a^2b^2x^3+15\,a^3bx)\sqrt{bx^2+a}}{768\,b^2},\ \frac{15\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(48\,b^4x^7+136\,ab^3x^5+118\,a^2b^2x^3+15\,a^3bx)\sqrt{bx^2+a}}{384\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(48*b^4*x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 + a)
)/b^2, 1/384*(15*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*
x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A]
time = 7.37, size = 150, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{b^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*a**(5/2)*x**3/(384*sqrt(1 + b
*x**2/a)) + 127*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*sqrt(a)*b**2*
x**7/(48*sqrt(1 + b*x**2/a)) - 5*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2
)) + b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.81, size = 77, normalized size = 0.69

$$\frac{5\,a^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{384}\left(2\left(4\left(6\,b^2x^2+17\,ab\right)x^2+59\,a^2\right)x^2+\frac{15\,a^3}{b}\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/384*(2*(4*(6*b
^2*x^2 + 17*a*b)*x^2 + 59*a^2)*x^2 + 15*a^3/b)*sqrt(b*x^2 + a)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(b\,x^2+a\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(5/2), x)
```

3.399 $\int (a + bx^2)^{5/2}\, dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax(a+bx^2)^{3/2}+\frac{1}{6}x(a+bx^2)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

```
[Out] 5/24*a*x*(b*x^2+a)^(3/2)+1/6*x*(b*x^2+a)^(5/2)+5/16*a^3*arctanh(x*b^(1/2)/(
b*x^2+a)^(1/2))/b^(1/2)+5/16*a^2*x*(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 223, 212}

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax(a+bx^2)^{3/2}+\frac{1}{6}x(a+bx^2)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^{5/2} dx &= \frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{6}(5a)\int \left(a+bx^2\right)^{3/2} dx \\
&= \frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \sqrt{a+bx^2}\, dx \\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}} dx \\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2} d\right. \\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 71, normalized size = 0.85

$$
\frac{1}{48}\sqrt{a+bx^2}\left(33a^2x+26abx^3+8b^2x^5\right)-\frac{5a^3\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5))/48 - (5*a^3*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Maple [A]
time = 0.04, size = 68, normalized size = 0.81

method	result	size
risch	$\frac{x\left(8b^2x^4+26ab\,x^2+33a^2\right)\sqrt{b\,x^2+a}}{48}+\frac{5a^3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}$	59
default	$\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(
1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.29, size = 58, normalized size = 0.69

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}x+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ax+\frac{5}{16}\sqrt{bx^2+a}\,a^2x+\frac{5\,a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*x + 5/24*(b*x^2 + a)^(3/2)*a*x + 5/16*sqrt(b*x^2 + a)
*a^2*x + 5/16*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 0.93, size = 146, normalized size = 1.74

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{96\,b},-\frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{48\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 + 33*
a^2*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 2.63, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}\,b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a)
/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqrt(
a))/(16*sqrt(b))
```

Giac [A]
time = 0.95, size = 63, normalized size = 0.75

$$-\frac{5\,a^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*x
^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

Mupad [B]
time = 4.46, size = 37, normalized size = 0.44

$$\frac{x\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2),x)
```

```
[Out] (x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(5/2)
```

3.400 $\int \frac{(a+bx^2)^{5/2}}{x^2} dx$

Optimal. Leaf size=83

$$\frac{15}{8}abx\sqrt{a+bx^2}+\frac{5}{4}bx\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{5/2}}{x}+\frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] 5/4*b*x*(b*x^2+a)^(3/2)-(b*x^2+a)^(5/2)/x+15/8*a^2*arctanh(x*b^(1/2)/(b*x^2
+a)^(1/2))*b^(1/2)+15/8*a*b*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 201, 223, 212}

$$\frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}+\frac{5}{4}bx\left(a+bx^2\right)^{3/2}+\frac{15}{8}abx\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^2,x]
```

```
[Out] (15*a*b*x*Sqrt[a + b*x^2])/8 + (5*b*x*(a + b*x^2)^(3/2))/4 - (a + b*x^2)^(5
/2)/x + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx &= -\frac{(a+bx^2)^{5/2}}{x}+(5b)\int (a+bx^2)^{3/2}\,dx\\
&= \frac{5}{4}bx(a+bx^2)^{3/2}-\frac{(a+bx^2)^{5/2}}{x}+\frac{1}{4}(15ab)\int \sqrt{a+bx^2}\,dx\\
&= \frac{15}{8}abx\sqrt{a+bx^2}+\frac{5}{4}bx(a+bx^2)^{3/2}-\frac{(a+bx^2)^{5/2}}{x}+\frac{1}{8}(15a^2b)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{15}{8}abx\sqrt{a+bx^2}+\frac{5}{4}bx(a+bx^2)^{3/2}-\frac{(a+bx^2)^{5/2}}{x}+\frac{1}{8}(15a^2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x\right.\\
&= \frac{15}{8}abx\sqrt{a+bx^2}+\frac{5}{4}bx(a+bx^2)^{3/2}-\frac{(a+bx^2)^{5/2}}{x}+\frac{15}{8}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 0.88

$$\frac{\sqrt{a+bx^2}\,(-8a^2+9abx^2+2b^2x^4)}{8x}-\frac{15}{8}a^2\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2 + 9*a*b*x^2 + 2*b^2*x^4))/(8*x) - (15*a^2*Sqrt[b]*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/8
```

Maple [A]
time = 0.04, size = 92, normalized size = 1.11

method	result	size
risch	$-\frac{\sqrt{b\,x^2+a}\,(-2b^2x^4-9ab\,x^2+8a^2)}{8x}+\frac{15a^2\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8}$	61

default	$-\frac{(bx^2+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{a}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x*(b*x^2+a)^(7/2)+6*b/a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^
(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/
2)))))
```

Maxima [A]
time = 0.43, size = 59, normalized size = 0.71

$$\frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}bx+\frac{15}{8}\sqrt{bx^2+a}\,abx+\frac{15}{8}a^2\sqrt{b}\ \text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*b*x + 15/8*sqrt(b*x^2 + a)*a*b*x + 15/8*a^2*sqrt(b)*a
rcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(5/2)/x
```

Fricas [A]
time = 1.38, size = 140, normalized size = 1.69

$$\left[\frac{15\,a^2\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(2\,b^2x^4+9\,abx^2-8\,a^2)\sqrt{bx^2+a}}{16\,x},\,-\frac{15\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^4+9\,abx^2-8\,a^2)\sqrt{bx^2+a}}{8\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/16*(15*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a))/x, -1/8*(15*a^2*sqrt(-b)*
x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*sqrt
(b*x^2 + a))/x]
```

Sympy [A]
time = 2.33, size = 117, normalized size = 1.41

$$-\frac{a^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}\,b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{b^3x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)/(x*sqrt(1 + b*x**2/a)) + a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 11
*sqrt(a)*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*a**2*sqrt(b)*asinh(sqrt(b)*x
/sqrt(a))/8 + b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.85, size = 87, normalized size = 1.05

$$-\frac{15}{16}\,a^2\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^3\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{8}\,\left(2\,b^2x^2+9\,ab\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*a^2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^3*sqrt(b)/((s
qrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*b^2*x^2 + 9*a*b)*sqrt(b*x^2 + a
)*x
```

Mupad [B]
time = 5.05, size = 40, normalized size = 0.48

$$-\frac{\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^2,x)
```

```
[Out] -((a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a
 + 1)^(5/2))
```

3.401 $\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=86

$$\frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b(a+bx^2)^{3/2}}{3x}-\frac{(a+bx^2)^{5/2}}{3x^3}+\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -5/3*b*(b*x^2+a)^(3/2)/x-1/3*(b*x^2+a)^(5/2)/x^3+5/2*a*b^(3/2)*arctanh(x*b^
(1/2)/(b*x^2+a)^(1/2))+5/2*b^2*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 201, 223, 212}

$$\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b(a+bx^2)^{3/2}}{3x}-\frac{(a+bx^2)^{5/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^4,x]
```

```
[Out] (5*b^2*x*Sqrt[a + b*x^2])/2 - (5*b*(a + b*x^2)^(3/2))/(3*x) - (a + b*x^2)^(
5/2)/(3*x^3) + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx = -\frac{(a+bx^2)^{5/2}}{3x^3}+\frac{1}{3}(5b)\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx$$
$$= -\frac{5b(a+bx^2)^{3/2}}{3x}-\frac{(a+bx^2)^{5/2}}{3x^3}+(5b^2)\int \sqrt{a+bx^2}\,dx$$
$$= \frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b(a+bx^2)^{3/2}}{3x}-\frac{(a+bx^2)^{5/2}}{3x^3}+\frac{1}{2}(5ab^2)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b(a+bx^2)^{3/2}}{3x}-\frac{(a+bx^2)^{5/2}}{3x^3}+\frac{1}{2}(5ab^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b(a+bx^2)^{3/2}}{3x}-\frac{(a+bx^2)^{5/2}}{3x^3}+\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Mathematica [A]
time = 0.11, size = 71, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\,(-2a^2-14abx^2+3b^2x^4)}{6x^3}-\frac{5}{2}ab^{3/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a^2 - 14*a*b*x^2 + 3*b^2*x^4))/(6*x^3) - (5*a*b^(3/2)*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/2
```

Maple [A]
time = 0.05, size = 116, normalized size = 1.35

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-3b^2x^4+14ab\,x^2+2a^2\right)}{6x^3}+\frac{5a\,b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2}$

default	$-\frac{(bx^2+a)^{\frac{7}{2}}}{3a\,x^3}+\frac{4b\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{a}\right)}{3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(7/2)+4/3*b/a*(-1/a/x*(b*x^2+a)^(7/2)+6*b/a*(1/6*x*(b*
x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*
a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.38, size = 84, normalized size = 0.98

$$\frac{5}{2}\sqrt{bx^2+a}\,b^2x+\frac{5\,(bx^2+a)^{\frac{3}{2}}b^2x}{3\,a}+\frac{5}{2}\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{4\,(bx^2+a)^{\frac{5}{2}}b}{3\,ax}-\frac{(bx^2+a)^{\frac{7}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(b*x^2 + a)*b^2*x + 5/3*(b*x^2 + a)^(3/2)*b^2*x/a + 5/2*a*b^(3/2)*a
rcsinh(b*x/sqrt(a*b)) - 4/3*(b*x^2 + a)^(5/2)*b/(a*x) - 1/3*(b*x^2 + a)^(7/
2)/(a*x^3)
```

Fricas [A]
time = 1.20, size = 141, normalized size = 1.64

$$\left[\frac{15\,ab^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(3\,b^2x^4-14\,abx^2-2\,a^2)\sqrt{bx^2+a}}{12\,x^3},\,-\frac{15\,a\sqrt{-b}\,bx^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(3\,b^2x^4-14\,abx^2-2\,a^2)\sqrt{bx^2+a}}{6\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(15*a*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(3*b^2*x^4 - 14*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^3, -1/6*(15*a*sqrt(-b)
*b*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*b^2*x^4 - 14*a*b*x^2 - 2*a^2
)*sqrt(b*x^2 + a))/x^3]
```

Sympy [**A**]
time = 2.11, size = 112, normalized size = 1.30

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{7ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}-\frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{4}+\frac{5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2}+\frac{b^{\frac{5}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**4,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 7*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/3 - 5*a*b**(3/2)*log(a/(b*x**2))/4 + 5*a*b**(3/2)*log(sqrt(a/(b*x**2)
+ 1) + 1)/2 + b**(5/2)*x**2*sqrt(a/(b*x**2) + 1)/2
```

Giac [**A**]
time = 0.92, size = 132, normalized size = 1.53

$$\frac{1}{2}\sqrt{bx^2+a}\,b^2x-\frac{5}{4}ab^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{3}{2}}-12\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{3}{2}}+7\,a^4b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^2*x - 5/4*a*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))
^2) + 2/3*(9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2) - 12*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a^3*b^(3/2) + 7*a^4*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^3
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^4,x)
```

```
[Out] int((a + b*x^2)^(5/2)/x^4, x)
```

3.402 $\int \frac{(a+bx^2)^{5/2}}{x^6} dx$

Optimal. Leaf size=82

$$-\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b(a+bx^2)^{3/2}}{3x^3}-\frac{(a+bx^2)^{5/2}}{5x^5}+b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/3*b*(b*x^2+a)^(3/2)/x^3-1/5*(b*x^2+a)^(5/2)/x^5+b^(5/2)*arctanh(x*b^(1/2
)/(b*x^2+a)^(1/2))-b^2*(b*x^2+a)^(1/2)/x
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {283, 223, 212}

$$b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b^2\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{5/2}}{5x^5}-\frac{b(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^6,x]
```

```
[Out] -((b^2*Sqrt[a + b*x^2])/x) - (b*(a + b*x^2)^(3/2))/(3*x^3) - (a + b*x^2)^(5
/2)/(5*x^5) + b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx &= -\frac{(a+bx^2)^{5/2}}{5x^5}+b\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx \\
&= -\frac{b(a+bx^2)^{3/2}}{3x^3}-\frac{(a+bx^2)^{5/2}}{5x^5}+b^2\int \frac{\sqrt{a+bx^2}}{x^2}\,dx \\
&= -\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b(a+bx^2)^{3/2}}{3x^3}-\frac{(a+bx^2)^{5/2}}{5x^5}+b^3\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= -\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b(a+bx^2)^{3/2}}{3x^3}-\frac{(a+bx^2)^{5/2}}{5x^5}+b^3\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right) \\
&= -\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b(a+bx^2)^{3/2}}{3x^3}-\frac{(a+bx^2)^{5/2}}{5x^5}+b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\,(-3a^2-11abx^2-23b^2x^4)}{15x^5}-b^{5/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2 - 11*a*b*x^2 - 23*b^2*x^4))/(15*x^5) - b^(5/2)*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(66) = 132.
time = 0.05, size = 140, normalized size = 1.71

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(23b^2x^4+11ab\,x^2+3a^2\right)}{15x^5}+b^{\frac{5}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)$

default	$-\frac{(bx^2+a)^{\frac{7}{2}}}{5a\,x^5}+\frac{2b\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{3a\,x^3}+\frac{4b\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/x^5*(b*x^2+a)^(7/2)+2/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(7/2)+4/3*b/a*(-1/
a/x*(b*x^2+a)^(7/2)+6*b/a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/
2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
)))))
```

Maxima [A]
time = 0.31, size = 104, normalized size = 1.27

$$\frac{2\,(bx^2+a)^{\frac{3}{2}}b^3x}{3\,a^2}+\frac{\sqrt{bx^2+a}\,b^3x}{a}+b^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{8\,(bx^2+a)^{\frac{5}{2}}b^2}{15\,a^2x}-\frac{2\,(bx^2+a)^{\frac{7}{2}}b}{15\,a^2x^3}-\frac{(bx^2+a)^{\frac{7}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x^2 + a)^(3/2)*b^3*x/a^2 + sqrt(b*x^2 + a)*b^3*x/a + b^(5/2)*arcsinh
(b*x/sqrt(a*b)) - 8/15*(b*x^2 + a)^(5/2)*b^2/(a^2*x) - 2/15*(b*x^2 + a)^(7/
2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2)/(a*x^5)
```

Fricas [A]
time = 1.06, size = 140, normalized size = 1.71

$$\left[\frac{15\,b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(23\,b^2x^4+11\,abx^2+3\,a^2)\sqrt{bx^2+a}}{30\,x^5},-\frac{15\,\sqrt{-b}\,b^2x^5\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(23\,b^2x^4+11\,abx^2+3\,a^2)\sqrt{bx^2+a}}{15\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(
23*b^2*x^4 + 11*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a))/x^5, -1/15*(15*sqrt(-b)*b
^2*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (23*b^2*x^4 + 11*a*b*x^2 + 3*a^
2)*sqrt(b*x^2 + a))/x^5]
```

Sympy [A]
time = 2.34, size = 105, normalized size = 1.28

$$-\frac{a^2\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{11ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{23b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15}-\frac{b^{\frac{5}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{5}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**6,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(15*x**2) - 23*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 - b**(5/2)*log(a/(b*x
**2))/2 + b**(5/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(66) = 132.
time = 0.98, size = 168, normalized size = 2.05

$$-\frac{1}{2}b^{\frac{5}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(45\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8ab^{\frac{5}{2}}-90\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{5}{2}}+140\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^3b^{\frac{5}{2}}-70\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^4b^{\frac{5}{2}}+23\,a^5b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*a*b^(5/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(5/2
) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(5/2) - 70*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*a^4*b^(5/2) + 23*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^6,x)
```

```
[Out] int((a + b*x^2)^(5/2)/x^6, x)
```

3.403 $\int \frac{(a+bx^2)^{5/2}}{x^8} dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

```
[Out] -1/7*(b*x^2+a)^(7/2)/a/x^7
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^8,x]
```

```
[Out] -1/7*(a + b*x^2)^(7/2)/(a*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^8} dx = -\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Mathematica [A]
time = 0.07, size = 21, normalized size = 1.00

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^8,x]
```

```
[Out] -1/7*(a + b*x^2)^(7/2)/(a*x^7)
```

Maple [A]
time = 0.07, size = 18, normalized size = 0.86

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7a\,x^{7}}$	18
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7a\,x^{7}}$	18
trager	$-\frac{\left(b^{3}x^{6}+3a\,b^{2}x^{4}+3a^{2}b\,x^{2}+a^{3}\right)\sqrt{b\,x^{2}+a}}{7a\,x^{7}}$	47
risch	$-\frac{\left(b^{3}x^{6}+3a\,b^{2}x^{4}+3a^{2}b\,x^{2}+a^{3}\right)\sqrt{b\,x^{2}+a}}{7a\,x^{7}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/7*(b*x^2+a)^(7/2)/a/x^7
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.81

$$-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{7\,ax^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="maxima")
[Out] -1/7*(b*x^2 + a)^(7/2)/(a*x^7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.
time = 0.97, size = 46, normalized size = 2.19

$$-\frac{\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{bx^{2}+a}}{7\,ax^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="fricas")
[Out] -1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/(a*x^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(17) = 34.
time = 0.62, size = 95, normalized size = 4.52

$$-\frac{a^{2}\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{7x^{6}}-\frac{3ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{7x^{4}}-\frac{3b^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{7x^{2}}-\frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**8,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 3*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(7*x**4) - 3*b**(5/2)*sqrt(a/(b*x**2) + 1)/(7*x**2) - b**(7/2)*sqrt(a/
(b*x**2) + 1)/(7*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(17) = 34.
time = 1.38, size = 113, normalized size = 5.38

$$\frac{2\left(7\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}b^{\frac{7}{2}}+35\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}a^2b^{\frac{7}{2}}+21\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^4b^{\frac{7}{2}}+a^6b^{\frac{7}{2}}\right)}{7\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/7*(7*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(7/2) + 35*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*a^2*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(7/2) + a^
6*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

Mupad [B]
time = 5.07, size = 71, normalized size = 3.38

$$-\frac{a^2\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{3\,b^2\,\sqrt{b\,x^2+a}}{7\,x^3}-\frac{b^3\,\sqrt{b\,x^2+a}}{7\,a\,x}-\frac{3\,a\,b\,\sqrt{b\,x^2+a}}{7\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^8,x)
```

```
[Out] - (a^2*(a + b*x^2)^(1/2))/(7*x^7) - (3*b^2*(a + b*x^2)^(1/2))/(7*x^3) - (b^
3*(a + b*x^2)^(1/2))/(7*a*x) - (3*a*b*(a + b*x^2)^(1/2))/(7*x^5)
```

3.404 $\int \frac{(a+bx^2)^{5/2}}{x^{10}} dx$

Optimal. Leaf size=44

$$-\frac{(a+bx^2)^{7/2}}{9ax^9}+\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}$$

```
[Out] -1/9*(b*x^2+a)^(7/2)/a/x^9+2/63*b*(b*x^2+a)^(7/2)/a^2/x^7
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 270}

$$\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}-\frac{(a+bx^2)^{7/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^10,x]
```

```
[Out] -1/9*(a + b*x^2)^(7/2)/(a*x^9) + (2*b*(a + b*x^2)^(7/2))/(63*a^2*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx &= -\frac{(a+bx^2)^{7/2}}{9ax^9}-\frac{(2b)\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx}{9a}\\ &= -\frac{(a+bx^2)^{7/2}}{9ax^9}+\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 31, normalized size = 0.70

$$\frac{(a+bx^2)^{7/2}\,(-7a+2bx^2)}{63a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^10,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-7*a + 2*b*x^2))/(63*a^2*x^9)
```

Maple [A]
time = 0.08, size = 37, normalized size = 0.84

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}\,(-2b\,x^2+7a)}{63x^9a^2}$	28
default	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{9a\,x^9}+\frac{2b(b\,x^2+a)^{\frac{7}{2}}}{63a^2x^7}$	37
trager	$-\frac{(-2b^4x^8+a\,b^3x^6+15a^2b^2x^4+19a^3b\,x^2+7a^4)\sqrt{b\,x^2+a}}{63x^9a^2}$	60
risch	$-\frac{(-2b^4x^8+a\,b^3x^6+15a^2b^2x^4+19a^3b\,x^2+7a^4)\sqrt{b\,x^2+a}}{63x^9a^2}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(b*x^2+a)^(7/2)/a/x^9+2/63*b*(b*x^2+a)^(7/2)/a^2/x^7
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.82

$$\frac{2\,(bx^2+a)^{\frac{7}{2}}b}{63\,a^2x^7}-\frac{(bx^2+a)^{\frac{7}{2}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 2/63*(b*x^2 + a)^(7/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(7/2)/(a*x^9)
```

Fricas [A]
time = 1.12, size = 60, normalized size = 1.36

$$\frac{(2\,b^4x^8-ab^3x^6-15\,a^2b^2x^4-19\,a^3bx^2-7\,a^4)\sqrt{bx^2+a}}{63\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(2*b^4*x^8 - a*b^3*x^6 - 15*a^2*b^2*x^4 - 19*a^3*b*x^2 - 7*a^4)*sqrt(b
*x^2 + a)/(a^2*x^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(37) = 74$.
time = 0.80, size = 121, normalized size = 2.75

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{9x^8}-\frac{19ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{63x^6}-\frac{5b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{21x^4}-\frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{63ax^2}+\frac{2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{63a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**10,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 19*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(63*x**6) - 5*b**(5/2)*sqrt(a/(b*x**2) + 1)/(21*x**4) - b**(7/2)*sqrt
(a/(b*x**2) + 1)/(63*a*x**2) + 2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(63*a**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(36) = 72$.
time = 1.29, size = 220, normalized size = 5.00

$$\frac{4\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{9}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{9}{2}}+189\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^3b^{\frac{9}{2}}+189\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^4b^{\frac{9}{2}}+27\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^5b^{\frac{9}{2}}+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^6b^{\frac{9}{2}}-a^7b^{\frac{9}{2}}\right)}{63\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 4/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(9/2) + 105*(sqrt(b)*x - sqrt(b
*x^2 + a))^12*a*b^(9/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)
+ 189*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2) + 189*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a^4*b^(9/2) + 27*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/2) +
 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2) - a^7*b^(9/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^9
```

Mupad [B]
time = 5.37, size = 91, normalized size = 2.07

$$\frac{2\,b^4\,\sqrt{b\,x^2+a}}{63\,a^2\,x}-\frac{5\,b^2\,\sqrt{b\,x^2+a}}{21\,x^5}-\frac{b^3\,\sqrt{b\,x^2+a}}{63\,a\,x^3}-\frac{a^2\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{19\,a\,b\,\sqrt{b\,x^2+a}}{63\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^10,x)
```

```
[Out] (2*b^4*(a + b*x^2)^(1/2))/(63*a^2*x) - (5*b^2*(a + b*x^2)^(1/2))/(21*x^5) -
 (b^3*(a + b*x^2)^(1/2))/(63*a*x^3) - (a^2*(a + b*x^2)^(1/2))/(9*x^9) - (19
*a*b*(a + b*x^2)^(1/2))/(63*x^7)
```

3.405 $\int \frac{(a+bx^2)^{5/2}}{x^{12}} dx$

Optimal. Leaf size=68

$$-\frac{(a+bx^2)^{7/2}}{11ax^{11}}+\frac{4b(a+bx^2)^{7/2}}{99a^2x^9}-\frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7}$$

```
[Out] -1/11*(b*x^2+a)^(7/2)/a/x^11+4/99*b*(b*x^2+a)^(7/2)/a^2/x^9-8/693*b^2*(b*x^
2+a)^(7/2)/a^3/x^7
```

Rubi [A]

time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$-\frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7}+\frac{4b(a+bx^2)^{7/2}}{99a^2x^9}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^12,x]
```

```
[Out] -1/11*(a + b*x^2)^(7/2)/(a*x^11) + (4*b*(a + b*x^2)^(7/2))/(99*a^2*x^9) - (
8*b^2*(a + b*x^2)^(7/2))/(693*a^3*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^{12}} dx &= -\frac{(a+bx^2)^{7/2}}{11ax^{11}}-\frac{(4b)\int \frac{(a+bx^2)^{5/2}}{x^{10}} dx}{11a} \\
&= -\frac{(a+bx^2)^{7/2}}{11ax^{11}}+\frac{4b(a+bx^2)^{7/2}}{99a^2x^9}+\frac{(8b^2)\int \frac{(a+bx^2)^{5/2}}{x^8} dx}{99a^2} \\
&= -\frac{(a+bx^2)^{7/2}}{11ax^{11}}+\frac{4b(a+bx^2)^{7/2}}{99a^2x^9}-\frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 42, normalized size = 0.62

$$\frac{(a+bx^2)^{7/2}\left(-63a^2+28abx^2-8b^2x^4\right)}{693a^3x^{11}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(5/2)/x^12,x]

[Out] ((a + b*x^2)^(7/2)*(-63*a^2 + 28*a*b*x^2 - 8*b^2*x^4))/(693*a^3*x^11)

Maple [A]
time = 0.12, size = 61, normalized size = 0.90

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}\left(8b^2x^4-28ab\,x^2+63a^2\right)}{693x^{11}a^3}$	39
default	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{11a\,x^{11}}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{9a\,x^9}+\frac{2b(b\,x^2+a)^{\frac{7}{2}}}{63a^2x^7}\right)}{11a}$	61
trager	$-\frac{\left(8b^5x^{10}-4a\,b^4x^8+3a^2b^3x^6+113a^3b^2x^4+161a^4b\,x^2+63a^5\right)\sqrt{b\,x^2+a}}{693x^{11}a^3}$	72
risch	$-\frac{\left(8b^5x^{10}-4a\,b^4x^8+3a^2b^3x^6+113a^3b^2x^4+161a^4b\,x^2+63a^5\right)\sqrt{b\,x^2+a}}{693x^{11}a^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(5/2)/x^12,x,method=_RETURNVERBOSE)

[Out] -1/11*(b*x^2+a)^(7/2)/a/x^11-4/11*b/a*(-1/9*(b*x^2+a)^(7/2)/a/x^9+2/63*b*(b*x^2+a)^(7/2)/a^2/x^7)

Maxima [A]
time = 0.30, size = 56, normalized size = 0.82

$$-\frac{8\,(bx^2+a)^{\frac{7}{2}}b^2}{693\,a^3x^7}+\frac{4\,(bx^2+a)^{\frac{7}{2}}b}{99\,a^2x^9}-\frac{(bx^2+a)^{\frac{7}{2}}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="maxima")

[Out] -8/693*(b*x^2 + a)^(7/2)*b^2/(a^3*x^7) + 4/99*(b*x^2 + a)^(7/2)*b/(a^2*x^9) - 1/11*(b*x^2 + a)^(7/2)/(a*x^11)

Fricas [A]
time = 0.88, size = 71, normalized size = 1.04

$$-\frac{\left(8\,b^5x^{10}-4\,ab^4x^8+3\,a^2b^3x^6+113\,a^3b^2x^4+161\,a^4bx^2+63\,a^5\right)\sqrt{bx^2+a}}{693\,a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(8*b^5*x^10 - 4*a*b^4*x^8 + 3*a^2*b^3*x^6 + 113*a^3*b^2*x^4 + 161*a^
4*b*x^2 + 63*a^5)*sqrt(b*x^2 + a)/(a^3*x^11)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(61) = 122$.
time = 1.08, size = 481, normalized size = 7.07

$$-\frac{63a^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^2\cdot(693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12})}-\frac{287a^6b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{498a^5b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{390a^4b^{\frac{15}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{115a^3b^{\frac{17}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{3a^2b^{\frac{19}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{12ab^{\frac{21}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{8b^{\frac{23}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**12,x)
```

```
[Out] -63*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**2*(693*a**5*b**4*x**8 + 1386*a**
4*b**5*x**10 + 693*a**3*b**6*x**12)) - 287*a**6*b**(11/2)*sqrt(a/(b*x**2) +
 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 498
*a**5*b**(13/2)*x**2*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b
**5*x**10 + 693*a**3*b**6*x**12) - 390*a**4*b**(15/2)*x**4*sqrt(a/(b*x**2)
+ 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 11
5*a**3*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*
b**5*x**10 + 693*a**3*b**6*x**12) - 3*a**2*b**(19/2)*x**8*sqrt(a/(b*x**2) +
 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 12*
a*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5
*x**10 + 693*a**3*b**6*x**12) - 8*b**(23/2)*x**12*sqrt(a/(b*x**2) + 1)/(693
*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(56) = 112$.
time = 0.78, size = 246, normalized size = 3.62

$$\frac{16\left(462\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}b^{\frac{11}{2}}+1155\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}ab^{\frac{11}{2}}+2541\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^2b^{\frac{11}{2}}+2079\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^3b^{\frac{11}{2}}+1485\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^4b^{\frac{11}{2}}+297\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^5b^{\frac{11}{2}}+55\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^6b^{\frac{11}{2}}-11\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^7b^{\frac{11}{2}}+a^8b^{\frac{11}{2}}\right)}{693\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 16/693*(462*(sqrt(b)*x - sqrt(b*x^2 + a))^16*b^(11/2) + 1155*(sqrt(b)*x - s
qrt(b*x^2 + a))^14*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b
^(11/2) + 2079*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(11/2) + 1485*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a^4*b^(11/2) + 297*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*a^5*b^(11/2) + 55*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(11/2) - 11*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^7*b^(11/2) + a^8*b^(11/2))/((sqrt(b)*x - sqrt(b
*x^2 + a))^2 - a)^11
```

Mupad [B]
time = 5.62, size = 111, normalized size = 1.63

$$\frac{4 b^4 \sqrt{b x^2+a}}{693 a^2 x^3}-\frac{113 b^2 \sqrt{b x^2+a}}{693 x^7}-\frac{b^3 \sqrt{b x^2+a}}{231 a x^5}-\frac{a^2 \sqrt{b x^2+a}}{11 x^{11}}-\frac{8 b^5 \sqrt{b x^2+a}}{693 a^3 x}-\frac{23 a b \sqrt{b x^2+a}}{99 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^12,x)
```

```
[Out] (4*b^4*(a + b*x^2)^(1/2))/(693*a^2*x^3) - (113*b^2*(a + b*x^2)^(1/2))/(693*
x^7) - (b^3*(a + b*x^2)^(1/2))/(231*a*x^5) - (a^2*(a + b*x^2)^(1/2))/(11*x^
11) - (8*b^5*(a + b*x^2)^(1/2))/(693*a^3*x) - (23*a*b*(a + b*x^2)^(1/2))/(9
9*x^9)
```

3.406 $\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=92

$$-\frac{(a+bx^2)^{7/2}}{13ax^{13}}+\frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}}-\frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9}+\frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7}$$

```
[Out] -1/13*(b*x^2+a)^(7/2)/a/x^13+6/143*b*(b*x^2+a)^(7/2)/a^2/x^11-8/429*b^2*(b*
x^2+a)^(7/2)/a^3/x^9+16/3003*b^3*(b*x^2+a)^(7/2)/a^4/x^7
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$\frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7}-\frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9}+\frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^14,x]
```

```
[Out] -1/13*(a + b*x^2)^(7/2)/(a*x^13) + (6*b*(a + b*x^2)^(7/2))/(143*a^2*x^11) -
 (8*b^2*(a + b*x^2)^(7/2))/(429*a^3*x^9) + (16*b^3*(a + b*x^2)^(7/2))/(3003
*a^4*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx = -\frac{(a+bx^2)^{7/2}}{13ax^{13}} - \frac{(6b)\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx}{13a}$$
$$= -\frac{(a+bx^2)^{7/2}}{13ax^{13}} + \frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}} + \frac{(24b^2)\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx}{143a^2}$$
$$= -\frac{(a+bx^2)^{7/2}}{13ax^{13}} + \frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9} - \frac{(16b^3)\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx}{429a^3}$$
$$= -\frac{(a+bx^2)^{7/2}}{13ax^{13}} + \frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9} + \frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7}$$

Mathematica [A]
time = 0.11, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{7/2}\left(-231a^3+126a^2bx^2-56ab^2x^4+16b^3x^6\right)}{3003a^4x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^14,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-231*a^3 + 126*a^2*b*x^2 - 56*a*b^2*x^4 + 16*b^3*x^6))/
(3003*a^4*x^13)
```

Maple [A]
time = 0.21, size = 85, normalized size = 0.92

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}\left(-16b^3x^6+56a\,b^2x^4-126a^2b\,x^2+231a^3\right)}{3003x^{13}a^4}$	50
trager	$-\frac{\left(-16b^6x^{12}+8a\,b^5x^{10}-6a^2b^4x^8+5a^3x^6b^3+371a^4b^2x^4+567a^5b\,x^2+231a^6\right)\sqrt{b\,x^2+a}}{3003x^{13}a^4}$	83
risch	$-\frac{\left(-16b^6x^{12}+8a\,b^5x^{10}-6a^2b^4x^8+5a^3x^6b^3+371a^4b^2x^4+567a^5b\,x^2+231a^6\right)\sqrt{b\,x^2+a}}{3003x^{13}a^4}$	83
default	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{13a\,x^{13}}-\frac{6b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{11a\,x^{11}}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{9a\,x^9}+\frac{2b(b\,x^2+a)^{\frac{7}{2}}}{63a^2x^7}\right)}{11a}\right)}{13a}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^14,x,method=_RETURNVERBOSE)
```

[Out] -1/13*(b*x^2+a)^(7/2)/a/x^13-6/13*b/a*(-1/11*(b*x^2+a)^(7/2)/a/x^11-4/11*b/a*(-1/9*(b*x^2+a)^(7/2)/a/x^9+2/63*b*(b*x^2+a)^(7/2)/a^2/x^7))

Maxima [A]
time = 0.36, size = 76, normalized size = 0.83

$$\frac{16\,(bx^2+a)^{\frac{7}{2}}b^3}{3003\,a^4x^7}-\frac{8\,(bx^2+a)^{\frac{7}{2}}b^2}{429\,a^3x^9}+\frac{6\,(bx^2+a)^{\frac{7}{2}}b}{143\,a^2x^{11}}-\frac{(bx^2+a)^{\frac{7}{2}}}{13\,ax^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="maxima")

[Out] 16/3003*(b*x^2 + a)^(7/2)*b^3/(a^4*x^7) - 8/429*(b*x^2 + a)^(7/2)*b^2/(a^3*x^9) + 6/143*(b*x^2 + a)^(7/2)*b/(a^2*x^11) - 1/13*(b*x^2 + a)^(7/2)/(a*x^13)

Fricas [A]
time = 1.17, size = 82, normalized size = 0.89

$$\frac{(16\,b^6x^{12}-8\,ab^5x^{10}+6\,a^2b^4x^8-5\,a^3b^3x^6-371\,a^4b^2x^4-567\,a^5bx^2-231\,a^6)\sqrt{bx^2+a}}{3003\,a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="fricas")

[Out] 1/3003*(16*b^6*x^12 - 8*a*b^5*x^10 + 6*a^2*b^4*x^8 - 5*a^3*b^3*x^6 - 371*a^4*b^2*x^4 - 567*a^5*b*x^2 - 231*a^6)*sqrt(b*x^2 + a)/(a^4*x^13)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(85) = 170.
time = 1.42, size = 721, normalized size = 7.84

$$-\frac{231a^9b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{1260a^8b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{2765a^7b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{3050a^6b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{1689a^5b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{376a^4b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}+\frac{5a^3b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}+\frac{30a^2b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}+\frac{40ab^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}+\frac{16b^{\frac{37}{2}}x^{18}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(5/2)/x**14,x)

[Out] -231*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1260*a**8*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 2765*a**7*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 3050*a**6*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1689*a**5*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**1

```
1*x**16 + 3003*a**4*b**12*x**18) - 376*a**4*b**(29/2)*x**10*sqrt(a/(b*x**2)
 + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16
 + 3003*a**4*b**12*x**18) + 5*a**3*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(30
03*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a
**4*b**12*x**18) + 30*a**2*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(3003*a**7*
b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**1
2*x**18) + 40*a*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12
+ 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) +
16*b**(37/2)*x**18*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b
**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(76) = 152$.
time = 0.73, size = 274, normalized size = 2.98

$$\frac{32\left(3003\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{18}b^{\frac{13}{2}}+9009\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{16}ab^{\frac{13}{2}}+18018\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{14}a^2b^{\frac{13}{2}}+16302\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}a^3b^{\frac{13}{2}}+10296\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a^4b^{\frac{13}{2}}+2288\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}a^5b^{\frac{13}{2}}+286\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{6}a^6b^{\frac{13}{2}}-78\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^7b^{\frac{13}{2}}+13\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{2}a^8b^{\frac{13}{2}}-a^9b^{\frac{13}{2}}\right)}{3003\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 32/3003*(3003*(sqrt(b)*x - sqrt(b*x^2 + a))^18*b^(13/2) + 9009*(sqrt(b)*x -
 sqrt(b*x^2 + a))^16*a*b^(13/2) + 18018*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^
2*b^(13/2) + 16302*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(13/2) + 10296*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(13/2) + 2288*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^5*b^(13/2) + 286*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(13/2) - 7
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(13/2) + 13*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^8*b^(13/2) - a^9*b^(13/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
^13
```

Mupad [B]
time = 5.98, size = 131, normalized size = 1.42

$$\frac{2\,b^4\,\sqrt{b\,x^2+a}}{1001\,a^2\,x^5}-\frac{53\,b^2\,\sqrt{b\,x^2+a}}{429\,x^9}-\frac{5\,b^3\,\sqrt{b\,x^2+a}}{3003\,a\,x^7}-\frac{a^2\,\sqrt{b\,x^2+a}}{13\,x^{13}}-\frac{8\,b^5\,\sqrt{b\,x^2+a}}{3003\,a^3\,x^3}+\frac{16\,b^6\,\sqrt{b\,x^2+a}}{3003\,a^4\,x}-\frac{27\,a\,b\,\sqrt{b\,x^2+a}}{143\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^14,x)
```

```
[Out] (2*b^4*(a + b*x^2)^(1/2))/(1001*a^2*x^5) - (53*b^2*(a + b*x^2)^(1/2))/(429*
x^9) - (5*b^3*(a + b*x^2)^(1/2))/(3003*a*x^7) - (a^2*(a + b*x^2)^(1/2))/(13
*x^13) - (8*b^5*(a + b*x^2)^(1/2))/(3003*a^3*x^3) + (16*b^6*(a + b*x^2)^(1/
2))/(3003*a^4*x) - (27*a*b*(a + b*x^2)^(1/2))/(143*x^11)
```

3.407 $\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=116

$$-\frac{(a+bx^2)^{7/2}}{15ax^{15}}+\frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}}-\frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}}+\frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9}-\frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7}$$

```
[Out] -1/15*(b*x^2+a)^(7/2)/a/x^15+8/195*b*(b*x^2+a)^(7/2)/a^2/x^13-16/715*b^2*(b
*x^2+a)^(7/2)/a^3/x^11+64/6435*b^3*(b*x^2+a)^(7/2)/a^4/x^9-128/45045*b^4*(b
*x^2+a)^(7/2)/a^5/x^7
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 270}

$$-\frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7}+\frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9}-\frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}}+\frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^16,x]
```

```
[Out] -1/15*(a + b*x^2)^(7/2)/(a*x^15) + (8*b*(a + b*x^2)^(7/2))/(195*a^2*x^13) -
 (16*b^2*(a + b*x^2)^(7/2))/(715*a^3*x^11) + (64*b^3*(a + b*x^2)^(7/2))/(64
35*a^4*x^9) - (128*b^4*(a + b*x^2)^(7/2))/(45045*a^5*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx = -\frac{(a+bx^2)^{7/2}}{15ax^{15}} - \frac{(8b)\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx}{15a}$$
$$= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} + \frac{(16b^2)\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx}{65a^2}$$
$$= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} - \frac{(64b^3)\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx}{715a^3}$$
$$= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} + \frac{(128b^4)\int \frac{(a}{}}{6435}$$
$$= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} - \frac{128b^4(a+b}{45045a^5}$$

Mathematica [A]
time = 0.12, size = 64, normalized size = 0.55

$$\frac{(a+bx^2)^{7/2}(-3003a^4+1848a^3bx^2-1008a^2b^2x^4+448ab^3x^6-128b^4x^8)}{45045a^5x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^16,x]
[Out] ((a + b*x^2)^(7/2)*(-3003*a^4 + 1848*a^3*b*x^2 - 1008*a^2*b^2*x^4 + 448*a*b
^3*x^6 - 128*b^4*x^8))/(45045*a^5*x^15)
```

Maple [A]
time = 0.42, size = 109, normalized size = 0.94

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}(128b^4x^8-448a\,b^3x^6+1008a^2b^2x^4-1848a^3b\,x^2+3003a^4)}{45045x^{15}a^5}$	61
trager	$-\frac{(128b^7x^{14}-64a\,b^6x^{12}+48a^2b^5x^{10}-40a^3b^4x^8+35a^4b^3x^6+4473a^5b^2x^4+7161a^6b\,x^2+3003a^7)\sqrt{b\,x^2+a}}{45045x^{15}a^5}$	94
risch	$-\frac{(128b^7x^{14}-64a\,b^6x^{12}+48a^2b^5x^{10}-40a^3b^4x^8+35a^4b^3x^6+4473a^5b^2x^4+7161a^6b\,x^2+3003a^7)\sqrt{b\,x^2+a}}{45045x^{15}a^5}$	94

default	$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{15a\,x^{15}}-\frac{8b\left(-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{13a\,x^{13}}-\frac{6b\left(-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{11a\,x^{11}}-\frac{4b\left(-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{9a\,x^{9}}+\frac{2b\left(b\,x^2+a\right)^{\frac{7}{2}}}{63a^2x^7}\right)}{11a}\right)}{13a}\right)}{15a}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(b*x^2+a)^(7/2)/a/x^15-8/15*b/a*(-1/13*(b*x^2+a)^(7/2)/a/x^13-6/13*b/
a*(-1/11*(b*x^2+a)^(7/2)/a/x^11-4/11*b/a*(-1/9*(b*x^2+a)^(7/2)/a/x^9+2/63*b
*(b*x^2+a)^(7/2)/a^2/x^7)))
```

Maxima [A]
time = 0.29, size = 96, normalized size = 0.83

$$-\frac{128\,(bx^2+a)^{\frac{7}{2}}b^4}{45045\,a^5x^7}+\frac{64\,(bx^2+a)^{\frac{7}{2}}b^3}{6435\,a^4x^9}-\frac{16\,(bx^2+a)^{\frac{7}{2}}b^2}{715\,a^3x^{11}}+\frac{8\,(bx^2+a)^{\frac{7}{2}}b}{195\,a^2x^{13}}-\frac{(bx^2+a)^{\frac{7}{2}}}{15\,ax^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] -128/45045*(b*x^2 + a)^(7/2)*b^4/(a^5*x^7) + 64/6435*(b*x^2 + a)^(7/2)*b^3/
(a^4*x^9) - 16/715*(b*x^2 + a)^(7/2)*b^2/(a^3*x^11) + 8/195*(b*x^2 + a)^(7/
2)*b/(a^2*x^13) - 1/15*(b*x^2 + a)^(7/2)/(a*x^15)
```

Fricas [A]
time = 1.72, size = 93, normalized size = 0.80

$$-\frac{(128\,b^7x^{14}-64\,ab^6x^{12}+48\,a^2b^5x^{10}-40\,a^3b^4x^8+35\,a^4b^3x^6+4473\,a^5b^2x^4+7161\,a^6bx^2+3003\,a^7)\sqrt{bx^2+a}}{45045\,a^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(128*b^7*x^14 - 64*a*b^6*x^12 + 48*a^2*b^5*x^10 - 40*a^3*b^4*x^8 +
 35*a^4*b^3*x^6 + 4473*a^5*b^2*x^4 + 7161*a^6*b*x^2 + 3003*a^7)*sqrt(b*x^2
+ a)/(a^5*x^15)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(109) = 218$.
time = 1.83, size = 1012, normalized size = 8.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**16,x)
```

```
[Out] -3003*a**11*b**(33/2)*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180
*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 450
45*a**5*b**20*x**22) - 19173*a**10*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(450
45*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 1
80180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 51135*a**9*b**(37/2)*x**
4*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 +
270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22)
 - 72905*a**8*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 +
 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**2
0 + 45045*a**5*b**20*x**22) - 58585*a**7*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1
)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**
18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 25151*a**6*b**(43/
2)*x**10*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x
**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20
*x**22) - 4501*a**5*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*
x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**
19*x**20 + 45045*a**5*b**20*x**22) - 35*a**4*b**(47/2)*x**14*sqrt(a/(b*x**2
) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**1
8*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 280*a**3*b**(
49/2)*x**16*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**1
7*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b*
*20*x**22) - 560*a**2*b**(51/2)*x**18*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**1
6*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b
**19*x**20 + 45045*a**5*b**20*x**22) - 448*a*b**(53/2)*x**20*sqrt(a/(b*x**2
) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**1
8*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 128*b**(55/2)
*x**22*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**
16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x
**22)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(96) = 192$.
time = 0.76, size = 300, normalized size = 2.59

$$\frac{256\left(18018\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{20}b^{\frac{15}{2}}+60060\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{18}ab^{\frac{15}{2}}+115830\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{16}a^2b^{\frac{15}{2}}+109395\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{14}a^3b^{\frac{15}{2}}+65065\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}a^4b^{\frac{15}{2}}+15015\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a^5b^{\frac{15}{2}}+1365\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}a^6b^{\frac{15}{2}}-455\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{6}a^7b^{\frac{15}{2}}+105\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^8b^{\frac{15}{2}}-15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{2}a^9b^{\frac{15}{2}}+a^{10}b^{\frac{15}{2}}\right)}{45045\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] 256/45045*(18018*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(15/2) + 60060*(sqrt(b)
*x - sqrt(b*x^2 + a))^18*a*b^(15/2) + 115830*(sqrt(b)*x - sqrt(b*x^2 + a))^
16*a^2*b^(15/2) + 109395*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*b^(15/2) + 65
```

```
065*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(15/2) + 15015*(sqrt(b)*x - sqrt
(b*x^2 + a))^10*a^5*b^(15/2) + 1365*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(
15/2) - 455*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^7*b^(15/2) + 105*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a^8*b^(15/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^9*b
^(15/2) + a^10*b^(15/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^15
```

Mupad [B]
time = 6.37, size = 151, normalized size = 1.30

$$\frac{8\,b^4\,\sqrt{b\,x^2+a}}{9009\,a^2\,x^7}-\frac{71\,b^2\,\sqrt{b\,x^2+a}}{715\,x^{11}}-\frac{b^3\,\sqrt{b\,x^2+a}}{1287\,a\,x^9}-\frac{a^2\,\sqrt{b\,x^2+a}}{15\,x^{15}}-\frac{16\,b^5\,\sqrt{b\,x^2+a}}{15015\,a^3\,x^5}+\frac{64\,b^6\,\sqrt{b\,x^2+a}}{45045\,a^4\,x^3}-\frac{128\,b^7\,\sqrt{b\,x^2+a}}{45045\,a^5\,x}-\frac{31\,a\,b\,\sqrt{b\,x^2+a}}{195\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^16,x)
```

```
[Out] (8*b^4*(a + b*x^2)^(1/2))/(9009*a^2*x^7) - (71*b^2*(a + b*x^2)^(1/2))/(715*
x^11) - (b^3*(a + b*x^2)^(1/2))/(1287*a*x^9) - (a^2*(a + b*x^2)^(1/2))/(15*
x^15) - (16*b^5*(a + b*x^2)^(1/2))/(15015*a^3*x^5) + (64*b^6*(a + b*x^2)^(1
/2))/(45045*a^4*x^3) - (128*b^7*(a + b*x^2)^(1/2))/(45045*a^5*x) - (31*a*b*
(a + b*x^2)^(1/2))/(195*x^13)
```

3.408 $\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=140

$$-\frac{(a+bx^2)^{7/2}}{17ax^{17}}+\frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}}-\frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}}+\frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}}-\frac{128b^4(a+bx^2)^{7/2}}{21879a^5x^9}+\frac{256b^5(a+bx^2)^{7/2}}{153153a^6x^7}$$

```
[Out] -1/17*(b*x^2+a)^(7/2)/a/x^17+2/51*b*(b*x^2+a)^(7/2)/a^2/x^15-16/663*b^2*(b*
x^2+a)^(7/2)/a^3/x^13+32/2431*b^3*(b*x^2+a)^(7/2)/a^4/x^11-128/21879*b^4*(b
*x^2+a)^(7/2)/a^5/x^9+256/153153*b^5*(b*x^2+a)^(7/2)/a^6/x^7
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$\frac{256b^5(a+bx^2)^{7/2}}{153153a^6x^7}-\frac{128b^4(a+bx^2)^{7/2}}{21879a^5x^9}+\frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}}-\frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}}+\frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}}-\frac{(a+bx^2)^{7/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^18,x]
```

```
[Out] -1/17*(a + b*x^2)^(7/2)/(a*x^17) + (2*b*(a + b*x^2)^(7/2))/(51*a^2*x^15) -
(16*b^2*(a + b*x^2)^(7/2))/(663*a^3*x^13) + (32*b^3*(a + b*x^2)^(7/2))/(243
1*a^4*x^11) - (128*b^4*(a + b*x^2)^(7/2))/(21879*a^5*x^9) + (256*b^5*(a + b
*x^2)^(7/2))/(153153*a^6*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx = -\frac{(a+bx^2)^{7/2}}{17ax^{17}} - \frac{(10b)\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx}{17a}$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} + \frac{(16b^2)\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx}{51a^2}$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}} - \frac{(32b^3)\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx}{221a^3}$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}} + \frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}} + \frac{(128b^4)\int \frac{(a+}{}}{2431a}$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}} + \frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}} - \frac{128b^4(a+bx}{21879a^5x}$$

$$= -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}} + \frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}} - \frac{128b^4(a+bx}{21879a^5x}$$

Mathematica [A]
time = 0.14, size = 75, normalized size = 0.54

$$\frac{(a+bx^2)^{7/2}\left(-9009a^5+6006a^4bx^2-3696a^3b^2x^4+2016a^2b^3x^6-896ab^4x^8+256b^5x^{10}\right)}{153153a^6x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^18,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-9009*a^5 + 6006*a^4*b*x^2 - 3696*a^3*b^2*x^4 + 2016*a^
2*b^3*x^6 - 896*a*b^4*x^8 + 256*b^5*x^10))/(153153*a^6*x^17)
```

Maple [A]
time = 0.99, size = 133, normalized size = 0.95

method	result	si
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}\left(-256b^5x^{10}+896a\,b^4x^8-2016a^2b^3x^6+3696a^3b^2x^4-6006a^4b\,x^2+9009a^5\right)}{153153x^{17}a^6}$	72
trager	$-\frac{\left(-256b^8x^{16}+128a\,b^7x^{14}-96a^2b^6x^{12}+80a^3b^5x^{10}-70a^4b^4x^8+63a^5b^3x^6+12705a^6b^2x^4+21021a^7b\,x^2+9009a^8\right)\sqrt{b\,x^2+a}}{153153x^{17}a^6}$	10
risch	$-\frac{\left(-256b^8x^{16}+128a\,b^7x^{14}-96a^2b^6x^{12}+80a^3b^5x^{10}-70a^4b^4x^8+63a^5b^3x^6+12705a^6b^2x^4+21021a^7b\,x^2+9009a^8\right)\sqrt{b\,x^2+a}}{153153x^{17}a^6}$	10

default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{17 a\,x^{17}}-\frac{10 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{15 a\,x^{15}}-\frac{8 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{13 a\,x^{13}}-\frac{6 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{11 a\,x^{11}}-\frac{4 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9 a\,x^{9}}+\frac{2 b \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63 a^{2} x^{7}}\right)}{11 a}\right)}{13 a}\right)}{15 a}\right)}{17 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -1/17*(b*x^2+a)^(7/2)/a/x^17-10/17*b/a*(-1/15*(b*x^2+a)^(7/2)/a/x^15-8/15*b
/a*(-1/13*(b*x^2+a)^(7/2)/a/x^13-6/13*b/a*(-1/11*(b*x^2+a)^(7/2)/a/x^11-4/1
1*b/a*(-1/9*(b*x^2+a)^(7/2)/a/x^9+2/63*b*(b*x^2+a)^(7/2)/a^2/x^7))))
```

Maxima [A]
time = 0.29, size = 116, normalized size = 0.83

$$\frac{256\,(bx^2+a)^{\frac{7}{2}}b^5}{153153\,a^6x^7}-\frac{128\,(bx^2+a)^{\frac{7}{2}}b^4}{21879\,a^5x^9}+\frac{32\,(bx^2+a)^{\frac{7}{2}}b^3}{2431\,a^4x^{11}}-\frac{16\,(bx^2+a)^{\frac{7}{2}}b^2}{663\,a^3x^{13}}+\frac{2\,(bx^2+a)^{\frac{7}{2}}b}{51\,a^2x^{15}}-\frac{(bx^2+a)^{\frac{7}{2}}}{17\,ax^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] 256/153153*(b*x^2 + a)^(7/2)*b^5/(a^6*x^7) - 128/21879*(b*x^2 + a)^(7/2)*b^
4/(a^5*x^9) + 32/2431*(b*x^2 + a)^(7/2)*b^3/(a^4*x^11) - 16/663*(b*x^2 + a)
^(7/2)*b^2/(a^3*x^13) + 2/51*(b*x^2 + a)^(7/2)*b/(a^2*x^15) - 1/17*(b*x^2 +
 a)^(7/2)/(a*x^17)
```

Fricas [A]
time = 1.15, size = 104, normalized size = 0.74

$$\frac{(256\,b^8x^{16}-128\,ab^7x^{14}+96\,a^2b^6x^{12}-80\,a^3b^5x^{10}+70\,a^4b^4x^8-63\,a^5b^3x^6-12705\,a^6b^2x^4-21021\,a^7bx^2-9009\,a^8)\sqrt{bx^2+a}}{153153\,a^6x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] 1/153153*(256*b^8*x^16 - 128*a*b^7*x^14 + 96*a^2*b^6*x^12 - 80*a^3*b^5*x^10
 + 70*a^4*b^4*x^8 - 63*a^5*b^3*x^6 - 12705*a^6*b^2*x^4 - 21021*a^7*b*x^2 -
9009*a^8)*sqrt(b*x^2 + a)/(a^6*x^17)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. $2(133) = 266$.
time = 2.36, size = 1346, normalized size = 9.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**18,x)
```

```
[Out] -9009*a**13*b**(51/2)*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 7657
65*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22
+ 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 66066*a**12*b**(53/2
)*x**2*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*
x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b
**29*x**24 + 153153*a**6*b**30*x**26) - 207900*a**11*b**(55/2)*x**4*sqrt(a/
(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 153153
0*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 1
53153*a**6*b**30*x**26) - 363888*a**10*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/
(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x
**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**
30*x**26) - 382550*a**9*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(153153*a**11*b
**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*
a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 241
524*a**8*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 7
65765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**
22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 84780*a**7*b**(63
/2)*x**12*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**
26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**
7*b**29*x**24 + 153153*a**6*b**30*x**26) - 12768*a**6*b**(65/2)*x**14*sqrt(
a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531
530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 +
 153153*a**6*b**30*x**26) + 63*a**5*b**(67/2)*x**16*sqrt(a/(b*x**2) + 1)/(1
53153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**
20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30
*x**26) + 630*a**4*b**(69/2)*x**18*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25
*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8
*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 1680*a*
*3*b**(71/2)*x**20*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*
a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 7
65765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 2016*a**2*b**(73/2)*x**
```

```
22*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**1
8 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29
*x**24 + 153153*a**6*b**30*x**26) + 1152*a*b**(75/2)*x**24*sqrt(a/(b*x**2)
+ 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b*
*27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**
6*b**30*x**26) + 256*b**(77/2)*x**26*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**
25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a*
*8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(116) = 232.
time = 1.54, size = 328, normalized size = 2.34

$$\frac{512\left(102102\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}b^{\frac{17}{2}}+364650\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}ab^{\frac{17}{2}}+692835\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^2b^{\frac{17}{2}}+668525\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^3b^{\frac{17}{2}}+384098\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^4b^{\frac{17}{2}}+89726\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^5b^{\frac{17}{2}}+6188\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^6b^{\frac{17}{2}}-2380\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^7b^{\frac{17}{2}}+680\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^8b^{\frac{17}{2}}-136\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^9b^{\frac{17}{2}}+17\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{10}b^{\frac{17}{2}}-a^{11}b^{\frac{17}{2}}\right)}{153153\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] 512/153153*(102102*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(17/2) + 364650*(sqrt
(b)*x - sqrt(b*x^2 + a))^20*a*b^(17/2) + 692835*(sqrt(b)*x - sqrt(b*x^2 + a
))^18*a^2*b^(17/2) + 668525*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(17/2) +
 384098*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*b^(17/2) + 89726*(sqrt(b)*x -
sqrt(b*x^2 + a))^12*a^5*b^(17/2) + 6188*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^
6*b^(17/2) - 2380*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(17/2) + 680*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*a^8*b^(17/2) - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*a^9*b^(17/2) + 17*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(17/2) - a^11*b^
(17/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^17
```

Mupad [B]
time = 6.87, size = 171, normalized size = 1.22

$$\frac{10\,b^4\,\sqrt{b\,x^2+a}}{21879\,a^2\,x^9}-\frac{55\,b^2\,\sqrt{b\,x^2+a}}{663\,x^{13}}-\frac{b^3\,\sqrt{b\,x^2+a}}{2431\,a\,x^{11}}-\frac{a^2\,\sqrt{b\,x^2+a}}{17\,x^{17}}-\frac{80\,b^5\,\sqrt{b\,x^2+a}}{153153\,a^3\,x^7}+\frac{32\,b^6\,\sqrt{b\,x^2+a}}{51051\,a^4\,x^5}-\frac{128\,b^7\,\sqrt{b\,x^2+a}}{153153\,a^5\,x^3}+\frac{256\,b^8\,\sqrt{b\,x^2+a}}{153153\,a^6\,x}-\frac{7\,a\,b\,\sqrt{b\,x^2+a}}{51\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^18,x)
```

```
[Out] (10*b^4*(a + b*x^2)^(1/2))/(21879*a^2*x^9) - (55*b^2*(a + b*x^2)^(1/2))/(66
3*x^13) - (b^3*(a + b*x^2)^(1/2))/(2431*a*x^11) - (a^2*(a + b*x^2)^(1/2))/(
17*x^17) - (80*b^5*(a + b*x^2)^(1/2))/(153153*a^3*x^7) + (32*b^6*(a + b*x^2
)^(1/2))/(51051*a^4*x^5) - (128*b^7*(a + b*x^2)^(1/2))/(153153*a^5*x^3) + (
256*b^8*(a + b*x^2)^(1/2))/(153153*a^6*x) - (7*a*b*(a + b*x^2)^(1/2))/(51*x
^15)
```

3.409 $\int x^{15}(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=161

$$-\frac{a^7(a+bx^2)^{11/2}}{11b^8}+\frac{7a^6(a+bx^2)^{13/2}}{13b^8}-\frac{7a^5(a+bx^2)^{15/2}}{5b^8}+\frac{35a^4(a+bx^2)^{17/2}}{17b^8}-\frac{35a^3(a+bx^2)^{19/2}}{19b^8}+\frac{a^2(a+bx^2)^{21}}{b^8}$$

```
[Out] -1/11*a^7*(b*x^2+a)^(11/2)/b^8+7/13*a^6*(b*x^2+a)^(13/2)/b^8-7/5*a^5*(b*x^2
+a)^(15/2)/b^8+35/17*a^4*(b*x^2+a)^(17/2)/b^8-35/19*a^3*(b*x^2+a)^(19/2)/b^
8+a^2*(b*x^2+a)^(21/2)/b^8-7/23*a*(b*x^2+a)^(23/2)/b^8+1/25*(b*x^2+a)^(25/2
)/b^8
```

Rubi [A]
time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$-\frac{a^7(a+bx^2)^{11/2}}{11b^8}+\frac{7a^6(a+bx^2)^{13/2}}{13b^8}-\frac{7a^5(a+bx^2)^{15/2}}{5b^8}+\frac{35a^4(a+bx^2)^{17/2}}{17b^8}-\frac{35a^3(a+bx^2)^{19/2}}{19b^8}+\frac{a^2(a+bx^2)^{21/2}}{b^8}+\frac{(a+bx^2)^{25/2}}{25b^8}-\frac{7a(a+bx^2)^{23/2}}{23b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(a + b*x^2)^(9/2),x]
```

```
[Out] -1/11*(a^7*(a + b*x^2)^(11/2))/b^8 + (7*a^6*(a + b*x^2)^(13/2))/(13*b^8) -
(7*a^5*(a + b*x^2)^(15/2))/(5*b^8) + (35*a^4*(a + b*x^2)^(17/2))/(17*b^8) -
 (35*a^3*(a + b*x^2)^(19/2))/(19*b^8) + (a^2*(a + b*x^2)^(21/2))/b^8 - (7*a
*(a + b*x^2)^(23/2))/(23*b^8) + (a + b*x^2)^(25/2)/(25*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{15}\left(a+bx^2\right)^{9/2}\,dx = \frac{1}{2}\text{Subst}\left(\int x^7(a+bx)^{9/2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^7(a+bx)^{9/2}}{b^7} + \frac{7a^6(a+bx)^{11/2}}{b^7} - \frac{21a^5(a+bx)^{13/2}}{b^7} + \frac{35a^4(a+bx}{b^7}\right.\right.$$
$$= -\frac{a^7\left(a+bx^2\right)^{11/2}}{11b^8} + \frac{7a^6\left(a+bx^2\right)^{13/2}}{13b^8} - \frac{7a^5\left(a+bx^2\right)^{15/2}}{5b^8} + \frac{35a^4\left(a+bx^2\right)^{17/2}}{17b^8} - 35$$

Mathematica [A]
time = 0.06, size = 94, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{11/2}\left(-2048a^7+11264a^6bx^2-36608a^5b^2x^4+91520a^4b^3x^6-194480a^3b^4x^8+369512a^2b^5x^{10}-646646ab^6x^{12}+1062347b^7x^{14}\right)}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(a + b*x^2)^(9/2),x]
[Out] ((a + b*x^2)^(11/2)*(-2048*a^7 + 11264*a^6*b*x^2 - 36608*a^5*b^2*x^4 + 9152
0*a^4*b^3*x^6 - 194480*a^3*b^4*x^8 + 369512*a^2*b^5*x^10 - 646646*a*b^6*x^1
2 + 1062347*b^7*x^14))/(26558675*b^8)
```

Maple [A]
time = 0.46, size = 178, normalized size = 1.11

method	result
gosper	$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(-1062347b^7x^{14}+646646a\,b^6x^{12}-369512a^2b^5x^{10}+194480a^3b^4x^8-91520a^4b^3x^6+36608a^5b^2x^4-11264a^6b\,x^2+2048a\right.}{26558675b^8}$
trager	$-\frac{\left(-1062347b^{12}x^{24}-4665089a\,b^{11}x^{22}-7759752a^2b^{10}x^{20}-5810090a^3b^9x^{18}-1659515a^4b^8x^{16}-429a^5b^7x^{14}+462a^6b^6x^{12}-504a^7b^5x\right.}{26558675b^8}$
risch	$-\frac{\left(-1062347b^{12}x^{24}-4665089a\,b^{11}x^{22}-7759752a^2b^{10}x^{20}-5810090a^3b^9x^{18}-1659515a^4b^8x^{16}-429a^5b^7x^{14}+462a^6b^6x^{12}-504a^7b^5x\right.}{26558675b^8}$

$$4a\left(\frac{x^2(}{}\right.$$

$$6a\left(\frac{x^4\left(b\,x^2+a\right)^{\frac{11}{2}}}{15b}-\right.$$

$$8a\left(\frac{x^6\left(b\,x^2+a\right)^{\frac{11}{2}}}{17b}-\right.$$

$$10a\left(\frac{x^8\left(b\,x^2+a\right)^{\frac{11}{2}}}{19b}-\frac{}{19b}\right.$$

$$12a\left(\frac{x^{10}\left(b\,x^2+a\right)^{\frac{11}{2}}}{21b}-\frac{}{21b}\right.$$

$$14a\left(\frac{x^{12}\left(b\,x^2+a\right)^{\frac{11}{2}}}{23b}-\frac{}{23b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/25*x^14*(b*x^2+a)^(11/2)/b-14/25*a/b*(1/23*x^12*(b*x^2+a)^(11/2)/b-12/23*
a/b*(1/21*x^10*(b*x^2+a)^(11/2)/b-10/21*a/b*(1/19*x^8*(b*x^2+a)^(11/2)/b-8/
19*a/b*(1/17*x^6*(b*x^2+a)^(11/2)/b-6/17*a/b*(1/15*x^4*(b*x^2+a)^(11/2)/b-4
/15*a/b*(1/13*x^2*(b*x^2+a)^(11/2)/b-2/143*a*(b*x^2+a)^(11/2)/b^2))))))
```

Maxima [A]
time = 0.29, size = 153, normalized size = 0.95

$$\frac{(bx^2+a)^{\frac{11}{2}}x^{14}}{25\,b}-\frac{14\,(bx^2+a)^{\frac{11}{2}}ax^{12}}{575\,b^2}+\frac{8\,(bx^2+a)^{\frac{11}{2}}a^2x^{10}}{575\,b^3}-\frac{16\,(bx^2+a)^{\frac{11}{2}}a^3x^8}{2185\,b^4}+\frac{128\,(bx^2+a)^{\frac{11}{2}}a^4x^6}{37145\,b^5}-\frac{256\,(bx^2+a)^{\frac{11}{2}}a^5x^4}{185725\,b^6}+\frac{1024\,(bx^2+a)^{\frac{11}{2}}a^6x^2}{2414425\,b^7}-\frac{2048\,(bx^2+a)^{\frac{11}{2}}a^7}{26558675\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/25*(b*x^2 + a)^(11/2)*x^14/b - 14/575*(b*x^2 + a)^(11/2)*a*x^12/b^2 + 8/5
75*(b*x^2 + a)^(11/2)*a^2*x^10/b^3 - 16/2185*(b*x^2 + a)^(11/2)*a^3*x^8/b^4
 + 128/37145*(b*x^2 + a)^(11/2)*a^4*x^6/b^5 - 256/185725*(b*x^2 + a)^(11/2)
*a^5*x^4/b^6 + 1024/2414425*(b*x^2 + a)^(11/2)*a^6*x^2/b^7 - 2048/26558675*
(b*x^2 + a)^(11/2)*a^7/b^8
```

Fricas [A]
time = 1.11, size = 145, normalized size = 0.90

$$\frac{\left(1062347\,b^{12}x^{24}+4665089\,ab^{11}x^{22}+7759752\,a^2b^{10}x^{20}+5810090\,a^3b^9x^{18}+1659515\,a^4b^8x^{16}+429\,a^5b^7x^{14}-462\,a^6b^6x^{12}+504\,a^7b^5x^{10}-560\,a^8b^4x^8+640\,a^9b^3x^6-768\,a^{10}b^2x^4+1024\,a^{11}bx^2-2048\,a^{12}\right)\sqrt{bx^2+a}}{26558675\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/26558675*(1062347*b^12*x^24 + 4665089*a*b^11*x^22 + 7759752*a^2*b^10*x^20
 + 5810090*a^3*b^9*x^18 + 1659515*a^4*b^8*x^16 + 429*a^5*b^7*x^14 - 462*a^6
*b^6*x^12 + 504*a^7*b^5*x^10 - 560*a^8*b^4*x^8 + 640*a^9*b^3*x^6 - 768*a^10
*b^2*x^4 + 1024*a^11*b*x^2 - 2048*a^12)*sqrt(b*x^2 + a)/b^8
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(150) = 300$.
time = 3.21, size = 301, normalized size = 1.87

$$\begin{cases} -\frac{2048a^{12}\sqrt{a+bx^2}}{26558675b^8}+\frac{1024a^{11}x^2\sqrt{a+bx^2}}{26558675b^7}-\frac{768a^{10}x^4\sqrt{a+bx^2}}{26558675b^6}+\frac{128a^9x^6\sqrt{a+bx^2}}{5311735b^5}-\frac{112a^8x^8\sqrt{a+bx^2}}{5311735b^4}+\frac{504a^7x^{10}\sqrt{a+bx^2}}{26558675b^3}-\frac{42a^6x^{12}\sqrt{a+bx^2}}{2414425b^2}+\frac{3a^5x^{14}\sqrt{a+bx^2}}{185725b}+\frac{2321a^4x^{16}\sqrt{a+bx^2}}{37145}+\frac{478a^3bx^{18}\sqrt{a+bx^2}}{2185}+\frac{168a^2b^2x^{20}\sqrt{a+bx^2}}{575}+\frac{101ab^3x^{22}\sqrt{a+bx^2}}{575}+\frac{b^4x^{24}\sqrt{a+bx^2}}{25} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^{16}}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2048*a**12*sqrt(a + b*x**2)/(26558675*b**8) + 1024*a**11*x**2*s
qrt(a + b*x**2)/(26558675*b**7) - 768*a**10*x**4*sqrt(a + b*x**2)/(26558675
*b**6) + 128*a**9*x**6*sqrt(a + b*x**2)/(5311735*b**5) - 112*a**8*x**8*sqrt
(a + b*x**2)/(5311735*b**4) + 504*a**7*x**10*sqrt(a + b*x**2)/(26558675*b**
3) - 42*a**6*x**12*sqrt(a + b*x**2)/(2414425*b**2) + 3*a**5*x**14*sqrt(a +
b*x**2)/(185725*b) + 2321*a**4*x**16*sqrt(a + b*x**2)/37145 + 478*a**3*b*x*
*18*sqrt(a + b*x**2)/2185 + 168*a**2*b**2*x**20*sqrt(a + b*x**2)/575 + 101*
a*b**3*x**22*sqrt(a + b*x**2)/575 + b**4*x**24*sqrt(a + b*x**2)/25, Ne(b, 0
)), (a**(9/2)*x**16/16, True))
```

Giac [A]
time = 1.18, size = 113, normalized size = 0.70

$$\frac{1062347\,(bx^2+a)^{\frac{25}{2}} - 8083075\,(bx^2+a)^{\frac{23}{2}}a + 26558675\,(bx^2+a)^{\frac{21}{2}}a^2 - 48923875\,(bx^2+a)^{\frac{19}{2}}a^3 + 54679625\,(bx^2+a)^{\frac{17}{2}}a^4 - 37182145\,(bx^2+a)^{\frac{15}{2}}a^5 + 14300825\,(bx^2+a)^{\frac{13}{2}}a^6 - 2414425\,(bx^2+a)^{\frac{11}{2}}a^7}{26558675\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/26558675*(1062347*(b*x^2 + a)^(25/2) - 8083075*(b*x^2 + a)^(23/2)*a + 265
58675*(b*x^2 + a)^(21/2)*a^2 - 48923875*(b*x^2 + a)^(19/2)*a^3 + 54679625*(
b*x^2 + a)^(17/2)*a^4 - 37182145*(b*x^2 + a)^(15/2)*a^5 + 14300825*(b*x^2 +
 a)^(13/2)*a^6 - 2414425*(b*x^2 + a)^(11/2)*a^7)/b^8
```

Mupad [B]
time = 4.84, size = 141, normalized size = 0.88

$$\sqrt{b\,x^2+a}\,\left(\frac{2321\,a^4\,x^{16}}{37145}-\frac{2048\,a^{12}}{26558675\,b^8}+\frac{b^4\,x^{24}}{25}+\frac{478\,a^3\,b\,x^{18}}{2185}+\frac{101\,a\,b^3\,x^{22}}{575}+\frac{3\,a^5\,x^{14}}{185725\,b}-\frac{42\,a^6\,x^{12}}{2414425\,b^2}+\frac{504\,a^7\,x^{10}}{26558675\,b^3}-\frac{112\,a^8\,x^8}{5311735\,b^4}+\frac{128\,a^9\,x^6}{5311735\,b^5}-\frac{768\,a^{10}\,x^4}{26558675\,b^6}+\frac{1024\,a^{11}\,x^2}{26558675\,b^7}+\frac{168\,a^2\,b^2\,x^{20}}{575}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((2321*a^4*x^16)/37145 - (2048*a^12)/(26558675*b^8) + (b^
4*x^24)/25 + (478*a^3*b*x^18)/2185 + (101*a*b^3*x^22)/575 + (3*a^5*x^14)/(1
85725*b) - (42*a^6*x^12)/(2414425*b^2) + (504*a^7*x^10)/(26558675*b^3) - (1
12*a^8*x^8)/(5311735*b^4) + (128*a^9*x^6)/(5311735*b^5) - (768*a^10*x^4)/(2
6558675*b^6) + (1024*a^11*x^2)/(26558675*b^7) + (168*a^2*b^2*x^20)/575)
```

3.410 $\int x^{13}(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=140

$$\frac{a^6(a+bx^2)^{11/2}}{11b^7}-\frac{6a^5(a+bx^2)^{13/2}}{13b^7}+\frac{a^4(a+bx^2)^{15/2}}{b^7}-\frac{20a^3(a+bx^2)^{17/2}}{17b^7}+\frac{15a^2(a+bx^2)^{19/2}}{19b^7}-\frac{2a(a+bx^2)^{21/2}}{7b^7}$$

```
[Out] 1/11*a^6*(b*x^2+a)^(11/2)/b^7-6/13*a^5*(b*x^2+a)^(13/2)/b^7+a^4*(b*x^2+a)^(
15/2)/b^7-20/17*a^3*(b*x^2+a)^(17/2)/b^7+15/19*a^2*(b*x^2+a)^(19/2)/b^7-2/7
*a*(b*x^2+a)^(21/2)/b^7+1/23*(b*x^2+a)^(23/2)/b^7
```

Rubi [A]
time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{a^6(a+bx^2)^{11/2}}{11b^7}-\frac{6a^5(a+bx^2)^{13/2}}{13b^7}+\frac{a^4(a+bx^2)^{15/2}}{b^7}-\frac{20a^3(a+bx^2)^{17/2}}{17b^7}+\frac{15a^2(a+bx^2)^{19/2}}{19b^7}+\frac{(a+bx^2)^{23/2}}{23b^7}-\frac{2a(a+bx^2)^{21/2}}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^6*(a + b*x^2)^(11/2))/(11*b^7) - (6*a^5*(a + b*x^2)^(13/2))/(13*b^7) + (
a^4*(a + b*x^2)^(15/2))/b^7 - (20*a^3*(a + b*x^2)^(17/2))/(17*b^7) + (15*a^
2*(a + b*x^2)^(19/2))/(19*b^7) - (2*a*(a + b*x^2)^(21/2))/(7*b^7) + (a + b*
x^2)^(23/2)/(23*b^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{13}\left(a+bx^2\right)^{9/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^6(a+bx)^{9/2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^6(a+bx)^{9/2}}{b^6}-\frac{6a^5(a+bx)^{11/2}}{b^6}+\frac{15a^4(a+bx)^{13/2}}{b^6}-\frac{20a^3(a+bx)^{15/}}{b^6}\right.\right.$$
$$=\frac{a^6(a+bx^2)^{11/2}}{11b^7}-\frac{6a^5(a+bx^2)^{13/2}}{13b^7}+\frac{a^4(a+bx^2)^{15/2}}{b^7}-\frac{20a^3(a+bx^2)^{17/2}}{17b^7}+\frac{15a^2(a}{}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 0.59

$$\frac{(a+bx^2)^{11/2}\left(1024a^6-5632a^5bx^2+18304a^4b^2x^4-45760a^3b^3x^6+97240a^2b^4x^8-184756ab^5x^{10}+323323b^6x^{12}\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(1024*a^6 - 5632*a^5*b*x^2 + 18304*a^4*b^2*x^4 - 45760*
a^3*b^3*x^6 + 97240*a^2*b^4*x^8 - 184756*a*b^5*x^10 + 323323*b^6*x^12))/(74
36429*b^7)
```

Maple [A]
time = 0.22, size = 154, normalized size = 1.10

method	result
gosper	$\frac{(b\,x^2+a)^{\frac{11}{2}}(323323b^6x^{12}-184756a\,b^5x^{10}+97240a^2b^4x^8-45760a^3x^6b^3+18304a^4b^2x^4-5632a^5b\,x^2+1024a^6)}{7436429b^7}$
trager	$\frac{(323323b^{11}x^{22}+1431859a\,b^{10}x^{20}+2406690a^2b^9x^{18}+1826110a^3b^8x^{16}+530959a^4b^7x^{14}+231a^5b^6x^{12}-252a^6b^5x^{10}+280a^7b^4x^8-320a^8b}{7436429b^7}$
risch	$\frac{(323323b^{11}x^{22}+1431859a\,b^{10}x^{20}+2406690a^2b^9x^{18}+1826110a^3b^8x^{16}+530959a^4b^7x^{14}+231a^5b^6x^{12}-252a^6b^5x^{10}+280a^7b^4x^8-320a^8b}{7436429b^7}$

default	$\frac{x^{12}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{23b}-\frac{12a\left(\frac{x^{10}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{21b}-\frac{10a\left(\frac{x^{8}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{19b}-\frac{8a\left(\frac{x^{6}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{17b}-\frac{6a\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{15b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{13b}-\frac{2a\left(b\,x^{2}\right.}{143}\right.}{15b}\right.}{17b}\right.}{19b}\right.}{21b}\right.}{23b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/23*x^12*(b*x^2+a)^(11/2)/b-12/23*a/b*(1/21*x^10*(b*x^2+a)^(11/2)/b-10/21*
a/b*(1/19*x^8*(b*x^2+a)^(11/2)/b-8/19*a/b*(1/17*x^6*(b*x^2+a)^(11/2)/b-6/17
*a/b*(1/15*x^4*(b*x^2+a)^(11/2)/b-4/15*a/b*(1/13*x^2*(b*x^2+a)^(11/2)/b-2/1
43*a*(b*x^2+a)^(11/2)/b^2)))))
```

Maxima [A]

time = 0.28, size = 133, normalized size = 0.95

$$\frac{(bx^2+a)^{\frac{11}{2}}x^{12}}{23\,b}-\frac{4\,(bx^2+a)^{\frac{11}{2}}ax^{10}}{161\,b^2}+\frac{40\,(bx^2+a)^{\frac{11}{2}}a^2x^8}{3059\,b^3}-\frac{320\,(bx^2+a)^{\frac{11}{2}}a^3x^6}{52003\,b^4}+\frac{128\,(bx^2+a)^{\frac{11}{2}}a^4x^4}{52003\,b^5}-\frac{512\,(bx^2+a)^{\frac{11}{2}}a^5x^2}{676039\,b^6}+\frac{1024\,(bx^2+a)^{\frac{11}{2}}a^6}{7436429\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/23*(b*x^2 + a)^(11/2)*x^12/b - 4/161*(b*x^2 + a)^(11/2)*a*x^10/b^2 + 40/3
059*(b*x^2 + a)^(11/2)*a^2*x^8/b^3 - 320/52003*(b*x^2 + a)^(11/2)*a^3*x^6/b
^4 + 128/52003*(b*x^2 + a)^(11/2)*a^4*x^4/b^5 - 512/676039*(b*x^2 + a)^(11/
2)*a^5*x^2/b^6 + 1024/7436429*(b*x^2 + a)^(11/2)*a^6/b^7
```

Fricas [**A**]
time = 1.08, size = 134, normalized size = 0.96

$$\frac{\left(323323\,b^{11}x^{22}+1431859\,ab^{10}x^{20}+2406690\,a^{2}b^{9}x^{18}+1826110\,a^{3}b^{8}x^{16}+530959\,a^{4}b^{7}x^{14}+231\,a^{5}b^{6}x^{12}-252\,a^{6}b^{5}x^{10}+280\,a^{7}b^{4}x^{8}-320\,a^{8}b^{3}x^{6}+384\,a^{9}b^{2}x^{4}-512\,a^{10}bx^{2}+1024\,a^{11}\right)\sqrt{bx^{2}+a}}{7436429\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7436429*(323323*b^11*x^22 + 1431859*a*b^10*x^20 + 2406690*a^2*b^9*x^18 +
1826110*a^3*b^8*x^16 + 530959*a^4*b^7*x^14 + 231*a^5*b^6*x^12 - 252*a^6*b^5
*x^10 + 280*a^7*b^4*x^8 - 320*a^8*b^3*x^6 + 384*a^9*b^2*x^4 - 512*a^10*b*x^
2 + 1024*a^11)*sqrt(b*x^2 + a)/b^7
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(129) = 258$.
time = 2.70, size = 277, normalized size = 1.98

$$\begin{cases} \frac{1024a^{11}\sqrt{a+bx^{2}}}{7436429b^{7}}-\frac{512a^{10}x^{2}\sqrt{a+bx^{2}}}{7436429b^{6}}+\frac{384a^{9}x^{4}\sqrt{a+bx^{2}}}{7436429b^{5}}-\frac{320a^{8}x^{6}\sqrt{a+bx^{2}}}{7436429b^{4}}+\frac{40a^{7}x^{8}\sqrt{a+bx^{2}}}{1062347b^{3}}-\frac{36a^{6}x^{10}\sqrt{a+bx^{2}}}{1062347b^{2}}+\frac{3a^{5}x^{12}\sqrt{a+bx^{2}}}{96577b}+\frac{3713a^{4}x^{14}\sqrt{a+bx^{2}}}{52003}+\frac{12770a^{3}bx^{16}\sqrt{a+bx^{2}}}{52003}+\frac{990a^{2}b^{2}x^{18}\sqrt{a+bx^{2}}}{3059}+\frac{31ab^{3}x^{20}\sqrt{a+bx^{2}}}{161}+\frac{b^{4}x^{22}\sqrt{a+bx^{2}}}{23} & \text{for}\; b \neq 0 \\ \frac{a^{\frac{9}{2}}x^{14}}{14} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((1024*a**11*sqrt(a + b*x**2)/(7436429*b**7) - 512*a**10*x**2*sqrt
(a + b*x**2)/(7436429*b**6) + 384*a**9*x**4*sqrt(a + b*x**2)/(7436429*b**5)
 - 320*a**8*x**6*sqrt(a + b*x**2)/(7436429*b**4) + 40*a**7*x**8*sqrt(a + b*
x**2)/(1062347*b**3) - 36*a**6*x**10*sqrt(a + b*x**2)/(1062347*b**2) + 3*a*
*5*x**12*sqrt(a + b*x**2)/(96577*b) + 3713*a**4*x**14*sqrt(a + b*x**2)/5200
3 + 12770*a**3*b*x**16*sqrt(a + b*x**2)/52003 + 990*a**2*b**2*x**18*sqrt(a
+ b*x**2)/3059 + 31*a*b**3*x**20*sqrt(a + b*x**2)/161 + b**4*x**22*sqrt(a +
 b*x**2)/23, Ne(b, 0)), (a**(9/2)*x**14/14, True))
```

Giac [**A**]
time = 1.00, size = 99, normalized size = 0.71

$$\frac{323323\,(bx^{2}+a)^{\frac{23}{2}}-2124694\,(bx^{2}+a)^{\frac{21}{2}}a+5870865\,(bx^{2}+a)^{\frac{19}{2}}a^{2}-8748740\,(bx^{2}+a)^{\frac{17}{2}}a^{3}+7436429\,(bx^{2}+a)^{\frac{15}{2}}a^{4}-3432198\,(bx^{2}+a)^{\frac{13}{2}}a^{5}+676039\,(bx^{2}+a)^{\frac{11}{2}}a^{6}}{7436429\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="giac")

[Out] 1/7436429*(323323*(b*x^2 + a)^(23/2) - 2124694*(b*x^2 + a)^(21/2)*a + 5870865*(b*x^2 + a)^(19/2)*a^2 - 8748740*(b*x^2 + a)^(17/2)*a^3 + 7436429*(b*x^2 + a)^(15/2)*a^4 - 3432198*(b*x^2 + a)^(13/2)*a^5 + 676039*(b*x^2 + a)^(11/2)*a^6)/b^7

Mupad [B]
time = 4.84, size = 130, normalized size = 0.93

$$\sqrt{b\,x^2+a}\,\left(\frac{1024\,a^{11}}{7436429\,b^7}+\frac{3713\,a^4\,x^{14}}{52003}+\frac{b^4\,x^{22}}{23}+\frac{12770\,a^3\,b\,x^{16}}{52003}+\frac{31\,a\,b^3\,x^{20}}{161}+\frac{3\,a^5\,x^{12}}{96577\,b}-\frac{36\,a^6\,x^{10}}{1062347\,b^2}+\frac{40\,a^7\,x^8}{1062347\,b^3}-\frac{320\,a^8\,x^6}{7436429\,b^4}+\frac{384\,a^9\,x^4}{7436429\,b^5}-\frac{512\,a^{10}\,x^2}{7436429\,b^6}+\frac{990\,a^2\,b^2\,x^{18}}{3059}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^13*(a + b*x^2)^(9/2),x)

[Out] (a + b*x^2)^(1/2)*((1024*a^11)/(7436429*b^7) + (3713*a^4*x^14)/52003 + (b^4*x^22)/23 + (12770*a^3*b*x^16)/52003 + (31*a*b^3*x^20)/161 + (3*a^5*x^12)/(96577*b) - (36*a^6*x^10)/(1062347*b^2) + (40*a^7*x^8)/(1062347*b^3) - (320*a^8*x^6)/(7436429*b^4) + (384*a^9*x^4)/(7436429*b^5) - (512*a^10*x^2)/(7436429*b^6) + (990*a^2*b^2*x^18)/3059)

3.411 $\int x^{11}(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=122

$$-\frac{a^5(a+bx^2)^{11/2}}{11b^6}+\frac{5a^4(a+bx^2)^{13/2}}{13b^6}-\frac{2a^3(a+bx^2)^{15/2}}{3b^6}+\frac{10a^2(a+bx^2)^{17/2}}{17b^6}-\frac{5a(a+bx^2)^{19/2}}{19b^6}+\frac{(a+bx^2)^{21/2}}{21b^6}$$

```
[Out] -1/11*a^5*(b*x^2+a)^(11/2)/b^6+5/13*a^4*(b*x^2+a)^(13/2)/b^6-2/3*a^3*(b*x^2
+a)^(15/2)/b^6+10/17*a^2*(b*x^2+a)^(17/2)/b^6-5/19*a*(b*x^2+a)^(19/2)/b^6+1
/21*(b*x^2+a)^(21/2)/b^6
```

Rubi [A]

time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$-\frac{a^5(a+bx^2)^{11/2}}{11b^6}+\frac{5a^4(a+bx^2)^{13/2}}{13b^6}-\frac{2a^3(a+bx^2)^{15/2}}{3b^6}+\frac{10a^2(a+bx^2)^{17/2}}{17b^6}+\frac{(a+bx^2)^{21/2}}{21b^6}-\frac{5a(a+bx^2)^{19/2}}{19b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^(9/2),x]
```

```
[Out] -1/11*(a^5*(a + b*x^2)^(11/2))/b^6 + (5*a^4*(a + b*x^2)^(13/2))/(13*b^6) -
(2*a^3*(a + b*x^2)^(15/2))/(3*b^6) + (10*a^2*(a + b*x^2)^(17/2))/(17*b^6) -
 (5*a*(a + b*x^2)^(19/2))/(19*b^6) + (a + b*x^2)^(21/2)/(21*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^2\right)^{9/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^5(a+bx)^{9/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}\right.\right.\\ &= -\frac{a^5(a+bx^2)^{11/2}}{11b^6}+\frac{5a^4(a+bx^2)^{13/2}}{13b^6}-\frac{2a^3(a+bx^2)^{15/2}}{3b^6}+\frac{10a^2(a+bx^2)^{17/2}}{17b^6}-5a(\end{aligned}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.59

$$\frac{(a+bx^2)^{11/2}\left(-256a^5+1408a^4bx^2-4576a^3b^2x^4+11440a^2b^3x^6-24310ab^4x^8+46189b^5x^{10}\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-256*a^5 + 1408*a^4*b*x^2 - 4576*a^3*b^2*x^4 + 11440*a
^2*b^3*x^6 - 24310*a*b^4*x^8 + 46189*b^5*x^10))/(969969*b^6)
```

Maple [A]
time = 0.13, size = 130, normalized size = 1.07

method	result
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}\left(-46189b^5x^{10}+24310a\,b^4x^8-11440a^2b^3x^6+4576a^3b^2x^4-1408a^4b\,x^2+256a^5\right)}{969969b^6}$
trager	$-\frac{(-46189b^{10}x^{20}-206635a\,b^9x^{18}-351780a^2b^8x^{16}-271414a^3b^7x^{14}-80773a^4b^6x^{12}-63a^5x^{10}b^5+70a^6b^4x^8-80a^7b^3x^6+96a^8b^2x^4-}{969969b^6}$
risch	$-\frac{(-46189b^{10}x^{20}-206635a\,b^9x^{18}-351780a^2b^8x^{16}-271414a^3b^7x^{14}-80773a^4b^6x^{12}-63a^5x^{10}b^5+70a^6b^4x^8-80a^7b^3x^6+96a^8b^2x^4-}{969969b^6}$
default	$\frac{x^{10}(b\,x^2+a)^{\frac{11}{2}}}{21b}-\frac{10a\left(\frac{x^8(b\,x^2+a)^{\frac{11}{2}}}{19b}-\frac{8a\left(\frac{x^6(b\,x^2+a)^{\frac{11}{2}}}{17b}-\frac{6a\left(\frac{x^4(b\,x^2+a)^{\frac{11}{2}}}{15b}-\frac{4a\left(\frac{x^2(b\,x^2+a)^{\frac{11}{2}}}{13b}-\frac{2a(b\,x^2+a)^{\frac{11}{2}}}{143b^2}\right)}{15b}\right)}{17b}\right)}{19b}\right)}{21b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*x^10*(b*x^2+a)^(11/2)/b-10/21*a/b*(1/19*x^8*(b*x^2+a)^(11/2)/b-8/19*a/
b*(1/17*x^6*(b*x^2+a)^(11/2)/b-6/17*a/b*(1/15*x^4*(b*x^2+a)^(11/2)/b-4/15*a
/b*(1/13*x^2*(b*x^2+a)^(11/2)/b-2/143*a*(b*x^2+a)^(11/2)/b^2))))
```

Maxima [A]
time = 0.29, size = 113, normalized size = 0.93

$$\frac{(bx^2+a)^{\frac{11}{2}}x^{10}}{21\,b}-\frac{10\,(bx^2+a)^{\frac{11}{2}}ax^8}{399\,b^2}+\frac{80\,(bx^2+a)^{\frac{11}{2}}a^2x^6}{6783\,b^3}-\frac{32\,(bx^2+a)^{\frac{11}{2}}a^3x^4}{6783\,b^4}+\frac{128\,(bx^2+a)^{\frac{11}{2}}a^4x^2}{88179\,b^5}-\frac{256\,(bx^2+a)^{\frac{11}{2}}a^5}{969969\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/21*(b*x^2 + a)^(11/2)*x^10/b - 10/399*(b*x^2 + a)^(11/2)*a*x^8/b^2 + 80/6
783*(b*x^2 + a)^(11/2)*a^2*x^6/b^3 - 32/6783*(b*x^2 + a)^(11/2)*a^3*x^4/b^4
 + 128/88179*(b*x^2 + a)^(11/2)*a^4*x^2/b^5 - 256/969969*(b*x^2 + a)^(11/2)
*a^5/b^6
```

Fricas [A]
time = 0.79, size = 123, normalized size = 1.01

$$\frac{(46189\,b^{10}x^{20}+206635\,ab^9x^{18}+351780\,a^2b^8x^{16}+271414\,a^3b^7x^{14}+80773\,a^4b^6x^{12}+63\,a^5b^5x^{10}-70\,a^6b^4x^8+80\,a^7b^3x^6-96\,a^8b^2x^4+128\,a^9bx^2-256\,a^{10})\sqrt{bx^2+a}}{969969\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/969969*(46189*b^10*x^20 + 206635*a*b^9*x^18 + 351780*a^2*b^8*x^16 + 27141
4*a^3*b^7*x^14 + 80773*a^4*b^6*x^12 + 63*a^5*b^5*x^10 - 70*a^6*b^4*x^8 + 80
*a^7*b^3*x^6 - 96*a^8*b^2*x^4 + 128*a^9*b*x^2 - 256*a^10)*sqrt(b*x^2 + a)/b
^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(112) = 224.
time = 2.18, size = 253, normalized size = 2.07

$$\begin{cases} -\frac{256a^{10}\sqrt{a+bx^2}}{969969b^6}+\frac{128a^9x^2\sqrt{a+bx^2}}{969969b^5}-\frac{32a^8x^4\sqrt{a+bx^2}}{323323b^4}+\frac{80a^7x^6\sqrt{a+bx^2}}{969969b^3}-\frac{10a^6x^8\sqrt{a+bx^2}}{138567b^2}+\frac{3a^5x^{10}\sqrt{a+bx^2}}{46189b}+\frac{1049a^4x^{12}\sqrt{a+bx^2}}{12597}+\frac{1898a^3bx^{14}\sqrt{a+bx^2}}{6783}+\frac{820a^2b^2x^{16}\sqrt{a+bx^2}}{2261}+\frac{85ab^3x^{18}\sqrt{a+bx^2}}{399}+\frac{b^4x^{20}\sqrt{a+bx^2}}{21} & \text{for } b\neq 0 \\ \frac{a^{\frac{9}{2}}x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-256*a**10*sqrt(a + b*x**2)/(969969*b**6) + 128*a**9*x**2*sqrt(a
 + b*x**2)/(969969*b**5) - 32*a**8*x**4*sqrt(a + b*x**2)/(323323*b**4) + 80
*a**7*x**6*sqrt(a + b*x**2)/(969969*b**3) - 10*a**6*x**8*sqrt(a + b*x**2)/(
138567*b**2) + 3*a**5*x**10*sqrt(a + b*x**2)/(46189*b) + 1049*a**4*x**12*sq
rt(a + b*x**2)/12597 + 1898*a**3*b*x**14*sqrt(a + b*x**2)/6783 + 820*a**2*b
**2*x**16*sqrt(a + b*x**2)/2261 + 85*a*b**3*x**18*sqrt(a + b*x**2)/399 + b*
*4*x**20*sqrt(a + b*x**2)/21, Ne(b, 0)), (a**(9/2)*x**12/12, True))
```

Giac [A]
time = 1.79, size = 85, normalized size = 0.70

$$\frac{46189\,(bx^2+a)^{\frac{21}{2}}-255255\,(bx^2+a)^{\frac{19}{2}}a+570570\,(bx^2+a)^{\frac{17}{2}}a^2-646646\,(bx^2+a)^{\frac{15}{2}}a^3+373065\,(bx^2+a)^{\frac{13}{2}}a^4-88179\,(bx^2+a)^{\frac{11}{2}}a^5}{969969\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/969969*(46189*(b*x^2 + a)^(21/2) - 255255*(b*x^2 + a)^(19/2)*a + 570570*(
b*x^2 + a)^(17/2)*a^2 - 646646*(b*x^2 + a)^(15/2)*a^3 + 373065*(b*x^2 + a)^
(13/2)*a^4 - 88179*(b*x^2 + a)^(11/2)*a^5)/b^6
```

Mupad [B]
time = 4.81, size = 119, normalized size = 0.98

$$\sqrt{b\,x^2+a}\,\left(\frac{1049\,a^4\,x^{12}}{12597}-\frac{256\,a^{10}}{969969\,b^6}+\frac{b^4\,x^{20}}{21}+\frac{1898\,a^3\,b\,x^{14}}{6783}+\frac{85\,a\,b^3\,x^{18}}{399}+\frac{3\,a^5\,x^{10}}{46189\,b}-\frac{10\,a^6\,x^8}{138567\,b^2}+\frac{80\,a^7\,x^6}{969969\,b^3}-\frac{32\,a^8\,x^4}{323323\,b^4}+\frac{128\,a^9\,x^2}{969969\,b^5}+\frac{820\,a^2\,b^2\,x^{16}}{2261}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((1049*a^4*x^12)/12597 - (256*a^10)/(969969*b^6) + (b^4*x
^20)/21 + (1898*a^3*b*x^14)/6783 + (85*a*b^3*x^18)/399 + (3*a^5*x^10)/(4618
9*b) - (10*a^6*x^8)/(138567*b^2) + (80*a^7*x^6)/(969969*b^3) - (32*a^8*x^4)
/(323323*b^4) + (128*a^9*x^2)/(969969*b^5) + (820*a^2*b^2*x^16)/2261)
```

3.412 $\int x^9(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=101

$$\frac{a^4(a+bx^2)^{11/2}}{11b^5}-\frac{4a^3(a+bx^2)^{13/2}}{13b^5}+\frac{2a^2(a+bx^2)^{15/2}}{5b^5}-\frac{4a(a+bx^2)^{17/2}}{17b^5}+\frac{(a+bx^2)^{19/2}}{19b^5}$$

```
[Out] 1/11*a^4*(b*x^2+a)^(11/2)/b^5-4/13*a^3*(b*x^2+a)^(13/2)/b^5+2/5*a^2*(b*x^2+
a)^(15/2)/b^5-4/17*a*(b*x^2+a)^(17/2)/b^5+1/19*(b*x^2+a)^(19/2)/b^5
```

Rubi [A]

time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{a^4(a+bx^2)^{11/2}}{11b^5}-\frac{4a^3(a+bx^2)^{13/2}}{13b^5}+\frac{2a^2(a+bx^2)^{15/2}}{5b^5}+\frac{(a+bx^2)^{19/2}}{19b^5}-\frac{4a(a+bx^2)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^4*(a + b*x^2)^(11/2))/(11*b^5) - (4*a^3*(a + b*x^2)^(13/2))/(13*b^5) + (
2*a^2*(a + b*x^2)^(15/2))/(5*b^5) - (4*a*(a + b*x^2)^(17/2))/(17*b^5) + (a
+ b*x^2)^(19/2)/(19*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^{9/2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^{9/2}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^4(a+bx)^{9/2}}{b^4}-\frac{4a^3(a+bx)^{11/2}}{b^4}+\frac{6a^2(a+bx)^{13/2}}{b^4}-\frac{4a(a+bx)^{15/2}}{b^4}+\right.\right.\\&=\frac{a^4(a+bx^2)^{11/2}}{11b^5}-\frac{4a^3(a+bx^2)^{13/2}}{13b^5}+\frac{2a^2(a+bx^2)^{15/2}}{5b^5}-\frac{4a(a+bx^2)^{17/2}}{17b^5}+\frac{(a+bx^2}{19b^5}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.60

$$\frac{(a+bx^2)^{11/2}\left(128a^4-704a^3bx^2+2288a^2b^2x^4-5720ab^3x^6+12155b^4x^8\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(128*a^4 - 704*a^3*b*x^2 + 2288*a^2*b^2*x^4 - 5720*a*b^
3*x^6 + 12155*b^4*x^8))/(230945*b^5)
```

Maple [A]
time = 0.08, size = 106, normalized size = 1.05

method	result
gosper	$\frac{(bx^2+a)^{\frac{11}{2}}(12155b^4x^8-5720ab^3x^6+2288a^2b^2x^4-704a^3bx^2+128a^4)}{230945b^5}$
default	$\frac{x^8(bx^2+a)^{\frac{11}{2}}}{19b}-\frac{8a\left(\frac{x^6(bx^2+a)^{\frac{11}{2}}}{17b}-\frac{6a\left(\frac{x^4(bx^2+a)^{\frac{11}{2}}}{15b}-\frac{4a\left(\frac{x^2(bx^2+a)^{\frac{11}{2}}}{13b}-\frac{2a(bx^2+a)^{\frac{11}{2}}}{143b^2}\right)}{15b}\right)}{17b}\right)}{19b}$
trager	$\frac{(12155b^9x^{18}+55055a\,b^8x^{16}+95238a^2b^7x^{14}+75086a^3b^6x^{12}+23063a^4b^5x^{10}+35a^5b^4x^8-40a^6b^3x^6+48a^7b^2x^4-64a^8b\,x^2+128a^9)\sqrt{b}}{230945b^5}$
risch	$\frac{(12155b^9x^{18}+55055a\,b^8x^{16}+95238a^2b^7x^{14}+75086a^3b^6x^{12}+23063a^4b^5x^{10}+35a^5b^4x^8-40a^6b^3x^6+48a^7b^2x^4-64a^8b\,x^2+128a^9)\sqrt{b}}{230945b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/19*x^8*(b*x^2+a)^(11/2)/b-8/19*a/b*(1/17*x^6*(b*x^2+a)^(11/2)/b-6/17*a/b*
(1/15*x^4*(b*x^2+a)^(11/2)/b-4/15*a/b*(1/13*x^2*(b*x^2+a)^(11/2)/b-2/143*a*
(b*x^2+a)^(11/2)/b^2)))
```

Maxima [A]
time = 0.31, size = 93, normalized size = 0.92

$$\frac{(bx^2+a)^{\frac{11}{2}}x^8}{19\,b}-\frac{8\,(bx^2+a)^{\frac{11}{2}}ax^6}{323\,b^2}+\frac{16\,(bx^2+a)^{\frac{11}{2}}a^2x^4}{1615\,b^3}-\frac{64\,(bx^2+a)^{\frac{11}{2}}a^3x^2}{20995\,b^4}+\frac{128\,(bx^2+a)^{\frac{11}{2}}a^4}{230945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/19*(b*x^2 + a)^(11/2)*x^8/b - 8/323*(b*x^2 + a)^(11/2)*a*x^6/b^2 + 16/161
5*(b*x^2 + a)^(11/2)*a^2*x^4/b^3 - 64/20995*(b*x^2 + a)^(11/2)*a^3*x^2/b^4
+ 128/230945*(b*x^2 + a)^(11/2)*a^4/b^5
```

Fricas [A]
time = 0.95, size = 112, normalized size = 1.11

$$\frac{\left(12155\,b^9x^{18}+55055\,ab^8x^{16}+95238\,a^2b^7x^{14}+75086\,a^3b^6x^{12}+23063\,a^4b^5x^{10}+35\,a^5b^4x^8-40\,a^6b^3x^6+48\,a^7b^2x^4-64\,a^8bx^2+128\,a^9\right)\sqrt{bx^2+a}}{230945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/230945*(12155*b^9*x^18 + 55055*a*b^8*x^16 + 95238*a^2*b^7*x^14 + 75086*a^
3*b^6*x^12 + 23063*a^4*b^5*x^10 + 35*a^5*b^4*x^8 - 40*a^6*b^3*x^6 + 48*a^7*
b^2*x^4 - 64*a^8*b*x^2 + 128*a^9)*sqrt(b*x^2 + a)/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(92) = 184$.
time = 1.79, size = 230, normalized size = 2.28

$$\begin{cases} \frac{128a^9\sqrt{a+bx^2}}{230945b^5} - \frac{64a^8x^2\sqrt{a+bx^2}}{230945b^4} + \frac{48a^7x^4\sqrt{a+bx^2}}{230945b^3} - \frac{8a^6x^6\sqrt{a+bx^2}}{46189b^2} + \frac{7a^5x^8\sqrt{a+bx^2}}{46189b} + \frac{23063a^4x^{10}\sqrt{a+bx^2}}{230945} + \frac{6826a^3bx^{12}\sqrt{a+bx^2}}{20995} + \frac{666a^2b^2x^{14}\sqrt{a+bx^2}}{1615} + \frac{77ab^3x^{16}\sqrt{a+bx^2}}{323} + \frac{b^4x^{18}\sqrt{a+bx^2}}{19} & \text{for}\: b \neq 0 \\ \frac{a^{\frac{9}{2}}x^{10}}{10} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((128*a**9*sqrt(a + b*x**2)/(230945*b**5) - 64*a**8*x**2*sqrt(a +
b*x**2)/(230945*b**4) + 48*a**7*x**4*sqrt(a + b*x**2)/(230945*b**3) - 8*a**
6*x**6*sqrt(a + b*x**2)/(46189*b**2) + 7*a**5*x**8*sqrt(a + b*x**2)/(46189*
b) + 23063*a**4*x**10*sqrt(a + b*x**2)/230945 + 6826*a**3*b*x**12*sqrt(a +
b*x**2)/20995 + 666*a**2*b**2*x**14*sqrt(a + b*x**2)/1615 + 77*a*b**3*x**16
*sqrt(a + b*x**2)/323 + b**4*x**18*sqrt(a + b*x**2)/19, Ne(b, 0)), (a**(9/2
)*x**10/10, True))
```

Giac [A]
time = 1.02, size = 71, normalized size = 0.70

$$\frac{12155\,(bx^2+a)^{\frac{19}{2}}-54340\,(bx^2+a)^{\frac{17}{2}}a+92378\,(bx^2+a)^{\frac{15}{2}}a^2-71060\,(bx^2+a)^{\frac{13}{2}}a^3+20995\,(bx^2+a)^{\frac{11}{2}}a^4}{230945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/230945*(12155*(b*x^2 + a)^(19/2) - 54340*(b*x^2 + a)^(17/2)*a + 92378*(b*
x^2 + a)^(15/2)*a^2 - 71060*(b*x^2 + a)^(13/2)*a^3 + 20995*(b*x^2 + a)^(11/
2)*a^4)/b^5
```

Mupad [B]
time = 4.74, size = 108, normalized size = 1.07

$$\sqrt{b\,x^2+a}\,\left(\frac{128\,a^9}{230945\,b^5}+\frac{23063\,a^4\,x^{10}}{230945}+\frac{b^4\,x^{18}}{19}+\frac{6826\,a^3\,b\,x^{12}}{20995}+\frac{77\,a\,b^3\,x^{16}}{323}+\frac{7\,a^5\,x^8}{46189\,b}-\frac{8\,a^6\,x^6}{46189\,b^2}+\frac{48\,a^7\,x^4}{230945\,b^3}-\frac{64\,a^8\,x^2}{230945\,b^4}+\frac{666\,a^2\,b^2\,x^{14}}{1615}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^9)/(230945*b^5) + (23063*a^4*x^10)/230945 + (b^4*
x^18)/19 + (6826*a^3*b*x^12)/20995 + (77*a*b^3*x^16)/323 + (7*a^5*x^8)/(461
89*b) - (8*a^6*x^6)/(46189*b^2) + (48*a^7*x^4)/(230945*b^3) - (64*a^8*x^2)/
(230945*b^4) + (666*a^2*b^2*x^14)/1615)
```

3.413 $\int x^7(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3(a+bx^2)^{11/2}}{11b^4}+\frac{3a^2(a+bx^2)^{13/2}}{13b^4}-\frac{a(a+bx^2)^{15/2}}{5b^4}+\frac{(a+bx^2)^{17/2}}{17b^4}$$

```
[Out] -1/11*a^3*(b*x^2+a)^(11/2)/b^4+3/13*a^2*(b*x^2+a)^(13/2)/b^4-1/5*a*(b*x^2+a
)^(15/2)/b^4+1/17*(b*x^2+a)^(17/2)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$-\frac{a^3(a+bx^2)^{11/2}}{11b^4}+\frac{3a^2(a+bx^2)^{13/2}}{13b^4}+\frac{(a+bx^2)^{17/2}}{17b^4}-\frac{a(a+bx^2)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(9/2),x]
```

```
[Out] -1/11*(a^3*(a + b*x^2)^(11/2))/b^4 + (3*a^2*(a + b*x^2)^(13/2))/(13*b^4) -
(a*(a + b*x^2)^(15/2))/(5*b^4) + (a + b*x^2)^(17/2)/(17*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{9/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{9/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}\right)\,dx\right.\\ &= -\frac{a^3(a+bx^2)^{11/2}}{11b^4}+\frac{3a^2(a+bx^2)^{13/2}}{13b^4}-\frac{a(a+bx^2)^{15/2}}{5b^4}+\frac{(a+bx^2)^{17/2}}{17b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.62

$$\frac{(a+bx^2)^{11/2}(-16a^3+88a^2bx^2-286ab^2x^4+715b^3x^6)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-16*a^3 + 88*a^2*b*x^2 - 286*a*b^2*x^4 + 715*b^3*x^6))
/(12155*b^4)
```

Maple [A]
time = 0.06, size = 82, normalized size = 1.02

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-715b^3x^6+286a\,b^2x^4-88a^2b\,x^2+16a^3)}{12155b^4}$	47
default	$\frac{x^6(b\,x^2+a)^{\frac{11}{2}}}{17b}-\frac{6a\left(\frac{x^4(b\,x^2+a)^{\frac{11}{2}}}{15b}-\frac{4a\left(\frac{x^2(b\,x^2+a)^{\frac{11}{2}}}{13b}-\frac{2a(b\,x^2+a)^{\frac{11}{2}}}{143b^2}\right)}{15b}\right)}{17b}$	82
trager	$-\frac{(-715b^8x^{16}-3289a\,b^7x^{14}-5808a^2b^6x^{12}-4714a^3b^5x^{10}-1515a^4b^4x^8-5a^5b^3x^6+6a^6b^2x^4-8a^7b\,x^2+16a^8)\sqrt{b\,x^2+a}}{12155b^4}$	102
risch	$-\frac{(-715b^8x^{16}-3289a\,b^7x^{14}-5808a^2b^6x^{12}-4714a^3b^5x^{10}-1515a^4b^4x^8-5a^5b^3x^6+6a^6b^2x^4-8a^7b\,x^2+16a^8)\sqrt{b\,x^2+a}}{12155b^4}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*x^6*(b*x^2+a)^(11/2)/b-6/17*a/b*(1/15*x^4*(b*x^2+a)^(11/2)/b-4/15*a/b*
(1/13*x^2*(b*x^2+a)^(11/2)/b-2/143*a*(b*x^2+a)^(11/2)/b^2))
```

Maxima [A]
time = 0.34, size = 73, normalized size = 0.91

$$\frac{(bx^2+a)^{\frac{11}{2}}x^6}{17\,b}-\frac{2\,(bx^2+a)^{\frac{11}{2}}ax^4}{85\,b^2}+\frac{8\,(bx^2+a)^{\frac{11}{2}}a^2x^2}{1105\,b^3}-\frac{16\,(bx^2+a)^{\frac{11}{2}}a^3}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/17*(b*x^2 + a)^(11/2)*x^6/b - 2/85*(b*x^2 + a)^(11/2)*a*x^4/b^2 + 8/1105*
(b*x^2 + a)^(11/2)*a^2*x^2/b^3 - 16/12155*(b*x^2 + a)^(11/2)*a^3/b^4
```

Fricas [A]
time = 0.86, size = 101, normalized size = 1.26

$$\frac{(715\,b^8x^{16}+3289\,ab^7x^{14}+5808\,a^2b^6x^{12}+4714\,a^3b^5x^{10}+1515\,a^4b^4x^8+5\,a^5b^3x^6-6\,a^6b^2x^4+8\,a^7bx^2-16\,a^8)\sqrt{bx^2+a}}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/12155*(715*b^8*x^16 + 3289*a*b^7*x^14 + 5808*a^2*b^6*x^12 + 4714*a^3*b^5*
x^10 + 1515*a^4*b^4*x^8 + 5*a^5*b^3*x^6 - 6*a^6*b^2*x^4 + 8*a^7*b*x^2 - 16*
a^8)*sqrt(b*x^2 + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(70) = 140.
time = 1.48, size = 204, normalized size = 2.55

$$\begin{cases} -\frac{16a^8\sqrt{a+bx^2}}{12155b^4}+\frac{8a^7x^2\sqrt{a+bx^2}}{12155b^3}-\frac{6a^6x^4\sqrt{a+bx^2}}{12155b^2}+\frac{a^5x^6\sqrt{a+bx^2}}{2431b}+\frac{303a^4x^8\sqrt{a+bx^2}}{2431}+\frac{4714a^3bx^{10}\sqrt{a+bx^2}}{12155}+\frac{528a^2b^2x^{12}\sqrt{a+bx^2}}{1105}+\frac{23ab^3x^{14}\sqrt{a+bx^2}}{85}+\frac{b^4x^{16}\sqrt{a+bx^2}}{17} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{9}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-16*a**8*sqrt(a + b*x**2)/(12155*b**4) + 8*a**7*x**2*sqrt(a + b*
x**2)/(12155*b**3) - 6*a**6*x**4*sqrt(a + b*x**2)/(12155*b**2) + a**5*x**6*
sqrt(a + b*x**2)/(2431*b) + 303*a**4*x**8*sqrt(a + b*x**2)/2431 + 4714*a**3
*b*x**10*sqrt(a + b*x**2)/12155 + 528*a**2*b**2*x**12*sqrt(a + b*x**2)/1105
 + 23*a*b**3*x**14*sqrt(a + b*x**2)/85 + b**4*x**16*sqrt(a + b*x**2)/17, Ne
(b, 0)), (a**(9/2)*x**8/8, True))
```

Giac [A]
time = 3.83, size = 57, normalized size = 0.71

$$\frac{715\,(bx^2+a)^{\frac{17}{2}}-2431\,(bx^2+a)^{\frac{15}{2}}a+2805\,(bx^2+a)^{\frac{13}{2}}a^2-1105\,(bx^2+a)^{\frac{11}{2}}a^3}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/12155*(715*(b*x^2 + a)^(17/2) - 2431*(b*x^2 + a)^(15/2)*a + 2805*(b*x^2 +
 a)^(13/2)*a^2 - 1105*(b*x^2 + a)^(11/2)*a^3)/b^4
```

Mupad [B]
time = 4.73, size = 97, normalized size = 1.21

$$\sqrt{b\,x^2+a}\,\left(\frac{303\,a^4\,x^8}{2431}-\frac{16\,a^8}{12155\,b^4}+\frac{b^4\,x^{16}}{17}+\frac{4714\,a^3\,b\,x^{10}}{12155}+\frac{23\,a\,b^3\,x^{14}}{85}+\frac{a^5\,x^6}{2431\,b}-\frac{6\,a^6\,x^4}{12155\,b^2}+\frac{8\,a^7\,x^2}{12155\,b^3}+\frac{528\,a^2\,b^2\,x^{12}}{1105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((303*a^4*x^8)/2431 - (16*a^8)/(12155*b^4) + (b^4*x^16)/1
7 + (4714*a^3*b*x^10)/12155 + (23*a*b^3*x^14)/85 + (a^5*x^6)/(2431*b) - (6*
a^6*x^4)/(12155*b^2) + (8*a^7*x^2)/(12155*b^3) + (528*a^2*b^2*x^12)/1105)
```

3.414 $\int x^5(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=59

$$\frac{a^2(a+bx^2)^{11/2}}{11b^3}-\frac{2a(a+bx^2)^{13/2}}{13b^3}+\frac{(a+bx^2)^{15/2}}{15b^3}$$

```
[Out] 1/11*a^2*(b*x^2+a)^(11/2)/b^3-2/13*a*(b*x^2+a)^(13/2)/b^3+1/15*(b*x^2+a)^(1
5/2)/b^3
```

Rubi [A]

time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{a^2(a+bx^2)^{11/2}}{11b^3}+\frac{(a+bx^2)^{15/2}}{15b^3}-\frac{2a(a+bx^2)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(11/2))/(11*b^3) - (2*a*(a + b*x^2)^(13/2))/(13*b^3) + (a
+ b*x^2)^(15/2)/(15*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{9/2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{9/2}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}\right)\,dx,x,x^2\right)\\&=\frac{a^2(a+bx^2)^{11/2}}{11b^3}-\frac{2a(a+bx^2)^{13/2}}{13b^3}+\frac{(a+bx^2)^{15/2}}{15b^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{(a+bx^2)^{11/2}\left(8a^2-44abx^2+143b^2x^4\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(9/2),x]
[Out] ((a + b*x^2)^(11/2)*(8*a^2 - 44*a*b*x^2 + 143*b^2*x^4))/(2145*b^3)
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.98

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(143b^{2}x^{4}-44ab\,x^{2}+8a^{2}\right)}{2145b^{3}}$	36
default	$\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{15b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{13b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{143b^{2}}\right)}{15b}$	58
trager	$\frac{\left(143b^{7}x^{14}+671a\,b^{6}x^{12}+1218a^{2}b^{5}x^{10}+1030a^{3}b^{4}x^{8}+355a^{4}b^{3}x^{6}+3a^{5}b^{2}x^{4}-4a^{6}b\,x^{2}+8a^{7}\right)\sqrt{b\,x^{2}+a}}{2145b^{3}}$	91
risch	$\frac{\left(143b^{7}x^{14}+671a\,b^{6}x^{12}+1218a^{2}b^{5}x^{10}+1030a^{3}b^{4}x^{8}+355a^{4}b^{3}x^{6}+3a^{5}b^{2}x^{4}-4a^{6}b\,x^{2}+8a^{7}\right)\sqrt{b\,x^{2}+a}}{2145b^{3}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
[Out] 1/15*x^4*(b*x^2+a)^(11/2)/b-4/15*a/b*(1/13*x^2*(b*x^2+a)^(11/2)/b-2/143*a*(
b*x^2+a)^(11/2)/b^2)
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.90

$$\frac{(bx^2+a)^{\frac{11}{2}}x^4}{15\,b}-\frac{4\,(bx^2+a)^{\frac{11}{2}}ax^2}{195\,b^2}+\frac{8\,(bx^2+a)^{\frac{11}{2}}a^2}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="maxima")
[Out] 1/15*(b*x^2 + a)^(11/2)*x^4/b - 4/195*(b*x^2 + a)^(11/2)*a*x^2/b^2 + 8/2145
*(b*x^2 + a)^(11/2)*a^2/b^3
```

Fricas [A]
time = 0.78, size = 90, normalized size = 1.53

$$\frac{\left(143\,b^7x^{14}+671\,ab^6x^{12}+1218\,a^2b^5x^{10}+1030\,a^3b^4x^8+355\,a^4b^3x^6+3\,a^5b^2x^4-4\,a^6bx^2+8\,a^7\right)\sqrt{bx^2+a}}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/2145*(143*b^7*x^14 + 671*a*b^6*x^12 + 1218*a^2*b^5*x^10 + 1030*a^3*b^4*x^
8 + 355*a^4*b^3*x^6 + 3*a^5*b^2*x^4 - 4*a^6*b*x^2 + 8*a^7)*sqrt(b*x^2 + a)/
b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(51) = 102$.
time = 1.21, size = 180, normalized size = 3.05

$$\begin{cases} \frac{8a^7\sqrt{a+bx^2}}{2145b^3} - \frac{4a^6x^2\sqrt{a+bx^2}}{2145b^2} + \frac{a^5x^4\sqrt{a+bx^2}}{715b} + \frac{71a^4x^6\sqrt{a+bx^2}}{429} + \frac{206a^3bx^8\sqrt{a+bx^2}}{429} + \frac{406a^2b^2x^{10}\sqrt{a+bx^2}}{715} + \frac{61ab^3x^{12}\sqrt{a+bx^2}}{195} + \frac{b^4x^{14}\sqrt{a+bx^2}}{15} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((8*a**7*sqrt(a + b*x**2)/(2145*b**3) - 4*a**6*x**2*sqrt(a + b*x**
2)/(2145*b**2) + a**5*x**4*sqrt(a + b*x**2)/(715*b) + 71*a**4*x**6*sqrt(a +
 b*x**2)/429 + 206*a**3*b*x**8*sqrt(a + b*x**2)/429 + 406*a**2*b**2*x**10*s
qrt(a + b*x**2)/715 + 61*a*b**3*x**12*sqrt(a + b*x**2)/195 + b**4*x**14*sqr
t(a + b*x**2)/15, Ne(b, 0)), (a**(9/2)*x**6/6, True))
```

Giac [A]
time = 1.63, size = 43, normalized size = 0.73

$$\frac{143\,(bx^2+a)^{\frac{15}{2}} - 330\,(bx^2+a)^{\frac{13}{2}}\,a + 195\,(bx^2+a)^{\frac{11}{2}}\,a^2}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/2145*(143*(b*x^2 + a)^(15/2) - 330*(b*x^2 + a)^(13/2)*a + 195*(b*x^2 + a)
^(11/2)*a^2)/b^3
```

Mupad [B]
time = 4.60, size = 86, normalized size = 1.46

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a^7}{2145\,b^3}+\frac{71\,a^4\,x^6}{429}+\frac{b^4\,x^{14}}{15}+\frac{206\,a^3\,b\,x^8}{429}+\frac{61\,a\,b^3\,x^{12}}{195}+\frac{a^5\,x^4}{715\,b}-\frac{4\,a^6\,x^2}{2145\,b^2}+\frac{406\,a^2\,b^2\,x^{10}}{715}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^7)/(2145*b^3) + (71*a^4*x^6)/429 + (b^4*x^14)/15 +
(206*a^3*b*x^8)/429 + (61*a*b^3*x^12)/195 + (a^5*x^4)/(715*b) - (4*a^6*x^2)
/(2145*b^2) + (406*a^2*b^2*x^10)/715)
```

3.415 $\int x^3(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=38

$$-\frac{a(a+bx^2)^{11/2}}{11b^2}+\frac{(a+bx^2)^{13/2}}{13b^2}$$

```
[Out] -1/11*a*(b*x^2+a)^(11/2)/b^2+1/13*(b*x^2+a)^(13/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {272, 45}

$$\frac{(a+bx^2)^{13/2}}{13b^2}-\frac{a(a+bx^2)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(9/2),x]
```

```
[Out] -1/11*(a*(a + b*x^2)^(11/2))/b^2 + (a + b*x^2)^(13/2)/(13*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{9/2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^{9/2}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^{9/2}}{b}+\frac{(a+bx)^{11/2}}{b}\right)\,dx,x,x^2\right)\\&=-\frac{a(a+bx^2)^{11/2}}{11b^2}+\frac{(a+bx^2)^{13/2}}{13b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.74

$$\frac{(a+bx^2)^{11/2}(-2a+11bx^2)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(9/2),x]
[Out] ((a + b*x^2)^(11/2)*(-2*a + 11*b*x^2))/(143*b^2)
```

Maple [A]
time = 0.05, size = 34, normalized size = 0.89

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-11b\,x^2+2a)}{143b^2}$	25
default	$\frac{x^2(b\,x^2+a)^{\frac{11}{2}}}{13b}-\frac{2a(b\,x^2+a)^{\frac{11}{2}}}{143b^2}$	34
trager	$-\frac{(-11b^6x^{12}-53a\,b^5x^{10}-100a^2b^4x^8-90a^3x^6b^3-35a^4b^2x^4-a^5b\,x^2+2a^6)\sqrt{b\,x^2+a}}{143b^2}$	80
risch	$-\frac{(-11b^6x^{12}-53a\,b^5x^{10}-100a^2b^4x^8-90a^3x^6b^3-35a^4b^2x^4-a^5b\,x^2+2a^6)\sqrt{b\,x^2+a}}{143b^2}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
[Out] 1/13*x^2*(b*x^2+a)^(11/2)/b-2/143*a*(b*x^2+a)^(11/2)/b^2
```

Maxima [A]
time = 0.33, size = 33, normalized size = 0.87

$$\frac{(bx^2+a)^{\frac{11}{2}}x^2}{13\,b}-\frac{2\,(bx^2+a)^{\frac{11}{2}}a}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="maxima")
[Out] 1/13*(b*x^2 + a)^(11/2)*x^2/b - 2/143*(b*x^2 + a)^(11/2)*a/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(30) = 60.
time = 0.60, size = 78, normalized size = 2.05

$$\frac{(11\,b^6x^{12}+53\,ab^5x^{10}+100\,a^2b^4x^8+90\,a^3b^3x^6+35\,a^4b^2x^4+a^5bx^2-2\,a^6)\sqrt{bx^2+a}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/143*(11*b^6*x^12 + 53*a*b^5*x^10 + 100*a^2*b^4*x^8 + 90*a^3*b^3*x^6 + 35*
a^4*b^2*x^4 + a^5*b*x^2 - 2*a^6)*sqrt(b*x^2 + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(31) = 62$.
time = 0.96, size = 156, normalized size = 4.11

$$\begin{cases} -\frac{2a^6\sqrt{a+bx^2}}{143b^2} + \frac{a^5x^2\sqrt{a+bx^2}}{143b} + \frac{35a^4x^4\sqrt{a+bx^2}}{143} + \frac{90a^3bx^6\sqrt{a+bx^2}}{143} + \frac{100a^2b^2x^8\sqrt{a+bx^2}}{143} + \frac{53ab^3x^{10}\sqrt{a+bx^2}}{143} + \frac{b^4x^{12}\sqrt{a+bx^2}}{13} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2*a**6*sqrt(a + b*x**2)/(143*b**2) + a**5*x**2*sqrt(a + b*x**2)
/(143*b) + 35*a**4*x**4*sqrt(a + b*x**2)/143 + 90*a**3*b*x**6*sqrt(a + b*x*
*2)/143 + 100*a**2*b**2*x**8*sqrt(a + b*x**2)/143 + 53*a*b**3*x**10*sqrt(a
+ b*x**2)/143 + b**4*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(9/2)*x**4/4
, True))
```

Giac [A]
time = 1.52, size = 29, normalized size = 0.76

$$\frac{11\,(bx^2+a)^{\frac{13}{2}} - 13\,(bx^2+a)^{\frac{11}{2}}a}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/143*(11*(b*x^2 + a)^(13/2) - 13*(b*x^2 + a)^(11/2)*a)/b^2
```

Mupad [B]
time = 4.74, size = 29, normalized size = 0.76

$$-\frac{13\,a\,(b\,x^2+a)^{11/2} - 11\,(b\,x^2+a)^{13/2}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(9/2),x)
```

```
[Out] -(13*a*(a + b*x^2)^(11/2) - 11*(a + b*x^2)^(13/2))/(143*b^2)
```

3.416 $\int x(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=18

$$\frac{(a+bx^2)^{11/2}}{11b}$$

[Out] 1/11*(b*x^2+a)^(11/2)/b

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\frac{(a+bx^2)^{11/2}}{11b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^(9/2),x]

[Out] (a + b*x^2)^(11/2)/(11*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{9/2}\,dx = \frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{(a+bx^2)^{11/2}}{11b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^(9/2),x]

[Out] (a + b*x^2)^(11/2)/(11*b)

Maple [A]
time = 0.04, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{11b}$	15
derivativedivides	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{11b}$	15
default	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{11b}$	15
trager	$\frac{\left(b^{5}x^{10}+5a\,b^{4}x^{8}+10a^{2}b^{3}x^{6}+10a^{3}b^{2}x^{4}+5a^{4}b\,x^{2}+a^{5}\right)\sqrt{b\,x^{2}+a}}{11b}$	66
risch	$\frac{\left(b^{5}x^{10}+5a\,b^{4}x^{8}+10a^{2}b^{3}x^{6}+10a^{3}b^{2}x^{4}+5a^{4}b\,x^{2}+a^{5}\right)\sqrt{b\,x^{2}+a}}{11b}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*(b*x^2+a)^(11/2)/b
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.78

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(11/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(14) = 28.
time = 0.76, size = 65, normalized size = 3.61

$$\frac{\left(b^{5}x^{10}+5\,ab^{4}x^{8}+10\,a^{2}b^{3}x^{6}+10\,a^{3}b^{2}x^{4}+5\,a^{4}bx^{2}+a^{5}\right)\sqrt{bx^{2}+a}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^
2 + a^5)*sqrt(b*x^2 + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(12) = 24.

time = 0.74, size = 133, normalized size = 7.39

$$\begin{cases} \frac{a^5\sqrt{a+bx^2}}{11b} + \frac{5a^4x^2\sqrt{a+bx^2}}{11} + \frac{10a^3bx^4\sqrt{a+bx^2}}{11} + \frac{10a^2b^2x^6\sqrt{a+bx^2}}{11} + \frac{5ab^3x^8\sqrt{a+bx^2}}{11} + \frac{b^4x^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((a**5*sqrt(a + b*x**2)/(11*b) + 5*a**4*x**2*sqrt(a + b*x**2)/11 +
 10*a**3*b*x**4*sqrt(a + b*x**2)/11 + 10*a**2*b**2*x**6*sqrt(a + b*x**2)/11
 + 5*a*b**3*x**8*sqrt(a + b*x**2)/11 + b**4*x**10*sqrt(a + b*x**2)/11, Ne(b
, 0)), (a**(9/2)*x**2/2, True))
```

Giac [A]
time = 1.39, size = 14, normalized size = 0.78

$$\frac{(bx^2+a)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/11*(b*x^2 + a)^(11/2)/b
```

Mupad [B]
time = 4.81, size = 14, normalized size = 0.78

$$\frac{(b\,x^2+a)^{11/2}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(11/2)/(11*b)
```

3.417 $\int \frac{(a+bx^2)^{9/2}}{x}\,dx$

Optimal. Leaf size=108

$$a^4\sqrt{a+bx^2}+\frac{1}{3}a^3\left(a+bx^2\right)^{3/2}+\frac{1}{5}a^2\left(a+bx^2\right)^{5/2}+\frac{1}{7}a\left(a+bx^2\right)^{7/2}+\frac{1}{9}\left(a+bx^2\right)^{9/2}-a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/3*a^3*(b*x^2+a)^(3/2)+1/5*a^2*(b*x^2+a)^(5/2)+1/7*a*(b*x^2+a)^(7/2)+1/9*(
b*x^2+a)^(9/2)-a^(9/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+a^4*(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 52, 65, 214}

$$a^{9/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a^4\sqrt{a+bx^2}+\frac{1}{3}a^3\left(a+bx^2\right)^{3/2}+\frac{1}{5}a^2\left(a+bx^2\right)^{5/2}+\frac{1}{7}a\left(a+bx^2\right)^{7/2}+\frac{1}{9}\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x,x]
```

```
[Out] a^4*Sqrt[a + b*x^2] + (a^3*(a + b*x^2)^(3/2))/3 + (a^2*(a + b*x^2)^(5/2))/5
 + (a*(a + b*x^2)^(7/2))/7 + (a + b*x^2)^(9/2)/9 - a^(9/2)*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{1}{2}a\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{1}{2}a^3\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x\right.$$

$$= \frac{1}{3}a^3\left(a+bx^2\right)^{3/2} + \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{1}{2}a^4\text{Subst}\left(\int\right.$$

$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3\left(a+bx^2\right)^{3/2} + \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} +$$

$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3\left(a+bx^2\right)^{3/2} + \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} +$$

$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3\left(a+bx^2\right)^{3/2} + \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} -$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 0.78

$$\frac{1}{315}\sqrt{a+bx^2}\left(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^6+35b^4x^8\right) - a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^2*b^2*x^4 + 185*a*b^3*x^6
 + 35*b^4*x^8))/315 - a^(9/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Maple [A]
time = 0.05, size = 95, normalized size = 0.88

method	result
default	$\frac{(b\,x^2+a)^{\frac{9}{2}}}{9}+a\left(\frac{(b\,x^2+a)^{\frac{7}{2}}}{7}+a\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2}}{x}\right.\right.\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(b*x^2+a)^(9/2)+a*(1/7*(b*x^2+a)^(7/2)+a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b
*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)))))
```

Maxima [A]
time = 0.31, size = 82, normalized size = 0.76

$$-a^{\frac{9}{2}}\,\text{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{9}\,(bx^2+a)^{\frac{9}{2}}+\frac{1}{7}\,(bx^2+a)^{\frac{7}{2}}a+\frac{1}{5}\,(bx^2+a)^{\frac{5}{2}}a^2+\frac{1}{3}\,(bx^2+a)^{\frac{3}{2}}a^3+\sqrt{bx^2+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(9/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/9*(b*x^2 + a)^(9/2) + 1/7*(b*x^2
 + a)^(7/2)*a + 1/5*(b*x^2 + a)^(5/2)*a^2 + 1/3*(b*x^2 + a)^(3/2)*a^3 + sqr
t(b*x^2 + a)*a^4
```

Fricas [A]
time = 1.10, size = 170, normalized size = 1.57

$$\left[\frac{1}{2}\,a^{\frac{9}{2}}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+\frac{1}{315}\,(35\,b^4x^8+185\,ab^3x^6+408\,a^2b^2x^4+506\,a^3bx^2+563\,a^4)\sqrt{bx^2+a},\ \sqrt{-a}\,a^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\frac{1}{315}\,(35\,b^4x^8+185\,ab^3x^6+408\,a^2b^2x^4+506\,a^3bx^2+563\,a^4)\sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(9/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/315*(3
5*b^4*x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sqrt
(b*x^2 + a), sqrt(-a)*a^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 1/315*(35*b^4*
x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sqrt(b*x^2
 + a)]
```

Sympy [A]
time = 10.72, size = 160, normalized size = 1.48

$$\frac{563a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}}{315}+\frac{a^{\frac{9}{2}}\log\left(\frac{bx^2}{a}\right)}{2}-a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)+\frac{506a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}{315}+\frac{136a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{105}+\frac{37a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}{63}+\frac{\sqrt{a}\,b^4x^8\sqrt{1+\frac{bx^2}{a}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x,x)
```

```
[Out] 563*a**(9/2)*sqrt(1 + b*x**2/a)/315 + a**(9/2)*log(b*x**2/a)/2 - a**(9/2)*l
og(sqrt(1 + b*x**2/a) + 1) + 506*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)/315 + 1
36*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)/105 + 37*a**(3/2)*b**3*x**6*sqrt(1
 + b*x**2/a)/63 + sqrt(a)*b**4*x**8*sqrt(1 + b*x**2/a)/9
```

Giac [A]
time = 1.02, size = 90, normalized size = 0.83

$$\frac{a^5 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{9}\left(bx^2+a\right)^{\frac{9}{2}} + \frac{1}{7}\left(bx^2+a\right)^{\frac{7}{2}}a + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}a^2 + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a^3 + \sqrt{bx^2+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="giac")
```

```
[Out] a^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/9*(b*x^2 + a)^(9/2) + 1/7
*(b*x^2 + a)^(7/2)*a + 1/5*(b*x^2 + a)^(5/2)*a^2 + 1/3*(b*x^2 + a)^(3/2)*a^
3 + sqrt(b*x^2 + a)*a^4
```

Mupad [B]
time = 5.31, size = 87, normalized size = 0.81

$$\frac{a\left(b\,x^2+a\right)^{7/2}}{7} + \frac{\left(b\,x^2+a\right)^{9/2}}{9} + a^4\,\sqrt{b\,x^2+a} + \frac{a^3\left(b\,x^2+a\right)^{3/2}}{3} + \frac{a^2\left(b\,x^2+a\right)^{5/2}}{5} + a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\;1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x,x)
```

```
[Out] a^(9/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (a*(a + b*x^2)^(7/2))/7 +
 (a + b*x^2)^(9/2)/9 + a^4*(a + b*x^2)^(1/2) + (a^3*(a + b*x^2)^(3/2))/3 +
(a^2*(a + b*x^2)^(5/2))/5
```

3.418 $\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx$

Optimal. Leaf size=118

$$\frac{9}{2}a^3b\sqrt{a+bx^2}+\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}-\frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a}\cdots}{}\right.$$

```
[Out] 3/2*a^2*b*(b*x^2+a)^(3/2)+9/10*a*b*(b*x^2+a)^(5/2)+9/14*b*(b*x^2+a)^(7/2)-1
/2*(b*x^2+a)^(9/2)/x^2-9/2*a^(7/2)*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))+9/2*a
^3*b*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 52, 65, 214}

$$-\frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{9}{2}a^3b\sqrt{a+bx^2}+\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^3,x]
```

```
[Out] (9*a^3*b*Sqrt[a + b*x^2])/2 + (3*a^2*b*(a + b*x^2)^(3/2))/2 + (9*a*b*(a + b
*x^2)^(5/2))/10 + (9*b*(a + b*x^2)^(7/2))/14 - (a + b*x^2)^(9/2)/(2*x^2) -
(9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^{9/2}}{2x^2} + \frac{1}{4}(9b)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx, x, x^2\right)\\
&= \frac{9}{14}b(a+bx^2)^{7/2} - \frac{(a+bx^2)^{9/2}}{2x^2} + \frac{1}{4}(9ab)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)\\
&= \frac{9}{10}ab(a+bx^2)^{5/2} + \frac{9}{14}b(a+bx^2)^{7/2} - \frac{(a+bx^2)^{9/2}}{2x^2} + \frac{1}{4}(9a^2b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx\right.\\
&= \frac{3}{2}a^2b(a+bx^2)^{3/2} + \frac{9}{10}ab(a+bx^2)^{5/2} + \frac{9}{14}b(a+bx^2)^{7/2} - \frac{(a+bx^2)^{9/2}}{2x^2} + \frac{1}{4}(9a^3b)\,\text{Sub}\\
&= \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b(a+bx^2)^{3/2} + \frac{9}{10}ab(a+bx^2)^{5/2} + \frac{9}{14}b(a+bx^2)^{7/2} - \frac{(a+bx^2)^{}}{2x^2}\\
&= \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b(a+bx^2)^{3/2} + \frac{9}{10}ab(a+bx^2)^{5/2} + \frac{9}{14}b(a+bx^2)^{7/2} - \frac{(a+bx^2)^{}}{2x^2}\\
&= \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b(a+bx^2)^{3/2} + \frac{9}{10}ab(a+bx^2)^{5/2} + \frac{9}{14}b(a+bx^2)^{7/2} - \frac{(a+bx^2)^{}}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 90, normalized size = 0.76

$$\frac{\sqrt{a+bx^2}\,(-35a^4+388a^3bx^2+156a^2b^2x^4+58ab^3x^6+10b^4x^8)}{70x^2} - \frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4 + 388*a^3*b*x^2 + 156*a^2*b^2*x^4 + 58*a*b^3*x^6
+ 10*b^4*x^8))/(70*x^2) - (9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Maple [A]
time = 0.09, size = 119, normalized size = 1.01

method	result
risch	$-\frac{a^4\sqrt{b\,x^2+a}}{2x^2}+\frac{b^4x^6\sqrt{b\,x^2+a}}{7}+\frac{29b^3a\,x^4\sqrt{b\,x^2+a}}{35}+\frac{78b^2a^2x^2\sqrt{b\,x^2+a}}{35}+\frac{194a^3b\sqrt{b\,x^2+a}}{35}-$
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{2a\,x^2}+\frac{9b\left(\frac{(b\,x^2+a)^{\frac{9}{2}}}{9}+a\left(\frac{(b\,x^2+a)^{\frac{7}{2}}}{7}+a\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right.\right.\right.\right.\right.}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(11/2)+9/2*b/a*(1/9*(b*x^2+a)^(9/2)+a*(1/7*(b*x^2+a)^(
7/2)+a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/
2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

Maxima [A]
time = 0.32, size = 106, normalized size = 0.90

$$-\frac{9}{2}\,a^{\frac{7}{2}}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{9}{14}\,(bx^2+a)^{\frac{7}{2}}b+\frac{(bx^2+a)^{\frac{9}{2}}b}{2\,a}+\frac{9}{10}\,(bx^2+a)^{\frac{5}{2}}ab+\frac{3}{2}\,(bx^2+a)^{\frac{3}{2}}a^2b+\frac{9}{2}\,\sqrt{bx^2+a}\,a^3b-\frac{(bx^2+a)^{\frac{11}{2}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="maxima")
```

```
[Out] -9/2*a^(7/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 9/14*(b*x^2 + a)^(7/2)*b + 1
/2*(b*x^2 + a)^(9/2)*b/a + 9/10*(b*x^2 + a)^(5/2)*a*b + 3/2*(b*x^2 + a)^(3/
2)*a^2*b + 9/2*sqrt(b*x^2 + a)*a^3*b - 1/2*(b*x^2 + a)^(11/2)/(a*x^2)
```

Fricas [A]
time = 1.26, size = 188, normalized size = 1.59

$$\left[\frac{315\,a^{\frac{7}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(10\,b^4x^8+58\,ab^3x^6+156\,a^2b^2x^4+388\,a^3bx^2-35\,a^4)\sqrt{bx^2+a}}{140\,x^2},\ \frac{315\,\sqrt{-a}\,a^3bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(10\,b^4x^8+58\,ab^3x^6+156\,a^2b^2x^4+388\,a^3bx^2-35\,a^4)\sqrt{bx^2+a}}{70\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*a^(7/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2 - 35*a^
4)*sqrt(b*x^2 + a))/x^2, 1/70*(315*sqrt(-a)*a^3*b*x^2*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) + (10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2
- 35*a^4)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A]
time = 10.31, size = 167, normalized size = 1.42

$$-\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}}{2x^2}+\frac{194a^{\frac{7}{2}}b\sqrt{1+\frac{bx^2}{a}}}{35}+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx^2}{a}\right)}{4}-\frac{9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2}+\frac{78a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx^2}{a}}}{35}+\frac{29a^{\frac{3}{2}}b^3x^4\sqrt{1+\frac{bx^2}{a}}}{35}+\frac{\sqrt{a}\,b^4x^6\sqrt{1+\frac{bx^2}{a}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**3,x)
```

```
[Out] -a**(9/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 194*a**(7/2)*b*sqrt(1 + b*x**2/a)/3
5 + 9*a**(7/2)*b*log(b*x**2/a)/4 - 9*a**(7/2)*b*log(sqrt(1 + b*x**2/a) + 1)
/2 + 78*a**(5/2)*b**2*x**2*sqrt(1 + b*x**2/a)/35 + 29*a**(3/2)*b**3*x**4*sq
rt(1 + b*x**2/a)/35 + sqrt(a)*b**4*x**6*sqrt(1 + b*x**2/a)/7
```

Giac [A]
time = 1.05, size = 116, normalized size = 0.98

$$\frac{\frac{315\,a^4b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\,(bx^2+a)^{\frac{7}{2}}b^2+28\,(bx^2+a)^{\frac{5}{2}}ab^2+70\,(bx^2+a)^{\frac{3}{2}}a^2b^2+280\,\sqrt{bx^2+a}\,a^3b^2-\frac{35\,\sqrt{bx^2+a}\,a^4b}{x^2}}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 10*(b*x^2 + a
)^(7/2)*b^2 + 28*(b*x^2 + a)^(5/2)*a*b^2 + 70*(b*x^2 + a)^(3/2)*a^2*b^2 + 2
80*sqrt(b*x^2 + a)*a^3*b^2 - 35*sqrt(b*x^2 + a)*a^4*b/x^2)/b
```

Mupad [B]
time = 5.45, size = 95, normalized size = 0.81

$$\frac{b\,(b\,x^2+a)^{7/2}}{7}+4\,a^3\,b\,\sqrt{b\,x^2+a}+a^2\,b\,\left(b\,x^2+a\right)^{3/2}-\frac{a^4\,\sqrt{b\,x^2+a}}{2\,x^2}+\frac{2\,a\,b\,(b\,x^2+a)^{5/2}}{5}+\frac{a^{7/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^3,x)
```

```
[Out] (b*(a + b*x^2)^(7/2))/7 + 4*a^3*b*(a + b*x^2)^(1/2) + a^2*b*(a + b*x^2)^(3/
2) - (a^4*(a + b*x^2)^(1/2))/(2*x^2) + (a^(7/2)*b*atan(((a + b*x^2)^(1/2)*1
i)/a^(1/2))*9i)/2 + (2*a*b*(a + b*x^2)^(5/2))/5
```

3.419 $\int \frac{(a+bx^2)^{9/2}}{x^5} dx$

Optimal. Leaf size=126

$$\frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}-\frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\right.$$

```
[Out] 21/8*a*b^2*(b*x^2+a)^(3/2)+63/40*b^2*(b*x^2+a)^(5/2)-9/8*b*(b*x^2+a)^(7/2)/
x^2-1/4*(b*x^2+a)^(9/2)/x^4-63/8*a^(5/2)*b^2*arctanh((b*x^2+a)^(1/2)/a^(1/2
))+63/8*a^2*b^2*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 52, 65, 214}

$$-\frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^5,x]
```

```
[Out] (63*a^2*b^2*Sqrt[a + b*x^2])/8 + (21*a*b^2*(a + b*x^2)^(3/2))/8 + (63*b^2*(
a + b*x^2)^(5/2))/40 - (9*b*(a + b*x^2)^(7/2))/(8*x^2) - (a + b*x^2)^(9/2)/
(4*x^4) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^{9/2}}{4x^4} + \frac{1}{8}(9b)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{9b(a+bx^2)^{7/2}}{8x^2} - \frac{(a+bx^2)^{9/2}}{4x^4} + \frac{1}{16}(63b^2)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)\\
&= \frac{63}{40}b^2(a+bx^2)^{5/2} - \frac{9b(a+bx^2)^{7/2}}{8x^2} - \frac{(a+bx^2)^{9/2}}{4x^4} + \frac{1}{16}(63ab^2)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,d\right.\\
&= \frac{21}{8}ab^2(a+bx^2)^{3/2} + \frac{63}{40}b^2(a+bx^2)^{5/2} - \frac{9b(a+bx^2)^{7/2}}{8x^2} - \frac{(a+bx^2)^{9/2}}{4x^4} + \frac{1}{16}(63a^2b^2)\,\text{S}\\
&= \frac{63}{8}a^2b^2\sqrt{a+bx^2} + \frac{21}{8}ab^2(a+bx^2)^{3/2} + \frac{63}{40}b^2(a+bx^2)^{5/2} - \frac{9b(a+bx^2)^{7/2}}{8x^2} - \frac{(a+bx^2}{4x^4}\\
&= \frac{63}{8}a^2b^2\sqrt{a+bx^2} + \frac{21}{8}ab^2(a+bx^2)^{3/2} + \frac{63}{40}b^2(a+bx^2)^{5/2} - \frac{9b(a+bx^2)^{7/2}}{8x^2} - \frac{(a+bx^2}{4x^4}\\
&= \frac{63}{8}a^2b^2\sqrt{a+bx^2} + \frac{21}{8}ab^2(a+bx^2)^{3/2} + \frac{63}{40}b^2(a+bx^2)^{5/2} - \frac{9b(a+bx^2)^{7/2}}{8x^2} - \frac{(a+bx^2}{4x^4}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 92, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\,(-10a^4-85a^3bx^2+288a^2b^2x^4+56ab^3x^6+8b^4x^8)}{40x^4} - \frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^5,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-10*a^4 - 85*a^3*b*x^2 + 288*a^2*b^2*x^4 + 56*a*b^3*x^6 +
 8*b^4*x^8))/(40*x^4) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Maple [A]
time = 0.08, size = 143, normalized size = 1.13

method	result
risch	$-\frac{a^{3} \sqrt{b\,x^{2}+a}\,\left(17 b\,x^{2}+2 a \right)}{8 x^{4}}+\frac{b^{4} x^{4} \sqrt{b\,x^{2}+a}}{5}+\frac{7 b^{3} a\,x^{2} \sqrt{b\,x^{2}+a}}{5}+\frac{36 a^{2} b^{2} \sqrt{b\,x^{2}+a}}{5}-\frac{63 b^{2} a^{\frac{5}{2}} \ln \left(\frac{2 a +2}{}\right.}{}$
default	$-\frac{\left(b\,x^{2}+a \right)^{\frac{11}{2}}}{4 a\,x^{4}}+\frac{7 b \left(-\frac{\left(b\,x^{2}+a \right)^{\frac{11}{2}}}{2 a\,x^{2}}+\frac{9 b \left(\frac{\left(b\,x^{2}+a \right)^{\frac{9}{2}}}{9}+a \left(\frac{\left(b\,x^{2}+a \right)^{\frac{7}{2}}}{7}+a \left(\frac{\left(b\,x^{2}+a \right)^{\frac{5}{2}}}{5}+a \left(\frac{\left(b\,x^{2}+a \right)^{\frac{3}{2}}}{3}+a \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln \left(2\right.\right.\right.\right.\right.}{2 a}\right.}{4 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(11/2)+7/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(11/2)+9/2*b/a*(1
/9*(b*x^2+a)^(9/2)+a*(1/7*(b*x^2+a)^(7/2)+a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*
x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)))))))
```

Maxima [A]
time = 0.30, size = 136, normalized size = 1.08

$$-\frac{63}{8}\,a^{\frac{5}{2}}b^{2}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{63}{40}\,\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}+\frac{7\,\left(bx^{2}+a\right)^{\frac{9}{2}}b^{2}}{8\,a^{2}}+\frac{9\,\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}{8\,a}+\frac{21}{8}\,\left(bx^{2}+a\right)^{\frac{3}{2}}ab^{2}+\frac{63}{8}\,\sqrt{bx^{2}+a}\,a^{2}b^{2}-\frac{7\,\left(bx^{2}+a\right)^{\frac{11}{2}}b}{8\,a^{2}x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="maxima")
```

```
[Out] -63/8*a^(5/2)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) + 63/40*(b*x^2 + a)^(5/2)*b
^2 + 7/8*(b*x^2 + a)^(9/2)*b^2/a^2 + 9/8*(b*x^2 + a)^(7/2)*b^2/a + 21/8*(b*
x^2 + a)^(3/2)*a*b^2 + 63/8*sqrt(b*x^2 + a)*a^2*b^2 - 7/8*(b*x^2 + a)^(11/2
)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(11/2)/(a*x^4)
```

Fricas [A]
time = 1.09, size = 192, normalized size = 1.52

$$\left[\frac{315\,a^{\frac{5}{2}}b^{2}x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(8\,b^{4}x^{8}+56\,ab^{3}x^{6}+288\,a^{2}b^{2}x^{4}-85\,a^{3}bx^{2}-10\,a^{4}\right)\sqrt{bx^{2}+a}}{80\,x^{4}},\ \frac{315\,\sqrt{-a}\,a^{2}b^{2}x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(8\,b^{4}x^{8}+56\,ab^{3}x^{6}+288\,a^{2}b^{2}x^{4}-85\,a^{3}bx^{2}-10\,a^{4}\right)\sqrt{bx^{2}+a}}{40\,x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/80*(315*a^(5/2)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b*x^2 - 10*a^4
)*sqrt(b*x^2 + a))/x^4, 1/40*(315*sqrt(-a)*a^2*b^2*x^4*arctan(sqrt(-a)/sqrt
(b*x^2 + a)) + (8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b*x^2 -
 10*a^4)*sqrt(b*x^2 + a))/x^4]
```

Sympy [A]
time = 9.77, size = 175, normalized size = 1.39

$$-\frac{63a^{\frac{5}{2}}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8}-\frac{a^5}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{19a^4\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{203a^3b^{\frac{3}{2}}}{40x\sqrt{\frac{a}{bx^2}+1}}+\frac{43a^2b^{\frac{5}{2}}x}{5\sqrt{\frac{a}{bx^2}+1}}+\frac{8ab^{\frac{7}{2}}x^3}{5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{9}{2}}x^5}{5\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**5,x)
```

```
[Out] -63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**5/(4*sqrt(b)*x**5*sqrt(
a/(b*x**2) + 1)) - 19*a**4*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) + 203*a**3
*b**(3/2)/(40*x*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)*x/(5*sqrt(a/(b*x**
2) + 1)) + 8*a*b**(7/2)*x**3/(5*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x**5/(5*sq
rt(a/(b*x**2) + 1))
```

Giac [A]
time = 1.12, size = 124, normalized size = 0.98

$$\frac{\frac{315\,a^3b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\,(bx^2+a)^{\frac{5}{2}}b^3+40\,(bx^2+a)^{\frac{3}{2}}ab^3+240\,\sqrt{bx^2+a}\,a^2b^3-\frac{5\left(17\,(bx^2+a)^{\frac{3}{2}}a^3b^3-15\,\sqrt{bx^2+a}\,a^4b^3\right)}{b^2x^4}}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/40*(315*a^3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*(b*x^2 + a)
^(5/2)*b^3 + 40*(b*x^2 + a)^(3/2)*a*b^3 + 240*sqrt(b*x^2 + a)*a^2*b^3 - 5*(
17*(b*x^2 + a)^(3/2)*a^3*b^3 - 15*sqrt(b*x^2 + a)*a^4*b^3)/(b^2*x^4))/b
```

Mupad [B]
time = 5.65, size = 132, normalized size = 1.05

$$\frac{\frac{15\,a^4\,b^2\,\sqrt{b\,x^2+a}}{8}-\frac{17\,a^3\,b^2\,{\left(b\,x^2+a\right)}^{3/2}}{8}}{{\left(b\,x^2+a\right)}^2-2\,a\,\left(b\,x^2+a\right)+a^2}+\frac{b^2\,{\left(b\,x^2+a\right)}^{5/2}}{5}+a\,b^2\,{\left(b\,x^2+a\right)}^{3/2}+6\,a^2\,b^2\,\sqrt{b\,x^2+a}+\frac{a^{5/2}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,63{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^5,x)
```

```
[Out] ((15*a^4*b^2*(a + b*x^2)^(1/2))/8 - (17*a^3*b^2*(a + b*x^2)^(3/2))/8)/((a +
 b*x^2)^2 - 2*a*(a + b*x^2) + a^2) + (b^2*(a + b*x^2)^(5/2))/5 + (a^(5/2)*b
^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*63i)/8 + a*b^2*(a + b*x^2)^(3/2) +
6*a^2*b^2*(a + b*x^2)^(1/2)
```

3.420 $\int \frac{(a+bx^2)^{9/2}}{x^7} dx$

Optimal. Leaf size=126

$$\frac{105}{16}ab^3\sqrt{a+bx^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}-\frac{21b^2(a+bx^2)^{5/2}}{16x^2}-\frac{3b(a+bx^2)^{7/2}}{8x^4}-\frac{(a+bx^2)^{9/2}}{6x^6}-\frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\right.$$

```
[Out] 35/16*b^3*(b*x^2+a)^(3/2)-21/16*b^2*(b*x^2+a)^(5/2)/x^2-3/8*b*(b*x^2+a)^(7/
2)/x^4-1/6*(b*x^2+a)^(9/2)/x^6-105/16*a^(3/2)*b^3*arctanh((b*x^2+a)^(1/2)/a
^(1/2))+105/16*a*b^3*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 52, 65, 214}

$$-\frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}+\frac{105}{16}ab^3\sqrt{a+bx^2}-\frac{21b^2(a+bx^2)^{5/2}}{16x^2}-\frac{(a+bx^2)^{9/2}}{6x^6}-\frac{3b(a+bx^2)^{7/2}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^7,x]
```

```
[Out] (105*a*b^3*Sqrt[a + b*x^2])/16 + (35*b^3*(a + b*x^2)^(3/2))/16 - (21*b^2*(a
 + b*x^2)^(5/2))/(16*x^2) - (3*b*(a + b*x^2)^(7/2))/(8*x^4) - (a + b*x^2)^(
9/2)/(6*x^6) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^{9/2}}{6x^6} + \frac{1}{4}(3b)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{3b(a+bx^2)^{7/2}}{8x^4} - \frac{(a+bx^2)^{9/2}}{6x^6} + \frac{1}{16}(21b^2)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{21b^2(a+bx^2)^{5/2}}{16x^2} - \frac{3b(a+bx^2)^{7/2}}{8x^4} - \frac{(a+bx^2)^{9/2}}{6x^6} + \frac{1}{32}(105b^3)\,\text{Subst}\left(\int \frac{(a+bx)^{3/}}{x}\right.\\
&= \frac{35}{16}b^3(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{16x^2} - \frac{3b(a+bx^2)^{7/2}}{8x^4} - \frac{(a+bx^2)^{9/2}}{6x^6} + \frac{1}{32}(105ab^3)\,\text{S}\\
&= \frac{105}{16}ab^3\sqrt{a+bx^2} + \frac{35}{16}b^3(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{16x^2} - \frac{3b(a+bx^2)^{7/2}}{8x^4} - \frac{(a+bx^2}{6x^6}\\
&= \frac{105}{16}ab^3\sqrt{a+bx^2} + \frac{35}{16}b^3(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{16x^2} - \frac{3b(a+bx^2)^{7/2}}{8x^4} - \frac{(a+bx^2}{6x^6}\\
&= \frac{105}{16}ab^3\sqrt{a+bx^2} + \frac{35}{16}b^3(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{16x^2} - \frac{3b(a+bx^2)^{7/2}}{8x^4} - \frac{(a+bx^2}{6x^6}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 92, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\,(-8a^4-50a^3bx^2-165a^2b^2x^4+208ab^3x^6+16b^4x^8)}{48x^6} - \frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^4 - 50*a^3*b*x^2 - 165*a^2*b^2*x^4 + 208*a*b^3*x^6 +
 16*b^4*x^8))/(48*x^6) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/16
```

Maple [A]
time = 0.08, size = 167, normalized size = 1.33

method	result
risch	$-\frac{a^{2} \sqrt{b\,x^{2}+a}\,\left(165 b^{2} x^{4}+50 a b\,x^{2}+8 a^{2}\right)}{48 x^{6}}+\frac{b^{4} x^{2} \sqrt{b\,x^{2}+a}}{3}+\frac{13 a\,b^{3} \sqrt{b\,x^{2}+a}}{3}-\frac{105 b^{3} a^{\frac{3}{2}} \ln \left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+\cdots}}{x}\right.}{16}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{6 a\,x^{6}}+\frac{5 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{4 a\,x^{4}}+\frac{7 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{2 a\,x^{2}}+\frac{9 b \left(\frac{\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{9}+a \left(\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7}+a \left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a \left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a \left(\sqrt{b\,x^{2}+\cdots}\right.\right.\right.\right.\right.}{2 a}\right.}{4 a}\right.}{6 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(11/2)+5/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(11/2)+7/4*b/a*(-
1/2/a/x^2*(b*x^2+a)^(11/2)+9/2*b/a*(1/9*(b*x^2+a)^(9/2)+a*(1/7*(b*x^2+a)^(7
/2)+a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2
)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))))
```

Maxima [A]
time = 0.27, size = 156, normalized size = 1.24

$$-\frac{105}{16}\,a^{\frac{3}{2}}b^{3}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{35}{16}\,(bx^{2}+a)^{\frac{3}{2}}b^{3}+\frac{35\,(bx^{2}+a)^{\frac{9}{2}}b^{3}}{48\,a^{3}}+\frac{15\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}{16\,a^{2}}+\frac{21\,(bx^{2}+a)^{\frac{5}{2}}b^{3}}{16\,a}+\frac{105}{16}\,\sqrt{bx^{2}+a}\,ab^{3}-\frac{35\,(bx^{2}+a)^{\frac{11}{2}}b^{2}}{48\,a^{3}x^{2}}-\frac{5\,(bx^{2}+a)^{\frac{11}{2}}b}{24\,a^{2}x^{4}}-\frac{(bx^{2}+a)^{\frac{11}{2}}}{6\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="maxima")
```

```
[Out] -105/16*a^(3/2)*b^3*arcsinh(a/(sqrt(a*b)*abs(x))) + 35/16*(b*x^2 + a)^(3/2)
*b^3 + 35/48*(b*x^2 + a)^(9/2)*b^3/a^3 + 15/16*(b*x^2 + a)^(7/2)*b^3/a^2 +
21/16*(b*x^2 + a)^(5/2)*b^3/a + 105/16*sqrt(b*x^2 + a)*a*b^3 - 35/48*(b*x^2
 + a)^(11/2)*b^2/(a^3*x^2) - 5/24*(b*x^2 + a)^(11/2)*b/(a^2*x^4) - 1/6*(b*x
^2 + a)^(11/2)/(a*x^6)
```

Fricas [A]
time = 1.06, size = 190, normalized size = 1.51

$$\left[\frac{315\,a^{\frac{3}{2}}b^{3}x^{6}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(16\,b^{4}x^{8}+208\,ab^{3}x^{6}-165\,a^{2}b^{2}x^{4}-50\,a^{3}bx^{2}-8\,a^{4})\sqrt{bx^{2}+a}}{96\,x^{6}}, \frac{315\,\sqrt{-a}\,ab^{3}x^{6}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(16\,b^{4}x^{8}+208\,ab^{3}x^{6}-165\,a^{2}b^{2}x^{4}-50\,a^{3}bx^{2}-8\,a^{4})\sqrt{bx^{2}+a}}{48\,x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(315*a^(3/2)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*b*x^2 - 8*a^
4)*sqrt(b*x^2 + a))/x^6, 1/48*(315*sqrt(-a)*a*b^3*x^6*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) + (16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*b*x^2
- 8*a^4)*sqrt(b*x^2 + a))/x^6]
```

Sympy [A]
time = 9.34, size = 175, normalized size = 1.39

$$-\frac{105a^{\frac{3}{2}}b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16}-\frac{a^{5}}{6\sqrt{b}\,x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{29a^{4}\sqrt{b}}{24x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{215a^{3}b^{\frac{3}{2}}}{48x^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{43a^{2}b^{\frac{5}{2}}}{48x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{14ab^{\frac{7}{2}}x}{3\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{\frac{9}{2}}x^{3}}{3\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**7,x)
```

```
[Out] -105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*x))/16 - a**5/(6*sqrt(b)*x**7*sqr
t(a/(b*x**2) + 1)) - 29*a**4*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1)) - 215*a
**3*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)/(48*x*sqrt(a
/(b*x**2) + 1)) + 14*a*b**(7/2)*x/(3*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x**3/
(3*sqrt(a/(b*x**2) + 1))
```

Giac [A]
time = 0.95, size = 124, normalized size = 0.98

$$\frac{\frac{315\,a^{2}b^{4}\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+16\,(bx^{2}+a)^{\frac{3}{2}}b^{4}+192\,\sqrt{bx^{2}+a}\,ab^{4}-\frac{165\,(bx^{2}+a)^{\frac{5}{2}}a^{2}b^{4}-280\,(bx^{2}+a)^{\frac{3}{2}}a^{3}b^{4}+123\,\sqrt{bx^{2}+a}\,a^{4}b^{4}}{b^{3}x^{6}}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(315*a^2*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 16*(b*x^2 + a
)^(3/2)*b^4 + 192*sqrt(b*x^2 + a)*a*b^4 - (165*(b*x^2 + a)^(5/2)*a^2*b^4 -
280*(b*x^2 + a)^(3/2)*a^3*b^4 + 123*sqrt(b*x^2 + a)*a^4*b^4)/(b^3*x^6))/b
```

Mupad [B]
time = 5.99, size = 149, normalized size = 1.18

$$\frac{\frac{41\,a^4\,b^3\,\sqrt{b\,x^2+a}}{16}-\frac{35\,a^3\,b^3\,{\left(b\,x^2+a\right)}^{3/2}}{6}+\frac{55\,a^2\,b^3\,{\left(b\,x^2+a\right)}^{5/2}}{16}}{3\,a\,{\left(b\,x^2+a\right)}^2-3\,a^2\,\left(b\,x^2+a\right)-{\left(b\,x^2+a\right)}^3+a^3}+\frac{b^3\,{\left(b\,x^2+a\right)}^{3/2}}{3}+4\,a\,b^3\,\sqrt{b\,x^2+a}+\frac{a^{3/2}\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,105{}\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^7,x)
```

```
[Out] ((41*a^4*b^3*(a + b*x^2)^(1/2))/16 - (35*a^3*b^3*(a + b*x^2)^(3/2))/6 + (55
*a^2*b^3*(a + b*x^2)^(5/2))/16)/(3*a*(a + b*x^2)^2 - 3*a^2*(a + b*x^2) - (a
 + b*x^2)^3 + a^3) + (b^3*(a + b*x^2)^(3/2))/3 + (a^(3/2)*b^3*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*105i)/16 + 4*a*b^3*(a + b*x^2)^(1/2)
```

3.421 $\int \frac{(a+bx^2)^{9/2}}{x^9} dx$

Optimal. Leaf size=128

$$\frac{315}{128}b^4\sqrt{a+bx^2}-\frac{105b^3(a+bx^2)^{3/2}}{128x^2}-\frac{21b^2(a+bx^2)^{5/2}}{64x^4}-\frac{3b(a+bx^2)^{7/2}}{16x^6}-\frac{(a+bx^2)^{9/2}}{8x^8}-\frac{315}{128}\sqrt{a}\,b^4\tanh^{-1}\Bigg($$

```
[Out] -105/128*b^3*(b*x^2+a)^(3/2)/x^2-21/64*b^2*(b*x^2+a)^(5/2)/x^4-3/16*b*(b*x^
2+a)^(7/2)/x^6-1/8*(b*x^2+a)^(9/2)/x^8-315/128*b^4*arctanh((b*x^2+a)^(1/2)/
a^(1/2))*a^(1/2)+315/128*b^4*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 52, 65, 214}

$$\frac{315}{128}b^4\sqrt{a+bx^2}-\frac{315}{128}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{105b^3(a+bx^2)^{3/2}}{128x^2}-\frac{21b^2(a+bx^2)^{5/2}}{64x^4}-\frac{(a+bx^2)^{9/2}}{8x^8}-\frac{3b(a+bx^2)^{7/2}}{16x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^9,x]
```

```
[Out] (315*b^4*Sqrt[a + b*x^2])/128 - (105*b^3*(a + b*x^2)^(3/2))/(128*x^2) - (21
*b^2*(a + b*x^2)^(5/2))/(64*x^4) - (3*b*(a + b*x^2)^(7/2))/(16*x^6) - (a +
b*x^2)^(9/2)/(8*x^8) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/1
28
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^{9/2}}{8x^8} + \frac{1}{16}(9b)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{3b(a+bx^2)^{7/2}}{16x^6} - \frac{(a+bx^2)^{9/2}}{8x^8} + \frac{1}{32}(21b^2)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{21b^2(a+bx^2)^{5/2}}{64x^4} - \frac{3b(a+bx^2)^{7/2}}{16x^6} - \frac{(a+bx^2)^{9/2}}{8x^8} + \frac{1}{128}(105b^3)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\right.$$
$$= -\frac{105b^3(a+bx^2)^{3/2}}{128x^2} - \frac{21b^2(a+bx^2)^{5/2}}{64x^4} - \frac{3b(a+bx^2)^{7/2}}{16x^6} - \frac{(a+bx^2)^{9/2}}{8x^8} + \frac{1}{256}(315b^4)$$
$$= \frac{315}{128}b^4\sqrt{a+bx^2} - \frac{105b^3(a+bx^2)^{3/2}}{128x^2} - \frac{21b^2(a+bx^2)^{5/2}}{64x^4} - \frac{3b(a+bx^2)^{7/2}}{16x^6} - \frac{(a+bx^2)^{9/2}}{8x^8}$$
$$= \frac{315}{128}b^4\sqrt{a+bx^2} - \frac{105b^3(a+bx^2)^{3/2}}{128x^2} - \frac{21b^2(a+bx^2)^{5/2}}{64x^4} - \frac{3b(a+bx^2)^{7/2}}{16x^6} - \frac{(a+bx^2)^{9/2}}{8x^8}$$
$$= \frac{315}{128}b^4\sqrt{a+bx^2} - \frac{105b^3(a+bx^2)^{3/2}}{128x^2} - \frac{21b^2(a+bx^2)^{5/2}}{64x^4} - \frac{3b(a+bx^2)^{7/2}}{16x^6} - \frac{(a+bx^2)^{9/2}}{8x^8}$$

Mathematica [A]

time = 0.14, size = 92, normalized size = 0.72

$$\frac{\sqrt{a+bx^2}\,(-16a^4-88a^3bx^2-210a^2b^2x^4-325ab^3x^6+128b^4x^8)}{128x^8}-\frac{315}{128}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^4 - 88*a^3*b*x^2 - 210*a^2*b^2*x^4 - 325*a*b^3*x^6
+ 128*b^4*x^8))/(128*x^8) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]])/128
```

Maple [A]
time = 0.09, size = 191, normalized size = 1.49

method	result
risch	$-\frac{a\sqrt{b\,x^2+a}\,(325b^3x^6+210a\,b^2x^4+88a^2b\,x^2+16a^3)}{128x^8}-\frac{315\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)b^4}{128}+b^4\sqrt{b\,x^2+a}$
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{8a\,x^8}+\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{6a\,x^6}+\frac{5b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{4a\,x^4}+\frac{7b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{2a\,x^2}+\frac{9b\left(\frac{(b\,x^2+a)^{\frac{9}{2}}}{9}+a\left(\frac{(b\,x^2+a)^{\frac{7}{2}}}{7}+a\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2}{}\right.\right.\right.\right.}{}\right.}{}\right)}{6a}\right)}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(11/2)+3/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(11/2)+5/6*b/a*(-
1/4/a/x^4*(b*x^2+a)^(11/2)+7/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(11/2)+9/2*b/a*(1/
9*(b*x^2+a)^(9/2)+a*(1/7*(b*x^2+a)^(7/2)+a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x
^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x
)))))))))
```

Maxima [A]
time = 0.28, size = 178, normalized size = 1.39

$$-\frac{315}{128}\sqrt{a}\,b^4\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{315}{128}\sqrt{bx^2+a}\,b^4+\frac{35\,(bx^2+a)^{\frac{9}{2}}b^4}{128\,a^4}+\frac{45\,(bx^2+a)^{\frac{7}{2}}b^4}{128\,a^3}+\frac{63\,(bx^2+a)^{\frac{5}{2}}b^4}{128\,a^2}+\frac{105\,(bx^2+a)^{\frac{3}{2}}b^4}{128\,a}-\frac{35\,(bx^2+a)^{\frac{11}{2}}b^3}{128\,a^4x^2}-\frac{5\,(bx^2+a)^{\frac{11}{2}}b^2}{64\,a^3x^4}-\frac{(bx^2+a)^{\frac{11}{2}}b}{16\,a^2x^6}-\frac{(bx^2+a)^{\frac{11}{2}}}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="maxima")
```

```
[Out] -315/128*sqrt(a)*b^4*arcsinh(a/(sqrt(a*b)*abs(x))) + 315/128*sqrt(b*x^2 + a
)*b^4 + 35/128*(b*x^2 + a)^(9/2)*b^4/a^4 + 45/128*(b*x^2 + a)^(7/2)*b^4/a^3
 + 63/128*(b*x^2 + a)^(5/2)*b^4/a^2 + 105/128*(b*x^2 + a)^(3/2)*b^4/a - 35/
128*(b*x^2 + a)^(11/2)*b^3/(a^4*x^2) - 5/64*(b*x^2 + a)^(11/2)*b^2/(a^3*x^4
) - 1/16*(b*x^2 + a)^(11/2)*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(11/2)/(a*x^8)
```

Fricas [A]
time = 1.40, size = 189, normalized size = 1.48

$$\left[\frac{315\,\sqrt{a}\,b^4x^8\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(128\,b^4x^8-325\,ab^3x^6-210\,a^2b^2x^4-88\,a^3bx^2-16\,a^4)\sqrt{bx^2+a}}{256\,x^8},\ \frac{315\,\sqrt{-a}\,b^4x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(128\,b^4x^8-325\,ab^3x^6-210\,a^2b^2x^4-88\,a^3bx^2-16\,a^4)\sqrt{bx^2+a}}{128\,x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/256*(315*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) + 2*(128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a^3*b*x^2 - 16
*a^4)*sqrt(b*x^2 + a))/x^8, 1/128*(315*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqr
t(b*x^2 + a)) + (128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a^3*b*x
^2 - 16*a^4)*sqrt(b*x^2 + a))/x^8]
```

Sympy [A]
time = 9.73, size = 173, normalized size = 1.35

$$-\frac{315\sqrt{a}\,b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128}-\frac{a^5}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{13a^4\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{149a^3b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{535a^2b^{\frac{5}{2}}}{128x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{197ab^{\frac{7}{2}}}{128x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{9}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**9,x)
```

```
[Out] -315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*x))/128 - a**5/(8*sqrt(b)*x**9*sqr
t(a/(b*x**2) + 1)) - 13*a**4*sqrt(b)/(16*x**7*sqrt(a/(b*x**2) + 1)) - 149*a
**3*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) - 535*a**2*b**(5/2)/(128*x**3*s
qrt(a/(b*x**2) + 1)) - 197*a*b**(7/2)/(128*x*sqrt(a/(b*x**2) + 1)) + b**(9/
2)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A]
time = 0.87, size = 122, normalized size = 0.95

$$\frac{\frac{315\,ab^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+128\,\sqrt{bx^2+a}\,b^5-\frac{325\,(bx^2+a)^{\frac{7}{2}}ab^5-765\,(bx^2+a)^{\frac{5}{2}}a^2b^5+643\,(bx^2+a)^{\frac{3}{2}}a^3b^5-187\,\sqrt{bx^2+a}\,a^4b^5}{b^4x^8}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/128*(315*a*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x^2
 + a)*b^5 - (325*(b*x^2 + a)^(7/2)*a*b^5 - 765*(b*x^2 + a)^(5/2)*a^2*b^5 +
643*(b*x^2 + a)^(3/2)*a^3*b^5 - 187*sqrt(b*x^2 + a)*a^4*b^5)/(b^4*x^8))/b
```

Mupad [B]
time = 6.20, size = 105, normalized size = 0.82

$$b^4\,\sqrt{b\,x^2+a}-\frac{325\,a\,{\left(b\,x^2+a\right)}^{7/2}}{128\,x^8}+\frac{187\,a^4\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{643\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^8}+\frac{765\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}{128\,x^8}+\frac{\sqrt{a}\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,315{}\mathrm{i}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^9,x)
```

```
[Out] b^4*(a + b*x^2)^(1/2) + (a^(1/2)*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3
15i)/128 - (325*a*(a + b*x^2)^(7/2))/(128*x^8) + (187*a^4*(a + b*x^2)^(1/2)
)/(128*x^8) - (643*a^3*(a + b*x^2)^(3/2))/(128*x^8) + (765*a^2*(a + b*x^2)^
(5/2))/(128*x^8)
```

3.422 $\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx$

Optimal. Leaf size=131

$$-\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3(a+bx^2)^{3/2}}{128x^4}-\frac{21b^2(a+bx^2)^{5/2}}{160x^6}-\frac{9b(a+bx^2)^{7/2}}{80x^8}-\frac{(a+bx^2)^{9/2}}{10x^{10}}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}$$

```
[Out] -21/128*b^3*(b*x^2+a)^(3/2)/x^4-21/160*b^2*(b*x^2+a)^(5/2)/x^6-9/80*b*(b*x^
2+a)^(7/2)/x^8-1/10*(b*x^2+a)^(9/2)/x^10-63/256*b^5*arctanh((b*x^2+a)^(1/2)
/a^(1/2))/a^(1/2)-63/256*b^4*(b*x^2+a)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 43, 65, 214}

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}-\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3(a+bx^2)^{3/2}}{128x^4}-\frac{21b^2(a+bx^2)^{5/2}}{160x^6}-\frac{(a+bx^2)^{9/2}}{10x^{10}}-\frac{9b(a+bx^2)^{7/2}}{80x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^11,x]
```

```
[Out] (-63*b^4*Sqrt[a + b*x^2])/(256*x^2) - (21*b^3*(a + b*x^2)^(3/2))/(128*x^4)
- (21*b^2*(a + b*x^2)^(5/2))/(160*x^6) - (9*b*(a + b*x^2)^(7/2))/(80*x^8) -
 (a + b*x^2)^(9/2)/(10*x^10) - (63*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2
56*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)^{9/2}}{10x^{10}} + \frac{1}{20}(9b)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{9b(a+bx^2)^{7/2}}{80x^8} - \frac{(a+bx^2)^{9/2}}{10x^{10}} + \frac{1}{160}\left(63b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{21b^2(a+bx^2)^{5/2}}{160x^6} - \frac{9b(a+bx^2)^{7/2}}{80x^8} - \frac{(a+bx^2)^{9/2}}{10x^{10}} + \frac{1}{64}\left(21b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\right.$$

$$= -\frac{21b^3(a+bx^2)^{3/2}}{128x^4} - \frac{21b^2(a+bx^2)^{5/2}}{160x^6} - \frac{9b(a+bx^2)^{7/2}}{80x^8} - \frac{(a+bx^2)^{9/2}}{10x^{10}} + \frac{1}{256}\left(63b^4\right)\text{S}$$

$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3(a+bx^2)^{3/2}}{128x^4} - \frac{21b^2(a+bx^2)^{5/2}}{160x^6} - \frac{9b(a+bx^2)^{7/2}}{80x^8} - \frac{(a+bx^2)}{10x^{10}}$$

$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3(a+bx^2)^{3/2}}{128x^4} - \frac{21b^2(a+bx^2)^{5/2}}{160x^6} - \frac{9b(a+bx^2)^{7/2}}{80x^8} - \frac{(a+bx^2)}{10x^{10}}$$

$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3(a+bx^2)^{3/2}}{128x^4} - \frac{21b^2(a+bx^2)^{5/2}}{160x^6} - \frac{9b(a+bx^2)^{7/2}}{80x^8} - \frac{(a+bx^2)}{10x^{10}}$$

Mathematica [A]
time = 0.16, size = 92, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\left(-128a^4-656a^3bx^2-1368a^2b^2x^4-1490ab^3x^6-965b^4x^8\right)}{1280x^{10}} - \frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-128*a^4 - 656*a^3*b*x^2 - 1368*a^2*b^2*x^4 - 1490*a*b^3*
x^6 - 965*b^4*x^8))/(1280*x^10) - (63*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/(256*Sqrt[a])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(103) = 206$.

time = 0.12, size = 215, normalized size = 1.64

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(965 b^{4} x^{8}+1490 a\,b^{3} x^{6}+1368 a^{2} b^{2} x^{4}+656 a^{3} b\,x^{2}+128 a^{4}\right)}{1280 x^{10}}-\frac{63 b^{5} \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{256 \sqrt{a}}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{10 a\,x^{10}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{8 a\,x^{8}}+\frac{3 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{6 a\,x^{6}}+\frac{5 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{4 a\,x^{4}}+\frac{7 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{2 a\,x^{2}}+\frac{9 b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{9}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7}+a\left(\frac{\left(b\,x\right.}{}\right.\right.\right)}{}\right)}{}\right)}{8}\right)}{10 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/a/x^10*(b*x^2+a)^(11/2)+1/10*b/a*(-1/8/a/x^8*(b*x^2+a)^(11/2)+3/8*b/a
*(-1/6/a/x^6*(b*x^2+a)^(11/2)+5/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(11/2)+7/4*b/a*
(-1/2/a/x^2*(b*x^2+a)^(11/2)+9/2*b/a*(1/9*(b*x^2+a)^(9/2)+a*(1/7*(b*x^2+a)^
(7/2)+a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1
/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))))))
```

Maxima [A]
time = 0.30, size = 201, normalized size = 1.53

$$-\frac{63\,b^{5}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,\sqrt{a}}+\frac{7\,(bx^2+a)^{\frac{9}{2}}b^5}{256\,a^5}+\frac{9\,(bx^2+a)^{\frac{7}{2}}b^5}{256\,a^4}+\frac{63\,(bx^2+a)^{\frac{5}{2}}b^5}{1280\,a^3}+\frac{21\,(bx^2+a)^{\frac{3}{2}}b^5}{256\,a^2}+\frac{63\,\sqrt{bx^2+a}\,b^5}{256\,a}-\frac{7\,(bx^2+a)^{\frac{11}{2}}b^4}{256\,a^5x^2}-\frac{(bx^2+a)^{\frac{11}{2}}b^3}{128\,a^4x^4}-\frac{(bx^2+a)^{\frac{11}{2}}b^2}{160\,a^3x^6}-\frac{(bx^2+a)^{\frac{11}{2}}b}{80\,a^2x^8}-\frac{(bx^2+a)^{\frac{11}{2}}}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="maxima")
```

```
[Out] -63/256*b^5*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 7/256*(b*x^2 + a)^(9/2)
*b^5/a^5 + 9/256*(b*x^2 + a)^(7/2)*b^5/a^4 + 63/1280*(b*x^2 + a)^(5/2)*b^5/
a^3 + 21/256*(b*x^2 + a)^(3/2)*b^5/a^2 + 63/256*sqrt(b*x^2 + a)*b^5/a - 7/2
56*(b*x^2 + a)^(11/2)*b^4/(a^5*x^2) - 1/128*(b*x^2 + a)^(11/2)*b^3/(a^4*x^4
) - 1/160*(b*x^2 + a)^(11/2)*b^2/(a^3*x^6) - 1/80*(b*x^2 + a)^(11/2)*b/(a^2
*x^8) - 1/10*(b*x^2 + a)^(11/2)/(a*x^10)
```

Fricas [A]
time = 1.45, size = 202, normalized size = 1.54

$$\left[\frac{315\,\sqrt{a}\,b^5x^{10}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(965\,ab^4x^8+1490\,a^2b^3x^6+1368\,a^3b^2x^4+656\,a^4bx^2+128\,a^5)\sqrt{bx^2+a}}{2560\,ax^{10}},\ \frac{315\,\sqrt{-a}\,b^5x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(965\,ab^4x^8+1490\,a^2b^3x^6+1368\,a^3b^2x^4+656\,a^4bx^2+128\,a^5)\sqrt{bx^2+a}}{1280\,ax^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/2560*(315*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(965*a*b^4*x^8 + 1490*a^2*b^3*x^6 + 1368*a^3*b^2*x^4 + 656*a^4*b
*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10), 1/1280*(315*sqrt(-a)*b^5*x^10*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) - (965*a*b^4*x^8 + 1490*a^2*b^3*x^6 + 1368*a
^3*b^2*x^4 + 656*a^4*b*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10)]
```

Sympy [A]
time = 10.67, size = 153, normalized size = 1.17

$$-\frac{a^4\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{10x^9}-\frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{80x^7}-\frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{160x^5}-\frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{128x^3}-\frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{256x}-\frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{256\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**11,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(10*x**9) - 41*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(80*x**7) - 171*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(160*x**5) - 1
49*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(128*x**3) - 193*b**(9/2)*sqrt(a/(b*x**2
) + 1)/(256*x) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*x))/(256*sqrt(a))
```

Giac [A]
time = 1.19, size = 121, normalized size = 0.92

$$\frac{\frac{315\,b^6\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{965\,(bx^2+a)^{\frac{9}{2}}b^6-2370\,(bx^2+a)^{\frac{7}{2}}ab^6+2688\,(bx^2+a)^{\frac{5}{2}}a^2b^6-1470\,(bx^2+a)^{\frac{3}{2}}a^3b^6+315\,\sqrt{bx^2+a}\,a^4b^6}{b^5x^{10}}}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(315*b^6*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (965*(b*x^2 + a
)^(9/2)*b^6 - 2370*(b*x^2 + a)^(7/2)*a*b^6 + 2688*(b*x^2 + a)^(5/2)*a^2*b^6
 - 1470*(b*x^2 + a)^(3/2)*a^3*b^6 + 315*sqrt(b*x^2 + a)*a^4*b^6)/(b^5*x^10)
)/b
```

Mupad [B]
time = 6.62, size = 106, normalized size = 0.81

$$\frac{237\,a\,{\left(b\,x^2+a\right)}^{7/2}}{128\,x^{10}}-\frac{193\,{\left(b\,x^2+a\right)}^{9/2}}{256\,x^{10}}-\frac{63\,a^4\,\sqrt{b\,x^2+a}}{256\,x^{10}}+\frac{147\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^{10}}-\frac{21\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}{10\,x^{10}}+\frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,63{}\mathrm{i}}{256\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^11,x)
```

```
[Out] (b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*63i)/(256*a^(1/2)) - (193*(a + b*
x^2)^(9/2))/(256*x^10) + (237*a*(a + b*x^2)^(7/2))/(128*x^10) - (63*a^4*(a
+ b*x^2)^(1/2))/(256*x^10) + (147*a^3*(a + b*x^2)^(3/2))/(128*x^10) - (21*a
^2*(a + b*x^2)^(5/2))/(10*x^10)
```

3.423 $\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx$

Optimal. Leaf size=155

$$-\frac{21b^4\sqrt{a+bx^2}}{512x^4}-\frac{21b^5\sqrt{a+bx^2}}{1024ax^2}-\frac{7b^3(a+bx^2)^{3/2}}{128x^6}-\frac{21b^2(a+bx^2)^{5/2}}{320x^8}-\frac{3b(a+bx^2)^{7/2}}{40x^{10}}-\frac{(a+bx^2)^{9/2}}{12x^{12}}+\frac{21b^6}{}$$

```
[Out] -7/128*b^3*(b*x^2+a)^(3/2)/x^6-21/320*b^2*(b*x^2+a)^(5/2)/x^8-3/40*b*(b*x^2
+a)^(7/2)/x^10-1/12*(b*x^2+a)^(9/2)/x^12+21/1024*b^6*arctanh((b*x^2+a)^(1/2
)/a^(1/2))/a^(3/2)-21/512*b^4*(b*x^2+a)^(1/2)/x^4-21/1024*b^5*(b*x^2+a)^(1/
2)/a/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {272, 43, 44, 65, 214}

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}-\frac{21b^5\sqrt{a+bx^2}}{1024ax^2}-\frac{21b^4\sqrt{a+bx^2}}{512x^4}-\frac{7b^3(a+bx^2)^{3/2}}{128x^6}-\frac{21b^2(a+bx^2)^{5/2}}{320x^8}-\frac{(a+bx^2)^{9/2}}{12x^{12}}-\frac{3b(a+bx^2)^{7/2}}{40x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^13,x]
```

```
[Out] (-21*b^4*Sqrt[a + b*x^2])/(512*x^4) - (21*b^5*Sqrt[a + b*x^2])/(1024*a*x^2)
 - (7*b^3*(a + b*x^2)^(3/2))/(128*x^6) - (21*b^2*(a + b*x^2)^(5/2))/(320*x^
8) - (3*b*(a + b*x^2)^(7/2))/(40*x^10) - (a + b*x^2)^(9/2)/(12*x^12) + (21*
b^6*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(1024*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)^{9/2}}{12x^{12}} + \frac{1}{8}(3b)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{3b(a+bx^2)^{7/2}}{40x^{10}} - \frac{(a+bx^2)^{9/2}}{12x^{12}} + \frac{1}{80}\left(21b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/2}}{40x^{10}} - \frac{(a+bx^2)^{9/2}}{12x^{12}} + \frac{1}{128}\left(21b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/}}{x^4}\right.$$

$$= -\frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/2}}{40x^{10}} - \frac{(a+bx^2)^{9/2}}{12x^{12}} + \frac{1}{256}\left(21b^4\right)\text{Su}$$

$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/2}}{40x^{10}} - \frac{(a+bx^2)^{9}}{12x^{12}}$$

$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx}{40x^{10}}$$

$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx}{40x^{10}}$$

$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx}{40x^{10}}$$

Mathematica [A]
time = 0.16, size = 106, normalized size = 0.68

$$\frac{\sqrt{a+bx^2}\left(-1280a^5-6272a^4bx^2-12144a^3b^2x^4-11432a^2b^3x^6-4910ab^4x^8-315b^5x^{10}\right)}{15360ax^{12}} + \frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^13,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5 - 6272*a^4*b*x^2 - 12144*a^3*b^2*x^4 - 11432*a^
2*b^3*x^6 - 4910*a*b^4*x^8 - 315*b^5*x^10))/(15360*a*x^12) + (21*b^6*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(1024*a^(3/2))
```

Maple [A]
time = 0.21, size = 239, normalized size = 1.54

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(315 b^{5} x^{10}+4910 a\,b^{4} x^{8}+11432 a^{2} b^{3} x^{6}+12144 a^{3} b^{2} x^{4}+6272 a^{4} b\,x^{2}+1280 a^{5}\right)}{15360 x^{12} a}+\frac{21 b^{6} \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{1024 a^{\frac{3}{2}}}$

$$7b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{2a\,x^2}+\frac{9b\left(\frac{\left(b\,x^2+a\right)^{\frac{9}{2}}}{9}+a\left(\right.\right.}{}\right.$$

$$5b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{4a\,x^4}+\frac{}{}\right.$$

$$3b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{6a\,x^6}+\frac{}{}\right.$$

$$b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{8a\,x^8}+\frac{}{}\right.$$

$$b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{10a\,x^{10}}+\frac{}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/a/x^12*(b*x^2+a)^(11/2)-1/12*b/a*(-1/10/a/x^10*(b*x^2+a)^(11/2)+1/10*
b/a*(-1/8/a/x^8*(b*x^2+a)^(11/2)+3/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(11/2)+5/6*b
/a*(-1/4/a/x^4*(b*x^2+a)^(11/2)+7/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(11/2)+9/2*b/
a*(1/9*(b*x^2+a)^(9/2)+a*(1/7*(b*x^2+a)^(7/2)+a*(1/5*(b*x^2+a)^(5/2)+a*(1/3
*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/
2))/x))))))))))
```

Maxima [A]
time = 0.28, size = 221, normalized size = 1.43

$$\frac{21\,b^6\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{1024\,a^{\frac{3}{2}}}-\frac{7\,(bx^2+a)^{\frac{9}{2}}b^6}{3072\,a^6}-\frac{3\,(bx^2+a)^{\frac{7}{2}}b^6}{1024\,a^5}-\frac{21\,(bx^2+a)^{\frac{5}{2}}b^6}{5120\,a^4}-\frac{7\,(bx^2+a)^{\frac{3}{2}}b^6}{1024\,a^3}-\frac{21\,\sqrt{bx^2+a}\,b^6}{1024\,a^2}+\frac{7\,(bx^2+a)^{\frac{11}{2}}b^5}{3072\,a^6x^2}+\frac{(bx^2+a)^{\frac{11}{2}}b^4}{1536\,a^5x^4}+\frac{(bx^2+a)^{\frac{11}{2}}b^3}{1920\,a^4x^6}+\frac{(bx^2+a)^{\frac{11}{2}}b^2}{960\,a^3x^8}+\frac{(bx^2+a)^{\frac{11}{2}}b}{120\,a^2x^{10}}-\frac{(bx^2+a)^{\frac{11}{2}}}{12\,ax^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="maxima")
```

```
[Out] 21/1024*b^6*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 7/3072*(b*x^2 + a)^(9/2
)*b^6/a^6 - 3/1024*(b*x^2 + a)^(7/2)*b^6/a^5 - 21/5120*(b*x^2 + a)^(5/2)*b^
6/a^4 - 7/1024*(b*x^2 + a)^(3/2)*b^6/a^3 - 21/1024*sqrt(b*x^2 + a)*b^6/a^2
+ 7/3072*(b*x^2 + a)^(11/2)*b^5/(a^6*x^2) + 1/1536*(b*x^2 + a)^(11/2)*b^4/(
a^5*x^4) + 1/1920*(b*x^2 + a)^(11/2)*b^3/(a^4*x^6) + 1/960*(b*x^2 + a)^(11/
2)*b^2/(a^3*x^8) + 1/120*(b*x^2 + a)^(11/2)*b/(a^2*x^10) - 1/12*(b*x^2 + a)
^(11/2)/(a*x^12)
```

Fricas [A]
time = 1.91, size = 223, normalized size = 1.44

$$\left[\frac{315\,\sqrt{a}\,b^6x^{12}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(315\,ab^5x^{10}+4910\,a^2b^4x^8+11432\,a^3b^3x^6+12144\,a^4b^2x^4+6272\,a^5bx^2+1280\,a^6)\sqrt{bx^2+a}}{30720\,a^2x^{12}},-\frac{315\,\sqrt{-a}\,b^6x^{12}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(315\,ab^5x^{10}+4910\,a^2b^4x^8+11432\,a^3b^3x^6+12144\,a^4b^2x^4+6272\,a^5bx^2+1280\,a^6)\sqrt{bx^2+a}}{15360\,a^2x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/30720*(315*sqrt(a)*b^6*x^12*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 2*(315*a*b^5*x^10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 12144*
a^4*b^2*x^4 + 6272*a^5*b*x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12), -1/15
360*(315*sqrt(-a)*b^6*x^12*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (315*a*b^5*x^
10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 12144*a^4*b^2*x^4 + 6272*a^5*b*
x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12)]
```

Sympy [A]
time = 47.16, size = 204, normalized size = 1.32

$$-\frac{a^5}{12\sqrt{b}\,x^{13}\sqrt{\frac{a}{bx^2}+1}}-\frac{59a^4\sqrt{b}}{120x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{1151a^3b^{\frac{3}{2}}}{960x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{2947a^2b^{\frac{5}{2}}}{1920x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{8171ab^{\frac{7}{2}}}{7680x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{1045b^{\frac{9}{2}}}{3072x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{21b^{\frac{11}{2}}}{1024ax\sqrt{\frac{a}{bx^2}+1}}+\frac{21b^6\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{1024a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**13,x)
```

```
[Out] -a**5/(12*sqrt(b)*x**13*sqrt(a/(b*x**2) + 1)) - 59*a**4*sqrt(b)/(120*x**11*
sqrt(a/(b*x**2) + 1)) - 1151*a**3*b**(3/2)/(960*x**9*sqrt(a/(b*x**2) + 1))
- 2947*a**2*b**(5/2)/(1920*x**7*sqrt(a/(b*x**2) + 1)) - 8171*a*b**(7/2)/(76
80*x**5*sqrt(a/(b*x**2) + 1)) - 1045*b**(9/2)/(3072*x**3*sqrt(a/(b*x**2) +
1)) - 21*b**(11/2)/(1024*a*x*sqrt(a/(b*x**2) + 1)) + 21*b**6*asinh(sqrt(a)/
(sqrt(b)*x))/(1024*a**(3/2))
```

Giac [A]
time = 2.56, size = 143, normalized size = 0.92

$$-\frac{\frac{315\,b^7\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{315\,(bx^2+a)^{\frac{11}{2}}b^7+3335\,(bx^2+a)^{\frac{9}{2}}ab^7-5058\,(bx^2+a)^{\frac{7}{2}}a^2b^7+4158\,(bx^2+a)^{\frac{5}{2}}a^3b^7-1785\,(bx^2+a)^{\frac{3}{2}}a^4b^7+315\,\sqrt{bx^2+a}\,a^5b^7}{ab^6x^{12}}}{15360\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/15360*(315*b^7*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x
^2 + a)^(11/2)*b^7 + 3335*(b*x^2 + a)^(9/2)*a*b^7 - 5058*(b*x^2 + a)^(7/2)*
a^2*b^7 + 4158*(b*x^2 + a)^(5/2)*a^3*b^7 - 1785*(b*x^2 + a)^(3/2)*a^4*b^7 +
 315*sqrt(b*x^2 + a)*a^5*b^7)/(a*b^6*x^12))/b
```

Mupad [B]
time = 6.68, size = 123, normalized size = 0.79

$$\frac{843\,a\,{\left(b\,x^2+a\right)}^{7/2}}{2560\,x^{12}}-\frac{667\,{\left(b\,x^2+a\right)}^{9/2}}{3072\,x^{12}}-\frac{21\,a^4\,\sqrt{b\,x^2+a}}{1024\,x^{12}}+\frac{119\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}{1024\,x^{12}}-\frac{693\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}{2560\,x^{12}}-\frac{21\,{\left(b\,x^2+a\right)}^{11/2}}{1024\,a\,x^{12}}-\frac{b^6\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,21{}\mathrm{i}}{1024\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^13,x)
```

```
[Out] (843*a*(a + b*x^2)^(7/2))/(2560*x^12) - (b^6*atan(((a + b*x^2)^(1/2)*1i)/a^
(1/2))*21i)/(1024*a^(3/2)) - (667*(a + b*x^2)^(9/2))/(3072*x^12) - (21*a^4*
(a + b*x^2)^(1/2))/(1024*x^12) + (119*a^3*(a + b*x^2)^(3/2))/(1024*x^12) -
(693*a^2*(a + b*x^2)^(5/2))/(2560*x^12) - (21*(a + b*x^2)^(11/2))/(1024*a*x
^12)
```

3.424 $\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx$

Optimal. Leaf size=179

$$-\frac{3b^4\sqrt{a+bx^2}}{256x^6}-\frac{3b^5\sqrt{a+bx^2}}{1024ax^4}+\frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2}-\frac{3b^3(a+bx^2)^{3/2}}{128x^8}-\frac{3b^2(a+bx^2)^{5/2}}{80x^{10}}-\frac{3b(a+bx^2)^{7/2}}{56x^{12}}-\frac{(a+bx^2)^{9/2}}{14x^{14}}$$

```
[Out] -3/128*b^3*(b*x^2+a)^(3/2)/x^8-3/80*b^2*(b*x^2+a)^(5/2)/x^10-3/56*b*(b*x^2+
a)^(7/2)/x^12-1/14*(b*x^2+a)^(9/2)/x^14-9/2048*b^7*arctanh((b*x^2+a)^(1/2)/
a^(1/2))/a^(5/2)-3/256*b^4*(b*x^2+a)^(1/2)/x^6-3/1024*b^5*(b*x^2+a)^(1/2)/a
/x^4+9/2048*b^6*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 44, 65, 214}

$$-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}+\frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2}-\frac{3b^5\sqrt{a+bx^2}}{1024ax^4}-\frac{3b^4\sqrt{a+bx^2}}{256x^6}-\frac{3b^3(a+bx^2)^{3/2}}{128x^8}-\frac{3b^2(a+bx^2)^{5/2}}{80x^{10}}-\frac{(a+bx^2)^{9/2}}{14x^{14}}-\frac{3b(a+bx^2)^{7/2}}{56x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^15,x]
```

```
[Out] (-3*b^4*Sqrt[a + b*x^2])/(256*x^6) - (3*b^5*Sqrt[a + b*x^2])/(1024*a*x^4) +
 (9*b^6*Sqrt[a + b*x^2])/(2048*a^2*x^2) - (3*b^3*(a + b*x^2)^(3/2))/(128*x^
8) - (3*b^2*(a + b*x^2)^(5/2))/(80*x^10) - (3*b*(a + b*x^2)^(7/2))/(56*x^12
) - (a + b*x^2)^(9/2)/(14*x^14) - (9*b^7*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(2048*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^8}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)^{9/2}}{14x^{14}} + \frac{1}{28}(9b)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{3b(a+bx^2)^{7/2}}{56x^{12}} - \frac{(a+bx^2)^{9/2}}{14x^{14}} + \frac{1}{16}(3b^2)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{3b^2(a+bx^2)^{5/2}}{80x^{10}} - \frac{3b(a+bx^2)^{7/2}}{56x^{12}} - \frac{(a+bx^2)^{9/2}}{14x^{14}} + \frac{1}{32}(3b^3)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx,\right.$$

$$= -\frac{3b^3(a+bx^2)^{3/2}}{128x^8} - \frac{3b^2(a+bx^2)^{5/2}}{80x^{10}} - \frac{3b(a+bx^2)^{7/2}}{56x^{12}} - \frac{(a+bx^2)^{9/2}}{14x^{14}} + \frac{1}{256}(9b^4)\,\text{Subst}$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^3(a+bx^2)^{3/2}}{128x^8} - \frac{3b^2(a+bx^2)^{5/2}}{80x^{10}} - \frac{3b(a+bx^2)^{7/2}}{56x^{12}} - \frac{(a+bx^2)^{9/2}}{14x^{14}} +$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} - \frac{3b^3(a+bx^2)^{3/2}}{128x^8} - \frac{3b^2(a+bx^2)^{5/2}}{80x^{10}} - \frac{3b(a+bx^2)^{7/2}}{56x^{12}}$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3(a+bx^2)^{3/2}}{128x^8} - \frac{3b^2(a+bx^2)^{5/2}}{80x^{10}}$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3(a+bx^2)^{3/2}}{128x^8} - \frac{3b^2(a+bx^2)^{5/2}}{80x^{10}}$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3(a+bx^2)^{3/2}}{128x^8} - \frac{3b^2(a+bx^2)^{5/2}}{80x^{10}}$$

Mathematica [A]
time = 0.19, size = 117, normalized size = 0.65

$$\frac{\sqrt{a+bx^2}\,(-5120a^6-24320a^5bx^2-44928a^4b^2x^4-39056a^3b^3x^6-14168a^2b^4x^8-210ab^5x^{10}+315b^6x^{12})}{71680a^2x^{14}} - \frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^15,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-5120*a^6 - 24320*a^5*b*x^2 - 44928*a^4*b^2*x^4 - 39056*a
^3*b^3*x^6 - 14168*a^2*b^4*x^8 - 210*a*b^5*x^10 + 315*b^6*x^12))/(71680*a^2
*x^14) - (9*b^7*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2048*a^(5/2))
```

Maple [A]
time = 0.42, size = 263, normalized size = 1.47

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-315 b^{6} x^{12}+210 a\,b^{5} x^{10}+14168 a^{2} b^{4} x^{8}+39056 a^{3} x^{6} b^{3}+44928 a^{4} b^{2} x^{4}+24320 a^{5} b\,x^{2}+5120 a^{6}\right)}{71680 x^{14} a^{2}}-\frac{9 b^{7} \ln\left(\frac{2a+2\sqrt{a}}{}\right.}{20}$

$$7b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{2a\,x^2}+\cdots\right.$$

$$5b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{4a\,x^4}+\cdots\right.$$

$$3b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{6a\,x^6}+\cdots\right.$$

$$b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{8a\,x^8}+\cdots\right.$$

$$b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{10a\,x^{10}}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14/a/x^14*(b*x^2+a)^(11/2)-3/14*b/a*(-1/12/a/x^12*(b*x^2+a)^(11/2)-1/12*
b/a*(-1/10/a/x^10*(b*x^2+a)^(11/2)+1/10*b/a*(-1/8/a/x^8*(b*x^2+a)^(11/2)+3/
8*b/a*(-1/6/a/x^6*(b*x^2+a)^(11/2)+5/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(11/2)+7/4
*b/a*(-1/2/a/x^2*(b*x^2+a)^(11/2)+9/2*b/a*(1/9*(b*x^2+a)^(9/2)+a*(1/7*(b*x^
2+a)^(7/2)+a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)
-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))))))))
```

Maxima [A]
time = 0.30, size = 241, normalized size = 1.35

$$-\frac{9\,b^{7}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2048\,a^{\frac{5}{2}}}+\frac{(bx^{2}+a)^{\frac{9}{2}}b^{7}}{2048\,a^{7}}+\frac{9\,(bx^{2}+a)^{\frac{7}{2}}b^{7}}{14336\,a^{6}}+\frac{9\,(bx^{2}+a)^{\frac{5}{2}}b^{7}}{10240\,a^{5}}+\frac{3\,(bx^{2}+a)^{\frac{3}{2}}b^{7}}{2048\,a^{4}}+\frac{9\,\sqrt{bx^{2}+a}\,b^{7}}{2048\,a^{3}}-\frac{(bx^{2}+a)^{\frac{11}{2}}b^{6}}{2048\,a^{7}x^{2}}-\frac{(bx^{2}+a)^{\frac{11}{2}}b^{5}}{7168\,a^{6}x^{4}}-\frac{(bx^{2}+a)^{\frac{11}{2}}b^{4}}{8960\,a^{5}x^{6}}-\frac{(bx^{2}+a)^{\frac{11}{2}}b^{3}}{4480\,a^{4}x^{8}}-\frac{(bx^{2}+a)^{\frac{11}{2}}b^{2}}{560\,a^{3}x^{10}}+\frac{(bx^{2}+a)^{\frac{11}{2}}b}{56\,a^{2}x^{12}}-\frac{(bx^{2}+a)^{\frac{11}{2}}}{14\,ax^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="maxima")
```

```
[Out] -9/2048*b^7*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2048*(b*x^2 + a)^(9/2
)*b^7/a^7 + 9/14336*(b*x^2 + a)^(7/2)*b^7/a^6 + 9/10240*(b*x^2 + a)^(5/2)*b
^7/a^5 + 3/2048*(b*x^2 + a)^(3/2)*b^7/a^4 + 9/2048*sqrt(b*x^2 + a)*b^7/a^3
- 1/2048*(b*x^2 + a)^(11/2)*b^6/(a^7*x^2) - 1/7168*(b*x^2 + a)^(11/2)*b^5/(
a^6*x^4) - 1/8960*(b*x^2 + a)^(11/2)*b^4/(a^5*x^6) - 1/4480*(b*x^2 + a)^(11
/2)*b^3/(a^4*x^8) - 1/560*(b*x^2 + a)^(11/2)*b^2/(a^3*x^10) + 1/56*(b*x^2 +
 a)^(11/2)*b/(a^2*x^12) - 1/14*(b*x^2 + a)^(11/2)/(a*x^14)
```

Fricas [A]
time = 1.14, size = 245, normalized size = 1.37

$$\left[\frac{315\,\sqrt{a}\,b^{7}x^{14}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(315\,ab^{6}x^{12}-210\,a^{2}b^{5}x^{10}-14168\,a^{3}b^{4}x^{8}-39056\,a^{4}b^{3}x^{6}-44928\,a^{5}b^{2}x^{4}-24320\,a^{6}bx^{2}-5120\,a^{7})\sqrt{bx^{2}+a}}{143360\,a^{3}x^{14}},\ \frac{315\,\sqrt{-a}\,b^{7}x^{14}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(315\,ab^{6}x^{12}-210\,a^{2}b^{5}x^{10}-14168\,a^{3}b^{4}x^{8}-39056\,a^{4}b^{3}x^{6}-44928\,a^{5}b^{2}x^{4}-24320\,a^{6}bx^{2}-5120\,a^{7})\sqrt{bx^{2}+a}}{71680\,a^{3}x^{14}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="fricas")
```

```
[Out] [1/143360*(315*sqrt(a)*b^7*x^14*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 39056
*a^4*b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^2 +
 a))/(a^3*x^14), 1/71680*(315*sqrt(-a)*b^7*x^14*arctan(sqrt(-a)/sqrt(b*x^2
+ a)) + (315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 39056*a^4*
b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^2 + a))/
(a^3*x^14)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**15,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.12, size = 160, normalized size = 0.89

$$\frac{\frac{315\,b^8\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{315\,(bx^2+a)^{\frac{13}{2}}b^8-2100\,(bx^2+a)^{\frac{11}{2}}ab^8-8393\,(bx^2+a)^{\frac{9}{2}}a^2b^8+9216\,(bx^2+a)^{\frac{7}{2}}a^3b^8-5943\,(bx^2+a)^{\frac{5}{2}}a^4b^8+2100\,(bx^2+a)^{\frac{3}{2}}a^5b^8-315\,\sqrt{bx^2+a}\,a^6b^8}{a^2b^7x^{14}}}{71680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="giac")
```

```
[Out] 1/71680*(315*b^8*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*
x^2 + a)^(13/2)*b^8 - 2100*(b*x^2 + a)^(11/2)*a*b^8 - 8393*(b*x^2 + a)^(9/2
)*a^2*b^8 + 9216*(b*x^2 + a)^(7/2)*a^3*b^8 - 5943*(b*x^2 + a)^(5/2)*a^4*b^8
 + 2100*(b*x^2 + a)^(3/2)*a^5*b^8 - 315*sqrt(b*x^2 + a)*a^6*b^8)/(a^2*b^7*x
^14))/b
```

Mupad [B]
time = 7.23, size = 140, normalized size = 0.78

$$\frac{9\,a\,{\left(b\,x^2+a\right)}^{7/2}}{70\,x^{14}}-\frac{1199\,{\left(b\,x^2+a\right)}^{9/2}}{10240\,x^{14}}-\frac{9\,a^4\,\sqrt{b\,x^2+a}}{2048\,x^{14}}+\frac{15\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}{512\,x^{14}}-\frac{849\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}{10240\,x^{14}}-\frac{15\,{\left(b\,x^2+a\right)}^{11/2}}{512\,a\,x^{14}}+\frac{9\,{\left(b\,x^2+a\right)}^{13/2}}{2048\,a^2\,x^{14}}+\frac{b^7\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,9{}\mathrm{i}}{2048\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^15,x)
```

```
[Out] (b^7*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*9i)/(2048*a^(5/2)) - (1199*(a + b
*x^2)^(9/2))/(10240*x^14) + (9*a*(a + b*x^2)^(7/2))/(70*x^14) - (9*a^4*(a +
 b*x^2)^(1/2))/(2048*x^14) + (15*a^3*(a + b*x^2)^(3/2))/(512*x^14) - (849*a
^2*(a + b*x^2)^(5/2))/(10240*x^14) - (15*(a + b*x^2)^(11/2))/(512*a*x^14) +
 (9*(a + b*x^2)^(13/2))/(2048*a^2*x^14)
```

3.425 $\int x^6(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=202

$$\frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7($$

```
[Out] 3/256*a^3*x^7*(b*x^2+a)^(3/2)+3/128*a^2*x^7*(b*x^2+a)^(5/2)+9/224*a*x^7*(b*
x^2+a)^(7/2)+1/16*x^7*(b*x^2+a)^(9/2)-45/32768*a^8*arctanh(x*b^(1/2)/(b*x^2
+a)^(1/2))/b^(7/2)+45/32768*a^7*x*(b*x^2+a)^(1/2)/b^3-15/16384*a^6*x^3*(b*x
^2+a)^(1/2)/b^2+3/4096*a^5*x^5*(b*x^2+a)^(1/2)/b+9/2048*a^4*x^7*(b*x^2+a)^(
1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 212}

$$-\frac{45a^8\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}}+\frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^(9/2),x]
```

```
[Out] (45*a^7*x*Sqrt[a + b*x^2])/(32768*b^3) - (15*a^6*x^3*Sqrt[a + b*x^2])/(1638
4*b^2) + (3*a^5*x^5*Sqrt[a + b*x^2])/(4096*b) + (9*a^4*x^7*Sqrt[a + b*x^2])
/2048 + (3*a^3*x^7*(a + b*x^2)^(3/2))/256 + (3*a^2*x^7*(a + b*x^2)^(5/2))/1
28 + (9*a*x^7*(a + b*x^2)^(7/2))/224 + (x^7*(a + b*x^2)^(9/2))/16 - (45*a^8
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32768*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^6\left(a+bx^2\right)^{9/2}\,dx = \frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{16}(9a)\int x^6\left(a+bx^2\right)^{7/2}\,dx$$
$$= \frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{32}\left(9a^2\right)\int x^6\left(a+bx^2\right)^{5/2}\,dx$$
$$= \frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{128}\left(15a^3\right)\int x^6(a$$
$$= \frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)$$
$$= \frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^7$$
$$= \frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}$$
$$= -\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+$$
$$= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a$$
$$= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a$$
$$= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a$$

Mathematica [A]
time = 0.15, size = 129, normalized size = 0.64

$$\frac{\sqrt{a+bx^2}\left(315a^7x-210a^6bx^3+168a^5b^2x^5+32624a^4b^3x^7+98432a^3b^4x^9+119040a^2b^5x^{11}+66560ab^6x^{13}+14336b^7x^{15}\right)}{229376b^3}+\frac{45a^8\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{32768b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(315*a^7*x - 210*a^6*b*x^3 + 168*a^5*b^2*x^5 + 32624*a^4*b
^3*x^7 + 98432*a^3*b^4*x^9 + 119040*a^2*b^5*x^11 + 66560*a*b^6*x^13 + 14336
*b^7*x^15))/(229376*b^3) + (45*a^8*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(32
768*b^(7/2))
```

Maple [A]
time = 0.07, size = 170, normalized size = 0.84

method	result
risch	$\frac{x\left(14336 b^{7} x^{14}+66560 a\, b^{6} x^{12}+119040 a^{2} b^{5} x^{10}+98432 a^{3} b^{4} x^{8}+32624 a^{4} b^{3} x^{6}+168 a^{5} b^{2} x^{4}-210 a^{6} b\, x^{2}+315 a^{7}\right) \sqrt{b\, x^{2}+a}}{229376 b^{3}}-\frac{45 a^{8}}{}$

$3a$ $5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\cdots\right.}{\cdots}\right.$

$7a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\cdots\right.$

$9a\left(\frac{x\left(b\,x^2+a\right)^{\frac{7}{2}}}{8}+\cdots\right.$

$a\left(\frac{x\left(b\,x^2+a\right)^{\frac{9}{2}}}{10}+\cdots\right.$ 10

$3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{11}{2}}}{12b}-\cdots\right.$ $12b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x^5*(b*x^2+a)^(11/2)/b-5/16*a/b*(1/14*x^3*(b*x^2+a)^(11/2)/b-3/14*a/b*
(1/12*x*(b*x^2+a)^(11/2)/b-1/12*a/b*(1/10*x*(b*x^2+a)^(9/2)+9/10*a*(1/8*x*(
b*x^2+a)^(7/2)+7/8*a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/
4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))
))
```

Maxima [A]
time = 0.30, size = 161, normalized size = 0.80

$$\frac{(bx^2+a)^{\frac{11}{2}}x^5}{16\,b}-\frac{5\,(bx^2+a)^{\frac{11}{2}}ax^3}{224\,b^2}+\frac{5\,(bx^2+a)^{\frac{11}{2}}a^2x}{896\,b^3}-\frac{(bx^2+a)^{\frac{9}{2}}a^3x}{1792\,b^3}-\frac{9\,(bx^2+a)^{\frac{7}{2}}a^4x}{14336\,b^3}-\frac{3\,(bx^2+a)^{\frac{5}{2}}a^5x}{4096\,b^3}-\frac{15\,(bx^2+a)^{\frac{3}{2}}a^6x}{16384\,b^3}-\frac{45\,\sqrt{bx^2+a}\,a^7x}{32768\,b^3}-\frac{45\,a^8\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{32768\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/16*(b*x^2 + a)^(11/2)*x^5/b - 5/224*(b*x^2 + a)^(11/2)*a*x^3/b^2 + 5/896*
(b*x^2 + a)^(11/2)*a^2*x/b^3 - 1/1792*(b*x^2 + a)^(9/2)*a^3*x/b^3 - 9/14336
*(b*x^2 + a)^(7/2)*a^4*x/b^3 - 3/4096*(b*x^2 + a)^(5/2)*a^5*x/b^3 - 15/1638
4*(b*x^2 + a)^(3/2)*a^6*x/b^3 - 45/32768*sqrt(b*x^2 + a)*a^7*x/b^3 - 45/327
68*a^8*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Fricas [A]
time = 1.13, size = 255, normalized size = 1.26

$$\left[\frac{315\,a^8\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(14336\,b^8x^{15}+66560\,ab^7x^{13}+119040\,a^2b^6x^{11}+98432\,a^3b^5x^9+32624\,a^4b^4x^7+168\,a^5b^3x^5-210\,a^6b^2x^3+315\,a^7bx)\sqrt{bx^2+a}}{458752\,b^4},\ \frac{315\,a^8\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(14336\,b^8x^{15}+66560\,ab^7x^{13}+119040\,a^2b^6x^{11}+98432\,a^3b^5x^9+32624\,a^4b^4x^7+168\,a^5b^3x^5-210\,a^6b^2x^3+315\,a^7bx)\sqrt{bx^2+a}}{229376\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/458752*(315*a^8*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11 + 98432*a^3*b^
5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^3 + 315*a^7*b*x
)*sqrt(b*x^2 + a))/b^4, 1/229376*(315*a^8*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11 + 984
32*a^3*b^5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^3 + 31
5*a^7*b*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.04, size = 133, normalized size = 0.66

$$\frac{45\,a^8\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{32768\,b^{\frac{7}{2}}}+\frac{1}{229376}\left(\frac{315\,a^7}{b^3}-2\left(\frac{105\,a^6}{b^2}-4\left(\frac{21\,a^5}{b}+2\left(2039\,a^4+8\left(769\,a^3b+2\left(465\,a^2b^2+4\left(14\,b^4x^2+65\,ab^3\right)x^2\right)x^2\right)x^2\right)x^2\right)x^2\right)x^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 45/32768*a^8*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2) + 1/229376*(315
*a^7/b^3 - 2*(105*a^6/b^2 - 4*(21*a^5/b + 2*(2039*a^4 + 8*(769*a^3*b + 2*(4
65*a^2*b^2 + 4*(14*b^4*x^2 + 65*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*x^2)*sqrt(b
*x^2 + a)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,{\left(b\,x^2+a\right)}^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^6*(a + b*x^2)^(9/2), x)
```

3.426 $\int x^4(a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=178

$$-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}a$$

```
[Out] 3/128*a^3*x^5*(b*x^2+a)^(3/2)+3/80*a^2*x^5*(b*x^2+a)^(5/2)+3/56*a*x^5*(b*x^
2+a)^(7/2)+1/14*x^5*(b*x^2+a)^(9/2)+9/2048*a^7*arctanh(x*b^(1/2)/(b*x^2+a)^
(1/2))/b^(5/2)-9/2048*a^6*x*(b*x^2+a)^(1/2)/b^2+3/1024*a^5*x^3*(b*x^2+a)^(1
/2)/b+3/256*a^4*x^5*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 212}

$$\frac{9a^7\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}}-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(9/2),x]
```

```
[Out] (-9*a^6*x*Sqrt[a + b*x^2])/(2048*b^2) + (3*a^5*x^3*Sqrt[a + b*x^2])/(1024*b
) + (3*a^4*x^5*Sqrt[a + b*x^2])/256 + (3*a^3*x^5*(a + b*x^2)^(3/2))/128 + (
3*a^2*x^5*(a + b*x^2)^(5/2))/80 + (3*a*x^5*(a + b*x^2)^(7/2))/56 + (x^5*(a
+ b*x^2)^(9/2))/14 + (9*a^7*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2048*b^(
5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{9/2}\,dx = \frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{1}{14}(9a)\int x^4\left(a+bx^2\right)^{7/2}\,dx$$

$$= \frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{1}{8}\left(3a^2\right)\int x^4\left(a+bx^2\right)^{5/2}\,dx$$

$$= \frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{1}{16}\left(3a^3\right)\int x^4\left(a+b\right.$$

$$= \frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

$$= \frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7}$$

$$= \frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}$$

$$= -\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+$$

$$= -\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+$$

$$= -\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+$$

Mathematica [A]
time = 0.14, size = 118, normalized size = 0.66

$$\frac{\sqrt{a+bx^2}\left(-315a^6x+210a^5bx^3+14168a^4b^2x^5+39056a^3b^3x^7+44928a^2b^4x^9+24320ab^5x^{11}+5120b^6x^{13}\right)}{71680b^2}-\frac{9a^7\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2048b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(9/2),x]
```

[Out] (Sqrt[a + b*x^2]*(-315*a^6*x + 210*a^5*b*x^3 + 14168*a^4*b^2*x^5 + 39056*a^3*b^3*x^7 + 44928*a^2*b^4*x^9 + 24320*a*b^5*x^11 + 5120*b^6*x^13))/(71680*b^2) - (9*a^7*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2048*b^(5/2))

Maple [A]
time = 0.06, size = 146, normalized size = 0.82

method	result
risch	$-\frac{x\left(-5120 b^{6} x^{12}-24320 a\,b^{5} x^{10}-44928 a^{2} b^{4} x^{8}-39056 a^{3} x^{6} b^{3}-14168 a^{4} b^{2} x^{4}-210 a^{5} b\,x^{2}+315 a^{6}\right)\sqrt{b\,x^{2}+a}}{71680 b^{2}}+\frac{9 a^{7} \ln\left(x\sqrt{b}+\right.}{204}$

$$3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{11}{2}}}{12b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{9}{2}}}{10}+\frac{9a\left(\frac{x\left(b\,x^2+a\right)^{\frac{7}{2}}}{8}+\frac{7a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\,\ldots}{\ldots}\right.}{\ldots}\right.}{6}\right.}{8}\right.}{10}\right.}{12b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*x^3*(b*x^2+a)^(11/2)/b-3/14*a/b*(1/12*x*(b*x^2+a)^(11/2)/b-1/12*a/b*(1
/10*x*(b*x^2+a)^(9/2)+9/10*a*(1/8*x*(b*x^2+a)^(7/2)+7/8*a*(1/6*x*(b*x^2+a)^
(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/
2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))))
```

Maxima [A]
time = 0.29, size = 141, normalized size = 0.79

$$\frac{(bx^2+a)^{\frac{11}{2}}x^3}{14\,b}-\frac{(bx^2+a)^{\frac{11}{2}}ax}{56\,b^2}+\frac{(bx^2+a)^{\frac{9}{2}}a^2x}{560\,b^2}+\frac{9\,(bx^2+a)^{\frac{7}{2}}a^3x}{4480\,b^2}+\frac{3\,(bx^2+a)^{\frac{5}{2}}a^4x}{1280\,b^2}+\frac{3\,(bx^2+a)^{\frac{3}{2}}a^5x}{1024\,b^2}+\frac{9\,\sqrt{bx^2+a}\,a^6x}{2048\,b^2}+\frac{9\,a^7\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2048\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/14*(b*x^2 + a)^(11/2)*x^3/b - 1/56*(b*x^2 + a)^(11/2)*a*x/b^2 + 1/560*(b*
x^2 + a)^(9/2)*a^2*x/b^2 + 9/4480*(b*x^2 + a)^(7/2)*a^3*x/b^2 + 3/1280*(b*x
^2 + a)^(5/2)*a^4*x/b^2 + 3/1024*(b*x^2 + a)^(3/2)*a^5*x/b^2 + 9/2048*sqrt(
b*x^2 + a)*a^6*x/b^2 + 9/2048*a^7*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 1.30, size = 234, normalized size = 1.31

$$\left[\frac{315\,a^7\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(5120\,b^7x^{13}+24320\,ab^6x^{11}+44928\,a^2b^5x^9+39056\,a^3b^4x^7+14168\,a^4b^3x^5+210\,a^5b^2x^3-315\,a^6bx)\sqrt{bx^2+a}}{143360\,b^3},\ -\frac{315\,a^7\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(5120\,b^7x^{13}+24320\,ab^6x^{11}+44928\,a^2b^5x^9+39056\,a^3b^4x^7+14168\,a^4b^3x^5+210\,a^5b^2x^3-315\,a^6bx)\sqrt{bx^2+a}}{71680\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/143360*(315*a^7*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(5120*b^7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4*x
^7 + 14168*a^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))/b^
3, -1/71680*(315*a^7*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (5120*b^
7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4*x^7 + 14168*a
^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(9/2),x)
```

[Out] Timed out

Giac [A]
time = 0.89, size = 119, normalized size = 0.67

$$-\frac{9\,a^{7}\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{2048\,b^{\frac{5}{2}}}-\frac{1}{71680}\left(\frac{315\,a^{6}}{b^{2}}-2\left(\frac{105\,a^{5}}{b}+4\left(1771\,a^{4}+2\left(2441\,a^{3}b+8\left(351\,a^{2}b^{2}+10\left(4\,b^{4}x^{2}+19\,ab^{3}\right)x^{2}\right)x^{2}\right)x^{2}\right)x^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2048*a^7*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/71680*(315*a
^6/b^2 - 2*(105*a^5/b + 4*(1771*a^4 + 2*(2441*a^3*b + 8*(351*a^2*b^2 + 10*(
4*b^4*x^2 + 19*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(b\,x^2+a\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(9/2), x)
```

3.427 $\int x^2(a+bx^2)^{9/2}\, dx$

Optimal. Leaf size=154

$$\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}$$

```
[Out] 7/128*a^3*x^3*(b*x^2+a)^(3/2)+21/320*a^2*x^3*(b*x^2+a)^(5/2)+3/40*a*x^3*(b*
x^2+a)^(7/2)+1/12*x^3*(b*x^2+a)^(9/2)-21/1024*a^6*arctanh(x*b^(1/2)/(b*x^2+
a)^(1/2))/b^(3/2)+21/1024*a^5*x*(b*x^2+a)^(1/2)/b+21/512*a^4*x^3*(b*x^2+a)^
(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 212}

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}}+\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(9/2),x]
```

```
[Out] (21*a^5*x*Sqrt[a + b*x^2])/(1024*b) + (21*a^4*x^3*Sqrt[a + b*x^2])/512 + (7
*a^3*x^3*(a + b*x^2)^(3/2))/128 + (21*a^2*x^3*(a + b*x^2)^(5/2))/320 + (3*a
*x^3*(a + b*x^2)^(7/2))/40 + (x^3*(a + b*x^2)^(9/2))/12 - (21*a^6*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2(a+bx^2)^{9/2}\,dx = \frac{1}{12}x^3(a+bx^2)^{9/2} + \frac{1}{4}(3a)\int x^2(a+bx^2)^{7/2}\,dx$$
$$= \frac{3}{40}ax^3(a+bx^2)^{7/2} + \frac{1}{12}x^3(a+bx^2)^{9/2} + \frac{1}{40}(21a^2)\int x^2(a+bx^2)^{5/2}\,dx$$
$$= \frac{21}{320}a^2x^3(a+bx^2)^{5/2} + \frac{3}{40}ax^3(a+bx^2)^{7/2} + \frac{1}{12}x^3(a+bx^2)^{9/2} + \frac{1}{64}(21a^3)\int x^2(a+$$
$$= \frac{7}{128}a^3x^3(a+bx^2)^{3/2} + \frac{21}{320}a^2x^3(a+bx^2)^{5/2} + \frac{3}{40}ax^3(a+bx^2)^{7/2} + \frac{1}{12}x^3(a+bx^2)^{9/}$$
$$= \frac{21}{512}a^4x^3\sqrt{a+bx^2} + \frac{7}{128}a^3x^3(a+bx^2)^{3/2} + \frac{21}{320}a^2x^3(a+bx^2)^{5/2} + \frac{3}{40}ax^3(a+bx^2)$$
$$= \frac{21a^5x\sqrt{a+bx^2}}{1024b} + \frac{21}{512}a^4x^3\sqrt{a+bx^2} + \frac{7}{128}a^3x^3(a+bx^2)^{3/2} + \frac{21}{320}a^2x^3(a+bx^2)^{5/}$$
$$= \frac{21a^5x\sqrt{a+bx^2}}{1024b} + \frac{21}{512}a^4x^3\sqrt{a+bx^2} + \frac{7}{128}a^3x^3(a+bx^2)^{3/2} + \frac{21}{320}a^2x^3(a+bx^2)^{5/}$$
$$= \frac{21a^5x\sqrt{a+bx^2}}{1024b} + \frac{21}{512}a^4x^3\sqrt{a+bx^2} + \frac{7}{128}a^3x^3(a+bx^2)^{3/2} + \frac{21}{320}a^2x^3(a+bx^2)^{5/}$$

Mathematica [A]
time = 0.13, size = 107, normalized size = 0.69

$$\frac{\sqrt{a+bx^2}\,(315a^5x+4910a^4bx^3+11432a^3b^2x^5+12144a^2b^3x^7+6272ab^4x^9+1280b^5x^{11})}{15360b}+\frac{21a^6\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{1024b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(315*a^5*x + 4910*a^4*b*x^3 + 11432*a^3*b^2*x^5 + 12144*a^
2*b^3*x^7 + 6272*a*b^4*x^9 + 1280*b^5*x^11))/(15360*b) + (21*a^6*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/(1024*b^(3/2))
```

Maple [A]
time = 0.06, size = 122, normalized size = 0.79

method	result
risch	$\frac{x\left(1280 b^{5} x^{10}+6272 a\,b^{4} x^{8}+12144 a^{2} b^{3} x^{6}+11432 a^{3} b^{2} x^{4}+4910 a^{4} b\,x^{2}+315 a^{5}\right) \sqrt{b\,x^{2}+a}}{15360 b}-\frac{21 a^{6} \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{1024 b^{\frac{3}{2}}}$

default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{12b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{10}+\frac{9a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8}+\frac{7a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8}\right)}{10}\right)}{12b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x*(b*x^2+a)^(11/2)/b-1/12*a/b*(1/10*x*(b*x^2+a)^(9/2)+9/10*a*(1/8*x*(b
*x^2+a)^(7/2)+7/8*a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4
*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))
```

Maxima [A]
time = 0.33, size = 121, normalized size = 0.79

$$\frac{(bx^2+a)^{\frac{11}{2}}x}{12\,b}-\frac{(bx^2+a)^{\frac{9}{2}}ax}{120\,b}-\frac{3\,(bx^2+a)^{\frac{7}{2}}a^2x}{320\,b}-\frac{7\,(bx^2+a)^{\frac{5}{2}}a^3x}{640\,b}-\frac{7\,(bx^2+a)^{\frac{3}{2}}a^4x}{512\,b}-\frac{21\,\sqrt{bx^2+a}\,a^5x}{1024\,b}-\frac{21\,a^6\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^(11/2)*x/b - 1/120*(b*x^2 + a)^(9/2)*a*x/b - 3/320*(b*x^2
+ a)^(7/2)*a^2*x/b - 7/640*(b*x^2 + a)^(5/2)*a^3*x/b - 7/512*(b*x^2 + a)^(3
/2)*a^4*x/b - 21/1024*sqrt(b*x^2 + a)*a^5*x/b - 21/1024*a^6*arcsinh(b*x/sqr
t(a*b))/b^(3/2)
```

Fricas [A]
time = 1.37, size = 211, normalized size = 1.37

$$\left[\frac{315\,a^6\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(1280\,b^6x^{11}+6272\,ab^5x^9+12144\,a^2b^4x^7+11432\,a^3b^3x^5+4910\,a^4b^2x^3+315\,a^5bx)\sqrt{bx^2+a}}{30720\,b^2},\ \frac{315\,a^6\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(1280\,b^6x^{11}+6272\,ab^5x^9+12144\,a^2b^4x^7+11432\,a^3b^3x^5+4910\,a^4b^2x^3+315\,a^5bx)\sqrt{bx^2+a}}{15360\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(315*a^6*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(1280*b^6*x^11 + 6272*a*b^5*x^9 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^5
+ 4910*a^4*b^2*x^3 + 315*a^5*b*x)*sqrt(b*x^2 + a))/b^2, 1/15360*(315*a^6*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (1280*b^6*x^11 + 6272*a*b^5*x^9
 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^5 + 4910*a^4*b^2*x^3 + 315*a^5*b*x)*
sqrt(b*x^2 + a))/b^2]
```

Sympy [A]
time = 46.71, size = 204, normalized size = 1.32

$$\frac{21a^{\frac{11}{2}}x}{1024b\sqrt{1+\frac{bx^2}{a}}}+\frac{1045a^{\frac{9}{2}}x^3}{3072\sqrt{1+\frac{bx^2}{a}}}+\frac{8171a^{\frac{7}{2}}bx^5}{7680\sqrt{1+\frac{bx^2}{a}}}+\frac{2947a^{\frac{5}{2}}b^2x^7}{1920\sqrt{1+\frac{bx^2}{a}}}+\frac{1151a^{\frac{3}{2}}b^3x^9}{960\sqrt{1+\frac{bx^2}{a}}}+\frac{59\sqrt{a}\,b^4x^{11}}{120\sqrt{1+\frac{bx^2}{a}}}-\frac{21a^6\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1024b^{\frac{3}{2}}}+\frac{b^5x^{13}}{12\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(9/2),x)
```

```
[Out] 21*a**(11/2)*x/(1024*b*sqrt(1 + b*x**2/a)) + 1045*a**(9/2)*x**3/(3072*sqrt(
1 + b*x**2/a)) + 8171*a**(7/2)*b*x**5/(7680*sqrt(1 + b*x**2/a)) + 2947*a**(
5/2)*b**2*x**7/(1920*sqrt(1 + b*x**2/a)) + 1151*a**(3/2)*b**3*x**9/(960*sqr
```

```
t(1 + b*x**2/a)) + 59*sqrt(a)*b**4*x**11/(120*sqrt(1 + b*x**2/a)) - 21*a**6
*asinh(sqrt(b)*x/sqrt(a))/(1024*b**(3/2)) + b**5*x**13/(12*sqrt(a)*sqrt(1 +
 b*x**2/a))
```

Giac [A]
time = 0.92, size = 105, normalized size = 0.68

$$\frac{21\,a^6\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{1024\,b^{\frac{3}{2}}}+\frac{1}{15360}\left(\frac{315\,a^5}{b}+2\left(2455\,a^4+4\left(1429\,a^3b+2\left(759\,a^2b^2+8\left(10\,b^4x^2+49\,ab^3\right)x^2\right)x^2\right)x^2\right)x^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 21/1024*a^6*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/15360*(315*a
^5/b + 2*(2455*a^4 + 4*(1429*a^3*b + 2*(759*a^2*b^2 + 8*(10*b^4*x^2 + 49*a*
b^3)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^2+a\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(9/2), x)
```

3.428 $\int (a+bx^2)^{9/2}\,dx$

Optimal. Leaf size=122

$$\frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x\left(a+bx^2\right)^{9/2}+\frac{63a^5\,\mathrm{t}}{}$$

```
[Out] 21/128*a^3*x*(b*x^2+a)^(3/2)+21/160*a^2*x*(b*x^2+a)^(5/2)+9/80*a*x*(b*x^2+a
)^(7/2)+1/10*x*(b*x^2+a)^(9/2)+63/256*a^5*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2)
)/b^(1/2)+63/256*a^4*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 223, 212}

$$\frac{63a^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}}+\frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2),x]
```

```
[Out] (63*a^4*x*Sqrt[a + b*x^2])/256 + (21*a^3*x*(a + b*x^2)^(3/2))/128 + (21*a^2
*x*(a + b*x^2)^(5/2))/160 + (9*a*x*(a + b*x^2)^(7/2))/80 + (x*(a + b*x^2)^(
9/2))/10 + (63*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx^2)^{9/2}\,dx = \frac{1}{10}x(a+bx^2)^{9/2} + \frac{1}{10}(9a)\int (a+bx^2)^{7/2}\,dx$$
$$= \frac{9}{80}ax(a+bx^2)^{7/2} + \frac{1}{10}x(a+bx^2)^{9/2} + \frac{1}{80}(63a^2)\int (a+bx^2)^{5/2}\,dx$$
$$= \frac{21}{160}a^2x(a+bx^2)^{5/2} + \frac{9}{80}ax(a+bx^2)^{7/2} + \frac{1}{10}x(a+bx^2)^{9/2} + \frac{1}{32}(21a^3)\int (a+bx^2)^{3/}$$
$$= \frac{21}{128}a^3x(a+bx^2)^{3/2} + \frac{21}{160}a^2x(a+bx^2)^{5/2} + \frac{9}{80}ax(a+bx^2)^{7/2} + \frac{1}{10}x(a+bx^2)^{9/2} + \frac{}{1}$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x(a+bx^2)^{3/2} + \frac{21}{160}a^2x(a+bx^2)^{5/2} + \frac{9}{80}ax(a+bx^2)^{7/2} +$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x(a+bx^2)^{3/2} + \frac{21}{160}a^2x(a+bx^2)^{5/2} + \frac{9}{80}ax(a+bx^2)^{7/2} +$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x(a+bx^2)^{3/2} + \frac{21}{160}a^2x(a+bx^2)^{5/2} + \frac{9}{80}ax(a+bx^2)^{7/2} +$$

Mathematica [A]
time = 0.12, size = 93, normalized size = 0.76

$$\frac{\sqrt{a+bx^2}\,(965a^4x+1490a^3bx^3+1368a^2b^2x^5+656ab^3x^7+128b^4x^9)}{1280} - \frac{63a^5\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{256\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2),x]
[Out] (Sqrt[a + b*x^2]*(965*a^4*x + 1490*a^3*b*x^3 + 1368*a^2*b^2*x^5 + 656*a*b^3
*x^7 + 128*b^4*x^9))/1280 - (63*a^5*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2
56*Sqrt[b])
```

Maple [A]
time = 0.04, size = 100, normalized size = 0.82

method	result
risch	$\frac{x(128b^4x^8+656a\,b^3x^6+1368a^2b^2x^4+1490a^3b\,x^2+965a^4)\sqrt{b\,x^2+a}}{1280}+\frac{63a^5\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{256\sqrt{b}}$

default	$\frac{x(bx^2+a)^{\frac{9}{2}}}{10}+\frac{9a\left(\frac{x(bx^2+a)^{\frac{7}{2}}}{8}+\frac{7a\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8}\right)}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x*(b*x^2+a)^(9/2)+9/10*a*(1/8*x*(b*x^2+a)^(7/2)+7/8*a*(1/6*x*(b*x^2+a)
^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1
/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.27, size = 88, normalized size = 0.72

$$\frac{1}{10}\left(bx^2+a\right)^{\frac{9}{2}}x+\frac{9}{80}\left(bx^2+a\right)^{\frac{7}{2}}ax+\frac{21}{160}\left(bx^2+a\right)^{\frac{5}{2}}a^2x+\frac{21}{128}\left(bx^2+a\right)^{\frac{3}{2}}a^3x+\frac{63}{256}\sqrt{bx^2+a}\,a^4x+\frac{63\,a^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(9/2)*x + 9/80*(b*x^2 + a)^(7/2)*a*x + 21/160*(b*x^2 + a)^
(5/2)*a^2*x + 21/128*(b*x^2 + a)^(3/2)*a^3*x + 63/256*sqrt(b*x^2 + a)*a^4*x
 + 63/256*a^5*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 1.34, size = 190, normalized size = 1.56

$$\left[\frac{315\,a^5\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(128\,b^5x^9+656\,ab^4x^7+1368\,a^2b^3x^5+1490\,a^3b^2x^3+965\,a^4bx)\sqrt{bx^2+a}}{2560\,b},\ -\frac{315\,a^5\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(128\,b^5x^9+656\,ab^4x^7+1368\,a^2b^3x^5+1490\,a^3b^2x^3+965\,a^4bx)\sqrt{bx^2+a}}{1280\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490*a^3*b^2*x^3 + 965*
a^4*b*x)*sqrt(b*x^2 + a))/b, -1/1280*(315*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490*a^3
*b^2*x^3 + 965*a^4*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 10.32, size = 151, normalized size = 1.24

$$\frac{193a^{\frac{9}{2}}x\sqrt{1+\frac{bx^2}{a}}}{256}+\frac{149a^{\frac{7}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{128}+\frac{171a^{\frac{5}{2}}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{160}+\frac{41a^{\frac{3}{2}}b^3x^7\sqrt{1+\frac{bx^2}{a}}}{80}+\frac{\sqrt{a}\,b^4x^9\sqrt{1+\frac{bx^2}{a}}}{10}+\frac{63a^5\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2),x)
```

```
[Out] 193*a**(9/2)*x*sqrt(1 + b*x**2/a)/256 + 149*a**(7/2)*b*x**3*sqrt(1 + b*x**2
/a)/128 + 171*a**(5/2)*b**2*x**5*sqrt(1 + b*x**2/a)/160 + 41*a**(3/2)*b**3*
x**7*sqrt(1 + b*x**2/a)/80 + sqrt(a)*b**4*x**9*sqrt(1 + b*x**2/a)/10 + 63*a
**5*asinh(sqrt(b)*x/sqrt(a))/(256*sqrt(b))
```

Giac [A]
time = 1.28, size = 91, normalized size = 0.75

$$-\frac{63\,a^5\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,\sqrt{b}}+\frac{1}{1280}\left(965\,a^4+2\left(745\,a^3b+4\left(171\,a^2b^2+2\left(8\,b^4x^2+41\,ab^3\right)x^2\right)x^2\right)x^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -63/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/1280*(965*a^
4 + 2*(745*a^3*b + 4*(171*a^2*b^2 + 2*(8*b^4*x^2 + 41*a*b^3)*x^2)*x^2)*x^2)
*sqrt(b*x^2 + a)*x
```

Mupad [B]
time = 4.62, size = 37, normalized size = 0.30

$$\frac{x\left(b\,x^2+a\right)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{9}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2),x)
```

```
[Out] (x*(a + b*x^2)^(9/2)*hypergeom([-9/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(9/2)
```

3.429 $\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx$

Optimal. Leaf size=123

$$\frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}+\frac{21}{16}abx\left(a+bx^2\right)^{5/2}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{315}{128}a^4\sqrt{b}$$

```
[Out] 105/64*a^2*b*x*(b*x^2+a)^(3/2)+21/16*a*b*x*(b*x^2+a)^(5/2)+9/8*b*x*(b*x^2+a
)^(7/2)-(b*x^2+a)^(9/2)/x+315/128*a^4*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^
(1/2)+315/128*a^3*b*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 201, 223, 212}

$$\frac{315}{128}a^4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}+\frac{21}{16}abx\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^2,x]
```

```
[Out] (315*a^3*b*x*Sqrt[a + b*x^2])/128 + (105*a^2*b*x*(a + b*x^2)^(3/2))/64 + (2
1*a*b*x*(a + b*x^2)^(5/2))/16 + (9*b*x*(a + b*x^2)^(7/2))/8 - (a + b*x^2)^(
9/2)/x + (315*a^4*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/128
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx = -\frac{(a+bx^2)^{9/2}}{x} + (9b)\int \left(a+bx^2\right)^{7/2}\,dx$$
$$= \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{(a+bx^2)^{9/2}}{x} + \frac{1}{8}(63ab)\int \left(a+bx^2\right)^{5/2}\,dx$$
$$= \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{(a+bx^2)^{9/2}}{x} + \frac{1}{16}\left(105a^2b\right)\int \left(a+bx^2\right)^{3/2}\,dx$$
$$= \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{(a+bx^2)^{9/2}}{x} + \frac{1}{64}(315$$
$$= \frac{315}{128}a^3bx\sqrt{a+bx^2} + \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} -$$
$$= \frac{315}{128}a^3bx\sqrt{a+bx^2} + \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} -$$
$$= \frac{315}{128}a^3bx\sqrt{a+bx^2} + \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} -$$

Mathematica [A]
time = 0.12, size = 95, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\,\left(-128a^4+325a^3bx^2+210a^2b^2x^4+88ab^3x^6+16b^4x^8\right)}{128x} - \frac{315}{128}a^4\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-128*a^4 + 325*a^3*b*x^2 + 210*a^2*b^2*x^4 + 88*a*b^3*x^6
 + 16*b^4*x^8))/(128*x) - (315*a^4*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/128
```

Maple [A]
time = 0.06, size = 124, normalized size = 1.01

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-16 b^{4} x^{8}-88 a\,b^{3} x^{6}-210 a^{2} b^{2} x^{4}-325 a^{3} b\,x^{2}+128 a^{4}\right)}{128 x}+\frac{315 a^{4} \sqrt{b}\,\ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{128}$

default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{a x}+\frac{10 b \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{9}{2}}}{10}+\frac{9 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8}+\frac{7 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x}\right.}{2 \sqrt{b}}\right.}{4}\right.}{6}\right.}{8}\right.}{10}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x*(b*x^2+a)^(11/2)+10*b/a*(1/10*x*(b*x^2+a)^(9/2)+9/10*a*(1/8*x*(b*x^2
+a)^(7/2)+7/8*a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(
1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 0.74

$$\frac{9}{8}\left(bx^2+a\right)^{\frac{7}{2}}bx+\frac{21}{16}\left(bx^2+a\right)^{\frac{5}{2}}abx+\frac{105}{64}\left(bx^2+a\right)^{\frac{3}{2}}a^2bx+\frac{315}{128}\sqrt{bx^2+a}\,a^3bx+\frac{315}{128}a^4\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{9}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="maxima")
```

```
[Out] 9/8*(b*x^2 + a)^(7/2)*b*x + 21/16*(b*x^2 + a)^(5/2)*a*b*x + 105/64*(b*x^2 +
 a)^(3/2)*a^2*b*x + 315/128*sqrt(b*x^2 + a)*a^3*b*x + 315/128*a^4*sqrt(b)*a
rcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(9/2)/x
```

Fricas [A]
time = 0.91, size = 184, normalized size = 1.50

$$\left[\frac{315\,a^4\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(16\,b^4x^8+88\,ab^3x^6+210\,a^2b^2x^4+325\,a^3bx^2-128\,a^4)\sqrt{bx^2+a}}{256\,x},\ -\frac{315\,a^4\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(16\,b^4x^8+88\,ab^3x^6+210\,a^2b^2x^4+325\,a^3bx^2-128\,a^4)\sqrt{bx^2+a}}{128\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*a^4*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 - 128*a^4)*
sqrt(b*x^2 + a))/x, -1/128*(315*a^4*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 - 128
*a^4)*sqrt(b*x^2 + a))/x]
```

Sympy [A]
time = 9.83, size = 173, normalized size = 1.41

$$-\frac{a^{\frac{9}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{197a^{\frac{7}{2}}bx}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{535a^{\frac{5}{2}}b^2x^3}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{149a^{\frac{3}{2}}b^3x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{13\sqrt{a}\,b^4x^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{315a^4\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128}+\frac{b^5x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**2,x)
```

```
[Out] -a**(9/2)/(x*sqrt(1 + b*x**2/a)) + 197*a**(7/2)*b*x/(128*sqrt(1 + b*x**2/a)
) + 535*a**(5/2)*b**2*x**3/(128*sqrt(1 + b*x**2/a)) + 149*a**(3/2)*b**3*x**
5/(64*sqrt(1 + b*x**2/a)) + 13*sqrt(a)*b**4*x**7/(16*sqrt(1 + b*x**2/a)) +
315*a**4*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/128 + b**5*x**9/(8*sqrt(a)*sqrt(1
 + b*x**2/a))
```

Giac [A]
time = 0.90, size = 115, normalized size = 0.93

$$-\frac{315}{256}\,a^4\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^5\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{128}\left(325\,a^3b+2\left(105\,a^2b^2+4\left(2\,b^4x^2+11\,ab^3\right)x^2\right)x^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="giac")
```

```
[Out] -315/256*a^4*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^5*sqrt(b)/(
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/128*(325*a^3*b + 2*(105*a^2*b^2 +
4*(2*b^4*x^2 + 11*a*b^3)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

Mupad [B]
time = 6.03, size = 40, normalized size = 0.33

$$-\frac{\left(b\,x^2+a\right)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{9}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^2,x)
```

```
[Out] -((a + b*x^2)^(9/2)*hypergeom([-9/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a
 + 1)^(9/2))
```

3.430 $\int \frac{(a+bx^2)^{9/2}}{x^4} dx$

Optimal. Leaf size=128

$$\frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{35}{8}ab^2x(a+bx^2)^{3/2}+\frac{7}{2}b^2x(a+bx^2)^{5/2}-\frac{3b(a+bx^2)^{7/2}}{x}-\frac{(a+bx^2)^{9/2}}{3x^3}+\frac{105}{16}a^3b^{3/2}\tanh$$

```
[Out] 35/8*a*b^2*x*(b*x^2+a)^(3/2)+7/2*b^2*x*(b*x^2+a)^(5/2)-3*b*(b*x^2+a)^(7/2)/
x-1/3*(b*x^2+a)^(9/2)/x^3+105/16*a^3*b^(3/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))+105/16*a^2*b^2*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 201, 223, 212}

$$\frac{105}{16}a^3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{7}{2}b^2x(a+bx^2)^{5/2}+\frac{35}{8}ab^2x(a+bx^2)^{3/2}-\frac{3b(a+bx^2)^{7/2}}{x}-\frac{(a+bx^2)^{9/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^4,x]
```

```
[Out] (105*a^2*b^2*x*Sqrt[a + b*x^2])/16 + (35*a*b^2*x*(a + b*x^2)^(3/2))/8 + (7*
b^2*x*(a + b*x^2)^(5/2))/2 - (3*b*(a + b*x^2)^(7/2))/x - (a + b*x^2)^(9/2)/
(3*x^3) + (105*a^3*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/16
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx &= -\frac{(a+bx^2)^{9/2}}{3x^3}+(3b)\int \frac{(a+bx^2)^{7/2}}{x^2}\,dx\\
&= -\frac{3b(a+bx^2)^{7/2}}{x}-\frac{(a+bx^2)^{9/2}}{3x^3}+(21b^2)\int (a+bx^2)^{5/2}\,dx\\
&= \frac{7}{2}b^2x(a+bx^2)^{5/2}-\frac{3b(a+bx^2)^{7/2}}{x}-\frac{(a+bx^2)^{9/2}}{3x^3}+\frac{1}{2}(35ab^2)\int (a+bx^2)^{3/2}\,dx\\
&= \frac{35}{8}ab^2x(a+bx^2)^{3/2}+\frac{7}{2}b^2x(a+bx^2)^{5/2}-\frac{3b(a+bx^2)^{7/2}}{x}-\frac{(a+bx^2)^{9/2}}{3x^3}+\frac{1}{8}(105a^2b^2)\\
&= \frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{35}{8}ab^2x(a+bx^2)^{3/2}+\frac{7}{2}b^2x(a+bx^2)^{5/2}-\frac{3b(a+bx^2)^{7/2}}{x}-\frac{(a+\\
&= \frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{35}{8}ab^2x(a+bx^2)^{3/2}+\frac{7}{2}b^2x(a+bx^2)^{5/2}-\frac{3b(a+bx^2)^{7/2}}{x}-\frac{(a+\\
&= \frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{35}{8}ab^2x(a+bx^2)^{3/2}+\frac{7}{2}b^2x(a+bx^2)^{5/2}-\frac{3b(a+bx^2)^{7/2}}{x}-\frac{(a+
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 95, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-16a^4-208a^3bx^2+165a^2b^2x^4+50ab^3x^6+8b^4x^8)}{48x^3}-\frac{105}{16}a^3b^{3/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^4 - 208*a^3*b*x^2 + 165*a^2*b^2*x^4 + 50*a*b^3*x^6
+ 8*b^4*x^8))/(48*x^3) - (105*a^3*b^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/16
```

Maple [A]
time = 0.06, size = 148, normalized size = 1.16

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-8 b^{4} x^{8}-50 a\,b^{3} x^{6}-165 a^{2} b^{2} x^{4}+208 a^{3} b\,x^{2}+16 a^{4}\right)}{48 x^{3}}+\frac{105 a^{3} b^{\frac{3}{2}} \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16}$

$$8b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{a x}+\frac{10b\left(\frac{x\left(b\,x^2+a\right)^{\frac{9}{2}}}{10}+\frac{9a\left(\frac{x\left(b\,x^2+a\right)^{\frac{7}{2}}}{8}+\frac{7a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\cdots\right)}{\cdots}\right)}{6}\right)}{8}\right)}{10}\right)}{a}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(11/2)+8/3*b/a*(-1/a/x*(b*x^2+a)^(11/2)+10*b/a*(1/10*x
*(b*x^2+a)^(9/2)+9/10*a*(1/8*x*(b*x^2+a)^(7/2)+7/8*a*(1/6*x*(b*x^2+a)^(5/2)
+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))))))))
```

Maxima [A]
time = 0.32, size = 120, normalized size = 0.94

$$\frac{7}{2}\left(bx^2+a\right)^{\frac{5}{2}}b^2x+\frac{3\left(bx^2+a\right)^{\frac{7}{2}}b^2x}{a}+\frac{35}{8}\left(bx^2+a\right)^{\frac{3}{2}}ab^2x+\frac{105}{16}\sqrt{bx^2+a}\,a^2b^2x+\frac{105}{16}a^3b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{8\left(bx^2+a\right)^{\frac{9}{2}}b}{3\,ax}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="maxima")
```

```
[Out] 7/2*(b*x^2 + a)^(5/2)*b^2*x + 3*(b*x^2 + a)^(7/2)*b^2*x/a + 35/8*(b*x^2 + a
)^(3/2)*a*b^2*x + 105/16*sqrt(b*x^2 + a)*a^2*b^2*x + 105/16*a^3*b^(3/2)*arc
sinh(b*x/sqrt(a*b)) - 8/3*(b*x^2 + a)^(9/2)*b/(a*x) - 1/3*(b*x^2 + a)^(11/2
)/(a*x^3)
```

Fricas [A]
time = 0.82, size = 189, normalized size = 1.48

$$\left[\frac{315\,a^3b^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,b^4x^8+50\,ab^3x^6+165\,a^2b^2x^4-208\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{96\,x^3},\,-\frac{315\,a^3\sqrt{-b}\,bx^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(8\,b^4x^8+50\,ab^3x^6+165\,a^2b^2x^4-208\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{48\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(315*a^3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^2 - 16*a^4)*s
qrt(b*x^2 + a))/x^3, -1/48*(315*a^3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^2 -
16*a^4)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A]
time = 9.39, size = 175, normalized size = 1.37

$$-\frac{a^{\frac{9}{2}}}{3x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{14a^{\frac{7}{2}}b}{3x\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2x}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{215a^{\frac{3}{2}}b^3x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{29\sqrt{a}\,b^4x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{105a^3b^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16}+\frac{b^5x^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**4,x)
```

```
[Out] -a**(9/2)/(3*x**3*sqrt(1 + b*x**2/a)) - 14*a**(7/2)*b/(3*x*sqrt(1 + b*x**2/
a)) - 43*a**(5/2)*b**2*x/(48*sqrt(1 + b*x**2/a)) + 215*a**(3/2)*b**3*x**3/(
48*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**4*x**5/(24*sqrt(1 + b*x**2/a)) + 105
*a**3*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/16 + b**5*x**7/(6*sqrt(a)*sqrt(1 +
b*x**2/a))
```

Giac [A]
time = 0.62, size = 160, normalized size = 1.25

$$-\frac{105}{32}\,a^3b^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{48}\,\left(165\,a^2b^2+2\,\left(4\,b^4x^2+25\,ab^3\right)x^2\right)\sqrt{bx^2+a}\,x+\frac{2\,\left(15\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^4b^{\frac{3}{2}}-24\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^5b^{\frac{3}{2}}+13\,a^6b^{\frac{3}{2}}\right)}{3\,\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="giac")
```

```
[Out] -105/32*a^3*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/48*(165*a^2*b^
2 + 2*(4*b^4*x^2 + 25*a*b^3)*x^2)*sqrt(b*x^2 + a)*x + 2/3*(15*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*a^4*b^(3/2) - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(
3/2) + 13*a^6*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{9/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^4,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^4, x)
```

3.431 $\int \frac{(a+bx^2)^{9/2}}{x^6} dx$

Optimal. Leaf size=129

$$\frac{63}{8}ab^3x\sqrt{a+bx^2}+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}+\frac{63}{8}a^2b^{5/2}\tanh^{-1}\left(\right.$$

```
[Out] 21/4*b^3*x*(b*x^2+a)^(3/2)-21/5*b^2*(b*x^2+a)^(5/2)/x-3/5*b*(b*x^2+a)^(7/2)
/x^3-1/5*(b*x^2+a)^(9/2)/x^5+63/8*a^2*b^(5/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))+63/8*a*b^3*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 201, 223, 212}

$$\frac{63}{8}a^2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}+\frac{63}{8}ab^3x\sqrt{a+bx^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^6,x]
```

```
[Out] (63*a*b^3*x*Sqrt[a + b*x^2])/8 + (21*b^3*x*(a + b*x^2)^(3/2))/4 - (21*b^2*(
a + b*x^2)^(5/2))/(5*x) - (3*b*(a + b*x^2)^(7/2))/(5*x^3) - (a + b*x^2)^(9/
2)/(5*x^5) + (63*a^2*b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx = -\frac{(a+bx^2)^{9/2}}{5x^5} + \frac{1}{5}(9b)\int \frac{(a+bx^2)^{7/2}}{x^4}\,dx$$

$$= -\frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{5x^5} + \frac{1}{5}(21b^2)\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx$$

$$= -\frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{5x^5} + (21b^3)\int (a+bx^2)^{3/2}\,dx$$

$$= \frac{21}{4}b^3x(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{5x^5} + \frac{1}{4}(63ab^3)\int$$

$$= \frac{63}{8}ab^3x\sqrt{a+bx^2} + \frac{21}{4}b^3x(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx}{5x^5}$$

$$= \frac{63}{8}ab^3x\sqrt{a+bx^2} + \frac{21}{4}b^3x(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx}{5x^5}$$

$$= \frac{63}{8}ab^3x\sqrt{a+bx^2} + \frac{21}{4}b^3x(a+bx^2)^{3/2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{3b(a+bx^2)^{7/2}}{5x^3} - \frac{(a+bx}{5x^5}$$

Mathematica [A]
time = 0.15, size = 95, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-8a^4-56a^3bx^2-288a^2b^2x^4+85ab^3x^6+10b^4x^8)}{40x^5} - \frac{63}{8}a^2b^{5/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^4 - 56*a^3*b*x^2 - 288*a^2*b^2*x^4 + 85*a*b^3*x^6 +
10*b^4*x^8))/(40*x^5) - (63*a^2*b^(5/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/8
```

Maple [A]
time = 0.07, size = 172, normalized size = 1.33

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-10 b^{4} x^{8}-85 a\,b^{3} x^{6}+288 a^{2} b^{2} x^{4}+56 a^{3} b\,x^{2}+8 a^{4}\right)}{40 x^{5}}+\frac{63 a^{2} b^{\frac{5}{2}} \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8}$

$3a$

$5a \left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4} + \right.$

$7a \left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6} + \right.$

$9a \left(\frac{x\left(b\,x^2+a\right)^{\frac{7}{2}}}{8} + \right.$

$10b \left(\frac{x\left(b\,x^2+a\right)^{\frac{9}{2}}}{10} + \right.$ 1

$8b \left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{a x} + \right.$ a

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/x^5*(b*x^2+a)^(11/2)+6/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(11/2)+8/3*b/a*(-
1/a/x*(b*x^2+a)^(11/2)+10*b/a*(1/10*x*(b*x^2+a)^(9/2)+9/10*a*(1/8*x*(b*x^2+
a)^(7/2)+7/8*a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1
/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))))
```

Maxima [A]
time = 0.34, size = 140, normalized size = 1.09

$$\frac{21}{4}\left(bx^2+a\right)^{\frac{3}{2}}b^3x+\frac{18\left(bx^2+a\right)^{\frac{7}{2}}b^3x}{5\,a^2}+\frac{21\left(bx^2+a\right)^{\frac{5}{2}}b^3x}{5\,a}+\frac{63}{8}\sqrt{bx^2+a}\,ab^3x+\frac{63}{8}a^2b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{16\left(bx^2+a\right)^{\frac{9}{2}}b^2}{5\,a^2x}-\frac{2\left(bx^2+a\right)^{\frac{11}{2}}b}{5\,a^2x^3}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="maxima")
```

```
[Out] 21/4*(b*x^2 + a)^(3/2)*b^3*x + 18/5*(b*x^2 + a)^(7/2)*b^3*x/a^2 + 21/5*(b*x
^2 + a)^(5/2)*b^3*x/a + 63/8*sqrt(b*x^2 + a)*a*b^3*x + 63/8*a^2*b^(5/2)*arc
sinh(b*x/sqrt(a*b)) - 16/5*(b*x^2 + a)^(9/2)*b^2/(a^2*x) - 2/5*(b*x^2 + a)^
(11/2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(11/2)/(a*x^5)
```

Fricas [A]
time = 0.99, size = 191, normalized size = 1.48

$$\left[\frac{315\,a^2b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(10\,b^4x^8+85\,ab^3x^6-288\,a^2b^2x^4-56\,a^3bx^2-8\,a^4\right)\sqrt{bx^2+a}}{80\,x^5},\;-\frac{315\,a^2\sqrt{-b}\,b^2x^5\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(10\,b^4x^8+85\,ab^3x^6-288\,a^2b^2x^4-56\,a^3bx^2-8\,a^4\right)\sqrt{bx^2+a}}{40\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(315*a^2*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 - 8*a^4)*sq
rt(b*x^2 + a))/x^5, -1/40*(315*a^2*sqrt(-b)*b^2*x^5*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - (10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 -
 8*a^4)*sqrt(b*x^2 + a))/x^5]
```

Sympy [A]
time = 9.74, size = 175, normalized size = 1.36

$$-\frac{a^{\frac{9}{2}}}{5x^5\sqrt{1+\frac{bx^2}{a}}}-\frac{8a^{\frac{7}{2}}b}{5x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2}{5x\sqrt{1+\frac{bx^2}{a}}}-\frac{203a^{\frac{3}{2}}b^3x}{40\sqrt{1+\frac{bx^2}{a}}}+\frac{19\sqrt{a}\,b^4x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{63a^2b^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{b^5x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**6,x)
```

```
[Out] -a**(9/2)/(5*x**5*sqrt(1 + b*x**2/a)) - 8*a**(7/2)*b/(5*x**3*sqrt(1 + b*x**
2/a)) - 43*a**(5/2)*b**2/(5*x*sqrt(1 + b*x**2/a)) - 203*a**(3/2)*b**3*x/(40
*sqrt(1 + b*x**2/a)) + 19*sqrt(a)*b**4*x**3/(8*sqrt(1 + b*x**2/a)) + 63*a**
2*b**(5/2)*asinh(sqrt(b)*x/sqrt(a))/8 + b**5*x**5/(4*sqrt(a)*sqrt(1 + b*x**
2/a))
```

Giac [A]
time = 0.90, size = 200, normalized size = 1.55

$$-\frac{63}{16}\,a^{2}b^{\frac{5}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{1}{8}\left(2\,b^{4}x^{2}+17\,ab^{3}\right)\sqrt{bx^{2}+a}\,x+\frac{4\left(25\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{8}a^{3}b^{\frac{5}{2}}-75\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{6}a^{4}b^{\frac{5}{2}}+105\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{4}a^{5}b^{\frac{5}{2}}-65\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}a^{6}b^{\frac{5}{2}}+18\,a^{7}b^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="giac")
```

```
[Out] -63/16*a^2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/8*(2*b^4*x^2 +
17*a*b^3)*sqrt(b*x^2 + a)*x + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b
^(5/2) - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(5/2) + 105*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*a^5*b^(5/2) - 65*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(
5/2) + 18*a^7*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{9/2}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^6,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^6, x)
```

3.432 $\int \frac{(a+bx^2)^{9/2}}{x^8} dx$

Optimal. Leaf size=126

$$\frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3(a+bx^2)^{3/2}}{x}-\frac{3b^2(a+bx^2)^{5/2}}{5x^3}-\frac{9b(a+bx^2)^{7/2}}{35x^5}-\frac{(a+bx^2)^{9/2}}{7x^7}+\frac{9}{2}ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -3*b^3*(b*x^2+a)^(3/2)/x-3/5*b^2*(b*x^2+a)^(5/2)/x^3-9/35*b*(b*x^2+a)^(7/2)
/x^5-1/7*(b*x^2+a)^(9/2)/x^7+9/2*a*b^(7/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2
))+9/2*b^4*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 201, 223, 212}

$$\frac{9}{2}ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3(a+bx^2)^{3/2}}{x}-\frac{3b^2(a+bx^2)^{5/2}}{5x^3}-\frac{(a+bx^2)^{9/2}}{7x^7}-\frac{9b(a+bx^2)^{7/2}}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^8,x]
```

```
[Out] (9*b^4*x*Sqrt[a + b*x^2])/2 - (3*b^3*(a + b*x^2)^(3/2))/x - (3*b^2*(a + b*x
^2)^(5/2))/(5*x^3) - (9*b*(a + b*x^2)^(7/2))/(35*x^5) - (a + b*x^2)^(9/2)/(
7*x^7) + (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx = -\frac{(a+bx^2)^{9/2}}{7x^7}+\frac{1}{7}(9b)\int \frac{(a+bx^2)^{7/2}}{x^6}\,dx$$
$$= -\frac{9b(a+bx^2)^{7/2}}{35x^5}-\frac{(a+bx^2)^{9/2}}{7x^7}+\frac{1}{5}(9b^2)\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx$$
$$= -\frac{3b^2(a+bx^2)^{5/2}}{5x^3}-\frac{9b(a+bx^2)^{7/2}}{35x^5}-\frac{(a+bx^2)^{9/2}}{7x^7}+(3b^3)\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx$$
$$= -\frac{3b^3(a+bx^2)^{3/2}}{x}-\frac{3b^2(a+bx^2)^{5/2}}{5x^3}-\frac{9b(a+bx^2)^{7/2}}{35x^5}-\frac{(a+bx^2)^{9/2}}{7x^7}+(9b^4)\int \sqrt{a+b$$
$$= \frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3(a+bx^2)^{3/2}}{x}-\frac{3b^2(a+bx^2)^{5/2}}{5x^3}-\frac{9b(a+bx^2)^{7/2}}{35x^5}-\frac{(a+bx^2)^{9/2}}{7x^7}+$$
$$= \frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3(a+bx^2)^{3/2}}{x}-\frac{3b^2(a+bx^2)^{5/2}}{5x^3}-\frac{9b(a+bx^2)^{7/2}}{35x^5}-\frac{(a+bx^2)^{9/2}}{7x^7}+$$
$$= \frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3(a+bx^2)^{3/2}}{x}-\frac{3b^2(a+bx^2)^{5/2}}{5x^3}-\frac{9b(a+bx^2)^{7/2}}{35x^5}-\frac{(a+bx^2)^{9/2}}{7x^7}+$$

Mathematica [A]
time = 0.16, size = 93, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-10a^4-58a^3bx^2-156a^2b^2x^4-388ab^3x^6+35b^4x^8)}{70x^7}-\frac{9}{2}ab^{7/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^8,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-10*a^4 - 58*a^3*b*x^2 - 156*a^2*b^2*x^4 - 388*a*b^3*x^6
+ 35*b^4*x^8))/(70*x^7) - (9*a*b^(7/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/2
```

Maple [A]
time = 0.07, size = 196, normalized size = 1.56

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-35b^{4}x^{8}+388a\,b^{3}x^{6}+156a^{2}b^{2}x^{4}+58a^{3}b\,x^{2}+10a^{4}\right)}{70x^{7}}+\frac{9a\,b^{\frac{7}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2}$

$5a$

$7a\left(\frac{x\left(bx^2+a\right)^{\frac{5}{2}}}{6}+\right.$

$9a\left(\frac{x\left(bx^2+a\right)^{\frac{7}{2}}}{8}+\right.$

$10b\left(\frac{x\left(bx^2+a\right)^{\frac{9}{2}}}{10}+\right.$

$8b\left(-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{ax}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/a/x^7*(b*x^2+a)^(11/2)+4/7*b/a*(-1/5/a/x^5*(b*x^2+a)^(11/2)+6/5*b/a*(-
1/3/a/x^3*(b*x^2+a)^(11/2)+8/3*b/a*(-1/a/x*(b*x^2+a)^(11/2)+10*b/a*(1/10*x*
(b*x^2+a)^(9/2)+9/10*a*(1/8*x*(b*x^2+a)^(7/2)+7/8*a*(1/6*x*(b*x^2+a)^(5/2)+
5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))))))))))
```

Maxima [A]
time = 0.32, size = 160, normalized size = 1.27

$$\frac{9}{2}\sqrt{bx^2+a}\,b^4x+\frac{72\,(bx^2+a)^{\frac{7}{2}}b^4x}{35\,a^3}+\frac{12\,(bx^2+a)^{\frac{5}{2}}b^4x}{5\,a^2}+\frac{3\,(bx^2+a)^{\frac{3}{2}}b^4x}{a}+\frac{9}{2}\,ab^{\frac{7}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{64\,(bx^2+a)^{\frac{9}{2}}b^3}{35\,a^3x}-\frac{8\,(bx^2+a)^{\frac{11}{2}}b^2}{35\,a^3x^3}-\frac{4\,(bx^2+a)^{\frac{11}{2}}b}{35\,a^2x^5}-\frac{(bx^2+a)^{\frac{11}{2}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="maxima")
```

```
[Out] 9/2*sqrt(b*x^2 + a)*b^4*x + 72/35*(b*x^2 + a)^(7/2)*b^4*x/a^3 + 12/5*(b*x^2
 + a)^(5/2)*b^4*x/a^2 + 3*(b*x^2 + a)^(3/2)*b^4*x/a + 9/2*a*b^(7/2)*arcsinh
(b*x/sqrt(a*b)) - 64/35*(b*x^2 + a)^(9/2)*b^3/(a^3*x) - 8/35*(b*x^2 + a)^(1
1/2)*b^2/(a^3*x^3) - 4/35*(b*x^2 + a)^(11/2)*b/(a^2*x^5) - 1/7*(b*x^2 + a)^
(11/2)/(a*x^7)
```

Fricas [A]
time = 0.89, size = 187, normalized size = 1.48

$$\left[\frac{315\,ab^{\frac{7}{2}}x^7\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(35\,b^4x^8-388\,ab^3x^6-156\,a^2b^2x^4-58\,a^3bx^2-10\,a^4)\sqrt{bx^2+a}}{140\,x^7},-\frac{315\,a\sqrt{-b}\,b^3x^7\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(35\,b^4x^8-388\,ab^3x^6-156\,a^2b^2x^4-58\,a^3bx^2-10\,a^4)\sqrt{bx^2+a}}{70\,x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/140*(315*a*b^(7/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 - 10*a^4)*s
qrt(b*x^2 + a))/x^7, -1/70*(315*a*sqrt(-b)*b^3*x^7*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 -
 10*a^4)*sqrt(b*x^2 + a))/x^7]
```

Sympy [A]
time = 10.21, size = 167, normalized size = 1.33

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6}-\frac{29a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^4}-\frac{78a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^2}-\frac{194ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{35}-\frac{9ab^{\frac{7}{2}}\log\left(\frac{a}{bx^2}\right)}{4}+\frac{9ab^{\frac{7}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2}+\frac{b^{\frac{9}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**8,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 29*a**3*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(35*x**4) - 78*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*x**2) - 194*
a*b**(7/2)*sqrt(a/(b*x**2) + 1)/35 - 9*a*b**(7/2)*log(a/(b*x**2))/4 + 9*a*b
**(7/2)*log(sqrt(a/(b*x**2) + 1) + 1)/2 + b**(9/2)*x**2*sqrt(a/(b*x**2) + 1
)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(100) = 200.
time = 0.72, size = 240, normalized size = 1.90

$$\frac{1}{2}\sqrt{bx^2+a}\,b^4x-\frac{9}{4}ab^{\frac{7}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{4\left(175\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}a^2b^{\frac{7}{2}}-700\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a^3b^{\frac{7}{2}}+1575\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}a^4b^{\frac{7}{2}}-1820\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{6}a^5b^{\frac{7}{2}}+1337\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^6b^{\frac{7}{2}}-504\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{2}a^7b^{\frac{7}{2}}+97a^8b^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^4*x - 9/4*a*b^(7/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))
^2) + 4/35*(175*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2) - 700*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8
*a^4*b^(7/2) - 1820*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(7/2) + 1337*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(7/2) - 504*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a^7*b^(7/2) + 97*a^8*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{9/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^8,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^8, x)
```

3.433 $\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx$

Optimal. Leaf size=124

$$-\frac{b^4\sqrt{a+bx^2}}{x}-\frac{b^3(a+bx^2)^{3/2}}{3x^3}-\frac{b^2(a+bx^2)^{5/2}}{5x^5}-\frac{b(a+bx^2)^{7/2}}{7x^7}-\frac{(a+bx^2)^{9/2}}{9x^9}+b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/3*b^3*(b*x^2+a)^(3/2)/x^3-1/5*b^2*(b*x^2+a)^(5/2)/x^5-1/7*b*(b*x^2+a)^(7
/2)/x^7-1/9*(b*x^2+a)^(9/2)/x^9+b^(9/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))-
b^4*(b*x^2+a)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {283, 223, 212}

$$b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b^4\sqrt{a+bx^2}}{x}-\frac{b^3(a+bx^2)^{3/2}}{3x^3}-\frac{b^2(a+bx^2)^{5/2}}{5x^5}-\frac{(a+bx^2)^{9/2}}{9x^9}-\frac{b(a+bx^2)^{7/2}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^10,x]
```

```
[Out] -((b^4*Sqrt[a + b*x^2])/x) - (b^3*(a + b*x^2)^(3/2))/(3*x^3) - (b^2*(a + b*
x^2)^(5/2))/(5*x^5) - (b*(a + b*x^2)^(7/2))/(7*x^7) - (a + b*x^2)^(9/2)/(9*
x^9) + b^(9/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx &= -\frac{(a+bx^2)^{9/2}}{9x^9} + b\int \frac{(a+bx^2)^{7/2}}{x^8}\,dx \\
&= -\frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^2\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx \\
&= -\frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^3\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx \\
&= -\frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^4\int \frac{\sqrt{a+bx^2}}{x^2}\,dx \\
&= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^5 \int \\
&= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^5\mathrm{S} \\
&= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^{9/2}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 90, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\,(-35a^4-185a^3bx^2-408a^2b^2x^4-506ab^3x^6-563b^4x^8)}{315x^9} - b^{9/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^10,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4 - 185*a^3*b*x^2 - 408*a^2*b^2*x^4 - 506*a*b^3*x^6
 - 563*b^4*x^8))/(315*x^9) - b^(9/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(100) = 200.
time = 0.09, size = 220, normalized size = 1.77

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(563b^4x^8+506a\,b^3x^6+408a^2b^2x^4+185a^3b\,x^2+35a^4)}{315x^9}+b^{\frac{9}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)$

$$9a\left(\frac{x\left(bx^2+a\right)^{\frac{7}{2}}}{8}\right.$$

$$10b\left(\frac{x\left(bx^2+a\right)^{\frac{9}{2}}}{10}+\right.$$

$$8b\left(-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{ax}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/a/x^9*(b*x^2+a)^(11/2)+2/9*b/a*(-1/7/a/x^7*(b*x^2+a)^(11/2)+4/7*b/a*(-
1/5/a/x^5*(b*x^2+a)^(11/2)+6/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(11/2)+8/3*b/a*(-1
/a/x*(b*x^2+a)^(11/2)+10*b/a*(1/10*x*(b*x^2+a)^(9/2)+9/10*a*(1/8*x*(b*x^2+a
)^(7/2)+7/8*a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/
2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))))))
```

Maxima [A]
time = 0.30, size = 180, normalized size = 1.45

$$\frac{16\,(bx^2+a)^{\frac{7}{2}}b^5x}{35\,a^4}+\frac{8\,(bx^2+a)^{\frac{5}{2}}b^5x}{15\,a^3}+\frac{2\,(bx^2+a)^{\frac{3}{2}}b^5x}{3\,a^2}+\frac{\sqrt{bx^2+a}\,b^5x}{a}+b^{\frac{9}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{128\,(bx^2+a)^{\frac{9}{2}}b^4}{315\,a^4x}-\frac{16\,(bx^2+a)^{\frac{11}{2}}b^3}{315\,a^4x^3}-\frac{8\,(bx^2+a)^{\frac{11}{2}}b^2}{315\,a^3x^5}-\frac{2\,(bx^2+a)^{\frac{11}{2}}b}{63\,a^2x^7}-\frac{(bx^2+a)^{\frac{11}{2}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="maxima")
```

```
[Out] 16/35*(b*x^2 + a)^(7/2)*b^5*x/a^4 + 8/15*(b*x^2 + a)^(5/2)*b^5*x/a^3 + 2/3*
(b*x^2 + a)^(3/2)*b^5*x/a^2 + sqrt(b*x^2 + a)*b^5*x/a + b^(9/2)*arcsinh(b*x
/sqrt(a*b)) - 128/315*(b*x^2 + a)^(9/2)*b^4/(a^4*x) - 16/315*(b*x^2 + a)^(1
1/2)*b^3/(a^4*x^3) - 8/315*(b*x^2 + a)^(11/2)*b^2/(a^3*x^5) - 2/63*(b*x^2 +
 a)^(11/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(11/2)/(a*x^9)
```

Fricas [A]
time = 1.09, size = 184, normalized size = 1.48

$$\left[\frac{315\,b^{\frac{9}{2}}x^9\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(563\,b^4x^8+506\,ab^3x^6+408\,a^2b^2x^4+185\,a^3bx^2+35\,a^4)\sqrt{bx^2+a}}{630\,x^9},-\frac{315\,\sqrt{-b}\,b^4x^9\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(563\,b^4x^8+506\,ab^3x^6+408\,a^2b^2x^4+185\,a^3bx^2+35\,a^4)\sqrt{bx^2+a}}{315\,x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/630*(315*b^(9/2)*x^9*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2
*(563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*x^2 + 35*a^4)*s
qrt(b*x^2 + a))/x^9, -1/315*(315*sqrt(-b)*b^4*x^9*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) + (563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*x^2
+ 35*a^4)*sqrt(b*x^2 + a))/x^9]
```

Sympy [A]
time = 10.94, size = 160, normalized size = 1.29

$$-\frac{a^4\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{9x^8}-\frac{37a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{63x^6}-\frac{136a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{105x^4}-\frac{506ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{315x^2}-\frac{563b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315}-\frac{b^{\frac{9}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{9}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**10,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 37*a**3*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(63*x**6) - 136*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(105*x**4) - 50
6*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(315*x**2) - 563*b**(9/2)*sqrt(a/(b*x**2)
 + 1)/315 - b**(9/2)*log(a/(b*x**2))/2 + b**(9/2)*log(sqrt(a/(b*x**2) + 1)
+ 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(100) = 200.
time = 0.86, size = 276, normalized size = 2.23

$$-\frac{1}{2}\, b^{\frac{9}{2}} \log\left(\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2}\right) + \frac{2 \left(1575 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{16} a b^{\frac{9}{2}} - 6300 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{14} a^{2} b^{\frac{9}{2}} + 21000 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{12} a^{3} b^{\frac{9}{2}} - 31500 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{10} a^{4} b^{\frac{9}{2}} + 39438 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{8} a^{5} b^{\frac{9}{2}} - 26292 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{6} a^{6} b^{\frac{9}{2}} + 13968 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{4} a^{7} b^{\frac{9}{2}} - 3492 \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2} a^{8} b^{\frac{9}{2}} + 563 a^{9} b^{\frac{9}{2}}\right)}{315 \left(\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2} - a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/2*b^(9/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/315*(1575*(sqrt(b)*x
- sqrt(b*x^2 + a))^16*a*b^(9/2) - 6300*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2
*b^(9/2) + 21000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(9/2) - 31500*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(9/2) + 39438*(sqrt(b)*x - sqrt(b*x^2 + a
))^8*a^5*b^(9/2) - 26292*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(9/2) + 1396
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(9/2) - 3492*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^8*b^(9/2) + 563*a^9*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
 a)^9
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{9/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^10,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^10, x)
```

3.434 $\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

```
[Out] -1/11*(b*x^2+a)^(11/2)/a/x^11
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^12,x]
```

```
[Out] -1/11*(a + b*x^2)^(11/2)/(a*x^11)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx = -\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Mathematica [A]
time = 0.11, size = 21, normalized size = 1.00

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^12,x]
```

```
[Out] -1/11*(a + b*x^2)^(11/2)/(a*x^11)
```

Maple [A]
time = 0.14, size = 18, normalized size = 0.86

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{11a\,x^{11}}$	18
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{11a\,x^{11}}$	18
trager	$-\frac{(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)\sqrt{b\,x^2+a}}{11a\,x^{11}}$	69
risch	$-\frac{(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)\sqrt{b\,x^2+a}}{11a\,x^{11}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^12,x,method=_RETURNVERBOSE)
[Out] -1/11*(b*x^2+a)^(11/2)/a/x^11
```

Maxima [A]
time = 0.34, size = 17, normalized size = 0.81

$$-\frac{(bx^2+a)^{\frac{11}{2}}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="maxima")
[Out] -1/11*(b*x^2 + a)^(11/2)/(a*x^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.
time = 1.09, size = 68, normalized size = 3.24

$$-\frac{(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{bx^2+a}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="fricas")
[Out] -1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x
^2 + a^5)*sqrt(b*x^2 + a)/(a*x^11)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(17) = 34$.
time = 1.33, size = 150, normalized size = 7.14

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{11x^{10}}-\frac{5a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^8}-\frac{10a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^6}-\frac{10ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^4}-\frac{5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^2}-\frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{11a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**12,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(11*x**10) - 5*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(11*x**8) - 10*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(11*x**6) - 10*
a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(11*x**4) - 5*b**(9/2)*sqrt(a/(b*x**2) + 1)
/(11*x**2) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(11*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(17) = 34.
time = 0.78, size = 167, normalized size = 7.95

$$\frac{2\left(11\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}b^{\frac{11}{2}}+165\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^2b^{\frac{11}{2}}+462\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^4b^{\frac{11}{2}}+330\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^6b^{\frac{11}{2}}+55\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^8b^{\frac{11}{2}}+a^{10}b^{\frac{11}{2}}\right)}{11\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="giac")
```

```
[Out] 2/11*(11*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(11/2) + 165*(sqrt(b)*x - sqrt(
b*x^2 + a))^16*a^2*b^(11/2) + 462*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(1
1/2) + 330*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(11/2) + 55*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*a^8*b^(11/2) + a^10*b^(11/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^11
```

Mupad [B]
time = 6.28, size = 111, normalized size = 5.29

$$-\frac{a^4\sqrt{b\,x^2+a}}{11\,x^{11}}-\frac{5\,b^4\sqrt{b\,x^2+a}}{11\,x^3}-\frac{10\,a\,b^3\sqrt{b\,x^2+a}}{11\,x^5}-\frac{5\,a^3\,b\sqrt{b\,x^2+a}}{11\,x^9}-\frac{b^5\sqrt{b\,x^2+a}}{11\,a\,x}-\frac{10\,a^2\,b^2\sqrt{b\,x^2+a}}{11\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^12,x)
```

```
[Out] - (a^4*(a + b*x^2)^(1/2))/(11*x^11) - (5*b^4*(a + b*x^2)^(1/2))/(11*x^3) -
(10*a*b^3*(a + b*x^2)^(1/2))/(11*x^5) - (5*a^3*b*(a + b*x^2)^(1/2))/(11*x^9
) - (b^5*(a + b*x^2)^(1/2))/(11*a*x) - (10*a^2*b^2*(a + b*x^2)^(1/2))/(11*x
^7)
```

3.435 $\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx$

Optimal. Leaf size=44

$$-\frac{(a+bx^2)^{11/2}}{13ax^{13}}+\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}$$

[Out] -1/13*(b*x^2+a)^(11/2)/a/x^13+2/143*b*(b*x^2+a)^(11/2)/a^2/x^11

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 270}

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}-\frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(9/2)/x^14,x]

[Out] -1/13*(a + b*x^2)^(11/2)/(a*x^13) + (2*b*(a + b*x^2)^(11/2))/(143*a^2*x^11)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx &= -\frac{(a+bx^2)^{11/2}}{13ax^{13}}-\frac{(2b)\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx}{13a}\\ &= -\frac{(a+bx^2)^{11/2}}{13ax^{13}}+\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 31, normalized size = 0.70

$$\frac{(a+bx^2)^{11/2}(-11a+2bx^2)}{143a^2x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^14,x]
[Out] ((a + b*x^2)^(11/2)*(-11*a + 2*b*x^2))/(143*a^2*x^13)
```

Maple [A]
time = 0.30, size = 37, normalized size = 0.84

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-2b\,x^2+11a)}{143x^{13}a^2}$	28
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b(b\,x^2+a)^{\frac{11}{2}}}{143a^2x^{11}}$	37
trager	$-\frac{(-2b^6x^{12}+a\,b^5x^{10}+35a^2b^4x^8+90a^3x^6b^3+100a^4b^2x^4+53a^5b\,x^2+11a^6)\sqrt{b\,x^2+a}}{143x^{13}a^2}$	82
risch	$-\frac{(-2b^6x^{12}+a\,b^5x^{10}+35a^2b^4x^8+90a^3x^6b^3+100a^4b^2x^4+53a^5b\,x^2+11a^6)\sqrt{b\,x^2+a}}{143x^{13}a^2}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^14,x,method=_RETURNVERBOSE)
[Out] -1/13*(b*x^2+a)^(11/2)/a/x^13+2/143*b*(b*x^2+a)^(11/2)/a^2/x^11
```

Maxima [A]
time = 0.34, size = 36, normalized size = 0.82

$$\frac{2\,(bx^2+a)^{\frac{11}{2}}b}{143\,a^2x^{11}}-\frac{(bx^2+a)^{\frac{11}{2}}}{13\,ax^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="maxima")
[Out] 2/143*(b*x^2 + a)^(11/2)*b/(a^2*x^11) - 1/13*(b*x^2 + a)^(11/2)/(a*x^13)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.
time = 1.53, size = 82, normalized size = 1.86

$$\frac{(2\,b^6x^{12}-ab^5x^{10}-35\,a^2b^4x^8-90\,a^3b^3x^6-100\,a^4b^2x^4-53\,a^5bx^2-11\,a^6)\sqrt{bx^2+a}}{143\,a^2x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/143*(2*b^6*x^12 - a*b^5*x^10 - 35*a^2*b^4*x^8 - 90*a^3*b^3*x^6 - 100*a^4*
b^2*x^4 - 53*a^5*b*x^2 - 11*a^6)*sqrt(b*x^2 + a)/(a^2*x^13)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(37) = 74$.
time = 1.65, size = 175, normalized size = 3.98

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{13x^{12}}-\frac{53a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^{10}}-\frac{100a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^8}-\frac{90ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^6}-\frac{35b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^4}-\frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{143ax^2}+\frac{2b^{\frac{13}{2}}\sqrt{\frac{a}{bx^2}+1}}{143a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**14,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(13*x**12) - 53*a**3*b**(3/2)*sqrt(a/(b*
x**2) + 1)/(143*x**10) - 100*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(143*x**8)
- 90*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(143*x**6) - 35*b**(9/2)*sqrt(a/(b*x**
2) + 1)/(143*x**4) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(143*a*x**2) + 2*b**(13
/2)*sqrt(a/(b*x**2) + 1)/(143*a**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(36) = 72$.
time = 0.71, size = 328, normalized size = 7.45

$$\frac{4\left(143\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}b^{\frac{13}{2}}+429\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}ab^{\frac{13}{2}}+2145\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^2b^{\frac{13}{2}}+3003\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^3b^{\frac{13}{2}}+6006\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^4b^{\frac{13}{2}}+4290\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^5b^{\frac{13}{2}}+4290\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^6b^{\frac{13}{2}}+1430\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^7b^{\frac{13}{2}}+715\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^8b^{\frac{13}{2}}+65\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^9b^{\frac{13}{2}}+13\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{10}b^{\frac{13}{2}}-a^{11}b^{\frac{13}{2}}\right)}{143\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="giac")
```

```
[Out] 4/143*(143*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(13/2) + 429*(sqrt(b)*x - sqr
t(b*x^2 + a))^20*a*b^(13/2) + 2145*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^2*b^(
13/2) + 3003*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(13/2) + 6006*(sqrt(b)*
x - sqrt(b*x^2 + a))^14*a^4*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a))^1
2*a^5*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*b^(13/2) + 1430*
(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(13/2) + 715*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*a^8*b^(13/2) + 65*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^9*b^(13/2) + 13
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(13/2) - a^11*b^(13/2))/((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)^13
```

Mupad [B]
time = 6.85, size = 131, normalized size = 2.98

$$\frac{2\,b^6\,\sqrt{b\,x^2+a}}{143\,a^2\,x}-\frac{35\,b^4\,\sqrt{b\,x^2+a}}{143\,x^5}-\frac{90\,a\,b^3\,\sqrt{b\,x^2+a}}{143\,x^7}-\frac{53\,a^3\,b\,\sqrt{b\,x^2+a}}{143\,x^{11}}-\frac{b^5\,\sqrt{b\,x^2+a}}{143\,a\,x^3}-\frac{a^4\,\sqrt{b\,x^2+a}}{13\,x^{13}}-\frac{100\,a^2\,b^2\,\sqrt{b\,x^2+a}}{143\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^14,x)
```

```
[Out] (2*b^6*(a + b*x^2)^(1/2))/(143*a^2*x) - (35*b^4*(a + b*x^2)^(1/2))/(143*x^5
) - (90*a*b^3*(a + b*x^2)^(1/2))/(143*x^7) - (53*a^3*b*(a + b*x^2)^(1/2))/(
143*x^11) - (b^5*(a + b*x^2)^(1/2))/(143*a*x^3) - (a^4*(a + b*x^2)^(1/2))/(
13*x^13) - (100*a^2*b^2*(a + b*x^2)^(1/2))/(143*x^9)
```

3.436 $\int \frac{(a+bx^2)^{9/2}}{x^{16}} dx$

Optimal. Leaf size=68

$$-\frac{(a+bx^2)^{11/2}}{15ax^{15}}+\frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}}-\frac{8b^2(a+bx^2)^{11/2}}{2145a^3x^{11}}$$

```
[Out] -1/15*(b*x^2+a)^(11/2)/a/x^15+4/195*b*(b*x^2+a)^(11/2)/a^2/x^13-8/2145*b^2*
(b*x^2+a)^(11/2)/a^3/x^11
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$-\frac{8b^2(a+bx^2)^{11/2}}{2145a^3x^{11}}+\frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^16,x]
```

```
[Out] -1/15*(a + b*x^2)^(11/2)/(a*x^15) + (4*b*(a + b*x^2)^(11/2))/(195*a^2*x^13)
 - (8*b^2*(a + b*x^2)^(11/2))/(2145*a^3*x^11)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^{16}} dx &= -\frac{(a+bx^2)^{11/2}}{15ax^{15}}-\frac{(4b)\int \frac{(a+bx^2)^{9/2}}{x^{14}} dx}{15a}\\
&= -\frac{(a+bx^2)^{11/2}}{15ax^{15}}+\frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}}+\frac{(8b^2)\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx}{195a^2}\\
&= -\frac{(a+bx^2)^{11/2}}{15ax^{15}}+\frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}}-\frac{8b^2(a+bx^2)^{11/2}}{2145a^3x^{11}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 42, normalized size = 0.62

$$\frac{(a+bx^2)^{11/2}(-143a^2+44abx^2-8b^2x^4)}{2145a^3x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^16,x]
[Out] ((a + b*x^2)^(11/2)*(-143*a^2 + 44*a*b*x^2 - 8*b^2*x^4))/(2145*a^3*x^15)
```

Maple [A]
time = 0.62, size = 61, normalized size = 0.90

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(8b^2x^4-44ab\,x^2+143a^2)}{2145x^{15}a^3}$	39
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{15a\,x^{15}}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b(b\,x^2+a)^{\frac{11}{2}}}{143a^2x^{11}}\right)}{15a}$	61
trager	$-\frac{(8b^7x^{14}-4a\,b^6x^{12}+3a^2b^5x^{10}+355a^3b^4x^8+1030a^4b^3x^6+1218a^5b^2x^4+671a^6b\,x^2+143a^7)\sqrt{b\,x^2+a}}{2145x^{15}a^3}$	94
risch	$-\frac{(8b^7x^{14}-4a\,b^6x^{12}+3a^2b^5x^{10}+355a^3b^4x^8+1030a^4b^3x^6+1218a^5b^2x^4+671a^6b\,x^2+143a^7)\sqrt{b\,x^2+a}}{2145x^{15}a^3}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^16,x,method=_RETURNVERBOSE)
[Out] -1/15*(b*x^2+a)^(11/2)/a/x^15-4/15*b/a*(-1/13*(b*x^2+a)^(11/2)/a/x^13+2/143
*b*(b*x^2+a)^(11/2)/a^2/x^11)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.82

$$-\frac{8\,(bx^2+a)^{\frac{11}{2}}b^2}{2145\,a^3x^{11}}+\frac{4\,(bx^2+a)^{\frac{11}{2}}b}{195\,a^2x^{13}}-\frac{(bx^2+a)^{\frac{11}{2}}}{15\,ax^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="maxima")
[Out] -8/2145*(b*x^2 + a)^(11/2)*b^2/(a^3*x^11) + 4/195*(b*x^2 + a)^(11/2)*b/(a^2
*x^13) - 1/15*(b*x^2 + a)^(11/2)/(a*x^15)
```

Fricas [A]
time = 1.66, size = 93, normalized size = 1.37

$$-\frac{(8\,b^7x^{14}-4\,ab^6x^{12}+3\,a^2b^5x^{10}+355\,a^3b^4x^8+1030\,a^4b^3x^6+1218\,a^5b^2x^4+671\,a^6bx^2+143\,a^7)\sqrt{bx^2+a}}{2145\,a^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/2145*(8*b^7*x^14 - 4*a*b^6*x^12 + 3*a^2*b^5*x^10 + 355*a^3*b^4*x^8 + 103
0*a^4*b^3*x^6 + 1218*a^5*b^2*x^4 + 671*a^6*b*x^2 + 143*a^7)*sqrt(b*x^2 + a)
/(a^3*x^15)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(61) = 122$.
time = 2.09, size = 604, normalized size = 8.88

$$-\frac{143a^9b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^6\cdot(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12})}-\frac{957a^8b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^4\cdot(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12})}-\frac{2703a^7b^{\frac{13}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^2\cdot(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12})}-\frac{4137a^6b^{\frac{15}{2}}\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{3633a^5b^{\frac{17}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{1743a^4b^{\frac{19}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{357a^3b^{\frac{21}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{3a^2b^{\frac{23}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{12ab^{\frac{25}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{8b^{\frac{27}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**16,x)
```

```
[Out] -143*a**9*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**6*(2145*a**5*b**4*x**8 + 4290*a
**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 957*a**8*b**(11/2)*sqrt(a/(b*x**2
) + 1)/(x**4*(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x
**12)) - 2703*a**7*b**(13/2)*sqrt(a/(b*x**2) + 1)/(x**2*(2145*a**5*b**4*x**
8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 4137*a**6*b**(15/2)*sqr
t(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b
**6*x**12) - 3633*a**5*b**(17/2)*x**2*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*
x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 1743*a**4*b**(19/2)*x
**4*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145
*a**3*b**6*x**12) - 357*a**3*b**(21/2)*x**6*sqrt(a/(b*x**2) + 1)/(2145*a**5
*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 3*a**2*b**(23/2
)*x**8*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2
145*a**3*b**6*x**12) - 12*a*b**(25/2)*x**10*sqrt(a/(b*x**2) + 1)/(2145*a**5
*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 8*b**(27/2)*x**
12*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*
a**3*b**6*x**12)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(56) = 112$.
time = 0.67, size = 354, normalized size = 5.21

$$\frac{16\left(1430\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}b^{\frac{15}{2}}+6435\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}ab^{\frac{15}{2}}+24453\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}a^2b^{\frac{15}{2}}+45045\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^3b^{\frac{15}{2}}+70785\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^4b^{\frac{15}{2}}+64350\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^5b^{\frac{15}{2}}+50050\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^6b^{\frac{15}{2}}+21450\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^7b^{\frac{15}{2}}+7800\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^8b^{\frac{15}{2}}+975\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^9b^{\frac{15}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{10}b^{\frac{15}{2}}-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{11}b^{\frac{15}{2}}+a^{12}b^{\frac{15}{2}}\right)}{2145\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="giac")
```

```
[Out] 16/2145*(1430*(sqrt(b)*x - sqrt(b*x^2 + a))^24*b^(15/2) + 6435*(sqrt(b)*x -
 sqrt(b*x^2 + a))^22*a*b^(15/2) + 24453*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^
2*b^(15/2) + 45045*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^3*b^(15/2) + 70785*(s
```

```
qrt(b)*x - sqrt(b*x^2 + a))^16*a^4*b^(15/2) + 64350*(sqrt(b)*x - sqrt(b*x^2
 + a))^14*a^5*b^(15/2) + 50050*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^6*b^(15/2
) + 21450*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^7*b^(15/2) + 7800*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*a^8*b^(15/2) + 975*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^9*
b^(15/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^10*b^(15/2) - 15*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^11*b^(15/2) + a^12*b^(15/2))/((sqrt(b)*x - sqrt(b*
x^2 + a))^2 - a)^15
```

Mupad [B]
time = 7.42, size = 151, normalized size = 2.22

$$\frac{4\,b^6\,\sqrt{b\,x^2+a}}{2145\,a^2\,x^3}-\frac{71\,b^4\,\sqrt{b\,x^2+a}}{429\,x^7}-\frac{206\,a\,b^3\,\sqrt{b\,x^2+a}}{429\,x^9}-\frac{61\,a^3\,b\,\sqrt{b\,x^2+a}}{195\,x^{13}}-\frac{b^5\,\sqrt{b\,x^2+a}}{715\,a\,x^5}-\frac{a^4\,\sqrt{b\,x^2+a}}{15\,x^{15}}-\frac{8\,b^7\,\sqrt{b\,x^2+a}}{2145\,a^3\,x}-\frac{406\,a^2\,b^2\,\sqrt{b\,x^2+a}}{715\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^16,x)
```

```
[Out] (4*b^6*(a + b*x^2)^(1/2))/(2145*a^2*x^3) - (71*b^4*(a + b*x^2)^(1/2))/(429*
x^7) - (206*a*b^3*(a + b*x^2)^(1/2))/(429*x^9) - (61*a^3*b*(a + b*x^2)^(1/2
))/(195*x^13) - (b^5*(a + b*x^2)^(1/2))/(715*a*x^5) - (a^4*(a + b*x^2)^(1/2
))/(15*x^15) - (8*b^7*(a + b*x^2)^(1/2))/(2145*a^3*x) - (406*a^2*b^2*(a + b
*x^2)^(1/2))/(715*x^11)
```

3.437 $\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx$

Optimal. Leaf size=92

$$-\frac{(a+bx^2)^{11/2}}{17ax^{17}}+\frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}}-\frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}}+\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}$$

```
[Out] -1/17*(b*x^2+a)^(11/2)/a/x^17+2/85*b*(b*x^2+a)^(11/2)/a^2/x^15-8/1105*b^2*(
b*x^2+a)^(11/2)/a^3/x^13+16/12155*b^3*(b*x^2+a)^(11/2)/a^4/x^11
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {277, 270}

$$\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}-\frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}}+\frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^18,x]
```

```
[Out] -1/17*(a + b*x^2)^(11/2)/(a*x^17) + (2*b*(a + b*x^2)^(11/2))/(85*a^2*x^15)
- (8*b^2*(a + b*x^2)^(11/2))/(1105*a^3*x^13) + (16*b^3*(a + b*x^2)^(11/2))/
(12155*a^4*x^11)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx = -\frac{(a+bx^2)^{11/2}}{17ax^{17}} - \frac{(6b)\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx}{17a}$$
$$= -\frac{(a+bx^2)^{11/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} + \frac{(8b^2)\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx}{85a^2}$$
$$= -\frac{(a+bx^2)^{11/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} - \frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}} - \frac{(16b^3)\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx}{1105a^3}$$
$$= -\frac{(a+bx^2)^{11/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} - \frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}} + \frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}$$

Mathematica [A]
time = 0.15, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{11/2}(-715a^3+286a^2bx^2-88ab^2x^4+16b^3x^6)}{12155a^4x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^18,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-715*a^3 + 286*a^2*b*x^2 - 88*a*b^2*x^4 + 16*b^3*x^6))
/(12155*a^4*x^17)
```

Maple [A]
time = 1.54, size = 85, normalized size = 0.92

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-16b^3x^6+88a\,b^2x^4-286a^2b\,x^2+715a^3)}{12155x^{17}a^4}$	50
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{17a\,x^{17}}-\frac{6b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{15a\,x^{15}}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b(b\,x^2+a)^{\frac{11}{2}}}{143a^2x^{11}}\right)}{15a}\right)}{17a}$	85
trager	$-\frac{(-16b^8x^{16}+8a\,b^7x^{14}-6a^2b^6x^{12}+5a^3b^5x^{10}+1515a^4b^4x^8+4714a^5b^3x^6+5808a^6b^2x^4+3289a^7b\,x^2+715a^8)\sqrt{b\,x^2+a}}{12155a^4x^{17}}$	105
risch	$-\frac{(-16b^8x^{16}+8a\,b^7x^{14}-6a^2b^6x^{12}+5a^3b^5x^{10}+1515a^4b^4x^8+4714a^5b^3x^6+5808a^6b^2x^4+3289a^7b\,x^2+715a^8)\sqrt{b\,x^2+a}}{12155a^4x^{17}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -1/17*(b*x^2+a)^(11/2)/a/x^17-6/17*b/a*(-1/15*(b*x^2+a)^(11/2)/a/x^15-4/15*
b/a*(-1/13*(b*x^2+a)^(11/2)/a/x^13+2/143*b*(b*x^2+a)^(11/2)/a^2/x^11))
```

Maxima [A]
time = 0.30, size = 76, normalized size = 0.83

$$\frac{16\,(bx^2+a)^{\frac{11}{2}}\,b^3}{12155\,a^4x^{11}} - \frac{8\,(bx^2+a)^{\frac{11}{2}}\,b^2}{1105\,a^3x^{13}} + \frac{2\,(bx^2+a)^{\frac{11}{2}}\,b}{85\,a^2x^{15}} - \frac{(bx^2+a)^{\frac{11}{2}}}{17\,ax^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="maxima")
```

```
[Out] 16/12155*(b*x^2 + a)^(11/2)*b^3/(a^4*x^11) - 8/1105*(b*x^2 + a)^(11/2)*b^2/
(a^3*x^13) + 2/85*(b*x^2 + a)^(11/2)*b/(a^2*x^15) - 1/17*(b*x^2 + a)^(11/2)
/(a*x^17)
```

Fricas [A]
time = 1.45, size = 104, normalized size = 1.13

$$\frac{(16\,b^8x^{16} - 8\,ab^7x^{14} + 6\,a^2b^6x^{12} - 5\,a^3b^5x^{10} - 1515\,a^4b^4x^8 - 4714\,a^5b^3x^6 - 5808\,a^6b^2x^4 - 3289\,a^7bx^2 - 715\,a^8)\sqrt{bx^2+a}}{12155\,a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="fricas")
```

```
[Out] 1/12155*(16*b^8*x^16 - 8*a*b^7*x^14 + 6*a^2*b^6*x^12 - 5*a^3*b^5*x^10 - 151
5*a^4*b^4*x^8 - 4714*a^5*b^3*x^6 - 5808*a^6*b^2*x^4 - 3289*a^7*b*x^2 - 715*
a^8)*sqrt(b*x^2 + a)/(a^4*x^17)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(85) = 170$.
time = 2.61, size = 867, normalized size = 9.42

$$-\frac{715a^{11}b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} - \frac{5434a^{10}b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} - \frac{17820a^9b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} - \frac{32720a^8b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} - \frac{36370a^7b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} - \frac{24500a^6b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} - \frac{9268a^5b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} - \frac{1520a^4b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} + \frac{5a^3b^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} + \frac{30a^2b^{\frac{37}{2}}x^{18}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} + \frac{40ab^{\frac{39}{2}}x^{20}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}} + \frac{16b^{\frac{41}{2}}x^{22}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**18,x)
```

```
[Out] -715*a**11*b**(19/2)*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a*
*6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 5434*a*
*10*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6
*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 17820*a**
9*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b
**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 32720*a**8*
b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**
10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 36370*a**7*b*
*(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10
```

```
*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 24500*a**6*b**(
29/2)*x**10*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*
x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 9268*a**5*b**(31
/2)*x**12*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x*
*18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 1520*a**4*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**1
8 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 5*a**3*b**(35/2)*x**
16*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 3
6465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 30*a**2*b**(37/2)*x**18*s
qrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465
*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 40*a*b**(39/2)*x**20*sqrt(a/(
b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b
**11*x**20 + 12155*a**4*b**12*x**22) + 16*b**(41/2)*x**22*sqrt(a/(b*x**2) +
 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b**11*x**2
0 + 12155*a**4*b**12*x**22)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(76) = 152.
time = 0.81, size = 382, normalized size = 4.15

$$\frac{32\left(12155\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{26}b^{\frac{17}{2}}+65637\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{24}ab^{\frac{17}{2}}+233376\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{22}a^2b^{\frac{17}{2}}+466752\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{20}a^3b^{\frac{17}{2}}+692835\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{18}a^4b^{\frac{17}{2}}+668525\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{16}a^5b^{\frac{17}{2}}+486200\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{14}a^6b^{\frac{17}{2}}+221000\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}a^7b^{\frac{17}{2}}+71825\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a^8b^{\frac{17}{2}}+9775\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}a^9b^{\frac{17}{2}}+680\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{6}a^{10}b^{\frac{17}{2}}-136\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^{11}b^{\frac{17}{2}}+17\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{2}a^{12}b^{\frac{17}{2}}-a^{13}b^{\frac{17}{2}}\right)}{12155\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="giac")
```

```
[Out] 32/12155*(12155*(sqrt(b)*x - sqrt(b*x^2 + a))^26*b^(17/2) + 65637*(sqrt(b)*
x - sqrt(b*x^2 + a))^24*a*b^(17/2) + 233376*(sqrt(b)*x - sqrt(b*x^2 + a))^2
2*a^2*b^(17/2) + 466752*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^3*b^(17/2) + 692
835*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^4*b^(17/2) + 668525*(sqrt(b)*x - sqr
t(b*x^2 + a))^16*a^5*b^(17/2) + 486200*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^6
*b^(17/2) + 221000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^7*b^(17/2) + 71825*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^8*b^(17/2) + 9775*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^9*b^(17/2) + 680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^10*b^(17/2) -
136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^11*b^(17/2) + 17*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a^12*b^(17/2) - a^13*b^(17/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^17
```

Mupad [**B**]
time = 8.17, size = 171, normalized size = 1.86

$$\frac{6\,b^6\,\sqrt{b\,x^2+a}}{12155\,a^2\,x^5}-\frac{303\,b^4\,\sqrt{b\,x^2+a}}{2431\,x^9}-\frac{4714\,a\,b^3\,\sqrt{b\,x^2+a}}{12155\,x^{11}}-\frac{23\,a^3\,b\,\sqrt{b\,x^2+a}}{85\,x^{15}}-\frac{b^5\,\sqrt{b\,x^2+a}}{2431\,a\,x^7}-\frac{a^4\,\sqrt{b\,x^2+a}}{17\,x^{17}}-\frac{8\,b^7\,\sqrt{b\,x^2+a}}{12155\,a^3\,x^3}+\frac{16\,b^8\,\sqrt{b\,x^2+a}}{12155\,a^4\,x}-\frac{528\,a^2\,b^2\,\sqrt{b\,x^2+a}}{1105\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^18,x)
```

```
[Out] (6*b^6*(a + b*x^2)^(1/2))/(12155*a^2*x^5) - (303*b^4*(a + b*x^2)^(1/2))/(24
31*x^9) - (4714*a*b^3*(a + b*x^2)^(1/2))/(12155*x^11) - (23*a^3*b*(a + b*x^
2)^(1/2))/(85*x^15) - (b^5*(a + b*x^2)^(1/2))/(2431*a*x^7) - (a^4*(a + b*x^
2)^(1/2))/(17*x^17) - (8*b^7*(a + b*x^2)^(1/2))/(12155*a^3*x^3) + (16*b^8*(
a + b*x^2)^(1/2))/(12155*a^4*x) - (528*a^2*b^2*(a + b*x^2)^(1/2))/(1105*x^1
3)
```

3.438 $\int \frac{(a+bx^2)^{9/2}}{x^{20}} dx$

Optimal. Leaf size=116

$$-\frac{(a+bx^2)^{11/2}}{19ax^{19}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}}$$

```
[Out] -1/19*(b*x^2+a)^(11/2)/a/x^19+8/323*b*(b*x^2+a)^(11/2)/a^2/x^17-16/1615*b^2
*(b*x^2+a)^(11/2)/a^3/x^15+64/20995*b^3*(b*x^2+a)^(11/2)/a^4/x^13-128/23094
5*b^4*(b*x^2+a)^(11/2)/a^5/x^11
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^20,x]
```

```
[Out] -1/19*(a + b*x^2)^(11/2)/(a*x^19) + (8*b*(a + b*x^2)^(11/2))/(323*a^2*x^17)
 - (16*b^2*(a + b*x^2)^(11/2))/(1615*a^3*x^15) + (64*b^3*(a + b*x^2)^(11/2)
)/(20995*a^4*x^13) - (128*b^4*(a + b*x^2)^(11/2))/(230945*a^5*x^11)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx = -\frac{(a+bx^2)^{11/2}}{19ax^{19}} - \frac{(8b)\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx}{19a}$$

$$= -\frac{(a+bx^2)^{11/2}}{19ax^{19}} + \frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}} + \frac{(48b^2)\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx}{323a^2}$$

$$= -\frac{(a+bx^2)^{11/2}}{19ax^{19}} + \frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}} - \frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}} - \frac{(64b^3)\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx}{1615a^3}$$

$$= -\frac{(a+bx^2)^{11/2}}{19ax^{19}} + \frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}} - \frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}} + \frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}} + \frac{(128b^4) \ldots}{20 \ldots}$$

$$= -\frac{(a+bx^2)^{11/2}}{19ax^{19}} + \frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}} - \frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}} + \frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}} - \frac{128b^4(a \ldots}{23094 \ldots}$$

Mathematica [A]

time = 0.18, size = 64, normalized size = 0.55

$$\frac{(a+bx^2)^{11/2}\left(-12155a^4+5720a^3bx^2-2288a^2b^2x^4+704ab^3x^6-128b^4x^8\right)}{230945a^5x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^20,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-12155*a^4 + 5720*a^3*b*x^2 - 2288*a^2*b^2*x^4 + 704*a
*b^3*x^6 - 128*b^4*x^8))/(230945*a^5*x^19)
```

Maple [A]

time = 4.02, size = 109, normalized size = 0.94

method	result
gosper	$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(128b^4x^8-704a\,b^3x^6+2288a^2b^2x^4-5720a^3b\,x^2+12155a^4\right)}{230945x^{19}a^5}$
default	$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{19a\,x^{19}}-\frac{8b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{17a\,x^{17}}-\frac{6b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{15a\,x^{15}}-\frac{4b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b\left(b\,x^2+a\right)^{\frac{11}{2}}}{143a^2x^{11}}\right)}{15a}\right)}{17a}\right)}{19a}$
trager	$-\frac{\left(128b^9x^{18}-64a\,b^8x^{16}+48b^7x^{14}a^2-40b^6x^{12}a^3+35a^4b^5x^{10}+23063a^5b^4x^8+75086a^6b^3x^6+95238a^7b^2x^4+55055a^8b\,x^2+12155a^9\right)\sqrt{\ldots}}{230945a^5x^{19}}$

risch	$-\frac{\left(128b^9x^{18}-64a\,b^8x^{16}+48b^7x^{14}a^2-40b^6x^{12}a^3+35a^4b^5x^{10}+23063a^5b^4x^8+75086a^6b^3x^6+95238a^7b^2x^4+55055a^8b\,x^2+12155a^9\right)\sqrt{}}{230945a^5x^{19}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/19*(b*x^2+a)^(11/2)/a/x^19-8/19*b/a*(-1/17*(b*x^2+a)^(11/2)/a/x^17-6/17*
b/a*(-1/15*(b*x^2+a)^(11/2)/a/x^15-4/15*b/a*(-1/13*(b*x^2+a)^(11/2)/a/x^13+
2/143*b*(b*x^2+a)^(11/2)/a^2/x^11)))
```

Maxima [A]
time = 0.29, size = 96, normalized size = 0.83

$$-\frac{128\,(bx^2+a)^{\frac{11}{2}}b^4}{230945\,a^5x^{11}}+\frac{64\,(bx^2+a)^{\frac{11}{2}}b^3}{20995\,a^4x^{13}}-\frac{16\,(bx^2+a)^{\frac{11}{2}}b^2}{1615\,a^3x^{15}}+\frac{8\,(bx^2+a)^{\frac{11}{2}}b}{323\,a^2x^{17}}-\frac{(bx^2+a)^{\frac{11}{2}}}{19\,ax^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="maxima")
```

```
[Out] -128/230945*(b*x^2 + a)^(11/2)*b^4/(a^5*x^11) + 64/20995*(b*x^2 + a)^(11/2)
*b^3/(a^4*x^13) - 16/1615*(b*x^2 + a)^(11/2)*b^2/(a^3*x^15) + 8/323*(b*x^2
+ a)^(11/2)*b/(a^2*x^17) - 1/19*(b*x^2 + a)^(11/2)/(a*x^19)
```

Fricas [A]
time = 1.52, size = 115, normalized size = 0.99

$$-\frac{\left(128\,b^9x^{18}-64\,ab^8x^{16}+48\,a^2b^7x^{14}-40\,a^3b^6x^{12}+35\,a^4b^5x^{10}+23063\,a^5b^4x^8+75086\,a^6b^3x^6+95238\,a^7b^2x^4+55055\,a^8bx^2+12155\,a^9\right)\sqrt{bx^2+a}}{230945\,a^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/230945*(128*b^9*x^18 - 64*a*b^8*x^16 + 48*a^2*b^7*x^14 - 40*a^3*b^6*x^12
 + 35*a^4*b^5*x^10 + 23063*a^5*b^4*x^8 + 75086*a^6*b^3*x^6 + 95238*a^7*b^2*
x^4 + 55055*a^8*b*x^2 + 12155*a^9)*sqrt(b*x^2 + a)/(a^5*x^19)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. $2(109) = 218$.
time = 3.23, size = 1182, normalized size = 10.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**20,x)
```

```
[Out] -12155*a**13*b**(33/2)*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 9237
80*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 +
```

```
230945*a**5*b**20*x**26) - 103675*a**12*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)
/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x*
*22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 388388*a**11*b**
(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**
17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5
*b**20*x**26) - 834988*a**10*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(230945*a*
*9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 92378
0*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 1127210*a**9*b**(41/2)*x**8
*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 +
1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**2
6) - 978810*a**8*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x*
*18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**1
9*x**24 + 230945*a**5*b**20*x**26) - 534060*a**7*b**(45/2)*x**12*sqrt(a/(b*
x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**
7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 167436
*a**6*b**(47/2)*x**14*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 92378
0*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 2
30945*a**5*b**20*x**26) - 23091*a**5*b**(49/2)*x**16*sqrt(a/(b*x**2) + 1)/(
230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**2
2 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 35*a**4*b**(51/2)*
x**18*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**
20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20
*x**26) - 280*a**3*b**(53/2)*x**20*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*
x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b*
*19*x**24 + 230945*a**5*b**20*x**26) - 560*a**2*b**(55/2)*x**22*sqrt(a/(b*x
**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7
*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 448*a*b
**(57/2)*x**24*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*
b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a
**5*b**20*x**26) - 128*b**(59/2)*x**26*sqrt(a/(b*x**2) + 1)/(230945*a**9*b*
*16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**
6*b**19*x**24 + 230945*a**5*b**20*x**26)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(96) = 192.
time = 0.69, size = 408, normalized size = 3.52

$$\frac{256\left(92378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{28}b^{\frac{19}{2}}+554268\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{26}ab^{\frac{19}{2}}+1939938\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}a^2b^{\frac{19}{2}}+4018443\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}a^3b^{\frac{19}{2}}+5866003\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}a^4b^{\frac{19}{2}}+5773625\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^5b^{\frac{19}{2}}+4094025\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^6b^{\frac{19}{2}}+1889550\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^7b^{\frac{19}{2}}+581400\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^8b^{\frac{19}{2}}+80750\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^9b^{\frac{19}{2}}+3876\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^{10}b^{\frac{19}{2}}-969\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^{11}b^{\frac{19}{2}}+171\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{12}b^{\frac{19}{2}}-19\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{13}b^{\frac{19}{2}}+a^{14}b^{\frac{19}{2}}\right)}{230945\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="giac")
```

```
[Out] 256/230945*(92378*(sqrt(b)*x - sqrt(b*x^2 + a))^28*b^(19/2) + 554268*(sqrt(
b)*x - sqrt(b*x^2 + a))^26*a*b^(19/2) + 1939938*(sqrt(b)*x - sqrt(b*x^2 + a
))^24*a^2*b^(19/2) + 4018443*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^3*b^(19/2)
```

```
+ 5866003*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^4*b^(19/2) + 5773625*(sqrt(b)*
x - sqrt(b*x^2 + a))^18*a^5*b^(19/2) + 4094025*(sqrt(b)*x - sqrt(b*x^2 + a)
)^16*a^6*b^(19/2) + 1889550*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^7*b^(19/2) +
 581400*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^8*b^(19/2) + 80750*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^9*b^(19/2) + 3876*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^1
0*b^(19/2) - 969*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^11*b^(19/2) + 171*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*a^12*b^(19/2) - 19*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^13*b^(19/2) + a^14*b^(19/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^19
```

Mupad [B]
time = 8.84, size = 191, normalized size = 1.65

$$\frac{8\,b^6\,\sqrt{b\,x^2+a}}{46189\,a^2\,x^7}-\frac{23063\,b^4\,\sqrt{b\,x^2+a}}{230945\,x^{11}}-\frac{6826\,a\,b^3\,\sqrt{b\,x^2+a}}{20995\,x^{13}}-\frac{77\,a^3\,b\,\sqrt{b\,x^2+a}}{323\,x^{17}}-\frac{7\,b^5\,\sqrt{b\,x^2+a}}{46189\,a\,x^9}-\frac{a^4\,\sqrt{b\,x^2+a}}{19\,x^{19}}-\frac{48\,b^7\,\sqrt{b\,x^2+a}}{230945\,a^3\,x^5}+\frac{64\,b^8\,\sqrt{b\,x^2+a}}{230945\,a^4\,x^3}-\frac{128\,b^9\,\sqrt{b\,x^2+a}}{230945\,a^5\,x}-\frac{666\,a^2\,b^2\,\sqrt{b\,x^2+a}}{1615\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^20,x)
```

```
[Out] (8*b^6*(a + b*x^2)^(1/2))/(46189*a^2*x^7) - (23063*b^4*(a + b*x^2)^(1/2))/(
230945*x^11) - (6826*a*b^3*(a + b*x^2)^(1/2))/(20995*x^13) - (77*a^3*b*(a +
 b*x^2)^(1/2))/(323*x^17) - (7*b^5*(a + b*x^2)^(1/2))/(46189*a*x^9) - (a^4*
(a + b*x^2)^(1/2))/(19*x^19) - (48*b^7*(a + b*x^2)^(1/2))/(230945*a^3*x^5)
+ (64*b^8*(a + b*x^2)^(1/2))/(230945*a^4*x^3) - (128*b^9*(a + b*x^2)^(1/2))
/(230945*a^5*x) - (666*a^2*b^2*(a + b*x^2)^(1/2))/(1615*x^15)
```

3.439 $\int \frac{(a+bx^2)^{9/2}}{x^{22}} dx$

Optimal. Leaf size=140

$$-\frac{(a+bx^2)^{11/2}}{21ax^{21}}+\frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}}-\frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}}+\frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}}-\frac{128b^4(a+bx^2)^{11/2}}{88179a^5x^{13}}+\frac{256b^5(a+bx^2)^{11/2}}{969969a^6x^{11}}$$

```
[Out] -1/21*(b*x^2+a)^(11/2)/a/x^21+10/399*b*(b*x^2+a)^(11/2)/a^2/x^19-80/6783*b^
2*(b*x^2+a)^(11/2)/a^3/x^17+32/6783*b^3*(b*x^2+a)^(11/2)/a^4/x^15-128/88179
*b^4*(b*x^2+a)^(11/2)/a^5/x^13+256/969969*b^5*(b*x^2+a)^(11/2)/a^6/x^11
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$\frac{256b^5(a+bx^2)^{11/2}}{969969a^6x^{11}}-\frac{128b^4(a+bx^2)^{11/2}}{88179a^5x^{13}}+\frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}}-\frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}}+\frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}}-\frac{(a+bx^2)^{11/2}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^22,x]
```

```
[Out] -1/21*(a + b*x^2)^(11/2)/(a*x^21) + (10*b*(a + b*x^2)^(11/2))/(399*a^2*x^19
) - (80*b^2*(a + b*x^2)^(11/2))/(6783*a^3*x^17) + (32*b^3*(a + b*x^2)^(11/2
))/(6783*a^4*x^15) - (128*b^4*(a + b*x^2)^(11/2))/(88179*a^5*x^13) + (256*b
^5*(a + b*x^2)^(11/2))/(969969*a^6*x^11)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx = -\frac{(a+bx^2)^{11/2}}{21ax^{21}} - \frac{(10b)\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx}{21a}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} + \frac{(80b^2)\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx}{399a^2}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}} - \frac{(160b^3)\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx}{2261a^3}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}} + \frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}} + \frac{(128b^4)\int}{6}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}} + \frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}} - \frac{128b^4(a}{8817}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}} + \frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}} - \frac{128b^4(a}{8817}$$

Mathematica [A]
time = 0.18, size = 75, normalized size = 0.54

$$\frac{(a+bx^2)^{11/2}\left(-46189a^5+24310a^4bx^2-11440a^3b^2x^4+4576a^2b^3x^6-1408ab^4x^8+256b^5x^{10}\right)}{969969a^6x^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^22,x]
[Out] ((a + b*x^2)^(11/2)*(-46189*a^5 + 24310*a^4*b*x^2 - 11440*a^3*b^2*x^4 + 457
6*a^2*b^3*x^6 - 1408*a*b^4*x^8 + 256*b^5*x^10))/(969969*a^6*x^21)
```

Maple [A]
time = 10.29, size = 133, normalized size = 0.95

method	result
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-256b^5x^{10}+1408a\,b^4x^8-4576a^2b^3x^6+11440a^3b^2x^4-24310a^4b\,x^2+46189a^5)}{969969x^{21}a^6}$
trager	$-\frac{(-256b^{10}x^{20}+128a\,b^9x^{18}-96a^2b^8x^{16}+80a^3b^7x^{14}-70a^4b^6x^{12}+63a^5b^5x^{10}+80773a^6b^4x^8+271414a^7b^3x^6+351780a^8b^2x^4+206635a}{969969a^6x^{21}}$
risch	$-\frac{(-256b^{10}x^{20}+128a\,b^9x^{18}-96a^2b^8x^{16}+80a^3b^7x^{14}-70a^4b^6x^{12}+63a^5b^5x^{10}+80773a^6b^4x^8+271414a^7b^3x^6+351780a^8b^2x^4+206635a}{969969a^6x^{21}}$

default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{21a\,x^{21}}-\frac{10b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{19a\,x^{19}}-\frac{8b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{17a\,x^{17}}-\frac{6b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{15a\,x^{15}}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b(b\,x^2+a)^{\frac{11}{2}}}{143a^2x^{11}}\right)}{15a}\right)}{17a}\right)}{19a}\right)}{21a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^22,x,method=_RETURNVERBOSE)
[Out] -1/21*(b*x^2+a)^(11/2)/a/x^21-10/21*b/a*(-1/19*(b*x^2+a)^(11/2)/a/x^19-8/19
*b/a*(-1/17*(b*x^2+a)^(11/2)/a/x^17-6/17*b/a*(-1/15*(b*x^2+a)^(11/2)/a/x^15
-4/15*b/a*(-1/13*(b*x^2+a)^(11/2)/a/x^13+2/143*b*(b*x^2+a)^(11/2)/a^2/x^11)
)))
```

Maxima [A]
time = 0.32, size = 116, normalized size = 0.83

$$\frac{256\,(bx^2+a)^{\frac{11}{2}}b^5}{969969\,a^6x^{11}}-\frac{128\,(bx^2+a)^{\frac{11}{2}}b^4}{88179\,a^5x^{13}}+\frac{32\,(bx^2+a)^{\frac{11}{2}}b^3}{6783\,a^4x^{15}}-\frac{80\,(bx^2+a)^{\frac{11}{2}}b^2}{6783\,a^3x^{17}}+\frac{10\,(bx^2+a)^{\frac{11}{2}}b}{399\,a^2x^{19}}-\frac{(bx^2+a)^{\frac{11}{2}}}{21\,ax^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="maxima")
[Out] 256/969969*(b*x^2 + a)^(11/2)*b^5/(a^6*x^11) - 128/88179*(b*x^2 + a)^(11/2)
*b^4/(a^5*x^13) + 32/6783*(b*x^2 + a)^(11/2)*b^3/(a^4*x^15) - 80/6783*(b*x^
2 + a)^(11/2)*b^2/(a^3*x^17) + 10/399*(b*x^2 + a)^(11/2)*b/(a^2*x^19) - 1/2
1*(b*x^2 + a)^(11/2)/(a*x^21)
```

Fricas [A]
time = 2.03, size = 126, normalized size = 0.90

$$\frac{(256\,b^{10}x^{20}-128\,ab^9x^{18}+96\,a^2b^8x^{16}-80\,a^3b^7x^{14}+70\,a^4b^6x^{12}-63\,a^5b^5x^{10}-80773\,a^6b^4x^8-271414\,a^7b^3x^6-351780\,a^8b^2x^4-206635\,a^9bx^2-46189\,a^{10})\sqrt{bx^2+a}}{969969\,a^6x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="fricas")

[Out] 1/969969*(256*b^10*x^20 - 128*a*b^9*x^18 + 96*a^2*b^8*x^16 - 80*a^3*b^7*x^1
4 + 70*a^4*b^6*x^12 - 63*a^5*b^5*x^10 - 80773*a^6*b^4*x^8 - 271414*a^7*b^3*
x^6 - 351780*a^8*b^2*x^4 - 206635*a^9*b*x^2 - 46189*a^10)*sqrt(b*x^2 + a)/(
a^6*x^21)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(133) = 266$.
time = 3.92, size = 1540, normalized size = 11.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(9/2)/x**22,x)

[Out] -46189*a**15*b**(51/2)*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 484
9845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**2
6 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 437580*a**14*b**(
53/2)*x**2*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b
26*x22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*
a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 1846845*a**13*b**(55/2)*x**4*
sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22
+ 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*
x**28 + 969969*a**6*b**30*x**30) - 4558554*a**12*b**(57/2)*x**6*sqrt(a/(b*x
2) + 1)/(969969*a11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a
9*b27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969
969*a**6*b**30*x**30) - 7252938*a**11*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(
969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x
24 + 9699690*a8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b*
*30*x**30) - 7715232*a**10*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(969969*a**
11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 969
9690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30)
 - 5487650*a**9*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x*
*20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b
28*x26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 2516940*
a**8*b**(65/2)*x**14*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 48498
45*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26
+ 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 675513*a**7*b**(67/
2)*x**16*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**
26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a*
*7*b**29*x**28 + 969969*a**6*b**30*x**30) - 80836*a**6*b**(69/2)*x**18*sqrt
(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 96
99690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**2
8 + 969969*a**6*b**30*x**30) + 63*a**5*b**(71/2)*x**20*sqrt(a/(b*x**2) + 1)
/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27

```
*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*
b**30*x**30) + 630*a**4*b**(73/2)*x**22*sqrt(a/(b*x**2) + 1)/(969969*a**11*
b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 969969
0*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) +
1680*a**3*b**(75/2)*x**24*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 +
4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x
**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 2016*a**2*b**(
77/2)*x**26*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*
b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845
*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 1152*a*b**(79/2)*x**28*sqrt(
a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 969
9690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28
 + 969969*a**6*b**30*x**30) + 256*b**(81/2)*x**30*sqrt(a/(b*x**2) + 1)/(969
969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**2
4 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30
*x**30)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(116) = 232$.

time = 0.67, size = 436, normalized size = 3.11

$$\frac{512 \left(646646 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{30} b^{\frac{21}{2}}+4157010 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{28} a b^{\frac{21}{2}}+14549535 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{26} a^2 b^{\frac{21}{2}}+30715685 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{24} a^3 b^{\frac{21}{2}}+44618574 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{22} a^4 b^{\frac{21}{2}}+44265858 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{20} a^5 b^{\frac{21}{2}}+31009615 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{18} a^6 b^{\frac{21}{2}}+14346045 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{16} a^7 b^{\frac{21}{2}}+4273290 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{14} a^8 b^{\frac{21}{2}}+592382 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{12} a^9 b^{\frac{21}{2}}+20349 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{10} a^{10} b^{\frac{21}{2}}-5985 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{8} a^{11} b^{\frac{21}{2}}+1330 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{6} a^{12} b^{\frac{21}{2}}-210 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{4} a^{13} b^{\frac{21}{2}}+21 \left(\sqrt{b} x-\sqrt{b x^2+a}\right)^{2} a^{14} b^{\frac{21}{2}}-a^{15} b^{\frac{21}{2}}\right)}{969969 \left(\left(\sqrt{b} x-\sqrt{b x^2+a}\right)^2-a\right)^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="giac")
```

```
[Out] 512/969969*(646646*(sqrt(b)*x - sqrt(b*x^2 + a))^30*b^(21/2) + 4157010*(sqr
t(b)*x - sqrt(b*x^2 + a))^28*a*b^(21/2) + 14549535*(sqrt(b)*x - sqrt(b*x^2
+ a))^26*a^2*b^(21/2) + 30715685*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^3*b^(21
/2) + 44618574*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^4*b^(21/2) + 44265858*(sq
rt(b)*x - sqrt(b*x^2 + a))^20*a^5*b^(21/2) + 31009615*(sqrt(b)*x - sqrt(b*x
^2 + a))^18*a^6*b^(21/2) + 14346045*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^7*b^
(21/2) + 4273290*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^8*b^(21/2) + 592382*(sq
rt(b)*x - sqrt(b*x^2 + a))^12*a^9*b^(21/2) + 20349*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^10*b^(21/2) - 5985*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^11*b^(21/2)
 + 1330*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^12*b^(21/2) - 210*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*a^13*b^(21/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^14*b^
(21/2) - a^15*b^(21/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^21
```

Mupad [B]

time = 9.63, size = 211, normalized size = 1.51

$$\frac{10\,b^6\,\sqrt{b\,x^2+a}}{138567\,a^2\,x^9}-\frac{1049\,b^4\,\sqrt{b\,x^2+a}}{12597\,x^{13}}-\frac{1898\,a\,b^3\,\sqrt{b\,x^2+a}}{6783\,x^{15}}-\frac{85\,a^3\,b\,\sqrt{b\,x^2+a}}{399\,x^{19}}-\frac{3\,b^5\,\sqrt{b\,x^2+a}}{46189\,a\,x^{11}}-\frac{a^4\,\sqrt{b\,x^2+a}}{21\,x^{21}}-\frac{80\,b^7\,\sqrt{b\,x^2+a}}{969969\,a^3\,x^7}+\frac{32\,b^8\,\sqrt{b\,x^2+a}}{323323\,a^4\,x^5}-\frac{128\,b^9\,\sqrt{b\,x^2+a}}{969969\,a^5\,x^3}+\frac{256\,b^{10}\,\sqrt{b\,x^2+a}}{969969\,a^6\,x}-\frac{820\,a^2\,b^2\,\sqrt{b\,x^2+a}}{2261\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^22,x)
```

```
[Out] (10*b^6*(a + b*x^2)^(1/2))/(138567*a^2*x^9) - (1049*b^4*(a + b*x^2)^(1/2))/
(12597*x^13) - (1898*a*b^3*(a + b*x^2)^(1/2))/(6783*x^15) - (85*a^3*b*(a +
b*x^2)^(1/2))/(399*x^19) - (3*b^5*(a + b*x^2)^(1/2))/(46189*a*x^11) - (a^4*
(a + b*x^2)^(1/2))/(21*x^21) - (80*b^7*(a + b*x^2)^(1/2))/(969969*a^3*x^7)
+ (32*b^8*(a + b*x^2)^(1/2))/(323323*a^4*x^5) - (128*b^9*(a + b*x^2)^(1/2))
/(969969*a^5*x^3) + (256*b^10*(a + b*x^2)^(1/2))/(969969*a^6*x) - (820*a^2*
b^2*(a + b*x^2)^(1/2))/(2261*x^17)
```

3.440 $\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx$

Optimal. Leaf size=164

$$-\frac{(a+bx^2)^{11/2}}{23ax^{23}}+\frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}}-\frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}}+\frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}}-\frac{128b^4(a+bx^2)^{11/2}}{52003a^5x^{15}}+\frac{512b^5(a+bx}{676039a^6}$$

```
[Out] -1/23*(b*x^2+a)^(11/2)/a/x^23+4/161*b*(b*x^2+a)^(11/2)/a^2/x^21-40/3059*b^2
*(b*x^2+a)^(11/2)/a^3/x^19+320/52003*b^3*(b*x^2+a)^(11/2)/a^4/x^17-128/5200
3*b^4*(b*x^2+a)^(11/2)/a^5/x^15+512/676039*b^5*(b*x^2+a)^(11/2)/a^6/x^13-10
24/7436429*b^6*(b*x^2+a)^(11/2)/a^7/x^11
```

Rubi [A]

time = 0.05, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$-\frac{1024b^6(a+bx^2)^{11/2}}{7436429a^7x^{11}}+\frac{512b^5(a+bx^2)^{11/2}}{676039a^6x^{13}}-\frac{128b^4(a+bx^2)^{11/2}}{52003a^5x^{15}}+\frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}}-\frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}}+\frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}}-\frac{(a+bx^2)^{11/2}}{23ax^{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^24,x]
```

```
[Out] -1/23*(a + b*x^2)^(11/2)/(a*x^23) + (4*b*(a + b*x^2)^(11/2))/(161*a^2*x^21)
 - (40*b^2*(a + b*x^2)^(11/2))/(3059*a^3*x^19) + (320*b^3*(a + b*x^2)^(11/2
))/(52003*a^4*x^17) - (128*b^4*(a + b*x^2)^(11/2))/(52003*a^5*x^15) + (512*
b^5*(a + b*x^2)^(11/2))/(676039*a^6*x^13) - (1024*b^6*(a + b*x^2)^(11/2))/(
7436429*a^7*x^11)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx = -\frac{(a+bx^2)^{11/2}}{23ax^{23}} - \frac{(12b)\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx}{23a}$$

$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} + \frac{(40b^2)\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx}{161a^2}$$

$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} - \frac{(320b^3)\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx}{3059a^3}$$

$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} + \frac{(1920b^4)}{5}$$

$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} - \frac{128b^4(a}{5200}$$

$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} - \frac{128b^4(a}{5200}$$

$$= -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} - \frac{128b^4(a}{5200}$$

Mathematica [A]
time = 0.23, size = 86, normalized size = 0.52

$$\frac{(a+bx^2)^{11/2}(-323323a^6+184756a^5bx^2-97240a^4b^2x^4+45760a^3b^3x^6-18304a^2b^4x^8+5632ab^5x^{10}-1024b^6x^{12})}{7436429a^7x^{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^24,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-323323*a^6 + 184756*a^5*b*x^2 - 97240*a^4*b^2*x^4 + 4
5760*a^3*b^3*x^6 - 18304*a^2*b^4*x^8 + 5632*a*b^5*x^10 - 1024*b^6*x^12))/(7
436429*a^7*x^23)
```

Maple [A]
time = 26.68, size = 157, normalized size = 0.96

method	result
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(1024b^6x^{12}-5632a\,b^5x^{10}+18304a^2b^4x^8-45760a^3x^6b^3+97240a^4b^2x^4-184756a^5b\,x^2+323323a^6)}{7436429x^{23}a^7}$
trager	$-\frac{(1024b^{11}x^{22}-512a\,b^{10}x^{20}+384a^2b^9x^{18}-320a^3b^8x^{16}+280a^4b^7x^{14}-252a^5b^6x^{12}+231a^6b^5x^{10}+530959a^7b^4x^8+1826110a^8b^3x^6+240}{7436429a^7x^{23}}$
risch	$-\frac{(1024b^{11}x^{22}-512a\,b^{10}x^{20}+384a^2b^9x^{18}-320a^3b^8x^{16}+280a^4b^7x^{14}-252a^5b^6x^{12}+231a^6b^5x^{10}+530959a^7b^4x^8+1826110a^8b^3x^6+240}{7436429a^7x^{23}}$

default	$-\frac{(b x^2+a)^{\frac{11}{2}}}{23a x^{23}}-\frac{12b\left(-\frac{(b x^2+a)^{\frac{11}{2}}}{21a x^{21}}-\frac{10b\left(-\frac{(b x^2+a)^{\frac{11}{2}}}{19a x^{19}}-\frac{8b\left(-\frac{(b x^2+a)^{\frac{11}{2}}}{17a x^{17}}-\frac{6b\left(-\frac{(b x^2+a)^{\frac{11}{2}}}{15a x^{15}}-\frac{4b\left(-\frac{(b x^2+a)^{\frac{11}{2}}}{13a x^{13}}+\frac{2b(b x^2+a)^{\frac{11}{2}}}{143a^2x^{11}}\right.}{15a}\right.}{17a}\right.}{19a}\right.}{21a}\right.}{23a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^24,x,method=_RETURNVERBOSE)
```

```
[Out] -1/23*(b*x^2+a)^(11/2)/a/x^23-12/23*b/a*(-1/21*(b*x^2+a)^(11/2)/a/x^21-10/2
1*b/a*(-1/19*(b*x^2+a)^(11/2)/a/x^19-8/19*b/a*(-1/17*(b*x^2+a)^(11/2)/a/x^1
7-6/17*b/a*(-1/15*(b*x^2+a)^(11/2)/a/x^15-4/15*b/a*(-1/13*(b*x^2+a)^(11/2)/
a/x^13+2/143*b*(b*x^2+a)^(11/2)/a^2/x^11)))))
```

Maxima [A]

time = 0.31, size = 136, normalized size = 0.83

$$-\frac{1024\,(bx^2+a)^{\frac{11}{2}}b^6}{7436429\,a^7x^{11}}+\frac{512\,(bx^2+a)^{\frac{11}{2}}b^5}{676039\,a^6x^{13}}-\frac{128\,(bx^2+a)^{\frac{11}{2}}b^4}{52003\,a^5x^{15}}+\frac{320\,(bx^2+a)^{\frac{11}{2}}b^3}{52003\,a^4x^{17}}-\frac{40\,(bx^2+a)^{\frac{11}{2}}b^2}{3059\,a^3x^{19}}+\frac{4\,(bx^2+a)^{\frac{11}{2}}b}{161\,a^2x^{21}}-\frac{(bx^2+a)^{\frac{11}{2}}}{23\,ax^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1024/7436429*(b*x^2 + a)^(11/2)*b^6/(a^7*x^11) + 512/676039*(b*x^2 + a)^(1
1/2)*b^5/(a^6*x^13) - 128/52003*(b*x^2 + a)^(11/2)*b^4/(a^5*x^15) + 320/520
03*(b*x^2 + a)^(11/2)*b^3/(a^4*x^17) - 40/3059*(b*x^2 + a)^(11/2)*b^2/(a^3*
x^19) + 4/161*(b*x^2 + a)^(11/2)*b/(a^2*x^21) - 1/23*(b*x^2 + a)^(11/2)/(a*
x^23)
```

Fricas [A]
time = 1.61, size = 137, normalized size = 0.84

$$-\frac{\left(1024\,b^{11}x^{22}-512\,ab^{10}x^{20}+384\,a^2b^9x^{18}-320\,a^3b^8x^{16}+280\,a^4b^7x^{14}-252\,a^5b^6x^{12}+231\,a^6b^5x^{10}+530959\,a^7b^4x^8+1826110\,a^8b^3x^6+2406690\,a^9b^2x^4+1431859\,a^{10}bx^2+323323\,a^{11}\right)\sqrt{bx^2+a}}{7436429\,a^7x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/7436429*(1024*b^11*x^22 - 512*a*b^10*x^20 + 384*a^2*b^9*x^18 - 320*a^3*b
^8*x^16 + 280*a^4*b^7*x^14 - 252*a^5*b^6*x^12 + 231*a^6*b^5*x^10 + 530959*a
^7*b^4*x^8 + 1826110*a^8*b^3*x^6 + 2406690*a^9*b^2*x^4 + 1431859*a^10*b*x^2
 + 323323*a^11)*sqrt(b*x^2 + a)/(a^7*x^23)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1950 vs. $2(156) = 312$.
time = 4.82, size = 1950, normalized size = 11.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**24,x)
```

```
[Out] -323323*a**17*b**(73/2)*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 4
4618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b
**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 74364
29*a**7*b**42*x**34) - 3371797*a**16*b**(75/2)*x**2*sqrt(a/(b*x**2) + 1)/(7
436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**
38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618
574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 15847689*a**15*b**(77/2)
*x**4*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**3
7*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 11154
6435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**3
```

4) - 44210595*a**14*b**(79/2)*x**6*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**3
6*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728
580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x*
*32 + 7436429*a**7*b**42*x**34) - 81074994*a**13*b**(81/2)*x**8*sqrt(a/(b*x
2) + 1)/(7436429*a13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 1115464
35*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x
30 + 44618574*a8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 102129258*a*
*12*b**(83/2)*x**10*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618
574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39
*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a
7*b42*x**34) - 89502546*a**11*b**(85/2)*x**12*sqrt(a/(b*x**2) + 1)/(743
6429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38
*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 4461857
4*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 53885062*a**10*b**(87/2)*x
14*sqrt(a/(b*x2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37
*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546
435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34
) - 21329935*a**9*b**(89/2)*x**16*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36
*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 1487285
80*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**
32 + 7436429*a**7*b**42*x**34) - 5012953*a**8*b**(91/2)*x**18*sqrt(a/(b*x**
2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435
*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**
30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 531157*a**7*b*
*(93/2)*x**20*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a*
*12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28
 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b*
*42*x**34) - 231*a**6*b**(95/2)*x**22*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b
36*x22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148
728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41
*x**32 + 7436429*a**7*b**42*x**34) - 2772*a**5*b**(97/2)*x**24*sqrt(a/(b*x*
*2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 11154643
5*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x*
*30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 9240*a**4*b**
(99/2)*x**26*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**
12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28
 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**
42*x**34) - 14784*a**3*b**(101/2)*x**28*sqrt(a/(b*x**2) + 1)/(7436429*a**13
*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 1
48728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**
41*x**32 + 7436429*a**7*b**42*x**34) - 12672*a**2*b**(103/2)*x**30*sqrt(a/(
b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 1115
46435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**4
0*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 5632*a*b*
*(105/2)*x**32*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a

```
**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**2
8 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b
**42*x**34) - 1024*b**(107/2)*x**34*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**
36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 14872
8580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x
**32 + 7436429*a**7*b**42*x**34)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(136) = 272$.
time = 0.61, size = 462, normalized size = 2.82

$$\frac{2048\left(4249388\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{32}b^{\frac{23}{2}}+28683369\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{30}ab^{\frac{23}{2}}+100922965\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{28}a^2b^{\frac{23}{2}}+215656441\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{26}a^3b^{\frac{23}{2}}+313006057\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}a^4b^{\frac{23}{2}}+311653979\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}a^5b^{\frac{23}{2}}+216800507\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}a^6b^{\frac{23}{2}}+100105775\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^7b^{\frac{23}{2}}+29173683\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^8b^{\frac{23}{2}}+4004231\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^9b^{\frac{23}{2}}+100947\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^{10}b^{\frac{23}{2}}-33649\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^{11}b^{\frac{23}{2}}+8855\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^{12}b^{\frac{23}{2}}-1771\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^{13}b^{\frac{23}{2}}+253\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{14}b^{\frac{23}{2}}-23\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{15}b^{\frac{23}{2}}+a^{16}b^{\frac{23}{2}}\right)}{7436429\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="giac")
```

```
[Out] 2048/7436429*(4249388*(sqrt(b)*x - sqrt(b*x^2 + a))^32*b^(23/2) + 28683369*
(sqrt(b)*x - sqrt(b*x^2 + a))^30*a*b^(23/2) + 100922965*(sqrt(b)*x - sqrt(b
*x^2 + a))^28*a^2*b^(23/2) + 215656441*(sqrt(b)*x - sqrt(b*x^2 + a))^26*a^3
*b^(23/2) + 313006057*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^4*b^(23/2) + 31165
3979*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^5*b^(23/2) + 216800507*(sqrt(b)*x -
 sqrt(b*x^2 + a))^20*a^6*b^(23/2) + 100105775*(sqrt(b)*x - sqrt(b*x^2 + a))
^18*a^7*b^(23/2) + 29173683*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^8*b^(23/2) +
 4004231*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^9*b^(23/2) + 100947*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*a^10*b^(23/2) - 33649*(sqrt(b)*x - sqrt(b*x^2 + a))^1
0*a^11*b^(23/2) + 8855*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^12*b^(23/2) - 1771
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^13*b^(23/2) + 253*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a^14*b^(23/2) - 23*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^15*b^(23/2)
+ a^16*b^(23/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^23
```

Mupad [**B**]
time = 10.47, size = 231, normalized size = 1.41

$$\frac{36\,b^6\,\sqrt{b\,x^2+a}}{1062347\,a^2\,x^{11}}-\frac{3713\,b^4\,\sqrt{b\,x^2+a}}{52003\,x^{15}}-\frac{12770\,a\,b^3\,\sqrt{b\,x^2+a}}{52003\,x^{17}}-\frac{31\,a^3\,b\,\sqrt{b\,x^2+a}}{161\,x^{21}}-\frac{3\,b^5\,\sqrt{b\,x^2+a}}{96577\,a\,x^{13}}-\frac{a^4\,\sqrt{b\,x^2+a}}{23\,x^{23}}-\frac{40\,b^7\,\sqrt{b\,x^2+a}}{1062347\,a^3\,x^9}+\frac{320\,b^8\,\sqrt{b\,x^2+a}}{7436429\,a^4\,x^7}-\frac{384\,b^9\,\sqrt{b\,x^2+a}}{7436429\,a^5\,x^5}+\frac{512\,b^{10}\,\sqrt{b\,x^2+a}}{7436429\,a^6\,x^3}-\frac{1024\,b^{11}\,\sqrt{b\,x^2+a}}{7436429\,a^7\,x}-\frac{990\,a^2\,b^2\,\sqrt{b\,x^2+a}}{3059\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^24,x)
```

```
[Out] (36*b^6*(a + b*x^2)^(1/2))/(1062347*a^2*x^11) - (3713*b^4*(a + b*x^2)^(1/2)
)/(52003*x^15) - (12770*a*b^3*(a + b*x^2)^(1/2))/(52003*x^17) - (31*a^3*b*(
a + b*x^2)^(1/2))/(161*x^21) - (3*b^5*(a + b*x^2)^(1/2))/(96577*a*x^13) - (
a^4*(a + b*x^2)^(1/2))/(23*x^23) - (40*b^7*(a + b*x^2)^(1/2))/(1062347*a^3*
x^9) + (320*b^8*(a + b*x^2)^(1/2))/(7436429*a^4*x^7) - (384*b^9*(a + b*x^2)
^(1/2))/(7436429*a^5*x^5) + (512*b^10*(a + b*x^2)^(1/2))/(7436429*a^6*x^3)
- (1024*b^11*(a + b*x^2)^(1/2))/(7436429*a^7*x) - (990*a^2*b^2*(a + b*x^2)^
(1/2))/(3059*x^19)
```

3.441 $\int x^5 \sqrt{9+4x^2}\, dx$

Optimal. Leaf size=46

$$\frac{27}{64}\left(9+4x^2\right)^{3/2}-\frac{9}{160}\left(9+4x^2\right)^{5/2}+\frac{1}{448}\left(9+4x^2\right)^{7/2}$$

```
[Out] 27/64*(4*x^2+9)^(3/2)-9/160*(4*x^2+9)^(5/2)+1/448*(4*x^2+9)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{1}{448}\left(4x^2+9\right)^{7/2}-\frac{9}{160}\left(4x^2+9\right)^{5/2}+\frac{27}{64}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[9 + 4*x^2],x]
```

```
[Out] (27*(9 + 4*x^2)^(3/2))/64 - (9*(9 + 4*x^2)^(5/2))/160 + (9 + 4*x^2)^(7/2)/4
48
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{9+4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{9+4x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{9+4x}-\frac{9}{8}(9+4x)^{3/2}+\frac{1}{16}(9+4x)^{5/2}\right)dx,x,x^2\right)\\ &= \frac{27}{64}\left(9+4x^2\right)^{3/2}-\frac{9}{160}\left(9+4x^2\right)^{5/2}+\frac{1}{448}\left(9+4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(9+4x^2\right)^{3/2}\left(27-18x^2+10x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[9 + 4*x^2],x]
[Out] ((9 + 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))/280
```

Maple [A]
time = 0.06, size = 41, normalized size = 0.89

method	result	size
gosper	$\frac{\left(4x^2+9\right)^{\frac{3}{2}}\left(10x^4-18x^2+27\right)}{280}$	24
trager	$\left(\frac{1}{7}x^6+\frac{9}{140}x^4-\frac{27}{140}x^2+\frac{243}{280}\right)\sqrt{4x^2+9}$	28
risch	$\frac{\left(40x^6+18x^4-54x^2+243\right)\sqrt{4x^2+9}}{280}$	29
meijerg	$-\frac{2187\left(\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\left(1+\frac{4x^2}{9}\right)^{\frac{3}{2}}\left(\frac{80}{27}x^4-\frac{16}{3}x^2+8\right)}{105}\right)}{256\sqrt{\pi}}$	38
default	$\frac{x^4\left(4x^2+9\right)^{\frac{3}{2}}}{28}-\frac{9x^2\left(4x^2+9\right)^{\frac{3}{2}}}{140}+\frac{27\left(4x^2+9\right)^{\frac{3}{2}}}{280}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/28*x^4*(4*x^2+9)^(3/2)-9/140*x^2*(4*x^2+9)^(3/2)+27/280*(4*x^2+9)^(3/2)
```

Maxima [A]
time = 0.49, size = 40, normalized size = 0.87

$$\frac{1}{28}\left(4\,x^2+9\right)^{\frac{3}{2}}x^4-\frac{9}{140}\left(4\,x^2+9\right)^{\frac{3}{2}}x^2+\frac{27}{280}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/28*(4*x^2 + 9)^(3/2)*x^4 - 9/140*(4*x^2 + 9)^(3/2)*x^2 + 27/280*(4*x^2 +
9)^(3/2)
```

Fricas [A]
time = 1.18, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6+18\,x^4-54\,x^2+243\right)\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(4*x^2 + 9)
```

Sympy [A]
time = 0.33, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{4x^2+9}}{7}+\frac{9x^4\sqrt{4x^2+9}}{140}-\frac{27x^2\sqrt{4x^2+9}}{140}+\frac{243\sqrt{4x^2+9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(4*x**2+9)**(1/2),x)
```

```
[Out] x**6*sqrt(4*x**2 + 9)/7 + 9*x**4*sqrt(4*x**2 + 9)/140 - 27*x**2*sqrt(4*x**2
 + 9)/140 + 243*sqrt(4*x**2 + 9)/280
```

Giac [A]
time = 0.61, size = 34, normalized size = 0.74

$$\frac{1}{448}\left(4\,x^2+9\right)^{\frac{7}{2}}-\frac{9}{160}\left(4\,x^2+9\right)^{\frac{5}{2}}+\frac{27}{64}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 + 9)^(7/2) - 9/160*(4*x^2 + 9)^(5/2) + 27/64*(4*x^2 + 9)^(3/2)
```

Mupad [B]
time = 4.55, size = 25, normalized size = 0.54

$$\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2\,x^6}{7}+\frac{9\,x^4}{70}-\frac{27\,x^2}{70}+\frac{243}{140}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*((9*x^4)/70 - (27*x^2)/70 + (2*x^6)/7 + 243/140)
```

3.442 $\int x^4\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 729/512*arcsinh(2/3*x)-81/256*x*(4*x^2+9)^(1/2)+3/32*x^3*(4*x^2+9)^(1/2)+1/
6*x^5*(4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {285, 327, 221}

$$-\frac{81}{256}\sqrt{4x^2+9}\,x+\frac{1}{6}\sqrt{4x^2+9}\,x^5+\frac{3}{32}\sqrt{4x^2+9}\,x^3+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[9 + 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[9 + 4*x^2])/256 + (3*x^3*Sqrt[9 + 4*x^2])/32 + (x^5*Sqrt[9 + 4*
x^2])/6 + (729*ArcSinh[(2*x)/3])/512
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\sqrt{9+4x^2}\,dx = \frac{1}{6}x^5\sqrt{9+4x^2}+\frac{3}{2}\int\frac{x^4}{\sqrt{9+4x^2}}\,dx$$
$$=\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}-\frac{81}{32}\int\frac{x^2}{\sqrt{9+4x^2}}\,dx$$
$$=-\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{256}\int\frac{1}{\sqrt{9+4x^2}}\,dx$$
$$=-\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.78

$$\frac{1}{768}x\sqrt{9+4x^2}\left(-243+72x^2+128x^4\right)-\frac{729}{512}\log\left(-2x+\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 - (729*Log[-2*x + Sqrt[9
+ 4*x^2]])/512
```

Maple [A]
time = 0.11, size = 46, normalized size = 0.73

method	result	size
risch	$\frac{x(128x^4+72x^2-243)\sqrt{4x^2+9}}{768}+\frac{729\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{512}$	32
trager	$\frac{x(128x^4+72x^2-243)\sqrt{4x^2+9}}{768}+\frac{729\ln\left(2x+\sqrt{4x^2+9}\right)}{512}$	42
meijerg	$-\frac{729\left(\frac{\sqrt{\pi}\,x\left(-\frac{640}{81}x^4-\frac{40}{9}x^2+15\right)\sqrt{1+\frac{4x^2}{9}}}{90}-\frac{\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{4}\right)}{128\sqrt{\pi}}$	43
default	$\frac{x^3(4x^2+9)^{\frac{3}{2}}}{24}-\frac{9x(4x^2+9)^{\frac{3}{2}}}{128}+\frac{729\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{512}+\frac{81x\sqrt{4x^2+9}}{256}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*x^3*(4*x^2+9)^(3/2)-9/128*x*(4*x^2+9)^(3/2)+729/512*arcsinh(2/3*x)+81/
256*x*(4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.48, size = 45, normalized size = 0.71

$$\frac{1}{24}\left(4\,x^{2}+9\right)^{\frac{3}{2}}x^{3}-\frac{9}{128}\left(4\,x^{2}+9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{4\,x^{2}+9}\,x+\frac{729}{512}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^2 + 9)^(3/2)*x^3 - 9/128*(4*x^2 + 9)^(3/2)*x + 81/256*sqrt(4*x^2
+ 9)*x + 729/512*arcsinh(2/3*x)
```

Fricas [A]
time = 1.09, size = 42, normalized size = 0.67

$$\frac{1}{768}\left(128\,x^{5}+72\,x^{3}-243\,x\right)\sqrt{4\,x^{2}+9}-\frac{729}{512}\,\log\left(-2\,x+\sqrt{4\,x^{2}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(4*x^2 + 9) - 729/512*log(-2*x + sqrt(
4*x^2 + 9))
```

Sympy [A]
time = 4.53, size = 75, normalized size = 1.19

$$\frac{2x^{7}}{3\sqrt{4x^{2}+9}}+\frac{15x^{5}}{8\sqrt{4x^{2}+9}}-\frac{27x^{3}}{64\sqrt{4x^{2}+9}}-\frac{729x}{256\sqrt{4x^{2}+9}}+\frac{729\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**2+9)**(1/2),x)
```

```
[Out] 2*x**7/(3*sqrt(4*x**2 + 9)) + 15*x**5/(8*sqrt(4*x**2 + 9)) - 27*x**3/(64*sq
rt(4*x**2 + 9)) - 729*x/(256*sqrt(4*x**2 + 9)) + 729*asinh(2*x/3)/512
```

Giac [A]
time = 0.61, size = 43, normalized size = 0.68

$$\frac{1}{768}\left(8\left(16\,x^{2}+9\right)x^{2}-243\right)\sqrt{4\,x^{2}+9}\,x-\frac{729}{512}\,\log\left(-2\,x+\sqrt{4\,x^{2}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 + 9)*x^2 - 243)*sqrt(4*x^2 + 9)*x - 729/512*log(-2*x + sqr
t(4*x^2 + 9))
```

Mupad [B]
time = 0.03, size = 30, normalized size = 0.48

$$\frac{729\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{512}+\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2\,x^5}{3}+\frac{3\,x^3}{8}-\frac{81\,x}{64}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2 + 9)^(1/2),x)
```

```
[Out] (729*asinh((2*x)/3))/512 + ((x^2 + 9/4)^(1/2)*((3*x^3)/8 - (81*x)/64 + (2*x
^5)/3))/2
```

3.443 $\int x^3\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=31

$$-\frac{3}{16}\left(9+4x^2\right)^{3/2}+\frac{1}{80}\left(9+4x^2\right)^{5/2}$$

```
[Out] -3/16*(4*x^2+9)^(3/2)+1/80*(4*x^2+9)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{1}{80}\left(4x^2+9\right)^{5/2}-\frac{3}{16}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[9 + 4*x^2],x]
```

```
[Out] (-3*(9 + 4*x^2)^(3/2))/16 + (9 + 4*x^2)^(5/2)/80
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{9+4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{9+4x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{9}{4}\sqrt{9+4x}+\frac{1}{4}(9+4x)^{3/2}\right)dx,x,x^2\right)\\ &= -\frac{3}{16}\left(9+4x^2\right)^{3/2}+\frac{1}{80}\left(9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(-3+2x^2\right)\left(9+4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[9 + 4*x^2],x]
[Out] ((-3 + 2*x^2)*(9 + 4*x^2)^(3/2))/40
```

Maple [A]
time = 0.04, size = 27, normalized size = 0.87

method	result	size
gosper	$\frac{(4x^2+9)^{\frac{3}{2}}(2x^2-3)}{40}$	19
trager	$\left(\frac{1}{5}x^4+\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{4x^2+9}$	23
risch	$\frac{(8x^4+6x^2-27)\sqrt{4x^2+9}}{40}$	24
default	$\frac{x^2(4x^2+9)^{\frac{3}{2}}}{20}-\frac{3(4x^2+9)^{\frac{3}{2}}}{40}$	27
meijerg	$-\frac{243\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(1+\frac{4x^2}{9}\right)^{\frac{3}{2}}\left(-\frac{4x^2}{3}+2\right)}{15}\right)}{64\sqrt{\pi}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/20*x^2*(4*x^2+9)^(3/2)-3/40*(4*x^2+9)^(3/2)
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.84

$$\frac{1}{20}\left(4x^2+9\right)^{\frac{3}{2}}x^2-\frac{3}{40}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/20*(4*x^2 + 9)^(3/2)*x^2 - 3/40*(4*x^2 + 9)^(3/2)
```

Fricas [A]
time = 1.11, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8x^4+6x^2-27\right)\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 + 6*x^2 - 27)*sqrt(4*x^2 + 9)
```

Sympy [A]
time = 0.15, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4x^2+9}}{5}+\frac{3x^2\sqrt{4x^2+9}}{20}-\frac{27\sqrt{4x^2+9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 + 9)/5 + 3*x**2*sqrt(4*x**2 + 9)/20 - 27*sqrt(4*x**2 + 9)/
40
```

Giac [A]
time = 0.65, size = 23, normalized size = 0.74

$$\frac{1}{80}\left(4\,x^2+9\right)^{\frac{5}{2}}-\frac{3}{16}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 + 9)^(5/2) - 3/16*(4*x^2 + 9)^(3/2)
```

Mupad [B]
time = 0.02, size = 20, normalized size = 0.65

$$\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2\,x^4}{5}+\frac{3\,x^2}{10}-\frac{27}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*((3*x^2)/10 + (2*x^4)/5 - 27/20)
```

3.444 $\int x^2 \sqrt{9+4x^2}\,dx$

Optimal. Leaf size=45

$$\frac{9}{32}x\sqrt{9+4x^2}+\frac{1}{4}x^3\sqrt{9+4x^2}-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] -81/64*arcsinh(2/3*x)+9/32*x*(4*x^2+9)^(1/2)+1/4*x^3*(4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {285, 327, 221}

$$\frac{9}{32}\sqrt{4x^2+9}\,x+\frac{1}{4}\sqrt{4x^2+9}\,x^3-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9*x*Sqrt[9 + 4*x^2])/32 + (x^3*Sqrt[9 + 4*x^2])/4 - (81*ArcSinh[(2*x)/3])/
64
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\sqrt{9+4x^2}\,dx = \frac{1}{4}x^3\sqrt{9+4x^2} + \frac{9}{4}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx$$
$$= \frac{9}{32}x\sqrt{9+4x^2} + \frac{1}{4}x^3\sqrt{9+4x^2} - \frac{81}{32}\int \frac{1}{\sqrt{9+4x^2}}\,dx$$
$$= \frac{9}{32}x\sqrt{9+4x^2} + \frac{1}{4}x^3\sqrt{9+4x^2} - \frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.98

$$\frac{1}{32}x\sqrt{9+4x^2}\,(9+8x^2) + \frac{81}{64}\log\left(-2x+\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 + 4*x^2],x]
[Out] (x*Sqrt[9 + 4*x^2]*(9 + 8*x^2))/32 + (81*Log[-2*x + Sqrt[9 + 4*x^2]])/64
```

Maple [A]
time = 0.08, size = 32, normalized size = 0.71

method	result	size
risch	$\frac{x(8x^2+9)\sqrt{4x^2+9}}{32} - \frac{81\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{64}$	27
default	$\frac{x(4x^2+9)^{\frac{3}{2}}}{16} - \frac{81\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{64} - \frac{9x\sqrt{4x^2+9}}{32}$	32
meijerg	$-\frac{81\left(-\frac{\sqrt{\pi}\,x\left(\frac{8x^2}{3}+3\right)\sqrt{1+\frac{4x^2}{9}}}{9}+\frac{\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{2}\right)}{32\sqrt{\pi}}$	38
trager	$\frac{x(8x^2+9)\sqrt{4x^2+9}}{32} + \frac{81\ln\left(2x-\sqrt{4x^2+9}\right)}{64}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/16*x*(4*x^2+9)^(3/2)-81/64*arcsinh(2/3*x)-9/32*x*(4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.61, size = 31, normalized size = 0.69

$$\frac{1}{16}\left(4\,x^2+9\right)^{\frac{3}{2}}x - \frac{9}{32}\sqrt{4\,x^2+9}\,x - \frac{81}{64}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/16*(4*x^2 + 9)^(3/2)*x - 9/32*sqrt(4*x^2 + 9)*x - 81/64*arcsinh(2/3*x)
```

Fricas [A]
time = 1.11, size = 37, normalized size = 0.82

$$\frac{1}{32}\left(8\,x^{3}+9\,x\right)\sqrt{4\,x^{2}+9}+\frac{81}{64}\,\log\left(-2\,x+\sqrt{4\,x^{2}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/32*(8*x^3 + 9*x)*sqrt(4*x^2 + 9) + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A]
time = 1.64, size = 54, normalized size = 1.20

$$\frac{x^{5}}{\sqrt{4x^{2}+9}}+\frac{27x^{3}}{8\sqrt{4x^{2}+9}}+\frac{81x}{32\sqrt{4x^{2}+9}}-\frac{81\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**2+9)**(1/2),x)
[Out] x**5/sqrt(4*x**2 + 9) + 27*x**3/(8*sqrt(4*x**2 + 9)) + 81*x/(32*sqrt(4*x**2
 + 9)) - 81*asinh(2*x/3)/64
```

Giac [A]
time = 0.62, size = 36, normalized size = 0.80

$$\frac{1}{32}\left(8\,x^{2}+9\right)\sqrt{4\,x^{2}+9}\,x+\frac{81}{64}\,\log\left(-2\,x+\sqrt{4\,x^{2}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/32*(8*x^2 + 9)*sqrt(4*x^2 + 9)*x + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.51

$$\frac{\left(x^{3}+\frac{9\,x}{8}\right)\,\sqrt{x^{2}+\frac{9}{4}}}{2}-\frac{81\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2 + 9)^(1/2),x)
[Out] (((9*x)/8 + x^3)*(x^2 + 9/4)^(1/2))/2 - (81*asinh((2*x)/3))/64
```

3.445 $\int x\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\left(9+4x^2\right)^{3/2}$$

```
[Out] 1/12*(4*x^2+9)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9 + 4*x^2)^(3/2)/12
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{9+4x^2}\,dx=\frac{1}{12}\left(9+4x^2\right)^{3/2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{12}\left(9+4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9 + 4*x^2)^(3/2)/12
```

Maple [A]
time = 0.04, size = 12, normalized size = 0.80

method	result	size
gosper	$\frac{(4x^2+9)^{\frac{3}{2}}}{12}$	12
derivativedivides	$\frac{(4x^2+9)^{\frac{3}{2}}}{12}$	12
default	$\frac{(4x^2+9)^{\frac{3}{2}}}{12}$	12
risch	$\frac{(4x^2+9)^{\frac{3}{2}}}{12}$	12
trager	$\left(\frac{x^2}{3}+\frac{3}{4}\right)\sqrt{4x^2+9}$	18
meijerg	$-\frac{27\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{8x^2}{9}\right)\sqrt{1+\frac{4x^2}{9}}}{3}\right)}{16\sqrt{\pi}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*(4*x^2+9)^(3/2)
```

Maxima [A]
time = 0.29, size = 11, normalized size = 0.73

$$\frac{1}{12}\,\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

Fricas [A]
time = 1.20, size = 11, normalized size = 0.73

$$\frac{1}{12}\,\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 0.06, size = 27, normalized size = 1.80

$$\frac{x^2\sqrt{4x^2+9}}{3}+\frac{3\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2+9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 + 9)/3 + 3*sqrt(4*x**2 + 9)/4
```

Giac [A]
time = 0.62, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

Mupad [B]
time = 0.02, size = 16, normalized size = 1.07

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{4x^2}{3}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2 + 9)^(1/2),x)
```

```
[Out] ((x^2 + 9/4)^(1/2)*((4*x^2)/3 + 3))/2
```

3.446 $\int \sqrt{9+4x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 9/4*arcsinh(2/3*x)+1/2*x*(4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {201, 221}

$$\frac{1}{2}\sqrt{4x^2+9}\,x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{9+4x^2}\,dx&=\frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{2}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\&=\frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.37

$$\frac{1}{2}x\sqrt{9+4x^2}-\frac{9}{4}\log\left(-2x+\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/2 - (9*Log[-2*x + Sqrt[9 + 4*x^2]])/4
```

Maple [A]
time = 0.07, size = 20, normalized size = 0.74

method	result	size
default	$\frac{9\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{4}+\frac{x\sqrt{4x^{2}+9}}{2}$	20
risch	$\frac{9\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{4}+\frac{x\sqrt{4x^{2}+9}}{2}$	20
meijerg	$-\frac{9\left(-\frac{4\sqrt{\pi}\,x\sqrt{1+\frac{4x^{2}}{9}}}{3}-2\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}}$	31
trager	$\frac{x\sqrt{4x^{2}+9}}{2}-\frac{9\ln\left(2x-\sqrt{4x^{2}+9}\right)}{4}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 9/4*arcsinh(2/3*x)+1/2*x*(4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.51, size = 19, normalized size = 0.70

$$\frac{1}{2}\,\sqrt{4\,x^{2}+9}\,x+\frac{9}{4}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x + 9/4*arcsinh(2/3*x)
```

Fricas [A]
time = 1.25, size = 29, normalized size = 1.07

$$\frac{1}{2}\,\sqrt{4\,x^{2}+9}\,x-\frac{9}{4}\,\log\left(-2\,x+\sqrt{4\,x^{2}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A]
time = 0.07, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2),x)
[Out] x*sqrt(4*x**2 + 9)/2 + 9*asinh(2*x/3)/4
```

Giac [A]
time = 0.54, size = 29, normalized size = 1.07

$$\frac{1}{2}\,\sqrt{4\,x^2+9}\,x-\frac{9}{4}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.59

$$\frac{9\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{4}+x\,\sqrt{x^2+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2),x)
[Out] (9*asinh((2*x)/3))/4 + x*(x^2 + 9/4)^(1/2)
```

3.447 $\int \frac{\sqrt{9+4x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{9+4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

```
[Out] -3*arctanh(1/3*(4*x^2+9)^(1/2))+(4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {272, 52, 65, 213}

$$\sqrt{4x^2+9}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9+4x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{9+4x^2}+\frac{9}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= \sqrt{9+4x^2}+\frac{9}{4}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\ &= \sqrt{9+4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 1.00

$$\sqrt{9+4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x,x]
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.83

method	result	size
default	$\sqrt{4x^2+9}-3\,\text{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)$	25
trager	$\sqrt{4x^2+9}+3\ln\left(\frac{\sqrt{4x^2+9}-3}{x}\right)$	29
meijerg	$-\frac{3\left(4\sqrt{\pi}-4\sqrt{\pi}\sqrt{1+\frac{4x^2}{9}}+4\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)-2(2+2\ln(x)-2\ln(3))\sqrt{\pi}\right)}{4\sqrt{\pi}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (4*x^2+9)^(1/2)-3*arctanh(3/(4*x^2+9)^(1/2))
```

Maxima [A]
time = 0.51, size = 19, normalized size = 0.63

$$\sqrt{4\,x^2+9}-3\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(4*x^2 + 9) - 3*arcsinh(3/2/abs(x))
```

Fricas [A]
time = 1.18, size = 44, normalized size = 1.47

$$\sqrt{4\,x^2+9}-3\,\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)+3\,\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(4*x^2 + 9) - 3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 3*log(-2*x + sqrt(4*x
^2 + 9) - 3)
```

Sympy [A]
time = 0.59, size = 39, normalized size = 1.30

$$\frac{2x}{\sqrt{1+\frac{9}{4x^2}}}-3\operatorname{asinh}\left(\frac{3}{2x}\right)+\frac{9}{2x\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x,x)
```

```
[Out] 2*x/sqrt(1 + 9/(4*x**2)) - 3*asinh(3/(2*x)) + 9/(2*x*sqrt(1 + 9/(4*x**2)))
```

Giac [A]
time = 0.57, size = 38, normalized size = 1.27

$$\sqrt{4\,x^2+9}-\frac{3}{2}\,\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{3}{2}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(4*x^2 + 9) - 3/2*log(sqrt(4*x^2 + 9) + 3) + 3/2*log(sqrt(4*x^2 + 9) -
3)
```

Mupad [B]
time = 0.03, size = 22, normalized size = 0.73

$$2\sqrt{x^2+\frac{9}{4}}-3\,\mathrm{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x,x)
```

```
[Out] 2*(x^2 + 9/4)^(1/2) - 3*atanh((2*(x^2 + 9/4)^(1/2))/3)
```

3.448 $\int \frac{\sqrt{9+4x^2}}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{\sqrt{9+4x^2}}{x}+2\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 2*arcsinh(2/3*x)-(4*x^2+9)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {283, 221}

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 + 4*x^2]/x) + 2*ArcSinh[(2*x)/3]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x^2}\,dx &= -\frac{\sqrt{9+4x^2}}{x}+4\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{\sqrt{9+4x^2}}{x}+2\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 1.40

$$-\frac{\sqrt{9+4x^2}}{x}-2\log\left(-2x+\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 + 4*x^2]/x) - 2*Log[-2*x + Sqrt[9 + 4*x^2]]
```

Maple [A]
time = 0.08, size = 34, normalized size = 1.36

method	result	size
risch	$2\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$	22
trager	$-\frac{\sqrt{4x^2+9}}{x}-2\ln\left(\sqrt{4x^2+9}-2x\right)$	32
meijerg	$-\frac{\frac{6\sqrt{\pi}\,\sqrt{1+\frac{4x^2}{9}}}{x}-4\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{2\sqrt{\pi}}$	33
default	$-\frac{(4x^2+9)^{\frac{3}{2}}}{9x}+2\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)+\frac{4x\sqrt{4x^2+9}}{9}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/x*(4*x^2+9)^(3/2)+2*arcsinh(2/3*x)+4/9*x*(4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.49, size = 21, normalized size = 0.84

$$-\frac{\sqrt{4\,x^2+9}}{x}+2\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(4*x^2 + 9)/x + 2*arcsinh(2/3*x)
```

Fricas [A]
time = 1.50, size = 35, normalized size = 1.40

$$-\frac{2\,x\log\left(-2\,x+\sqrt{4\,x^2+9}\right)+2\,x+\sqrt{4\,x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
[Out] -(2*x*log(-2*x + sqrt(4*x^2 + 9)) + 2*x + sqrt(4*x^2 + 9))/x
```

Sympy [A]
time = 0.08, size = 19, normalized size = 0.76

$$2\operatorname{asinh}\left(\frac{2x}{3}\right) - \frac{\sqrt{4x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**2,x)
[Out] 2*asinh(2*x/3) - sqrt(4*x**2 + 9)/x
```

Giac [A]
time = 0.51, size = 40, normalized size = 1.60

$$\frac{36}{\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9} - 2\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
[Out] 36/((2*x - sqrt(4*x^2 + 9))^2 - 9) - 2*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$2\operatorname{asinh}\left(\frac{2\,x}{3}\right) - \frac{2\,\sqrt{x^2+\frac{9}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^2,x)
[Out] 2*asinh((2*x)/3) - (2*(x^2 + 9/4)^(1/2))/x
```

3.449 $\int \frac{\sqrt{9+4x^2}}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{9+4x^2}}{2x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

```
[Out] -2/3*arctanh(1/3*(4*x^2+9)^(1/2))-1/2*(4*x^2+9)^(1/2)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 43, 65, 213}

$$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^3,x]
```

```
[Out] -1/2*Sqrt[9 + 4*x^2]/x^2 - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9+4x}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{2x^2} + \text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\ &= -\frac{\sqrt{9+4x^2}}{2x^2} - \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.00

$$-\frac{\sqrt{9+4x^2}}{2x^2} - \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^3,x]
[Out] -1/2*Sqrt[9 + 4*x^2]/x^2 - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/3
```

Maple [A]
time = 0.09, size = 41, normalized size = 1.05

method	result	size
risch	$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{3}$	30
trager	$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2\ln\left(\frac{\sqrt{4x^2+9}+3}{x}\right)}{3}$	34
default	$-\frac{(4x^2+9)^{\frac{3}{2}}}{18x^2}+\frac{2\sqrt{4x^2+9}}{9}-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{3}$	41

meijerg	$-\frac{-\frac{9\sqrt{\pi}\,\left(8+\frac{16x^2}{9}\right)}{16x^2}+\frac{9\sqrt{\pi}\,\sqrt{1+\frac{4x^2}{9}}}{2x^2}+2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)-(-1+2\ln(x)-2\ln(3))\sqrt{\pi}+\frac{9\sqrt{\pi}}{2x^2}}{3\sqrt{\pi}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18/x^2*(4*x^2+9)^(3/2)+2/9*(4*x^2+9)^(1/2)-2/3*arctanh(3/(4*x^2+9)^(1/2)
)
```

Maxima [A]
time = 0.52, size = 35, normalized size = 0.90

$$\frac{2}{9}\sqrt{4\,x^2+9}-\frac{(4\,x^2+9)^{\frac{3}{2}}}{18\,x^2}-\frac{2}{3}\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(4*x^2 + 9) - 1/18*(4*x^2 + 9)^(3/2)/x^2 - 2/3*arcsinh(3/2/abs(x))
```

Fricas [A]
time = 1.63, size = 57, normalized size = 1.46

$$-\frac{4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)+3\,\sqrt{4\,x^2+9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2 +
 9) - 3) + 3*sqrt(4*x^2 + 9))/x^2
```

Sympy [A]
time = 0.80, size = 24, normalized size = 0.62

$$-\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}-\frac{\sqrt{1+\frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**3,x)
```

```
[Out] -2*asinh(3/(2*x))/3 - sqrt(1 + 9/(4*x**2))/x
```

Giac [A]
time = 0.51, size = 43, normalized size = 1.10

$$-\frac{\sqrt{4\,x^2+9}}{2\,x^2}-\frac{1}{3}\,\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{1}{3}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(4*x^2 + 9)/x^2 - 1/3*log(sqrt(4*x^2 + 9) + 3) + 1/3*log(sqrt(4*x^
2 + 9) - 3)
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.64

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)}{3}-\frac{\sqrt{x^2+\frac{9}{4}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^3,x)
```

```
[Out] - (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/3 - (x^2 + 9/4)^(1/2)/x^2
```

3.450 $\int \frac{\sqrt{9+4x^2}}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{(9+4x^2)^{3/2}}{27x^3}$$

```
[Out] -1/27*(4*x^2+9)^(3/2)/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{270\}$

$$-\frac{(4x^2+9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^4,x]
```

```
[Out] -1/27*(9 + 4*x^2)^(3/2)/x^3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{9+4x^2}}{x^4}\,dx = -\frac{(9+4x^2)^{3/2}}{27x^3}$$

Mathematica [A]
time = 0.03, size = 18, normalized size = 1.00

$$-\frac{(9+4x^2)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^4,x]
```

```
[Out] -1/27*(9 + 4*x^2)^(3/2)/x^3
```

Maple [A]
time = 0.05, size = 15, normalized size = 0.83

method	result	size
gosper	$-\frac{(4x^2+9)^{\frac{3}{2}}}{27x^3}$	15
default	$-\frac{(4x^2+9)^{\frac{3}{2}}}{27x^3}$	15
trager	$-\frac{(4x^2+9)^{\frac{3}{2}}}{27x^3}$	15
meijerg	$-\frac{\left(1+\frac{4x^2}{9}\right)^{\frac{3}{2}}}{x^3}$	15
risch	$-\frac{16x^4+72x^2+81}{27x^3\sqrt{4x^2+9}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/27*(4*x^2+9)^(3/2)/x^3
```

Maxima [A]
time = 0.61, size = 14, normalized size = 0.78

$$-\frac{(4\,x^2+9)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/27*(4*x^2 + 9)^(3/2)/x^3
```

Fricas [A]
time = 1.22, size = 20, normalized size = 1.11

$$-\frac{8\,x^3+(4\,x^2+9)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/27*(8*x^3 + (4*x^2 + 9)^(3/2))/x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.
time = 0.45, size = 34, normalized size = 1.89

$$-\frac{8\sqrt{1+\frac{9}{4x^2}}}{27}-\frac{2\sqrt{1+\frac{9}{4x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**4,x)
```

```
[Out] -8*sqrt(1 + 9/(4*x**2))/27 - 2*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.
time = 0.58, size = 42, normalized size = 2.33

$$\frac{16\left(\left(2x-\sqrt{4x^2+9}\right)^4+27\right)}{\left(\left(2x-\sqrt{4x^2+9}\right)^2-9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 16*((2*x - sqrt(4*x^2 + 9))^4 + 27)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

Mupad [B]
time = 0.03, size = 27, normalized size = 1.50

$$-\frac{18\sqrt{x^2+\frac{9}{4}}+8x^2\sqrt{x^2+\frac{9}{4}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^4,x)
```

```
[Out] -(18*(x^2 + 9/4)^(1/2) + 8*x^2*(x^2 + 9/4)^(1/2))/(27*x^3)
```

3.451 $\int \frac{\sqrt{9+4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{9+4x^2}}{4x^4}-\frac{\sqrt{9+4x^2}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

```
[Out] 2/27*arctanh(1/3*(4*x^2+9)^(1/2))-1/4*(4*x^2+9)^(1/2)/x^4-1/18*(4*x^2+9)^(1
/2)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 44, 65, 213}

$$-\frac{\sqrt{4x^2+9}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^5,x]
```

```
[Out] -1/4*Sqrt[9 + 4*x^2]/x^4 - Sqrt[9 + 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 + 4
*x^2]/3])/27
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{9+4x^2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9+4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4} - \frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4} - \frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4} - \frac{\sqrt{9+4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 0.81

$$\frac{(-9-2x^2)\sqrt{9+4x^2}}{36x^4} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^5,x]
[Out] ((-9 - 2*x^2)*Sqrt[9 + 4*x^2])/(36*x^4) + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27
```

Maple [A]
time = 0.09, size = 55, normalized size = 0.96

method	result
trager	$-\frac{(2x^2+9)\sqrt{4x^2+9}}{36x^4}-\frac{2\ln\left(\frac{\sqrt{4x^2+9}-3}{x}\right)}{27}$
risch	$-\frac{8x^4+54x^2+81}{36x^4\sqrt{4x^2+9}}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}$
default	$-\frac{(4x^2+9)^{\frac{3}{2}}}{36x^4}+\frac{(4x^2+9)^{\frac{3}{2}}}{162x^2}-\frac{2\sqrt{4x^2+9}}{81}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}$
meijerg	$-\frac{4\left(-\frac{81\sqrt{\pi}\left(\frac{16}{81}x^4+\frac{32}{9}x^2+8\right)}{128x^4}+\frac{81\sqrt{\pi}\left(8+\frac{16x^2}{9}\right)\sqrt{1+\frac{4x^2}{9}}}{128x^4}-\frac{\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)}{2}+\frac{\left(\frac{1}{2}+2\ln(x)-2\ln(3)\right)\sqrt{\pi}}{4}+\cdots\right.}{27\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/36/x^4*(4*x^2+9)^(3/2)+1/162/x^2*(4*x^2+9)^(3/2)-2/81*(4*x^2+9)^(1/2)+2/
27*arctanh(3/(4*x^2+9)^(1/2))
```

Maxima [A]
time = 0.49, size = 49, normalized size = 0.86

$$-\frac{2}{81}\sqrt{4\,x^2+9}+\frac{(4\,x^2+9)^{\frac{3}{2}}}{162\,x^2}-\frac{(4\,x^2+9)^{\frac{3}{2}}}{36\,x^4}+\frac{2}{27}\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(4*x^2 + 9) + 1/162*(4*x^2 + 9)^(3/2)/x^2 - 1/36*(4*x^2 + 9)^(3/2
)/x^4 + 2/27*arcsinh(3/2/abs(x))
```

Fricas [A]
time = 1.45, size = 64, normalized size = 1.12

$$\frac{8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}\,(2\,x^2+9)}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^2
+ 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 + 9))/x^4
```

Sympy [A]
time = 1.86, size = 63, normalized size = 1.11

$$\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{27}-\frac{1}{9x\sqrt{1+\frac{9}{4x^2}}}-\frac{3}{4x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{9}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**5,x)
```

```
[Out] 2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 3/(4*x**3*sqrt(1 + 9/(
4*x**2))) - 9/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Giac [A]
time = 0.60, size = 55, normalized size = 0.96

$$-\frac{(4x^2+9)^{\frac{3}{2}}+9\sqrt{4x^2+9}}{72x^4}+\frac{1}{27}\log\left(\sqrt{4x^2+9}+3\right)-\frac{1}{27}\log\left(\sqrt{4x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((4*x^2 + 9)^(3/2) + 9*sqrt(4*x^2 + 9))/x^4 + 1/27*log(sqrt(4*x^2 + 9
) + 3) - 1/27*log(sqrt(4*x^2 + 9) - 3)
```

Mupad [B]
time = 0.03, size = 45, normalized size = 0.79

$$\frac{2\operatorname{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{27}+\frac{\sqrt{x^2+\frac{9}{4}}\left(\frac{2}{3x^2}-\frac{1}{x^4}\right)}{2}-\frac{4\sqrt{x^2+\frac{9}{4}}}{9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^5,x)
```

```
[Out] (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/27 + ((x^2 + 9/4)^(1/2)*(2/(3*x^2) - 1/x
^4))/2 - (4*(x^2 + 9/4)^(1/2))/(9*x^2)
```

3.452 $\int x^5 \sqrt{9-4x^2}\, dx$

Optimal. Leaf size=46

$$-\frac{27}{64}\left(9-4x^2\right)^{3/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{1}{448}\left(9-4x^2\right)^{7/2}$$

```
[Out] -27/64*(-4*x^2+9)^(3/2)+9/160*(-4*x^2+9)^(5/2)-1/448*(-4*x^2+9)^(7/2)
```

Rubi [A]

time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$-\frac{1}{448}\left(9-4x^2\right)^{7/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{27}{64}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-27*(9 - 4*x^2)^(3/2))/64 + (9*(9 - 4*x^2)^(5/2))/160 - (9 - 4*x^2)^(7/2)/
448
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{9-4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{9-4x}\,x^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16}\sqrt{9-4x}-\frac{9}{8}(9-4x)^{3/2}+\frac{1}{16}(9-4x)^{5/2}\right)dx,x,x^2\right)\\
&= -\frac{27}{64}\left(9-4x^2\right)^{3/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{1}{448}\left(9-4x^2\right)^{7/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.70

$$\frac{1}{280}\sqrt{9-4x^2}\left(-243-54x^2-18x^4+40x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[9 - 4*x^2],x]
```

```
[Out] (Sqrt[9 - 4*x^2]*(-243 - 54*x^2 - 18*x^4 + 40*x^6))/280
```

Maple [A]
time = 0.07, size = 41, normalized size = 0.89

method	result	size
trager	$\left(\frac{1}{7}x^6-\frac{9}{140}x^4-\frac{27}{140}x^2-\frac{243}{280}\right)\sqrt{-4x^2+9}$	28
gosper	$\frac{(2x-3)(2x+3)(10x^4+18x^2+27)\sqrt{-4x^2+9}}{280}$	34
risch	$-\frac{(40x^6-18x^4-54x^2-243)(4x^2-9)}{280\sqrt{-4x^2+9}}$	36
meijerg	$\frac{\frac{729\sqrt{\pi}}{280}-\frac{729\sqrt{\pi}\left(1-\frac{4x^2}{9}\right)^{\frac{3}{2}}\left(\frac{80}{27}x^4+\frac{16}{3}x^2+8\right)}{2240}}{\sqrt{\pi}}$	38
default	$-\frac{x^4(-4x^2+9)^{\frac{3}{2}}}{28}-\frac{9x^2(-4x^2+9)^{\frac{3}{2}}}{140}-\frac{27(-4x^2+9)^{\frac{3}{2}}}{280}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/28*x^4*(-4*x^2+9)^(3/2)-9/140*x^2*(-4*x^2+9)^(3/2)-27/280*(-4*x^2+9)^(3/
2)
```

Maxima [A]
time = 0.50, size = 40, normalized size = 0.87

$$-\frac{1}{28}\left(-4x^2+9\right)^{\frac{3}{2}}x^4-\frac{9}{140}\left(-4x^2+9\right)^{\frac{3}{2}}x^2-\frac{27}{280}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/28*(-4*x^2 + 9)^(3/2)*x^4 - 9/140*(-4*x^2 + 9)^(3/2)*x^2 - 27/280*(-4*x^
2 + 9)^(3/2)
```

Fricas [A]
time = 1.03, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40x^6-18x^4-54x^2-243\right)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(-4*x^2 + 9)
```

Sympy [A]
time = 0.33, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{9-4x^2}}{7}-\frac{9x^4\sqrt{9-4x^2}}{140}-\frac{27x^2\sqrt{9-4x^2}}{140}-\frac{243\sqrt{9-4x^2}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**6*sqrt(9 - 4*x**2)/7 - 9*x**4*sqrt(9 - 4*x**2)/140 - 27*x**2*sqrt(9 - 4*
x**2)/140 - 243*sqrt(9 - 4*x**2)/280
```

Giac [A]
time = 0.57, size = 52, normalized size = 1.13

$$\frac{1}{448}\left(4x^2-9\right)^3\sqrt{-4x^2+9}+\frac{9}{160}\left(4x^2-9\right)^2\sqrt{-4x^2+9}-\frac{27}{64}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 - 9)^3*sqrt(-4*x^2 + 9) + 9/160*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9)
 - 27/64*(-4*x^2 + 9)^(3/2)
```

Mupad [B]
time = 4.53, size = 28, normalized size = 0.61

$$-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(-\frac{4x^6}{7}+\frac{9x^4}{35}+\frac{27x^2}{35}+\frac{243}{70}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*((27*x^2)/35 + (9*x^4)/35 - (4*x^6)/7 + 243/70))/2
```

3.453 $\int x^4\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 729/512*arcsin(2/3*x)-81/256*x*(-4*x^2+9)^(1/2)-3/32*x^3*(-4*x^2+9)^(1/2)+1
/6*x^5*(-4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {285, 327, 222}

$$\frac{729}{512}\text{ArcSin}\left(\frac{2x}{3}\right)-\frac{81}{256}\sqrt{9-4x^2}\,x+\frac{1}{6}\sqrt{9-4x^2}\,x^5-\frac{3}{32}\sqrt{9-4x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[9 - 4*x^2])/256 - (3*x^3*Sqrt[9 - 4*x^2])/32 + (x^5*Sqrt[9 - 4*
x^2])/6 + (729*ArcSin[(2*x)/3])/512
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\sqrt{9-4x^2}\,dx = \frac{1}{6}x^5\sqrt{9-4x^2}+\frac{3}{2}\int \frac{x^4}{\sqrt{9-4x^2}}\,dx$$
$$= -\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{81}{32}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$$
$$= -\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{256}\int \frac{1}{\sqrt{9-4x^2}}\,dx$$
$$= -\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A]
time = 0.11, size = 52, normalized size = 0.83

$$\frac{1}{768}x\sqrt{9-4x^2}\left(-243-72x^2+128x^4\right)+\frac{729}{256}\tan^{-1}\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 + (729*ArcTan[(2*x)/(-3 +
 Sqrt[9 - 4*x^2])])/256
```

Maple [A]
time = 0.16, size = 46, normalized size = 0.73

method	result	size
risch	$-\frac{x\left(128x^4-72x^2-243\right)\left(4x^2-9\right)}{768\sqrt{-4x^2+9}}+\frac{729\arcsin\left(\frac{2x}{3}\right)}{512}$	39
default	$-\frac{x^3\left(-4x^2+9\right)^{\frac{3}{2}}}{24}-\frac{9x\left(-4x^2+9\right)^{\frac{3}{2}}}{128}+\frac{729\arcsin\left(\frac{2x}{3}\right)}{512}+\frac{81x\sqrt{-4x^2+9}}{256}$	46
meijerg	$\frac{729i\left(\frac{i\sqrt{\pi}\,x\left(-\frac{640}{81}x^4+\frac{40}{9}x^2+15\right)\sqrt{1-\frac{4x^2}{9}}}{90}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{4}\right)}{128\sqrt{\pi}}$	46
trager	$\frac{x\left(128x^4-72x^2-243\right)\sqrt{-4x^2+9}}{768}-\frac{729\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{512}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*x^3*(-4*x^2+9)^(3/2)-9/128*x*(-4*x^2+9)^(3/2)+729/512*arcsin(2/3*x)+8
1/256*x*(-4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.51, size = 45, normalized size = 0.71

$$-\frac{1}{24}\left(-4\,x^2+9\right)^{\frac{3}{2}}x^3-\frac{9}{128}\left(-4\,x^2+9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{-4\,x^2+9}\,x+\frac{729}{512}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(-4*x^2 + 9)^(3/2)*x^3 - 9/128*(-4*x^2 + 9)^(3/2)*x + 81/256*sqrt(-4*
x^2 + 9)*x + 729/512*arcsin(2/3*x)
```

Fricas [A]
time = 1.31, size = 45, normalized size = 0.71

$$\frac{1}{768}\left(128\,x^5-72\,x^3-243\,x\right)\sqrt{-4\,x^2+9}-\frac{729}{256}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(-4*x^2 + 9) - 729/256*arctan(1/2*(sqr
t(-4*x^2 + 9) - 3)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.55, size = 165, normalized size = 2.62

$$\begin{cases} \frac{2ix^7}{3\sqrt{4x^2-9}}-\frac{15ix^5}{8\sqrt{4x^2-9}}-\frac{27ix^3}{64\sqrt{4x^2-9}}+\frac{729ix}{256\sqrt{4x^2-9}}-\frac{729i\,\operatorname{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for}\ |x^2|>\frac{9}{4} \\ -\frac{2x^7}{3\sqrt{9-4x^2}}+\frac{15x^5}{8\sqrt{9-4x^2}}+\frac{27x^3}{64\sqrt{9-4x^2}}-\frac{729x}{256\sqrt{9-4x^2}}+\frac{729\,\operatorname{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((2*I*x**7/(3*sqrt(4*x**2 - 9)) - 15*I*x**5/(8*sqrt(4*x**2 - 9)) -
 27*I*x**3/(64*sqrt(4*x**2 - 9)) + 729*I*x/(256*sqrt(4*x**2 - 9)) - 729*I*a
cosh(2*x/3)/512, Abs(x**2) > 9/4), (-2*x**7/(3*sqrt(9 - 4*x**2)) + 15*x**5/
(8*sqrt(9 - 4*x**2)) + 27*x**3/(64*sqrt(9 - 4*x**2)) - 729*x/(256*sqrt(9 -
4*x**2)) + 729*asin(2*x/3)/512, True))
```

Giac [A]
time = 0.53, size = 33, normalized size = 0.52

$$\frac{1}{768}\left(8\left(16\,x^2-9\right)x^2-243\right)\sqrt{-4\,x^2+9}\,x+\frac{729}{512}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(-4*x^2 + 9)*x + 729/512*arcsin(2/3*x)
```

Mupad [B]
time = 4.56, size = 32, normalized size = 0.51

$$\frac{729\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{512}-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(-\frac{2\,x^5}{3}+\frac{3\,x^3}{8}+\frac{81\,x}{64}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(9 - 4*x^2)^(1/2),x)
```

```
[Out] (729*asin((2*x)/3))/512 - ((9/4 - x^2)^(1/2)*((81*x)/64 + (3*x^3)/8 - (2*x^
5)/3))/2
```

3.454 $\int x^3\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=31

$$-\frac{3}{16}\left(9-4x^2\right)^{3/2}+\frac{1}{80}\left(9-4x^2\right)^{5/2}$$

[Out] -3/16*(-4*x^2+9)^(3/2)+1/80*(-4*x^2+9)^(5/2)

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{1}{80}\left(9-4x^2\right)^{5/2}-\frac{3}{16}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[9 - 4*x^2],x]

[Out] (-3*(9 - 4*x^2)^(3/2))/16 + (9 - 4*x^2)^(5/2)/80

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{9-4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int\sqrt{9-4x}\,x\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{9}{4}\sqrt{9-4x}-\frac{1}{4}(9-4x)^{3/2}\right)dx,x,x^2\right)\\&=-\frac{3}{16}\left(9-4x^2\right)^{3/2}+\frac{1}{80}\left(9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(9-4x^2\right)^{3/2}\left(-3-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[9 - 4*x^2],x]
[Out] ((9 - 4*x^2)^(3/2)*(-3 - 2*x^2))/40
```

Maple [A]
time = 0.05, size = 27, normalized size = 0.87

method	result	size
trager	$\left(\frac{1}{5}x^4-\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{-4x^2+9}$	23
default	$-\frac{x^2\left(-4x^2+9\right)^{\frac{3}{2}}}{20}-\frac{3\left(-4x^2+9\right)^{\frac{3}{2}}}{40}$	27
gosper	$\frac{(2x-3)(2x+3)\left(2x^2+3\right)\sqrt{-4x^2+9}}{40}$	29
risch	$-\frac{\left(8x^4-6x^2-27\right)\left(4x^2-9\right)}{40\sqrt{-4x^2+9}}$	31
meijerg	$-\frac{243\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(1-\frac{4x^2}{9}\right)^{\frac{3}{2}}\left(2+\frac{4x^2}{3}\right)}{15}\right)}{64\sqrt{\pi}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/20*x^2*(-4*x^2+9)^(3/2)-3/40*(-4*x^2+9)^(3/2)
```

Maxima [A]
time = 0.55, size = 26, normalized size = 0.84

$$-\frac{1}{20}\left(-4\,x^2+9\right)^{\frac{3}{2}}x^2-\frac{3}{40}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/20*(-4*x^2 + 9)^(3/2)*x^2 - 3/40*(-4*x^2 + 9)^(3/2)
```

Fricas [A]
time = 0.90, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8\,x^4-6\,x^2-27\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 6*x^2 - 27)*sqrt(-4*x^2 + 9)
```

Sympy [A]
time = 0.15, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{9-4x^2}}{5}-\frac{3x^2\sqrt{9-4x^2}}{20}-\frac{27\sqrt{9-4x^2}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(9 - 4*x**2)/5 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**2)/
40
```

Giac [A]
time = 0.73, size = 32, normalized size = 1.03

$$\frac{1}{80}\left(4x^2-9\right)^2\sqrt{-4x^2+9}-\frac{3}{16}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) - 3/16*(-4*x^2 + 9)^(3/2)
```

Mupad [B]
time = 0.02, size = 23, normalized size = 0.74

$$-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(-\frac{4\,x^4}{5}+\frac{3\,x^2}{5}+\frac{27}{10}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*((3*x^2)/5 - (4*x^4)/5 + 27/10))/2
```

3.455 $\int x^2 \sqrt{9-4x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{9}{32}x\sqrt{9-4x^2}+\frac{1}{4}x^3\sqrt{9-4x^2}+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

[Out] 81/64*arcsin(2/3*x)-9/32*x*(-4*x^2+9)^(1/2)+1/4*x^3*(-4*x^2+9)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {285, 327, 222}

$$\frac{81}{64}\text{ArcSin}\left(\frac{2x}{3}\right)-\frac{9}{32}\sqrt{9-4x^2}\,x+\frac{1}{4}\sqrt{9-4x^2}\,x^3$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[9 - 4*x^2],x]

[Out] (-9*x*Sqrt[9 - 4*x^2])/32 + (x^3*Sqrt[9 - 4*x^2])/4 + (81*ArcSin[(2*x)/3])/64

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\sqrt{9-4x^2}\,dx = \frac{1}{4}x^3\sqrt{9-4x^2} + \frac{9}{4}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$$
$$= -\frac{9}{32}x\sqrt{9-4x^2} + \frac{1}{4}x^3\sqrt{9-4x^2} + \frac{81}{32}\int \frac{1}{\sqrt{9-4x^2}}\,dx$$
$$= -\frac{9}{32}x\sqrt{9-4x^2} + \frac{1}{4}x^3\sqrt{9-4x^2} + \frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A]
time = 0.08, size = 47, normalized size = 1.04

$$\frac{1}{32}x\sqrt{9-4x^2}\left(-9+8x^2\right) + \frac{81}{32}\tan^{-1}\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2]*(-9 + 8*x^2))/32 + (81*ArcTan[(2*x)/(-3 + Sqrt[9 - 4*x^2
])])/32
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.71

method	result	size
default	$-\frac{x\left(-4x^2+9\right)^{\frac{3}{2}}}{16}+\frac{81\arcsin\left(\frac{2x}{3}\right)}{64}+\frac{9x\sqrt{-4x^2+9}}{32}$	32
risch	$-\frac{x\left(8x^2-9\right)\left(4x^2-9\right)}{32\sqrt{-4x^2+9}}+\frac{81\arcsin\left(\frac{2x}{3}\right)}{64}$	34
meijerg	$-\frac{81i\left(-\frac{i\sqrt{\pi}\,x\left(-\frac{8x^2}{3}+3\right)\sqrt{1-\frac{4x^2}{9}}}{9}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{2}\right)}{32\sqrt{\pi}}$	41
trager	$\frac{x\left(8x^2-9\right)\sqrt{-4x^2+9}}{32}+\frac{81\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{64}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*x*(-4*x^2+9)^(3/2)+81/64*arcsin(2/3*x)+9/32*x*(-4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.50, size = 31, normalized size = 0.69

$$-\frac{1}{16}\left(-4\,x^2+9\right)^{\frac{3}{2}}x + \frac{9}{32}\sqrt{-4\,x^2+9}\,x + \frac{81}{64}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(-4*x^2 + 9)^(3/2)*x + 9/32*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

Fricas [A]
time = 0.86, size = 40, normalized size = 0.89

$$\frac{1}{32}\left(8\,x^{3}-9\,x\right)\sqrt{-4\,x^{2}+9}-\frac{81}{32}\arctan\left(\frac{\sqrt{-4\,x^{2}+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 9*x)*sqrt(-4*x^2 + 9) - 81/32*arctan(1/2*(sqrt(-4*x^2 + 9) -
3)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.73, size = 122, normalized size = 2.71

$$\begin{cases} \frac{ix^{5}}{\sqrt{4x^{2}-9}}-\frac{27ix^{3}}{8\sqrt{4x^{2}-9}}+\frac{81ix}{32\sqrt{4x^{2}-9}}-\frac{81i\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for}\ |x^{2}|>\frac{9}{4} \\ -\frac{x^{5}}{\sqrt{9-4x^{2}}}+\frac{27x^{3}}{8\sqrt{9-4x^{2}}}-\frac{81x}{32\sqrt{9-4x^{2}}}+\frac{81\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/sqrt(4*x**2 - 9) - 27*I*x**3/(8*sqrt(4*x**2 - 9)) + 81*I*
x/(32*sqrt(4*x**2 - 9)) - 81*I*acosh(2*x/3)/64, Abs(x**2) > 9/4), (-x**5/sq
rt(9 - 4*x**2) + 27*x**3/(8*sqrt(9 - 4*x**2)) - 81*x/(32*sqrt(9 - 4*x**2))
+ 81*asin(2*x/3)/64, True))
```

Giac [A]
time = 0.64, size = 26, normalized size = 0.58

$$\frac{1}{32}\left(8\,x^{2}-9\right)\sqrt{-4\,x^{2}+9}\,x+\frac{81}{64}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*(8*x^2 - 9)*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

Mupad [B]
time = 0.03, size = 27, normalized size = 0.60

$$\frac{81\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{64}-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{9\,x}{8}-x^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(9 - 4*x^2)^(1/2),x)
[Out] (81*asin((2*x)/3))/64 - ((9/4 - x^2)^(1/2)*((9*x)/8 - x^3))/2
```

3.456 $\int x\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

[Out] -1/12*(-4*x^2+9)^(3/2)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[9 - 4*x^2],x]

[Out] -1/12*(9 - 4*x^2)^(3/2)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{9-4x^2}\,dx=-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[9 - 4*x^2],x]

[Out] -1/12*(9 - 4*x^2)^(3/2)

Maple [A]
time = 0.04, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{(-4x^2+9)^{\frac{3}{2}}}{12}$	12
default	$-\frac{(-4x^2+9)^{\frac{3}{2}}}{12}$	12
trager	$\left(\frac{x^2}{3}-\frac{3}{4}\right)\sqrt{-4x^2+9}$	18
risch	$-\frac{(4x^2-9)^2}{12\sqrt{-4x^2+9}}$	21
gosper	$\frac{(2x-3)(2x+3)\sqrt{-4x^2+9}}{12}$	22
meijerg	$\frac{\frac{9\sqrt{\pi}}{4}-\frac{9\sqrt{\pi}\left(-\frac{8x^2}{9}+2\right)\sqrt{1-\frac{4x^2}{9}}}{8}}{\sqrt{\pi}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-4*x^2+9)^(3/2)
```

Maxima [A]
time = 0.32, size = 11, normalized size = 0.73

$$-\frac{1}{12}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-4*x^2 + 9)^(3/2)
```

Fricas [A]
time = 0.72, size = 18, normalized size = 1.20

$$\frac{1}{12}\left(4\,x^2-9\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 - 9)*sqrt(-4*x^2 + 9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.
time = 0.07, size = 27, normalized size = 1.80

$$\frac{x^2\sqrt{9-4x^2}}{3}-\frac{3\sqrt{9-4x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**2*sqrt(9 - 4*x**2)/3 - 3*sqrt(9 - 4*x**2)/4
```

Giac [A]
time = 0.56, size = 11, normalized size = 0.73

$$-\frac{1}{12}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(-4*x^2 + 9)^(3/2)
```

Mupad [B]
time = 0.02, size = 18, normalized size = 1.20

$$\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{4\,x^2}{3}-3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(9 - 4*x^2)^(1/2),x)
```

```
[Out] ((9/4 - x^2)^(1/2)*((4*x^2)/3 - 3))/2
```

3.457 $\int \sqrt{9-4x^2}\, dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 9/4*arcsin(2/3*x)+1/2*x*(-4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {201, 222}

$$\frac{9}{4}\text{ArcSin}\left(\frac{2x}{3}\right)+\frac{1}{2}\sqrt{9-4x^2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{9-4x^2}\, dx &= \frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{2}\int \frac{1}{\sqrt{9-4x^2}}\, dx\\ &= \frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 1.52

$$\frac{1}{2}x\sqrt{9-4x^2}-\frac{9}{2}\tan^{-1}\left(\frac{\sqrt{9-4x^2}}{3+2x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 - 4*x^2],x]

[Out] (x*Sqrt[9 - 4*x^2])/2 - (9*ArcTan[Sqrt[9 - 4*x^2]/(3 + 2*x)])/2

Maple [A]
time = 0.11, size = 20, normalized size = 0.74

method	result	size
default	$\frac{9 \arcsin\left(\frac{2x}{3}\right)}{4}+\frac{x\sqrt{-4x^{2}+9}}{2}$	20
risch	$-\frac{x\left(4x^{2}-9\right)}{2\sqrt{-4x^{2}+9}}+\frac{9 \arcsin\left(\frac{2x}{3}\right)}{4}$	27
meijerg	$\frac{9i\left(-\frac{4i\sqrt{\pi}\,x\sqrt{1-\frac{4x^{2}}{9}}}{3}-2i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}}$	34
trager	$\frac{x\sqrt{-4x^{2}+9}}{2}-\frac{9\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-4x^{2}+9}+2x\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)

[Out] 9/4*arcsin(2/3*x)+1/2*x*(-4*x^2+9)^(1/2)

Maxima [A]
time = 0.60, size = 19, normalized size = 0.70

$$\frac{1}{2}\,\sqrt{-4\,x^{2}+9}\,x+\frac{9}{4}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+9)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)

Fricas [A]
time = 1.36, size = 32, normalized size = 1.19

$$\frac{1}{2}\,\sqrt{-4\,x^{2}+9}\,x-\frac{9}{2}\,\arctan\left(\frac{\sqrt{-4\,x^{2}+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+9)^(1/2),x, algorithm="fricas")

```
[Out] 1/2*sqrt(-4*x^2 + 9)*x - 9/2*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A]
time = 0.07, size = 22, normalized size = 0.81

$$\frac{x\sqrt{9-4x^2}}{2}+\frac{9\,\mathrm{asin}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2),x)
[Out] x*sqrt(9 - 4*x**2)/2 + 9*asin(2*x/3)/4
```

Giac [A]
time = 0.55, size = 19, normalized size = 0.70

$$\frac{1}{2}\,\sqrt{-4\,x^2+9}\,x+\frac{9}{4}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

Mupad [B]
time = 0.02, size = 18, normalized size = 0.67

$$\frac{9\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{4}+x\,\sqrt{\frac{9}{4}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2),x)
[Out] (9*asin((2*x)/3))/4 + x*(9/4 - x^2)^(1/2)
```

3.458 $\int \frac{\sqrt{9-4x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{9-4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -3*arctanh(1/3*(-4*x^2+9)^(1/2))+(-4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {272, 52, 65, 212}

$$\sqrt{9-4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x,x]
```

```
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9-4x^2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9-4x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{9-4x^2}+\frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\ &= \sqrt{9-4x^2}-\frac{9}{4}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\ &= \sqrt{9-4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 1.00

$$\sqrt{9-4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x,x]
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.83

method	result	size
default	$\sqrt{-4x^2+9}-3\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)$	25
trager	$\sqrt{-4x^2+9}-3\ln\left(\frac{\sqrt{-4x^2+9}+3}{x}\right)$	29
meijerg	$-\frac{3\left(4\sqrt{\pi}-4\sqrt{\pi}\sqrt{1-\frac{4x^2}{9}}+4\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)-2(2+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}\right)}{4\sqrt{\pi}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-4*x^2+9)^(1/2)-3*arctanh(3/(-4*x^2+9)^(1/2))
```

Maxima [A]
time = 0.57, size = 35, normalized size = 1.17

$$\sqrt{-4\,x^2+9} - 3\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-4*x^2 + 9) - 3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A]
time = 1.01, size = 28, normalized size = 0.93

$$\sqrt{-4\,x^2+9} + 3\,\log\left(\frac{\sqrt{-4\,x^2+9} - 3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(-4*x^2 + 9) + 3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.67, size = 75, normalized size = 2.50

$$\begin{cases} i\sqrt{4x^2-9} - 3\log\left(x\right) + \frac{3\log\left(x^2\right)}{2} + 3i\,\mathrm{asin}\left(\frac{3}{2x}\right) & \text{for } |x^2| > \frac{9}{4} \\ \sqrt{9-4x^2} + \frac{3\log\left(x^2\right)}{2} - 3\log\left(\sqrt{1-\frac{4x^2}{9}} + 1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(4*x**2 - 9) - 3*log(x) + 3*log(x**2)/2 + 3*I*asin(3/(2*x)
), Abs(x**2) > 9/4), (sqrt(9 - 4*x**2) + 3*log(x**2)/2 - 3*log(sqrt(1 - 4*x
**2/9) + 1), True))
```

Giac [A]
time = 0.57, size = 40, normalized size = 1.33

$$\sqrt{-4\,x^2+9}-\frac{3}{2}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{3}{2}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(-4*x^2 + 9) - 3/2*log(sqrt(-4*x^2 + 9) + 3) + 3/2*log(-sqrt(-4*x^2 + 9
) + 3)
```

Mupad [B]
time = 4.55, size = 32, normalized size = 1.07

$$3\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)+2\,\sqrt{\frac{9}{4}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x,x)
```

```
[Out] 3*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2) + 2*(9/4 - x^2)^(1/2)
```

3.459 $\int \frac{\sqrt{9-4x^2}}{x^2} \, dx$

Optimal. Leaf size=25

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] -2*arcsin(2/3*x)-(-4*x^2+9)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {283, 222}

$$-2\text{ArcSin}\left(\frac{2x}{3}\right)-\frac{\sqrt{9-4x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - 4*x^2]/x) - 2*ArcSin[(2*x)/3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9-4x^2}}{x^2}\,dx &= -\frac{\sqrt{9-4x^2}}{x}-4\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 1.56

$$-\frac{\sqrt{9-4x^2}}{x}+4\tan^{-1}\left(\frac{\sqrt{9-4x^2}}{3+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^2,x]
[Out] -(Sqrt[9 - 4*x^2]/x) + 4*ArcTan[Sqrt[9 - 4*x^2]/(3 + 2*x)]
```

Maple [A]
time = 0.13, size = 34, normalized size = 1.36

method	result	size
risch	$\frac{4x^2-9}{x\sqrt{-4x^2+9}}-2\arcsin\left(\frac{2x}{3}\right)$	28
default	$-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{9x}-2\arcsin\left(\frac{2x}{3}\right)-\frac{4x\sqrt{-4x^2+9}}{9}$	34
meijerg	$-\frac{i\left(-\frac{6i\sqrt{\pi}\,\sqrt{1-\frac{4x^2}{9}}}{x}-4i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{2\sqrt{\pi}}$	36
trager	$-\frac{\sqrt{-4x^2+9}}{x}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-4x^2+9}\right)$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/9/x*(-4*x^2+9)^(3/2)-2*arcsin(2/3*x)-4/9*x*(-4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.52, size = 21, normalized size = 0.84

$$-\frac{\sqrt{-4\,x^2+9}}{x}-2\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
[Out] -sqrt(-4*x^2 + 9)/x - 2*arcsin(2/3*x)
```

Fricas [A]
time = 1.04, size = 36, normalized size = 1.44

$$\frac{4\,x\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)-\sqrt{-4\,x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (4*x*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x) - sqrt(-4*x^2 + 9))/x
```

Sympy [A]
time = 0.08, size = 20, normalized size = 0.80

$$-2\operatorname{asin}\left(\frac{2x}{3}\right)-\frac{\sqrt{9-4x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**2,x)
```

```
[Out] -2*asin(2*x/3) - sqrt(9 - 4*x**2)/x
```

Giac [A]
time = 0.50, size = 39, normalized size = 1.56

$$\frac{2\,x}{\sqrt{-4\,x^2+9}-3}-\frac{\sqrt{-4\,x^2+9}-3}{2\,x}-2\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*x/(sqrt(-4*x^2 + 9) - 3) - 1/2*(sqrt(-4*x^2 + 9) - 3)/x - 2*arcsin(2/3*x)
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$-2\operatorname{asin}\left(\frac{2\,x}{3}\right)-\frac{2\,\sqrt{\frac{9}{4}-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^2,x)
```

```
[Out] - 2*asin((2*x)/3) - (2*(9/4 - x^2)^(1/2))/x
```

3.460 $\int \frac{\sqrt{9-4x^2}}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{9-4x^2}}{2x^2}+\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] 2/3*arctanh(1/3*(-4*x^2+9)^(1/2))-1/2*(-4*x^2+9)^(1/2)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 43, 65, 212}

$$\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^3,x]
```

```
[Out] -1/2*Sqrt[9 - 4*x^2]/x^2 + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9-4x^2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9-4x}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{2x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\ &= -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.00

$$-\frac{\sqrt{9-4x^2}}{2x^2} + \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^3,x]
[Out] -1/2*Sqrt[9 - 4*x^2]/x^2 + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/3
```

Maple [A]
time = 0.12, size = 41, normalized size = 1.05

method	result	size
trager	$-\frac{\sqrt{-4x^2+9}}{2x^2}+\frac{2\ln\left(\frac{\sqrt{-4x^2+9}+3}{x}\right)}{3}$	34
risch	$\frac{4x^2-9}{2x^2\sqrt{-4x^2+9}}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{3}$	37
default	$-\frac{(-4x^2+9)^{\frac{3}{2}}}{18x^2}-\frac{2\sqrt{-4x^2+9}}{9}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{3}$	41

meijerg	$\frac{\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)}{16x^2}-\frac{9\sqrt{\pi}\sqrt{1-\frac{4x^2}{9}}}{2x^2}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)-(-1+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}-\frac{9\sqrt{\pi}}{2x^2}}{3\sqrt{\pi}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18/x^2*(-4*x^2+9)^(3/2)-2/9*(-4*x^2+9)^(1/2)+2/3*arctanh(3/(-4*x^2+9)^(1
/2))
```

Maxima [A]
time = 0.50, size = 51, normalized size = 1.31

$$-\frac{2}{9}\sqrt{-4x^2+9}-\frac{(-4x^2+9)^{\frac{3}{2}}}{18x^2}+\frac{2}{3}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(-4*x^2 + 9) - 1/18*(-4*x^2 + 9)^(3/2)/x^2 + 2/3*log(6*sqrt(-4*x^2
 + 9)/abs(x) + 18/abs(x))
```

Fricas [A]
time = 1.09, size = 38, normalized size = 0.97

$$-\frac{4x^2\log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)+3\sqrt{-4x^2+9}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) + 3*sqrt(-4*x^2 + 9))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.86, size = 97, normalized size = 2.49

$$\begin{cases} \frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{3}+\frac{1}{x\sqrt{-1+\frac{9}{4x^2}}}-\frac{9}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{1}{|x^2|}>\frac{4}{9} \\ -\frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{3}-\frac{i}{x\sqrt{1-\frac{9}{4x^2}}}+\frac{9i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**3,x)
```

```
[Out] Piecewise((2*acosh(3/(2*x))/3 + 1/(x*sqrt(-1 + 9/(4*x**2))) - 9/(4*x**3*sqr
t(-1 + 9/(4*x**2))), 1/Abs(x**2) > 4/9), (-2*I*asin(3/(2*x))/3 - I/(x*sqrt(
1 - 9/(4*x**2))) + 9*I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Giac [**A**]
time = 0.58, size = 45, normalized size = 1.15

$$-\frac{\sqrt{-4\,x^2+9}}{2\,x^2}+\frac{1}{3}\,\log\left(\sqrt{-4\,x^2+9}+3\right)-\frac{1}{3}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-4*x^2 + 9)/x^2 + 1/3*log(sqrt(-4*x^2 + 9) + 3) - 1/3*log(-sqrt(-
4*x^2 + 9) + 3)
```

Mupad [**B**]
time = 4.66, size = 35, normalized size = 0.90

$$-\frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{3}-\frac{\sqrt{\frac{9}{4}-x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^3,x)
```

```
[Out] - (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2))/3 - (9/4 - x^2)^(1/2
)/x^2
```

3.461 $\int \frac{\sqrt{9-4x^2}}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

[Out] -1/27*(-4*x^2+9)^(3/2)/x^3

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{270\}$

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 - 4*x^2]/x^4,x]

[Out] -1/27*(9 - 4*x^2)^(3/2)/x^3

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{9-4x^2}}{x^4}\,dx = -\frac{(9-4x^2)^{3/2}}{27x^3}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 - 4*x^2]/x^4,x]

[Out] -1/27*(9 - 4*x^2)^(3/2)/x^3

Maple [A]
time = 0.05, size = 15, normalized size = 0.83

method	result	size
default	$-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{27x^3}$	15
meijerg	$-\frac{\left(1-\frac{4x^2}{9}\right)^{\frac{3}{2}}}{x^3}$	15
trager	$\frac{\left(4x^2-9\right)\sqrt{-4x^2+9}}{27x^3}$	22
gosper	$\frac{(2x-3)(2x+3)\sqrt{-4x^2+9}}{27x^3}$	25
risch	$-\frac{16x^4-72x^2+81}{27x^3\sqrt{-4x^2+9}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/27*(-4*x^2+9)^(3/2)/x^3
```

Maxima [A]
time = 0.59, size = 14, normalized size = 0.78

$$-\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/27*(-4*x^2 + 9)^(3/2)/x^3
```

Fricas [A]
time = 1.02, size = 21, normalized size = 1.17

$$\frac{\left(4\,x^2-9\right)\sqrt{-4\,x^2+9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/27*(4*x^2 - 9)*sqrt(-4*x^2 + 9)/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 76, normalized size = 4.22

$$\begin{cases} \frac{8\sqrt{-1+\frac{9}{4x^2}}}{27}-\frac{2\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for}\: \frac{1}{|x^2|}>\frac{4}{9} \\ \frac{8i\sqrt{1-\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**4,x)
```

```
[Out] Piecewise((8*sqrt(-1 + 9/(4*x**2))/27 - 2*sqrt(-1 + 9/(4*x**2))/(3*x**2), 1
/Abs(x**2) > 4/9), (8*I*sqrt(1 - 9/(4*x**2))/27 - 2*I*sqrt(1 - 9/(4*x**2))/
(3*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(14) = 28$.
time = 0.52, size = 73, normalized size = 4.06

$$-\frac{2x^3\left(\frac{3\left(\sqrt{-4x^2+9}-3\right)^2}{x^2}-4\right)}{27\left(\sqrt{-4x^2+9}-3\right)^3}+\frac{\sqrt{-4x^2+9}-3}{18x}-\frac{\left(\sqrt{-4x^2+9}-3\right)^3}{216x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2/27*x^3*(3*(sqrt(-4*x^2 + 9) - 3)^2/x^2 - 4)/(sqrt(-4*x^2 + 9) - 3)^3 + 1
/18*(sqrt(-4*x^2 + 9) - 3)/x - 1/216*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

Mupad [B]
time = 0.03, size = 31, normalized size = 1.72

$$\frac{8x^2\sqrt{\frac{9}{4}-x^2}-18\sqrt{\frac{9}{4}-x^2}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^4,x)
```

```
[Out] (8*x^2*(9/4 - x^2)^(1/2) - 18*(9/4 - x^2)^(1/2))/(27*x^3)
```

3.462 $\int \frac{\sqrt{9-4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{9-4x^2}}{4x^4}+\frac{\sqrt{9-4x^2}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] 2/27*arctanh(1/3*(-4*x^2+9)^(1/2))-1/4*(-4*x^2+9)^(1/2)/x^4+1/18*(-4*x^2+9)
^(1/2)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 44, 65, 212}

$$\frac{\sqrt{9-4x^2}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^5,x]
```

```
[Out] -1/4*Sqrt[9 - 4*x^2]/x^4 + Sqrt[9 - 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 - 4
*x^2]/3])/27
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{9-4x^2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9-4x}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^2}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4}+\frac{\sqrt{9-4x^2}}{18x^2}-\frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4}+\frac{\sqrt{9-4x^2}}{18x^2}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{9-4x^2}\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4}+\frac{\sqrt{9-4x^2}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 0.81

$$\frac{\sqrt{9-4x^2}\,(-9+2x^2)}{36x^4}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^5,x]
[Out] (Sqrt[9 - 4*x^2]*(-9 + 2*x^2))/(36*x^4) + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

Maple [A]
time = 0.10, size = 55, normalized size = 0.96

method	result
trager	$\frac{(2x^2-9)\sqrt{-4x^2+9}}{36x^4}-\frac{2\ln\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)}{27}$
risch	$-\frac{8x^4-54x^2+81}{36x^4\sqrt{-4x^2+9}}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}$
default	$-\frac{(-4x^2+9)^{\frac{3}{2}}}{36x^4}-\frac{(-4x^2+9)^{\frac{3}{2}}}{162x^2}-\frac{2\sqrt{-4x^2+9}}{81}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}$
meijerg	$-\frac{4\left(-\frac{81\sqrt{\pi}\left(\frac{16}{81}x^4-\frac{32}{9}x^2+8\right)}{128x^4}+\frac{81\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)\sqrt{1-\frac{4x^2}{9}}}{128x^4}-\frac{\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)}{2}+\frac{\left(\frac{1}{2}+2\ln(x)-2\ln(3)+i\pi\right)\sqrt{\pi}}{4}\right.}{27\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/36/x^4*(-4*x^2+9)^(3/2)-1/162/x^2*(-4*x^2+9)^(3/2)-2/81*(-4*x^2+9)^(1/2)
+2/27*arctanh(3/(-4*x^2+9)^(1/2))
```

Maxima [A]
time = 0.50, size = 65, normalized size = 1.14

$$-\frac{2}{81}\sqrt{-4x^2+9}-\frac{(-4x^2+9)^{\frac{3}{2}}}{162x^2}-\frac{(-4x^2+9)^{\frac{3}{2}}}{36x^4}+\frac{2}{27}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(-4*x^2 + 9) - 1/162*(-4*x^2 + 9)^(3/2)/x^2 - 1/36*(-4*x^2 + 9)^(
3/2)/x^4 + 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A]
time = 1.57, size = 45, normalized size = 0.79

$$-\frac{8x^4\log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)-3(2x^2-9)\sqrt{-4x^2+9}}{108x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/108*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 - 9)*sqrt(-4*x^2 + 9
))/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 1.95, size = 139, normalized size = 2.44

$$\begin{cases} \frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}} + \frac{3}{4x^3\sqrt{-1+\frac{9}{4x^2}}} - \frac{9}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{27} + \frac{i}{9x\sqrt{1-\frac{9}{4x^2}}} - \frac{3i}{4x^3\sqrt{1-\frac{9}{4x^2}}} + \frac{9i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**5,x)
```

```
[Out] Piecewise((2*acosh(3/(2*x))/27 - 1/(9*x*sqrt(-1 + 9/(4*x**2))) + 3/(4*x**3*
sqrt(-1 + 9/(4*x**2))) - 9/(8*x**5*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2) > 4/
9), (-2*I*asin(3/(2*x))/27 + I/(9*x*sqrt(1 - 9/(4*x**2))) - 3*I/(4*x**3*sqr
t(1 - 9/(4*x**2))) + 9*I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A]
time = 0.58, size = 57, normalized size = 1.00

$$-\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}} + 9\,\sqrt{-4\,x^2+9}}{72\,x^4} + \frac{1}{27}\,\log\left(\sqrt{-4\,x^2+9} + 3\right) - \frac{1}{27}\,\log\left(-\sqrt{-4\,x^2+9} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((-4*x^2 + 9)^(3/2) + 9*sqrt(-4*x^2 + 9))/x^4 + 1/27*log(sqrt(-4*x^2
+ 9) + 3) - 1/27*log(-sqrt(-4*x^2 + 9) + 3)
```

Mupad [B]
time = 0.03, size = 49, normalized size = 0.86

$$\frac{\sqrt{\frac{9}{4}-x^2}}{9\,x^2} - \frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1} - \frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{27} - \frac{\sqrt{\frac{9}{4}-x^2}}{2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^5,x)
```

```
[Out] (9/4 - x^2)^(1/2)/(9*x^2) - (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2)
)/2))/27 - (9/4 - x^2)^(1/2)/(2*x^4)
```

3.463 $\int x^5\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=46

$$\frac{27}{64}\left(-9+4x^2\right)^{3/2}+\frac{9}{160}\left(-9+4x^2\right)^{5/2}+\frac{1}{448}\left(-9+4x^2\right)^{7/2}$$

```
[Out] 27/64*(4*x^2-9)^(3/2)+9/160*(4*x^2-9)^(5/2)+1/448*(4*x^2-9)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{1}{448}\left(4x^2-9\right)^{7/2}+\frac{9}{160}\left(4x^2-9\right)^{5/2}+\frac{27}{64}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (27*(-9 + 4*x^2)^(3/2))/64 + (9*(-9 + 4*x^2)^(5/2))/160 + (-9 + 4*x^2)^(7/2
)/448
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{-9+4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2\sqrt{-9+4x}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{-9+4x}+\frac{9}{8}(-9+4x)^{3/2}+\frac{1}{16}(-9+4x)^{5/2}\right)dx,x,x^2\right)\\&=\frac{27}{64}\left(-9+4x^2\right)^{3/2}+\frac{9}{160}\left(-9+4x^2\right)^{5/2}+\frac{1}{448}\left(-9+4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(-9+4x^2\right)^{3/2}\left(27+18x^2+10x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-9 + 4*x^2],x]
[Out] ((-9 + 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))/280
```

Maple [A]
time = 0.08, size = 41, normalized size = 0.89

method	result	size
trager	$\left(\frac{1}{7}x^6-\frac{9}{140}x^4-\frac{27}{140}x^2-\frac{243}{280}\right)\sqrt{4x^2-9}$	28
risch	$\frac{(40x^6-18x^4-54x^2-243)\sqrt{4x^2-9}}{280}$	29
gosper	$\frac{(2x-3)(2x+3)(10x^4+18x^2+27)\sqrt{4x^2-9}}{280}$	34
default	$\frac{x^4(4x^2-9)^{\frac{3}{2}}}{28}+\frac{9x^2(4x^2-9)^{\frac{3}{2}}}{140}+\frac{27(4x^2-9)^{\frac{3}{2}}}{280}$	41
meijerg	$\frac{2187\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\left(1-\frac{4x^2}{9}\right)^{\frac{3}{2}}\left(\frac{80}{27}x^4+\frac{16}{3}x^2+8\right)}{105}\right)}{256\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/28*x^4*(4*x^2-9)^(3/2)+9/140*x^2*(4*x^2-9)^(3/2)+27/280*(4*x^2-9)^(3/2)
```

Maxima [A]
time = 0.48, size = 40, normalized size = 0.87

$$\frac{1}{28}\left(4\,x^2-9\right)^{\frac{3}{2}}x^4+\frac{9}{140}\left(4\,x^2-9\right)^{\frac{3}{2}}x^2+\frac{27}{280}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/28*(4*x^2 - 9)^(3/2)*x^4 + 9/140*(4*x^2 - 9)^(3/2)*x^2 + 27/280*(4*x^2 -
9)^(3/2)
```

Fricas [A]
time = 1.62, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6-18\,x^4-54\,x^2-243\right)\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(4*x^2 - 9)
```

Sympy [A]
time = 0.33, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{4x^2-9}}{7}-\frac{9x^4\sqrt{4x^2-9}}{140}-\frac{27x^2\sqrt{4x^2-9}}{140}-\frac{243\sqrt{4x^2-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(4*x**2-9)**(1/2),x)
```

```
[Out] x**6*sqrt(4*x**2 - 9)/7 - 9*x**4*sqrt(4*x**2 - 9)/140 - 27*x**2*sqrt(4*x**2
 - 9)/140 - 243*sqrt(4*x**2 - 9)/280
```

Giac [A]
time = 0.58, size = 34, normalized size = 0.74

$$\frac{1}{448}\left(4\,x^2-9\right)^{\frac{7}{2}}+\frac{9}{160}\left(4\,x^2-9\right)^{\frac{5}{2}}+\frac{27}{64}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 - 9)^(7/2) + 9/160*(4*x^2 - 9)^(5/2) + 27/64*(4*x^2 - 9)^(3/2)
```

Mupad [B]
time = 5.31, size = 28, normalized size = 0.61

$$-\sqrt{4\,x^2-9}\,\left(-\frac{x^6}{7}+\frac{9\,x^4}{140}+\frac{27\,x^2}{140}+\frac{243}{280}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2 - 9)^(1/2),x)
```

```
[Out] -(4*x^2 - 9)^(1/2)*((27*x^2)/140 + (9*x^4)/140 - x^6/7 + 243/280)
```

3.464 $\int x^4 \sqrt{-9+4x^2}\, dx$

Optimal. Leaf size=72

$$-\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

```
[Out] -729/512*arctanh(2*x/(4*x^2-9)^(1/2))-81/256*x*(4*x^2-9)^(1/2)-3/32*x^3*(4*
x^2-9)^(1/2)+1/6*x^5*(4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 212}

$$-\frac{81}{256}\sqrt{4x^2-9}\,x-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{6}\sqrt{4x^2-9}\,x^5-\frac{3}{32}\sqrt{4x^2-9}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[-9 + 4*x^2])/256 - (3*x^3*Sqrt[-9 + 4*x^2])/32 + (x^5*Sqrt[-9 +
 4*x^2])/6 - (729*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/512
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{-9+4x^2}\,dx &= \frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{3}{2}\int\frac{x^4}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{81}{32}\int\frac{x^2}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{256}\int\frac{1}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{256}\text{Subst}\left(\int\frac{1}{1-4x^2}\right.\\ &= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4}}\right.\end{aligned}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.68

$$\frac{1}{768}x\sqrt{-9+4x^2}\left(-243-72x^2+128x^4\right)+\frac{729}{512}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 + (729*Log[-2*x + Sqrt[-
9 + 4*x^2]])/512
```

Maple [A]
time = 0.11, size = 61, normalized size = 0.85

method	result	size
trager	$\frac{x(128x^4-72x^2-243)\sqrt{4x^2-9}}{768}-\frac{729\ln\left(\sqrt{4x^2-9}+2x\right)}{512}$	42
risch	$\frac{x(128x^4-72x^2-243)\sqrt{4x^2-9}}{768}-\frac{729\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{1024}$	47
default	$\frac{x^3(4x^2-9)^{\frac{3}{2}}}{24}+\frac{9x(4x^2-9)^{\frac{3}{2}}}{128}+\frac{81x\sqrt{4x^2-9}}{256}-\frac{729\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{1024}$	61

meijerg	$\frac{729i\sqrt{\text{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{i\sqrt{\pi}\,x\left(-\frac{640}{81}x^4+\frac{40}{9}x^2+15\right)\sqrt{1-\frac{4x^2}{9}}}{90}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{4}\right)}{128\sqrt{\pi}\,\sqrt{-\text{signum}\left(-1+\frac{4x^2}{9}\right)}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*x^3*(4*x^2-9)^(3/2)+9/128*x*(4*x^2-9)^(3/2)+81/256*x*(4*x^2-9)^(1/2)-7
29/1024*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

Maxima [A]
time = 0.50, size = 57, normalized size = 0.79

$$\frac{1}{24}\left(4x^2-9\right)^{\frac{3}{2}}x^3+\frac{9}{128}\left(4x^2-9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{4x^2-9}\,x-\frac{729}{512}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^2 - 9)^(3/2)*x^3 + 9/128*(4*x^2 - 9)^(3/2)*x + 81/256*sqrt(4*x^2
- 9)*x - 729/512*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A]
time = 1.13, size = 42, normalized size = 0.58

$$\frac{1}{768}\left(128x^5-72x^3-243x\right)\sqrt{4x^2-9}+\frac{729}{512}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(4*x^2 - 9) + 729/512*log(-2*x + sqrt(
4*x^2 - 9))
```

Sympy [C] Result contains complex when optimal does not.
time = 4.59, size = 165, normalized size = 2.29

$$\begin{cases} \frac{2x^7}{3\sqrt{4x^2-9}}-\frac{15x^5}{8\sqrt{4x^2-9}}-\frac{27x^3}{64\sqrt{4x^2-9}}+\frac{729x}{256\sqrt{4x^2-9}}-\frac{729\,\text{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for } |x^2|>\frac{9}{4} \\ -\frac{2ix^7}{3\sqrt{9-4x^2}}+\frac{15ix^5}{8\sqrt{9-4x^2}}+\frac{27ix^3}{64\sqrt{9-4x^2}}-\frac{729ix}{256\sqrt{9-4x^2}}+\frac{729i\,\text{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*x**7/(3*sqrt(4*x**2 - 9)) - 15*x**5/(8*sqrt(4*x**2 - 9)) - 27*
x**3/(64*sqrt(4*x**2 - 9)) + 729*x/(256*sqrt(4*x**2 - 9)) - 729*acosh(2*x/3
)/512, Abs(x**2) > 9/4), (-2*I*x**7/(3*sqrt(9 - 4*x**2)) + 15*I*x**5/(8*sqr
t(9 - 4*x**2)) + 27*I*x**3/(64*sqrt(9 - 4*x**2)) - 729*I*x/(256*sqrt(9 - 4*
x**2)) + 729*I*asin(2*x/3)/512, True))
```

Giac [A]
time = 0.54, size = 44, normalized size = 0.61

$$\frac{1}{768}\left(8\left(16\,x^2-9\right)x^2-243\right)\sqrt{4\,x^2-9}\,x+\frac{729}{512}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(4*x^2 - 9)*x + 729/512*log(abs(-2*x +
 sqrt(4*x^2 - 9)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{4\,x^2-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^4*(4*x^2 - 9)^(1/2), x)
```

3.465 $\int x^3\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{3}{16}\left(-9+4x^2\right)^{3/2}+\frac{1}{80}\left(-9+4x^2\right)^{5/2}$$

```
[Out] 3/16*(4*x^2-9)^(3/2)+1/80*(4*x^2-9)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{1}{80}\left(4x^2-9\right)^{5/2}+\frac{3}{16}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (3*(-9 + 4*x^2)^(3/2))/16 + (-9 + 4*x^2)^(5/2)/80
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{-9+4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{-9+4x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{9}{4}\sqrt{-9+4x}+\frac{1}{4}(-9+4x)^{3/2}\right)dx,x,x^2\right)\\ &= \frac{3}{16}\left(-9+4x^2\right)^{3/2}+\frac{1}{80}\left(-9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}(3+2x^2)\left(-9+4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-9 + 4*x^2],x]
[Out] ((3 + 2*x^2)*(-9 + 4*x^2)^(3/2))/40
```

Maple [A]
time = 0.06, size = 27, normalized size = 0.87

method	result	size
trager	$\left(\frac{1}{5}x^4-\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{4x^2-9}$	23
risch	$\frac{(8x^4-6x^2-27)\sqrt{4x^2-9}}{40}$	24
default	$\frac{x^2(4x^2-9)^{\frac{3}{2}}}{20}+\frac{3(4x^2-9)^{\frac{3}{2}}}{40}$	27
gosper	$\frac{(2x-3)(2x+3)(2x^2+3)\sqrt{4x^2-9}}{40}$	29
meijerg	$-\frac{243\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(1-\frac{4x^2}{9}\right)^{\frac{3}{2}}\left(2+\frac{4x^2}{3}\right)}{15}\right)}{64\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/20*x^2*(4*x^2-9)^(3/2)+3/40*(4*x^2-9)^(3/2)
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.84

$$\frac{1}{20}\left(4x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/20*(4*x^2 - 9)^(3/2)*x^2 + 3/40*(4*x^2 - 9)^(3/2)
```

Fricas [A]
time = 1.27, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8x^4-6x^2-27\right)\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 6*x^2 - 27)*sqrt(4*x^2 - 9)
```

Sympy [A]
time = 0.15, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4x^2-9}}{5}-\frac{3x^2\sqrt{4x^2-9}}{20}-\frac{27\sqrt{4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 - 9)/5 - 3*x**2*sqrt(4*x**2 - 9)/20 - 27*sqrt(4*x**2 - 9)/
40
```

Giac [A]
time = 0.56, size = 23, normalized size = 0.74

$$\frac{1}{80}\left(4\,x^2-9\right)^{\frac{5}{2}}+\frac{3}{16}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 - 9)^(5/2) + 3/16*(4*x^2 - 9)^(3/2)
```

Mupad [B]
time = 5.51, size = 23, normalized size = 0.74

$$-\sqrt{4\,x^2-9}\,\left(-\frac{x^4}{5}+\frac{3\,x^2}{20}+\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2 - 9)^(1/2),x)
```

```
[Out] -(4*x^2 - 9)^(1/2)*((3*x^2)/20 - x^4/5 + 27/40)
```

3.466 $\int x^2\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

```
[Out] -81/64*arctanh(2*x/(4*x^2-9)^(1/2))-9/32*x*(4*x^2-9)^(1/2)+1/4*x^3*(4*x^2-9
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 212}

$$-\frac{9}{32}\sqrt{4x^2-9}\,x-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{4}\sqrt{4x^2-9}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-9*x*Sqrt[-9 + 4*x^2])/32 + (x^3*Sqrt[-9 + 4*x^2])/4 - (81*ArcTanh[(2*x)/S
qrt[-9 + 4*x^2]])/64
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{-9+4x^2}\,dx &= \frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{9}{4}\int\frac{x^2}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{32}\int\frac{1}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{32}\text{Subst}\left(\int\frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\ &= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.81

$$\frac{1}{32}x\sqrt{-9+4x^2}\left(-9+8x^2\right)+\frac{81}{64}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-9 + 4*x^2],x]
[Out] (x*Sqrt[-9 + 4*x^2]*(-9 + 8*x^2))/32 + (81*Log[-2*x + Sqrt[-9 + 4*x^2]])/64
```

Maple [A]
time = 0.10, size = 47, normalized size = 0.87

method	result	size
trager	$\frac{x(8x^2-9)\sqrt{4x^2-9}}{32}-\frac{81\ln\left(\sqrt{4x^2-9}+2x\right)}{64}$	37
risch	$\frac{x(8x^2-9)\sqrt{4x^2-9}}{32}-\frac{81\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{128}$	42
default	$\frac{x(4x^2-9)^{\frac{3}{2}}}{16}+\frac{9x\sqrt{4x^2-9}}{32}-\frac{81\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{128}$	47
meijerg	$-\frac{81i\sqrt{\text{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{i\sqrt{\pi}\,x\left(-\frac{8x^2}{3}+3\right)\sqrt{1-\frac{4x^2}{9}}}{9}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{2}\right)}{32\sqrt{\pi}\,\sqrt{-\text{signum}\left(-1+\frac{4x^2}{9}\right)}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x*(4*x^2-9)^(3/2)+9/32*x*(4*x^2-9)^(1/2)-81/128*ln(x*4^(1/2)+(4*x^2-9)
^(1/2))*4^(1/2)
```

Maxima [A]
time = 0.49, size = 43, normalized size = 0.80

$$\frac{1}{16}\left(4\,x^2-9\right)^{\frac{3}{2}}x+\frac{9}{32}\,\sqrt{4\,x^2-9}\,x-\frac{81}{64}\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^2 - 9)^(3/2)*x + 9/32*sqrt(4*x^2 - 9)*x - 81/64*log(8*x + 4*sqrt(
4*x^2 - 9))
```

Fricas [A]
time = 0.89, size = 37, normalized size = 0.69

$$\frac{1}{32}\left(8\,x^3-9\,x\right)\sqrt{4\,x^2-9}+\frac{81}{64}\,\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 9*x)*sqrt(4*x^2 - 9) + 81/64*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.64, size = 122, normalized size = 2.26

$$\begin{cases} \frac{x^5}{\sqrt{4x^2-9}}-\frac{27x^3}{8\sqrt{4x^2-9}}+\frac{81x}{32\sqrt{4x^2-9}}-\frac{81\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for } |x^2|>\frac{9}{4} \\ -\frac{ix^5}{\sqrt{9-4x^2}}+\frac{27ix^3}{8\sqrt{9-4x^2}}-\frac{81ix}{32\sqrt{9-4x^2}}+\frac{81i\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((x**5/sqrt(4*x**2 - 9) - 27*x**3/(8*sqrt(4*x**2 - 9)) + 81*x/(32*
sqrt(4*x**2 - 9)) - 81*acosh(2*x/3)/64, Abs(x**2) > 9/4), (-I*x**5/sqrt(9 -
 4*x**2) + 27*I*x**3/(8*sqrt(9 - 4*x**2)) - 81*I*x/(32*sqrt(9 - 4*x**2)) +
81*I*asin(2*x/3)/64, True))
```

Giac [A]
time = 0.52, size = 37, normalized size = 0.69

$$\frac{1}{32}\left(8\,x^2-9\right)\sqrt{4\,x^2-9}\,x+\frac{81}{64}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/32*(8*x^2 - 9)*sqrt(4*x^2 - 9)*x + 81/64*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\sqrt{4\,x^2-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2 - 9)^(1/2),x)
[Out] int(x^2*(4*x^2 - 9)^(1/2), x)
```

3.467 $\int x\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\left(-9+4x^2\right)^{3/2}$$

```
[Out] 1/12*(4*x^2-9)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-9 + 4*x^2],x]
[Out] (-9 + 4*x^2)^(3/2)/12
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{-9+4x^2}\,dx = \frac{1}{12}\left(-9+4x^2\right)^{3/2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{12}\left(-9+4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-9 + 4*x^2],x]
[Out] (-9 + 4*x^2)^(3/2)/12
```

Maple [A]
time = 0.06, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$\frac{(4x^2-9)^{\frac{3}{2}}}{12}$	12
default	$\frac{(4x^2-9)^{\frac{3}{2}}}{12}$	12
risch	$\frac{(4x^2-9)^{\frac{3}{2}}}{12}$	12
trager	$\left(\frac{x^2}{3}-\frac{3}{4}\right)\sqrt{4x^2-9}$	18
gosper	$\frac{(2x-3)(2x+3)\sqrt{4x^2-9}}{12}$	22
meijerg	$\frac{27\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(-\frac{8x^2}{9}+2\right)\sqrt{1-\frac{4x^2}{9}}}{3}\right)}{16\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(4*x^2-9)^(3/2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

Fricas [A]
time = 1.09, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

time = 0.07, size = 27, normalized size = 1.80

$$\frac{x^2\sqrt{4x^2-9}}{3} - \frac{3\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2-9)**(1/2),x)
[Out] x**2*sqrt(4*x**2 - 9)/3 - 3*sqrt(4*x**2 - 9)/4
```

Giac [A]
time = 0.50, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

Mupad [B]
time = 0.06, size = 11, normalized size = 0.73

$$\frac{\left(4x^2-9\right)^{3/2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2 - 9)^(1/2),x)
[Out] (4*x^2 - 9)^(3/2)/12
```

3.468 $\int \sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{-9+4x^2}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

[Out] -9/4*arctanh(2*x/(4*x^2-9)^(1/2))+1/2*x*(4*x^2-9)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 223, 212}

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 + 4*x^2],x]

[Out] (x*Sqrt[-9 + 4*x^2])/2 - (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/4

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{-9+4x^2}\,dx = \frac{1}{2}x\sqrt{-9+4x^2} - \frac{9}{2}\int \frac{1}{\sqrt{-9+4x^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{-9+4x^2} - \frac{9}{2}\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)$$
$$= \frac{1}{2}x\sqrt{-9+4x^2} - \frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.03

$$\frac{1}{2}x\sqrt{-9+4x^2} + \frac{9}{4}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2],x]
[Out] (x*Sqrt[-9 + 4*x^2])/2 + (9*Log[-2*x + Sqrt[-9 + 4*x^2]])/4
```

Maple [A]
time = 0.08, size = 35, normalized size = 0.97

method	result	size
trager	$\frac{x\sqrt{4x^2-9}}{2} - \frac{9\ln\left(\sqrt{4x^2-9}+2x\right)}{4}$	30
default	$\frac{x\sqrt{4x^2-9}}{2} - \frac{9\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{8}$	35
risch	$\frac{x\sqrt{4x^2-9}}{2} - \frac{9\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{8}$	35
meijerg	$\frac{9i\sqrt{\text{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{4i\sqrt{\pi}\,x\sqrt{1-\frac{4x^2}{9}}}{3}-2i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}\,\sqrt{-\text{signum}\left(-1+\frac{4x^2}{9}\right)}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(4*x^2-9)^(1/2)-9/8*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

Maxima [A]
time = 0.49, size = 31, normalized size = 0.86

$$\frac{1}{2}\sqrt{4x^2-9}\,x - \frac{9}{4}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(4*x^2 - 9)*x - 9/4*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A]
time = 1.31, size = 29, normalized size = 0.81

$$\frac{1}{2}\sqrt{4x^2-9}\,x+\frac{9}{4}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(4*x^2 - 9)*x + 9/4*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A]
time = 0.07, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\operatorname{acosh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2),x)
[Out] x*sqrt(4*x**2 - 9)/2 - 9*acosh(2*x/3)/4
```

Giac [A]
time = 0.69, size = 30, normalized size = 0.83

$$\frac{1}{2}\sqrt{4x^2-9}\,x+\frac{9}{4}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(4*x^2 - 9)*x + 9/4*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

Mupad [B]
time = 5.03, size = 29, normalized size = 0.81

$$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(2x+\sqrt{4x^2-9}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2),x)
[Out] (x*(4*x^2 - 9)^(1/2))/2 - (9*log(2*x + (4*x^2 - 9)^(1/2)))/4
```

3.469 $\int \frac{\sqrt{-9+4x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{-9+4x^2}-3\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

```
[Out] -3*arctan(1/3*(4*x^2-9)^(1/2))+(4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 52, 65, 209}

$$\sqrt{4x^2-9}-3\text{ArcTan}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-9+4x^2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9+4x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{-9+4x^2} - \frac{9}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\ &= \sqrt{-9+4x^2} - \frac{9}{4}\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\ &= \sqrt{-9+4x^2} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 1.00

$$\sqrt{-9+4x^2} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x,x]
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

Maple [A]
time = 0.14, size = 25, normalized size = 0.83

method	result
default	$\sqrt{4x^2-9}+3\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)$
trager	$\sqrt{4x^2-9}+3\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{4x^2-9}-3\,\text{RootOf}\left(\_Z^2+1\right)}{x}\right)$
meijerg	$-\frac{3\sqrt{\text{signum}\left(-1+\frac{4x^2}{9}\right)}\left(4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{1-\frac{4x^2}{9}}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)-2(2+2\ln(x)-2\ln(3)+i\pi)\sqrt{\right.}{4\sqrt{\pi}\,\sqrt{-\text{signum}\left(-1+\frac{4x^2}{9}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (4*x^2-9)^(1/2)+3*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A]
time = 0.56, size = 19, normalized size = 0.63

$$\sqrt{4\,x^2-9}+3\,\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(4*x^2 - 9) + 3*arcsin(3/2/abs(x))
```

Fricas [A]
time = 1.25, size = 28, normalized size = 0.93

$$\sqrt{4\,x^2-9}-6\,\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(4*x^2 - 9) - 6*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.67, size = 80, normalized size = 2.67

$$\begin{cases}\sqrt{4x^2-9}-3i\log\left(x\right)+\frac{3i\log\left(x^2\right)}{2}+3\,\mathrm{asin}\left(\frac{3}{2x}\right) & \text{for } |x^2|>\frac{9}{4}\\ i\sqrt{9-4x^2}+\frac{3i\log\left(x^2\right)}{2}-3i\log\left(\sqrt{1-\frac{4x^2}{9}}+1\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x,x)
```

```
[Out] Piecewise((sqrt(4*x**2 - 9) - 3*I*log(x) + 3*I*log(x**2)/2 + 3*asin(3/(2*x)
), Abs(x**2) > 9/4), (I*sqrt(9 - 4*x**2) + 3*I*log(x**2)/2 - 3*I*log(sqrt(1
 - 4*x**2/9) + 1), True))
```

Giac [A]
time = 0.75, size = 24, normalized size = 0.80

$$\sqrt{4\,x^2-9}-3\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(4*x^2 - 9) - 3*arctan(1/3*sqrt(4*x^2 - 9))
```

Mupad [B]
time = 5.34, size = 24, normalized size = 0.80

$$\sqrt{4\,x^2-9}-3\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x,x)
[Out] (4*x^2 - 9)^(1/2) - 3*atan((4*x^2 - 9)^(1/2)/3)
```

3.470 $\int \frac{\sqrt{-9+4x^2}}{x^2} \, dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{-9+4x^2}}{x}+2\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

```
[Out] 2*arctanh(2*x/(4*x^2-9)^(1/2))-(4*x^2-9)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {283, 223, 212}

$$2\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)-\frac{\sqrt{4x^2-9}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 + 4*x^2]/x) + 2*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx = -\frac{\sqrt{-9+4x^2}}{x} + 4\int \frac{1}{\sqrt{-9+4x^2}}\,dx$$
$$= -\frac{\sqrt{-9+4x^2}}{x} + 4\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)$$
$$= -\frac{\sqrt{-9+4x^2}}{x} + 2\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 1.03

$$-\frac{\sqrt{-9+4x^2}}{x} - 2\log\left(-2x+\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^2,x]
[Out] -(Sqrt[-9 + 4*x^2]/x) - 2*Log[-2*x + Sqrt[-9 + 4*x^2]]
```

Maple [A]
time = 0.10, size = 48, normalized size = 1.41

method	result	size
trager	$-\frac{\sqrt{4x^2-9}}{x}+2\ln\left(\sqrt{4x^2-9}+2x\right)$	32
risch	$-\frac{\sqrt{4x^2-9}}{x}+\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}$	36
default	$\frac{(4x^2-9)^{\frac{3}{2}}}{9x}-\frac{4x\sqrt{4x^2-9}}{9}+\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}$	48
meijerg	$-\frac{i\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{6i\sqrt{\pi}\sqrt{1-\frac{4x^2}{9}}}{x}-4i\sqrt{\pi}\arcsin\left(\frac{2x}{3}\right)\right)}{2\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/9/x*(4*x^2-9)^(3/2)-4/9*x*(4*x^2-9)^(1/2)+ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4
^(1/2)
```

Maxima [A]
time = 0.54, size = 33, normalized size = 0.97

$$-\frac{\sqrt{4\,x^2-9}}{x} + 2\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
[Out] -sqrt(4*x^2 - 9)/x + 2*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A]
time = 1.24, size = 35, normalized size = 1.03

$$-\frac{2\,x\log\left(-2\,x+\sqrt{4\,x^2-9}\right)+2\,x+\sqrt{4\,x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
[Out] -(2*x*log(-2*x + sqrt(4*x^2 - 9)) + 2*x + sqrt(4*x^2 - 9))/x
```

Sympy [A]
time = 0.08, size = 19, normalized size = 0.56

$$2\operatorname{acosh}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**2,x)
[Out] 2*acosh(2*x/3) - sqrt(4*x**2 - 9)/x
```

Giac [A]
time = 0.80, size = 44, normalized size = 1.29

$$-\frac{36}{\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9}-\log\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
[Out] -36/((2*x - sqrt(4*x^2 - 9))^2 + 9) - log((2*x - sqrt(4*x^2 - 9))^2)
```

Mupad [B]
time = 5.55, size = 39, normalized size = 1.15

$$-\frac{\sqrt{4\,x^2-9}}{x}-\frac{2\,\mathrm{asin}\left(\frac{2\,x}{3}\right)\,\sqrt{4\,x^2-9}}{3\,\sqrt{1-\frac{4\,x^2}{9}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^2,x)
[Out] - (4*x^2 - 9)^(1/2)/x - (2*asin((2*x)/3)*(4*x^2 - 9)^(1/2))/(3*(1 - (4*x^2)
/9)^(1/2))
```

3.471 $\int \frac{\sqrt{-9+4x^2}}{x^3} \, dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{-9+4x^2}}{2x^2}+\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

[Out] 2/3*arctan(1/3*(4*x^2-9)^(1/2))-1/2*(4*x^2-9)^(1/2)/x^2

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 43, 65, 209}

$$\frac{2}{3}\text{ArcTan}\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 + 4*x^2]/x^3,x]

[Out] -1/2*Sqrt[-9 + 4*x^2]/x^2 + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/3

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9+4x}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{-9+4x^2}}{2x^2} + \text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{-9+4x^2}}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\ &= -\frac{\sqrt{-9+4x^2}}{2x^2} + \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.00

$$-\frac{\sqrt{-9+4x^2}}{2x^2} + \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^3,x]
[Out] -1/2*Sqrt[-9 + 4*x^2]/x^2 + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/3
```

Maple [A]
time = 0.13, size = 41, normalized size = 1.05

method	result
risch	$-\frac{\sqrt{4x^2-9}}{2x^2} - \frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{3}$
default	$\frac{(4x^2-9)^{\frac{3}{2}}}{18x^2} - \frac{2\sqrt{4x^2-9}}{9} - \frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{3}$
trager	$-\frac{\sqrt{4x^2-9}}{2x^2} - \frac{2\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{4x^2-9}-3\,\text{RootOf}\left(\_Z^2+1\right)}{x}\right)}{3}$

meijerg	$\frac{\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)}{16x^2}-\frac{9\sqrt{\pi}\sqrt{1-\frac{4x^2}{9}}}{2x^2}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)-(-1+2\ln(x)-2\ln\right.}{3\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18/x^2*(4*x^2-9)^(3/2)-2/9*(4*x^2-9)^(1/2)-2/3*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A]
time = 0.51, size = 35, normalized size = 0.90

$$-\frac{2}{9}\sqrt{4x^2-9}+\frac{(4x^2-9)^{\frac{3}{2}}}{18x^2}-\frac{2}{3}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(4*x^2 - 9) + 1/18*(4*x^2 - 9)^(3/2)/x^2 - 2/3*arcsin(3/2/abs(x))
```

Fricas [A]
time = 0.99, size = 38, normalized size = 0.97

$$\frac{8x^2\arctan\left(-\frac{2}{3}x+\frac{1}{3}\sqrt{4x^2-9}\right)-3\sqrt{4x^2-9}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) - 3*sqrt(4*x^2 - 9))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.83, size = 97, normalized size = 2.49

$$\begin{cases}\frac{2i\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{3}+\frac{i}{x\sqrt{-1+\frac{9}{4x^2}}}-\frac{9i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{1}{|x^2|}>\frac{4}{9}\\ -\frac{2\,\mathrm{asin}\left(\frac{3}{2x}\right)}{3}-\frac{1}{x\sqrt{1-\frac{9}{4x^2}}}+\frac{9}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**3,x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/3 + I/(x*sqrt(-1 + 9/(4*x**2))) - 9*I/(4*x**3
*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2) > 4/9), (-2*asin(3/(2*x))/3 - 1/(x*sqr
t(1 - 9/(4*x**2))) + 9/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A]
time = 0.80, size = 29, normalized size = 0.74

$$-\frac{\sqrt{4\,x^2-9}}{2\,x^2}+\frac{2}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(4*x^2 - 9)/x^2 + 2/3*arctan(1/3*sqrt(4*x^2 - 9))
```

Mupad [B]
time = 5.33, size = 29, normalized size = 0.74

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{3}-\frac{\sqrt{4\,x^2-9}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^3,x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/3 - (4*x^2 - 9)^(1/2)/(2*x^2)
```

3.472 $\int \frac{\sqrt{-9+4x^2}}{x^4} \, dx$

Optimal. Leaf size=18

$$\frac{(-9+4x^2)^{3/2}}{27x^3}$$

```
[Out] 1/27*(4*x^2-9)^(3/2)/x^3
```

Rubi [A]

time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{(4x^2-9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^4,x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-9+4x^2}}{x^4} \, dx = \frac{(-9+4x^2)^{3/2}}{27x^3}$$

Mathematica [A]

time = 0.02, size = 18, normalized size = 1.00

$$\frac{(-9+4x^2)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^4,x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

Maple [A]
time = 0.06, size = 15, normalized size = 0.83

method	result	size
default	$\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{27x^3}$	15
trager	$\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{27x^3}$	15
gosper	$\frac{(2x-3)(2x+3)\sqrt{4x^2-9}}{27x^3}$	25
risch	$\frac{16x^4-72x^2+81}{27x^3\sqrt{4x^2-9}}$	27
meijerg	$-\frac{\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,\left(1-\frac{4x^2}{9}\right)^{\frac{3}{2}}}{\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,x^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/27*(4*x^2-9)^(3/2)/x^3
```

Maxima [A]
time = 0.55, size = 14, normalized size = 0.78

$$\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/27*(4*x^2 - 9)^(3/2)/x^3
```

Fricas [A]
time = 0.89, size = 20, normalized size = 1.11

$$\frac{8\,x^3+\left(4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/27*(8*x^3 + (4*x^2 - 9)^(3/2))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 76, normalized size = 4.22

$$\begin{cases} \frac{8i\sqrt{-1+\frac{9}{4x^2}}}{27} - \frac{2i\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ \frac{8\sqrt{1-\frac{9}{4x^2}}}{27} - \frac{2\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**4,x)
```

```
[Out] Piecewise((8*I*sqrt(-1 + 9/(4*x**2))/27 - 2*I*sqrt(-1 + 9/(4*x**2))/(3*x**2
), 1/Abs(x**2) > 4/9), (8*sqrt(1 - 9/(4*x**2))/27 - 2*sqrt(1 - 9/(4*x**2))/
(3*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.
time = 0.61, size = 42, normalized size = 2.33

$$\frac{16\left(\left(2x-\sqrt{4x^2-9}\right)^4+27\right)}{\left(\left(2x-\sqrt{4x^2-9}\right)^2+9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 16*((2*x - sqrt(4*x^2 - 9))^4 + 27)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

Mupad [B]
time = 5.10, size = 31, normalized size = 1.72

$$\frac{4x^2\sqrt{4x^2-9}-9\sqrt{4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^4,x)
```

```
[Out] (4*x^2*(4*x^2 - 9)^(1/2) - 9*(4*x^2 - 9)^(1/2))/(27*x^3)
```

3.473 $\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

```
[Out] 2/27*arctan(1/3*(4*x^2-9)^(1/2))-1/4*(4*x^2-9)^(1/2)/x^4+1/18*(4*x^2-9)^(1/
2)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 43, 44, 65, 209}

$$\frac{2}{27}\text{ArcTan}\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}}{18x^2}-\frac{\sqrt{4x^2-9}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^5,x]
```

```
[Out] -1/4*Sqrt[-9 + 4*x^2]/x^4 + Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/27
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9+4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{9}\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 0.81

$$\frac{(-9+2x^2)\sqrt{-9+4x^2}}{36x^4}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^5,x]
[Out] ((-9 + 2*x^2)*Sqrt[-9 + 4*x^2])/(36*x^4) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/2
7
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.96

method	result
risch	$\frac{8x^4-54x^2+81}{36x^4\sqrt{4x^2-9}}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}$
trager	$\frac{(2x^2-9)\sqrt{4x^2-9}}{36x^4}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{4x^2-9}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)}{x}\right)}{27}$
default	$\frac{(4x^2-9)^{\frac{3}{2}}}{36x^4}+\frac{(4x^2-9)^{\frac{3}{2}}}{162x^2}-\frac{2\sqrt{4x^2-9}}{81}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}$
meijerg	$-\frac{4\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{81\sqrt{\pi}\left(\frac{16}{81}x^4-\frac{32}{9}x^2+8\right)}{128x^4}+\frac{81\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)\sqrt{1-\frac{4x^2}{9}}}{128x^4}-\frac{\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)}{2}\right.}{27\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36/x^4*(4*x^2-9)^(3/2)+1/162/x^2*(4*x^2-9)^(3/2)-2/81*(4*x^2-9)^(1/2)-2/2
7*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A]
time = 0.61, size = 49, normalized size = 0.86

$$-\frac{2}{81}\sqrt{4x^2-9}+\frac{(4x^2-9)^{\frac{3}{2}}}{162x^2}+\frac{(4x^2-9)^{\frac{3}{2}}}{36x^4}-\frac{2}{27}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(4*x^2 - 9) + 1/162*(4*x^2 - 9)^(3/2)/x^2 + 1/36*(4*x^2 - 9)^(3/2
)/x^4 - 2/27*arcsin(3/2/abs(x))
```

Fricas [A]
time = 0.79, size = 45, normalized size = 0.79

$$\frac{16x^4\arctan\left(-\frac{2}{3}x+\frac{1}{3}\sqrt{4x^2-9}\right)+3\sqrt{4x^2-9}(2x^2-9)}{108x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*x
^2 - 9))/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 1.90, size = 139, normalized size = 2.44

$$\begin{cases} \frac{2i \operatorname{acosh}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{-1 + \frac{9}{4x^2}}} + \frac{3i}{4x^3\sqrt{-1 + \frac{9}{4x^2}}} - \frac{9i}{8x^5\sqrt{-1 + \frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{2 \operatorname{asin}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{1 - \frac{9}{4x^2}}} - \frac{3}{4x^3\sqrt{1 - \frac{9}{4x^2}}} + \frac{9}{8x^5\sqrt{1 - \frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**5,x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + 3*I/(4*x
**3*sqrt(-1 + 9/(4*x**2))) - 9*I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2
) > 4/9), (-2*asin(3/(2*x))/27 + 1/(9*x*sqrt(1 - 9/(4*x**2))) - 3/(4*x**3*s
qrt(1 - 9/(4*x**2))) + 9/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A]
time = 0.69, size = 41, normalized size = 0.72

$$\frac{(4\,x^2-9)^{\frac{3}{2}} - 9\,\sqrt{4\,x^2-9}}{72\,x^4} + \frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/72*((4*x^2 - 9)^(3/2) - 9*sqrt(4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqrt(4*x
^2 - 9))
```

Mupad [B]
time = 5.19, size = 43, normalized size = 0.75

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{27} - \frac{\frac{\sqrt{4\,x^2-9}}{8} - \frac{(4\,x^2-9)^{3/2}}{72}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^5,x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/27 - ((4*x^2 - 9)^(1/2)/8 - (4*x^2 - 9)^(3/2)
/72)/x^4
```

3.474 $\int x^5\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{27}{64}\left(-9-4x^2\right)^{3/2}-\frac{9}{160}\left(-9-4x^2\right)^{5/2}-\frac{1}{448}\left(-9-4x^2\right)^{7/2}$$

[Out] -27/64*(-4*x^2-9)^(3/2)-9/160*(-4*x^2-9)^(5/2)-1/448*(-4*x^2-9)^(7/2)

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$-\frac{1}{448}\left(-4x^2-9\right)^{7/2}-\frac{9}{160}\left(-4x^2-9\right)^{5/2}-\frac{27}{64}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^5*Sqrt[-9 - 4*x^2],x]

[Out] (-27*(-9 - 4*x^2)^(3/2))/64 - (9*(-9 - 4*x^2)^(5/2))/160 - (-9 - 4*x^2)^(7/2)/448

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{-9-4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int\sqrt{-9-4x}\,x^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{-9-4x}+\frac{9}{8}(-9-4x)^{3/2}+\frac{1}{16}(-9-4x)^{5/2}\right)dx,x,x^2\right)\\ &= -\frac{27}{64}\left(-9-4x^2\right)^{3/2}-\frac{9}{160}\left(-9-4x^2\right)^{5/2}-\frac{1}{448}\left(-9-4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(-9-4x^2\right)^{3/2}\left(-27+18x^2-10x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-9 - 4*x^2],x]
```

```
[Out] ((-9 - 4*x^2)^(3/2)*(-27 + 18*x^2 - 10*x^4))/280
```

Maple [A]
time = 0.05, size = 41, normalized size = 0.89

method	result	size
gosper	$-\frac{\left(10x^4-18x^2+27\right)\left(-4x^2-9\right)^{\frac{3}{2}}}{280}$	24
trager	$\left(\frac{1}{7}x^6+\frac{9}{140}x^4-\frac{27}{140}x^2+\frac{243}{280}\right)\sqrt{-4x^2-9}$	28
risch	$-\frac{\left(40x^6+18x^4-54x^2+243\right)\left(4x^2+9\right)}{280\sqrt{-4x^2-9}}$	36
meijerg	$-\frac{2187i\left(\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\left(1+\frac{4x^2}{9}\right)^{\frac{3}{2}}\left(\frac{80}{27}x^4-\frac{16}{3}x^2+8\right)}{105}\right)}{256\sqrt{\pi}}$	39
default	$-\frac{x^4\left(-4x^2-9\right)^{\frac{3}{2}}}{28}+\frac{9x^2\left(-4x^2-9\right)^{\frac{3}{2}}}{140}-\frac{27\left(-4x^2-9\right)^{\frac{3}{2}}}{280}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/28*x^4*(-4*x^2-9)^(3/2)+9/140*x^2*(-4*x^2-9)^(3/2)-27/280*(-4*x^2-9)^(3/
2)
```

Maxima [A]
time = 0.49, size = 40, normalized size = 0.87

$$-\frac{1}{28}\left(-4\,x^2-9\right)^{\frac{3}{2}}x^4+\frac{9}{140}\left(-4\,x^2-9\right)^{\frac{3}{2}}x^2-\frac{27}{280}\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/28*(-4*x^2 - 9)^(3/2)*x^4 + 9/140*(-4*x^2 - 9)^(3/2)*x^2 - 27/280*(-4*x^
2 - 9)^(3/2)
```

Fricas [A]
time = 1.25, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^{6}+18\,x^{4}-54\,x^{2}+243\right)\sqrt{-4\,x^{2}-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(-4*x^2 - 9)
```

Sympy [A]
time = 0.34, size = 68, normalized size = 1.48

$$\frac{x^{6}\sqrt{-4x^{2}-9}}{7}+\frac{9x^{4}\sqrt{-4x^{2}-9}}{140}-\frac{27x^{2}\sqrt{-4x^{2}-9}}{140}+\frac{243\sqrt{-4x^{2}-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-4*x**2-9)**(1/2),x)
```

```
[Out] x**6*sqrt(-4*x**2 - 9)/7 + 9*x**4*sqrt(-4*x**2 - 9)/140 - 27*x**2*sqrt(-4*x
**2 - 9)/140 + 243*sqrt(-4*x**2 - 9)/280
```

Giac [C] Result contains complex when optimal does not.
time = 0.66, size = 34, normalized size = 0.74

$$\frac{1}{448}i\left(4\,x^{2}+9\right)^{\frac{7}{2}}-\frac{9}{160}i\left(4\,x^{2}+9\right)^{\frac{5}{2}}+\frac{27}{64}i\left(4\,x^{2}+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*I*(4*x^2 + 9)^(7/2) - 9/160*I*(4*x^2 + 9)^(5/2) + 27/64*I*(4*x^2 + 9)
^(3/2)
```

Mupad [B]
time = 5.13, size = 27, normalized size = 0.59

$$\sqrt{-4\,x^{2}-9}\,\left(\frac{x^{6}}{7}+\frac{9\,x^{4}}{140}-\frac{27\,x^{2}}{140}+\frac{243}{280}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] (- 4*x^2 - 9)^(1/2)*((9*x^4)/140 - (27*x^2)/140 + x^6/7 + 243/280)
```

3.475 $\int x^4 \sqrt{-9-4x^2}\, dx$

Optimal. Leaf size=72

$$-\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

```
[Out] -729/512*arctan(2*x/(-4*x^2-9)^(1/2))-81/256*x*(-4*x^2-9)^(1/2)+3/32*x^3*(-
4*x^2-9)^(1/2)+1/6*x^5*(-4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 209}

$$-\frac{729}{512}\text{ArcTan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{81}{256}\sqrt{-4x^2-9}\,x+\frac{1}{6}\sqrt{-4x^2-9}\,x^5+\frac{3}{32}\sqrt{-4x^2-9}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[-9 - 4*x^2])/256 + (3*x^3*Sqrt[-9 - 4*x^2])/32 + (x^5*Sqrt[-9 -
 4*x^2])/6 - (729*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/512
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{-9-4x^2}\,dx &= \frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{3}{2}\int\frac{x^4}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}+\frac{81}{32}\int\frac{x^2}{\sqrt{-9-4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{256}\int\frac{1}{\sqrt{-9-4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{256}\text{Subst}\left(\int\frac{1}{1+4x^2}\right.\\ &= -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x}}\right.\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 53, normalized size = 0.74

$$\frac{1}{768}x\sqrt{-9-4x^2}\left(-243+72x^2+128x^4\right)-\frac{729}{512}i\log\left(-2ix+\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 - 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 - ((729*I)/512)*Log[(-2*
I)*x + Sqrt[-9 - 4*x^2]]
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.76

method	result	size
meijerg	$-\frac{729i\left(\frac{\sqrt{\pi}\,x\left(-\frac{640}{81}x^4-\frac{40}{9}x^2+15\right)\sqrt{1+\frac{4x^2}{9}}}{90}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{4}\right)}{128\sqrt{\pi}}$	44
risch	$-\frac{x\left(128x^4+72x^2-243\right)\left(4x^2+9\right)}{768\sqrt{-4x^2-9}}-\frac{729\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{512}$	48

default	$-\frac{x^3(-4x^2-9)^{\frac{3}{2}}}{24}+\frac{9x(-4x^2-9)^{\frac{3}{2}}}{128}-\frac{729\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{512}+\frac{81x\sqrt{-4x^2-9}}{256}$	55
trager	$\frac{x(128x^4+72x^2-243)\sqrt{-4x^2-9}}{768}+\frac{729\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{512}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*x^3*(-4*x^2-9)^(3/2)+9/128*x*(-4*x^2-9)^(3/2)-729/512*arctan(2*x/(-4*
x^2-9)^(1/2))+81/256*x*(-4*x^2-9)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 45, normalized size = 0.62

$$-\frac{1}{24}\left(-4x^2-9\right)^{\frac{3}{2}}x^3+\frac{9}{128}\left(-4x^2-9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{-4x^2-9}\,x+\frac{729}{512}i\,\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(-4*x^2 - 9)^(3/2)*x^3 + 9/128*(-4*x^2 - 9)^(3/2)*x + 81/256*sqrt(-4*
x^2 - 9)*x + 729/512*I*arcsinh(2/3*x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.23, size = 72, normalized size = 1.00

$$\frac{1}{768}\left(128x^5+72x^3-243x\right)\sqrt{-4x^2-9}-\frac{729}{1024}i\log\left(-\frac{4\left(2x+i\sqrt{-4x^2-9}\right)}{x}\right)+\frac{729}{1024}i\log\left(-\frac{4\left(2x-i\sqrt{-4x^2-9}\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(-4*x^2 - 9) - 729/1024*I*log(-4*(2*x
+ I*sqrt(-4*x^2 - 9))/x) + 729/1024*I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.57, size = 83, normalized size = 1.15

$$\frac{2ix^7}{3\sqrt{4x^2+9}}+\frac{15ix^5}{8\sqrt{4x^2+9}}-\frac{27ix^3}{64\sqrt{4x^2+9}}-\frac{729ix}{256\sqrt{4x^2+9}}+\frac{729i\operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-4*x**2-9)**(1/2),x)
```

[Out] 2*I*x**7/(3*sqrt(4*x**2 + 9)) + 15*I*x**5/(8*sqrt(4*x**2 + 9)) - 27*I*x**3/(64*sqrt(4*x**2 + 9)) - 729*I*x/(256*sqrt(4*x**2 + 9)) + 729*I*asinh(2*x/3)/512

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-4*x^2 - 9)*x^4, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{-4\,x^2-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(- 4*x^2 - 9)^(1/2),x)

[Out] int(x^4*(- 4*x^2 - 9)^(1/2), x)

3.476 $\int x^3\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{3}{16}\left(-9-4x^2\right)^{3/2}+\frac{1}{80}\left(-9-4x^2\right)^{5/2}$$

```
[Out] 3/16*(-4*x^2-9)^(3/2)+1/80*(-4*x^2-9)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{1}{80}\left(-4x^2-9\right)^{5/2}+\frac{3}{16}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (3*(-9 - 4*x^2)^(3/2))/16 + (-9 - 4*x^2)^(5/2)/80
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{-9-4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int\sqrt{-9-4x}\,x\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{9}{4}\sqrt{-9-4x}-\frac{1}{4}(-9-4x)^{3/2}\right)\,dx,x,x^2\right)\\&=\frac{3}{16}\left(-9-4x^2\right)^{3/2}+\frac{1}{80}\left(-9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(-9-4x^2\right)^{3/2}\left(3-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-9 - 4*x^2],x]
```

```
[Out] ((-9 - 4*x^2)^(3/2)*(3 - 2*x^2))/40
```

Maple [A]
time = 0.04, size = 27, normalized size = 0.87

method	result	size
gosper	$-\frac{(2x^2-3)(-4x^2-9)^{\frac{3}{2}}}{40}$	19
trager	$\left(\frac{1}{5}x^4+\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{-4x^2-9}$	23
default	$-\frac{x^2(-4x^2-9)^{\frac{3}{2}}}{20}+\frac{3(-4x^2-9)^{\frac{3}{2}}}{40}$	27
risch	$-\frac{(8x^4+6x^2-27)(4x^2+9)}{40\sqrt{-4x^2-9}}$	31
meijerg	$-\frac{243i\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(1+\frac{4x^2}{9}\right)^{\frac{3}{2}}\left(-\frac{4x^2}{3}+2\right)}{15}\right)}{64\sqrt{\pi}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*x^2*(-4*x^2-9)^(3/2)+3/40*(-4*x^2-9)^(3/2)
```

Maxima [A]
time = 0.51, size = 26, normalized size = 0.84

$$-\frac{1}{20}\left(-4\,x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*(-4*x^2 - 9)^(3/2)*x^2 + 3/40*(-4*x^2 - 9)^(3/2)
```

Fricas [A]
time = 1.12, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8\,x^4+6\,x^2-27\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 + 6*x^2 - 27)*sqrt(-4*x^2 - 9)
```

Sympy [A]
time = 0.15, size = 49, normalized size = 1.58

$$\frac{x^4\sqrt{-4x^2-9}}{5}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(-4*x**2 - 9)/5 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2 -
9)/40
```

Giac [C] Result contains complex when optimal does not.
time = 0.57, size = 23, normalized size = 0.74

$$\frac{1}{80}i\left(4x^2+9\right)^{\frac{5}{2}}-\frac{3}{16}i\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*I*(4*x^2 + 9)^(5/2) - 3/16*I*(4*x^2 + 9)^(3/2)
```

Mupad [B]
time = 5.15, size = 22, normalized size = 0.71

$$\sqrt{-4x^2-9}\left(\frac{x^4}{5}+\frac{3x^2}{20}-\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] (- 4*x^2 - 9)^(1/2)*((3*x^2)/20 + x^4/5 - 27/40)
```

3.477 $\int x^2\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=54

$$\frac{9}{32}x\sqrt{-9-4x^2}+\frac{1}{4}x^3\sqrt{-9-4x^2}+\frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

```
[Out] 81/64*arctan(2*x/(-4*x^2-9)^(1/2))+9/32*x*(-4*x^2-9)^(1/2)+1/4*x^3*(-4*x^2-
9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 223, 209}

$$\frac{81}{64}\text{ArcTan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)+\frac{9}{32}\sqrt{-4x^2-9}\,x+\frac{1}{4}\sqrt{-4x^2-9}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (9*x*Sqrt[-9 - 4*x^2])/32 + (x^3*Sqrt[-9 - 4*x^2])/4 + (81*ArcTan[(2*x)/Sqr
t[-9 - 4*x^2]])/64
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{-9-4x^2}\,dx &= \frac{1}{4}x^3\sqrt{-9-4x^2}-\frac{9}{4}\int\frac{x^2}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{9}{32}x\sqrt{-9-4x^2}+\frac{1}{4}x^3\sqrt{-9-4x^2}+\frac{81}{32}\int\frac{1}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{9}{32}x\sqrt{-9-4x^2}+\frac{1}{4}x^3\sqrt{-9-4x^2}+\frac{81}{32}\text{Subst}\left(\int\frac{1}{1+4x^2}\,dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\ &= \frac{9}{32}x\sqrt{-9-4x^2}+\frac{1}{4}x^3\sqrt{-9-4x^2}+\frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 48, normalized size = 0.89

$$\frac{1}{32}x\sqrt{-9-4x^2}\left(9+8x^2\right)+\frac{81}{64}i\log\left(-2ix+\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 - 4*x^2]*(9 + 8*x^2))/32 + ((81*I)/64)*Log[(-2*I)*x + Sqrt[-9 -
4*x^2]]
```

Maple [A]
time = 0.12, size = 41, normalized size = 0.76

method	result	size
meijerg	$-\frac{81i\left(-\frac{\sqrt{\pi}\,x\left(\frac{8x^2}{3}+3\right)\sqrt{1+\frac{4x^2}{9}}}{9}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}\right)}{32\sqrt{\pi}}$	39
default	$-\frac{x\left(-4x^2-9\right)^{\frac{3}{2}}}{16}+\frac{81\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{64}-\frac{9x\sqrt{-4x^2-9}}{32}$	41
risch	$-\frac{x\left(8x^2+9\right)\left(4x^2+9\right)}{32\sqrt{-4x^2-9}}+\frac{81\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{64}$	43

trager	$\frac{x(8x^2+9)\sqrt{-4x^2-9}}{32}+\frac{81\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{64}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*x*(-4*x^2-9)^(3/2)+81/64*arctan(2*x/(-4*x^2-9)^(1/2))-9/32*x*(-4*x^2-
9)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 31, normalized size = 0.57

$$-\frac{1}{16}\left(-4\,x^2-9\right)^{\frac{3}{2}}x-\frac{9}{32}\sqrt{-4\,x^2-9}\,x-\frac{81}{64}i\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(-4*x^2 - 9)^(3/2)*x - 9/32*sqrt(-4*x^2 - 9)*x - 81/64*I*arcsinh(2/3*
x)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.01, size = 67, normalized size = 1.24

$$\frac{1}{32}\left(8\,x^3+9\,x\right)\sqrt{-4\,x^2-9}+\frac{81}{128}i\,\log\left(-\frac{4\left(2\,x+i\,\sqrt{-4\,x^2-9}\right)}{x}\right)-\frac{81}{128}i\,\log\left(-\frac{4\left(2\,x-i\,\sqrt{-4\,x^2-9}\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 + 9*x)*sqrt(-4*x^2 - 9) + 81/128*I*log(-4*(2*x + I*sqrt(-4*x^2
- 9))/x) - 81/128*I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.65, size = 61, normalized size = 1.13

$$\frac{ix^5}{\sqrt{4x^2+9}}+\frac{27ix^3}{8\sqrt{4x^2+9}}+\frac{81ix}{32\sqrt{4x^2+9}}-\frac{81i\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-4*x**2-9)**(1/2),x)
```

```
[Out] I*x**5/sqrt(4*x**2 + 9) + 27*I*x**3/(8*sqrt(4*x**2 + 9)) + 81*I*x/(32*sqrt(
4*x**2 + 9)) - 81*I*asinh(2*x/3)/64
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\sqrt{-4\,x^2-9}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^2*(- 4*x^2 - 9)^(1/2), x)
```

3.478 $\int x\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{12}\left(-9-4x^2\right)^{3/2}$$

```
[Out] -1/12*(-4*x^2-9)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/12*(-9 - 4*x^2)^(3/2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{-9-4x^2}\,dx = -\frac{1}{12}\left(-9-4x^2\right)^{3/2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{12}\left(-9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/12*(-9 - 4*x^2)^(3/2)
```

Maple [A]
time = 0.04, size = 12, normalized size = 0.80

method	result	size
gosper	$-\frac{(-4x^2-9)^{\frac{3}{2}}}{12}$	12
derivativedivides	$-\frac{(-4x^2-9)^{\frac{3}{2}}}{12}$	12
default	$-\frac{(-4x^2-9)^{\frac{3}{2}}}{12}$	12
trager	$\left(\frac{x^2}{3}+\frac{3}{4}\right)\sqrt{-4x^2-9}$	18
risch	$-\frac{(4x^2+9)^2}{12\sqrt{-4x^2-9}}$	21
meijerg	$-\frac{27i\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{8x^2}{9}\right)\sqrt{1+\frac{4x^2}{9}}}{3}\right)}{16\sqrt{\pi}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-4*x^2-9)^(3/2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$-\frac{1}{12}\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-4*x^2 - 9)^(3/2)
```

Fricas [A]
time = 1.10, size = 18, normalized size = 1.20

$$\frac{1}{12}\left(4\,x^2+9\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 + 9)*sqrt(-4*x^2 - 9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

time = 0.07, size = 31, normalized size = 2.07

$$\frac{x^2\sqrt{-4x^2-9}}{3}+\frac{3\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x**2-9)**(1/2),x)
[Out] x**2*sqrt(-4*x**2 - 9)/3 + 3*sqrt(-4*x**2 - 9)/4
```

Giac [C] Result contains complex when optimal does not.
time = 0.53, size = 11, normalized size = 0.73

$$\frac{1}{12}i\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/12*I*(4*x^2 + 9)^(3/2)
```

Mupad [B]
time = 0.07, size = 11, normalized size = 0.73

$$-\frac{\left(-4\,x^2-9\right)^{3/2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(- 4*x^2 - 9)^(1/2),x)
[Out] -(- 4*x^2 - 9)^(3/2)/12
```

3.479 $\int \sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

[Out] -9/4*arctan(2*x/(-4*x^2-9)^(1/2))+1/2*x*(-4*x^2-9)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {201, 223, 209}

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}\text{ArcTan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 - 4*x^2],x]

[Out] (x*Sqrt[-9 - 4*x^2])/2 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{-9-4x^2}\,dx = \frac{1}{2}x\sqrt{-9-4x^2} - \frac{9}{2}\int \frac{1}{\sqrt{-9-4x^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{-9-4x^2} - \frac{9}{2}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)$$
$$= \frac{1}{2}x\sqrt{-9-4x^2} - \frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 41, normalized size = 1.14

$$\frac{1}{2}x\sqrt{-9-4x^2} - \frac{9}{4}i\log\left(-2ix+\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2],x]
[Out] (x*Sqrt[-9 - 4*x^2])/2 - ((9*I)/4)*Log[(-2*I)*x + Sqrt[-9 - 4*x^2]]
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.81

method	result	size
default	$-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}+\frac{x\sqrt{-4x^2-9}}{2}$	29
meijerg	$-\frac{9i\left(-\frac{4\sqrt{\pi}\,x\sqrt{1+\frac{4x^2}{9}}}{3}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}}$	32
risch	$-\frac{x(4x^2+9)}{2\sqrt{-4x^2-9}}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$	36
trager	$\frac{x\sqrt{-4x^2-9}}{2}-\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -9/4*arctan(2*x/(-4*x^2-9)^(1/2))+1/2*x*(-4*x^2-9)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 19, normalized size = 0.53

$$\frac{1}{2}\sqrt{-4\,x^2-9}\,x+\frac{9}{4}i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 - 9)*x + 9/4*I*arcsinh(2/3*x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.95, size = 59, normalized size = 1.64

$$\frac{1}{2}\sqrt{-4x^2-9}\,x-\frac{9}{8}i\log\left(-\frac{4\left(2x+i\sqrt{-4x^2-9}\right)}{x}\right)+\frac{9}{8}i\log\left(-\frac{4\left(2x-i\sqrt{-4x^2-9}\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-4*x^2 - 9)*x - 9/8*I*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) + 9/8*I
*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A]
time = 0.16, size = 34, normalized size = 0.94

$$\frac{x\sqrt{-4x^2-9}}{2}-\frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2),x)
```

```
[Out] x*sqrt(-4*x**2 - 9)/2 - 9*atan(2*x/sqrt(-4*x**2 - 9))/4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9), x)
```

Mupad [B]
time = 5.03, size = 28, normalized size = 0.78

$$\frac{x\sqrt{-4x^2-9}}{2}-\frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2),x)
```

```
[Out] (x*(- 4*x^2 - 9)^(1/2))/2 - (9*atan((2*x)/(- 4*x^2 - 9)^(1/2)))/4
```

3.480 $\int \frac{\sqrt{-9-4x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{-9-4x^2}-3\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

[Out] -3*arctan(1/3*(-4*x^2-9)^(1/2))+(-4*x^2-9)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {272, 52, 65, 210}

$$\sqrt{-4x^2-9}-3\text{ArcTan}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 - 4*x^2]/x,x]

[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-9-4x^2}}{x}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{\sqrt{-9-4x}}{x}\,dx, x, x^2\right)\\
&= \sqrt{-9-4x^2} - \frac{9}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\
&= \sqrt{-9-4x^2} + \frac{9}{4}\mathrm{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\
&= \sqrt{-9-4x^2} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 1.00

$$\sqrt{-9-4x^2} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x,x]
[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.83

method	result	size
default	$\sqrt{-4x^2-9}+3\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)$	25
trager	$\sqrt{-4x^2-9}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{-4x^2-9}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)}{x}\right)$	42
meijerg	$-\frac{3i\left(4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{1+\frac{4x^2}{9}}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)-2(2+2\ln(x)-2\ln(3))\sqrt{\pi}\right)}{4\sqrt{\pi}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-4*x^2-9)^(1/2)+3*arctan(3/(-4*x^2-9)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 35, normalized size = 1.17

$$\sqrt{-4\,x^2-9}+3i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-4*x^2 - 9) + 3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.65, size = 52, normalized size = 1.73

$$\sqrt{-4\,x^2-9}-\frac{3}{2}i\,\log\left(-\frac{6\left(i\,\sqrt{-4\,x^2-9}-3\right)}{x}\right)+\frac{3}{2}i\,\log\left(-\frac{6\left(-i\,\sqrt{-4\,x^2-9}-3\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-4*x^2 - 9) - 3/2*I*log(-6*(I*sqrt(-4*x^2 - 9) - 3)/x) + 3/2*I*log(-6*
(-I*sqrt(-4*x^2 - 9) - 3)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 44, normalized size = 1.47

$$\frac{2ix}{\sqrt{1+\frac{9}{4x^2}}}-3i\,\text{asinh}\left(\frac{3}{2x}\right)+\frac{9i}{2x\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*x/sqrt(1 + 9/(4*x**2)) - 3*I*asinh(3/(2*x)) + 9*I/(2*x*sqrt(1 + 9/(4*x*
*2)))
```

Giac [A]
time = 0.53, size = 24, normalized size = 0.80

$$\sqrt{-4\,x^2-9}-3\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-4*x^2 - 9) - 3*arctan(1/3*sqrt(-4*x^2 - 9))
```

Mupad [B]
time = 4.73, size = 24, normalized size = 0.80

$$\sqrt{-4\,x^2-9}-3\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x,x)
```

```
[Out] (- 4*x^2 - 9)^(1/2) - 3*atan((- 4*x^2 - 9)^(1/2)/3)
```

3.481 $\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{-9-4x^2}}{x}-2\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

```
[Out] -2*arctan(2*x/(-4*x^2-9)^(1/2))-1/x*(-4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {283, 223, 209}

$$-2\text{ArcTan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{\sqrt{-4x^2-9}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 - 4*x^2]/x) - 2*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx = -\frac{\sqrt{-9-4x^2}}{x} - 4\int \frac{1}{\sqrt{-9-4x^2}}\,dx$$
$$= -\frac{\sqrt{-9-4x^2}}{x} - 4\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)$$
$$= -\frac{\sqrt{-9-4x^2}}{x} - 2\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 39, normalized size = 1.15

$$-\frac{\sqrt{-9-4x^2}}{x} - 2i\log\left(-2ix + \sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 - 4*x^2]/x) - (2*I)*Log[(-2*I)*x + Sqrt[-9 - 4*x^2]]
```

Maple [A]
time = 0.12, size = 43, normalized size = 1.26

method	result	size
meijerg	$-\frac{i\left(\frac{6\sqrt{\pi}\,\sqrt{1+\frac{4x^2}{9}}}{x}-4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)\right)}{2\sqrt{\pi}}$	34
risch	$\frac{4x^2+9}{x\sqrt{-4x^2-9}} - 2\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$	37
default	$\frac{(-4x^2-9)^{\frac{3}{2}}}{9x} - 2\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right) + \frac{4x\sqrt{-4x^2-9}}{9}$	43
trager	$-\frac{\sqrt{-4x^2-9}}{x} + 2\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(2\,\text{RootOf}\left(\_Z^2+1\right)x+\sqrt{-4x^2-9}\right)$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9/x*(-4*x^2-9)^(3/2)-2*arctan(2*x/(-4*x^2-9)^(1/2))+4/9*x*(-4*x^2-9)^(1/2
)
```

Maximā [C] Result contains complex when optimal does not.
time = 0.52, size = 21, normalized size = 0.62

$$-\frac{\sqrt{-4\,x^2-9}}{x}+2i\ \mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-4*x^2 - 9)/x + 2*I*arcsinh(2/3*x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.88, size = 64, normalized size = 1.88

$$\frac{-i\,x\log\left(-\frac{4\left(2\,x+i\,\sqrt{-4\,x^2-9}\right)}{x}\right)+i\,x\log\left(-\frac{4\left(2\,x-i\,\sqrt{-4\,x^2-9}\right)}{x}\right)-\sqrt{-4\,x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (-I*x*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) + I*x*log(-4*(2*x - I*sqrt(-4*x^
2 - 9))/x) - sqrt(-4*x^2 - 9))/x
```

Sympy [A]
time = 0.18, size = 32, normalized size = 0.94

$$-2\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{\sqrt{-4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**2,x)
```

```
[Out] -2*atan(2*x/sqrt(-4*x**2 - 9)) - sqrt(-4*x**2 - 9)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9)/x^2, x)
```

Mupad [B]
time = 4.77, size = 41, normalized size = 1.21

$$-\frac{\sqrt{-4\,x^2-9}}{x}-\frac{\mathrm{asin}\left(\frac{x\,2\mathrm{i}}{3}\right)\,\sqrt{-4\,x^2-9}\,2\mathrm{i}}{3\,\sqrt{\frac{4\,x^2}{9}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^2,x)
```

```
[Out] - (- 4*x^2 - 9)^(1/2)/x - (asin((x*2i)/3)*(- 4*x^2 - 9)^(1/2)*2i)/(3*((4*x^
2)/9 + 1)^(1/2))
```

3.482 $\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{-9-4x^2}}{2x^2}-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

```
[Out] -2/3*arctan(1/3*(-4*x^2-9)^(1/2))-1/2*(-4*x^2-9)^(1/2)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 43, 65, 210}

$$-\frac{2}{3}\text{ArcTan}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)-\frac{\sqrt{-4x^2-9}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^3,x]
```

```
[Out] -1/2*Sqrt[-9 - 4*x^2]/x^2 - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9-4x}}{x^2}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{-9-4x^2}}{2x^2}-\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{-9-4x^2}}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{-9-4x^2}\right)\\ &= -\frac{\sqrt{-9-4x^2}}{2x^2}-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$-\frac{\sqrt{-9-4x^2}}{2x^2}-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^3,x]
[Out] -1/2*Sqrt[-9 - 4*x^2]/x^2 - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/3
```

Maple [A]
time = 0.14, size = 41, normalized size = 1.05

method	result	si
risch	$\frac{4x^2+9}{2x^2\sqrt{-4x^2-9}}+\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}$	37
default	$\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{18x^2}+\frac{2\sqrt{-4x^2-9}}{9}+\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}$	41
trager	$-\frac{\sqrt{-4x^2-9}}{2x^2}-\frac{2\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{-4x^2-9}+3\,\text{RootOf}\left(\_Z^2+1\right)}{x}\right)}{3}$	47

meijerg	$-\frac{i\left(-\frac{9\sqrt{\pi}\left(8+\frac{16x^2}{9}\right)}{16x^2}+\frac{9\sqrt{\pi}\sqrt{1+\frac{4x^2}{9}}}{2x^2}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)-(-1+2\ln(x)-2\ln(3))\sqrt{\pi}+\frac{9\sqrt{\pi}}{2x^2}\right)}{3\sqrt{\pi}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18/x^2*(-4*x^2-9)^(3/2)+2/9*(-4*x^2-9)^(1/2)+2/3*arctan(3/(-4*x^2-9)^(1/2
))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.58, size = 51, normalized size = 1.31

$$\frac{2}{9}\sqrt{-4x^2-9}+\frac{(-4x^2-9)^{\frac{3}{2}}}{18x^2}+\frac{2}{3}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(-4*x^2 - 9) + 1/18*(-4*x^2 - 9)^(3/2)/x^2 + 2/3*I*log(6*sqrt(4*x^2
 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.76, size = 65, normalized size = 1.67

$$\frac{-2i\,x^2\log\left(-\frac{4\left(i\sqrt{-4x^2-9}-3\right)}{3x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\sqrt{-4x^2-9}-3\right)}{3x}\right)-3\sqrt{-4x^2-9}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(-2*I*x^2*log(-4/3*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/3*(-I*s
qrt(-4*x^2 - 9) - 3)/x) - 3*sqrt(-4*x^2 - 9))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.82, size = 27, normalized size = 0.69

$$-\frac{2i\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}-\frac{i\sqrt{1+\frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**3,x)
[Out] -2*I*asinh(3/(2*x))/3 - I*sqrt(1 + 9/(4*x**2))/x
```

Giac [A]
time = 0.53, size = 29, normalized size = 0.74

$$-\frac{\sqrt{-4\,x^2-9}}{2\,x^2}-\frac{2}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
[Out] -1/2*sqrt(-4*x^2 - 9)/x^2 - 2/3*arctan(1/3*sqrt(-4*x^2 - 9))
```

Mupad [B]
time = 4.79, size = 29, normalized size = 0.74

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{3}-\frac{\sqrt{-4\,x^2-9}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^3,x)
[Out] - (2*atan((- 4*x^2 - 9)^(1/2)/3))/3 - (- 4*x^2 - 9)^(1/2)/(2*x^2)
```

3.483 $\int \frac{\sqrt{-9-4x^2}}{x^4} dx$

Optimal. Leaf size=18

$$\frac{(-9-4x^2)^{3/2}}{27x^3}$$

```
[Out] 1/27*(-4*x^2-9)^(3/2)/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{(-4x^2-9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^4,x]
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-9-4x^2}}{x^4} dx = \frac{(-9-4x^2)^{3/2}}{27x^3}$$

Mathematica [A]
time = 0.03, size = 18, normalized size = 1.00

$$\frac{(-9-4x^2)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^4,x]
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

Maple [A]
time = 0.04, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{27x^3}$	15
default	$\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{27x^3}$	15
meijerg	$-\frac{i\left(1+\frac{4x^2}{9}\right)^{\frac{3}{2}}}{x^3}$	16
trager	$-\frac{\left(4x^2+9\right)\sqrt{-4x^2-9}}{27x^3}$	22
risch	$\frac{16x^4+72x^2+81}{27x^3\sqrt{-4x^2-9}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/27*(-4*x^2-9)^(3/2)/x^3
```

Maxima [A]
time = 0.52, size = 14, normalized size = 0.78

$$\frac{\left(-4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/27*(-4*x^2 - 9)^(3/2)/x^3
```

Fricas [A]
time = 0.97, size = 14, normalized size = 0.78

$$\frac{\left(-4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/27*(-4*x^2 - 9)^(3/2)/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 37, normalized size = 2.06

$$-\frac{8i\sqrt{1+\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{1+\frac{9}{4x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**4,x)
[Out] -8*I*sqrt(1 + 9/(4*x**2))/27 - 2*I*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(-4*x^2 - 9)/x^4, x)
```

Mupad [B]
time = 4.74, size = 31, normalized size = 1.72

$$-\frac{4\,x^2\,\sqrt{-4\,x^2-9}+9\,\sqrt{-4\,x^2-9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^4,x)
[Out] -(4*x^2*(- 4*x^2 - 9)^(1/2) + 9*(- 4*x^2 - 9)^(1/2))/(27*x^3)
```

3.484 $\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-9-4x^2}}{4x^4}-\frac{\sqrt{-9-4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

```
[Out] 2/27*arctan(1/3*(-4*x^2-9)^(1/2))-1/4*(-4*x^2-9)^(1/2)/x^4-1/18*(-4*x^2-9)^
(1/2)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {272, 43, 44, 65, 210}

$$\frac{2}{27}\text{ArcTan}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)-\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{\sqrt{-4x^2-9}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^5,x]
```

```
[Out] -1/4*Sqrt[-9 - 4*x^2]/x^4 - Sqrt[-9 - 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/27
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9-4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 0.81

$$\frac{\sqrt{-9-4x^2}\,(-9-2x^2)}{36x^4} + \frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^5,x]
[Out] (Sqrt[-9 - 4*x^2]*(-9 - 2*x^2))/(36*x^4) + (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/2
7
```

Maple [A]
time = 0.11, size = 55, normalized size = 0.96

method	result
risch	$\frac{8x^4+54x^2+81}{36x^4\sqrt{-4x^2-9}}-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}$
trager	$-\frac{(2x^2+9)\sqrt{-4x^2-9}}{36x^4}-\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{-4x^2-9}-3\operatorname{RootOf}\left(\_Z^2+1\right)}{x}\right)}{27}$
default	$\frac{(-4x^2-9)^{\frac{3}{2}}}{36x^4}-\frac{(-4x^2-9)^{\frac{3}{2}}}{162x^2}-\frac{2\sqrt{-4x^2-9}}{81}-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}$
meijerg	$-\frac{4i\left(-\frac{81\sqrt{\pi}\left(\frac{16}{81}x^4+\frac{32}{9}x^2+8\right)}{128x^4}+\frac{81\sqrt{\pi}\left(8+\frac{16x^2}{9}\right)\sqrt{1+\frac{4x^2}{9}}}{128x^4}-\frac{\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)}{2}+\frac{\left(\frac{1}{2}+2\ln(x)-2\ln(3)\right)\sqrt{\pi}}{4}+\cdots\right.}{27\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36/x^4*(-4*x^2-9)^(3/2)-1/162/x^2*(-4*x^2-9)^(3/2)-2/81*(-4*x^2-9)^(1/2)-
2/27*arctan(3/(-4*x^2-9)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 65, normalized size = 1.14

$$-\frac{2}{81}\sqrt{-4x^2-9}-\frac{(-4x^2-9)^{\frac{3}{2}}}{162x^2}+\frac{(-4x^2-9)^{\frac{3}{2}}}{36x^4}-\frac{2}{27}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(-4*x^2 - 9) - 1/162*(-4*x^2 - 9)^(3/2)/x^2 + 1/36*(-4*x^2 - 9)^(
3/2)/x^4 - 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.66, size = 72, normalized size = 1.26

$$\frac{-4i\,x^4\log\left(-\frac{4\left(i\sqrt{-4x^2-9}+3\right)}{27x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\sqrt{-4x^2-9}+3\right)}{27x}\right)-3(2x^2+9)\sqrt{-4x^2-9}}{108x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(-4*I*x^4*log(-4/27*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/27*(
-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 + 9)*sqrt(-4*x^2 - 9))/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 2.35, size = 68, normalized size = 1.19

$$\frac{2i\,\text{asinh}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{1+\frac{9}{4x^2}}} - \frac{3i}{4x^3\sqrt{1+\frac{9}{4x^2}}} - \frac{9i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**5,x)
```

```
[Out] 2*I*asinh(3/(2*x))/27 - I/(9*x*sqrt(1 + 9/(4*x**2))) - 3*I/(4*x**3*sqrt(1 +
 9/(4*x**2))) - 9*I/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Giac [C] Result contains complex when optimal does not.
time = 0.47, size = 43, normalized size = 0.75

$$-\frac{i\,(4\,x^2+9)^{\frac{3}{2}} + 9\,\sqrt{-4\,x^2-9}}{72\,x^4} + \frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*(I*(4*x^2 + 9)^(3/2) + 9*sqrt(-4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqrt
(-4*x^2 - 9))
```

Mupad [B]
time = 4.69, size = 43, normalized size = 0.75

$$\frac{2\,\text{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{27} - \frac{\frac{\sqrt{-4\,x^2-9}}{8} - \frac{(-4\,x^2-9)^{3/2}}{72}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^5,x)
```

```
[Out] (2*atan((- 4*x^2 - 9)^(1/2)/3))/27 - ((- 4*x^2 - 9)^(1/2)/8 - (- 4*x^2 - 9)
^(3/2)/72)/x^4
```

3.485 $\int \frac{x^5}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sqrt{a+bx^2}}{b^3}-\frac{2a(a+bx^2)^{3/2}}{3b^3}+\frac{(a+bx^2)^{5/2}}{5b^3}$$

```
[Out] -2/3*a*(b*x^2+a)^(3/2)/b^3+1/5*(b*x^2+a)^(5/2)/b^3+a^2*(b*x^2+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{a^2\sqrt{a+bx^2}}{b^3}+\frac{(a+bx^2)^{5/2}}{5b^3}-\frac{2a(a+bx^2)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*Sqrt[a + b*x^2])/b^3 - (2*a*(a + b*x^2)^(3/2))/(3*b^3) + (a + b*x^2)^(
5/2)/(5*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)\,dx,x,x^2\right)\\ &= \frac{a^2\sqrt{a+bx^2}}{b^3}-\frac{2a(a+bx^2)^{3/2}}{3b^3}+\frac{(a+bx^2)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\,(8a^2-4abx^2+3b^2x^4)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(8*a^2 - 4*a*b*x^2 + 3*b^2*x^4))/(15*b^3)
```

Maple [A]
time = 0.04, size = 58, normalized size = 1.04

method	result	size
gosper	$\frac{\sqrt{b\,x^2+a}\,(3b^2x^4-4ab\,x^2+8a^2)}{15b^3}$	36
trager	$\frac{\sqrt{b\,x^2+a}\,(3b^2x^4-4ab\,x^2+8a^2)}{15b^3}$	36
risch	$\frac{\sqrt{b\,x^2+a}\,(3b^2x^4-4ab\,x^2+8a^2)}{15b^3}$	36
default	$\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*x^4*(b*x^2+a)^(1/2)/b-4/5*a/b*(1/3*x^2*(b*x^2+a)^(1/2)/b-2/3*a*(b*x^2+a
)^(1/2)/b^2)
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.95

$$\frac{\sqrt{bx^2+a}\,x^4}{5\,b}-\frac{4\,\sqrt{bx^2+a}\,ax^2}{15\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/5*sqrt(b*x^2 + a)*x^4/b - 4/15*sqrt(b*x^2 + a)*a*x^2/b^2 + 8/15*sqrt(b*x^
2 + a)*a^2/b^3
```

Fricas [A]
time = 0.72, size = 35, normalized size = 0.62

$$\frac{(3\,b^2x^4-4\,abx^2+8\,a^2)\sqrt{bx^2+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*x^4 - 4*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/b^3
```

Sympy [A]
time = 0.25, size = 68, normalized size = 1.21

$$\begin{cases} \frac{8a^2\sqrt{a+bx^2}}{15b^3} - \frac{4ax^2\sqrt{a+bx^2}}{15b^2} + \frac{x^4\sqrt{a+bx^2}}{5b} & \text{for } b \neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*a*x**2*sqrt(a + b*x**2)/(1
5*b**2) + x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), (x**6/(6*sqrt(a)), True))
```

Giac [A]
time = 0.48, size = 46, normalized size = 0.82

$$\frac{\sqrt{bx^2+a}\,a^2}{b^3} + \frac{3\,(bx^2+a)^{\frac{5}{2}} - 10\,(bx^2+a)^{\frac{3}{2}}a}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)*a^2/b^3 + 1/15*(3*(b*x^2 + a)^(5/2) - 10*(b*x^2 + a)^(3/2)*
a)/b^3
```

Mupad [B]
time = 4.67, size = 36, normalized size = 0.64

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a^2}{15\,b^3}+\frac{x^4}{5\,b}-\frac{4\,a\,x^2}{15\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^2)/(15*b^3) + x^4/(5*b) - (4*a*x^2)/(15*b^2))
```

3.486 $\int \frac{x^4}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=73

$$-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

```
[Out] 3/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-3/8*a*x*(b*x^2+a)^(1/2)/
b^2+1/4*x^3*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2],x]
```

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx = \frac{x^3\sqrt{a+bx^2}}{4b} - \frac{(3a)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{(3a^2)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^2}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\,(-3ax+2bx^3)}{8b^2} - \frac{3a^2\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a*x + 2*b*x^3))/(8*b^2) - (3*a^2*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A]
time = 0.03, size = 63, normalized size = 0.86

method	result	size
risch	$-\frac{x(-2b\,x^2+3a)\sqrt{b\,x^2+a}}{8b^2}+\frac{3a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}$	51
default	$\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^3*(b*x^2+a)^(1/2)/b-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.70

$$\frac{\sqrt{bx^2+a}\,x^3}{4\,b}-\frac{3\,\sqrt{bx^2+a}\,ax}{8\,b^2}+\frac{3\,a^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*x^3/b - 3/8*sqrt(b*x^2 + a)*a*x/b^2 + 3/8*a^2*arcsinh(b
*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 0.89, size = 124, normalized size = 1.70

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(2\,b^2x^3-3\,abx)\sqrt{bx^2+a}}{16\,b^3},-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^3-3\,abx)\sqrt{bx^2+a}}{8\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 2.49, size = 95, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x*
*2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*sqr
t(1 + b*x**2/a))
```

Giac [A]
time = 0.46, size = 54, normalized size = 0.74

$$\frac{1}{8}\,\sqrt{bx^2+a}\,x\left(\frac{2\,x^2}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(1/2), x)
```

3.487 $\int \frac{x^3}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{a\sqrt{a+bx^2}}{b^2}+\frac{(a+bx^2)^{3/2}}{3b^2}$$

```
[Out] 1/3*(b*x^2+a)^(3/2)/b^2-a*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{(a+bx^2)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx,x,x^2\right)\\ &= -\frac{a\sqrt{a+bx^2}}{b^2}+\frac{(a+bx^2)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.75

$$\frac{(-2a + bx^2)\sqrt{a + bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2],x]
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

Maple [A]
time = 0.04, size = 34, normalized size = 0.94

method	result	size
gosper	$-\frac{\sqrt{b\,x^2+a}\,(-b\,x^2+2a)}{3b^2}$	25
trager	$-\frac{\sqrt{b\,x^2+a}\,(-b\,x^2+2a)}{3b^2}$	25
risch	$-\frac{\sqrt{b\,x^2+a}\,(-b\,x^2+2a)}{3b^2}$	25
default	$\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^2*(b*x^2+a)^(1/2)/b-2/3*a*(b*x^2+a)^(1/2)/b^2
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.92

$$\frac{\sqrt{bx^2 + a}\,x^2}{3\,b} - \frac{2\,\sqrt{bx^2 + a}\,a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(b*x^2 + a)*x^2/b - 2/3*sqrt(b*x^2 + a)*a/b^2
```

Fricas [A]
time = 1.19, size = 23, normalized size = 0.64

$$\frac{\sqrt{bx^2 + a}\,(bx^2 - 2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

Sympy [A]
time = 0.21, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2} + \frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne
(b, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A]
time = 0.51, size = 30, normalized size = 0.83

$$\frac{(bx^2+a)^{\frac{3}{2}}}{3\,b^2} - \frac{\sqrt{bx^2+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)/b^2 - sqrt(b*x^2 + a)*a/b^2
```

Mupad [B]
time = 4.73, size = 24, normalized size = 0.67

$$-\frac{\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(1/2),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

3.488 $\int \frac{x^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 223, 212}

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$
$$= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$
$$= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 1.04

$$\frac{x\sqrt{a+bx^2}}{2b} + \frac{a\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) + (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3
/2))
```

Maple [A]
time = 0.03, size = 39, normalized size = 0.80

method	result	size
default	$\frac{x\sqrt{b\,x^2+a}}{2b} - \frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	39
risch	$\frac{x\sqrt{b\,x^2+a}}{2b} - \frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.63

$$\frac{\sqrt{bx^2+a}\,x}{2\,b} - \frac{a\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b - 1/2*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 1.06, size = 93, normalized size = 1.90

$$\left[\frac{2\sqrt{bx^2+a}\,bx+a\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b^2},\frac{\sqrt{bx^2+a}\,bx+a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b^2]
```

Sympy [A]
time = 1.08, size = 42, normalized size = 0.86

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{a\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)
)
```

Giac [A]
time = 0.48, size = 40, normalized size = 0.82

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}+\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

Mupad [B]
time = 4.82, size = 56, normalized size = 1.14

$$\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{if } b=0 \\ \frac{x\sqrt{b\,x^2+a}}{2\,b}-\frac{a\ln\left(2\sqrt{b}\,x+2\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, x^3/(3*a^(1/2)), b ~= 0, (x*(a + b*x^2)^(1/2))/(2*b) - (a
*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)))
```

3.489 $\int \frac{x}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

```
[Out] (b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}}{b}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.93

method	result	size
gosper	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
derivativedivides	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
default	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
trager	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
risch	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (b*x^2+a)^(1/2)/b
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] sqrt(b*x^2 + a)/b
```

Fricas [A]
time = 1.19, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2 + a)/b
```

Sympy [A]
time = 0.17, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

Giac [A]
time = 0.45, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b
```

Mupad [B]
time = 4.58, size = 13, normalized size = 0.87

$$\frac{\sqrt{b\,x^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)/b
```

3.490 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.84

method	result	size
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 0.97, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 0.43, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [A]
time = 0.51, size = 37, normalized size = 1.48

$$\frac{1}{2}\sqrt{bx^2+a}\,x-\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

Mupad [B]
time = 0.12, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

3.491 $\int \frac{1}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {272, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2]),x]
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.16

method	result	size
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.68

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)

Fricas [A]
time = 1.13, size = 60, normalized size = 2.40

$$\left[\frac{\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a))/a]

Sympy [A]
time = 0.45, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**2+a)**(1/2),x)

[Out] -asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)

Giac [A]
time = 0.52, size = 22, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")

[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)

Mupad [B]
time = 4.87, size = 19, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/2)),x)
[Out] -atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.492 $\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

[Out] -(b*x^2+a)^(1/2)/a/x

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{270\}$

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a + b*x^2]),x]

[Out] -(Sqrt[a + b*x^2]/(a*x))

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{ax}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a + b*x^2]),x]

[Out] -(Sqrt[a + b*x^2]/(a*x))

Maple [A]
time = 0.03, size = 18, normalized size = 0.95

method	result	size
gosper	$-\frac{\sqrt{b\,x^{2}+a}}{ax}$	18
default	$-\frac{\sqrt{b\,x^{2}+a}}{ax}$	18
trager	$-\frac{\sqrt{b\,x^{2}+a}}{ax}$	18
risch	$-\frac{\sqrt{b\,x^{2}+a}}{ax}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(b*x^2+a)^(1/2)/a/x
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -sqrt(b*x^2 + a)/(a*x)
```

Fricas [A]
time = 0.82, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*x^2 + a)/(a*x)
```

Sympy [A]
time = 0.31, size = 19, normalized size = 1.00

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Giac [A]
time = 0.52, size = 30, normalized size = 1.58

$$\frac{2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B]
time = 4.60, size = 17, normalized size = 0.89

$$-\frac{\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] -(a + b*x^2)^(1/2)/(a*x)
```

3.493 $\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{a+bx^2}}{2ax^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[Out] 1/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 44, 65, 214}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2]/(a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3
/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a}\\ &= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a}\\ &= -\frac{\sqrt{a+bx^2}}{2ax^2} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{2ax^2} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2]/(a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3
/2))
```

Maple [A]
time = 0.04, size = 48, normalized size = 0.96

method	result	size
default	$-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}$	48

risch	$-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.72

$$\frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^{2}+a}}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)/(a*x^2)
```

Fricas [A]
time = 0.91, size = 105, normalized size = 2.10

$$\left[\frac{\sqrt{a}\,bx^{2}\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\,\sqrt{bx^{2}+a}\,a}{4\,a^{2}x^{2}},-\frac{\sqrt{-a}\,bx^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\sqrt{bx^{2}+a}\,a}{2\,a^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

Sympy [A]
time = 1.15, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx^{2}}+1}}{2ax}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**
(3/2))
```

Giac [A]
time = 0.54, size = 51, normalized size = 1.02

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{bx^2+a}\,b}{ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)*b
/(a*x^2))/b
```

Mupad [B]
time = 4.75, size = 38, normalized size = 0.76

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^2)^(1/2)/(2*a*x
^2)
```

3.494 $\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=44

$$-\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}$$

```
[Out] -1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {277, 270}

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^2]/(a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx &= -\frac{\sqrt{a+bx^2}}{3ax^3}-\frac{(2b)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\ &= -\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\,(-a+2bx^2)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2]),x]
[Out] (Sqrt[a + b*x^2]*(-a + 2*b*x^2))/(3*a^2*x^3)
```

Maple [A]
time = 0.03, size = 37, normalized size = 0.84

method	result	size
gosper	$-\frac{\sqrt{b\,x^2+a}\,(-2b\,x^2+a)}{3a^2x^3}$	26
trager	$-\frac{\sqrt{b\,x^2+a}\,(-2b\,x^2+a)}{3a^2x^3}$	26
risch	$-\frac{\sqrt{b\,x^2+a}\,(-2b\,x^2+a)}{3a^2x^3}$	26
default	$-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b*(b*x^2+a)^(1/2)/a^2/x
```

Maxima [A]
time = 0.31, size = 36, normalized size = 0.82

$$\frac{2\,\sqrt{bx^2+a}\,b}{3\,a^2x}-\frac{\sqrt{bx^2+a}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(b*x^2 + a)*b/(a^2*x) - 1/3*sqrt(b*x^2 + a)/(a*x^3)
```

Fricas [A]
time = 0.92, size = 27, normalized size = 0.61

$$\frac{(2\,bx^2-a)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A]
time = 0.40, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a**2)
```

Giac [A]
time = 0.65, size = 55, normalized size = 1.25

$$\frac{4\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3
```

Mupad [B]
time = 4.62, size = 25, normalized size = 0.57

$$-\frac{\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

3.495 $\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{a+bx^2}}{4ax^4}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

```
[Out] -3/8*b^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)-1/4*(b*x^2+a)^(1/2)/a/x^4
+3/8*b*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 44, 65, 214}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{\sqrt{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/4*Sqrt[a + b*x^2]/(a*x^4) + (3*b*Sqrt[a + b*x^2])/(8*a^2*x^2) - (3*b^2*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4}-\frac{(3b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{8a}\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}+\frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{16a^2}\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}+\frac{(3b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{8a^2}\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\,(-2a+3bx^2)}{8a^2x^4}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a + 3*b*x^2))/(8*a^2*x^4) - (3*b^2*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(8*a^(5/2))
```

Maple [A]
time = 0.04, size = 72, normalized size = 0.97

method	result	size

risch	$-\frac{\sqrt{b\,x^{2}+a}\,(-3b\,x^{2}+2a)}{8a^{2}x^{4}}-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{8a^{\frac{5}{2}}}$	60
default	$-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*x^2+a)^(1/2)/a/x^4-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2
)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.76

$$-\frac{3\,b^{2}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{3\,\sqrt{bx^{2}+a}\,b}{8\,a^{2}x^{2}}-\frac{\sqrt{bx^{2}+a}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/8*sqrt(b*x^2 + a)*b/(a^2
*x^2) - 1/4*sqrt(b*x^2 + a)/(a*x^4)
```

Fricas [A]
time = 1.21, size = 135, normalized size = 1.82

$$\left[\frac{3\,\sqrt{a}\,b^{2}x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(3\,abx^{2}-2\,a^{2})\sqrt{bx^{2}+a}}{16\,a^{3}x^{4}},\ \frac{3\,\sqrt{-a}\,b^{2}x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(3\,abx^{2}-2\,a^{2})\sqrt{bx^{2}+a}}{8\,a^{3}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/(a^3*x^4), 1/8*(3*sqrt(-a)*b^2*x
^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/
(a^3*x^4)]
```

Sympy [A]
time = 2.56, size = 97, normalized size = 1.31

$$-\frac{1}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] -1/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)/(8*a*x**3*sqrt(a/(b*x**2
) + 1)) + 3*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**2*asinh(sqrt(a)
/(sqrt(b)*x))/(8*a**(5/2))
```

Giac [A]
time = 0.66, size = 75, normalized size = 1.01

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,(bx^2+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx^2+a}\,ab^3}{a^2b^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)
^(3/2)*b^3 - 5*sqrt(b*x^2 + a)*a*b^3)/(a^2*b^2*x^4))/b
```

Mupad [B]
time = 4.73, size = 57, normalized size = 0.77

$$\frac{3\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{5\,\sqrt{b\,x^2+a}}{8\,a\,x^4}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(1/2)),x)
```

```
[Out] (3*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (5*(a + b*x^2)^(1/2))/(8*a*x^4) - (3*b^
2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

3.496 $\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{(a+bx^2)^{3/2}}{3b^3}$$

```
[Out] 1/3*(b*x^2+a)^(3/2)/b^3-a^2/b^3/(b*x^2+a)^(1/2)-2*a*(b*x^2+a)^(1/2)/b^3
```

Rubi [A]

time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$-\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{(a+bx^2)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2/(b^3*Sqrt[a + b*x^2])) - (2*a*Sqrt[a + b*x^2])/b^3 + (a + b*x^2)^(3/2
)/(3*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx, x, x^2\right)\\
&= -\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{(a+bx^2)^{3/2}}{3b^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.69

$$\frac{-8a^2-4abx^2+b^2x^4}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(3/2),x]
[Out] (-8*a^2 - 4*a*b*x^2 + b^2*x^4)/(3*b^3*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 57, normalized size = 1.04

method	result	size
gosper	$-\frac{-b^2x^4+4ab\,x^2+8a^2}{3\sqrt{b\,x^2+a}\,b^3}$	36
trager	$-\frac{-b^2x^4+4ab\,x^2+8a^2}{3\sqrt{b\,x^2+a}\,b^3}$	36
risch	$-\frac{(-b\,x^2+5a)\sqrt{b\,x^2+a}}{3b^3}-\frac{a^2}{b^3\sqrt{b\,x^2+a}}$	43
default	$\frac{x^4}{3\sqrt{b\,x^2+a}\,b}-\frac{4a\left(\frac{x^2}{\sqrt{b\,x^2+a}\,b}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)}{3b}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^4/(b*x^2+a)^(1/2)/b-4/3*a/b*(x^2/(b*x^2+a)^(1/2)/b+2*a/b^2/(b*x^2+a)^
(1/2))
```

Maxima [A]
time = 0.29, size = 53, normalized size = 0.96

$$\frac{x^4}{3\sqrt{bx^2+a}\,b}-\frac{4\,ax^2}{3\sqrt{bx^2+a}\,b^2}-\frac{8\,a^2}{3\sqrt{bx^2+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] 1/3*x^4/(sqrt(b*x^2 + a)*b) - 4/3*a*x^2/(sqrt(b*x^2 + a)*b^2) - 8/3*a^2/(sq
rt(b*x^2 + a)*b^3)
```

Fricas [A]
time = 1.18, size = 46, normalized size = 0.84

$$\frac{(b^2x^4-4\,abx^2-8\,a^2)\sqrt{bx^2+a}}{3\,(b^4x^2+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 - 4*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a)/(b^4*x^2 + a*b^3)
```

Sympy [A]
time = 0.30, size = 68, normalized size = 1.24

$$\begin{cases} -\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x**2/(3*b**2*sqrt(a + b*
x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(3/2)), True))
```

Giac [A]
time = 0.81, size = 52, normalized size = 0.95

$$-\frac{a^2}{\sqrt{bx^2+a}\,b^3} + \frac{(bx^2+a)^{\frac{3}{2}}b^6 - 6\sqrt{bx^2+a}\,ab^6}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -a^2/(sqrt(b*x^2 + a)*b^3) + 1/3*((b*x^2 + a)^(3/2)*b^6 - 6*sqrt(b*x^2 + a)
*a*b^6)/b^9
```

Mupad [B]
time = 4.72, size = 41, normalized size = 0.75

$$-\frac{6\,a\,(b\,x^2+a) - (b\,x^2+a)^2 + 3\,a^2}{3\,b^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(3/2),x)
```

```
[Out] -(6*a*(a + b*x^2) - (a + b*x^2)^2 + 3*a^2)/(3*b^3*(a + b*x^2)^(1/2))
```

3.497 $\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[Out] -3/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-x^3/b/(b*x^2+a)^(1/2)+3/2
*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {294, 327, 223, 212}

$$-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{x^3}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(3/2),x]
```

```
[Out] -(x^3/(b*Sqrt[a + b*x^2])) + (3*x*Sqrt[a + b*x^2])/(2*b^2) - (3*a*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx &= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b}\\ &= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{(3a)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\ &= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{(3a)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\ &= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 0.88

$$\frac{3ax+bx^3}{2b^2\sqrt{a+bx^2}}+\frac{3a\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x + b*x^3)/(2*b^2*Sqrt[a + b*x^2]) + (3*a*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(2*b^(5/2))
```

Maple [A]
time = 0.05, size = 61, normalized size = 0.90

method	result	size
risch	$\frac{x\sqrt{b\,x^2+a}}{2b^2}+\frac{ax}{b^2\sqrt{b\,x^2+a}}-\frac{3a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{5}{2}}}$	54

default	$\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.33, size = 49, normalized size = 0.72

$$\frac{x^3}{2\sqrt{bx^2+a}\,b}+\frac{3\,ax}{2\sqrt{bx^2+a}\,b^2}-\frac{3\,a\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(sqrt(b*x^2 + a)*b) + 3/2*a*x/(sqrt(b*x^2 + a)*b^2) - 3/2*a*arcsinh
(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 1.12, size = 159, normalized size = 2.34

$$\left[\frac{3\,(abx^2+a^2)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(b^2x^3+3\,abx)\sqrt{bx^2+a}}{4\,(b^4x^2+ab^3)},\ \frac{3\,(abx^2+a^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(b^2x^3+3\,abx)\sqrt{bx^2+a}}{2\,(b^4x^2+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2*(3*(a*
b*x^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (b^2*x^3 + 3*a*b
*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [A]
time = 1.69, size = 71, normalized size = 1.04

$$\frac{3\sqrt{a}\,x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\,\text{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b
**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))
```

Giac [**A**]
time = 0.66, size = 51, normalized size = 0.75

$$\frac{x\left(\frac{x^2}{b}+\frac{3\,a}{b^2}\right)}{2\,\sqrt{bx^2+a}}+\frac{3\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*(x^2/b + 3*a/b^2)/sqrt(b*x^2 + a) + 3/2*a*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(5/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(3/2), x)
```

3.498 $\int \frac{x^3}{(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=32

$$\frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}$$

```
[Out] a/b^2/(b*x^2+a)^(1/2)+(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(3/2),x]
```

```
[Out] a/(b^2*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^2)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)\,dx,x,x^2\right)\\
&= \frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.75

$$\frac{2a+bx^2}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a + b*x^2)/(b^2*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.04, size = 33, normalized size = 1.03

method	result	size
gosper	$\frac{b\,x^{2}+2a}{\sqrt{b\,x^{2}+a}\,b^{2}}$	23
trager	$\frac{b\,x^{2}+2a}{\sqrt{b\,x^{2}+a}\,b^{2}}$	23
risch	$\frac{a}{b^{2}\sqrt{b\,x^{2}+a}}+\frac{\sqrt{b\,x^{2}+a}}{b^{2}}$	29
default	$\frac{x^{2}}{\sqrt{b\,x^{2}+a}\,b}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^2/(b*x^2+a)^(1/2)/b+2*a/b^2/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.27, size = 32, normalized size = 1.00

$$\frac{x^2}{\sqrt{bx^2+a}\,b}+\frac{2\,a}{\sqrt{bx^2+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(b*x^2 + a)*b) + 2*a/(sqrt(b*x^2 + a)*b^2)
```

Fricas [A]
time = 1.14, size = 34, normalized size = 1.06

$$\frac{(bx^2+2\,a)\sqrt{bx^2+a}}{b^3x^2+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

Sympy [A]
time = 0.25, size = 41, normalized size = 1.28

$$\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0
)), (x**4/(4*a**(3/2)), True))
```

Giac [A]
time = 0.61, size = 32, normalized size = 1.00

$$\frac{\frac{\sqrt{bx^2+a}}{b} + \frac{a}{\sqrt{bx^2+a}\,b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(b*x^2 + a)/b + a/(sqrt(b*x^2 + a)*b))/b
```

Mupad [B]
time = 4.70, size = 22, normalized size = 0.69

$$\frac{b\,x^2+2\,a}{b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(3/2),x)
```

```
[Out] (2*a + b*x^2)/(b^2*(a + b*x^2)^(1/2))
```

3.499 $\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{x}{b\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-x/b/(b*x^2+a)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {294, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*x^2)^(3/2),x]

[Out] -(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/b^(3/2)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx = -\frac{x}{b\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}$$
$$= -\frac{x}{b\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}$$
$$= -\frac{x}{b\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 1.07

$$-\frac{x}{b\sqrt{a+bx^2}} - \frac{\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(3/2),x]
[Out] -(x/(b*Sqrt[a + b*x^2])) - Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/b^(3/2)
```

Maple [A]
time = 0.03, size = 37, normalized size = 0.86

method	result	size
default	$-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 29, normalized size = 0.67

$$-\frac{x}{\sqrt{bx^2+a}\,b} + \frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/(sqrt(b*x^2 + a)*b) + arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 1.47, size = 130, normalized size = 3.02

$$\left[-\frac{2\sqrt{bx^2+a}\,bx-(bx^2+a)\sqrt{b}\,\log\left(-2\,bx^2-2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,(b^3x^2+ab^2)},-\frac{\sqrt{bx^2+a}\,bx+(bx^2+a)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b^3x^2+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(b*x^2 + a)*b*x - (b*x^2 + a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a))/(b^3*x^2 + a*b^2), -(sqrt(b*x^2 + a)*b*x + (b*x^2
+ a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(b^3*x^2 + a*b^2)]
```

Sympy [A]
time = 0.77, size = 37, normalized size = 0.86

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.52, size = 39, normalized size = 0.91

$$-\frac{x}{\sqrt{bx^2+a}\,b}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -x/(sqrt(b*x^2 + a)*b) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B]
time = 0.09, size = 36, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}-\frac{x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(3/2),x)
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(3/2) - x/(b*(a + b*x^2)^(1/2))
```

3.500 $\int \frac{x}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{b\sqrt{a+bx^2}}$$

```
[Out] -1/b/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(3/2),x]
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{3/2}}\,dx = -\frac{1}{b\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(3/2),x]
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

Maple [A]
time = 0.03, size = 15, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15
derivativedivides	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15
default	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15
trager	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.88

$$-\frac{1}{\sqrt{bx^{2}+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(b*x^2 + a)*b)
```

Fricas [A]
time = 0.88, size = 24, normalized size = 1.50

$$-\frac{\sqrt{bx^{2}+a}}{b^{2}x^{2}+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)/(b^2*x^2 + a*b)
```

Sympy [A]
time = 0.24, size = 24, normalized size = 1.50

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^{2}}} & \text{for}\ b\neq 0 \\ \frac{x^{2}}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

Giac [A]
time = 0.53, size = 14, normalized size = 0.88

$$-\frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(b*x^2 + a)*b)
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.88

$$-\frac{1}{b\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(3/2),x)
```

```
[Out] -1/(b*(a + b*x^2)^(1/2))
```

3.501 $\int \frac{1}{(a+bx^2)^{3/2}} \, dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+bx^2}}$$

[Out] x/a/(b*x^2+a)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {197}

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-3/2),x]

[Out] x/(a*Sqrt[a + b*x^2])

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/2}} \, dx = \frac{x}{a\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(-3/2),x]

[Out] x/(a*Sqrt[a + b*x^2])

Maple [A]
time = 0.03, size = 15, normalized size = 0.94

method	result	size
gosper	$\frac{x}{a\sqrt{b\,x^2+a}}$	15
default	$\frac{x}{a\sqrt{b\,x^2+a}}$	15
trager	$\frac{x}{a\sqrt{b\,x^2+a}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/a/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.32, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

Fricas [A]
time = 0.66, size = 23, normalized size = 1.44

$$\frac{\sqrt{bx^2+a}\,x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

Sympy [A]
time = 0.29, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.50, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2 + a}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.88

$$\frac{x}{a\sqrt{b\, x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(3/2),x)
```

```
[Out] x/(a*(a + b*x^2)^(1/2))
```

3.502 $\int \frac{1}{x(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=41

$$\frac{1}{a\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+1/a/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 53, 65, 214}

$$\frac{1}{a\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{1}{a\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}\\ &= \frac{1}{a\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}\\ &= \frac{1}{a\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 1.00

$$\frac{1}{a\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(3/2)),x]
[Out] 1/(a*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(3/2)
```

Maple [A]
time = 0.04, size = 43, normalized size = 1.05

method	result	size
default	$\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [A]
time = 0.33, size = 31, normalized size = 0.76

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{1}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 1/(sqrt(b*x^2 + a)*a)
```

Fricas [A]
time = 0.68, size = 126, normalized size = 3.07

$$\left[\frac{(bx^2+a)\sqrt{a}\,\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)+2\sqrt{bx^2+a}\,a}{2\,(a^2bx^2+a^3)},\ \frac{(bx^2+a)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\,a}{a^2bx^2+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x^2 + a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3), ((b*x^2 + a)*sqrt(-a)*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(34) = 68$.
time = 0.79, size = 184, normalized size = 4.49

$$\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**
2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/
(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) +
2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2)
+ 2*a**(7/2)*b*x**2)
```

Giac [A]
time = 0.50, size = 39, normalized size = 0.95

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{1}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/(sqrt(b*x^2 + a)*a)
```

Mupad [B]
time = 4.78, size = 33, normalized size = 0.80

$$\frac{1}{a\,\sqrt{b\,x^2+a}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(3/2)),x)
[Out] 1/(a*(a + b*x^2)^(1/2)) - atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(3/2)
```

3.503 $\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{ax\sqrt{a+bx^2}}-\frac{2bx}{a^2\sqrt{a+bx^2}}$$

```
[Out] -1/a/x/(b*x^2+a)^(1/2)-2*b*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 197}

$$-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(3/2)),x]
[Out] -(1/(a*x*Sqrt[a + b*x^2])) - (2*b*x)/(a^2*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{1}{ax\sqrt{a+bx^2}}-\frac{(2b)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{a}\\ &= -\frac{1}{ax\sqrt{a+bx^2}}-\frac{2bx}{a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 28, normalized size = 0.74

$$\frac{-a-2bx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-a - 2*b*x^2)/(a^2*x*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 35, normalized size = 0.92

method	result	size
gosper	$-\frac{2b\,x^{2}+a}{x\sqrt{b\,x^{2}+a}\,a^{2}}$	26
trager	$-\frac{2b\,x^{2}+a}{x\sqrt{b\,x^{2}+a}\,a^{2}}$	26
default	$-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}$	35
risch	$-\frac{\sqrt{b\,x^{2}+a}}{a^{2}x}-\frac{bx}{a^{2}\sqrt{b\,x^{2}+a}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x/(b*x^2+a)^(1/2)-2*b*x/a^2/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.27, size = 34, normalized size = 0.89

$$-\frac{2\,bx}{\sqrt{bx^{2}+a}\,a^{2}}-\frac{1}{\sqrt{bx^{2}+a}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*x/(sqrt(b*x^2 + a)*a^2) - 1/(sqrt(b*x^2 + a)*a*x)
```

Fricas [A]
time = 0.64, size = 35, normalized size = 0.92

$$-\frac{(2\,bx^{2}+a)\sqrt{bx^{2}+a}}{a^{2}bx^{3}+a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*b*x^2 + a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

Sympy [A]
time = 0.41, size = 46, normalized size = 1.21

$$-\frac{1}{a\sqrt{b}\,x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] -1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2)
+ 1))
```

Giac [A]
time = 0.50, size = 50, normalized size = 1.32

$$-\frac{bx}{\sqrt{bx^2+a}\,a^2}+\frac{2\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -b*x/(sqrt(b*x^2 + a)*a^2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a)*a)
```

Mupad [B]
time = 4.64, size = 35, normalized size = 0.92

$$-\frac{\sqrt{b\,x^2+a}\,\left(\frac{1}{a}+\frac{2\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(3/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(1/a + (2*b*x^2)/a^2))/(a*x + b*x^3)
```

3.504 $\int \frac{1}{x^3(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{3b}{2a^2\sqrt{a+bx^2}}-\frac{1}{2ax^2\sqrt{a+bx^2}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] 3/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)-3/2*b/a^2/(b*x^2+a)^(1/2)-1/
2/a/x^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {272, 44, 53, 65, 214}

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3b}{2a^2\sqrt{a+bx^2}}-\frac{1}{2ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-3*b)/(2*a^2*Sqrt[a + b*x^2]) - 1/(2*a*x^2*Sqrt[a + b*x^2]) + (3*b*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3 (a+bx^2)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{1}{ax^2\sqrt{a+bx^2}} + \frac{3\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}\\ &= \frac{1}{ax^2\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{2a^2x^2} - \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^2}\\ &= \frac{1}{ax^2\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{2a^2x^2} - \frac{3\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^2}\\ &= \frac{1}{ax^2\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{2a^2x^2} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 60, normalized size = 0.87

$$\frac{-a-3bx^2}{2a^2x^2\sqrt{a+bx^2}} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-a - 3*b*x^2)/(2*a^2*x^2*Sqrt[a + b*x^2]) + (3*b*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(2*a^(5/2))
```

Maple [A]
time = 0.06, size = 67, normalized size = 0.97

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}}{2 a^{2} x^{2}}-\frac{b}{a^{2} \sqrt{b\,x^{2}+a}}+\frac{3 b \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{5}{2}}}$	63
default	$-\frac{1}{2 a\,x^{2} \sqrt{b\,x^{2}+a}}-\frac{3 b \left(\frac{1}{a \sqrt{b\,x^{2}+a}}-\frac{\ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2 a}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2
*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.74

$$\frac{3\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{3\,b}{2\,\sqrt{bx^{2}+a}\,a^{2}}-\frac{1}{2\,\sqrt{bx^{2}+a}\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 3/2*b/(sqrt(b*x^2 + a)*a^2) -
 1/2/(sqrt(b*x^2 + a)*a*x^2)
```

Fricas [A]
time = 0.79, size = 171, normalized size = 2.48

$$\left[\frac{3\,(b^{2}x^{4}+abx^{2})\sqrt{a}\,\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\,(3\,abx^{2}+a^{2})\sqrt{bx^{2}+a}}{4\,(a^{3}bx^{4}+a^{4}x^{2})},\,-\frac{3\,(b^{2}x^{4}+abx^{2})\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(3\,abx^{2}+a^{2})\sqrt{bx^{2}+a}}{2\,(a^{3}bx^{4}+a^{4}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b^2*x^4 + a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a)
 + 2*a)/x^2) - 2*(3*a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2),
-1/2*(3*(b^2*x^4 + a*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*
a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [A]
time = 1.73, size = 73, normalized size = 1.06

$$-\frac{1}{2a\sqrt{b}\,x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*
x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))
```

Giac [A]
time = 0.47, size = 72, normalized size = 1.04

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^2}-\frac{3\,(bx^2+a)b-2\,ab}{2\left((bx^2+a)^{\frac{3}{2}}-\sqrt{bx^2+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/2*(3*(b*x^2 + a)
*b - 2*a*b)/(((b*x^2 + a)^(3/2) - sqrt(b*x^2 + a)*a)*a^2)
```

Mupad [B]
time = 4.94, size = 53, normalized size = 0.77

$$\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{1}{2\,a\,x^2\,\sqrt{b\,x^2+a}}-\frac{3\,b}{2\,a^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(3/2)),x)
```

```
[Out] (3*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) - 1/(2*a*x^2*(a + b*x^2)
^(1/2)) - (3*b)/(2*a^2*(a + b*x^2)^(1/2))
```

3.505 $\int \frac{1}{x^4(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{3ax^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\sqrt{a+bx^2}}$$

```
[Out] -1/3/a/x^3/(b*x^2+a)^(1/2)+4/3*b/a^2/x/(b*x^2+a)^(1/2)+8/3*b^2*x/a^3/(b*x^2
+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {277, 197}

$$\frac{8b^2x}{3a^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/3*1/(a*x^3*Sqrt[a + b*x^2]) + (4*b)/(3*a^2*x*Sqrt[a + b*x^2]) + (8*b^2*x
)/(3*a^3*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{1}{3ax^3\sqrt{a+bx^2}}-\frac{(4b)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\ &= -\frac{1}{3ax^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}+\frac{\left(8b^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}\\ &= -\frac{1}{3ax^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 0.64

$$\frac{-a^2+4abx^2+8b^2x^4}{3a^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(3/2)),x]
[Out] (-a^2 + 4*a*b*x^2 + 8*b^2*x^4)/(3*a^3*x^3*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.06, size = 59, normalized size = 0.89

method	result	size
gosper	$-\frac{-8b^2x^4-4ab\,x^2+a^2}{3x^3\sqrt{b\,x^2+a}\,a^3}$	37
trager	$-\frac{-8b^2x^4-4ab\,x^2+a^2}{3x^3\sqrt{b\,x^2+a}\,a^3}$	37
risch	$-\frac{\sqrt{b\,x^2+a}\,(-5b\,x^2+a)}{3a^3x^3}+\frac{b^2x}{a^3\sqrt{b\,x^2+a}}$	44
default	$-\frac{1}{3a\,x^3\sqrt{b\,x^2+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)}{3a}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3/a/x^3/(b*x^2+a)^(1/2)-4/3*b/a*(-1/a/x/(b*x^2+a)^(1/2)-2*b*x/a^2/(b*x^2
+a)^(1/2))
```

Maxima [A]
time = 0.35, size = 54, normalized size = 0.82

$$\frac{8\,b^2x}{3\,\sqrt{bx^2+a}\,a^3}+\frac{4\,b}{3\,\sqrt{bx^2+a}\,a^2x}-\frac{1}{3\,\sqrt{bx^2+a}\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] 8/3*b^2*x/(sqrt(b*x^2 + a)*a^3) + 4/3*b/(sqrt(b*x^2 + a)*a^2*x) - 1/3/(sqrt
(b*x^2 + a)*a*x^3)
```

Fricas [A]
time = 0.82, size = 50, normalized size = 0.76

$$\frac{(8\,b^2x^4+4\,abx^2-a^2)\sqrt{bx^2+a}}{3\,(a^3bx^5+a^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*b^2*x^4 + 4*a*b*x^2 - a^2)*sqrt(b*x^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(60) = 120$.
time = 0.58, size = 233, normalized size = 3.53

$$-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] -a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 +
3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4
*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 8*b
**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3
*a**3*b**6*x**6)
```

Giac [A]
time = 0.49, size = 106, normalized size = 1.61

$$\frac{b^2x}{\sqrt{bx^2+a}\,a^3}-\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-12\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+5\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] b^2*x/(sqrt(b*x^2 + a)*a^3) - 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2
) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 5*a^2*b^(3/2))/(((sqrt(b
)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

Mupad [B]
time = 5.12, size = 38, normalized size = 0.58

$$\frac{-a^2+4\,a\,b\,x^2+8\,b^2\,x^4}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(3/2)),x)
```

```
[Out] (8*b^2*x^4 - a^2 + 4*a*b*x^2)/(3*a^3*x^3*(a + b*x^2)^(1/2))
```

3.506 $\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{x^5}{3b(a+bx^2)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

[Out] -1/3*x^5/b/(b*x^2+a)^(3/2)-5/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)-5/3*x^3/b^2/(b*x^2+a)^(1/2)+5/2*x*(b*x^2+a)^(1/2)/b^3

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {294, 327, 223, 212}

$$-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}-\frac{x^5}{3b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b*x^2)^(5/2),x]

[Out] -1/3*x^5/(b*(a + b*x^2)^(3/2)) - (5*x^3)/(3*b^2*Sqrt[a + b*x^2]) + (5*x*Sqrt[a + b*x^2])/(2*b^3) - (5*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}+\frac{5\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{3b} \\
&= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b^2} \\
&= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{(5a)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^3} \\
&= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{(5a)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^3} \\
&= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 72, normalized size = 0.79

$$\frac{15a^2x+20abx^3+3b^2x^5}{6b^3\left(a+bx^2\right)^{3/2}}+\frac{5a\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/2),x]
```

```
[Out] (15*a^2*x + 20*a*b*x^3 + 3*b^2*x^5)/(6*b^3*(a + b*x^2)^(3/2)) + (5*a*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Maple [A]
time = 0.10, size = 83, normalized size = 0.91

method	result
default	$\frac{x^{5}}{2 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}-\frac{5 a \left(-\frac{x^{3}}{3 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{-\frac{x}{b \sqrt{b \,x^{2}+a}}+\frac{\ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2 b}$
risch	$\frac{x \sqrt{b \,x^{2}+a}}{2 b^{3}}-\frac{5 a \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{2 b^{\frac{7}{2}}}-\frac{a^{2} \sqrt{\left(x -\frac{\sqrt{-a b}}{b}\right)^{2} b +2 \sqrt{-a b}\, \left(x -\frac{\sqrt{-a b}}{b}\right)}}{12 b^{4} \sqrt{-a b}\, \left(x -\frac{\sqrt{-a b}}{b}\right)^{2}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^5/b/(b*x^2+a)^(3/2)-5/2*a/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*
x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]

time = 0.29, size = 89, normalized size = 0.98

$$\frac{x^{5}}{2 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} b} + \frac{5 \, a x {\left(\frac{3 \, x^{2}}{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b} + \frac{2 \, a}{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b^{2}}\right)}}{6 \, b} + \frac{5 \, a x}{6 \, \sqrt{b x^{2} + a} b^{3}} - \frac{5 \, a \operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^5/((b*x^2 + a)^(3/2)*b) + 5/6*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/
((b*x^2 + a)^(3/2)*b^2))/b + 5/6*a*x/(sqrt(b*x^2 + a)*b^3) - 5/2*a*arcsinh(
b*x/sqrt(a*b))/b^(7/2)
```

Fricas [A]

time = 0.73, size = 227, normalized size = 2.49

$$\left[\frac{15 \, {\left(a b^{2} x^{4} + 2 \, a^{2} b x^{2} + a^{3}\right)} \sqrt{b} \log\left(-2 \, b x^{2} + 2 \, \sqrt{b x^{2} + a} \sqrt{b} x - a\right) + 2 \, {\left(3 \, b^{3} x^{5} + 20 \, a b^{2} x^{3} + 15 \, a^{2} b x\right)} \sqrt{b x^{2} + a}}{12 \, {\left(b^{6} x^{4} + 2 \, a b^{5} x^{2} + a^{2} b^{4}\right)}}, \frac{15 \, {\left(a b^{2} x^{4} + 2 \, a^{2} b x^{2} + a^{3}\right)} \sqrt{-b} \arctan\left(\frac{\sqrt{-b} x}{\sqrt{b x^{2} + a}}\right) + {\left(3 \, b^{3} x^{5} + 20 \, a b^{2} x^{3} + 15 \, a^{2} b x\right)} \sqrt{b x^{2} + a}}{6 \, {\left(b^{6} x^{4} + 2 \, a b^{5} x^{2} + a^{2} b^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) + 2*(3*b^3*x^5 + 20*a*b^2*x^3 + 15*a^2*b*x)*sqrt(b*x
^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(15*(a*b^2*x^4 + 2*a^2*b*x^
```

```
2 + a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*b^3*x^5 + 20*a*b^
2*x^3 + 15*a^2*b*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(83) = 166$.
time = 2.73, size = 367, normalized size = 4.03

$$-\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^3}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{38}b^{\frac{49}{2}}x^5}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/2),x)
```

```
[Out] -15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2
)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2
/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/
(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt
(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x*
*2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47/2)
*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**
2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sqrt(
1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.48, size = 65, normalized size = 0.71

$$\frac{\left(x^2\left(\frac{3\,x^2}{b}+\frac{20\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)x}{6\,(bx^2+a)^{\frac{3}{2}}}+\frac{5\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(x^2*(3*x^2/b + 20*a/b^2) + 15*a^2/b^3)*x/(b*x^2 + a)^(3/2) + 5/2*a*log
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{(b\,x^2+a)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^6/(a + b*x^2)^(5/2), x)
```

3.507 $\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}$$

```
[Out] -1/3*a^2/b^3/(b*x^2+a)^(3/2)+2*a/b^3/(b*x^2+a)^(1/2)+(b*x^2+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$-\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*a^2/(b^3*(a + b*x^2)^(3/2)) + (2*a)/(b^3*Sqrt[a + b*x^2]) + Sqrt[a + b
*x^2]/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx,x,x^2\right)\\
&= -\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.72

$$\frac{8a^2+12abx^2+3b^2x^4}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(5/2),x]
[Out] (8*a^2 + 12*a*b*x^2 + 3*b^2*x^4)/(3*b^3*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.05, size = 57, normalized size = 1.06

method	result	size
gosper	$\frac{3b^2x^4+12ab\,x^2+8a^2}{3(b\,x^2+a)^{\frac{3}{2}}\,b^3}$	36
trager	$\frac{3b^2x^4+12ab\,x^2+8a^2}{3(b\,x^2+a)^{\frac{3}{2}}\,b^3}$	36
default	$\frac{x^4}{(b\,x^2+a)^{\frac{3}{2}}\,b}-\frac{4a\left(-\frac{x^2}{\left(b\,x^2+a\right)^{\frac{3}{2}}b}-\frac{2a}{3b^2\left(b\,x^2+a\right)^{\frac{3}{2}}}\right)}{b}$	57
risch	$\frac{\sqrt{b\,x^2+a}}{b^3}+\frac{\sqrt{b\,x^2+a}\,(6b\,x^2+5a)a}{3b^3(b^2x^4+2ab\,x^2+a^2)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] x^4/(b*x^2+a)^(3/2)/b-4*a/b*(-x^2/(b*x^2+a)^(3/2)/b-2/3*a/b^2/(b*x^2+a)^(3/
2))
```

Maxima [A]
time = 0.32, size = 52, normalized size = 0.96

$$\frac{x^4}{(bx^2+a)^{\frac{3}{2}}b}+\frac{4\,ax^2}{(bx^2+a)^{\frac{3}{2}}b^2}+\frac{8\,a^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] x^4/((b*x^2 + a)^(3/2)*b) + 4*a*x^2/((b*x^2 + a)^(3/2)*b^2) + 8/3*a^2/((b*x
^2 + a)^(3/2)*b^3)
```

Fricas [A]
time = 0.64, size = 58, normalized size = 1.07

$$\frac{\left(3\,b^2x^4+12\,abx^2+8\,a^2\right)\sqrt{bx^2+a}}{3\,\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*x^4 + 12*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^5*x^4 + 2*a*b^4*x^2
 + a^2*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(48) = 96$.
time = 0.36, size = 138, normalized size = 2.56

$$\begin{cases} \frac{8a^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{12abx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{3b^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)
) + 12*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2))
+ 3*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)), N
e(b, 0)), (x**6/(6*a**(5/2)), True))
```

Giac [A]
time = 0.49, size = 44, normalized size = 0.81

$$\frac{\sqrt{bx^2+a}}{b^3} + \frac{6(bx^2+a)a - a^2}{3(bx^2+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b^3 + 1/3*(6*(b*x^2 + a)*a - a^2)/((b*x^2 + a)^(3/2)*b^3)
```

Mupad [B]
time = 5.20, size = 38, normalized size = 0.70

$$\frac{2a(bx^2+a) + (bx^2+a)^2 - \frac{a^2}{3}}{b^3(bx^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*a*(a + b*x^2) + (a + b*x^2)^2 - a^2/3)/(b^3*(a + b*x^2)^(3/2))
```

3.508 $\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}-\frac{x}{b^2\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] -1/3*x^3/b/(b*x^2+a)^(3/2)+arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-x/b^2
/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {294, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{x}{b^2\sqrt{a+bx^2}}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*x^3/(b*(a + b*x^2)^(3/2)) - x/(b^2*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]]/b^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx &= -\frac{x^3}{3b(a+bx^2)^{3/2}}+\frac{\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx}{b}\\ &= -\frac{x^3}{3b(a+bx^2)^{3/2}}-\frac{x}{b^2\sqrt{a+bx^2}}+\frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}\\ &= -\frac{x^3}{3b(a+bx^2)^{3/2}}-\frac{x}{b^2\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^2}\\ &= -\frac{x^3}{3b(a+bx^2)^{3/2}}-\frac{x}{b^2\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 58, normalized size = 0.91

$$\frac{-3ax-4bx^3}{3b^2(a+bx^2)^{3/2}}-\frac{\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a*x - 4*b*x^3)/(3*b^2*(a + b*x^2)^(3/2)) - Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/b^(5/2)
```

Maple [A]
time = 0.04, size = 59, normalized size = 0.92

method	result	size
default	$-\frac{x^3}{3b(b\,x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)
+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.30, size = 65, normalized size = 1.02

$$-\frac{1}{3}\,x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)-\frac{x}{3\,\sqrt{bx^2+a}\,b^2}+\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - 1/3*x/
(sqrt(b*x^2 + a)*b^2) + arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 0.82, size = 199, normalized size = 3.11

$$\left[\frac{3\,(b^2x^4+2\,abx^2+a^2)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(4\,b^2x^3+3\,abx)\sqrt{bx^2+a}}{6\,(b^5x^4+2\,ab^4x^2+a^2b^3)},\,-\frac{3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(4\,b^2x^3+3\,abx)\sqrt{bx^2+a}}{3\,(b^5x^4+2\,ab^4x^2+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*
b^4*x^2 + a^2*b^3), -1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) + (4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*x^4
+ 2*a*b^4*x^2 + a^2*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(54) = 108$.
time = 1.44, size = 303, normalized size = 4.73

$$\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*
b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a
)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*
a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1
+ b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a
) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3
/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a))
```

Giac [A]
time = 0.46, size = 51, normalized size = 0.80

$$-\frac{x\left(\frac{4\,x^2}{b}+\frac{3\,a}{b^2}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(4*x^2/b + 3*a/b^2)/(b*x^2 + a)^(3/2) - log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(5/2), x)
```

3.509 $\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}$$

```
[Out] 1/3*a/b^2/(b*x^2+a)^(3/2)-1/b^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(5/2),x]
```

```
[Out] a/(3*b^2*(a + b*x^2)^(3/2)) - 1/(b^2*Sqrt[a + b*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)\,dx,x,x^2\right)\\ &= \frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.78

$$\frac{-2a-3bx^2}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(5/2),x]
[Out] (-2*a - 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.04, size = 34, normalized size = 0.94

method	result	size
gosper	$-\frac{3b\,x^2+2a}{3(b\,x^2+a)^{\frac{3}{2}}\,b^2}$	25
trager	$-\frac{3b\,x^2+2a}{3(b\,x^2+a)^{\frac{3}{2}}\,b^2}$	25
default	$-\frac{x^2}{(b\,x^2+a)^{\frac{3}{2}}\,b}-\frac{2a}{3b^2(b\,x^2+a)^{\frac{3}{2}}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -x^2/(b*x^2+a)^(3/2)/b-2/3*a/b^2/(b*x^2+a)^(3/2)
```

Maxima [A]
time = 0.30, size = 33, normalized size = 0.92

$$-\frac{x^2}{(bx^2+a)^{\frac{3}{2}}b}-\frac{2\,a}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -x^2/((b*x^2 + a)^(3/2)*b) - 2/3*a/((b*x^2 + a)^(3/2)*b^2)
```

Fricas [A]
time = 0.69, size = 47, normalized size = 1.31

$$-\frac{(3\,bx^2+2\,a)\sqrt{bx^2+a}}{3\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(31) = 62$.

time = 0.35, size = 92, normalized size = 2.56

$$\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2))
- 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b
, 0)), (x**4/(4*a**(5/2)), True))
```

Giac [A]

time = 0.48, size = 24, normalized size = 0.67

$$-\frac{3\,bx^2+2\,a}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*x^2 + 2*a)/((b*x^2 + a)^(3/2)*b^2)
```

Mupad [B]

time = 5.17, size = 24, normalized size = 0.67

$$-\frac{3\,b\,x^2+2\,a}{3\,b^2\,(b\,x^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(5/2),x)
```

```
[Out] -(2*a + 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

3.510 $\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=21

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*x^3/a/(b*x^2+a)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{270\}$

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/2),x]
```

```
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Mathematica [A]
time = 0.04, size = 21, normalized size = 1.00

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/2),x]
```

```
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.
time = 0.03, size = 54, normalized size = 2.57

method	result	size
gosper	$\frac{x^{3}}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	18
trager	$\frac{x^{3}}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	18
default	$-\frac{x}{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}b}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(b*x^2+a)^(3/2)/b+1/2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+
a)^(1/2))
```

Maxima [A]
time = 0.27, size = 34, normalized size = 1.62

$$-\frac{x}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{x}{3\sqrt{bx^{2}+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*x/((b*x^2 + a)^(3/2)*b) + 1/3*x/(sqrt(b*x^2 + a)*a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.69, size = 37, normalized size = 1.76

$$\frac{\sqrt{bx^{2}+a}\,x^{3}}{3\left(ab^{2}x^{4}+2\,a^{2}bx^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*x^3/(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.
time = 0.37, size = 44, normalized size = 2.10

$$\frac{x^{3}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{3}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/2),x)
[Out] x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.49, size = 17, normalized size = 0.81

$$\frac{x^3}{3\left(bx^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*x^3/((b*x^2 + a)^(3/2)*a)
```

Mupad [B]
time = 5.13, size = 17, normalized size = 0.81

$$\frac{x^3}{3\,a\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(5/2),x)
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

3.511 $\int \frac{x}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{3b\,(a+bx^2)^{3/2}}$$

[Out] -1/3/b/(b*x^2+a)^(3/2)

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$-\frac{1}{3b\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^2)^(5/2),x]

[Out] -1/3*1/(b*(a + b*x^2)^(3/2))

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3b\,(a+bx^2)^{3/2}}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{3b\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^2)^(5/2),x]

[Out] -1/3*1/(b*(a + b*x^2)^(3/2))

Maple [A]
time = 0.03, size = 15, normalized size = 0.83

method	result	size
gosper	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15
derivativedivides	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15
default	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15
trager	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b/(b*x^2+a)^(3/2)
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.78

$$-\frac{1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3/((b*x^2 + a)^(3/2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.
time = 0.90, size = 35, normalized size = 1.94

$$-\frac{\sqrt{bx^2+a}}{3\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(b*x^2 + a)/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.
time = 0.33, size = 46, normalized size = 2.56

$$\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(b
, 0)), (x**2/(2*a**(5/2)), True))
```

Giac [A]
time = 0.63, size = 14, normalized size = 0.78

$$-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3/((b*x^2 + a)^(3/2)*b)
```

Mupad [B]
time = 4.94, size = 14, normalized size = 0.78

$$-\frac{1}{3\,b\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(5/2),x)
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2))
```

3.512 $\int \frac{1}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=39

$$\frac{x}{3a(a+bx^2)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}$$

```
[Out] 1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {198, 197}

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{5/2}}\,dx &= \frac{x}{3a(a+bx^2)^{3/2}}+\frac{2\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{3a}\\ &= \frac{x}{3a(a+bx^2)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.74

$$\frac{3ax+2bx^3}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/2),x]
```

```
[Out] (3*a*x + 2*b*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.03, size = 32, normalized size = 0.82

method	result	size
gosper	$\frac{x\left(2 b \,x^{2}+3 a \right)}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{2}}$	26
trager	$\frac{x\left(2 b \,x^{2}+3 a \right)}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{2}}$	26
default	$\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.29, size = 31, normalized size = 0.79

$$\frac{2\,x}{3\,\sqrt{bx^2+a}\,a^2}+\frac{x}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

Fricas [A]
time = 1.41, size = 47, normalized size = 1.21

$$\frac{(2\,bx^3+3\,ax)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(32) = 64$.

time = 0.41, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)
) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*
x**2/a))
```

Giac [A]
time = 1.10, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

Mupad [B]
time = 4.89, size = 28, normalized size = 0.72

$$\frac{2\,x\,(b\,x^2+a)+a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*x*(a + b*x^2) + a*x)/(3*a^2*(a + b*x^2)^(3/2))
```

3.513 $\int \frac{1}{x(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=59

$$\frac{1}{3a\,(a+bx^2)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 1/3/a/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/a^2/(b*x^2
+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 53, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{1}{a^2\sqrt{a+bx^2}}+\frac{1}{3a\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*(a + b*x^2)^(3/2)) + 1/(a^2*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, x^2\right)\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)}{2a}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}} + \frac{1}{a^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a^2}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}} + \frac{1}{a^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{a^2 b}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}} + \frac{1}{a^2\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 0.92

$$\frac{4a+3bx^2}{3a^2\left(a+bx^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (4*a + 3*b*x^2)/(3*a^2*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]/a^(5/2)
```

Maple [A]
time = 0.05, size = 62, normalized size = 1.05

method	result	size
default	$\frac{1}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*
(b*x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.76

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{1}{\sqrt{bx^2+a}\,a^2}+\frac{1}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/(sqrt(b*x^2 + a)*a^2) + 1/3/((b*
x^2 + a)^(3/2)*a)
```

Fricas [A]
time = 1.76, size = 197, normalized size = 3.34

$$\left[\frac{3\,(b^2x^4+2\,abx^2+a^2)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,abx^2+4\,a^2)\sqrt{bx^2+a}}{6\,(a^3b^2x^4+2\,a^4bx^2+a^5)},\ \frac{3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,abx^2+4\,a^2)\sqrt{bx^2+a}}{3\,(a^3b^2x^4+2\,a^4bx^2+a^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4
+ 2*a^4*b*x^2 + a^5), 1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a)*arctan(sq
rt(-a)/sqrt(b*x^2 + a)) + (3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4
 + 2*a^4*b*x^2 + a^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(51) = 102$.
time = 1.38, size = 740, normalized size = 12.54

$$\frac{8a^7\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{3a^7\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{6a^7\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{14a^6bx^2\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{9a^6bx^2\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{18a^6bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{6a^5b^2x^4\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{9a^5b^2x^4\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{18a^5b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{3a^4b^3x^6\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{6a^4b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)
*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 1
8*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a*
*7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(
15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a
)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)
*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**
2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sq
rt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**
2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a*
*(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x
**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(sqr
t(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2
*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2
) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) -
 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b
*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)
```

Giac [A]

time = 1.33, size = 50, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,bx^2+4\,a}{3\,(bx^2+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(3*b*x^2 + 4*a)/((b*x
^2 + a)^(3/2)*a^2)
```

Mupad [B]

time = 5.20, size = 47, normalized size = 0.80

$$\frac{\frac{b\,x^2+a}{a^2}+\frac{1}{3\,a}}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(5/2)),x)
```

```
[Out] ((a + b*x^2)/a^2 + 1/(3*a))/(a + b*x^2)^(3/2) - atanh((a + b*x^2)^(1/2)/a^(
1/2))/a^(5/2)
```

3.514 $\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{ax\left(a+bx^2\right)^{3/2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{8bx}{3a^3\sqrt{a+bx^2}}$$

```
[Out] -1/a/x/(b*x^2+a)^(3/2)-4/3*b*x/a^2/(b*x^2+a)^(3/2)-8/3*b*x/a^3/(b*x^2+a)^(1
/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {277, 198, 197}

$$-\frac{8bx}{3a^3\sqrt{a+bx^2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(3/2))) - (4*b*x)/(3*a^2*(a + b*x^2)^(3/2)) - (8*b*x)/
(3*a^3*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^2)^{5/2}}\,dx = -\frac{1}{ax (a+bx^2)^{3/2}} - \frac{(4b)\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{a}$$
$$= -\frac{1}{ax (a+bx^2)^{3/2}} - \frac{4bx}{3a^2 (a+bx^2)^{3/2}} - \frac{(8b)\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{3a^2}$$
$$= -\frac{1}{ax (a+bx^2)^{3/2}} - \frac{4bx}{3a^2 (a+bx^2)^{3/2}} - \frac{8bx}{3a^3\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 0.70

$$\frac{-3a^2 - 12abx^2 - 8b^2x^4}{3a^3x (a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/2)),x]
[Out] (-3*a^2 - 12*a*b*x^2 - 8*b^2*x^4)/(3*a^3*x*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.06, size = 56, normalized size = 0.93

method	result	size
gosper	$-\frac{8b^2x^4+12ab\,x^2+3a^2}{3x(b\,x^2+a)^{\frac{3}{2}}a^3}$	39
trager	$-\frac{8b^2x^4+12ab\,x^2+3a^2}{3x(b\,x^2+a)^{\frac{3}{2}}a^3}$	39
default	$-\frac{1}{ax(b\,x^2+a)^{\frac{3}{2}}} - \frac{4b\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}} + \frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}$	56
risch	$-\frac{\sqrt{b\,x^2+a}}{a^3x} - \frac{\sqrt{b\,x^2+a}\;x(5b\,x^2+6a)b}{3a^3(b^2x^4+2ab\,x^2+a^2)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/a/x/(b*x^2+a)^(3/2)-4*b/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(
1/2))
```

Maxima [A]
time = 0.36, size = 50, normalized size = 0.83

$$-\frac{8\,bx}{3\,\sqrt{bx^2+a}\,a^3} - \frac{4\,bx}{3\,(bx^2+a)^{\frac{3}{2}}a^2} - \frac{1}{(bx^2+a)^{\frac{3}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*b*x/((b*x^2 + a)^(3/2)*a^2) - 1/((b*x^
2 + a)^(3/2)*a*x)
```

Fricas [A]
time = 1.27, size = 59, normalized size = 0.98

$$-\frac{(8\,b^2x^4+12\,abx^2+3\,a^2)\sqrt{bx^2+a}}{3\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*b^2*x^4 + 12*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a)/(a^3*b^2*x^5 + 2*a^4*
b*x^3 + a^5*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(56) = 112$.
time = 0.61, size = 165, normalized size = 2.75

$$-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] -3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a
**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*
a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/
(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)
```

Giac [A]
time = 1.58, size = 64, normalized size = 1.07

$$-\frac{x\left(\frac{5\,b^2x^2}{a^3}+\frac{6\,b}{a^2}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(5*b^2*x^2/a^3 + 6*b/a^2)/(b*x^2 + a)^(3/2) + 2*sqrt(b)/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)*a^2)
```

Mupad [B]
time = 5.18, size = 42, normalized size = 0.70

$$\frac{4\,a\,\left(b\,x^2+a\right)-8\,{\left(b\,x^2+a\right)}^2+a^2}{3\,a^3\,x\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(5/2)),x)
[Out] (4*a*(a + b*x^2) - 8*(a + b*x^2)^2 + a^2)/(3*a^3*x*(a + b*x^2)^(3/2))
```

3.515 $\int \frac{1}{x^3(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{5b}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{2ax^2\left(a+bx^2\right)^{3/2}}-\frac{5b}{2a^3\sqrt{a+bx^2}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] -5/6*b/a^2/(b*x^2+a)^(3/2)-1/2/a/x^2/(b*x^2+a)^(3/2)+5/2*b*arctanh((b*x^2+a
)^(1/2)/a^(1/2))/a^(7/2)-5/2*b/a^3/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 44, 53, 65, 214}

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5b}{2a^3\sqrt{a+bx^2}}-\frac{5b}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{2ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-5*b)/(6*a^2*(a + b*x^2)^(3/2)) - 1/(2*a*x^2*(a + b*x^2)^(3/2)) - (5*b)/(2
*a^3*Sqrt[a + b*x^2]) + (5*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^2\right)}{6a}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}+\frac{5\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3x^2}-\frac{(5b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.}{4a^3}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3x^2}-\frac{5\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a}\right.}{2a^3}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3x^2}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.08, size = 71, normalized size = 0.81

$$\frac{-3a^2-20abx^2-15b^2x^4}{6a^3x^2\left(a+bx^2\right)^{3/2}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(5/2)),x]
[Out] (-3*a^2 - 20*a*b*x^2 - 15*b^2*x^4)/(6*a^3*x^2*(a + b*x^2)^(3/2)) + (5*b*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Maple [A]
time = 0.09, size = 86, normalized size = 0.98

method	result
default	$-\frac{1}{2a\,x^2(b\,x^2+a)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)}{2a}$
risch	$-\frac{\sqrt{b\,x^2+a}}{2a^3x^2}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}}{12a^3\left(x-\frac{\sqrt{-ab}}{b}\right)^2}-\frac{13b\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-}}}{12a^3\sqrt{-ab}\,\left(x-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2/(b*x^2+a)^(3/2)-5/2*b/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a
)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Maxima [A]
time = 0.35, size = 66, normalized size = 0.75

$$\frac{5\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{7}{2}}}-\frac{5\,b}{2\,\sqrt{bx^2+a}\,a^3}-\frac{5\,b}{6\,(bx^2+a)^{\frac{3}{2}}a^2}-\frac{1}{2\,(bx^2+a)^{\frac{3}{2}}ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/2*b/(sqrt(b*x^2 + a)*a^3) -
 5/6*b/((b*x^2 + a)^(3/2)*a^2) - 1/2/((b*x^2 + a)^(3/2)*a*x^2)
```

Fricas [A]
time = 0.96, size = 241, normalized size = 2.74

$$\left[\frac{15\,(b^3x^6+2\,ab^2x^4+a^2bx^2)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(15\,ab^2x^4+20\,a^2bx^2+3\,a^3)\sqrt{bx^2+a}}{12\,(a^4b^2x^6+2\,a^5bx^4+a^6x^2)}, -\frac{15\,(b^3x^6+2\,ab^2x^4+a^2bx^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(15\,ab^2x^4+20\,a^2bx^2+3\,a^3)\sqrt{bx^2+a}}{6\,(a^4b^2x^6+2\,a^5bx^4+a^6x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(15*a*b^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*sq
rt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(b^3*x^6 + 2
*a*b^2*x^4 + a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*a*b
^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4
+ a^6*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(82) = 164$.
time = 2.79, size = 864, normalized size = 9.82

$$-\frac{6a^{17}\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{46a^{16}bx^2\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{15a^{16}bx^2\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}+\frac{30a^{16}bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{70a^{15}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{45a^{15}b^2x^4\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}+\frac{90a^{15}b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{30a^{14}b^3x^6\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{45a^{14}b^3x^6\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}+\frac{90a^{14}b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{15a^{13}b^4x^8\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}+\frac{30a^{13}b^4x^8\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] -6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 + b*x
**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 +
12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2
+ 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) +
30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/
2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*b**
2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**
(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x**2/
a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a
**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a*
*(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*
b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*
a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a*
*14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sqrt(
1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*
b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*
```

```
a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2
)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)
*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x*
*8)
```

Giac [A]
time = 0.96, size = 73, normalized size = 0.83

$$-\frac{5\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^3}-\frac{6\,(bx^2+a)b+ab}{3\,(bx^2+a)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx^2+a}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) - 1/3*(6*(b*x^2 + a)
*b + a*b)/((b*x^2 + a)^(3/2)*a^3) - 1/2*sqrt(b*x^2 + a)/(a^3*x^2)
```

Mupad [B]
time = 5.25, size = 73, normalized size = 0.83

$$\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}-\frac{1}{2\,a\,x^2\,\left(b\,x^2+a\right)^{3/2}}-\frac{10\,b}{3\,a^2\,\left(b\,x^2+a\right)^{3/2}}-\frac{5\,b^2\,x^2}{2\,a^3\,\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(5/2)),x)
```

```
[Out] (5*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - 1/(2*a*x^2*(a + b*x^2)
^(3/2)) - (10*b)/(3*a^2*(a + b*x^2)^(3/2)) - (5*b^2*x^2)/(2*a^3*(a + b*x^2)
^(3/2))
```

3.516 $\int \frac{1}{x^4(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2b}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{16b^2x}{3a^4\sqrt{a+bx^2}}$$

```
[Out] -1/3/a/x^3/(b*x^2+a)^(3/2)+2*b/a^2/x/(b*x^2+a)^(3/2)+8/3*b^2*x/a^3/(b*x^2+a
)^(3/2)+16/3*b^2*x/a^4/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 198, 197}

$$\frac{16b^2x}{3a^4\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2b}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/3*1/(a*x^3*(a + b*x^2)^(3/2)) + (2*b)/(a^2*x*(a + b*x^2)^(3/2)) + (8*b^2
*x)/(3*a^3*(a + b*x^2)^(3/2)) + (16*b^2*x)/(3*a^4*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^2)^{5/2}}\,dx = -\frac{1}{3ax^3 (a+bx^2)^{3/2}} - \frac{(2b)\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx}{a}$$
$$= -\frac{1}{3ax^3 (a+bx^2)^{3/2}} + \frac{2b}{a^2x (a+bx^2)^{3/2}} + \frac{(8b^2)\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{a^2}$$
$$= -\frac{1}{3ax^3 (a+bx^2)^{3/2}} + \frac{2b}{a^2x (a+bx^2)^{3/2}} + \frac{8b^2x}{3a^3 (a+bx^2)^{3/2}} + \frac{(16b^2)\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{3a^3}$$
$$= -\frac{1}{3ax^3 (a+bx^2)^{3/2}} + \frac{2b}{a^2x (a+bx^2)^{3/2}} + \frac{8b^2x}{3a^3 (a+bx^2)^{3/2}} + \frac{16b^2x}{3a^4\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.07, size = 53, normalized size = 0.62

$$\frac{-a^3+6a^2bx^2+24ab^2x^4+16b^3x^6}{3a^4x^3 (a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-a^3 + 6*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x^6)/(3*a^4*x^3*(a + b*x^2)^(3/
2))
```

Maple [A]
time = 0.08, size = 80, normalized size = 0.93

method	result	size
gosper	$-\frac{-16b^3x^6-24a\,b^2x^4-6a^2b\,x^2+a^3}{3x^3(b\,x^2+a)^{\frac{3}{2}}a^4}$	48
trager	$-\frac{-16b^3x^6-24a\,b^2x^4-6a^2b\,x^2+a^3}{3x^3(b\,x^2+a)^{\frac{3}{2}}a^4}$	48
risch	$-\frac{\sqrt{b\,x^2+a}\,(-8b\,x^2+a)}{3a^4x^3}+\frac{\sqrt{b\,x^2+a}\,x(8b\,x^2+9a)b^2}{3a^4(b^2x^4+2ab\,x^2+a^2)}$	75
default	$-\frac{1}{3a\,x^3(b\,x^2+a)^{\frac{3}{2}}}-\frac{2b\left(-\frac{1}{ax(b\,x^2+a)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}\right)}{a}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3/(b*x^2+a)^(3/2)-2*b/a*(-1/a/x/(b*x^2+a)^(3/2)-4*b/a*(1/3*x/a/(b*
x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 0.84

$$\frac{16\,b^2x}{3\,\sqrt{bx^2+a}\,a^4}+\frac{8\,b^2x}{3\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{2\,b}{(bx^2+a)^{\frac{3}{2}}a^2x}-\frac{1}{3\,(bx^2+a)^{\frac{3}{2}}ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/3*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*b^2*x/((b*x^2 + a)^(3/2)*a^3) + 2*b/
((b*x^2 + a)^(3/2)*a^2*x) - 1/3/((b*x^2 + a)^(3/2)*a*x^3)
```

Fricas [A]
time = 1.27, size = 72, normalized size = 0.84

$$\frac{(16\,b^3x^6+24\,ab^2x^4+6\,a^2bx^2-a^3)\sqrt{bx^2+a}}{3\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(16*b^3*x^6 + 24*a*b^2*x^4 + 6*a^2*b*x^2 - a^3)*sqrt(b*x^2 + a)/(a^4*b^
2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(80) = 160.
time = 0.85, size = 354, normalized size = 4.12

$$-\frac{a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{5a^3b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{30a^2b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{40ab^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{16b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] -a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4
+ 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(21/2)*x**2*sqrt(a/(b*
x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a*
*4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x
**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 40*a*b**
(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*
a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(b*x**2) +
1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12
*x**8)
```

Giac [A]
time = 0.98, size = 121, normalized size = 1.41

$$\frac{x\left(\frac{8\,b^3x^2}{a^4}+\frac{9\,b^2}{a^3}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{4\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+4\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(8*b^3*x^2/a^4 + 9*b^2/a^3)/(b*x^2 + a)^(3/2) - 4/3*(3*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*b^(3/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 4
*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^3)
```

Mupad [B]
time = 5.00, size = 78, normalized size = 0.91

$$-\frac{6\,a^2\,(b\,x^2+a)-24\,a\,(b\,x^2+a)^2+16\,(b\,x^2+a)^3+a^3}{(b\,x^2+a)^{3/2}\,\left(\frac{3\,a^5\,x}{b}-\frac{3\,a^4\,x\,(b\,x^2+a)}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(5/2)),x)
```

```
[Out] -(6*a^2*(a + b*x^2) - 24*a*(a + b*x^2)^2 + 16*(a + b*x^2)^3 + a^3)/((a + b*
x^2)^(3/2)*((3*a^5*x)/b - (3*a^4*x*(a + b*x^2))/b))
```

3.517 $\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{3x^3}{b^4\sqrt{a+bx^2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}$$

```
[Out] -1/7*x^9/b/(b*x^2+a)^(7/2)-9/35*x^7/b^2/(b*x^2+a)^(5/2)-3/5*x^5/b^3/(b*x^2+
a)^(3/2)-9/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(11/2)-3*x^3/b^4/(b*x^2
+a)^(1/2)+9/2*x*(b*x^2+a)^(1/2)/b^5
```

Rubi [A]
time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {294, 327, 223, 212}

$$-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{3x^3}{b^4\sqrt{a+bx^2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*x^9/(b*(a + b*x^2)^(7/2)) - (9*x^7)/(35*b^2*(a + b*x^2)^(5/2)) - (3*x^
5)/(5*b^3*(a + b*x^2)^(3/2)) - (3*x^3)/(b^4*Sqrt[a + b*x^2]) + (9*x*Sqrt[a
+ b*x^2])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx &= -\frac{x^9}{7b(a+bx^2)^{7/2}} + \frac{9\int \frac{x^8}{(a+bx^2)^{7/2}}\,dx}{7b} \\
&= -\frac{x^9}{7b(a+bx^2)^{7/2}} - \frac{9x^7}{35b^2(a+bx^2)^{5/2}} + \frac{9\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx}{5b^2} \\
&= -\frac{x^9}{7b(a+bx^2)^{7/2}} - \frac{9x^7}{35b^2(a+bx^2)^{5/2}} - \frac{3x^5}{5b^3(a+bx^2)^{3/2}} + \frac{3\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx}{b^3} \\
&= -\frac{x^9}{7b(a+bx^2)^{7/2}} - \frac{9x^7}{35b^2(a+bx^2)^{5/2}} - \frac{3x^5}{5b^3(a+bx^2)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9\int \frac{x^2}{\sqrt{a+bx}}}{b^4} \\
&= -\frac{x^9}{7b(a+bx^2)^{7/2}} - \frac{9x^7}{35b^2(a+bx^2)^{5/2}} - \frac{3x^5}{5b^3(a+bx^2)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} \\
&= -\frac{x^9}{7b(a+bx^2)^{7/2}} - \frac{9x^7}{35b^2(a+bx^2)^{5/2}} - \frac{3x^5}{5b^3(a+bx^2)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} \\
&= -\frac{x^9}{7b(a+bx^2)^{7/2}} - \frac{9x^7}{35b^2(a+bx^2)^{5/2}} - \frac{3x^5}{5b^3(a+bx^2)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 94, normalized size = 0.72

$$\frac{315a^4x + 1050a^3bx^3 + 1218a^2b^2x^5 + 528ab^3x^7 + 35b^4x^9}{70b^5(a+bx^2)^{7/2}} + \frac{9a\log\left(-\sqrt{b}\,x + \sqrt{a+bx^2}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^(9/2),x]
```

```
[Out] (315*a^4*x + 1050*a^3*b*x^3 + 1218*a^2*b^2*x^5 + 528*a*b^3*x^7 + 35*b^4*x^9
)/(70*b^5*(a + b*x^2)^(7/2)) + (9*a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2
*b^(11/2))
```

Maple [A]
time = 0.13, size = 127, normalized size = 0.97

method	result
default	$\frac{x^{9}}{2 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}-\frac{9 a \left(-\frac{x^{7}}{7 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5 \left(b \,x^{2}+a \right)^{\frac{5}{2}} b}+\frac{-\frac{x^{3}}{3 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{-\frac{x}{b \sqrt{b \,x^{2}+a}}+\frac{\ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)}{2 b}$
risch	$\frac{x \sqrt{b \,x^{2}+a}}{2 b^{5}}-\frac{9 a \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{2 b^{\frac{11}{2}}}-\frac{53 a^{2} \sqrt{\left(x +\frac{\sqrt{-a b}}{b}\right)^{2} b -2 \sqrt{-a b}\, \left(x +\frac{\sqrt{-a b}}{b}\right)}}{560 b^{7} \left(x +\frac{\sqrt{-a b}}{b}\right)^{3}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^9/b/(b*x^2+a)^(7/2)-9/2*a/b*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5
/(b*x^2+a)^(5/2)/b+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2
)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(105) = 210$.
time = 0.34, size = 285, normalized size = 2.18

$$\frac{x^{9}}{2 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} b} + \frac{9 \, {\left(\frac{35 \, x^{6}}{{\left(b x^{2} + a\right)}^{\frac{7}{2}} b} + \frac{70 \, a x^{4}}{{\left(b x^{2} + a\right)}^{\frac{7}{2}} b^{2}} + \frac{56 \, a^{2} x^{2}}{{\left(b x^{2} + a\right)}^{\frac{7}{2}} b^{3}} + \frac{16 \, a^{3}}{{\left(b x^{2} + a\right)}^{\frac{7}{2}} b^{4}}\right)} a x}{70 \, b} + \frac{3 \, a x {\left(\frac{15 \, x^{4}}{{\left(b x^{2} + a\right)}^{\frac{5}{2}} b} + \frac{20 \, a x^{2}}{{\left(b x^{2} + a\right)}^{\frac{5}{2}} b^{2}} + \frac{8 \, a^{2}}{{\left(b x^{2} + a\right)}^{\frac{5}{2}} b^{3}}\right)}}{10 \, b^{2}} + \frac{3 \, a x {\left(\frac{3 \, x^{2}}{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b} + \frac{2 \, a}{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b^{2}}\right)}}{2 \, b^{3}} + \frac{9 \, a^{2} x^{3}}{2 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} b^{4}} - \frac{417 \, a x}{70 \, \sqrt{b x^{2} + a} b^{5}} - \frac{51 \, a^{2} x}{70 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} b^{5}} + \frac{261 \, a^{3} x}{70 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} b^{5}} - \frac{9 \, a \operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^9/((b*x^2 + a)^(7/2)*b) + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x
^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((
b*x^2 + a)^(7/2)*b^4))*a*x/b + 3/10*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*
a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 + 3/2*a*
```

```
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 + 9/2*a^2
*x^3/((b*x^2 + a)^(5/2)*b^4) - 417/70*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*a^2
*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*a^3*x/((b*x^2 + a)^(5/2)*b^5) - 9/2*a*a
rcsinh(b*x/sqrt(a*b))/b^(11/2)
```

Fricas [A]
time = 1.48, size = 359, normalized size = 2.74

$$\left[\frac{315\,(ab^4x^8 + 4\,a^2b^3x^6 + 6\,a^3b^2x^4 + 4\,a^4bx^2 + a^5)\sqrt{b}\,\log\left(-2\,bx^2 + 2\,\sqrt{bx^2 + a}\,\sqrt{b}\,x - a\right) + 2\,(35\,b^5x^9 + 528\,ab^4x^7 + 1218\,a^2b^3x^5 + 1050\,a^3b^2x^3 + 315\,a^4bx)\sqrt{bx^2 + a}}{140\,(b^{10}x^8 + 4\,ab^9x^6 + 6\,a^2b^8x^4 + 4\,a^3b^7x^2 + a^4b^6)}, \frac{315\,(ab^4x^8 + 4\,a^2b^3x^6 + 6\,a^3b^2x^4 + 4\,a^4bx^2 + a^5)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2 + a}}\right) + (35\,b^5x^9 + 528\,ab^4x^7 + 1218\,a^2b^3x^5 + 1050\,a^3b^2x^3 + 315\,a^4bx)\sqrt{bx^2 + a}}{70\,(b^{10}x^8 + 4\,ab^9x^6 + 6\,a^2b^8x^4 + 4\,a^3b^7x^2 + a^4b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(35*b^5*x^9 +
528*a*b^4*x^7 + 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(b*x
^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
, 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (35*b^5*x^9 + 528*a*b^4*x^7
+ 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(b*x^2 + a))/(b^10
*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3181 vs. $2(122) = 244$.
time = 24.90, size = 3181, normalized size = 24.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] -315*a**(311/2)*b**66*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(3
09/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1
 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a*
*(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x*
*8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a)
+ 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*a**(309/2)*b**6
7*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2
)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) +
1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(14
9/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x
**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)
*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a**(307/2)*b**68*x**4*sqrt(1 +
 b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x*
*2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)
```

```
*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(
1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a*
*(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**
12*sqrt(1 + b*x**2/a)) - 6300*a**(305/2)*b**69*x**6*sqrt(1 + b*x**2/a)*asin
h(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**
(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**
4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) +
 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(15
3/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x
**2/a)) - 4725*a**(303/2)*b**70*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqr
t(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145
/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x*
*2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2
)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt
(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*
a**(301/2)*b**71*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(
309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(
1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a
**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x
**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a)
 + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 315*a**(299/2)*b**7
2*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/
2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) +
 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(1
49/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*
x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2
)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 315*a**155*b**(133/2)*x/(70*a**(30
9/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1
+ b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**
(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**
8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) +
 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 1995*a**154*b**(135/2
)*x**3/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(14
5/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x
**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/
2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqr
t(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 5313
*a**153*b**(137/2)*x**5/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*
a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*
x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a
) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**
(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 +
b*x**2/a)) + 7647*a**152*b**(139/2)*x**7/(70*a**(309/2)*b**(143/2)*sqrt(1 +
 b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(3
05/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*
```

```
sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 4
20*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2
)*x**12*sqrt(1 + b*x**2/a)) + 6323*a**151*b**(141/2)*x**9/(70*a**(309/2)*b*
*(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**
2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)
*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(
1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**
(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + ...
```

Giac [A]
time = 0.91, size = 91, normalized size = 0.69

$$\frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b}+\frac{528\,a}{b^2}\right)+\frac{1218\,a^2}{b^3}\right)x^2+\frac{1050\,a^3}{b^4}\right)x^2+\frac{315\,a^4}{b^5}\right)x}{70\,(bx^2+a)^{\frac{7}{2}}}+\frac{9\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
+ 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(11/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^10/(a + b*x^2)^(9/2), x)
```

3.518 $\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}$$

```
[Out] -1/7*a^4/b^5/(b*x^2+a)^(7/2)+4/5*a^3/b^5/(b*x^2+a)^(5/2)-2*a^2/b^5/(b*x^2+a
)^(3/2)+4*a/b^5/(b*x^2+a)^(1/2)+(b*x^2+a)^(1/2)/b^5
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$-\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*a^4/(b^5*(a + b*x^2)^(7/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - (2*a
^2)/(b^5*(a + b*x^2)^(3/2)) + (4*a)/(b^5*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]
/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{9/2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{9/2}} - \frac{4a^3}{b^4(a+bx)^{7/2}} + \frac{6a^2}{b^4(a+bx)^{5/2}} - \frac{4a}{b^4(a+bx)^{3/2}} + \frac{1}{b^4\sqrt{a+}}\right.\right.\\ &= -\frac{a^4}{7b^5(a+bx^2)^{7/2}} + \frac{4a^3}{5b^5(a+bx^2)^{5/2}} - \frac{2a^2}{b^5(a+bx^2)^{3/2}} + \frac{4a}{b^5\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.65

$$\frac{128a^4+448a^3bx^2+560a^2b^2x^4+280ab^3x^6+35b^4x^8}{35b^5(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^(9/2),x]
```

```
[Out] (128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^4*x^8)/(3
5*b^5*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.10, size = 105, normalized size = 1.12

method	result	size
gosper	$\frac{35b^4x^8+280a\,b^3x^6+560a^2b^2x^4+448a^3b\,x^2+128a^4}{35(b\,x^2+a)^{\frac{7}{2}}b^5}$	58
trager	$\frac{35b^4x^8+280a\,b^3x^6+560a^2b^2x^4+448a^3b\,x^2+128a^4}{35(b\,x^2+a)^{\frac{7}{2}}b^5}$	58
risch	$\frac{\sqrt{b\,x^2+a}}{b^5}+\frac{\sqrt{b\,x^2+a}\,(140b^3x^6+350a\,b^2x^4+308a^2b\,x^2+93a^3)a}{35b^5(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$	104
default	$\frac{x^8}{b(b\,x^2+a)^{\frac{7}{2}}}-\frac{8a\left(-\frac{x^6}{b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^4}{3b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^2}{5b\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^2\left(b\,x^2+a\right)^{\frac{7}{2}}}\right)}{3b}\right)}{b}\right)}{b}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^8/b/(b*x^2+a)^(7/2)-8*a/b*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^
2+a)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2)))
)
```

Maxima [A]
time = 0.30, size = 92, normalized size = 0.98

$$\frac{x^8}{(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,ax^6}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{16\,a^2x^4}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{64\,a^3x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^4}+\frac{128\,a^4}{35\,(bx^2+a)^{\frac{7}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] x^8/((b*x^2 + a)^(7/2)*b) + 8*a*x^6/((b*x^2 + a)^(7/2)*b^2) + 16*a^2*x^4/((
b*x^2 + a)^(7/2)*b^3) + 64/5*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) + 128/35*a^4/(
(b*x^2 + a)^(7/2)*b^5)
```

Fricas [A]
time = 0.91, size = 102, normalized size = 1.09

$$\frac{(35\,b^4x^8+280\,ab^3x^6+560\,a^2b^2x^4+448\,a^3bx^2+128\,a^4)\sqrt{bx^2+a}}{35\,(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^
4)*sqrt(b*x^2 + a)/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 +
 a^4*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(87) = 174.
time = 0.88, size = 454, normalized size = 4.83

$$\begin{cases} \frac{128a^4}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}}+\frac{448a^3bx^2}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}}+\frac{560a^2b^2x^4}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}}+\frac{280ab^3x^6}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}}+\frac{35b^4x^8}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} & \text{for}\: b \neq 0 \\ \frac{x^{10}}{10a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((128*a**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqr
t(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*
x**2)) + 448*a**3*b*x**2/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**
2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a
 + b*x**2)) + 560*a**2*b**2*x**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*
b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**
6*sqrt(a + b*x**2)) + 280*a*b**3*x**6/(35*a**3*b**5*sqrt(a + b*x**2) + 105*
```

```
a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**
8*x**6*sqrt(a + b*x**2)) + 35*b**4*x**8/(35*a**3*b**5*sqrt(a + b*x**2) + 10
5*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b
**8*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**10/(10*a**(9/2)), True))
```

Giac [A]
time = 1.06, size = 72, normalized size = 0.77

$$\frac{\sqrt{bx^2+a}}{b^5}+\frac{140\,(bx^2+a)^3a-70\,(bx^2+a)^2a^2+28\,(bx^2+a)a^3-5\,a^4}{35\,(bx^2+a)^{\frac{7}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b^5 + 1/35*(140*(b*x^2 + a)^3*a - 70*(b*x^2 + a)^2*a^2 + 28
*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b^5)
```

Mupad [B]
time = 4.94, size = 80, normalized size = 0.85

$$\frac{\sqrt{b\,x^2+a}}{b^5}+\frac{4\,a}{b^5\,\sqrt{b\,x^2+a}}-\frac{2\,a^2}{b^5\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{4\,a^3}{5\,b^5\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{a^4}{7\,b^5\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)/b^5 + (4*a)/(b^5*(a + b*x^2)^(1/2)) - (2*a^2)/(b^5*(a + b
*x^2)^(3/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - a^4/(7*b^5*(a + b*x^2)^(
7/2))
```

3.519 $\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}-\frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{x}{b^4\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

```
[Out] -1/7*x^7/b/(b*x^2+a)^(7/2)-1/5*x^5/b^2/(b*x^2+a)^(5/2)-1/3*x^3/b^3/(b*x^2+a
)^(3/2)+arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-x/b^4/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {294, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{x}{b^4\sqrt{a+bx^2}}-\frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*x^7/(b*(a + b*x^2)^(7/2)) - x^5/(5*b^2*(a + b*x^2)^(5/2)) - x^3/(3*b^3
*(a + b*x^2)^(3/2)) - x/(b^4*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]]/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx &= -\frac{x^7}{7b(a+bx^2)^{7/2}} + \frac{\int \frac{x^6}{(a+bx^2)^{7/2}}\,dx}{b} \\
&= -\frac{x^7}{7b(a+bx^2)^{7/2}} - \frac{x^5}{5b^2(a+bx^2)^{5/2}} + \frac{\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx}{b^2} \\
&= -\frac{x^7}{7b(a+bx^2)^{7/2}} - \frac{x^5}{5b^2(a+bx^2)^{5/2}} - \frac{x^3}{3b^3(a+bx^2)^{3/2}} + \frac{\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx}{b^3} \\
&= -\frac{x^7}{7b(a+bx^2)^{7/2}} - \frac{x^5}{5b^2(a+bx^2)^{5/2}} - \frac{x^3}{3b^3(a+bx^2)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{a+bx^2}}\,d}{b^4} \\
&= -\frac{x^7}{7b(a+bx^2)^{7/2}} - \frac{x^5}{5b^2(a+bx^2)^{5/2}} - \frac{x^3}{3b^3(a+bx^2)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx}\right.}{} \\
&= -\frac{x^7}{7b(a+bx^2)^{7/2}} - \frac{x^5}{5b^2(a+bx^2)^{5/2}} - \frac{x^3}{3b^3(a+bx^2)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{b^{9/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 80, normalized size = 0.75

$$
-\frac{x(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6)}{105b^4(a+bx^2)^{7/2}} - \frac{\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/105*(x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b^3*x^6))/(b^4*(a
+ b*x^2)^(7/2)) - Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/b^(9/2)
```

Maple [A]
time = 0.05, size = 103, normalized size = 0.97

method	result	size

default	$-\frac{x^{7}}{7b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5\left(b\,x^{2}+a\right)^{\frac{5}{2}} b}+\frac{-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/(b*x^2+a)^(5/2)/b+1/b*(-1/3*x^3/b/
(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(86) = 172$.
time = 0.29, size = 255, normalized size = 2.41

$$-\frac{1}{35}\left(\frac{35\,x^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}+\frac{70\,ax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{56\,a^{2}x^{2}}{(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{16\,a^{3}}{(bx^{2}+a)^{\frac{7}{2}}b^{4}}\right)x-\frac{x\left(\frac{15\,x^{4}}{(bx^{2}+a)^{\frac{5}{2}}b}+\frac{20\,ax^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{8\,a^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}\right)}{15\,b}-\frac{x\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{3\,b^{2}}-\frac{ax^{3}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}+\frac{139\,x}{105\,\sqrt{bx^{2}+a}\,b^{4}}+\frac{17\,ax}{105\,(bx^{2}+a)^{\frac{3}{2}}b^{4}}-\frac{29\,a^{2}x}{35\,(bx^{2}+a)^{\frac{5}{2}}b^{4}}+\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*x - 1/15
*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2
/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*
x^2 + a)^(3/2)*b^2))/b^2 - a*x^3/((b*x^2 + a)^(5/2)*b^3) + 139/105*x/(sqrt(
b*x^2 + a)*b^4) + 17/105*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*a^2*x/((b*x^2
+ a)^(5/2)*b^4) + arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Fricas [A]
time = 1.16, size = 331, normalized size = 3.12

$$\left[\frac{105\,(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4})\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,(176\,b^{4}x^{7}+406\,ab^{3}x^{5}+350\,a^{2}b^{2}x^{3}+105\,a^{3}bx)\sqrt{bx^{2}+a}}{210\,(b^{9}x^{8}+4\,ab^{8}x^{6}+6\,a^{2}b^{7}x^{4}+4\,a^{3}b^{6}x^{2}+a^{4}b^{5})},\ -\frac{105\,(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4})\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+(176\,b^{4}x^{7}+406\,ab^{3}x^{5}+350\,a^{2}b^{2}x^{3}+105\,a^{3}bx)\sqrt{bx^{2}+a}}{105\,(b^{9}x^{8}+4\,ab^{8}x^{6}+6\,a^{2}b^{7}x^{4}+4\,a^{3}b^{6}x^{2}+a^{4}b^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(176*b^4*x^7 + 406
*a*b^3*x^5 + 350*a^2*b^2*x^3 + 105*a^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a
*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(b^4*x^8 +
 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*arctan(sqrt(-b)*
```

```
x/sqrt(b*x^2 + a)) + (176*b^4*x^7 + 406*a*b^3*x^5 + 350*a^2*b^2*x^3 + 105*a
^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6
*x^2 + a^4*b^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2980 vs. $2(92) = 184$.
time = 13.19, size = 2980, normalized size = 28.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] 105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(2
05/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1
+ b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**
(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**
8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) +
 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b**46
*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)
*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1
575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105
/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x*
*2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)
*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt(1 +
 b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x*
*2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)
*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(
1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a*
*(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x*
*12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)*asi
nh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a*
*(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x*
*4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a)
+ 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(1
09/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b
*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/s
qrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(1
01/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*
x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197
/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sq
rt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 63
0*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a
**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqr
t(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100
```

```
*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)
*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/
a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/2)*b
**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(
99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a
) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b*
*(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 +
 b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(1
93/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(105*a*
*(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt
(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*
a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*
x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a
) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b**(93
/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(
101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b
*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(19
7/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*s
qrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 1
771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2
)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2
/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b
**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
 + b*x**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*b**(99/2)*sqrt(1
+ b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(
201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6
*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) +
630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111
/2)*x**12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**9/(105*a**(205/2)*b
**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**
2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)
*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(
1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a*
*(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) -...
```

Giac [A]
time = 0.81, size = 78, normalized size = 0.74

$$-\frac{\left(2\left(x^2\left(\frac{88\,x^2}{b}+\frac{203\,a}{b^2}\right)+\frac{175\,a^2}{b^3}\right)x^2+\frac{105\,a^3}{b^4}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(
b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{(b\,x^2+a)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^8/(a + b*x^2)^(9/2), x)
```

3.520 $\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=75

$$\frac{a^3}{7b^4(a+bx^2)^{7/2}}-\frac{3a^2}{5b^4(a+bx^2)^{5/2}}+\frac{a}{b^4(a+bx^2)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}$$

```
[Out] 1/7*a^3/b^4/(b*x^2+a)^(7/2)-3/5*a^2/b^4/(b*x^2+a)^(5/2)+a/b^4/(b*x^2+a)^(3/
2)-1/b^4/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{a^3}{7b^4(a+bx^2)^{7/2}}-\frac{3a^2}{5b^4(a+bx^2)^{5/2}}+\frac{a}{b^4(a+bx^2)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(9/2),x]
```

```
[Out] a^3/(7*b^4*(a + b*x^2)^(7/2)) - (3*a^2)/(5*b^4*(a + b*x^2)^(5/2)) + a/(b^4*
(a + b*x^2)^(3/2)) - 1/(b^4*Sqrt[a + b*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{9/2}}+\frac{3a^2}{b^3(a+bx)^{7/2}}-\frac{3a}{b^3(a+bx)^{5/2}}+\frac{1}{b^3(a+bx)^{3/2}}\right)dx,x,\right.\\ &= \frac{a^3}{7b^4(a+bx^2)^{7/2}}-\frac{3a^2}{5b^4(a+bx^2)^{5/2}}+\frac{a}{b^4(a+bx^2)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.67

$$\frac{-16a^3 - 56a^2bx^2 - 70ab^2x^4 - 35b^3x^6}{35b^4(a + bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(9/2),x]
```

```
[Out] (-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6)/(35*b^4*(a + b*x^2)^(7
/2))
```

Maple [A]
time = 0.05, size = 82, normalized size = 1.09

method	result	size
gosper	$-\frac{35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3}{35(b\,x^2+a)^{\frac{7}{2}}b^4}$	47
trager	$-\frac{35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3}{35(b\,x^2+a)^{\frac{7}{2}}b^4}$	47
default	$-\frac{x^6}{b(b\,x^2+a)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^4}{3b(b\,x^2+a)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^2}{5b(b\,x^2+a)^{\frac{7}{2}}}-\frac{2a}{35b^2(b\,x^2+a)^{\frac{7}{2}}}\right)}{3b}\right)}{b}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(7/2)+4/3*a/b*(-1/5*x^2/
b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2)))
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.97

$$-\frac{x^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{2\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,a^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{16\,a^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -x^6/((b*x^2 + a)^(7/2)*b) - 2*a*x^4/((b*x^2 + a)^(7/2)*b^2) - 8/5*a^2*x^2/
((b*x^2 + a)^(7/2)*b^3) - 16/35*a^3/((b*x^2 + a)^(7/2)*b^4)
```

Fricas [A]
time = 1.48, size = 91, normalized size = 1.21

$$-\frac{\left(35\, b^{3} x^{6} + 70\, a b^{2} x^{4} + 56\, a^{2} b x^{2} + 16\, a^{3}\right) \sqrt{b x^{2} + a}}{35 \, \left(b^{8} x^{8} + 4\, a b^{7} x^{6} + 6\, a^{2} b^{6} x^{4} + 4\, a^{3} b^{5} x^{2} + a^{4} b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^2 + a)/(
b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(68) = 136.
time = 0.89, size = 364, normalized size = 4.85

$$\begin{cases} - \frac{16 a^{3}}{35 a^{3} b^{4} \sqrt{a + b x^{2}} + 105 a^{2} b^{5} x^{2} \sqrt{a + b x^{2}} + 105 a b^{6} x^{4} \sqrt{a + b x^{2}} + 35 b^{7} x^{6} \sqrt{a + b x^{2}}} - \frac{56 a^{2} b x^{2}}{35 a^{3} b^{4} \sqrt{a + b x^{2}} + 105 a^{2} b^{5} x^{2} \sqrt{a + b x^{2}} + 105 a b^{6} x^{4} \sqrt{a + b x^{2}} + 35 b^{7} x^{6} \sqrt{a + b x^{2}}} - \frac{70 a b^{2} x^{4}}{35 a^{3} b^{4} \sqrt{a + b x^{2}} + 105 a^{2} b^{5} x^{2} \sqrt{a + b x^{2}} + 105 a b^{6} x^{4} \sqrt{a + b x^{2}} + 35 b^{7} x^{6} \sqrt{a + b x^{2}}} - \frac{35 b^{3} x^{6}}{35 a^{3} b^{4} \sqrt{a + b x^{2}} + 105 a^{2} b^{5} x^{2} \sqrt{a + b x^{2}} + 105 a b^{6} x^{4} \sqrt{a + b x^{2}} + 35 b^{7} x^{6} \sqrt{a + b x^{2}}} & \text{for}\: b \neq 0 \\ \frac{x^{8}}{8 a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sqr
t(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b*
x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2
*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a
+ b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*
x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqr
t(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**
5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*s
qrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))
```

Giac [A]
time = 0.81, size = 55, normalized size = 0.73

$$-\frac{35 \, {\left(b x^{2} + a\right)}^{3} - 35 \, {\left(b x^{2} + a\right)}^{2} a + 21 \, {\left(b x^{2} + a\right)} a^{2} - 5 \, a^{3}}{35 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 - 5*a^3)/
((b*x^2 + a)^(7/2)*b^4)
```

Mupad [B]
time = 4.88, size = 63, normalized size = 0.84

$$\frac{a}{b^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{1}{b^4\,\sqrt{b\,x^2+a}}-\frac{3\,a^2}{5\,b^4\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{a^3}{7\,b^4\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(9/2),x)
```

```
[Out] a/(b^4*(a + b*x^2)^(3/2)) - 1/(b^4*(a + b*x^2)^(1/2)) - (3*a^2)/(5*b^4*(a +
 b*x^2)^(5/2)) + a^3/(7*b^4*(a + b*x^2)^(7/2))
```

3.521 $\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=21

$$\frac{x^7}{7a\,(a+bx^2)^{7/2}}$$

[Out] 1/7*x^7/a/(b*x^2+a)^(7/2)

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{x^7}{7a\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b*x^2)^(9/2),x]

[Out] x^7/(7*a*(a + b*x^2)^(7/2))

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx=\frac{x^7}{7a\,(a+bx^2)^{7/2}}$$

Mathematica [A]
time = 0.08, size = 21, normalized size = 1.00

$$\frac{x^7}{7a\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b*x^2)^(9/2),x]

[Out] x^7/(7*a*(a + b*x^2)^(7/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(17) = 34$.

time = 0.04, size = 144, normalized size = 6.86

method	result
gosper	$\frac{x^7}{7a(b\,x^2+a)^{\frac{7}{2}}}$
trager	$\frac{x^7}{7a(b\,x^2+a)^{\frac{7}{2}}}$
default	$-\frac{x^5}{2b(b\,x^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}\right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/
6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^
2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(17) = 34$.

time = 0.31, size = 103, normalized size = 4.90

$$-\frac{x^{5}}{2\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{5\,ax^{3}}{8\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}+\frac{x}{14\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}+\frac{x}{7\sqrt{bx^{2}+a}\,ab^{3}}+\frac{3\,ax}{56\left(bx^{2}+a\right)^{\frac{5}{2}}b^{3}}-\frac{15\,a^{2}x}{56\left(bx^{2}+a\right)^{\frac{7}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 1/14*x
/((b*x^2 + a)^(3/2)*b^3) + 1/7*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*a*x/((b*x^2
 + a)^(5/2)*b^3) - 15/56*a^2*x/((b*x^2 + a)^(7/2)*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(17) = 34$.
time = 1.45, size = 59, normalized size = 2.81

$$\frac{\sqrt{bx^{2}+a}\,x^{7}}{7\left(ab^{4}x^{8}+4\,a^{2}b^{3}x^{6}+6\,a^{3}b^{2}x^{4}+4\,a^{4}bx^{2}+a^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*x^7/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*
b*x^2 + a^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(15) = 30$.
time = 0.73, size = 95, normalized size = 4.52

$$\frac{x^{7}}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^{2}}{a}}+21a^{\frac{7}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+21a^{\frac{5}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+7a^{\frac{3}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))
```

Giac [A]
time = 0.84, size = 17, normalized size = 0.81

$$\frac{x^{7}}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

Mupad [B]
time = 4.76, size = 68, normalized size = 3.24

$$\frac{x}{7\,a\,b^3\,\sqrt{b\,x^2+a}}-\frac{3\,x}{7\,b^3\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{a^2\,x}{7\,b^3\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{3\,a\,x}{7\,b^3\,{\left(b\,x^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(9/2),x)
```

```
[Out] x/(7*a*b^3*(a + b*x^2)^(1/2)) - (3*x)/(7*b^3*(a + b*x^2)^(3/2)) - (a^2*x)/(
7*b^3*(a + b*x^2)^(7/2)) + (3*a*x)/(7*b^3*(a + b*x^2)^(5/2))
```

3.522 $\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/7*a^2/b^3/(b*x^2+a)^(7/2)+2/5*a/b^3/(b*x^2+a)^(5/2)-1/3/b^3/(b*x^2+a)^(3
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$-\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*a^2/(b^3*(a + b*x^2)^(7/2)) + (2*a)/(5*b^3*(a + b*x^2)^(5/2)) - 1/(3*b
^3*(a + b*x^2)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{9/2}}-\frac{2a}{b^2(a+bx)^{7/2}}+\frac{1}{b^2(a+bx)^{5/2}}\right)\,dx,x,x^2\right)\\
&= -\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{-8a^2-28abx^2-35b^2x^4}{105b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4)/(105*b^3*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.05, size = 58, normalized size = 0.98

method	result	size
gosper	$-\frac{35b^2x^4+28ab\,x^2+8a^2}{105\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}$	36
trager	$-\frac{35b^2x^4+28ab\,x^2+8a^2}{105\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}$	36
default	$-\frac{x^4}{3b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^2}{5b\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^2\left(b\,x^2+a\right)^{\frac{7}{2}}}\right)}{3b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^4/b/(b*x^2+a)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(
b*x^2+a)^(7/2))
```

Maxima [A]
time = 0.36, size = 53, normalized size = 0.90

$$-\frac{x^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{4\,ax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*x^4/((b*x^2 + a)^(7/2)*b) - 4/15*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 8/105
*a^2/((b*x^2 + a)^(7/2)*b^3)
```

Fricas [A]
time = 1.40, size = 80, normalized size = 1.36

$$-\frac{\left(35\,b^2x^4+28\,abx^2+8\,a^2\right)\sqrt{bx^2+a}}{105\left(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^7*x^8 + 4*a*b^6
*x^6 + 6*a^2*b^5*x^4 + 4*a^3*b^4*x^2 + a^4*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(53) = 106$.
time = 0.87, size = 272, normalized size = 4.61

$$\begin{cases} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{35b^2x^4}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqr
t(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b
*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*
sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a
+ b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x
**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqr
t(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True))
```

Giac [A]
time = 0.56, size = 41, normalized size = 0.69

$$-\frac{35\,(bx^2+a)^2 - 42\,(bx^2+a)a + 15\,a^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/((b*x^2 + a)^(7/2)*b^
3)
```

Mupad [B]
time = 4.81, size = 41, normalized size = 0.69

$$-\frac{35\,(b\,x^2+a)^2 - 42\,a\,(b\,x^2+a) + 15\,a^2}{105\,b^3\,(b\,x^2+a)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(9/2),x)
```

```
[Out] -(35*(a + b*x^2)^2 - 42*a*(a + b*x^2) + 15*a^2)/(105*b^3*(a + b*x^2)^(7/2))
```

3.523 $\int \frac{x^4}{(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=44

$$\frac{x^5}{5a\left(a+bx^2\right)^{7/2}}+\frac{2bx^7}{35a^2\left(a+bx^2\right)^{7/2}}$$

```
[Out] 1/5*x^5/a/(b*x^2+a)^(7/2)+2/35*b*x^7/a^2/(b*x^2+a)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$\frac{2bx^7}{35a^2\left(a+bx^2\right)^{7/2}}+\frac{x^5}{5a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(9/2),x]
```

```
[Out] x^5/(5*a*(a + b*x^2)^(7/2)) + (2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^2\right)^{9/2}} dx &= \frac{x^5}{5a\left(a+bx^2\right)^{7/2}}+\frac{(2b)\int \frac{x^6}{(a+bx^2)^{9/2}} dx}{5a}\\ &= \frac{x^5}{5a\left(a+bx^2\right)^{7/2}}+\frac{2bx^7}{35a^2\left(a+bx^2\right)^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 31, normalized size = 0.70

$$\frac{7ax^5+2bx^7}{35a^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(9/2),x]
```

```
[Out] (7*a*x^5 + 2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(36) = 72.
time = 0.04, size = 120, normalized size = 2.73

method	result	size
gosper	$\frac{x^{5}\left(2 b \,x^{2}+7 a \right)}{35 \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{2}}$	28
trager	$\frac{x^{5}\left(2 b \,x^{2}+7 a \right)}{35 \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{2}}$	28
default	$-\frac{x^{3}}{4 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{3 a \left(-\frac{x}{6 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{a \left(\frac{x}{7 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b \,x^{2}+a}}\right)}{7 a}}{a}\right)}{6 b}\right)}{4 b}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x
/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^
(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(36) = 72.
time = 0.31, size = 85, normalized size = 1.93

$$-\frac{x^3}{4\,(bx^2+a)^{\frac{7}{2}}b}+\frac{3\,x}{140\,(bx^2+a)^{\frac{5}{2}}b^2}+\frac{2\,x}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{x}{35\,(bx^2+a)^{\frac{3}{2}}ab^2}-\frac{3\,ax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/((b*x^2 + a)^(7/2)*b) + 3/140*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*x/(
sqrt(b*x^2 + a)*a^2*b^2) + 1/35*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*a*x/((b*
x^2 + a)^(7/2)*b^2)
```

Fricas [A]
time = 1.23, size = 71, normalized size = 1.61

$$\frac{\left(2\,bx^7+7\,ax^5\right)\sqrt{bx^2+a}}{35\,\left(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b*x^7 + 7*a*x^5)*sqrt(b*x^2 + a)/(a^2*b^4*x^8 + 4*a^3*b^3*x^6 + 6*a
^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(37) = 74.
time = 0.78, size = 199, normalized size = 4.52

$$\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] 7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*
x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(
9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a)
+ 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.54, size = 29, normalized size = 0.66

$$\frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\,\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

Mupad [B]
time = 4.88, size = 68, normalized size = 1.55

$$\frac{2\,x}{35\,a^2\,b^2\,\sqrt{b\,x^2+a}}-\frac{8\,x}{35\,b^2\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{x}{35\,a\,b^2\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{a\,x}{7\,b^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(9/2),x)
```

```
[Out] (2*x)/(35*a^2*b^2*(a + b*x^2)^(1/2)) - (8*x)/(35*b^2*(a + b*x^2)^(5/2)) + x
/(35*a*b^2*(a + b*x^2)^(3/2)) + (a*x)/(7*b^2*(a + b*x^2)^(7/2))
```

3.524 $\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=38

$$\frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}$$

```
[Out] 1/7*a/b^2/(b*x^2+a)^(7/2)-1/5/b^2/(b*x^2+a)^(5/2)
```

Rubi [A]

time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(9/2),x]
```

```
[Out] a/(7*b^2*(a + b*x^2)^(7/2)) - 1/(5*b^2*(a + b*x^2)^(5/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{9/2}}+\frac{1}{b(a+bx)^{7/2}}\right)\,dx,x,x^2\right)\\ &= \frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.74

$$\frac{-2a - 7bx^2}{35b^2 (a + bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(9/2),x]
[Out] (-2*a - 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.05, size = 34, normalized size = 0.89

method	result	size
gosper	$-\frac{7b\,x^2+2a}{35(b\,x^2+a)^{\frac{7}{2}}\,b^2}$	25
trager	$-\frac{7b\,x^2+2a}{35(b\,x^2+a)^{\frac{7}{2}}\,b^2}$	25
default	$-\frac{x^2}{5b(b\,x^2+a)^{\frac{7}{2}}} - \frac{2a}{35b^2(b\,x^2+a)^{\frac{7}{2}}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
[Out] -1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.87

$$-\frac{x^2}{5\,(bx^2 + a)^{\frac{7}{2}}b} - \frac{2\,a}{35\,(bx^2 + a)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
[Out] -1/5*x^2/((b*x^2 + a)^(7/2)*b) - 2/35*a/((b*x^2 + a)^(7/2)*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.
time = 1.01, size = 69, normalized size = 1.82

$$-\frac{(7\,bx^2 + 2\,a)\sqrt{bx^2 + a}}{35\,(b^6x^8 + 4\,ab^5x^6 + 6\,a^2b^4x^4 + 4\,a^3b^3x^2 + a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(7*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^
4 + 4*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(32) = 64$.
time = 0.86, size = 180, normalized size = 4.74

$$\begin{cases} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{7bx^2}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a
+ b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2
)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a +
b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2))
, Ne(b, 0)), (x**4/(4*a**(9/2)), True))
```

Giac [A]
time = 0.62, size = 24, normalized size = 0.63

$$-\frac{7\,bx^2+2\,a}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2)
```

Mupad [B]
time = 4.82, size = 24, normalized size = 0.63

$$-\frac{7\,b\,x^2+2\,a}{35\,b^2\,(b\,x^2+a)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(9/2),x)
```

```
[Out] -(2*a + 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

3.525 $\int \frac{x^2}{(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=68

$$\frac{x^3}{3a(a+bx^2)^{7/2}}+\frac{4bx^5}{15a^2(a+bx^2)^{7/2}}+\frac{8b^2x^7}{105a^3(a+bx^2)^{7/2}}$$

```
[Out] 1/3*x^3/a/(b*x^2+a)^(7/2)+4/15*b*x^5/a^2/(b*x^2+a)^(7/2)+8/105*b^2*x^7/a^3/
(b*x^2+a)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 270}

$$\frac{8b^2x^7}{105a^3(a+bx^2)^{7/2}}+\frac{4bx^5}{15a^2(a+bx^2)^{7/2}}+\frac{x^3}{3a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(9/2),x]
```

```
[Out] x^3/(3*a*(a + b*x^2)^(7/2)) + (4*b*x^5)/(15*a^2*(a + b*x^2)^(7/2)) + (8*b^2
*x^7)/(105*a^3*(a + b*x^2)^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx = \frac{x^3}{3a\,(a+bx^2)^{7/2}} + \frac{(4b)\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx}{3a}$$
$$= \frac{x^3}{3a\,(a+bx^2)^{7/2}} + \frac{4bx^5}{15a^2\,(a+bx^2)^{7/2}} + \frac{(8b^2)\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx}{15a^2}$$
$$= \frac{x^3}{3a\,(a+bx^2)^{7/2}} + \frac{4bx^5}{15a^2\,(a+bx^2)^{7/2}} + \frac{8b^2x^7}{105a^3\,(a+bx^2)^{7/2}}$$

Mathematica [A]
time = 0.08, size = 42, normalized size = 0.62

$$\frac{35a^2x^3 + 28abx^5 + 8b^2x^7}{105a^3\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(9/2),x]
[Out] (35*a^2*x^3 + 28*a*b*x^5 + 8*b^2*x^7)/(105*a^3*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.04, size = 96, normalized size = 1.41

method	result	size
gosper	$\frac{x^3(8b^2x^4+28ab\,x^2+35a^2)}{105(b\,x^2+a)^{\frac{7}{2}}\,a^3}$	39
trager	$\frac{x^3(8b^2x^4+28ab\,x^2+35a^2)}{105(b\,x^2+a)^{\frac{7}{2}}\,a^3}$	39
default	$-\frac{x}{6b(b\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{6b}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
[Out] -1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b
*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.03

$$-\frac{x}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,x}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,x}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{x}{35\,(bx^2+a)^{\frac{5}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*x/((b*x^2 + a)^(7/2)*b) + 8/105*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*x/((
b*x^2 + a)^(3/2)*a^2*b) + 1/35*x/((b*x^2 + a)^(5/2)*a*b)
```

Fricas [A]
time = 0.93, size = 82, normalized size = 1.21

$$\frac{(8\,b^2x^7+28\,abx^5+35\,a^2x^3)\sqrt{bx^2+a}}{105\,(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*b^2*x^7 + 28*a*b*x^5 + 35*a^2*x^3)*sqrt(b*x^2 + a)/(a^3*b^4*x^8 +
4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(61) = 122$.
time = 0.96, size = 517, normalized size = 7.60

$$\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}+\frac{63a^4bx^5}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}+\frac{36a^3b^2x^7}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}+\frac{8a^2b^3x^9}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] 35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(
1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))
+ 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13
/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/
a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420
*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 +
b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(1
7/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a)
 + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqr
t(1 + b*x**2/a))
```

Giac [A]
time = 0.54, size = 43, normalized size = 0.63

$$\frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\,(bx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

Mupad [B]
time = 4.76, size = 70, normalized size = 1.03

$$\frac{8\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}}-\frac{x}{7\,b\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{4\,x}{105\,a^2\,b\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{x}{35\,a\,b\,{\left(b\,x^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(9/2),x)
```

```
[Out] (8*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - x/(7*b*(a + b*x^2)^(7/2)) + (4*x)/(10
5*a^2*b*(a + b*x^2)^(3/2)) + x/(35*a*b*(a + b*x^2)^(5/2))
```

3.526 $\int \frac{x}{(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=18

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

[Out] -1/7/b/(b*x^2+a)^(7/2)

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^2)^(9/2),x]

[Out] -1/7*1/(b*(a + b*x^2)^(7/2))

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{9/2}} dx = -\frac{1}{7b(a+bx^2)^{7/2}}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^2)^(9/2),x]

[Out] -1/7*1/(b*(a + b*x^2)^(7/2))

Maple [A]
time = 0.04, size = 15, normalized size = 0.83

method	result	size
gosper	$-\frac{1}{7b(b\,x^2+a)^{\frac{7}{2}}}$	15
derivativedivides	$-\frac{1}{7b(b\,x^2+a)^{\frac{7}{2}}}$	15
default	$-\frac{1}{7b(b\,x^2+a)^{\frac{7}{2}}}$	15
trager	$-\frac{1}{7b(b\,x^2+a)^{\frac{7}{2}}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
[Out] -1/7/b/(b*x^2+a)^(7/2)
```

Maxima [A]
time = 0.32, size = 14, normalized size = 0.78

$$-\frac{1}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="maxima")
[Out] -1/7/((b*x^2 + a)^(7/2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.
time = 1.00, size = 57, normalized size = 3.17

$$-\frac{\sqrt{bx^2+a}}{7\,(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="fricas")
[Out] -1/7*sqrt(b*x^2 + a)/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2
 + a^4*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(15) = 30.
time = 0.85, size = 90, normalized size = 5.00

$$\begin{cases} -\frac{1}{7a^3b\sqrt{a+bx^2}+21a^2b^2x^2\sqrt{a+bx^2}+21ab^3x^4\sqrt{a+bx^2}+7b^4x^6\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x**
2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(b,
 0)), (x**2/(2*a**(9/2)), True))
```

Giac [A]
time = 0.53, size = 14, normalized size = 0.78

$$-\frac{1}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b)
```

Mupad [B]
time = 4.60, size = 14, normalized size = 0.78

$$-\frac{1}{7\,b\,(b\,x^2+a)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(9/2),x)
```

```
[Out] -1/(7*b*(a + b*x^2)^(7/2))
```

3.527 $\int \frac{1}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=77

$$\frac{x}{7a\,(a+bx^2)^{7/2}}+\frac{6x}{35a^2\,(a+bx^2)^{5/2}}+\frac{8x}{35a^3\,(a+bx^2)^{3/2}}+\frac{16x}{35a^4\sqrt{a+bx^2}}$$

```
[Out] 1/7*x/a/(b*x^2+a)^(7/2)+6/35*x/a^2/(b*x^2+a)^(5/2)+8/35*x/a^3/(b*x^2+a)^(3/
2)+16/35*x/a^4/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {198, 197}

$$\frac{16x}{35a^4\sqrt{a+bx^2}}+\frac{8x}{35a^3\,(a+bx^2)^{3/2}}+\frac{6x}{35a^2\,(a+bx^2)^{5/2}}+\frac{x}{7a\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-9/2),x]
```

```
[Out] x/(7*a*(a + b*x^2)^(7/2)) + (6*x)/(35*a^2*(a + b*x^2)^(5/2)) + (8*x)/(35*a^
3*(a + b*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{x}{7a(a+bx^2)^{7/2}} + \frac{6\int \frac{1}{(a+bx^2)^{7/2}}\,dx}{7a}$$
$$= \frac{x}{7a(a+bx^2)^{7/2}} + \frac{6x}{35a^2(a+bx^2)^{5/2}} + \frac{24\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{35a^2}$$
$$= \frac{x}{7a(a+bx^2)^{7/2}} + \frac{6x}{35a^2(a+bx^2)^{5/2}} + \frac{8x}{35a^3(a+bx^2)^{3/2}} + \frac{16\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{35a^3}$$
$$= \frac{x}{7a(a+bx^2)^{7/2}} + \frac{6x}{35a^2(a+bx^2)^{5/2}} + \frac{8x}{35a^3(a+bx^2)^{3/2}} + \frac{16x}{35a^4\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.66

$$\frac{35a^3x + 70a^2bx^3 + 56ab^2x^5 + 16b^3x^7}{35a^4(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-9/2),x]
```

```
[Out] (35*a^3*x + 70*a^2*b*x^3 + 56*a*b^2*x^5 + 16*b^3*x^7)/(35*a^4*(a + b*x^2)^(
7/2))
```

Maple [A]
time = 0.04, size = 74, normalized size = 0.96

method	result	size
gosper	$\frac{x\left(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3\right)}{35\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$	48
trager	$\frac{x\left(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3\right)}{35\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$	48
default	$\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^
2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.27, size = 61, normalized size = 0.79

$$\frac{16\,x}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,x}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,x}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{x}{7\,(bx^2+a)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*x/(sqrt(b*x^2 + a)*a^4) + 8/35*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*x/((b
*x^2 + a)^(5/2)*a^2) + 1/7*x/((b*x^2 + a)^(7/2)*a)
```

Fricas [A]
time = 1.02, size = 91, normalized size = 1.18

$$\frac{(16\,b^3x^7+56\,ab^2x^5+70\,a^2bx^3+35\,a^3x)\sqrt{bx^2+a}}{35\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*sqrt(b*x^2 + a)/
(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. 2(70) = 140.
time = 1.03, size = 1265, normalized size = 16.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/2),x)
```

```
[Out] 35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**
3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 21
0*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1
+ b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
```

```
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*
x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**
2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(
27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x*
*2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/
2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(
1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*
b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 +
 b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x
**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a
**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b
*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13
/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqr
t(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2
)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a
))
```

Giac [**A**]
time = 0.58, size = 55, normalized size = 0.71

$$\frac{\left(2\left(4x^2\left(\frac{2b^3x^2}{a^4}+\frac{7b^2}{a^3}\right)+\frac{35b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2
 + a)^(7/2)
```

Mupad [**B**]
time = 4.60, size = 61, normalized size = 0.79

$$\frac{16\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{6\,x}{35\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{x}{7\,a\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(9/2),x)
```

```
[Out] (16*x)/(35*a^4*(a + b*x^2)^(1/2)) + (8*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*x
)/(35*a^2*(a + b*x^2)^(5/2)) + x/(7*a*(a + b*x^2)^(7/2))
```

3.528 $\int \frac{1}{x(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=95

$$\frac{1}{7a(a+bx^2)^{7/2}}+\frac{1}{5a^2(a+bx^2)^{5/2}}+\frac{1}{3a^3(a+bx^2)^{3/2}}+\frac{1}{a^4\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[Out] 1/7/a/(b*x^2+a)^(7/2)+1/5/a^2/(b*x^2+a)^(5/2)+1/3/a^3/(b*x^2+a)^(3/2)-arcta
nh((b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)+1/a^4/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 53, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{1}{a^4\sqrt{a+bx^2}}+\frac{1}{3a^3(a+bx^2)^{3/2}}+\frac{1}{5a^2(a+bx^2)^{5/2}}+\frac{1}{7a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] 1/(7*a*(a + b*x^2)^(7/2)) + 1/(5*a^2*(a + b*x^2)^(5/2)) + 1/(3*a^3*(a + b*x
^2)^(3/2)) + 1/(a^4*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(
9/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{9/2}}\,dx,x,x^2\right)\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{7/2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)}{2a^3}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{a^4\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \ldots\right.}{\ldots}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{a^4\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \ldots\right.}{\ldots}\\
&= \frac{1}{7a\left(a+bx^2\right)^{7/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{a^4\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\sqrt{\ldots}\right.}{a^{9/\ldots}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 76, normalized size = 0.80

$$\frac{176a^3+406a^2bx^2+350ab^2x^4+105b^3x^6}{105a^4\left(a+bx^2\right)^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (176*a^3 + 406*a^2*b*x^2 + 350*a*b^2*x^4 + 105*b^3*x^6)/(105*a^4*(a + b*x^2
)^(7/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(9/2)
```

Maple [A]
time = 0.08, size = 100, normalized size = 1.05

method	result	size
default	$\frac{1}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}}{a}}{a}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/a/(b*x^2+a)^(7/2)+1/a*(1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)
+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
)
```

Maxima [A]
time = 0.31, size = 73, normalized size = 0.77

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{9}{2}}}+\frac{1}{\sqrt{bx^2+a}\,a^4}+\frac{1}{3\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{1}{5\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{1}{7\,(bx^2+a)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 1/(sqrt(b*x^2 + a)*a^4) + 1/3/((b*
x^2 + a)^(3/2)*a^3) + 1/5/((b*x^2 + a)^(5/2)*a^2) + 1/7/((b*x^2 + a)^(7/2)*
a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(75) = 150.
time = 1.85, size = 329, normalized size = 3.46

$$\left[\frac{105\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(105\,ab^3x^6+350\,a^2b^2x^4+406\,a^3bx^2+176\,a^4)\sqrt{bx^2+a}}{210\,(a^5b^4x^8+4\,a^6b^3x^6+6\,a^7b^2x^4+4\,a^8bx^2+a^9)},\ \frac{105\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(105\,ab^3x^6+350\,a^2b^2x^4+406\,a^3bx^2+176\,a^4)\sqrt{bx^2+a}}{105\,(a^5b^4x^8+4\,a^6b^3x^6+6\,a^7b^2x^4+4\,a^8bx^2+a^9)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(105*a*b^3*x^6
 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^2 + a))/(a^5*b^4*x^8
 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9), 1/105*(105*(b^4*x^8
+ 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a)*arctan(sqrt(-a)
/sqrt(b*x^2 + a)) + (105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*
a^4)*sqrt(b*x^2 + a))/(a^5*b^4*x^8 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*
b*x^2 + a^9)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5250 vs. $2(85) = 170$.
time = 3.90, size = 5250, normalized size = 55.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(9/2),x)
```

```
[Out] 352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*
a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8
 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2
)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*
a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(
71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100
*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x*
*12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/
2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x**2/
a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b*
*5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a
**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20
) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*
x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/
2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25
200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*
x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(210*a
**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/
2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 441
00*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*
x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a**31
*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)/(2
```

```
10*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**
(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b
**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 4725*a
**30*b**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*
a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8
 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2
)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*
a**(53/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(
210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a*
*(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10
+ 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*
b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23630
*a**29*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210*a*
*(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2
)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4410
0*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x
**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a**29
*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2/a)
/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*
a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**1
0 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2
)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 220
50*a**28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9
450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*
x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(
59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 +
210*a**(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a) +
 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 2...
```

Giac [**A**]

time = 0.54, size = 81, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}+\frac{105\,(bx^2+a)^3+35\,(bx^2+a)^2a+21\,(bx^2+a)a^2+15\,a^3}{105\,(bx^2+a)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + 1/105*(105*(b*x^2 + a)^3
+ 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2)*a^4)
```

Mupad [B]
time = 4.85, size = 75, normalized size = 0.79

$$\frac{\frac{b\,x^2+a}{5\,a^2}+\frac{1}{7\,a}+\frac{{\left(b\,x^2+a\right)}^2}{3\,a^3}+\frac{{\left(b\,x^2+a\right)}^3}{a^4}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((a + b*x^2)/(5*a^2) + 1/(7*a) + (a + b*x^2)^2/(3*a^3) + (a + b*x^2)^3/a^4)
/(a + b*x^2)^(7/2) - atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(9/2)
```

3.529 $\int \frac{1}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{1}{ax\left(a+bx^2\right)^{7/2}}-\frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}}-\frac{128bx}{35a^5\sqrt{a+bx^2}}$$

```
[Out] -1/a/x/(b*x^2+a)^(7/2)-8/7*b*x/a^2/(b*x^2+a)^(7/2)-48/35*b*x/a^3/(b*x^2+a)^
(5/2)-64/35*b*x/a^4/(b*x^2+a)^(3/2)-128/35*b*x/a^5/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {277, 198, 197}

$$-\frac{128bx}{35a^5\sqrt{a+bx^2}}-\frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}}-\frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(7/2))) - (8*b*x)/(7*a^2*(a + b*x^2)^(7/2)) - (48*b*x)
/(35*a^3*(a + b*x^2)^(5/2)) - (64*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (128*b*
x)/(35*a^5*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^2)^{9/2}}\,dx = -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{(8b)\int \frac{1}{(a+bx^2)^{9/2}}\,dx}{a}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{(48b)\int \frac{1}{(a+bx^2)^{7/2}}\,dx}{7a^2}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{(192b)\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{35a^3}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{64bx}{35a^4(a+bx^2)^{3/2}} - \frac{(12}{}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{64bx}{35a^4(a+bx^2)^{3/2}} - \frac{}{35a}$$

Mathematica [A]
time = 0.09, size = 64, normalized size = 0.64

$$\frac{-35a^4 - 280a^3bx^2 - 560a^2b^2x^4 - 448ab^3x^6 - 128b^4x^8}{35a^5x(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*b^4*x^8)/(
35*a^5*x*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.10, size = 98, normalized size = 0.98

method	result	size
gosper	$-\frac{128b^4x^8+448a\,b^3x^6+560a^2b^2x^4+280a^3b\,x^2+35a^4}{35x(b\,x^2+a)^{\frac{7}{2}}a^5}$	61
trager	$-\frac{128b^4x^8+448a\,b^3x^6+560a^2b^2x^4+280a^3b\,x^2+35a^4}{35x(b\,x^2+a)^{\frac{7}{2}}a^5}$	61
default	$-\frac{1}{ax(b\,x^2+a)^{\frac{7}{2}}}-\frac{8b\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{a}$	98

risch	$-\frac{\sqrt{b\,x^{2}+a}}{a^{5} x}-\frac{\sqrt{b\,x^{2}+a}\;x \left(93 b^{3} x^{6}+308 a\,b^{2} x^{4}+350 a^{2} b\,x^{2}+140 a^{3}\right) b}{35 \left(b^{4} x^{8}+4 a\,b^{3} x^{6}+6 a^{2} b^{2} x^{4}+4 a^{3} b\,x^{2}+a^{4}\right) a^{5}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2
+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.30, size = 82, normalized size = 0.82

$$-\frac{128\,bx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,bx}{35\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{48\,bx}{35\,(bx^2+a)^{\frac{5}{2}}a^3}-\frac{8\,bx}{7\,(bx^2+a)^{\frac{7}{2}}a^2}-\frac{1}{(bx^2+a)^{\frac{7}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/
35*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*b*x/((b*x^2 + a)^(7/2)*a^2) - 1/((b*x^
2 + a)^(7/2)*a*x)
```

Fricas [A]
time = 1.45, size = 103, normalized size = 1.03

$$-\frac{(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4)\sqrt{bx^2+a}}{35\,(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a
^4)*sqrt(b*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*
x^3 + a^9*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(97) = 194$.
time = 1.40, size = 400, normalized size = 4.00

$$-\frac{35a^4b^{\frac{33}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}-\frac{280a^3b^{\frac{35}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}-\frac{560a^2b^{\frac{37}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}-\frac{448ab^{\frac{39}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}-\frac{128b^{\frac{41}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] -35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 280*a
```

```
**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 560*a
**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 448*a
*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 +
 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b**(
41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*
a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
```

Giac [A]
time = 0.62, size = 90, normalized size = 0.90

$$-\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}}+\frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Mupad [B]
time = 4.71, size = 76, normalized size = 0.76

$$-\frac{\frac{1}{a^4}+\frac{128\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{29\,b\,x}{35\,a^4\,\left(b\,x^2+a\right)^{3/2}}-\frac{13\,b\,x}{35\,a^3\,\left(b\,x^2+a\right)^{5/2}}-\frac{b\,x}{7\,a^2\,\left(b\,x^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] - (1/a^4 + (128*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (29*b*x)/(35*a^4*(
a + b*x^2)^(3/2)) - (13*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (b*x)/(7*a^2*(a +
 b*x^2)^(7/2))
```

3.530 $\int \frac{1}{x^3(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{9b}{14a^2(a+bx^2)^{7/2}}-\frac{1}{2ax^2(a+bx^2)^{7/2}}-\frac{9b}{10a^3(a+bx^2)^{5/2}}-\frac{3b}{2a^4(a+bx^2)^{3/2}}-\frac{9b}{2a^5\sqrt{a+bx^2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}$$

```
[Out] -9/14*b/a^2/(b*x^2+a)^(7/2)-1/2/a/x^2/(b*x^2+a)^(7/2)-9/10*b/a^3/(b*x^2+a)^
(5/2)-3/2*b/a^4/(b*x^2+a)^(3/2)+9/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1
1/2)-9/2*b/a^5/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 44, 53, 65, 214}

$$\frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{9b}{2a^5\sqrt{a+bx^2}}-\frac{3b}{2a^4(a+bx^2)^{3/2}}-\frac{9b}{10a^3(a+bx^2)^{5/2}}-\frac{9b}{14a^2(a+bx^2)^{7/2}}-\frac{1}{2ax^2(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-9*b)/(14*a^2*(a + b*x^2)^(7/2)) - 1/(2*a*x^2*(a + b*x^2)^(7/2)) - (9*b)/(
10*a^3*(a + b*x^2)^(5/2)) - (3*b)/(2*a^4*(a + b*x^2)^(3/2)) - (9*b)/(2*a^5*
Sqrt[a + b*x^2]) + (9*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^2)^{9/2}} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{9/2}} \, dx, x, x^2\right)$$
$$= \frac{1}{7ax^2 (a+bx^2)^{7/2}} + \frac{9\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{7/2}} \, dx, x, x^2\right)}{14a}$$
$$= \frac{1}{7ax^2 (a+bx^2)^{7/2}} + \frac{9}{35a^2x^2 (a+bx^2)^{5/2}} + \frac{9\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}} \, dx, x, x^2\right)}{10a^2}$$
$$= \frac{1}{7ax^2 (a+bx^2)^{7/2}} + \frac{9}{35a^2x^2 (a+bx^2)^{5/2}} + \frac{3}{5a^3x^2 (a+bx^2)^{3/2}} + \frac{3\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\right.}{2a^3}$$
$$= \frac{1}{7ax^2 (a+bx^2)^{7/2}} + \frac{9}{35a^2x^2 (a+bx^2)^{5/2}} + \frac{3}{5a^3x^2 (a+bx^2)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} + \frac{9S}{}$$
$$= \frac{1}{7ax^2 (a+bx^2)^{7/2}} + \frac{9}{35a^2x^2 (a+bx^2)^{5/2}} + \frac{3}{5a^3x^2 (a+bx^2)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9}{}$$
$$= \frac{1}{7ax^2 (a+bx^2)^{7/2}} + \frac{9}{35a^2x^2 (a+bx^2)^{5/2}} + \frac{3}{5a^3x^2 (a+bx^2)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9}{}$$
$$= \frac{1}{7ax^2 (a+bx^2)^{7/2}} + \frac{9}{35a^2x^2 (a+bx^2)^{5/2}} + \frac{3}{5a^3x^2 (a+bx^2)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9}{}$$

Mathematica [A]
time = 0.09, size = 93, normalized size = 0.74

$$\frac{-35a^4 - 528a^3bx^2 - 1218a^2b^2x^4 - 1050ab^3x^6 - 315b^4x^8}{70a^5x^2 (a+bx^2)^{7/2}} + \frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-35*a^4 - 528*a^3*b*x^2 - 1218*a^2*b^2*x^4 - 1050*a*b^3*x^6 - 315*b^4*x^8)
/(70*a^5*x^2*(a + b*x^2)^(7/2)) + (9*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2
*a^(11/2))
```

Maple [A]
time = 0.13, size = 124, normalized size = 0.98

method	result
default	$-\frac{1}{2a\,x^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{9b\left(\frac{1}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{\frac{1}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}}{a}}{a}\right)}{2a}$
risch	$-\frac{\sqrt{b\,x^{2}+a}}{2a^{5}x^{2}}-\frac{19\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}b-2\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}}{280a^{4}\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)^{3}}+\frac{389\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}b-2}}{1120a^{5}\left(x+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(7/2)-9/2*b/a*(1/7/a/(b*x^2+a)^(7/2)+1/a*(1/5/a/(b*x^2
+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

Maxima [A]

time = 0.34, size = 96, normalized size = 0.76

$$\frac{9\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{11}{2}}}-\frac{9\,b}{2\,\sqrt{bx^2+a}\,a^5}-\frac{3\,b}{2\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{9\,b}{10\,(bx^2+a)^{\frac{5}{2}}a^3}-\frac{9\,b}{14\,(bx^2+a)^{\frac{7}{2}}a^2}-\frac{1}{2\,(bx^2+a)^{\frac{7}{2}}ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 9/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(11/2) - 9/2*b/(sqrt(b*x^2 + a)*a^5)
- 3/2*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*b/((b*x^2 + a)^(5/2)*a^3) - 9/14*b/(
(b*x^2 + a)^(7/2)*a^2) - 1/2/((b*x^2 + a)^(7/2)*a*x^2)
```

Fricas [A]

time = 1.36, size = 373, normalized size = 2.96

$$\left[\frac{315\,(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(315\,ab^4x^8+1050\,a^2b^3x^6+1218\,a^3b^2x^4+528\,a^4bx^2+35\,a^5)\sqrt{bx^2+a}}{140\,(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8b^2x^6+4\,a^9bx^4+a^{10}x^2)},\ -\frac{315\,(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(315\,ab^4x^8+1050\,a^2b^3x^6+1218\,a^3b^2x^4+528\,a^4bx^2+35\,a^5)\sqrt{bx^2+a}}{70\,(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8b^2x^6+4\,a^9bx^4+a^{10}x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")

[Out] [1/140*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2), -1/70*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5540 vs. $2(119) = 238$.
time = 7.46, size = 5540, normalized size = 43.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x**2+a)**(9/2),x)

[Out] -70*a**49*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 1476*a**48*b*x**2*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 315*a**48*b*x**2*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 630*a**48*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 9822*a**47*b**2*x**4*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**47*b**2*x**4*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x*

```
*10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(9
3/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 1
40*a**(87/2)*b**10*x**22) + 6300*a**47*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1
)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6
 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2
)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 63
00*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x
**22) - 33956*a**46*b**3*x**6*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 140
0*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**
8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/
2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 14
00*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 14175*a**46*b**3*x**
6*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(10
3/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 +
35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b
**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**
(87/2)*b**10*x**22) + 28350*a**46*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(14
0*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16
800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**
5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a*
*(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22)
 - 71940*a**45*b**4*x**8*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**
(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 2
9400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b*
*6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a*
*(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**45*b**4*x**8*log
(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*
b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280
*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x
**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2
)*b**10*x**22) + 75600*a**45*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(140*a**
(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a
**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**
12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/
2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 10
0260*a**44*b**5*x**10*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(10
5/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 2940
0*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*
x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(8
9/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**44*b**5*x**10*log(b
*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b*
*2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a*...
```

Giac [A]

time = 0.64, size = 104, normalized size = 0.83

$$-\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^5}-\frac{\sqrt{bx^2+a}}{2\,a^5x^2}-\frac{140\,(bx^2+a)^3b+35\,(bx^2+a)^2ab+14\,(bx^2+a)a^2b+5\,a^3b}{35\,(bx^2+a)^{\frac{7}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 + a
)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^2
+ a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

Mupad [B]
time = 4.96, size = 113, normalized size = 0.90

$$\frac{9\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{11/2}}-\frac{\frac{b}{7\,a}+\frac{3\,b\,{\left(b\,x^2+a\right)}^2}{5\,a^3}+\frac{3\,b\,{\left(b\,x^2+a\right)}^3}{a^4}-\frac{9\,b\,{\left(b\,x^2+a\right)}^4}{2\,a^5}+\frac{9\,b\,\left(b\,x^2+a\right)}{35\,a^2}}{a\,{\left(b\,x^2+a\right)}^{7/2}-{\left(b\,x^2+a\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(9/2)),x)
```

```
[Out] (9*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2)) - (b/(7*a) + (3*b*(a +
b*x^2)^2)/(5*a^3) + (3*b*(a + b*x^2)^3)/a^4 - (9*b*(a + b*x^2)^4)/(2*a^5) +
 (9*b*(a + b*x^2))/(35*a^2))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2))
```

3.531 $\int \frac{1}{x^4(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{3ax^3(a+bx^2)^{7/2}}+\frac{10b}{3a^2x(a+bx^2)^{7/2}}+\frac{80b^2x}{21a^3(a+bx^2)^{7/2}}+\frac{32b^2x}{7a^4(a+bx^2)^{5/2}}+\frac{128b^2x}{21a^5(a+bx^2)^{3/2}}+\frac{256b^2x}{21a^6\sqrt{a+bx^2}}$$

```
[Out] -1/3/a/x^3/(b*x^2+a)^(7/2)+10/3*b/a^2/x/(b*x^2+a)^(7/2)+80/21*b^2*x/a^3/(b*
x^2+a)^(7/2)+32/7*b^2*x/a^4/(b*x^2+a)^(5/2)+128/21*b^2*x/a^5/(b*x^2+a)^(3/2
)+256/21*b^2*x/a^6/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {277, 198, 197}

$$\frac{256b^2x}{21a^6\sqrt{a+bx^2}}+\frac{128b^2x}{21a^5(a+bx^2)^{3/2}}+\frac{32b^2x}{7a^4(a+bx^2)^{5/2}}+\frac{80b^2x}{21a^3(a+bx^2)^{7/2}}+\frac{10b}{3a^2x(a+bx^2)^{7/2}}-\frac{1}{3ax^3(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/3*1/(a*x^3*(a + b*x^2)^(7/2)) + (10*b)/(3*a^2*x*(a + b*x^2)^(7/2)) + (80
*b^2*x)/(21*a^3*(a + b*x^2)^(7/2)) + (32*b^2*x)/(7*a^4*(a + b*x^2)^(5/2)) +
 (128*b^2*x)/(21*a^5*(a + b*x^2)^(3/2)) + (256*b^2*x)/(21*a^6*Sqrt[a + b*x^
2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4 (a+bx^2)^{9/2}}\,dx &= -\frac{1}{3ax^3 (a+bx^2)^{7/2}} - \frac{(10b)\int \frac{1}{x^2(a+bx^2)^{9/2}}\,dx}{3a} \\
&= -\frac{1}{3ax^3 (a+bx^2)^{7/2}} + \frac{10b}{3a^2x (a+bx^2)^{7/2}} + \frac{(80b^2)\int \frac{1}{(a+bx^2)^{9/2}}\,dx}{3a^2} \\
&= -\frac{1}{3ax^3 (a+bx^2)^{7/2}} + \frac{10b}{3a^2x (a+bx^2)^{7/2}} + \frac{80b^2x}{21a^3 (a+bx^2)^{7/2}} + \frac{(160b^2)\int \frac{1}{(a+bx^2)^{7/2}}\,dx}{7a^3} \\
&= -\frac{1}{3ax^3 (a+bx^2)^{7/2}} + \frac{10b}{3a^2x (a+bx^2)^{7/2}} + \frac{80b^2x}{21a^3 (a+bx^2)^{7/2}} + \frac{32b^2x}{7a^4 (a+bx^2)^{5/2}} + \frac{(1}{} \\
&= -\frac{1}{3ax^3 (a+bx^2)^{7/2}} + \frac{10b}{3a^2x (a+bx^2)^{7/2}} + \frac{80b^2x}{21a^3 (a+bx^2)^{7/2}} + \frac{32b^2x}{7a^4 (a+bx^2)^{5/2}} + \frac{}{21} \\
&= -\frac{1}{3ax^3 (a+bx^2)^{7/2}} + \frac{10b}{3a^2x (a+bx^2)^{7/2}} + \frac{80b^2x}{21a^3 (a+bx^2)^{7/2}} + \frac{32b^2x}{7a^4 (a+bx^2)^{5/2}} + \frac{}{21}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 0.57

$$
\frac{-7a^5 + 70a^4bx^2 + 560a^3b^2x^4 + 1120a^2b^3x^6 + 896ab^4x^8 + 256b^5x^{10}}{21a^6x^3 (a+bx^2)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*a*b^4*x^8
 + 256*b^5*x^10)/(21*a^6*x^3*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.14, size = 122, normalized size = 0.92

method	result	size
gosper	$-\frac{-256b^5x^{10}-896a\,b^4x^8-1120a^2b^3x^6-560a^3b^2x^4-70a^4b\,x^2+7a^5}{21x^3(b\,x^2+a)^{\frac{7}{2}}a^6}$	72
trager	$-\frac{-256b^5x^{10}-896a\,b^4x^8-1120a^2b^3x^6-560a^3b^2x^4-70a^4b\,x^2+7a^5}{21x^3(b\,x^2+a)^{\frac{7}{2}}a^6}$	72
risch	$-\frac{\sqrt{b\,x^2+a}\,(-14b\,x^2+a)}{3a^6x^3}+\frac{\sqrt{b\,x^2+a}\,x(158b^3x^6+511a\,b^2x^4+560a^2b\,x^2+210a^3)b^2}{21a^6(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$	119

default	$-\frac{1}{3 a \,x^{3} \left(b \,x^{2}+a \right)^{\frac{7}{2}}}-\frac{10 b \left(-\frac{1}{a x \left(b \,x^{2}+a \right)^{\frac{7}{2}}}-\frac{8 b \left(\frac{x}{7 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b \,x^{2}+a}}\right)}{7 a}}{a}\right)}{a}\right)}{3 a}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3/(b*x^2+a)^(7/2)-10/3*b/a*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/
(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/
2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

Maxima [A]
time = 0.31, size = 108, normalized size = 0.82

$$\frac{256\,b^2x}{21\,\sqrt{bx^2+a}\,a^6}+\frac{128\,b^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5}+\frac{32\,b^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4}+\frac{80\,b^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3}+\frac{10\,b}{3\,(bx^2+a)^{\frac{7}{2}}a^2x}-\frac{1}{3\,(bx^2+a)^{\frac{7}{2}}ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 256/21*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*b^2*x/((b*x^2 + a)^(3/2)*a^5) +
 32/7*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*b^2*x/((b*x^2 + a)^(7/2)*a^3) +
 10/3*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3/((b*x^2 + a)^(7/2)*a*x^3)
```

Fricas [A]
time = 1.42, size = 116, normalized size = 0.88

$$\frac{(256\,b^5x^{10}+896\,ab^4x^8+1120\,a^2b^3x^6+560\,a^3b^2x^4+70\,a^4bx^2-7\,a^5)\sqrt{bx^2+a}}{21\,(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 + 7
0*a^4*b*x^2 - 7*a^5)*sqrt(b*x^2 + a)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*
b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(126) = 252.
time = 1.97, size = 668, normalized size = 5.06

$$-\frac{7a^6b^{\frac{51}{2}}\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{63a^5b^{\frac{53}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{630a^4b^{\frac{55}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{1680a^3b^{\frac{57}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{2016a^2b^{\frac{59}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{1152ab^{\frac{61}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{256b^{\frac{63}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] -7*a**6*b**(51/2)*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**
26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10
+ 21*a**6*b**30*x**12) + 63*a**5*b**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/(21*a*
*11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**2
8*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 630*a**4*b**(55/2)*x
**4*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*
a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**3
0*x**12) + 1680*a**3*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x*
*2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105
*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 2016*a**2*b**(59/2)*x**8*sqrt(a/
(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*
x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) +
1152*a*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**1
0*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x
**10 + 21*a**6*b**30*x**12) + 256*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(21*
a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b*
*28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)
```

Giac [A]
time = 0.63, size = 147, normalized size = 1.11

$$\frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\,(bx^2+a)^{\frac{7}{2}}}-\frac{4\left(6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^3
)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) - 15
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

Mupad [B]
time = 4.80, size = 97, normalized size = 0.73

$$\frac{\frac{128\,b}{21\,a^5}+\frac{256\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{1}{3\,a^2}+\frac{19\,b\,x^2}{21\,a^3}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{32\,b}{21\,a^4\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{b^2\,x}{7\,a^3\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((128*b)/(21*a^5) + (256*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) - (1/(3*a
^2) + (19*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5/2)) - (32*b)/(21*a^4*x*(a +
b*x^2)^(3/2)) + (b^2*x)/(7*a^3*(a + b*x^2)^(7/2))
```

3.532 $\int \frac{x^5}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{81}{64}\sqrt{9+4x^2}-\frac{3}{32}\left(9+4x^2\right)^{3/2}+\frac{1}{320}\left(9+4x^2\right)^{5/2}$$

```
[Out] -3/32*(4*x^2+9)^(3/2)+1/320*(4*x^2+9)^(5/2)+81/64*(4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{1}{320}\left(4x^2+9\right)^{5/2}-\frac{3}{32}\left(4x^2+9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[9 + 4*x^2],x]
```

```
[Out] (81*Sqrt[9 + 4*x^2])/64 - (3*(9 + 4*x^2)^(3/2))/32 + (9 + 4*x^2)^(5/2)/320
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{9+4x^2}}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{9+4x}}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{9+4x}}-\frac{9}{8}\sqrt{9+4x}+\frac{1}{16}(9+4x)^{3/2}\right)\,dx,x,x^2\right)\\&=\frac{81}{64}\sqrt{9+4x^2}-\frac{3}{32}\left(9+4x^2\right)^{3/2}+\frac{1}{320}\left(9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{9+4x^2}\,(27-6x^2+2x^4)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[9 + 4*x^2],x]
[Out] (Sqrt[9 + 4*x^2]*(27 - 6*x^2 + 2*x^4))/40
```

Maple [A]
time = 0.05, size = 41, normalized size = 0.89

method	result	size
trager	$\sqrt{4x^2+9}\,\left(\frac{1}{20}x^4-\frac{3}{20}x^2+\frac{27}{40}\right)$	23
gosper	$\frac{\sqrt{4x^2+9}\,(2x^4-6x^2+27)}{40}$	24
risch	$\frac{\sqrt{4x^2+9}\,(2x^4-6x^2+27)}{40}$	24
meijerg	$\frac{-\frac{81\sqrt{\pi}}{40}+\frac{81\sqrt{\pi}\,\left(\frac{32}{27}x^4-\frac{32}{9}x^2+16\right)\sqrt{1+\frac{4x^2}{9}}}{640}}{\sqrt{\pi}}$	38
default	$\frac{x^4\sqrt{4x^2+9}}{20}-\frac{3x^2\sqrt{4x^2+9}}{20}+\frac{27\sqrt{4x^2+9}}{40}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/20*x^4*(4*x^2+9)^(1/2)-3/20*x^2*(4*x^2+9)^(1/2)+27/40*(4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.50, size = 40, normalized size = 0.87

$$\frac{1}{20}\sqrt{4\,x^2+9}\,x^4-\frac{3}{20}\sqrt{4\,x^2+9}\,x^2+\frac{27}{40}\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/20*sqrt(4*x^2 + 9)*x^4 - 3/20*sqrt(4*x^2 + 9)*x^2 + 27/40*sqrt(4*x^2 + 9)
```

Fricas [A]
time = 1.05, size = 23, normalized size = 0.50

$$\frac{1}{40}\,(2\,x^4-6\,x^2+27)\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(2*x^4 - 6*x^2 + 27)*sqrt(4*x^2 + 9)
```

Sympy [A]
time = 0.23, size = 44, normalized size = 0.96

$$\frac{x^4\sqrt{4x^2+9}}{20} - \frac{3x^2\sqrt{4x^2+9}}{20} + \frac{27\sqrt{4x^2+9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 + 9)/20 - 3*x**2*sqrt(4*x**2 + 9)/20 + 27*sqrt(4*x**2 + 9)
/40
```

Giac [A]
time = 0.57, size = 34, normalized size = 0.74

$$\frac{1}{320}\left(4\,x^2+9\right)^{\frac{5}{2}} - \frac{3}{32}\left(4\,x^2+9\right)^{\frac{3}{2}} + \frac{81}{64}\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*(4*x^2 + 9)^(5/2) - 3/32*(4*x^2 + 9)^(3/2) + 81/64*sqrt(4*x^2 + 9)
```

Mupad [B]
time = 0.02, size = 21, normalized size = 0.46

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{x^4}{5}-\frac{3\,x^2}{5}+\frac{27}{10}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2 + 9)^(1/2),x)
```

```
[Out] ((x^2 + 9/4)^(1/2)*(x^4/5 - (3*x^2)/5 + 27/10))/2
```

3.533 $\int \frac{x^4}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{27}{128}x\sqrt{9+4x^2}+\frac{1}{16}x^3\sqrt{9+4x^2}+\frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 243/256*arcsinh(2/3*x)-27/128*x*(4*x^2+9)^(1/2)+1/16*x^3*(4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {327, 221}

$$-\frac{27}{128}\sqrt{4x^2+9}\,x+\frac{1}{16}\sqrt{4x^2+9}\,x^3+\frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[9 + 4*x^2],x]
```

```
[Out] (-27*x*Sqrt[9 + 4*x^2])/128 + (x^3*Sqrt[9 + 4*x^2])/16 + (243*ArcSinh[(2*x)
/3])/256
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{9+4x^2}}\,dx &= \frac{1}{16}x^3\sqrt{9+4x^2}-\frac{27}{16}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9+4x^2}+\frac{1}{16}x^3\sqrt{9+4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9+4x^2}+\frac{1}{16}x^3\sqrt{9+4x^2}+\frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.98

$$\frac{1}{128}x\sqrt{9+4x^2}\left(-27+8x^2\right)-\frac{243}{256}\log\left(-2x+\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[9 + 4*x^2],x]
[Out] (x*Sqrt[9 + 4*x^2]*(-27 + 8*x^2))/128 - (243*Log[-2*x + Sqrt[9 + 4*x^2]])/2
56
```

Maple [A]
time = 0.08, size = 34, normalized size = 0.76

method	result	size
risch	$\frac{x(8x^2-27)\sqrt{4x^2+9}}{128}+\frac{243\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{256}$	27
default	$\frac{243\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{256}-\frac{27x\sqrt{4x^2+9}}{128}+\frac{x^3\sqrt{4x^2+9}}{16}$	34
meijerg	$\frac{-\frac{27\sqrt{\pi}\,x\left(-\frac{40x^2}{9}+15\right)\sqrt{1+\frac{4x^2}{9}}}{640}+\frac{243\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{256}}{\sqrt{\pi}}$	38
trager	$\frac{x(8x^2-27)\sqrt{4x^2+9}}{128}-\frac{243\ln\left(2x-\sqrt{4x^2+9}\right)}{256}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 243/256*arcsinh(2/3*x)-27/128*x*(4*x^2+9)^(1/2)+1/16*x^3*(4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.59, size = 33, normalized size = 0.73

$$\frac{1}{16}\sqrt{4x^2+9}\,x^3-\frac{27}{128}\sqrt{4x^2+9}\,x+\frac{243}{256}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/16*sqrt(4*x^2 + 9)*x^3 - 27/128*sqrt(4*x^2 + 9)*x + 243/256*arcsinh(2/3*x
)
```

Fricas [A]
time = 1.37, size = 37, normalized size = 0.82

$$\frac{1}{128}\left(8\,x^3-27\,x\right)\sqrt{4\,x^2+9}-\frac{243}{256}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*x^3 - 27*x)*sqrt(4*x^2 + 9) - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A]
time = 0.16, size = 39, normalized size = 0.87

$$\frac{x^3\sqrt{4x^2+9}}{16}-\frac{27x\sqrt{4x^2+9}}{128}+\frac{243\operatorname{asinh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(4*x**2+9)**(1/2),x)
```

```
[Out] x**3*sqrt(4*x**2 + 9)/16 - 27*x*sqrt(4*x**2 + 9)/128 + 243*asinh(2*x/3)/256
```

Giac [A]
time = 0.77, size = 36, normalized size = 0.80

$$\frac{1}{128}\left(8\,x^2-27\right)\sqrt{4\,x^2+9}\,x-\frac{243}{256}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*(8*x^2 - 27)*sqrt(4*x^2 + 9)*x - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.56

$$\frac{243\operatorname{asinh}\left(\frac{2\,x}{3}\right)}{256}-\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{27\,x}{32}-\frac{x^3}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2 + 9)^(1/2),x)
```

```
[Out] (243*asinh((2*x)/3))/256 - ((x^2 + 9/4)^(1/2)*((27*x)/32 - x^3/4))/2
```

3.534 $\int \frac{x^3}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{9}{16}\sqrt{9+4x^2}+\frac{1}{48}\left(9+4x^2\right)^{3/2}$$

```
[Out] 1/48*(4*x^2+9)^(3/2)-9/16*(4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{1}{48}\left(4x^2+9\right)^{3/2}-\frac{9}{16}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[9 + 4*x^2],x]
```

```
[Out] (-9*Sqrt[9 + 4*x^2])/16 + (9 + 4*x^2)^(3/2)/48
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{9}{4\sqrt{9+4x}}+\frac{1}{4}\sqrt{9+4x}\right)\,dx,x,x^2\right)\\ &= -\frac{9}{16}\sqrt{9+4x^2}+\frac{1}{48}\left(9+4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(-9+2x^2\right)\sqrt{9+4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[9 + 4*x^2],x]
[Out] ((-9 + 2*x^2)*Sqrt[9 + 4*x^2])/24
```

Maple [A]
time = 0.03, size = 27, normalized size = 0.87

method	result	size
trager	$\sqrt{4x^2+9}\left(\frac{x^2}{12}-\frac{3}{8}\right)$	18
gosper	$\frac{\sqrt{4x^2+9}\,(2x^2-9)}{24}$	19
risch	$\frac{\sqrt{4x^2+9}\,(2x^2-9)}{24}$	19
default	$\frac{x^2\sqrt{4x^2+9}}{12}-\frac{3\sqrt{4x^2+9}}{8}$	27
meijerg	$\frac{\frac{9\sqrt{\pi}}{8}-\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)\sqrt{1+\frac{4x^2}{9}}}{64}}{\sqrt{\pi}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*x^2*(4*x^2+9)^(1/2)-3/8*(4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.54, size = 26, normalized size = 0.84

$$\frac{1}{12}\sqrt{4x^2+9}\,x^2-\frac{3}{8}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/12*sqrt(4*x^2 + 9)*x^2 - 3/8*sqrt(4*x^2 + 9)
```

Fricas [A]
time = 1.19, size = 18, normalized size = 0.58

$$\frac{1}{24}\sqrt{4x^2+9}\left(2x^2-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(4*x^2 + 9)*(2*x^2 - 9)
```

Sympy [A]
time = 0.10, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2+9}}{12}-\frac{3\sqrt{4x^2+9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**2+9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 + 9)/12 - 3*sqrt(4*x**2 + 9)/8
```

Giac [A]
time = 1.51, size = 23, normalized size = 0.74

$$\frac{1}{48}\left(4\,x^2+9\right)^{\frac{3}{2}}-\frac{9}{16}\,\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(4*x^2 + 9)^(3/2) - 9/16*sqrt(4*x^2 + 9)
```

Mupad [B]
time = 0.02, size = 15, normalized size = 0.48

$$\sqrt{x^2+\frac{9}{4}}\,\left(\frac{x^2}{6}-\frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*(x^2/6 - 3/4)
```

3.535 $\int \frac{x^2}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] -9/16*arcsinh(2/3*x)+1/8*x*(4*x^2+9)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {327, 221}

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[9 + 4*x^2],x]
[Out] (x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx &= \frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{8}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= \frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A]

time = 0.03, size = 37, normalized size = 1.37

$$\frac{1}{8}x\sqrt{9+4x^2}+\frac{9}{16}\log\left(-2x+\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[9 + 4*x^2],x]
[Out] (x*Sqrt[9 + 4*x^2])/8 + (9*Log[-2*x + Sqrt[9 + 4*x^2]])/16
```

Maple [A]
time = 0.07, size = 20, normalized size = 0.74

method	result	size
default	$-\frac{9 \operatorname{arcsinh}\left(\frac{2x}{3}\right)}{16}+\frac{x\sqrt{4x^2+9}}{8}$	20
risch	$-\frac{9 \operatorname{arcsinh}\left(\frac{2x}{3}\right)}{16}+\frac{x\sqrt{4x^2+9}}{8}$	20
meijerg	$\frac{\frac{3\sqrt{\pi}\, x\sqrt{1+\frac{4x^2}{9}}}{8}-\frac{9\sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{2x}{3}\right)}{16}}{\sqrt{\pi}}$	31
trager	$\frac{x\sqrt{4x^2+9}}{8}+\frac{9\ln\left(2x-\sqrt{4x^2+9}\right)}{16}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -9/16*arcsinh(2/3*x)+1/8*x*(4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.50, size = 19, normalized size = 0.70

$$\frac{1}{8}\sqrt{4x^2+9}\,x-\frac{9}{16}\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/8*sqrt(4*x^2 + 9)*x - 9/16*arcsinh(2/3*x)
```

Fricas [A]
time = 0.89, size = 29, normalized size = 1.07

$$\frac{1}{8}\sqrt{4x^2+9}\,x+\frac{9}{16}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [**A**]
time = 0.07, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2+9}}{8}-\frac{9\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*x**2+9)**(1/2),x)
```

```
[Out] x*sqrt(4*x**2 + 9)/8 - 9*asinh(2*x/3)/16
```

Giac [**A**]
time = 1.55, size = 29, normalized size = 1.07

$$\frac{1}{8}\sqrt{4\,x^2+9}\,x+\frac{9}{16}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [**B**]
time = 0.03, size = 17, normalized size = 0.63

$$\frac{x\,\sqrt{x^2+\frac{9}{4}}}{4}-\frac{9\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x*(x^2 + 9/4)^(1/2))/4 - (9*asinh((2*x)/3))/16
```

3.536 $\int \frac{x}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{4}\sqrt{9+4x^2}$$

[Out] 1/4*(4*x^2+9)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[9 + 4*x^2],x]

[Out] Sqrt[9 + 4*x^2]/4

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{9+4x^2}}\,dx = \frac{1}{4}\sqrt{9+4x^2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{4}\sqrt{9+4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[9 + 4*x^2],x]

[Out] Sqrt[9 + 4*x^2]/4

Maple [A]
time = 0.03, size = 12, normalized size = 0.80

method	result	size
gosper	$\frac{\sqrt{4x^2+9}}{4}$	12
derivativedivides	$\frac{\sqrt{4x^2+9}}{4}$	12
default	$\frac{\sqrt{4x^2+9}}{4}$	12
trager	$\frac{\sqrt{4x^2+9}}{4}$	12
risch	$\frac{\sqrt{4x^2+9}}{4}$	12
meijerg	$\frac{-\frac{3\sqrt{\pi}}{4}+\frac{3\sqrt{\pi}\,\sqrt{1+\frac{4x^2}{9}}}{4}}{\sqrt{\pi}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.30, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

Fricas [A]
time = 1.03, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2+9)**(1/2),x)
```

```
[Out] sqrt(4*x**2 + 9)/4
```

Giac [A]
time = 1.39, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.60

$$\frac{\sqrt{x^2+\frac{9}{4}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x^2 + 9/4)^(1/2)/2
```

3.537 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 1/2*arcsinh(2/3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{221\}$

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 2.00

$$-\frac{1}{2}\log\left(-2x+\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] -1/2*Log[-2*x + Sqrt[9 + 4*x^2]]
```

Maple [A]
time = 0.06, size = 7, normalized size = 0.70

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
trager	$\frac{\ln\left(2x+\sqrt{4x^2+9}\right)}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Maxima [A]
time = 0.50, size = 6, normalized size = 0.60

$$\frac{1}{2}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.
time = 0.90, size = 16, normalized size = 1.60

$$-\frac{1}{2}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.70

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2+9)**(1/2),x)
```

```
[Out] asinh(2*x/3)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(6) = 12$.
time = 1.28, size = 29, normalized size = 2.90

$$\frac{1}{2}\sqrt{4x^2+9}\,x - \frac{9}{4}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B]
time = 0.03, size = 6, normalized size = 0.60

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 9)^(1/2),x)
```

```
[Out] asinh((2*x)/3)/2
```

3.538 $\int \frac{1}{x\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

```
[Out] -1/3*arctanh(1/3*(4*x^2+9)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {272, 65, 213}

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)$$
$$= -\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Maple [A]
time = 0.07, size = 15, normalized size = 0.75

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{3}$	15
trager	$-\frac{\ln\left(\frac{\sqrt{4x^2+9}+3}{x}\right)}{3}$	19
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)+(2\ln(x)-2\ln(3))\sqrt{\pi}}{6\sqrt{\pi}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctanh(3/(4*x^2+9)^(1/2))
```

Maxima [A]
time = 0.50, size = 9, normalized size = 0.45

$$-\frac{1}{3}\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*arcsinh(3/2/abs(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.
time = 1.09, size = 35, normalized size = 1.75

$$-\frac{1}{3}\log\left(-2x+\sqrt{4x^2+9}+3\right)+\frac{1}{3}\log\left(-2x+\sqrt{4x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 1/3*log(-2*x + sqrt(4*x^2 + 9) - 3)
```

Sympy [A]
time = 0.44, size = 8, normalized size = 0.40

$$-\frac{\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x**2+9)**(1/2),x)
```

```
[Out] -asinh(3/(2*x))/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.
time = 1.18, size = 29, normalized size = 1.45

$$-\frac{1}{6}\log\left(\sqrt{4x^2+9}+3\right)+\frac{1}{6}\log\left(\sqrt{4x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*log(sqrt(4*x^2 + 9) + 3) + 1/6*log(sqrt(4*x^2 + 9) - 3)
```

Mupad [B]
time = 0.04, size = 12, normalized size = 0.60

$$-\frac{\operatorname{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] -atanh((2*(x^2 + 9/4)^(1/2))/3)/3
```

3.539 $\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{9+4x^2}}{9x}$$

```
[Out] -1/9*(4*x^2+9)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/9*Sqrt[9 + 4*x^2]/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{9+4x^2}}{9x}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$-\frac{\sqrt{9+4x^2}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/9*Sqrt[9 + 4*x^2]/x
```

Maple [A]
time = 0.03, size = 15, normalized size = 0.83

method	result	size
gosper	$-\frac{\sqrt{4x^2+9}}{9x}$	15
default	$-\frac{\sqrt{4x^2+9}}{9x}$	15
trager	$-\frac{\sqrt{4x^2+9}}{9x}$	15
meijerg	$-\frac{\sqrt{1+\frac{4x^2}{9}}}{3x}$	15
risch	$-\frac{\sqrt{4x^2+9}}{9x}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/9*(4*x^2+9)^(1/2)/x
```

Maxima [A]
time = 0.48, size = 14, normalized size = 0.78

$$-\frac{\sqrt{4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/9*sqrt(4*x^2 + 9)/x
```

Fricas [A]
time = 0.93, size = 18, normalized size = 1.00

$$-\frac{2\,x+\sqrt{4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/9*(2*x + sqrt(4*x^2 + 9))/x
```

Sympy [A]
time = 0.36, size = 15, normalized size = 0.83

$$-\frac{2\sqrt{1+\frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4*x**2+9)**(1/2),x)
```

```
[Out] -2*sqrt(1 + 9/(4*x**2))/9
```

Giac [A]
time = 0.80, size = 23, normalized size = 1.28

$$\frac{4}{\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 4/((2*x - sqrt(4*x^2 + 9))^2 - 9)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.67

$$-\frac{2\,\sqrt{x^2+\frac{9}{4}}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] -(2*(x^2 + 9/4)^(1/2))/(9*x)
```

3.540 $\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{9+4x^2}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

```
[Out] 2/27*arctanh(1/3*(4*x^2+9)^(1/2))-1/18*(4*x^2+9)^(1/2)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 44, 65, 213}

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{18x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/18*Sqrt[9 + 4*x^2]/x^2 + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\ &= -\frac{\sqrt{9+4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.00

$$-\frac{\sqrt{9+4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[9 + 4*x^2]),x]
[Out] -1/18*Sqrt[9 + 4*x^2]/x^2 + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27
```

Maple [A]
time = 0.08, size = 30, normalized size = 0.77

method	result	size
default	$-\frac{\sqrt{4x^2+9}}{18x^2}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}$	30
risch	$-\frac{\sqrt{4x^2+9}}{18x^2}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}$	30
trager	$-\frac{\sqrt{4x^2+9}}{18x^2}+\frac{2\ln\left(\frac{\sqrt{4x^2+9}+3}{x}\right)}{27}$	34

meijerg	$\frac{\frac{\sqrt{\pi}\left(8+\frac{16x^2}{9}\right)}{48x^2}-\frac{\sqrt{\pi}\sqrt{1+\frac{4x^2}{9}}}{6x^2}+\frac{2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)}{27}-\frac{(1+2\ln(x)-2\ln(3))\sqrt{\pi}}{27}-\frac{\sqrt{\pi}}{6x^2}}{\sqrt{\pi}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/18*(4*x^2+9)^(1/2)/x^2+2/27*arctanh(3/(4*x^2+9)^(1/2))
```

Maxima [A]
time = 0.50, size = 24, normalized size = 0.62

$$-\frac{\sqrt{4\,x^2+9}}{18\,x^2}+\frac{2}{27}\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/18*sqrt(4*x^2 + 9)/x^2 + 2/27*arcsinh(3/2/abs(x))
```

Fricas [A]
time = 1.04, size = 57, normalized size = 1.46

$$\frac{4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/54*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2 +
 9) - 3) - 3*sqrt(4*x^2 + 9))/x^2
```

Sympy [A]
time = 1.05, size = 44, normalized size = 1.13

$$\frac{2\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{27}-\frac{1}{9x\sqrt{1+\frac{9}{4x^2}}}-\frac{1}{4x^3\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4*x**2+9)**(1/2),x)
[Out] 2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 1/(4*x**3*sqrt(1 + 9/(
4*x**2)))
```

Giac [A]
time = 1.93, size = 43, normalized size = 1.10

$$-\frac{\sqrt{4\,x^2+9}}{18\,x^2}+\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}+3\right)-\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/18*sqrt(4*x^2 + 9)/x^2 + 1/27*log(sqrt(4*x^2 + 9) + 3) - 1/27*log(sqrt(4
*x^2 + 9) - 3)
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.64

$$\frac{2\,\mathrm{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{27}-\frac{\sqrt{x^2+\frac{9}{4}}}{9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(4*x^2 + 9)^(1/2)),x)
[Out] (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/27 - (x^2 + 9/4)^(1/2)/(9*x^2)
```

3.541 $\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{9+4x^2}}{27x^3}+\frac{8\sqrt{9+4x^2}}{243x}$$

```
[Out] -1/27*(4*x^2+9)^(1/2)/x^3+8/243*(4*x^2+9)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 270}

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/27*Sqrt[9 + 4*x^2]/x^3 + (8*Sqrt[9 + 4*x^2])/(243*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx &= -\frac{\sqrt{9+4x^2}}{27x^3}-\frac{8}{27}\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx\\ &= -\frac{\sqrt{9+4x^2}}{27x^3}+\frac{8\sqrt{9+4x^2}}{243x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.68

$$\frac{\sqrt{9+4x^2}\,(-9+8x^2)}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[9 + 4*x^2]),x]
[Out] (Sqrt[9 + 4*x^2]*(-9 + 8*x^2))/(243*x^3)
```

Maple [A]
time = 0.05, size = 30, normalized size = 0.81

method	result	size
gosper	$\frac{\sqrt{4x^2+9}\,(8x^2-9)}{243x^3}$	22
trager	$\frac{\sqrt{4x^2+9}\,(8x^2-9)}{243x^3}$	22
meijerg	$-\frac{\left(1-\frac{8x^2}{9}\right)\sqrt{1+\frac{4x^2}{9}}}{9x^3}$	22
risch	$\frac{32x^4+36x^2-81}{243x^3\sqrt{4x^2+9}}$	27
default	$-\frac{\sqrt{4x^2+9}}{27x^3}+\frac{8\sqrt{4x^2+9}}{243x}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/27*(4*x^2+9)^(1/2)/x^3+8/243*(4*x^2+9)^(1/2)/x
```

Maxima [A]
time = 0.63, size = 29, normalized size = 0.78

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 8/243*sqrt(4*x^2 + 9)/x - 1/27*sqrt(4*x^2 + 9)/x^3
```

Fricas [A]
time = 1.39, size = 28, normalized size = 0.76

$$\frac{16x^3+(8x^2-9)\sqrt{4x^2+9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/243*(16*x^3 + (8*x^2 - 9)*sqrt(4*x^2 + 9))/x^3
```

Sympy [A]
time = 0.64, size = 32, normalized size = 0.86

$$\frac{16\sqrt{1+\frac{9}{4x^2}}}{243}-\frac{2\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4*x**2+9)**(1/2),x)
```

```
[Out] 16*sqrt(1 + 9/(4*x**2))/243 - 2*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

Giac [A]
time = 1.17, size = 42, normalized size = 1.14

$$\frac{32\left(\left(2x-\sqrt{4x^2+9}\right)^2-3\right)}{\left(\left(2x-\sqrt{4x^2+9}\right)^2-9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 32*((2*x - sqrt(4*x^2 + 9))^2 - 3)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

Mupad [B]
time = 0.02, size = 19, normalized size = 0.51

$$\sqrt{x^2+\frac{9}{4}}\left(\frac{16}{243x}-\frac{2}{27x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*(16/(243*x) - 2/(27*x^3))
```

3.542 $\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

```
[Out] -2/81*arctanh(1/3*(4*x^2+9)^(1/2))-1/36*(4*x^2+9)^(1/2)/x^4+1/54*(4*x^2+9)^
(1/2)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 44, 65, 213}

$$\frac{\sqrt{4x^2+9}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{36x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/36*Sqrt[9 + 4*x^2]/x^4 + Sqrt[9 + 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 +
4*x^2]/3])/81
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}+\frac{1}{27}\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}+\frac{1}{54}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{9+4x^2}\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 0.81

$$\frac{(-3+2x^2)\sqrt{9+4x^2}}{108x^4}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[9 + 4*x^2]),x]
[Out] ((-3 + 2*x^2)*Sqrt[9 + 4*x^2])/(108*x^4) - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/8
1
```

Maple [A]
time = 0.09, size = 44, normalized size = 0.77

method	result
trager	$\frac{(2x^2-3)\sqrt{4x^2+9}}{108x^4}+\frac{2\ln\left(\frac{\sqrt{4x^2+9}-3}{x}\right)}{81}$
risch	$\frac{8x^4+6x^2-27}{108x^4\sqrt{4x^2+9}}-\frac{2\,\text{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{81}$

default	$-\frac{\sqrt{4x^2+9}}{36x^4}+\frac{\sqrt{4x^2+9}}{54x^2}-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{81}$
meijerg	$\frac{\frac{\sqrt{\pi}\,\left(-\frac{112}{81}x^4-\frac{32}{9}x^2+8\right)}{96x^4}-\frac{\sqrt{\pi}\,\left(-\frac{16x^2}{3}+8\right)\sqrt{1+\frac{4x^2}{9}}}{96x^4}-\frac{2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)}{81}+\frac{\left(\frac{7}{6}+2\ln(x)-2\ln(3)\right)\sqrt{\pi}}{81}-\frac{\sqrt{\pi}}{12x^4}+}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(4*x^2+9)^(1/2)/x^4+1/54*(4*x^2+9)^(1/2)/x^2-2/81*arctanh(3/(4*x^2+9)
^(1/2))
```

Maxima [A]
time = 0.50, size = 38, normalized size = 0.67

$$\frac{\sqrt{4\,x^2+9}}{54\,x^2}-\frac{\sqrt{4\,x^2+9}}{36\,x^4}-\frac{2}{81}\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(4*x^2 + 9)/x^2 - 1/36*sqrt(4*x^2 + 9)/x^4 - 2/81*arcsinh(3/2/abs(
x))
```

Fricas [A]
time = 1.44, size = 64, normalized size = 1.12

$$-\frac{8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}\,(2\,x^2-3)}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/324*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^2
 + 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 - 3))/x^4
```

Sympy [A]
time = 2.48, size = 63, normalized size = 1.11

$$-\frac{2\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{1+\frac{9}{4x^2}}}+\frac{1}{36x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4*x**2+9)**(1/2),x)
```

```
[Out] -2*asinh(3/(2*x))/81 + 1/(27*x*sqrt(1 + 9/(4*x**2))) + 1/(36*x**3*sqrt(1 +
9/(4*x**2))) - 1/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Giac [A]
time = 1.23, size = 55, normalized size = 0.96

$$\frac{(4\,x^2+9)^{\frac{3}{2}}-15\,\sqrt{4\,x^2+9}}{216\,x^4}-\frac{1}{81}\,\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{1}{81}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((4*x^2 + 9)^(3/2) - 15*sqrt(4*x^2 + 9))/x^4 - 1/81*log(sqrt(4*x^2 +
9) + 3) + 1/81*log(sqrt(4*x^2 + 9) - 3)
```

Mupad [B]
time = 0.03, size = 33, normalized size = 0.58

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2}{27\,x^2}-\frac{1}{9\,x^4}\right)}{2}-\frac{2\,\mathrm{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] ((x^2 + 9/4)^(1/2)*(2/(27*x^2) - 1/(9*x^4)))/2 - (2*atanh((2*(x^2 + 9/4)^(1
/2))/3))/81
```

3.543 $\int \frac{x^5}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{81}{64}\sqrt{9-4x^2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{1}{320}\left(9-4x^2\right)^{5/2}$$

```
[Out] 3/32*(-4*x^2+9)^(3/2)-1/320*(-4*x^2+9)^(5/2)-81/64*(-4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$-\frac{1}{320}\left(9-4x^2\right)^{5/2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{81}{64}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-81*Sqrt[9 - 4*x^2])/64 + (3*(9 - 4*x^2)^(3/2))/32 - (9 - 4*x^2)^(5/2)/320
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{9-4x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{9-4x}}-\frac{9}{8}\sqrt{9-4x}+\frac{1}{16}(9-4x)^{3/2}\right)\,dx,x,x^2\right)\\
&= -\frac{81}{64}\sqrt{9-4x^2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{1}{320}\left(9-4x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{9-4x^2}\left(-27-6x^2-2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[9 - 4*x^2],x]
[Out] (Sqrt[9 - 4*x^2]*(-27 - 6*x^2 - 2*x^4))/40
```

Maple [A]
time = 0.06, size = 41, normalized size = 0.89

method	result	size
trager	$\left(-\frac{1}{20}x^4-\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{-4x^2+9}$	23
risch	$\frac{(2x^4+6x^2+27)(4x^2-9)}{40\sqrt{-4x^2+9}}$	31
gosper	$\frac{(2x-3)(2x+3)(2x^4+6x^2+27)}{40\sqrt{-4x^2+9}}$	34
meijerg	$-\frac{243\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(\frac{32}{27}x^4+\frac{32}{9}x^2+16\right)\sqrt{1-\frac{4x^2}{9}}}{15}\right)}{128\sqrt{\pi}}$	38
default	$-\frac{x^4\sqrt{-4x^2+9}}{20}-\frac{3x^2\sqrt{-4x^2+9}}{20}-\frac{27\sqrt{-4x^2+9}}{40}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/20*x^4*(-4*x^2+9)^(1/2)-3/20*x^2*(-4*x^2+9)^(1/2)-27/40*(-4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.50, size = 40, normalized size = 0.87

$$-\frac{1}{20}\sqrt{-4x^2+9}\,x^4-\frac{3}{20}\sqrt{-4x^2+9}\,x^2-\frac{27}{40}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/20*sqrt(-4*x^2 + 9)*x^4 - 3/20*sqrt(-4*x^2 + 9)*x^2 - 27/40*sqrt(-4*x^2
+ 9)
```

Fricas [A]
time = 1.19, size = 23, normalized size = 0.50

$$-\frac{1}{40}\left(2x^4+6x^2+27\right)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(2*x^4 + 6*x^2 + 27)*sqrt(-4*x^2 + 9)
```

Sympy [A]
time = 0.24, size = 46, normalized size = 1.00

$$-\frac{x^4\sqrt{9-4x^2}}{20}-\frac{3x^2\sqrt{9-4x^2}}{20}-\frac{27\sqrt{9-4x^2}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**4*sqrt(9 - 4*x**2)/20 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**2
)/40
```

Giac [A]
time = 1.30, size = 43, normalized size = 0.93

$$-\frac{1}{320}\left(4x^2-9\right)^2\sqrt{-4x^2+9}+\frac{3}{32}\left(-4x^2+9\right)^{\frac{3}{2}}-\frac{81}{64}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/320*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) + 3/32*(-4*x^2 + 9)^(3/2) - 81/64*sqr
t(-4*x^2 + 9)
```

Mupad [B]
time = 0.04, size = 23, normalized size = 0.50

$$-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{x^4}{5}+\frac{3\,x^2}{5}+\frac{27}{10}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*((3*x^2)/5 + x^4/5 + 27/10))/2
```

3.544 $\int \frac{x^4}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{27}{128}x\sqrt{9-4x^2}-\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 243/256*arcsin(2/3*x)-27/128*x*(-4*x^2+9)^(1/2)-1/16*x^3*(-4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {327, 222}

$$\frac{243}{256}\text{ArcSin}\left(\frac{2x}{3}\right)-\frac{27}{128}\sqrt{9-4x^2}\,x-\frac{1}{16}\sqrt{9-4x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[9 - 4*x^2],x]
[Out] (-27*x*Sqrt[9 - 4*x^2])/128 - (x^3*Sqrt[9 - 4*x^2])/16 + (243*ArcSin[(2*x)/
3])/256
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{9-4x^2}}\,dx &= -\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{27}{16}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9-4x^2}-\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9-4x^2}-\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 47, normalized size = 1.04

$$-\frac{1}{128}x\sqrt{9-4x^2}\,(27+8x^2)+\frac{243}{128}\tan^{-1}\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/128*(x*Sqrt[9 - 4*x^2]*(27 + 8*x^2)) + (243*ArcTan[(2*x)/(-3 + Sqrt[9 - 4*x^2])])/128
```

Maple [A]
time = 0.13, size = 34, normalized size = 0.76

method	result	size
default	$\frac{243\arcsin\left(\frac{2x}{3}\right)}{256}-\frac{27x\sqrt{-4x^2+9}}{128}-\frac{x^3\sqrt{-4x^2+9}}{16}$	34
risch	$\frac{x(8x^2+27)(4x^2-9)}{128\sqrt{-4x^2+9}}+\frac{243\arcsin\left(\frac{2x}{3}\right)}{256}$	34
meijerg	$-\frac{81i\left(-\frac{i\sqrt{\pi}\,x\left(\frac{40x^2}{9}+15\right)\sqrt{1-\frac{4x^2}{9}}}{30}+\frac{3i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{4}\right)}{64\sqrt{\pi}}$	41
trager	$-\frac{x(8x^2+27)\sqrt{-4x^2+9}}{128}+\frac{243\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{256}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 243/256*arcsin(2/3*x)-27/128*x*(-4*x^2+9)^(1/2)-1/16*x^3*(-4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.54, size = 33, normalized size = 0.73

$$-\frac{1}{16}\sqrt{-4\,x^2+9}\,x^3-\frac{27}{128}\sqrt{-4\,x^2+9}\,x+\frac{243}{256}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(-4*x^2 + 9)*x^3 - 27/128*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/3*x)
```

Fricas [A]
time = 1.51, size = 40, normalized size = 0.89

$$-\frac{1}{128}\left(8\,x^{3}+27\,x\right)\sqrt{-4\,x^{2}+9}-\frac{243}{128}\arctan\left(\frac{\sqrt{-4\,x^{2}+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/128*(8*x^3 + 27*x)*sqrt(-4*x^2 + 9) - 243/128*arctan(1/2*(sqrt(-4*x^2 +
9) - 3)/x)
```

Sympy [A]
time = 0.16, size = 39, normalized size = 0.87

$$-\frac{x^{3}\sqrt{9-4x^{2}}}{16}-\frac{27x\sqrt{9-4x^{2}}}{128}+\frac{243\,\mathrm{asin}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-4*x**2+9)**(1/2),x)
[Out] -x**3*sqrt(9 - 4*x**2)/16 - 27*x*sqrt(9 - 4*x**2)/128 + 243*asin(2*x/3)/256
```

Giac [A]
time = 0.99, size = 26, normalized size = 0.58

$$-\frac{1}{128}\left(8\,x^{2}+27\right)\sqrt{-4\,x^{2}+9}\,x+\frac{243}{256}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/128*(8*x^2 + 27)*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/3*x)
```

Mupad [B]
time = 0.03, size = 27, normalized size = 0.60

$$\frac{243\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{256}-\frac{\sqrt{\frac{9}{4}-x^{2}}\,\left(\frac{x^{3}}{4}+\frac{27\,x}{32}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(9 - 4*x^2)^(1/2),x)
[Out] (243*asin((2*x)/3))/256 - ((9/4 - x^2)^(1/2)*((27*x)/32 + x^3/4))/2
```

3.545 $\int \frac{x^3}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{9}{16}\sqrt{9-4x^2}+\frac{1}{48}\left(9-4x^2\right)^{3/2}$$

[Out] 1/48*(-4*x^2+9)^(3/2)-9/16*(-4*x^2+9)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{1}{48}\left(9-4x^2\right)^{3/2}-\frac{9}{16}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[9 - 4*x^2],x]

[Out] (-9*Sqrt[9 - 4*x^2])/16 + (9 - 4*x^2)^(3/2)/48

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{9-4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{9}{4\sqrt{9-4x}}-\frac{1}{4}\sqrt{9-4x}\right)\,dx,x,x^2\right)\\ &= -\frac{9}{16}\sqrt{9-4x^2}+\frac{1}{48}\left(9-4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\sqrt{9-4x^2}\left(-9-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[9 - 4*x^2],x]
[Out] (Sqrt[9 - 4*x^2]*(-9 - 2*x^2))/24
```

Maple [A]
time = 0.05, size = 27, normalized size = 0.87

method	result	size
trager	$\left(-\frac{x^2}{12}-\frac{3}{8}\right)\sqrt{-4x^2+9}$	18
risch	$\frac{(2x^2+9)(4x^2-9)}{24\sqrt{-4x^2+9}}$	26
default	$-\frac{x^2\sqrt{-4x^2+9}}{12}-\frac{3\sqrt{-4x^2+9}}{8}$	27
gosper	$\frac{(2x-3)(2x+3)(2x^2+9)}{24\sqrt{-4x^2+9}}$	29
meijerg	$\frac{\frac{9\sqrt{\pi}}{8}-\frac{9\sqrt{\pi}\left(8+\frac{16x^2}{9}\right)\sqrt{1-\frac{4x^2}{9}}}{64}}{\sqrt{\pi}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/12*x^2*(-4*x^2+9)^(1/2)-3/8*(-4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.84

$$-\frac{1}{12}\sqrt{-4x^2+9}\,x^2-\frac{3}{8}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/12*sqrt(-4*x^2 + 9)*x^2 - 3/8*sqrt(-4*x^2 + 9)
```

Fricas [A]
time = 0.95, size = 18, normalized size = 0.58

$$-\frac{1}{24}\left(2x^2+9\right)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*x^2 + 9)*sqrt(-4*x^2 + 9)
```

Sympy [A]
time = 0.10, size = 29, normalized size = 0.94

$$-\frac{x^2\sqrt{9-4x^2}}{12}-\frac{3\sqrt{9-4x^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**2*sqrt(9 - 4*x**2)/12 - 3*sqrt(9 - 4*x**2)/8
```

Giac [A]
time = 1.22, size = 23, normalized size = 0.74

$$\frac{1}{48}\left(-4\,x^2+9\right)^{\frac{3}{2}}-\frac{9}{16}\,\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(-4*x^2 + 9)^(3/2) - 9/16*sqrt(-4*x^2 + 9)
```

Mupad [B]
time = 0.02, size = 18, normalized size = 0.58

$$-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{x^2}{3}+\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*(x^2/3 + 3/2))/2
```

3.546 $\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 9/16*arcsin(2/3*x)-1/8*x*(-4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {327, 222}

$$\frac{9}{16}\text{ArcSin}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[9 - 4*x^2],x]
[Out] -1/8*(x*Sqrt[9 - 4*x^2]) + (9*ArcSin[(2*x)/3])/16
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx&=-\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{8}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\&=-\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 1.48

$$-\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{8}\tan^{-1}\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[9 - 4*x^2],x]
[Out] -1/8*(x*Sqrt[9 - 4*x^2]) + (9*ArcTan[(2*x)/(-3 + Sqrt[9 - 4*x^2])])/8
```

Maple [A]
time = 0.11, size = 20, normalized size = 0.74

method	result	size
default	$\frac{9\arcsin\left(\frac{2x}{3}\right)}{16}-\frac{x\sqrt{-4x^2+9}}{8}$	20
risch	$\frac{x\left(4x^2-9\right)}{8\sqrt{-4x^2+9}}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{16}$	27
meijerg	$\frac{9i\left(\frac{2i\sqrt{\pi}\,x\sqrt{1-\frac{4x^2}{9}}}{3}-i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{16\sqrt{\pi}}$	34
trager	$-\frac{x\sqrt{-4x^2+9}}{8}+\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{16}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 9/16*arcsin(2/3*x)-1/8*x*(-4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.52, size = 19, normalized size = 0.70

$$-\frac{1}{8}\sqrt{-4\,x^2+9}\,x+\frac{9}{16}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

Fricas [A]
time = 1.11, size = 32, normalized size = 1.19

$$-\frac{1}{8}\sqrt{-4\,x^2+9}\,x-\frac{9}{8}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 + 9)*x - 9/8*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A]
time = 0.08, size = 22, normalized size = 0.81

$$-\frac{x\sqrt{9-4x^2}}{8}+\frac{9\operatorname{asin}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+9)**(1/2),x)
[Out] -x*sqrt(9 - 4*x**2)/8 + 9*asin(2*x/3)/16
```

Giac [A]
time = 0.85, size = 19, normalized size = 0.70

$$-\frac{1}{8}\sqrt{-4x^2+9}\,x+\frac{9}{16}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

Mupad [B]
time = 0.02, size = 19, normalized size = 0.70

$$\frac{9\operatorname{asin}\left(\frac{2x}{3}\right)}{16}-\frac{x\sqrt{\frac{9}{4}-x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(9 - 4*x^2)^(1/2),x)
[Out] (9*asin((2*x)/3))/16 - (x*(9/4 - x^2)^(1/2))/4
```

3.547 $\int \frac{x}{\sqrt{9-4x^2}} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{4}\sqrt{9-4x^2}$$

```
[Out] -1/4*(-4*x^2+9)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[9 - 4*x^2],x]
[Out] -1/4*Sqrt[9 - 4*x^2]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{9-4x^2}} \, dx = -\frac{1}{4}\sqrt{9-4x^2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[9 - 4*x^2],x]
[Out] -1/4*Sqrt[9 - 4*x^2]
```

Maple [A]
time = 0.04, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{-4x^2+9}}{4}$	12
default	$-\frac{\sqrt{-4x^2+9}}{4}$	12
trager	$-\frac{\sqrt{-4x^2+9}}{4}$	12
risch	$\frac{4x^2-9}{4\sqrt{-4x^2+9}}$	19
gosper	$\frac{(2x-3)(2x+3)}{4\sqrt{-4x^2+9}}$	22
meijerg	$-\frac{3\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1-\frac{4x^2}{9}}\right)}{8\sqrt{\pi}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-4*x^2+9)^(1/2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

Fricas [A]
time = 1.05, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.80

$$-\frac{\sqrt{9-4x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x**2+9)**(1/2),x)
```

```
[Out] -sqrt(9 - 4*x**2)/4
```

Giac [A]
time = 1.30, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

Mupad [B]
time = 4.56, size = 11, normalized size = 0.73

$$-\frac{\sqrt{\frac{9}{4}-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(9 - 4*x^2)^(1/2),x)
```

```
[Out] -(9/4 - x^2)^(1/2)/2
```

3.548 $\int \frac{1}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

[Out] 1/2*arcsin(2/3*x)

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {222}

$$\frac{1}{2}\text{ArcSin}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[9 - 4*x^2],x]

[Out] ArcSin[(2*x)/3]/2

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 24, normalized size = 2.40

$$\frac{1}{2}i\log\left(-2ix+\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[9 - 4*x^2],x]

[Out] (I/2)*Log[(-2*I)*x + Sqrt[9 - 4*x^2]]

Maple [A]
time = 0.11, size = 7, normalized size = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{2x}{3}\right)}{2}$	7
meijerg	$\frac{\arcsin\left(\frac{2x}{3}\right)}{2}$	7
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsin(2/3*x)
```

Maxima [A]
time = 0.49, size = 6, normalized size = 0.60

$$\frac{1}{2}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsin(2/3*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.
time = 1.31, size = 19, normalized size = 1.90

$$-\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+9)**(1/2),x)
```

```
[Out] asin(2*x/3)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(6) = 12$.
time = 0.98, size = 19, normalized size = 1.90

$$\frac{1}{2}\sqrt{-4x^2+9}\,x + \frac{9}{4}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

Mupad [B]
time = 0.01, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9 - 4*x^2)^(1/2),x)
```

```
[Out] asin((2*x)/3)/2
```

3.549 $\int \frac{1}{x\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -1/3*arctanh(1/3*(-4*x^2+9)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {272, 65, 212}

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\right)$$
$$= -\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.75

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{3}$	15
trager	$-\frac{\ln\left(\frac{\sqrt{-4x^2+9}+3}{x}\right)}{3}$	19
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)+(2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}}{6\sqrt{\pi}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctanh(3/(-4*x^2+9)^(1/2))
```

Maxima [A]
time = 0.50, size = 25, normalized size = 1.25

$$-\frac{1}{3}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A]
time = 1.21, size = 18, normalized size = 0.90

$$\frac{1}{3}\log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 26, normalized size = 1.30

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ \frac{i\operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x**2+9)**(1/2),x)
[Out] Piecewise((-acosh(3/(2*x))/3, 1/Abs(x**2) > 4/9), (I*asin(3/(2*x))/3, True)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.
time = 1.20, size = 31, normalized size = 1.55

$$-\frac{1}{6}\log\left(\sqrt{-4x^2+9}+3\right)+\frac{1}{6}\log\left(-\sqrt{-4x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/6*log(sqrt(-4*x^2 + 9) + 3) + 1/6*log(-sqrt(-4*x^2 + 9) + 3)
```

Mupad [B]
time = 0.12, size = 20, normalized size = 1.00

$$\frac{\ln\left(\sqrt{\frac{9}{4x^2}-1}-\frac{3\sqrt{\frac{1}{x^2}}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(9 - 4*x^2)^(1/2)),x)
[Out] log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2)/3
```

3.550 $\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{9-4x^2}}{9x}$$

[Out] -1/9*(-4*x^2+9)^(1/2)/x

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[9 - 4*x^2]),x]

[Out] -1/9*Sqrt[9 - 4*x^2]/x

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{9-4x^2}}{9x}$$

Mathematica [A]
time = 0.03, size = 18, normalized size = 1.00

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[9 - 4*x^2]),x]

[Out] -1/9*Sqrt[9 - 4*x^2]/x

Maple [A]
time = 0.04, size = 15, normalized size = 0.83

method	result	size
default	$-\frac{\sqrt{-4x^2+9}}{9x}$	15
trager	$-\frac{\sqrt{-4x^2+9}}{9x}$	15
meijerg	$-\frac{\sqrt{1-\frac{4x^2}{9}}}{3x}$	15
risch	$\frac{4x^2-9}{9x\sqrt{-4x^2+9}}$	22
gosper	$\frac{(2x-3)(2x+3)}{9x\sqrt{-4x^2+9}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/9*(-4*x^2+9)^(1/2)/x
```

Maxima [A]
time = 0.50, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/9*sqrt(-4*x^2 + 9)/x
```

Fricas [A]
time = 1.27, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/9*sqrt(-4*x^2 + 9)/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.38, size = 36, normalized size = 2.00

$$\begin{cases} -\frac{i\sqrt{4x^2-9}}{9x} & \text{for } |x^2| > \frac{9}{4} \\ -\frac{\sqrt{9-4x^2}}{9x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(4*x**2 - 9)/(9*x), Abs(x**2) > 9/4), (-sqrt(9 - 4*x**2)/
(9*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 1.21, size = 33, normalized size = 1.83

$$\frac{2\,x}{9\,\left(\sqrt{-4\,x^2+9}-3\right)}-\frac{\sqrt{-4\,x^2+9}-3}{18\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*x/(sqrt(-4*x^2 + 9) - 3) - 1/18*(sqrt(-4*x^2 + 9) - 3)/x
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.78

$$-\frac{2\,\sqrt{\frac{9}{4}-x^2}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] -(2*(9/4 - x^2)^(1/2))/(9*x)
```

3.551 $\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -2/27*arctanh(1/3*(-4*x^2+9)^(1/2))-1/18*(-4*x^2+9)^(1/2)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 44, 65, 212}

$$-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -1/18*Sqrt[9 - 4*x^2]/x^2 - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{18x^2} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{18x^2} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\
&= -\frac{\sqrt{9-4x^2}}{18x^2} - \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.00

$$-\frac{\sqrt{9-4x^2}}{18x^2} - \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[9 - 4*x^2]),x]
[Out] -1/18*Sqrt[9 - 4*x^2]/x^2 - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

Maple [A]
time = 0.10, size = 30, normalized size = 0.77

method	result	si
default	$-\frac{\sqrt{-4x^2+9}}{18x^2}-\frac{2\,\text{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}$	30
trager	$-\frac{\sqrt{-4x^2+9}}{18x^2}+\frac{2\ln\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)}{27}$	34
risch	$\frac{4x^2-9}{18x^2\sqrt{-4x^2+9}}-\frac{2\,\text{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}$	37

meijerg	$-\frac{2\left(-\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)}{32x^2}+\frac{9\sqrt{\pi}\sqrt{1-\frac{4x^2}{9}}}{4x^2}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)-\frac{(1+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}}{2}+\frac{9\sqrt{\pi}}{4x^2}\right)}{27\sqrt{\pi}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*(-4*x^2+9)^(1/2)/x^2-2/27*arctanh(3/(-4*x^2+9)^(1/2))
```

Maxima [A]
time = 0.53, size = 40, normalized size = 1.03

$$-\frac{\sqrt{-4\,x^2+9}}{18\,x^2}-\frac{2}{27}\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(-4*x^2 + 9)/x^2 - 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x)
)
```

Fricas [A]
time = 1.87, size = 38, normalized size = 0.97

$$\frac{4\,x^2\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\,\sqrt{-4\,x^2+9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*sqrt(-4*x^2 + 9))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.10, size = 99, normalized size = 2.54

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{27}+\frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}}-\frac{1}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{1}{|x^2|}>\frac{4}{9} \\ \frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{1-\frac{9}{4x^2}}}+\frac{i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(3/(2*x))/27 + 1/(9*x*sqrt(-1 + 9/(4*x**2))) - 1/(4*x**3
*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2) > 4/9), (2*I*asin(3/(2*x))/27 - I/(9*x
*sqrt(1 - 9/(4*x**2))) + I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A]
time = 1.07, size = 45, normalized size = 1.15

$$-\frac{\sqrt{-4\,x^2+9}}{18\,x^2}-\frac{1}{27}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{1}{27}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(-4*x^2 + 9)/x^2 - 1/27*log(sqrt(-4*x^2 + 9) + 3) + 1/27*log(-sqr
t(-4*x^2 + 9) + 3)
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.90

$$\frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{27}-\frac{\sqrt{\frac{9}{4}-x^2}}{9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2))/27 - (9/4 - x^2)^(1/2)
/(9*x^2)
```

3.552 $\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{9-4x^2}}{27x^3}-\frac{8\sqrt{9-4x^2}}{243x}$$

```
[Out] -1/27*(-4*x^2+9)^(1/2)/x^3-8/243*(-4*x^2+9)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {277, 270}

$$-\frac{8\sqrt{9-4x^2}}{243x}-\frac{\sqrt{9-4x^2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[9 - 4*x^2]),x]
[Out] -1/27*Sqrt[9 - 4*x^2]/x^3 - (8*Sqrt[9 - 4*x^2])/(243*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx &= -\frac{\sqrt{9-4x^2}}{27x^3}+\frac{8}{27}\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx\\ &= -\frac{\sqrt{9-4x^2}}{27x^3}-\frac{8\sqrt{9-4x^2}}{243x}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 0.68

$$\frac{(-9-8x^2)\sqrt{9-4x^2}}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[9 - 4*x^2]),x]
```

```
[Out] ((-9 - 8*x^2)*Sqrt[9 - 4*x^2])/(243*x^3)
```

Maple [A]
time = 0.06, size = 30, normalized size = 0.81

method	result	size
trager	$-\frac{(8x^2+9)\sqrt{-4x^2+9}}{243x^3}$	22
meijerg	$-\frac{\left(1+\frac{8x^2}{9}\right)\sqrt{1-\frac{4x^2}{9}}}{9x^3}$	22
risch	$\frac{32x^4-36x^2-81}{243x^3\sqrt{-4x^2+9}}$	27
default	$-\frac{\sqrt{-4x^2+9}}{27x^3}-\frac{8\sqrt{-4x^2+9}}{243x}$	30
gosper	$\frac{(2x-3)(2x+3)(8x^2+9)}{243x^3\sqrt{-4x^2+9}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*(-4*x^2+9)^(1/2)/x^3-8/243*(-4*x^2+9)^(1/2)/x
```

Maxima [A]
time = 0.61, size = 29, normalized size = 0.78

$$-\frac{8\sqrt{-4x^2+9}}{243x}-\frac{\sqrt{-4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/243*sqrt(-4*x^2 + 9)/x - 1/27*sqrt(-4*x^2 + 9)/x^3
```

Fricas [A]
time = 1.03, size = 21, normalized size = 0.57

$$-\frac{(8x^2+9)\sqrt{-4x^2+9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(8*x^2 + 9)*sqrt(-4*x^2 + 9)/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.64, size = 80, normalized size = 2.16

$$\begin{cases} -\frac{16\sqrt{-1+\frac{9}{4x^2}}}{243} - \frac{2\sqrt{-1+\frac{9}{4x^2}}}{27x^2} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{16i\sqrt{1-\frac{9}{4x^2}}}{243} - \frac{2i\sqrt{1-\frac{9}{4x^2}}}{27x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-16*sqrt(-1 + 9/(4*x**2))/243 - 2*sqrt(-1 + 9/(4*x**2))/(27*x**2
), 1/Abs(x**2) > 4/9), (-16*I*sqrt(1 - 9/(4*x**2))/243 - 2*I*sqrt(1 - 9/(4*
x**2))/(27*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(29) = 58.
time = 1.07, size = 73, normalized size = 1.97

$$\frac{2\,x^3\left(\frac{9\left(\sqrt{-4\,x^2+9}-3\right)^2}{x^2}+4\right)}{243\left(\sqrt{-4\,x^2+9}-3\right)^3} - \frac{\sqrt{-4\,x^2+9}-3}{54\,x} - \frac{\left(\sqrt{-4\,x^2+9}-3\right)^3}{1944\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 2/243*x^3*(9*(sqrt(-4*x^2 + 9) - 3)^2/x^2 + 4)/(sqrt(-4*x^2 + 9) - 3)^3 - 1
/54*(sqrt(-4*x^2 + 9) - 3)/x - 1/1944*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

Mupad [B]
time = 0.02, size = 22, normalized size = 0.59

$$-\sqrt{\frac{9}{4}-x^2}\,\left(\frac{16}{243\,x}+\frac{2}{27\,x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] -(9/4 - x^2)^(1/2)*(16/(243*x) + 2/(27*x^3))
```

3.553 $\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -2/81*arctanh(1/3*(-4*x^2+9)^(1/2))-1/36*(-4*x^2+9)^(1/2)/x^4-1/54*(-4*x^2+
9)^(1/2)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 44, 65, 212}

$$-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{36x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -1/36*Sqrt[9 - 4*x^2]/x^4 - Sqrt[9 - 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 -
4*x^2]/3])/81
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^3}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{36x^4}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^2}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}+\frac{1}{27}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{1}{54}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{9-4x^2}\right)\\ &= -\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.81

$$\frac{\sqrt{9-4x^2}\,(-3-2x^2)}{108x^4}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[9 - 4*x^2]),x]
[Out] (Sqrt[9 - 4*x^2]*(-3 - 2*x^2))/(108*x^4) - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/8
1
```

Maple [A]
time = 0.09, size = 44, normalized size = 0.77

method	result
trager	$-\frac{(2x^2+3)\sqrt{-4x^2+9}}{108x^4}+\frac{2\ln\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)}{81}$
risch	$\frac{8x^4-6x^2-27}{108x^4\sqrt{-4x^2+9}}-\frac{2\,\text{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{81}$

default	$-\frac{\sqrt{-4x^2+9}}{36x^4}-\frac{\sqrt{-4x^2+9}}{54x^2}-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{81}$
meijerg	$\frac{\frac{\sqrt{\pi}\,\left(-\frac{112}{81}x^4+\frac{32}{9}x^2+8\right)}{96x^4}-\frac{\sqrt{\pi}\,\left(8+\frac{16x^2}{3}\right)\sqrt{1-\frac{4x^2}{9}}}{96x^4}-\frac{2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)}{81}+\frac{\left(\frac{7}{6}+2\ln(x)-2\ln(3)+i\pi\right)\sqrt{\pi}}{81}-\frac{\sqrt{\pi}}{12x^4}}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(-4*x^2+9)^(1/2)/x^4-1/54*(-4*x^2+9)^(1/2)/x^2-2/81*arctanh(3/(-4*x^2
+9)^(1/2))
```

Maxima [A]
time = 0.49, size = 54, normalized size = 0.95

$$-\frac{\sqrt{-4\,x^2+9}}{54\,x^2}-\frac{\sqrt{-4\,x^2+9}}{36\,x^4}-\frac{2}{81}\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(-4*x^2 + 9)/x^2 - 1/36*sqrt(-4*x^2 + 9)/x^4 - 2/81*log(6*sqrt(-4
*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A]
time = 1.43, size = 45, normalized size = 0.79

$$\frac{8\,x^4\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\,(2\,x^2+3)\sqrt{-4\,x^2+9}}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 + 3)*sqrt(-4*x^2 + 9)
)/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 2.54, size = 136, normalized size = 2.39

$$\begin{cases} -\frac{2\,\mathrm{acosh}\,\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{-1+\frac{9}{4x^2}}}-\frac{1}{36x^3\sqrt{-1+\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{1}{|x^2|}>\frac{4}{9} \\ \frac{2i\,\mathrm{asin}\,\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{1-\frac{9}{4x^2}}}+\frac{i}{36x^3\sqrt{1-\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(3/(2*x))/81 + 1/(27*x*sqrt(-1 + 9/(4*x**2))) - 1/(36*x*
*3*sqrt(-1 + 9/(4*x**2))) - 1/(8*x**5*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2) >
 4/9), (2*I*asin(3/(2*x))/81 - I/(27*x*sqrt(1 - 9/(4*x**2))) + I/(36*x**3*s
qrt(1 - 9/(4*x**2))) + I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A]
time = 0.97, size = 57, normalized size = 1.00

$$\frac{(-4x^2+9)^{\frac{3}{2}}-15\sqrt{-4x^2+9}}{216x^4}-\frac{1}{81}\log\left(\sqrt{-4x^2+9}+3\right)+\frac{1}{81}\log\left(-\sqrt{-4x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((-4*x^2 + 9)^(3/2) - 15*sqrt(-4*x^2 + 9))/x^4 - 1/81*log(sqrt(-4*x^2
 + 9) + 3) + 1/81*log(-sqrt(-4*x^2 + 9) + 3)
```

Mupad [B]
time = 4.51, size = 49, normalized size = 0.86

$$\frac{2\ln\left(\sqrt{\frac{9}{4x^2}-1}-\sqrt{\frac{9}{4x^2}}\right)}{81}-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{2}{27x^2}+\frac{1}{9x^4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] (2*log((9/(4*x^2) - 1)^(1/2) - (9/(4*x^2))^(1/2)))/81 - ((9/4 - x^2)^(1/2)*
(2/(27*x^2) + 1/(9*x^4)))/2
```

3.554 $\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{81}{64}\sqrt{-9+4x^2}+\frac{3}{32}\left(-9+4x^2\right)^{3/2}+\frac{1}{320}\left(-9+4x^2\right)^{5/2}$$

```
[Out] 3/32*(4*x^2-9)^(3/2)+1/320*(4*x^2-9)^(5/2)+81/64*(4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{1}{320}\left(4x^2-9\right)^{5/2}+\frac{3}{32}\left(4x^2-9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (81*Sqrt[-9 + 4*x^2])/64 + (3*(-9 + 4*x^2)^(3/2))/32 + (-9 + 4*x^2)^(5/2)/3
20
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{-9+4x}}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{-9+4x}}+\frac{9}{8}\sqrt{-9+4x}+\frac{1}{16}(-9+4x)^{3/2}\right)dx,x,x^2\right)\\&=\frac{81}{64}\sqrt{-9+4x^2}+\frac{3}{32}\left(-9+4x^2\right)^{3/2}+\frac{1}{320}\left(-9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{-9+4x^2}\left(27+6x^2+2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-9 + 4*x^2],x]
[Out] (Sqrt[-9 + 4*x^2]*(27 + 6*x^2 + 2*x^4))/40
```

Maple [A]
time = 0.06, size = 41, normalized size = 0.89

method	result	size
trager	$\left(\frac{1}{20}x^4+\frac{3}{20}x^2+\frac{27}{40}\right)\sqrt{4x^2-9}$	23
risch	$\frac{(2x^4+6x^2+27)\sqrt{4x^2-9}}{40}$	24
gosper	$\frac{(2x-3)(2x+3)(2x^4+6x^2+27)}{40\sqrt{4x^2-9}}$	34
default	$\frac{x^4\sqrt{4x^2-9}}{20}+\frac{3x^2\sqrt{4x^2-9}}{20}+\frac{27\sqrt{4x^2-9}}{40}$	41
meijerg	$-\frac{243\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(\frac{32}{27}x^4+\frac{32}{9}x^2+16\right)\sqrt{1-\frac{4x^2}{9}}}{15}\right)}{128\sqrt{\pi}\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/20*x^4*(4*x^2-9)^(1/2)+3/20*x^2*(4*x^2-9)^(1/2)+27/40*(4*x^2-9)^(1/2)
```

Maxima [A]
time = 0.49, size = 40, normalized size = 0.87

$$\frac{1}{20}\sqrt{4x^2-9}\,x^4+\frac{3}{20}\sqrt{4x^2-9}\,x^2+\frac{27}{40}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/20*sqrt(4*x^2 - 9)*x^4 + 3/20*sqrt(4*x^2 - 9)*x^2 + 27/40*sqrt(4*x^2 - 9)
```

Fricas [A]
time = 1.40, size = 23, normalized size = 0.50

$$\frac{1}{40}\left(2x^4+6x^2+27\right)\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(2*x^4 + 6*x^2 + 27)*sqrt(4*x^2 - 9)
```

Sympy [A]
time = 0.24, size = 44, normalized size = 0.96

$$\frac{x^4\sqrt{4x^2-9}}{20}+\frac{3x^2\sqrt{4x^2-9}}{20}+\frac{27\sqrt{4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 - 9)/20 + 3*x**2*sqrt(4*x**2 - 9)/20 + 27*sqrt(4*x**2 - 9)
/40
```

Giac [A]
time = 1.21, size = 34, normalized size = 0.74

$$\frac{1}{320}\left(4\,x^2-9\right)^{\frac{5}{2}}+\frac{3}{32}\left(4\,x^2-9\right)^{\frac{3}{2}}+\frac{81}{64}\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*(4*x^2 - 9)^(5/2) + 3/32*(4*x^2 - 9)^(3/2) + 81/64*sqrt(4*x^2 - 9)
```

Mupad [B]
time = 4.74, size = 22, normalized size = 0.48

$$\sqrt{4\,x^2-9}\,\left(\frac{x^4}{20}+\frac{3\,x^2}{20}+\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2 - 9)^(1/2),x)
```

```
[Out] (4*x^2 - 9)^(1/2)*((3*x^2)/20 + x^4/20 + 27/40)
```

3.555 $\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{27}{128}x\sqrt{-9+4x^2}+\frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

```
[Out] 243/256*arctanh(2*x/(4*x^2-9)^(1/2))+27/128*x*(4*x^2-9)^(1/2)+1/16*x^3*(4*x
^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 223, 212}

$$\frac{27}{128}\sqrt{4x^2-9}\,x+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{16}\sqrt{4x^2-9}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (27*x*Sqrt[-9 + 4*x^2])/128 + (x^3*Sqrt[-9 + 4*x^2])/16 + (243*ArcTanh[(2*x
)/Sqrt[-9 + 4*x^2]])/256
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx = \frac{1}{16}x^3\sqrt{-9+4x^2} + \frac{27}{16}\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx$$
$$= \frac{27}{128}x\sqrt{-9+4x^2} + \frac{1}{16}x^3\sqrt{-9+4x^2} + \frac{243}{128}\int \frac{1}{\sqrt{-9+4x^2}}\,dx$$
$$= \frac{27}{128}x\sqrt{-9+4x^2} + \frac{1}{16}x^3\sqrt{-9+4x^2} + \frac{243}{128}\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)$$
$$= \frac{27}{128}x\sqrt{-9+4x^2} + \frac{1}{16}x^3\sqrt{-9+4x^2} + \frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.81

$$\frac{1}{128}x\sqrt{-9+4x^2}\left(27+8x^2\right) - \frac{243}{256}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2]*(27 + 8*x^2))/128 - (243*Log[-2*x + Sqrt[-9 + 4*x^2]])/
256
```

Maple [A]
time = 0.10, size = 49, normalized size = 0.91

method	result	size
trager	$\frac{x(8x^2+27)\sqrt{4x^2-9}}{128} - \frac{243\ln\left(-\sqrt{4x^2-9}+2x\right)}{256}$	39
risch	$\frac{x(8x^2+27)\sqrt{4x^2-9}}{128} + \frac{243\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{512}$	42
default	$\frac{x^3\sqrt{4x^2-9}}{16} + \frac{27x\sqrt{4x^2-9}}{128} + \frac{243\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{512}$	49
meijerg	$-\frac{81i\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{i\sqrt{\pi}\,x\left(\frac{40x^2}{9}+15\right)\sqrt{1-\frac{4x^2}{9}}}{30}+\frac{3i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{4}\right)}{64\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x^3*(4*x^2-9)^(1/2)+27/128*x*(4*x^2-9)^(1/2)+243/512*ln(x*4^(1/2)+(4*x
^2-9)^(1/2))*4^(1/2)
```

Maxima [A]
time = 0.51, size = 45, normalized size = 0.83

$$\frac{1}{16}\sqrt{4x^2-9}\,x^3+\frac{27}{128}\sqrt{4x^2-9}\,x+\frac{243}{256}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(4*x^2 - 9)*x^3 + 27/128*sqrt(4*x^2 - 9)*x + 243/256*log(8*x + 4*s
qrt(4*x^2 - 9))
```

Fricas [A]
time = 1.42, size = 37, normalized size = 0.69

$$\frac{1}{128}\left(8x^3+27x\right)\sqrt{4x^2-9}-\frac{243}{256}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*x^3 + 27*x)*sqrt(4*x^2 - 9) - 243/256*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A]
time = 0.16, size = 39, normalized size = 0.72

$$\frac{x^3\sqrt{4x^2-9}}{16}+\frac{27x\sqrt{4x^2-9}}{128}+\frac{243\operatorname{acosh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(4*x**2-9)**(1/2),x)
```

```
[Out] x**3*sqrt(4*x**2 - 9)/16 + 27*x*sqrt(4*x**2 - 9)/128 + 243*acosh(2*x/3)/256
```

Giac [A]
time = 0.81, size = 37, normalized size = 0.69

$$\frac{1}{128}\left(8x^2+27\right)\sqrt{4x^2-9}\,x-\frac{243}{256}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*(8*x^2 + 27)*sqrt(4*x^2 - 9)*x - 243/256*log(abs(-2*x + sqrt(4*x^2 -
9)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\sqrt{4\,x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2 - 9)^(1/2),x)
[Out] int(x^4/(4*x^2 - 9)^(1/2), x)
```

3.556 $\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{9}{16}\sqrt{-9+4x^2}+\frac{1}{48}\left(-9+4x^2\right)^{3/2}$$

```
[Out] 1/48*(4*x^2-9)^(3/2)+9/16*(4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{1}{48}\left(4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (9*Sqrt[-9 + 4*x^2])/16 + (-9 + 4*x^2)^(3/2)/48
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{-9+4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{9}{4\sqrt{-9+4x}}+\frac{1}{4}\sqrt{-9+4x}\right)dx,x,x^2\right)\\ &= \frac{9}{16}\sqrt{-9+4x^2}+\frac{1}{48}\left(-9+4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(9+2x^2\right)\sqrt{-9+4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-9 + 4*x^2],x]
[Out] ((9 + 2*x^2)*Sqrt[-9 + 4*x^2])/24
```

Maple [A]
time = 0.05, size = 27, normalized size = 0.87

method	result	size
trager	$\left(\frac{x^2}{12}+\frac{3}{8}\right)\sqrt{4x^2-9}$	18
risch	$\frac{(2x^2+9)\sqrt{4x^2-9}}{24}$	19
default	$\frac{x^2\sqrt{4x^2-9}}{12}+\frac{3\sqrt{4x^2-9}}{8}$	27
gosper	$\frac{(2x-3)(2x+3)(2x^2+9)}{24\sqrt{4x^2-9}}$	29
meijerg	$\frac{27\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(8+\frac{16x^2}{9}\right)\sqrt{1-\frac{4x^2}{9}}}{6}\right)}{32\sqrt{\pi}\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*x^2*(4*x^2-9)^(1/2)+3/8*(4*x^2-9)^(1/2)
```

Maxima [A]
time = 0.49, size = 26, normalized size = 0.84

$$\frac{1}{12}\sqrt{4x^2-9}\,x^2+\frac{3}{8}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/12*sqrt(4*x^2 - 9)*x^2 + 3/8*sqrt(4*x^2 - 9)
```

Fricas [A]
time = 1.11, size = 18, normalized size = 0.58

$$\frac{1}{24}\sqrt{4x^2-9}\left(2x^2+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(4*x^2 - 9)*(2*x^2 + 9)
```

Sympy [A]
time = 0.11, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2-9}}{12}+\frac{3\sqrt{4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**2-9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 - 9)/12 + 3*sqrt(4*x**2 - 9)/8
```

Giac [A]
time = 0.92, size = 23, normalized size = 0.74

$$\frac{1}{48}\left(4\,x^2-9\right)^{\frac{3}{2}}+\frac{9}{16}\,\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(4*x^2 - 9)^(3/2) + 9/16*sqrt(4*x^2 - 9)
```

Mupad [B]
time = 4.86, size = 18, normalized size = 0.58

$$\frac{\left(2\,x^2+9\right)\,\sqrt{4\,x^2-9}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2 - 9)^(1/2),x)
```

```
[Out] ((2*x^2 + 9)*(4*x^2 - 9)^(1/2))/24
```

3.557 $\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=36

$$\frac{1}{8}x\sqrt{-9+4x^2}+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

```
[Out] 9/16*arctanh(2*x/(4*x^2-9)^(1/2))+1/8*x*(4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 223, 212}

$$\frac{1}{8}\sqrt{4x^2-9}\,x+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx = \frac{1}{8}x\sqrt{-9+4x^2} + \frac{9}{8}\int \frac{1}{\sqrt{-9+4x^2}}\,dx$$
$$= \frac{1}{8}x\sqrt{-9+4x^2} + \frac{9}{8}\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)$$
$$= \frac{1}{8}x\sqrt{-9+4x^2} + \frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 1.03

$$\frac{1}{8}x\sqrt{-9+4x^2} - \frac{9}{16}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-9 + 4*x^2],x]
[Out] (x*Sqrt[-9 + 4*x^2])/8 - (9*Log[-2*x + Sqrt[-9 + 4*x^2]])/16
```

Maple [A]
time = 0.09, size = 35, normalized size = 0.97

method	result	size
trager	$\frac{x\sqrt{4x^2-9}}{8}-\frac{9\ln\left(-\sqrt{4x^2-9}+2x\right)}{16}$	32
default	$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{32}$	35
risch	$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{32}$	35
meijerg	$\frac{9i\sqrt{-\text{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{2i\sqrt{\pi}\,x\sqrt{1-\frac{4x^2}{9}}}{3}-i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{16\sqrt{\pi}\,\sqrt{\text{signum}\left(-1+\frac{4x^2}{9}\right)}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8*x*(4*x^2-9)^(1/2)+9/32*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

Maxima [A]
time = 0.50, size = 31, normalized size = 0.86

$$\frac{1}{8}\sqrt{4x^2-9}\,x + \frac{9}{16}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/8*sqrt(4*x^2 - 9)*x + 9/16*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A]
time = 1.36, size = 29, normalized size = 0.81

$$\frac{1}{8}\sqrt{4x^2-9}\,x-\frac{9}{16}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(4*x^2 - 9)*x - 9/16*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A]
time = 0.07, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\operatorname{acosh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*x**2-9)**(1/2),x)
[Out] x*sqrt(4*x**2 - 9)/8 + 9*acosh(2*x/3)/16
```

Giac [A]
time = 0.80, size = 30, normalized size = 0.83

$$\frac{1}{8}\sqrt{4x^2-9}\,x-\frac{9}{16}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(4*x^2 - 9)*x - 9/16*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

Mupad [B]
time = 0.10, size = 29, normalized size = 0.81

$$\frac{9\ln\left(x+\frac{\sqrt{4x^2-9}}{2}\right)}{16}+\frac{x\sqrt{4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2 - 9)^(1/2),x)
[Out] (9*log(x + (4*x^2 - 9)^(1/2)/2))/16 + (x*(4*x^2 - 9)^(1/2))/8
```

3.558 $\int \frac{x}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{4}\sqrt{-9+4x^2}$$

[Out] 1/4*(4*x^2-9)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[-9 + 4*x^2],x]

[Out] Sqrt[-9 + 4*x^2]/4

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx = \frac{1}{4}\sqrt{-9+4x^2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{4}\sqrt{-9+4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[-9 + 4*x^2],x]

[Out] Sqrt[-9 + 4*x^2]/4

Maple [A]
time = 0.05, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\sqrt{4x^2-9}}{4}$	12
default	$\frac{\sqrt{4x^2-9}}{4}$	12
trager	$\frac{\sqrt{4x^2-9}}{4}$	12
risch	$\frac{\sqrt{4x^2-9}}{4}$	12
gosper	$\frac{(2x-3)(2x+3)}{4\sqrt{4x^2-9}}$	22
meijerg	$-\frac{3\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1-\frac{4x^2}{9}}\right)}{8\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(4*x^2-9)^(1/2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/4*sqrt(4*x^2 - 9)
```

Fricas [A]
time = 1.33, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(4*x^2 - 9)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2-9)**(1/2),x)
```

```
[Out] sqrt(4*x**2 - 9)/4
```

Giac [A]
time = 0.90, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

Mupad [B]
time = 0.14, size = 11, normalized size = 0.73

$$\frac{\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2 - 9)^(1/2),x)
```

```
[Out] (4*x^2 - 9)^(1/2)/4
```

3.559 $\int \frac{1}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

```
[Out] 1/2*arctanh(2*x/(4*x^2-9)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {223, 212}

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 + 4*x^2],x]
```

```
[Out] ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9+4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)\\ &= \frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.
time = 0.00, size = 43, normalized size = 2.26

$$-\frac{1}{4}\log\left(1-\frac{2x}{\sqrt{-9+4x^2}}\right)+\frac{1}{4}\log\left(1+\frac{2x}{\sqrt{-9+4x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 + 4*x^2],x]
[Out] -1/4*Log[1 - (2*x)/Sqrt[-9 + 4*x^2]] + Log[1 + (2*x)/Sqrt[-9 + 4*x^2]]/4
```

Maple [A]
time = 0.08, size = 22, normalized size = 1.16

method	result	size
trager	$\frac{\ln\left(\sqrt{4x^2-9}+2x\right)}{2}$	17
default	$\frac{\ln\left(x\sqrt{4}+\sqrt{4x^2-9}\right)\sqrt{4}}{4}$	22
meijerg	$\frac{\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\,\arcsin\left(\frac{2x}{3}\right)}{2\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

Maxima [A]
time = 0.48, size = 18, normalized size = 0.95

$$\frac{1}{2}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/2*log(8*x + 4*sqrt(4*x^2 - 9))
```

Fricas [A]
time = 1.74, size = 16, normalized size = 0.84

$$-\frac{1}{2}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [**A**]
time = 0.05, size = 7, normalized size = 0.37

$$\frac{\operatorname{acosh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2-9)**(1/2),x)
```

```
[Out] acosh(2*x/3)/2
```

Giac [**A**]
time = 0.79, size = 30, normalized size = 1.58

$$\frac{1}{2}\sqrt{4x^2-9}\,x+\frac{9}{4}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(4*x^2 - 9)*x + 9/4*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

Mupad [**B**]
time = 4.75, size = 16, normalized size = 0.84

$$\frac{\ln\left(2x+\sqrt{4x^2-9}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 9)^(1/2),x)
```

```
[Out] log(2*x + (4*x^2 - 9)^(1/2))/2
```

3.560 $\int \frac{1}{x\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

[Out] 1/3*arctan(1/3*(4*x^2-9)^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {272, 65, 209}

$$\frac{1}{3}\text{ArcTan}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[-9 + 4*x^2]),x]

[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\ &= \frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-9 + 4*x^2]),x]
[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

Maple [A]
time = 0.12, size = 15, normalized size = 0.75

method	result	size
default	$-\frac{\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{3}$	15
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{4x^2-9}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)}{x}\right)}{3}$	32
meijerg	$\frac{\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)+(2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}\right)}{6\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A]
time = 0.50, size = 9, normalized size = 0.45

$$-\frac{1}{3}\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/3*arcsin(3/2/abs(x))
```

Fricas [A]
time = 1.55, size = 18, normalized size = 0.90

$$\frac{2}{3}\arctan\left(-\frac{2}{3}x+\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 2/3*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 26, normalized size = 1.30

$$\begin{cases}\frac{i\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for}\ \frac{1}{|x^2|}>\frac{4}{9}\\ -\frac{\operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x**2-9)**(1/2),x)
[Out] Piecewise((I*acosh(3/(2*x))/3, 1/Abs(x**2) > 4/9), (-asin(3/(2*x))/3, True)
)
```

Giac [A]
time = 0.78, size = 14, normalized size = 0.70

$$\frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/3*arctan(1/3*sqrt(4*x^2 - 9))
```

Mupad [B]
time = 0.12, size = 20, normalized size = 1.00

$$\frac{\ln\left(\frac{\sqrt{4x^2-9}+3\mathrm{i}}{x}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4*x^2 - 9)^(1/2)),x)
[Out] (log(((4*x^2 - 9)^(1/2) + 3i)/x)*1i)/3
```

3.561 $\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{-9+4x^2}}{9x}$$

```
[Out] 1/9*(4*x^2-9)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{\sqrt{4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}}{9x}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{\sqrt{-9+4x^2}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

Maple [A]
time = 0.05, size = 15, normalized size = 0.83

method	result	size
default	$\frac{\sqrt{4x^2-9}}{9x}$	15
trager	$\frac{\sqrt{4x^2-9}}{9x}$	15
risch	$\frac{\sqrt{4x^2-9}}{9x}$	15
gosper	$\frac{(2x-3)(2x+3)}{9x\sqrt{4x^2-9}}$	25
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,\sqrt{1-\frac{4x^2}{9}}}{3\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,x}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/9*(4*x^2-9)^(1/2)/x
```

Maxima [A]
time = 0.50, size = 14, normalized size = 0.78

$$\frac{\sqrt{4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/9*sqrt(4*x^2 - 9)/x
```

Fricas [A]
time = 1.41, size = 18, normalized size = 1.00

$$\frac{2\,x+\sqrt{4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/9*(2*x + sqrt(4*x^2 - 9))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.38, size = 37, normalized size = 2.06

$$\begin{cases} \frac{2i\sqrt{-1+\frac{9}{4x^2}}}{9} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ \frac{2\sqrt{1-\frac{9}{4x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4*x**2-9)**(1/2),x)
[Out] Piecewise((2*I*sqrt(-1 + 9/(4*x**2))/9, 1/Abs(x**2) > 4/9), (2*sqrt(1 - 9/(
4*x**2))/9, True))
```

Giac [A]
time = 0.70, size = 23, normalized size = 1.28

$$\frac{4}{\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 4/((2*x - sqrt(4*x^2 - 9))^2 + 9)
```

Mupad [B]
time = 4.75, size = 14, normalized size = 0.78

$$\frac{\sqrt{4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(4*x^2 - 9)^(1/2)),x)
[Out] (4*x^2 - 9)^(1/2)/(9*x)
```

3.562 $\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

```
[Out] 2/27*arctan(1/3*(4*x^2-9)^(1/2))+1/18*(4*x^2-9)^(1/2)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 44, 65, 209}

$$\frac{2}{27}\text{ArcTan}\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}}{18x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/27
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{9}\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\ &= \frac{\sqrt{-9+4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.00

$$\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-9 + 4*x^2]),x]
[Out] Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/27
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.77

method	result
default	$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}$
risch	$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}$
trager	$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{2\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{4x^2-9}-3\,\text{RootOf}\left(\_Z^2+1\right)}{x}\right)}{27}$

meijerg	$-\frac{2\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)}{32x^2}+\frac{9\sqrt{\pi}\sqrt{1-\frac{4x^2}{9}}}{4x^2}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right)-\frac{(1+2\ln(x)-2\ln\ldots}{}\right.}{27\sqrt{\pi}\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/18*(4*x^2-9)^(1/2)/x^2-2/27*arctan(3/(4*x^2-9)^(1/2))
```

Maxima [A]
time = 0.53, size = 24, normalized size = 0.62

$$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{2}{27}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/18*sqrt(4*x^2 - 9)/x^2 - 2/27*arcsin(3/2/abs(x))
```

Fricas [A]
time = 2.35, size = 38, normalized size = 0.97

$$\frac{8x^2\arctan\left(-\frac{2}{3}x+\frac{1}{3}\sqrt{4x^2-9}\right)+3\sqrt{4x^2-9}}{54x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/54*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.08, size = 99, normalized size = 2.54

$$\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|}>\frac{4}{9}\\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{27}+\frac{1}{9x\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + I/(4*x**
3*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2) > 4/9), (-2*asin(3/(2*x))/27 + 1/(9*x
*sqrt(1 - 9/(4*x**2))) - 1/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A]
time = 0.91, size = 29, normalized size = 0.74

$$\frac{\sqrt{4\,x^2-9}}{18\,x^2}+\frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/18*sqrt(4*x^2 - 9)/x^2 + 2/27*arctan(1/3*sqrt(4*x^2 - 9))
```

Mupad [B]
time = 4.88, size = 29, normalized size = 0.74

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{27}+\frac{\sqrt{4\,x^2-9}}{18\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(4*x^2 - 9)^(1/2)),x)
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/27 + (4*x^2 - 9)^(1/2)/(18*x^2)
```

3.563 $\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{-9+4x^2}}{27x^3}+\frac{8\sqrt{-9+4x^2}}{243x}$$

```
[Out] 1/27*(4*x^2-9)^(1/2)/x^3+8/243*(4*x^2-9)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {277, 270}

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-9 + 4*x^2]),x]
[Out] Sqrt[-9 + 4*x^2]/(27*x^3) + (8*Sqrt[-9 + 4*x^2])/(243*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx&=\frac{\sqrt{-9+4x^2}}{27x^3}+\frac{8}{27}\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx\\&=\frac{\sqrt{-9+4x^2}}{27x^3}+\frac{8\sqrt{-9+4x^2}}{243x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.68

$$\frac{\sqrt{-9+4x^2}\,(9+8x^2)}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] (Sqrt[-9 + 4*x^2]*(9 + 8*x^2))/(243*x^3)
```

Maple [A]
time = 0.06, size = 30, normalized size = 0.81

method	result	size
trager	$\frac{(8x^2+9)\sqrt{4x^2-9}}{243x^3}$	22
risch	$\frac{32x^4-36x^2-81}{243x^3\sqrt{4x^2-9}}$	27
default	$\frac{\sqrt{4x^2-9}}{27x^3}+\frac{8\sqrt{4x^2-9}}{243x}$	30
gosper	$\frac{(2x-3)(2x+3)(8x^2+9)}{243x^3\sqrt{4x^2-9}}$	32
meijerg	$-\frac{\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(1+\frac{8x^2}{9}\right)\sqrt{1-\frac{4x^2}{9}}}{9\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\,x^3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(4*x^2-9)^(1/2)/x^3+8/243*(4*x^2-9)^(1/2)/x
```

Maxima [A]
time = 0.49, size = 29, normalized size = 0.78

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/243*sqrt(4*x^2 - 9)/x + 1/27*sqrt(4*x^2 - 9)/x^3
```

Fricas [A]
time = 1.59, size = 28, normalized size = 0.76

$$\frac{16x^3+(8x^2+9)\sqrt{4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/243*(16*x^3 + (8*x^2 + 9)*sqrt(4*x^2 - 9))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.63, size = 68, normalized size = 1.84

$$\begin{cases} \frac{8\sqrt{4x^2-9}}{243x} + \frac{\sqrt{4x^2-9}}{27x^3} & \text{for } |x^2| > \frac{9}{4} \\ \frac{8i\sqrt{9-4x^2}}{243x} + \frac{i\sqrt{9-4x^2}}{27x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((8*sqrt(4*x**2 - 9)/(243*x) + sqrt(4*x**2 - 9)/(27*x**3), Abs(x**
2) > 9/4), (8*I*sqrt(9 - 4*x**2)/(243*x) + I*sqrt(9 - 4*x**2)/(27*x**3), Tr
ue))
```

Giac [A]
time = 0.89, size = 42, normalized size = 1.14

$$\frac{32\left(\left(2x-\sqrt{4x^2-9}\right)^2+3\right)}{\left(\left(2x-\sqrt{4x^2-9}\right)^2+9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 32*((2*x - sqrt(4*x^2 - 9))^2 + 3)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

Mupad [B]
time = 4.87, size = 31, normalized size = 0.84

$$\frac{8x^2\sqrt{4x^2-9}+9\sqrt{4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(4*x^2 - 9)^(1/2)),x)
```

```
[Out] (8*x^2*(4*x^2 - 9)^(1/2) + 9*(4*x^2 - 9)^(1/2))/(243*x^3)
```

3.564 $\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{-9+4x^2}}{36x^4}+\frac{\sqrt{-9+4x^2}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

```
[Out] 2/81*arctan(1/3*(4*x^2-9)^(1/2))+1/36*(4*x^2-9)^(1/2)/x^4+1/54*(4*x^2-9)^(1
/2)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {272, 44, 65, 209}

$$\frac{2}{81}\text{ArcTan}\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}}{54x^2}+\frac{\sqrt{4x^2-9}}{36x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(36*x^4) + Sqrt[-9 + 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/81
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4}+\frac{\sqrt{-9+4x^2}}{54x^2}+\frac{1}{27}\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4}+\frac{\sqrt{-9+4x^2}}{54x^2}+\frac{1}{54}\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{-9+4x^2}\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4}+\frac{\sqrt{-9+4x^2}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.81

$$\frac{(3+2x^2)\sqrt{-9+4x^2}}{108x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-9 + 4*x^2]),x]
[Out] ((3 + 2*x^2)*Sqrt[-9 + 4*x^2])/(108*x^4) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/8
1
```

Maple [A]
time = 0.13, size = 44, normalized size = 0.77

method	result
risch	$\frac{8x^4-6x^2-27}{108x^4\sqrt{4x^2-9}}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{81}$
default	$\frac{\sqrt{4x^2-9}}{36x^4}+\frac{\sqrt{4x^2-9}}{54x^2}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{81}$

trager	$\frac{(2x^2+3)\sqrt{4x^2-9}}{108x^4}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{4x^2-9}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)}{x}\right)}{81}$
meijerg	$\frac{8\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{81\sqrt{\pi}\left(-\frac{112}{81}x^4+\frac{32}{9}x^2+8\right)}{256x^4}-\frac{81\sqrt{\pi}\left(8+\frac{16x^2}{3}\right)\sqrt{1-\frac{4x^2}{9}}}{256x^4}-\frac{3\sqrt{\pi}\ \ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{4x^2}{9}}}{2}\right.}{4}\right.}{243\sqrt{\pi}\ \sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(4*x^2-9)^(1/2)/x^4+1/54*(4*x^2-9)^(1/2)/x^2-2/81*arctan(3/(4*x^2-9)^(
1/2))
```

Maxima [A]
time = 0.50, size = 38, normalized size = 0.67

$$\frac{\sqrt{4\,x^2-9}}{54\,x^2}+\frac{\sqrt{4\,x^2-9}}{36\,x^4}-\frac{2}{81}\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(4*x^2 - 9)/x^2 + 1/36*sqrt(4*x^2 - 9)/x^4 - 2/81*arcsin(3/2/abs(x
))
```

Fricas [A]
time = 1.73, size = 45, normalized size = 0.79

$$\frac{16\,x^4\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}\,(2\,x^2+3)}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*x
^2 + 3))/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 2.49, size = 136, normalized size = 2.39

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{81} - \frac{i}{27x\sqrt{-1+\frac{9}{4x^2}}} + \frac{i}{36x^3\sqrt{-1+\frac{9}{4x^2}}} + \frac{i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{81} + \frac{1}{27x\sqrt{1-\frac{9}{4x^2}}} - \frac{1}{36x^3\sqrt{1-\frac{9}{4x^2}}} - \frac{1}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/81 - I/(27*x*sqrt(-1 + 9/(4*x**2))) + I/(36*x
**3*sqrt(-1 + 9/(4*x**2))) + I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2)
> 4/9), (-2*asin(3/(2*x))/81 + 1/(27*x*sqrt(1 - 9/(4*x**2))) - 1/(36*x**3*s
qrt(1 - 9/(4*x**2))) - 1/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Giac [A]
time = 1.10, size = 41, normalized size = 0.72

$$\frac{(4\,x^2-9)^{\frac{3}{2}}+15\,\sqrt{4\,x^2-9}}{216\,x^4}+\frac{2}{81}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((4*x^2 - 9)^(3/2) + 15*sqrt(4*x^2 - 9))/x^4 + 2/81*arctan(1/3*sqrt(4
*x^2 - 9))
```

Mupad [B]
time = 5.32, size = 57, normalized size = 1.00

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{81}+\frac{\frac{10\,\sqrt{4\,x^2-9}}{9}+\frac{2\,{(4\,x^2-9)}^{3/2}}{27}}{72\,x^2+{(4\,x^2-9)}^2-81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(4*x^2 - 9)^(1/2)),x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/81 + ((10*(4*x^2 - 9)^(1/2))/9 + (2*(4*x^2 -
9)^(3/2))/27)/(72*x^2 + (4*x^2 - 9)^2 - 81)
```

3.565 $\int \frac{x^5}{\sqrt{-9-4x^2}} \, dx$

Optimal. Leaf size=46

$$-\frac{81}{64}\sqrt{-9-4x^2}-\frac{3}{32}\left(-9-4x^2\right)^{3/2}-\frac{1}{320}\left(-9-4x^2\right)^{5/2}$$

```
[Out] -3/32*(-4*x^2-9)^(3/2)-1/320*(-4*x^2-9)^(5/2)-81/64*(-4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$-\frac{1}{320}\left(-4x^2-9\right)^{5/2}-\frac{3}{32}\left(-4x^2-9\right)^{3/2}-\frac{81}{64}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-81*Sqrt[-9 - 4*x^2])/64 - (3*(-9 - 4*x^2)^(3/2))/32 - (-9 - 4*x^2)^(5/2)/
320
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{-9-4x^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{-9-4x}} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{-9-4x}}+\frac{9}{8}\sqrt{-9-4x}+\frac{1}{16}(-9-4x)^{3/2}\right) dx, x, x^2\right)\\ &= -\frac{81}{64}\sqrt{-9-4x^2}-\frac{3}{32}\left(-9-4x^2\right)^{3/2}-\frac{1}{320}\left(-9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{-9-4x^2}\left(-27+6x^2-2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-9 - 4*x^2],x]
[Out] (Sqrt[-9 - 4*x^2]*(-27 + 6*x^2 - 2*x^4))/40
```

Maple [A]
time = 0.04, size = 41, normalized size = 0.89

method	result	size
trager	$\left(-\frac{1}{20}x^4+\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{-4x^2-9}$	23
gosper	$-\frac{(2x^4-6x^2+27)\sqrt{-4x^2-9}}{40}$	24
risch	$\frac{(4x^2+9)(2x^4-6x^2+27)}{40\sqrt{-4x^2-9}}$	31
meijerg	$-\frac{243i\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(\frac{32}{27}x^4-\frac{32}{9}x^2+16\right)\sqrt{1+\frac{4x^2}{9}}}{15}\right)}{128\sqrt{\pi}}$	39
default	$-\frac{x^4\sqrt{-4x^2-9}}{20}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/20*x^4*(-4*x^2-9)^(1/2)+3/20*x^2*(-4*x^2-9)^(1/2)-27/40*(-4*x^2-9)^(1/2)
```

Maxima [A]
time = 0.49, size = 40, normalized size = 0.87

$$-\frac{1}{20}\sqrt{-4x^2-9}x^4+\frac{3}{20}\sqrt{-4x^2-9}x^2-\frac{27}{40}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/20*sqrt(-4*x^2 - 9)*x^4 + 3/20*sqrt(-4*x^2 - 9)*x^2 - 27/40*sqrt(-4*x^2
- 9)
```

Fricas [A]
time = 1.00, size = 23, normalized size = 0.50

$$-\frac{1}{40}\left(2x^4-6x^2+27\right)\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(2*x^4 - 6*x^2 + 27)*sqrt(-4*x^2 - 9)
```

Sympy [A]
time = 0.24, size = 49, normalized size = 1.07

$$-\frac{x^4\sqrt{-4x^2-9}}{20}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**4*sqrt(-4*x**2 - 9)/20 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2
- 9)/40
```

Giac [C] Result contains complex when optimal does not.
time = 1.11, size = 34, normalized size = 0.74

$$-\frac{1}{320}i\left(4x^2+9\right)^{\frac{5}{2}}+\frac{3}{32}i\left(4x^2+9\right)^{\frac{3}{2}}-\frac{81}{64}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/320*I*(4*x^2 + 9)^(5/2) + 3/32*I*(4*x^2 + 9)^(3/2) - 81/64*sqrt(-4*x^2 -
 9)
```

Mupad [B]
time = 5.29, size = 23, normalized size = 0.50

$$-\sqrt{-4x^2-9}\left(\frac{x^4}{20}-\frac{3x^2}{20}+\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] -(- 4*x^2 - 9)^(1/2)*(x^4/20 - (3*x^2)/20 + 27/40)
```

3.566 $\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{27}{128}x\sqrt{-9-4x^2}-\frac{1}{16}x^3\sqrt{-9-4x^2}+\frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

```
[Out] 243/256*arctan(2*x/(-4*x^2-9)^(1/2))+27/128*x*(-4*x^2-9)^(1/2)-1/16*x^3*(-4
*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 223, 209}

$$\frac{243}{256}\text{ArcTan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)+\frac{27}{128}\sqrt{-4x^2-9}\,x-\frac{1}{16}\sqrt{-4x^2-9}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (27*x*Sqrt[-9 - 4*x^2])/128 - (x^3*Sqrt[-9 - 4*x^2])/16 + (243*ArcTan[(2*x)
/Sqrt[-9 - 4*x^2]])/256
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{16}x^3\sqrt{-9-4x^2} - \frac{27}{16}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx$$
$$= \frac{27}{128}x\sqrt{-9-4x^2} - \frac{1}{16}x^3\sqrt{-9-4x^2} + \frac{243}{128}\int \frac{1}{\sqrt{-9-4x^2}}\,dx$$
$$= \frac{27}{128}x\sqrt{-9-4x^2} - \frac{1}{16}x^3\sqrt{-9-4x^2} + \frac{243}{128}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)$$
$$= \frac{27}{128}x\sqrt{-9-4x^2} - \frac{1}{16}x^3\sqrt{-9-4x^2} + \frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 49, normalized size = 0.91

$$\frac{1}{128}\sqrt{-9-4x^2}\left(27x-8x^3\right) + \frac{243}{256}i\log\left(-2ix+\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (Sqrt[-9 - 4*x^2]*(27*x - 8*x^3))/128 + ((243*I)/256)*Log[(-2*I)*x + Sqrt[-
9 - 4*x^2]]
```

Maple [A]
time = 0.12, size = 43, normalized size = 0.80

method	result	size
meijerg	$-\frac{81i\left(-\frac{\sqrt{\pi}\,x\left(-\frac{40x^2}{9}+15\right)\sqrt{1+\frac{4x^2}{9}}}{30}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{4}\right)}{64\sqrt{\pi}}$	39
default	$\frac{243\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}+\frac{27x\sqrt{-4x^2-9}}{128}-\frac{x^3\sqrt{-4x^2-9}}{16}$	43
risch	$\frac{x(8x^2-27)(4x^2+9)}{128\sqrt{-4x^2-9}}+\frac{243\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$	43
trager	$-\frac{x(8x^2-27)\sqrt{-4x^2-9}}{128}+\frac{243\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(\text{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{256}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 243/256*arctan(2*x/(-4*x^2-9)^(1/2))+27/128*x*(-4*x^2-9)^(1/2)-1/16*x^3*(-4
*x^2-9)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 33, normalized size = 0.61

$$-\frac{1}{16}\sqrt{-4x^2-9}\,x^3+\frac{27}{128}\sqrt{-4x^2-9}\,x-\frac{243}{256}i\,\mathrm{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(-4*x^2 - 9)*x^3 + 27/128*sqrt(-4*x^2 - 9)*x - 243/256*I*arcsinh(
2/3*x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.76, size = 67, normalized size = 1.24

$$-\frac{1}{128}\left(8x^3-27x\right)\sqrt{-4x^2-9}+\frac{243}{512}i\log\left(-\frac{4\left(2x+i\sqrt{-4x^2-9}\right)}{x}\right)-\frac{243}{512}i\log\left(-\frac{4\left(2x-i\sqrt{-4x^2-9}\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(8*x^3 - 27*x)*sqrt(-4*x^2 - 9) + 243/512*I*log(-4*(2*x + I*sqrt(-4*
x^2 - 9))/x) - 243/512*I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A]
time = 0.25, size = 53, normalized size = 0.98

$$-\frac{x^3\sqrt{-4x^2-9}}{16}+\frac{27x\sqrt{-4x^2-9}}{128}+\frac{243\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**3*sqrt(-4*x**2 - 9)/16 + 27*x*sqrt(-4*x**2 - 9)/128 + 243*atan(2*x/sqrt
(-4*x**2 - 9))/256
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(-4*x^2 - 9), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\sqrt{-4\,x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^4/(- 4*x^2 - 9)^(1/2), x)
```

3.567 $\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{9}{16}\sqrt{-9-4x^2}+\frac{1}{48}\left(-9-4x^2\right)^{3/2}$$

```
[Out] 1/48*(-4*x^2-9)^(3/2)+9/16*(-4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{1}{48}\left(-4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (9*Sqrt[-9 - 4*x^2])/16 + (-9 - 4*x^2)^(3/2)/48
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{-9-4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{9}{4\sqrt{-9-4x}}-\frac{1}{4}\sqrt{-9-4x}\right)\,dx,x,x^2\right)\\ &= \frac{9}{16}\sqrt{-9-4x^2}+\frac{1}{48}\left(-9-4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\sqrt{-9-4x^2}\,(9-2x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-9 - 4*x^2],x]
[Out] (Sqrt[-9 - 4*x^2]*(9 - 2*x^2))/24
```

Maple [A]
time = 0.03, size = 27, normalized size = 0.87

method	result	size
trager	$\left(-\frac{x^2}{12}+\frac{3}{8}\right)\sqrt{-4x^2-9}$	18
gosper	$-\frac{(2x^2-9)\sqrt{-4x^2-9}}{24}$	19
risch	$\frac{(4x^2+9)(2x^2-9)}{24\sqrt{-4x^2-9}}$	26
default	$-\frac{x^2\sqrt{-4x^2-9}}{12}+\frac{3\sqrt{-4x^2-9}}{8}$	27
meijerg	$-\frac{27i\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)\sqrt{1+\frac{4x^2}{9}}}{6}\right)}{32\sqrt{\pi}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/12*x^2*(-4*x^2-9)^(1/2)+3/8*(-4*x^2-9)^(1/2)
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.84

$$-\frac{1}{12}\sqrt{-4\,x^2-9}\,x^2+\frac{3}{8}\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/12*sqrt(-4*x^2 - 9)*x^2 + 3/8*sqrt(-4*x^2 - 9)
```

Fricas [A]
time = 1.45, size = 18, normalized size = 0.58

$$-\frac{1}{24}\left(2\,x^2-9\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*x^2 - 9)*sqrt(-4*x^2 - 9)
```

Sympy [A]
time = 0.11, size = 31, normalized size = 1.00

$$-\frac{x^2\sqrt{-4x^2-9}}{12}+\frac{3\sqrt{-4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**2*sqrt(-4*x**2 - 9)/12 + 3*sqrt(-4*x**2 - 9)/8
```

Giac [C] Result contains complex when optimal does not.
time = 0.80, size = 23, normalized size = 0.74

$$-\frac{1}{48}i\left(4\,x^2+9\right)^{\frac{3}{2}}+\frac{9}{16}\,\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*I*(4*x^2 + 9)^(3/2) + 9/16*sqrt(-4*x^2 - 9)
```

Mupad [B]
time = 5.07, size = 18, normalized size = 0.58

$$-\frac{\left(2\,x^2-9\right)\,\sqrt{-4\,x^2-9}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] -((2*x^2 - 9)*(- 4*x^2 - 9)^(1/2))/24
```

3.568 $\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

```
[Out] -9/16*arctan(2*x/(-4*x^2-9)^(1/2))-1/8*x*(-4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 223, 209}

$$-\frac{9}{16}\text{ArcTan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{1}{8}\sqrt{-4x^2-9}\,x$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/8*(x*Sqrt[-9 - 4*x^2]) - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx &= -\frac{1}{8}x\sqrt{-9-4x^2} - \frac{9}{8}\int \frac{1}{\sqrt{-9-4x^2}}\,dx \\ &= -\frac{1}{8}x\sqrt{-9-4x^2} - \frac{9}{8}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right) \\ &= -\frac{1}{8}x\sqrt{-9-4x^2} - \frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 41, normalized size = 1.14

$$-\frac{1}{8}x\sqrt{-9-4x^2} - \frac{9}{16}i\log\left(-2ix + \sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-9 - 4*x^2],x]
[Out] -1/8*(x*Sqrt[-9 - 4*x^2]) - ((9*I)/16)*Log[(-2*I)*x + Sqrt[-9 - 4*x^2]]
```

Maple [A]
time = 0.11, size = 29, normalized size = 0.81

method	result	size
default	$-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16} - \frac{x\sqrt{-4x^2-9}}{8}$	29
meijerg	$-\frac{9i\left(\frac{2\sqrt{\pi}\,x\sqrt{1+\frac{4x^2}{9}}}{3} - \sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)\right)}{16\sqrt{\pi}}$	32
risch	$\frac{x(4x^2+9)}{8\sqrt{-4x^2-9}} - \frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$	36
trager	$-\frac{x\sqrt{-4x^2-9}}{8} - \frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{16}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] -9/16*arctan(2*x/(-4*x^2-9)^(1/2))-1/8*x*(-4*x^2-9)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 19, normalized size = 0.53

$$-\frac{1}{8}\sqrt{-4\,x^2-9}\,x + \frac{9}{16}i\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^2 - 9)*x + 9/16*I*arcsinh(2/3*x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.90, size = 59, normalized size = 1.64

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x-\frac{9}{32}i\log\left(-\frac{4\left(2x+i\sqrt{-4x^2-9}\right)}{x}\right)+\frac{9}{32}i\log\left(-\frac{4\left(2x-i\sqrt{-4x^2-9}\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 - 9)*x - 9/32*I*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) + 9/3
2*I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A]
time = 0.16, size = 36, normalized size = 1.00

$$-\frac{x\sqrt{-4x^2-9}}{8}-\frac{9\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x*sqrt(-4*x**2 - 9)/8 - 9*atan(2*x/sqrt(-4*x**2 - 9))/16
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-4*x^2 - 9), x)
```

Mupad [B]
time = 0.12, size = 31, normalized size = 0.86

$$-\frac{x\sqrt{-4x^2-9}}{8}+\frac{\ln\left(x-\frac{\sqrt{-4x^2-9}\,1\mathrm{i}}{2}\right)\,9\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] (log(x - ((- 4*x^2 - 9)^(1/2)*1i)/2)*9i)/16 - (x*(- 4*x^2 - 9)^(1/2))/8
```

3.569 $\int \frac{x}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{4}\sqrt{-9-4x^2}$$

```
[Out] -1/4*(-4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/4*Sqrt[-9 - 4*x^2]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{4}\sqrt{-9-4x^2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{4}\sqrt{-9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/4*Sqrt[-9 - 4*x^2]
```

Maple [A]
time = 0.03, size = 12, normalized size = 0.80

method	result	size
gosper	$-\frac{\sqrt{-4x^2-9}}{4}$	12
derivativedivides	$-\frac{\sqrt{-4x^2-9}}{4}$	12
default	$-\frac{\sqrt{-4x^2-9}}{4}$	12
trager	$-\frac{\sqrt{-4x^2-9}}{4}$	12
risch	$\frac{4x^2+9}{4\sqrt{-4x^2-9}}$	19
meijerg	$-\frac{3i\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+\frac{4x^2}{9}}\right)}{8\sqrt{\pi}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-4*x^2-9)^(1/2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

Fricas [A]
time = 0.92, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

Sympy [A]
time = 0.06, size = 14, normalized size = 0.93

$$-\frac{\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x**2-9)**(1/2),x)
```

```
[Out] -sqrt(-4*x**2 - 9)/4
```

Giac [A]
time = 1.06, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

Mupad [B]
time = 4.99, size = 11, normalized size = 0.73

$$-\frac{\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] -(- 4*x^2 - 9)^(1/2)/4
```

3.570 $\int \frac{1}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

```
[Out] 1/2*arctan(2*x/(-4*x^2-9)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {223, 209}

$$\frac{1}{2}\text{ArcTan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 - 4*x^2],x]
```

```
[Out] ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9-4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-9 - 4*x^2],x]

[Out] ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2

Maple [A]
time = 0.10, size = 16, normalized size = 0.84

method	result	size
meijerg	$-\frac{i \operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	8
default	$\frac{\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$	16
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*arctan(2*x/(-4*x^2-9)^(1/2))

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 6, normalized size = 0.32

$$-\frac{1}{2}i \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="maxima")

[Out] -1/2*I*arcsinh(2/3*x)

Fricas [C] Result contains complex when optimal does not.
time = 1.19, size = 47, normalized size = 2.47

$$\frac{1}{4}i \log\left(-\frac{4\left(2x+i\sqrt{-4x^2-9}\right)}{x}\right)-\frac{1}{4}i \log\left(-\frac{4\left(2x-i\sqrt{-4x^2-9}\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="fricas")

[Out] 1/4*I*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) - 1/4*I*log(-4*(2*x - I*sqrt(-4*
x^2 - 9))/x)

Sympy [A]
time = 0.14, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2-9)**(1/2),x)
[Out] atan(2*x/sqrt(-4*x**2 - 9))/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-4*x^2 - 9), x)
```

Mupad [B]
time = 0.11, size = 15, normalized size = 0.79

$$\frac{\operatorname{atan}\left(\frac{2\,x}{\sqrt{-4\,x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 4*x^2 - 9)^(1/2),x)
[Out] atan((2*x)/(- 4*x^2 - 9)^(1/2))/2
```

3.571 $\int \frac{1}{x\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

[Out] 1/3*arctan(1/3*(-4*x^2-9)^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {272, 65, 210}

$$\frac{1}{3}\text{ArcTan}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[-9 - 4*x^2]),x]

[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\right)$$
$$= \frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

Maple [A]
time = 0.10, size = 15, normalized size = 0.75

method	result	size
default	$-\frac{\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}$	15
trager	$\frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{-4x^2-9}+3\,\text{RootOf}\left(\_Z^2+1\right)}{x}\right)}{3}$	32
meijerg	$-\frac{i\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)+(2\ln(x)-2\ln(3))\sqrt{\pi}\right)}{6\sqrt{\pi}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctan(3/(-4*x^2-9)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 25, normalized size = 1.25

$$-\frac{1}{3}i\,\log\left(\frac{6\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.96, size = 43, normalized size = 2.15

$$-\frac{1}{6} i \log\left(-\frac{2\left(i\sqrt{-4x^2-9}+3\right)}{3x}\right)+\frac{1}{6} i \log\left(-\frac{2\left(-i\sqrt{-4x^2-9}+3\right)}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] -1/6*I*log(-2/3*(I*sqrt(-4*x^2 - 9) + 3)/x) + 1/6*I*log(-2/3*(-I*sqrt(-4*x^
2 - 9) + 3)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 8, normalized size = 0.40

$$\frac{i \operatorname{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x**2-9)**(1/2),x)
[Out] I*asinh(3/(2*x))/3
```

Giac [A]
time = 0.60, size = 14, normalized size = 0.70

$$\frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/3*arctan(1/3*sqrt(-4*x^2 - 9))
```

Mupad [B]
time = 5.27, size = 14, normalized size = 0.70

$$\frac{\operatorname{atan}\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- 4*x^2 - 9)^(1/2)),x)
[Out] atan((- 4*x^2 - 9)^(1/2)/3)/3
```

3.572 $\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{-9-4x^2}}{9x}$$

[Out] 1/9/x*(-4*x^2-9)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{270\}$

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[-9 - 4*x^2]),x]

[Out] Sqrt[-9 - 4*x^2]/(9*x)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-9-4x^2}}{9x}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{\sqrt{-9-4x^2}}{9x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[-9 - 4*x^2]),x]

[Out] Sqrt[-9 - 4*x^2]/(9*x)

Maple [A]
time = 0.03, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{\sqrt{-4x^2-9}}{9x}$	15
default	$\frac{\sqrt{-4x^2-9}}{9x}$	15
trager	$\frac{\sqrt{-4x^2-9}}{9x}$	15
meijerg	$\frac{i\sqrt{1+\frac{4x^2}{9}}}{3x}$	16
risch	$-\frac{4x^2+9}{9x\sqrt{-4x^2-9}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/9*(-4*x^2-9)^(1/2)/x
```

Maxima [A]
time = 0.50, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/9*sqrt(-4*x^2 - 9)/x
```

Fricas [A]
time = 1.31, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/9*sqrt(-4*x^2 - 9)/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.38, size = 15, normalized size = 0.83

$$\frac{2i\sqrt{1+\frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*sqrt(1 + 9/(4*x**2))/9
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-4*x^2 - 9)*x^2), x)
```

Mupad [B]
time = 5.04, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] (- 4*x^2 - 9)^(1/2)/(9*x)
```

3.573 $\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{-9-4x^2}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

```
[Out] -2/27*arctan(1/3*(-4*x^2-9)^(1/2))+1/18*(-4*x^2-9)^(1/2)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 44, 65, 210}

$$\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{2}{27}\text{ArcTan}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(18*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/27
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^2}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{18x^2}-\frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{18x^2}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{-9-4x^2}\right)\\ &= \frac{\sqrt{-9-4x^2}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{\sqrt{-9-4x^2}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-9 - 4*x^2]),x]
[Out] Sqrt[-9 - 4*x^2]/(18*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/27
```

Maple [A]
time = 0.11, size = 30, normalized size = 0.77

method	result	size
default	$\frac{\sqrt{-4x^2-9}}{18x^2}+\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}$	30
risch	$-\frac{4x^2+9}{18x^2\sqrt{-4x^2-9}}+\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}$	37
trager	$\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{2\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{-4x^2-9}+3\,\text{RootOf}\left(\_Z^2+1\right)}{x}\right)}{27}$	47

meijerg	$-\frac{2i\left(\frac{9\sqrt{\pi}\left(8+\frac{16x^2}{9}\right)}{32x^2}-\frac{9\sqrt{\pi}\sqrt{1+\frac{4x^2}{9}}}{4x^2}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)-\frac{(1+2\ln(x)-2\ln(3))\sqrt{\pi}}{2}-\frac{9\sqrt{\pi}}{4x^2}\right)}{27\sqrt{\pi}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(-4*x^2-9)^(1/2)/x^2+2/27*arctan(3/(-4*x^2-9)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 40, normalized size = 1.03

$$\frac{\sqrt{-4x^2-9}}{18x^2}+\frac{2}{27}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(-4*x^2 - 9)/x^2 + 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x)
)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.85, size = 65, normalized size = 1.67

$$\frac{-2i\,x^2\log\left(-\frac{4\left(i\sqrt{-4x^2-9}-3\right)}{27x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\sqrt{-4x^2-9}-3\right)}{27x}\right)+3\sqrt{-4x^2-9}}{54x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(-2*I*x^2*log(-4/27*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/27*(-
I*sqrt(-4*x^2 - 9) - 3)/x) + 3*sqrt(-4*x^2 - 9))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.09, size = 46, normalized size = 1.18

$$-\frac{2i\operatorname{asinh}\left(\frac{3}{2x}\right)}{27}+\frac{i}{9x\sqrt{1+\frac{9}{4x^2}}}+\frac{i}{4x^3\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-4*x**2-9)**(1/2),x)
```

```
[Out] -2*I*asinh(3/(2*x))/27 + I/(9*x*sqrt(1 + 9/(4*x**2))) + I/(4*x**3*sqrt(1 +
9/(4*x**2)))
```

Giac [A]
time = 0.53, size = 29, normalized size = 0.74

$$\frac{\sqrt{-4\,x^2-9}}{18\,x^2}-\frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/18*sqrt(-4*x^2 - 9)/x^2 - 2/27*arctan(1/3*sqrt(-4*x^2 - 9))
```

Mupad [B]
time = 5.18, size = 29, normalized size = 0.74

$$\frac{\sqrt{-4\,x^2-9}}{18\,x^2}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(- 4*x^2 - 9)^(1/2)),x)
[Out] (- 4*x^2 - 9)^(1/2)/(18*x^2) - (2*atan((- 4*x^2 - 9)^(1/2)/3))/27
```

3.574 $\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{-9-4x^2}}{27x^3}-\frac{8\sqrt{-9-4x^2}}{243x}$$

```
[Out] 1/27*(-4*x^2-9)^(1/2)/x^3-8/243/x*(-4*x^2-9)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 270}

$$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(27*x^3) - (8*Sqrt[-9 - 4*x^2])/(243*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx &= \frac{\sqrt{-9-4x^2}}{27x^3}-\frac{8}{27}\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx\\ &= \frac{\sqrt{-9-4x^2}}{27x^3}-\frac{8\sqrt{-9-4x^2}}{243x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.68

$$\frac{(9-8x^2)\sqrt{-9-4x^2}}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ((9 - 8*x^2)*Sqrt[-9 - 4*x^2])/(243*x^3)
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.81

method	result	size
gosper	$-\frac{(8x^2-9)\sqrt{-4x^2-9}}{243x^3}$	22
trager	$-\frac{(8x^2-9)\sqrt{-4x^2-9}}{243x^3}$	22
meijerg	$\frac{i\left(1-\frac{8x^2}{9}\right)\sqrt{1+\frac{4x^2}{9}}}{9x^3}$	23
risch	$\frac{32x^4+36x^2-81}{243x^3\sqrt{-4x^2-9}}$	27
default	$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(-4*x^2-9)^(1/2)/x^3-8/243*(-4*x^2-9)^(1/2)/x
```

Maxima [A]
time = 0.49, size = 29, normalized size = 0.78

$$-\frac{8\sqrt{-4\,x^2-9}}{243\,x}+\frac{\sqrt{-4\,x^2-9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/243*sqrt(-4*x^2 - 9)/x + 1/27*sqrt(-4*x^2 - 9)/x^3
```

Fricas [A]
time = 0.83, size = 21, normalized size = 0.57

$$-\frac{(8\,x^2-9)\sqrt{-4\,x^2-9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(8*x^2 - 9)*sqrt(-4*x^2 - 9)/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.63, size = 36, normalized size = 0.97

$$-\frac{16i\sqrt{1+\frac{9}{4x^2}}}{243}+\frac{2i\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-4*x**2-9)**(1/2),x)
```

```
[Out] -16*I*sqrt(1 + 9/(4*x**2))/243 + 2*I*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-4*x^2 - 9)*x^4), x)
```

Mupad [B]
time = 5.08, size = 31, normalized size = 0.84

$$-\frac{8\,x^2\,\sqrt{-4\,x^2-9}-9\,\sqrt{-4\,x^2-9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] -(8*x^2*(- 4*x^2 - 9)^(1/2) - 9*(- 4*x^2 - 9)^(1/2))/(243*x^3)
```

3.575 $\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

```
[Out] 2/81*arctan(1/3*(-4*x^2-9)^(1/2))+1/36*(-4*x^2-9)^(1/2)/x^4-1/54*(-4*x^2-9)
^(1/2)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {272, 44, 65, 210}

$$\frac{2}{81}\text{ArcTan}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)-\frac{\sqrt{-4x^2-9}}{54x^2}+\frac{\sqrt{-4x^2-9}}{36x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(36*x^4) - Sqrt[-9 - 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/81
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^3}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^2}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}+\frac{1}{27}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}-\frac{1}{54}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{-9-4x^2}\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.81

$$\frac{\sqrt{-9-4x^2}\,(3-2x^2)}{108x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-9 - 4*x^2]),x]
[Out] (Sqrt[-9 - 4*x^2]*(3 - 2*x^2))/(108*x^4) + (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/8
1
```

Maple [A]
time = 0.12, size = 44, normalized size = 0.77

method	result
risch	$\frac{8x^4+6x^2-27}{108x^4\sqrt{-4x^2-9}}-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{81}$
default	$\frac{\sqrt{-4x^2-9}}{36x^4}-\frac{\sqrt{-4x^2-9}}{54x^2}-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{81}$

trager	$-\frac{(2x^2-3)\sqrt{-4x^2-9}}{108x^4}+\frac{2\,\mathrm{RootOf}(\_Z^2+1)\ln\left(\frac{\sqrt{-4x^2-9}+3\,\mathrm{RootOf}(\_Z^2+1)}{x}\right)}{81}$
meijerg	$-\frac{8i\left(\frac{81\sqrt{\pi}\left(-\frac{112}{81}x^4-\frac{32}{9}x^2+8\right)}{256x^4}-\frac{81\sqrt{\pi}\left(-\frac{16x^2}{3}+8\right)\sqrt{1+\frac{4x^2}{9}}}{256x^4}-\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{4x^2}{9}}}{2}\right)}{4}+\frac{3\left(\frac{7}{6}+2\ln(x)-2\ln(3)\right)\sqrt{\pi}}{8}\right)}{243\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(-4*x^2-9)^(1/2)/x^4-1/54*(-4*x^2-9)^(1/2)/x^2-2/81*arctan(3/(-4*x^2-9
)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 54, normalized size = 0.95

$$-\frac{\sqrt{-4\,x^2-9}}{54\,x^2}+\frac{\sqrt{-4\,x^2-9}}{36\,x^4}-\frac{2}{81}i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(-4*x^2 - 9)/x^2 + 1/36*sqrt(-4*x^2 - 9)/x^4 - 2/81*I*log(6*sqrt(
4*x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.52, size = 72, normalized size = 1.26

$$\frac{-4i\,x^4\log\left(-\frac{4\left(i\,\sqrt{-4\,x^2-9}+3\right)}{81\,x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\,\sqrt{-4\,x^2-9}+3\right)}{81\,x}\right)-3\,(2\,x^2-3)\sqrt{-4\,x^2-9}}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(-4*I*x^4*log(-4/81*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/81*(
-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 - 3)*sqrt(-4*x^2 - 9))/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 2.53, size = 65, normalized size = 1.14

$$\frac{2i\operatorname{asinh}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{1+\frac{9}{4x^2}}}-\frac{i}{36x^3\sqrt{1+\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*asinh(3/(2*x))/81 - I/(27*x*sqrt(1 + 9/(4*x**2))) - I/(36*x**3*sqrt(1 +
 9/(4*x**2))) + I/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Giac [C] Result contains complex when optimal does not.
time = 0.50, size = 43, normalized size = 0.75

$$\frac{-i\,(4\,x^2+9)^{\frac{3}{2}}+15\,\sqrt{-4\,x^2-9}}{216\,x^4}+\frac{2}{81}\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*(-I*(4*x^2 + 9)^(3/2) + 15*sqrt(-4*x^2 - 9))/x^4 + 2/81*arctan(1/3*sq
rt(-4*x^2 - 9))
```

Mupad [B]
time = 5.07, size = 60, normalized size = 1.05

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{81}-\frac{\frac{10\,\sqrt{-4\,x^2-9}}{9}+\frac{2\,{\left(-4\,x^2-9\right)}^{3/2}}{27}}{72\,x^2-{\left(4\,x^2+9\right)}^2+81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] (2*atan((- 4*x^2 - 9)^(1/2)/3))/81 - ((10*(- 4*x^2 - 9)^(1/2))/9 + (2*(- 4*
x^2 - 9)^(3/2))/27)/(72*x^2 - (4*x^2 + 9)^2 + 81)
```

3.576 $\int \frac{1}{\sqrt{9+bx^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

```
[Out] arcsinh(1/3*x*b^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {221}

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + b*x^2],x]
[Out] ArcSinh[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx=\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.65

$$-\frac{\log\left(-\sqrt{b}\,x+\sqrt{9+bx^2}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + b*x^2],x]
```

```
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[9 + b*x^2]]/Sqrt[b])
```

Maple [A]
time = 0.04, size = 21, normalized size = 1.24

method	result	size
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{b}}{3}\right)}{\sqrt{b}}$	12
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+9}\right)}{\sqrt{b}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x*b^(1/2)+(b*x^2+9)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.30, size = 11, normalized size = 0.65

$$\frac{\operatorname{arsinh}\left(\frac{1}{3}\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*sqrt(b)*x)/sqrt(b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(11) = 22.
time = 1.71, size = 65, normalized size = 3.82

$$\left[\frac{\log\left(-\sqrt{b}\,x-\sqrt{bx^2+9}\right)}{\sqrt{b}},-\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+9}\sqrt{-b}-3\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(-sqrt(b)*x - sqrt(b*x^2 + 9))/sqrt(b), -2*sqrt(-b)*arctan((sqrt(b*x^2
+ 9)*sqrt(-b) - 3*sqrt(-b))/(b*x))/b]
```

Sympy [A]
time = 0.40, size = 14, normalized size = 0.82

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+9)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/3)/sqrt(b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(11) = 22.
time = 0.78, size = 35, normalized size = 2.06

$$\frac{1}{2}\sqrt{bx^2+9}\,x - \frac{9\log\left(-\sqrt{b}\,x+\sqrt{bx^2+9}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + 9)*x - 9/2*log(-sqrt(b)*x + sqrt(b*x^2 + 9))/sqrt(b)
```

Mupad [B]
time = 0.04, size = 11, normalized size = 0.65

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + 9)^(1/2),x)
```

```
[Out] asinh((b^(1/2)*x)/3)/b^(1/2)
```

3.577 $\int \frac{1}{\sqrt{9-bx^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

```
[Out] arcsin(1/3*x*b^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {222}

$$\frac{\text{ArcSin}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx = \frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.94

$$\frac{b\log\left(-\sqrt{-b}\,x+\sqrt{9-bx^2}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 - b*x^2],x]
```

```
[Out] (b*Log[-(Sqrt[-b]*x) + Sqrt[9 - b*x^2]])/(-b)^(3/2)
```

Maple [A]
time = 0.04, size = 21, normalized size = 1.24

method	result	size
meijerg	$\frac{\arcsin\left(\frac{x\sqrt{b}}{3}\right)}{\sqrt{b}}$	12
default	$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}+9}}\right)}{\sqrt{b}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/b^(1/2)*arctan(b^(1/2)*x/(-b*x^2+9)^(1/2))
```

Maxima [A]
time = 0.49, size = 11, normalized size = 0.65

$$\frac{\arcsin\left(\frac{1}{3}\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="maxima")
[Out] arcsin(1/3*sqrt(b)*x)/sqrt(b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 1.54, size = 58, normalized size = 3.41

$$\left[-\frac{\sqrt{-b}\,\log\left(-\sqrt{-b}\,x-\sqrt{-bx^{2}+9}\right)}{b},\,-\frac{2\,\arctan\left(\frac{\sqrt{-bx^{2}+9}-3}{\sqrt{b}\,x}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-b)*log(-sqrt(-b)*x - sqrt(-b*x^2 + 9))/b, -2*arctan((sqrt(-b*x^2 +
9) - 3)/(sqrt(b)*x))/sqrt(b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 37, normalized size = 2.18

$$\begin{cases} -\frac{i\,\mathrm{acosh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{for } |bx^2| > 9 \\ \frac{\mathrm{asin}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2) > 9), (asin(sqrt(b)*x
/3)/sqrt(b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(11) = 22.
time = 0.70, size = 41, normalized size = 2.41

$$\frac{1}{2}\sqrt{-bx^2+9}\,x - \frac{9\log\left(-\sqrt{-b}\,x+\sqrt{-bx^2+9}\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-b*x^2 + 9)*x - 9/2*log(-sqrt(-b)*x + sqrt(-b*x^2 + 9))/sqrt(-b)
```

Mupad [B]
time = 0.04, size = 15, normalized size = 0.88

$$\frac{\mathrm{asinh}\left(\frac{\sqrt{-b}\,x}{3}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9 - b*x^2)^(1/2),x)
```

```
[Out] asinh(((-b)^(1/2)*x)/3)/(-b)^(1/2)
```

3.578 $\int \frac{1}{\sqrt{-9+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2-9)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{-9+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 + b*x^2],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.84

method	result	size
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}-9}\right)}{\sqrt{b}}$	21
meijerg	$\frac{\sqrt{-\mathrm{signum}\left(-1+\frac{b\,x^{2}}{9}\right)}\,\arcsin\left(\frac{x\sqrt{b}}{3}\right)}{\sqrt{\mathrm{signum}\left(-1+\frac{b\,x^{2}}{9}\right)}\,\sqrt{b}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x*b^(1/2)+(b*x^2-9)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.96

$$\frac{\log\left(2\,bx+2\,\sqrt{bx^2-9}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + 2*sqrt(b*x^2 - 9)*sqrt(b))/sqrt(b)
```

Fricas [A]
time = 1.35, size = 57, normalized size = 2.28

$$\left[\frac{\log\left(2\,bx^2+2\,\sqrt{bx^2-9}\,\sqrt{b}\,x-9\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2-9}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - 9)*sqrt(b)*x - 9)/sqrt(b), -sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 - 9))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 37, normalized size = 1.48

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{for } |bx^2| > 9 \\ -\frac{i \operatorname{asin}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2) > 9), (-I*asin(sqrt(b)*x
/3)/sqrt(b), True))
```

Giac [A]
time = 0.77, size = 36, normalized size = 1.44

$$\frac{1}{2}\sqrt{bx^2-9}\,x+\frac{9\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2-9}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 - 9)*x + 9/2*log(abs(-sqrt(b)*x + sqrt(b*x^2 - 9)))/sqrt(b)
```

Mupad [B]
time = 0.08, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b\,x^2-9}+\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 - 9)^(1/2),x)
```

```
[Out] log((b*x^2 - 9)^(1/2) + b^(1/2)*x)/b^(1/2)
```

3.579 $\int \frac{1}{\sqrt{-9-bx^2}}\,dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9-bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2-9)^(1/2))/b^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {223, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-9-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.81

method	result	size
default	$\frac{\arctan\left(\frac{x\sqrt{b}}{\sqrt{-b\,x^{2}-9}}\right)}{\sqrt{b}}$	21
meijerg	$\frac{\sqrt{\mathrm{signum}\left(1+\frac{b\,x^{2}}{9}\right)}\,\mathrm{arcsinh}\left(\frac{x\sqrt{b}}{3}\right)}{\sqrt{-\mathrm{signum}\left(1+\frac{b\,x^{2}}{9}\right)}\,\sqrt{b}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2-9)^(1/2))/b^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.28, size = 12, normalized size = 0.46

$$-\frac{i\,\mathrm{arsinh}\left(\frac{1}{3}\,\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*arcsinh(1/3*sqrt(b)*x)/sqrt(b)
```

Fricas [A]
time = 1.15, size = 68, normalized size = 2.62

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx^2+2\,\sqrt{-bx^2-9}\,\sqrt{-b}\,x-9\right)}{2\,b},\,-\frac{\arctan\left(\frac{\sqrt{-bx^2-9}\,\sqrt{b}\,x}{bx^2+9}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - 9)*sqrt(-b)*x - 9)/b, -arctan
(sqrt(-b*x^2 - 9)*sqrt(b)*x/(b*x^2 + 9))/sqrt(b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 17, normalized size = 0.65

$$-\frac{i\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-9)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/3)/sqrt(b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.
time = 0.68, size = 42, normalized size = 1.62

$$\frac{1}{2}\sqrt{-bx^2-9}\,x+\frac{9\,\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2-9}\right|\right)}{2\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-b*x^2 - 9)*x + 9/2*log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - 9)))/sqrt(
-b)
```

Mupad [B]
time = 0.09, size = 25, normalized size = 0.96

$$\frac{\ln\left(\sqrt{-b\,x^2-9}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- b*x^2 - 9)^(1/2),x)
```

```
[Out] log((- b*x^2 - 9)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

3.580 $\int \frac{1}{\sqrt{\pi + bx^2}}\,dx$

Optimal. Leaf size=19

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

```
[Out] arcsinh(x*b^(1/2)/Pi^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{221\}$

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Pi + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\pi + bx^2}}\,dx = \frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.47

$$-\frac{\log\left(-\sqrt{b}\,x + \sqrt{\pi + bx^2}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Pi + b*x^2],x]
```

```
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[Pi + b*x^2]]/Sqrt[b])
```

Maple [A]
time = 0.05, size = 21, normalized size = 1.11

method	result	size
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{b}}{\sqrt{\pi}}\right)}{\sqrt{b}}$	14
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+\pi}\right)}{\sqrt{b}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x*b^(1/2)+(b*x^2+Pi)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.31, size = 13, normalized size = 0.68

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*x/sqrt(pi*b))/sqrt(b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.
time = 0.85, size = 59, normalized size = 3.11

$$\left[\frac{\log\left(-\pi - 2\,bx^2 - 2\,\sqrt{\pi + bx^2}\,\sqrt{b}\,x\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{\pi + bx^2}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-pi - 2*b*x^2 - 2*sqrt(pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(pi + b*x^2))/b]
```

Sympy [A]
time = 0.42, size = 17, normalized size = 0.89

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+pi)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(13) = 26$.
time = 0.61, size = 36, normalized size = 1.89

$$\frac{1}{2}\sqrt{\pi + bx^2}\,x - \frac{\pi \log\left(-\sqrt{b}\,x + \sqrt{\pi + bx^2}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(pi + b*x^2)*x - 1/2*pi*log(-sqrt(b)*x + sqrt(pi + b*x^2))/sqrt(b)
```

Mupad [B]
time = 5.12, size = 20, normalized size = 1.05

$$\frac{\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2 + \Pi}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi + b*x^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*x + (Pi + b*x^2)^(1/2))/b^(1/2)
```

3.581 $\int \frac{1}{\sqrt{\pi - bx^2}} \, dx$

Optimal. Leaf size=19

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

```
[Out] arcsin(x*b^(1/2)/Pi^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{222\}$

$$\frac{\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Pi - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\pi - bx^2}} \, dx = \frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.74

$$\frac{b \log\left(-\sqrt{-b}\,x + \sqrt{\pi - bx^2}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Pi - b*x^2],x]
```

```
[Out] (b*Log[-(Sqrt[-b]*x) + Sqrt[Pi - b*x^2]])/(-b)^(3/2)
```

Maple [A]
time = 0.04, size = 21, normalized size = 1.11

method	result	size
meijerg	$\frac{\arcsin\left(\frac{x\sqrt{b}}{\sqrt{\pi}}\right)}{\sqrt{b}}$	14
default	$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}+\pi}}\right)}{\sqrt{b}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/b^(1/2)*arctan(b^(1/2)*x/(-b*x^2+Pi)^(1/2))
```

Maxima [A]
time = 0.49, size = 13, normalized size = 0.68

$$\frac{\arcsin\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="maxima")
[Out] arcsin(b*x/sqrt(pi*b))/sqrt(b)
```

Fricas [A]
time = 1.62, size = 62, normalized size = 3.26

$$\left[-\frac{\sqrt{-b}\,\log\left(-\pi+2\,bx^{2}-2\,\sqrt{\pi-bx^{2}}\,\sqrt{-b}\,x\right)}{2\,b},-\frac{\arctan\left(-\frac{\sqrt{b}\,x}{\sqrt{\pi-bx^{2}}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-b)*log(-pi + 2*b*x^2 - 2*sqrt(pi - b*x^2)*sqrt(-b)*x)/b, -arcta
n(-sqrt(b)*x/sqrt(pi - b*x^2))/sqrt(b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 46, normalized size = 2.42

$$\begin{cases} -\frac{i \operatorname{acosh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for } \frac{|bx^2|}{\pi} > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (asin
(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(13) = 26.
time = 1.19, size = 43, normalized size = 2.26

$$\frac{1}{2}\sqrt{\pi - bx^2}\,x - \frac{\pi \log\left(\left|-\sqrt{-b}\,x + \sqrt{\pi - bx^2}\right|\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(pi - b*x^2)*x - 1/2*pi*log(abs(-sqrt(-b)*x + sqrt(pi - b*x^2)))/sq
rt(-b)
```

Mupad [B]
time = 0.10, size = 25, normalized size = 1.32

$$\frac{\ln\left(\sqrt{\Pi - b\,x^2} + \sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi - b*x^2)^(1/2),x)
```

```
[Out] log((Pi - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

3.582 $\int \frac{1}{\sqrt{-\pi + bx^2}} \, dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{-\pi + bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2-Pi)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{bx^2 - \pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Pi + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-\pi + bx^2}} \, dx &= \text{Subst}\left(\int \frac{1}{1 - bx^2} \, dx, x, \frac{x}{\sqrt{-\pi + bx^2}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{-\pi + bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-\pi+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Pi + b*x^2],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.04, size = 23, normalized size = 0.85

method	result	size
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2-\pi}\right)}{\sqrt{b}}$	23
meijerg	$\frac{\sqrt{\operatorname{signum}\left(1-\frac{x^2 b}{\pi}\right)}\,\arcsin\left(\frac{x\sqrt{b}}{\sqrt{\pi}}\right)}{\sqrt{-\operatorname{signum}\left(1-\frac{x^2 b}{\pi}\right)}\,\sqrt{b}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x*b^(1/2)+(b*x^2-Pi)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.96

$$\frac{\log\left(2\,bx+2\,\sqrt{-\pi+bx^2}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + 2*sqrt(-pi + b*x^2)*sqrt(b))/sqrt(b)
```

Fricas [A]
time = 1.46, size = 74, normalized size = 2.74

$$\left[\frac{\log\left(-\pi+2\,bx^2+2\,\sqrt{-\pi+bx^2}\,\sqrt{b}\,x\right)}{2\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(-\frac{\sqrt{-\pi+bx^2}\,\sqrt{-b}\,x}{\pi-bx^2}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-pi + 2*b*x^2 + 2*sqrt(-pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)*
arctan(-sqrt(-pi + b*x^2)*sqrt(-b)*x/(pi - b*x^2))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 46, normalized size = 1.70

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for } \frac{|bx^2|}{\pi} > 1 \\ -\frac{i \operatorname{asin}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-pi)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (-I*asin
(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

Giac [A]
time = 1.35, size = 41, normalized size = 1.52

$$\frac{1}{2}\sqrt{-\pi + bx^2}\,x + \frac{\pi \log\left(\left|-\sqrt{b}\,x + \sqrt{-\pi + bx^2}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-pi + b*x^2)*x + 1/2*pi*log(abs(-sqrt(b)*x + sqrt(-pi + b*x^2)))/s
qrt(b)
```

Mupad [B]
time = 0.13, size = 22, normalized size = 0.81

$$\frac{\ln\left(\sqrt{b\,x^2 - \Pi} + \sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 - Pi)^(1/2),x)
```

```
[Out] log((b*x^2 - Pi)^(1/2) + b^(1/2)*x)/b^(1/2)
```

3.583 $\int \frac{1}{\sqrt{-\pi - bx^2}}\, dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{-\pi - bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2-Pi)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {223, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\, x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Pi - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\pi - bx^2}}\, dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\, dx, x, \frac{x}{\sqrt{-\pi - bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{-\pi - bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-\pi-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Pi - b*x^2],x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.04, size = 23, normalized size = 0.82

method	result	size
default	$\frac{\arctan\left(\frac{x\sqrt{b}}{\sqrt{-b\,x^2-\pi}}\right)}{\sqrt{b}}$	23
meijerg	$\frac{\sqrt{\operatorname{signum}\left(1+\frac{x^2b}{\pi}\right)}\,\operatorname{arcsinh}\left(\frac{x\sqrt{b}}{\sqrt{\pi}}\right)}{\sqrt{-\operatorname{signum}\left(1+\frac{x^2b}{\pi}\right)}\,\sqrt{b}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-Pi)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x*b^(1/2)/(-b*x^2-Pi)^(1/2))/b^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.29, size = 14, normalized size = 0.50

$$-\frac{i\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="maxima")
[Out] -I*arcsinh(b*x/sqrt(pi*b))/sqrt(b)
```

Fricas [A]
time = 1.23, size = 74, normalized size = 2.64

$$\left[-\frac{\sqrt{-b}\,\log\left(-\pi-2\,bx^2+2\,\sqrt{-\pi-bx^2}\,\sqrt{-b}\,x\right)}{2\,b},-\frac{\arctan\left(\frac{\sqrt{-\pi-bx^2}\,\sqrt{b}\,x}{\pi+bx^2}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-pi - 2*b*x^2 + 2*sqrt(-pi - b*x^2)*sqrt(-b)*x)/b, -arct
an(sqrt(-pi - b*x^2)*sqrt(b)*x/(pi + b*x^2))/sqrt(b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.42, size = 20, normalized size = 0.71

$$-\frac{i\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-pi)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.
time = 2.02, size = 47, normalized size = 1.68

$$\frac{1}{2}\sqrt{-\pi-bx^2}\,x+\frac{\pi\log\left(\left|-\sqrt{-b}\,x+\sqrt{-\pi-bx^2}\right|\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-pi - b*x^2)*x + 1/2*pi*log(abs(-sqrt(-b)*x + sqrt(-pi - b*x^2)))/
sqrt(-b)
```

Mupad [B]
time = 5.09, size = 27, normalized size = 0.96

$$\frac{\ln\left(\sqrt{-b\,x^2-\Pi}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- Pi - b*x^2)^(1/2),x)
```

```
[Out] log((- Pi - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

3.584 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.00, size = 21, normalized size = 0.84

method	result	size
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.34, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 1.09, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [**A**]
time = 0.47, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [**A**]
time = 1.00, size = 37, normalized size = 1.48

$$\frac{1}{2}\sqrt{bx^2+a}\,x-\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

Mupad [**B**]
time = 0.00, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

3.585 $\int \frac{1}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {223, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - b*x^2],x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.81

method	result	size
default	$\frac{\arctan\left(\frac{x\sqrt{b}}{\sqrt{-b\,x^2+a}}\right)}{\sqrt{b}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.57, size = 13, normalized size = 0.50

$$\frac{\arcsin\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] arcsin(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 1.05, size = 72, normalized size = 2.77

$$\left[-\frac{\sqrt{-b}\,\log\left(2\,bx^2-2\,\sqrt{-bx^2+a}\,\sqrt{-b}\,x-a\right)}{2\,b},-\frac{\arctan\left(\frac{\sqrt{-bx^2+a}\,\sqrt{b}\,x}{bx^2-a}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*b*x^2 - 2*sqrt(-b*x^2 + a)*sqrt(-b)*x - a)/b, -arctan(
sqrt(-b*x^2 + a)*sqrt(b)*x/(b*x^2 - a))/sqrt(b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 46, normalized size = 1.77

$$\begin{cases} -\frac{i \operatorname{acosh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for } \left|\frac{bx^2}{a}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (asin(s
qrt(b)*x/sqrt(a))/sqrt(b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(20) = 40.
time = 0.91, size = 43, normalized size = 1.65

$$\frac{1}{2}\sqrt{-bx^2+a}\,x - \frac{a\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2+a}\right|\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(-b)*x + sqrt(-b*x^2 + a)))/sqr
t(-b)
```

Mupad [B]
time = 0.11, size = 25, normalized size = 0.96

$$\frac{\ln\left(\sqrt{a-b\,x^2}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(1/2),x)
```

```
[Out] log((a - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

3.586 $\int \frac{1}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2-a)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{-a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-a + b*x^2],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.03, size = 23, normalized size = 0.85

method	result	size
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}-a}\right)}{\sqrt{b}}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x*b^(1/2)+(b*x^2-a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.30, size = 26, normalized size = 0.96

$$\frac{\log\left(2\,bx+2\,\sqrt{bx^2-a}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + 2*sqrt(b*x^2 - a)*sqrt(b))/sqrt(b)
```

Fricas [A]
time = 1.21, size = 63, normalized size = 2.33

$$\left[\frac{\log\left(2\,bx^2+2\,\sqrt{bx^2-a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}},\,-\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2-a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 - a))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 46, normalized size = 1.70

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \left|\frac{bx^2}{a}\right| > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-a)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (-I*asin(s
qrt(b)*x/sqrt(a))/sqrt(b), True))
```

Giac [A]
time = 0.70, size = 41, normalized size = 1.52

$$\frac{1}{2}\sqrt{bx^2-a}\,x + \frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2-a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 - a)*x + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 - a)))/sqrt(b
)
```

Mupad [B]
time = 0.12, size = 22, normalized size = 0.81

$$\frac{\ln\left(\sqrt{b\,x^2-a}+\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 - a)^(1/2),x)
```

```
[Out] log((b*x^2 - a)^(1/2) + b^(1/2)*x)/b^(1/2)
```

3.587 $\int \frac{1}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2-a)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {223, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-a-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-a - b*x^2],x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.03, size = 23, normalized size = 0.82

method	result	size
default	$\frac{\arctan\left(\frac{x\sqrt{b}}{\sqrt{-b\,x^2-a}}\right)}{\sqrt{b}}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x*b^(1/2)/(-b*x^2-a)^(1/2))/b^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 14, normalized size = 0.50

$$-\frac{i\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] -I*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 1.50, size = 74, normalized size = 2.64

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx^2+2\,\sqrt{-bx^2-a}\,\sqrt{-b}\,x-a\right)}{2\,b},\,-\frac{\arctan\left(\frac{\sqrt{-bx^2-a}\,\sqrt{b}\,x}{bx^2+a}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - a)*sqrt(-b)*x - a)/b, -arctan
(sqrt(-b*x^2 - a)*sqrt(b)*x/(b*x^2 + a))/sqrt(b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 20, normalized size = 0.71

$$-\frac{i\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-a)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.
time = 0.76, size = 47, normalized size = 1.68

$$\frac{1}{2}\sqrt{-bx^2-a}\,x+\frac{a\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2-a}\right|\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-b*x^2 - a)*x + 1/2*a*log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - a)))/sqr
t(-b)
```

Mupad [B]
time = 5.13, size = 27, normalized size = 0.96

$$\frac{\ln\left(\sqrt{-b\,x^2-a}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- a - b*x^2)^(1/2),x)
```

```
[Out] log((- a - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

3.588 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=16

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {223, 209}

$$\text{ArcTan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2-x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Maple [A]
time = 0.04, size = 15, normalized size = 0.94

method	result	size
default	$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Maxima [A]
time = 0.53, size = 6, normalized size = 0.38

$$\arcsin\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x/a)
```

Fricas [A]
time = 1.21, size = 23, normalized size = 1.44

$$-2\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 19, normalized size = 1.19

$$\begin{cases} -i\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-x**2)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))
```

Giac [A]
time = 0.65, size = 28, normalized size = 1.75

$$\frac{1}{2}\,a^2\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a)+\frac{1}{2}\,\sqrt{a^2-x^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arcsin(x/a)*sgn(a) + 1/2*sqrt(a^2 - x^2)*x
```

Mupad [B]
time = 4.74, size = 14, normalized size = 0.88

$$\operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 - x^2)^(1/2),x)
```

```
[Out] atan(x/(a^2 - x^2)^(1/2))
```

3.589 $\int (cx)^{7/2}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=184

$$-\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{10a^{11/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+\cdots}{\left(\sqrt{a}+\cdots\right.}}}{231b^{9/4}\sqrt{\cdots}}$$

```
[Out] 4/77*a*c*(c*x)^(5/2)*(b*x^2+a)^(1/2)/b+2/11*(c*x)^(9/2)*(b*x^2+a)^(1/2)/c-2
0/231*a^2*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^2+10/231*a^(11/4)*c^(7/2)*(cos(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/
a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {285, 327, 335, 226}

$$\frac{10a^{11/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}-\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (-20*a^2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(231*b^2) + (4*a*c*(c*x)^(5/2)*Sqrt
[a + b*x^2])/(77*b) + (2*(c*x)^(9/2)*Sqrt[a + b*x^2])/(11*c) + (10*a^(11/4)
*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(231*b^(9/4)
*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx = \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c} + \frac{1}{11}(2a)\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b} + \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c} - \frac{(10a^2c^2)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx}{77b}$$

$$= -\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2} + \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b} + \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c} + \frac{(10a^3c^4)}{}$$

$$= -\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2} + \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b} + \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c} + \frac{(20a^3c^3)}{}$$

$$= -\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2} + \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b} + \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c} + \frac{10a^{11/4}c}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 103, normalized size = 0.56

$$\frac{2c^3\sqrt{cx}\,\sqrt{a+bx^2}\left(\sqrt{1+\frac{bx^2}{a}}\,(-5a^2+2abx^2+7b^2x^4)+5a^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 +
 7*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b
^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.12, size = 152, normalized size = 0.83

method	result
default	$\frac{2c^{3}\sqrt{cx}\left(21b^{4}x^{7}+5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\sqrt{-a}\right.}{231x\sqrt{bx^{2}+a}\,b^{3}}$
risch	$-\frac{2(-21b^{2}x^{4}-6abx^{2}+10a^{2})x\sqrt{bx^{2}+a}\,c^{4}}{231b^{2}\sqrt{cx}}+\frac{10a^{3}\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{231b^{3}\sqrt{bcx^{3}+acx}\sqrt{}}$
elliptic	$\frac{\sqrt{cx}\sqrt{cx(bx^{2}+a)}\left(\frac{2c^{3}x^{4}\sqrt{bcx^{3}+acx}}{11}+\frac{4ac^{3}x^{2}\sqrt{bcx^{3}+acx}}{77b}-\frac{20a^{2}c^{3}\sqrt{bcx^{3}+acx}}{231b^{2}}+\frac{10a^{3}c^{4}\sqrt{-ab}}{}\right.}{cx\sqrt{bx^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231*c^3/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(21*b^4*x^7+5*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2
^(1/2)*(-a*b)^(1/2)*a^3+27*a*b^3*x^5-4*a^2*b^2*x^3-10*a^3*b*x)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 76, normalized size = 0.41

$$\frac{2\left(10\,\sqrt{bc}\,a^3c^3\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(21\,b^3c^3x^4+6\,ab^2c^3x^2-10\,a^2bc^3\right)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{231\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(10*sqrt(b*c)*a^3*c^3*weierstrassPInverse(-4*a/b, 0, x) + (21*b^3*c^3
*x^4 + 6*a*b^2*c^3*x^2 - 10*a^2*b*c^3)*sqrt(b*x^2 + a)*sqrt(c*x))/b^3
```

Sympy [C] Result contains complex when optimal does not.
time = 17.36, size = 46, normalized size = 0.25

$$\frac{\sqrt{a}\,c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/2}\,\sqrt{b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(a + b*x^2)^(1/2),x)
[Out] int((c*x)^(7/2)*(a + b*x^2)^(1/2), x)
```

3.590 $\int (cx)^{5/2}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=301

$$\frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{4a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\cdots\right.}}}{15b^{7/4}\sqrt{a\cdots}}$$

```
[Out] 4/45*a*c*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b+2/9*(c*x)^(7/2)*(b*x^2+a)^(1/2)/c-4/
15*a^2*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/(a^(1/2)+x*b^(1/2))+4/15*a^(
9/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^
2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)-2/15*a^(9/4)*c^(5
/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*
x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(
1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {285, 327, 335, 311, 226, 1210}

$$-\frac{2a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}+\frac{4a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}-\frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}+\frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (4*a*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(45*b) + (2*(c*x)^(7/2)*Sqrt[a + b*x^2]
)/(9*c) - (4*a^2*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(15*b^(3/2)*(Sqrt[a] + Sqrt
[b]*x)) + (4*a^(9/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2]) - (2*a^(9/4)*c^(5/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx = \frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}+\frac{1}{9}(2a)\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{(2a^2c^2)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{15b}$$

$$= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{(4a^2c)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\right.}{15b}$$

$$= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{(4a^{5/2}c^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,\right.}{15b^{3/2}}$$

$$= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{4a^{9/4}c^{5/2}\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 85, normalized size = 0.28

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}\left(\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}-a\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{9b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hyper
geometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.07, size = 221, normalized size = 0.73

method	result

default	$-\frac{2c^2\sqrt{cx}\left(-5b^3x^6+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^3-\right.}{45x\sqrt{b\,x^2}}$
risch	$\frac{2x^2(5b\,x^2+2a)\sqrt{b\,x^2+a}\;c^3}{45b\sqrt{cx}}-\frac{2a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}}{}\right.}{15b^2\sqrt{bc}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(\frac{2c^2x^3\sqrt{bc\,x^3+acx}}{9}+\frac{4a\,c^2x\sqrt{bc\,x^3+acx}}{45b}-\frac{2a^2c^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2}{}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(-5*b^3*x^6+6*((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x
*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*a^3-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-7*a*b^2*x^4-2*a^2*b*x^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 71, normalized size = 0.24

$$\frac{2\left(6\sqrt{bc}\,a^2c^2\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(5\,b^2c^2x^3+2\,abc^2x\right)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(6*sqrt(b*c)*a^2*c^2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4
*a/b, 0, x)) + (5*b^2*c^2*x^3 + 2*a*b*c^2*x)*sqrt(b*x^2 + a)*sqrt(c*x))/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 5.19, size = 46, normalized size = 0.15

$$\frac{\sqrt{a}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{5/2}\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(5/2)*(a + b*x^2)^(1/2), x)
```

3.591 $\int (cx)^{3/2}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=153

$$\frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{c}}{\sqrt[4]{a}\sqrt{}}\right.\right.}{21b^{5/4}\sqrt{a+bx^2}}$$

```
[Out] 2/7*(c*x)^(5/2)*(b*x^2+a)^(1/2)/c+4/21*a*c*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b-2/
21*a^(7/4)*c^(3/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arc
tan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*
((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {285, 327, 335, 226}

$$-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^2}}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}+\frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (4*a*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(21*b) + (2*(c*x)^(5/2)*Sqrt[a + b*x^2])/
(7*c) - (2*a^(7/4)*c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
1/2])/(21*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx = \frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}+\frac{1}{7}(2a)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{4ac\sqrt{cx}\,\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{(2a^2c^2)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{21b}$$

$$= \frac{4ac\sqrt{cx}\,\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{(4a^2c)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{c\ldots}\right.}{21b}$$

$$= \frac{4ac\sqrt{cx}\,\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx\ldots}{\left(\sqrt{a}+\sqrt{\ldots}\right.}}}{21b^{5/4}\sqrt{a\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.98, size = 85, normalized size = 0.56

$$\frac{2c\sqrt{cx}\,\sqrt{a+bx^2}\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}}-a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{7b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hyperge
ometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.06, size = 138, normalized size = 0.90

method	result
default	$-\frac{2c\sqrt{cx}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a^2-\right.}{21x\sqrt{b\,x^2+a}\,b^2}$
risch	$\frac{2(3b\,x^2+2a)x\sqrt{b\,x^2+a}\,c^2}{21b\sqrt{cx}}-\frac{2a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\right.}{21b^2\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(\frac{2c\,x^2\sqrt{bc\,x^3+acx}}{7}+\frac{4ac\sqrt{bc\,x^3+acx}}{21b}-\frac{2a^2c^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2(}{}}}{}\right.}{cx\sqrt{b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(
1/2)*a^2-3*b^3*x^5-5*a*b^2*x^3-2*a^2*b*x)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 57, normalized size = 0.37

$$-\frac{2\left(2\,\sqrt{bc}\,a^{2}c\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(3\,b^{2}cx^{2}+2\,abc)\sqrt{bx^{2}+a}\,\sqrt{cx}\right)}{21\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21*(2*sqrt(b*c)*a^2*c*weierstrassPInverse(-4*a/b, 0, x) - (3*b^2*c*x^2 +
 2*a*b*c)*sqrt(b*x^2 + a)*sqrt(c*x))/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.39, size = 46, normalized size = 0.30

$$\frac{\sqrt{a}\,c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{3/2}\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(3/2)*(a + b*x^2)^(1/2), x)
```

3.592 $\int \sqrt{cx}\,\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=269

$$\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{4a\sqrt{cx}\,\sqrt{a+bx^2}}{5\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{4a^{5/4}\sqrt{c}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.\right.}{5b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] 2/5*(c*x)^(3/2)*(b*x^2+a)^(1/2)/c+4/5*a*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)
/(a^(1/2)+x*b^(1/2))-4/5*a^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*Ellip
ticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/
2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^
2+a)^(1/2)+2/5*a^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2
*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/
2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 335, 311, 226, 1210}

$$\frac{2a^{5/4}\sqrt{c}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}}-\frac{4a^{5/4}\sqrt{c}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{4a\sqrt{cx}\,\sqrt{a+bx^2}}{5\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[a + b*x^2],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*c) + (4*a*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*
Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) - (4*a^(5/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^2]) + (2*a^(5/4)*Sqr
t[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(3/4)*Sqrt[
a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx = \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{1}{5}(2a)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{(4a)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5c}$$

$$= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{\left(4a^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5\sqrt{b}} - \frac{\left(4a^{3/2}\right)\text{Subst}}{}$$

$$= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{4a\sqrt{cx}\sqrt{a+bx^2}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{4a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+}{\left(\sqrt{a}+\right.}}}{5b^{3/4}\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.11, size = 56, normalized size = 0.21

$$\frac{2x\sqrt{cx}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[a + b*x^2],x]
[Out] (2*x*Sqrt[c*x]*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/
a)])/(3*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.05, size = 205, normalized size = 0.76

method	result
default	$\frac{2\sqrt{cx}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2-\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}\right.}{5\sqrt{b\,x^2+a}\,bx}$

risch	$\frac{2x^2\sqrt{bx^2+a}\,c}{5\sqrt{cx}}+\frac{2a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\ldots}\right.}{\ldots}\right.}{5b\sqrt{bc\,x^3+acx}\ldots}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(bx^2+a\right)}\left(\frac{2x\sqrt{bc\,x^3+acx}}{5}+\frac{2ac\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-a\ldots}}}}{\ldots}\right.}{cx\sqrt{bx^2+a\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2
)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))*a^2+b^2*x^4+a*b*x^2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

[Out] integrate(sqrt(b*x^2 + a)*sqrt(c*x), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 48, normalized size = 0.18

$$\frac{2\left(\sqrt{bx^2+a}\,\sqrt{cx}\,bx-2\,\sqrt{bc}\,a\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(b*x^2 + a)*sqrt(c*x)*b*x - 2*sqrt(b*c)*a*weierstrassZeta(-4*a/b,
0, weierstrassPInverse(-4*a/b, 0, x)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.50, size = 46, normalized size = 0.17

$$\frac{\sqrt{a}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_p
olar(I*pi)/a)/(2*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x}\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(a + b*x^2)^(1/2), x)
```

3.593 $\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=126

$$\frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c}+\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}$$

```
[Out] 2/3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c+2/3*a^(3/4)*(cos(2*arctan(b^(1/4)*(c*x)^(
1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^
(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^
(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/
c^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {285, 335, 226}

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*c) + (2*a^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c}+\frac{1}{3}(2a)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx$$

$$= \frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c}+\frac{(4a)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3c}$$

$$= \frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c}+\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\right.}{3\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.19, size = 54, normalized size = 0.43

$$\frac{2x\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\,\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/(Sqrt
[c*x]*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.05, size = 119, normalized size = 0.94

method	result
default	$\frac{\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a}{3}+\frac{2b^2x^3}{3}+\frac{2abx}{3}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,b}$
risch	$\frac{2x\sqrt{b\,x^2+a}}{3\sqrt{cx}}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{3b\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx\,(b\,x^2+a)}\left(\frac{2\sqrt{bc\,x^3+acx}}{3c}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{Ellipt}}{3b\sqrt{bc\,x^3+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a+b^2*x^3+a*b
*x)/(c*x)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(c*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 42, normalized size = 0.33

$$\frac{2\left(2\sqrt{bc}\,a\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^2+a}\,\sqrt{cx}\,b\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*sqrt(b*c)*a*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^2 + a)*sqrt
(c*x)*b)/(b*c)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 46, normalized size = 0.37

$$\frac{\sqrt{a}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi
)/a)/(2*sqrt(c)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c*x)^(1/2), x)
```

3.594 $\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=263

$$-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}+\frac{4\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{4\sqrt[4]{a}\,\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)\right.}{c^{3/2}\sqrt{a+bx^2}}$$

```
[Out] -2*(b*x^2+a)^(1/2)/c/(c*x)^(1/2)+4*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^2/
(a^(1/2)+x*b^(1/2))-4*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*
EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(
a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(3/2)/(b*x^2+a
)^(1/2)+2*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(si
n(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {283, 335, 311, 226, 1210}

$$\frac{2\sqrt[4]{a}\,\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{a}\,\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}}+\frac{4\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(c*Sqrt[c*x]) + (4*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2])/
(c^2*(Sqrt[a] + Sqrt[b]*x)) - (4*a^(1/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(a^(1/4)*Sqrt[c])], 1/2])/(c^(3/2)*Sqrt[a + b*x^2]) + (2*a^(1/4)*b^(1/4)*
(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2
*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(c^(3/2)*Sqrt[a + b*x
^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}} + \frac{(2b)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}} + \frac{(4b)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c^3}$$

$$= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}} + \frac{\left(4\sqrt{a}\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c^2} - \frac{\left(4\sqrt{a}\sqrt{b}\right)\mathrm{Subst}\Big(}{}$$

$$= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}} + \frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{4\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E}{c^{3/2}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.21

$$-\frac{2x\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/((c
*x)^(3/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.07, size = 194, normalized size = 0.74

method	result
default	$\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}}{c\sqrt{cx}\,\sqrt{b\,x^2+a}}$

risch	$-\frac{2\sqrt{b\,x^2+a}}{c\sqrt{cx}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\cdots}\right.}{\cdots}\right.}{\sqrt{bc\,x^3+acx}\,c\sqrt{\cdots}}$
elliptic	$\frac{\sqrt{cx\left(b\,x^2+a\right)}\left(-\frac{2\left(c\,x^2b+ac\right)}{c^2\sqrt{x\left(c\,x^2b+ac\right)}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\cdots}{\cdots}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*
EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-b*x^2-a)/(
b*x^2+a)^(1/2)/c/(c*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 49, normalized size = 0.19

$$-\frac{2\left(2\sqrt{bc}\,x\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^2+a}\,\sqrt{cx}\right)}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*sqrt(b*c)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0,
 x)) + sqrt(b*x^2 + a)*sqrt(c*x))/(c^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.64, size = 49, normalized size = 0.19

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(3/2),x)
```

```
[Out] sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(
2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}}{{\left(c\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c*x)^(3/2), x)
```

3.595 $\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}+\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,c^{5/2}\sqrt{a+bx^2}}$$

```
[Out] -2/3*(b*x^2+a)^(1/2)/c/(c*x)^(3/2)+2/3*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)
/c^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {283, 335, 226}

$$\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(3*c*(c*x)^(3/2)) + (2*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(1/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{(2b)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{3c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3c^3}$$

$$= -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,c^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.44

$$-\frac{2x\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/(3*
(c*x)^(5/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.07, size = 120, normalized size = 0.95

method	result
default	$\frac{\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,x}{3}-\frac{2b\,x^{2}}{3}-\frac{2a}{3}}{\sqrt{b\,x^{2}+a}\,x\,c^{2}\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}}{3x\,c^{2}\sqrt{cx}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-ab}}}\right.}{3\sqrt{bc\,x^{3}+acx}\,c^{2}\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{cx\,(b\,x^{2}+a)}\left(-\frac{2\sqrt{bc\,x^{3}+acx}}{3c^{3}x^{2}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{El}}{3c^{2}\sqrt{bc\,x^{3}+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)/x*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x-b*x^2-a)/
c^2/(c*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 44, normalized size = 0.35

$$\frac{2\left(2\sqrt{bc}\,x^{2}\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^{2}+a}\,\sqrt{cx}\right)}{3\,c^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*sqrt(b*c)*x^2*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^2 + a)*sq
rt(c*x))/(c^3*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.47, size = 49, normalized size = 0.39

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(5/2),x)
```

```
[Out] sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(
2*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{{\left(c\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c*x)^(5/2), x)
```

3.596 $\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=303

$$-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}-\frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}}+\frac{4b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5ac^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\Big(2\tan^{-1}\Big(\cdots}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}$$

```
[Out] -2/5*(b*x^2+a)^(1/2)/c/(c*x)^(5/2)-4/5*b*(b*x^2+a)^(1/2)/a/c^3/(c*x)^(1/2)+
4/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/c^4/(a^(1/2)+x*b^(1/2))-4/5*b^(5/
4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1
/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/c^(7/2)/(b*x^2+a)^(1/2)+2/5*b^(5/4)*(cos(2*
arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1
/2))^2)^(1/2)/a^(3/4)/c^(7/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {283, 331, 335, 311, 226, 1210}

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}-\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}+\frac{4b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5ac^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(5*c*(c*x)^(5/2)) - (4*b*Sqrt[a + b*x^2])/(5*a*c^3*Sqr
t[c*x]) + (4*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*a*c^4*(Sqrt[a] + Sqrt[b]
*x)) - (4*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*
a^(3/4)*c^(7/2)*Sqrt[a + b*x^2]) + (2*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(
a^(1/4)*Sqrt[c])], 1/2])/(5*a^(3/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} + \frac{(2b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{5c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} + \frac{(2b^2)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5ac^4}$$

$$= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} + \frac{(4b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5ac^5}$$

$$= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} + \frac{(4b^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5\sqrt{a}\,c^4} - \frac{(4b^{3/2})\,\text{S}}{}$$

$$= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} + \frac{4b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5ac^4\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a}{\left(\sqrt{a}\right.}}}{5a^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.18

$$-\frac{2x\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(7/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/(5
*(c*x)^(7/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.07, size = 219, normalized size = 0.72

method	result

default	$\frac{\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2}{5}-\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}}{}}{x^2\sqrt{b\,x^2+a}\,c^3\sqrt{cx}\,a}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(2b\,x^2+a)}{5x^2a\,c^3\sqrt{cx}}+\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{Ell}}{}\right.}{5a\sqrt{bc\,x^3+}}$
elliptic	$\frac{\sqrt{cx\,(b\,x^2+a)}\left(-\frac{2\sqrt{bc\,x^3+acx}}{5c^4x^3}-\frac{4\left(c\,x^2b+ac\right)b}{5a\,c^4\sqrt{x\,(c\,x^2b+ac)}}+\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-}}{b}\right.}{\sqrt{-a}}}}{}\right.}{\sqrt{cx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/x^2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
a*b*x^2-2*b^2*x^4-3*a*b*x^2-a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/a
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 63, normalized size = 0.21

$$-\frac{2\left(2\sqrt{bc}\,bx^3\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+(2\,bx^2+a)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{5\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(2*sqrt(b*c)*b*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a
/b, 0, x)) + (2*b*x^2 + a)*sqrt(b*x^2 + a)*sqrt(c*x))/(a*c^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.36, size = 53, normalized size = 0.17

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(7/2),x)
```

```
[Out] sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}}{{\left(c\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c*x)^(7/2),x)
[Out] int((a + b*x^2)^(1/2)/(c*x)^(7/2), x)
```

3.597 $\int (cx)^{7/2} (a+bx^2)^{3/2} \, dx$

Optimal. Leaf size=212

$$-\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\frac{4a^{15/4}c^{7/2}\left(\sqrt{\right.}}{}$$

```
[Out] 2/15*(c*x)^(9/2)*(b*x^2+a)^(3/2)/c+8/385*a^2*c*(c*x)^(5/2)*(b*x^2+a)^(1/2)/
b+4/55*a*(c*x)^(9/2)*(b*x^2+a)^(1/2)/c-8/231*a^3*c^3*(c*x)^(1/2)*(b*x^2+a)^
(1/2)/b^2+4/231*a^(15/4)*c^(7/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*Ellip
ticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/
2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/
2)
```

Rubi [A]
time = 0.09, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {285, 327, 335, 226}

$$\frac{4a^{15/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}-\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (-8*a^3*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(231*b^2) + (8*a^2*c*(c*x)^(5/2)*Sqr
t[a + b*x^2])/(385*b) + (4*a*(c*x)^(9/2)*Sqrt[a + b*x^2])/(55*c) + (2*(c*x)
^(9/2)*(a + b*x^2)^(3/2))/(15*c) + (4*a^(15/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{7/2}\left(a+bx^2\right)^{3/2}\,dx = \frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\frac{1}{5}(2a)\int (cx)^{7/2}\sqrt{a+bx^2}\,dx$$

$$=\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\frac{1}{55}\left(4a^2\right)\int\frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx$$

$$=\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}-\frac{\left(4a^3c^2\right)\,\cdots}{\cdots}$$

$$=-\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)\cdots}{\cdots}$$

$$=-\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)\cdots}{\cdots}$$

$$=-\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 102, normalized size = 0.48

$$\frac{2c^3\sqrt{cx}\,\sqrt{a+bx^2}\left(-\left((5a-11bx^2)\,(a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}\right)+5a^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{165b^2\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 +
 (b*x^2)/a]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(165
*b^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.05, size = 163, normalized size = 0.77

method	result
default	$\frac{2c^3\sqrt{cx}\left(77b^5x^9+196a\,b^4x^7+10\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-}\right.}{1155x\sqrt{b\,x^2+a}\,b^3}$
risch	$-\frac{2(-77b^3x^6-119a\,b^2x^4-12a^2b\,x^2+20a^3)x\sqrt{b\,x^2+a}\,c^4}{1155b^2\sqrt{cx}}+\frac{4a^4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}}{231b^3\sqrt{bc\,x}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(\frac{2b\,c^3x^6\sqrt{bc\,x^3+acx}}{15}+\frac{34a\,c^3x^4\sqrt{bc\,x^3+acx}}{165}+\frac{8a^2c^3x^2\sqrt{bc\,x^3+acx}}{385b}-\frac{8a^3c^3\sqrt{bc}}{23}\right.}{cx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155*c^3/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(77*b^5*x^9+196*a*b^4*x^7+10*Ellip
ticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x
```

```
*b/(-a*b)^(1/2))^(1/2)*(-a*b)^(1/2)*a^4+131*a^2*b^3*x^5-8*a^3*b^2*x^3-20*a^
4*b*x)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 90, normalized size = 0.42

$$\frac{2\left(20\sqrt{bc}\,a^4c^3\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(77\,b^4c^3x^6+119\,ab^3c^3x^4+12\,a^2b^2c^3x^2-20\,a^3bc^3\right)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{1155\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(20*sqrt(b*c)*a^4*c^3*weierstrassPInverse(-4*a/b, 0, x) + (77*b^4*c^
3*x^6 + 119*a*b^3*c^3*x^4 + 12*a^2*b^2*c^3*x^2 - 20*a^3*b*c^3)*sqrt(b*x^2 +
 a)*sqrt(c*x))/b^3
```

Sympy [C] Result contains complex when optimal does not.
time = 27.91, size = 46, normalized size = 0.22

$$\frac{a^{\frac{3}{2}}c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((-3/2, 9/4), (13/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{7/2}\,{\left(b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)*(a + b*x^2)^(3/2), x)
```

3.598 $\int (cx)^{5/2} \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=329

$$\frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}-\frac{8a^3c^2\sqrt{cx}\,\sqrt{a+bx^2}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}+\frac{8a^{13/4}c^{5/2}\left(\sqrt{a}\right.}{}$$

```
[Out] 2/13*(c*x)^(7/2)*(b*x^2+a)^(3/2)/c+8/195*a^2*c*(c*x)^(3/2)*(b*x^2+a)^(1/2)/
b+4/39*a*(c*x)^(7/2)*(b*x^2+a)^(1/2)/c-8/65*a^3*c^2*(c*x)^(1/2)*(b*x^2+a)^(
1/2)/b^(3/2)/(a^(1/2)+x*b^(1/2))+8/65*a^(13/4)*c^(5/2)*(cos(2*arctan(b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a
^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
)),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)
/b^(7/4)/(b*x^2+a)^(1/2)-4/65*a^(13/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)
/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {285, 327, 335, 311, 226, 1210}

$$-\frac{4a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}}+\frac{8a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}}-\frac{8a^3c^2\sqrt{cx}\,\sqrt{a+bx^2}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (8*a^2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(195*b) + (4*a*(c*x)^(7/2)*Sqrt[a + b
*x^2])/(39*c) - (8*a^3*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(65*b^(3/2)*(Sqrt[a]
+ Sqrt[b]*x)) + (2*(c*x)^(7/2)*(a + b*x^2)^(3/2))/(13*c) + (8*a^(13/4)*c^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(65*b^(7/4)*Sqrt[
a + b*x^2]) - (4*a^(13/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt
[c])], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int (cx)^{5/2}\left(a+bx^2\right)^{3/2}dx = \frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}+\frac{1}{13}(6a)\int (cx)^{5/2}\sqrt{a+bx^2}\,dx$$

$$=\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}+\frac{1}{39}\left(4a^2\right)\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx$$

$$=\frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}-\frac{\left(4a^3c^2\right)\ldots}{\ldots}$$

$$=\frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}-\frac{\left(8a^3c\right)\mathrm{S}\ldots}{\ldots}$$

$$=\frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}-\frac{\left(8a^{7/2}c^2\ldots\right.}{\ldots}$$

$$=\frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}-\frac{8a^3c^2\sqrt{cx}\,\sqrt{a+bx^2}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(cx)^{7/\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 89, normalized size = 0.27

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{13b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*H
ypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.06, size = 232, normalized size = 0.71

method	result
default	$-\frac{2c^{2}\sqrt{cx}\left(-15b^{4}x^{8}-40a\,b^{3}x^{6}+12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.\right.}{195}$
risch	$\frac{2x^{2}\left(15b^{2}x^{4}+25ab\,x^{2}+4a^{2}\right)\sqrt{b\,x^{2}+a}\;c^{3}}{195b\sqrt{cx}}-\frac{4a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}}{}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(b\,x^{2}+a\right)}\left(\frac{2b\,c^{2}x^{5}\sqrt{bc\,x^{3}+acx}}{13}+\frac{10a\,c^{2}x^{3}\sqrt{bc\,x^{3}+acx}}{39}+\frac{8a^{2}c^{2}x\sqrt{bc\,x^{3}+acx}}{195b}-\frac{4a^{3}c^{3}\sqrt{-a}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/195*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(-15*b^4*x^8-40*a*b^3*x^6+12*(
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))*a^4-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
lipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4-29*a^2*b^2
*x^4-4*a^3*b*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 85, normalized size = 0.26

$$\frac{2\left(12\sqrt{bc}\,a^3c^2\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(15\,b^3c^2x^5+25\,ab^2c^2x^3+4\,a^2bc^2x\right)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{195\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] 2/195*(12*sqrt(b*c)*a^3*c^2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(
-4*a/b, 0, x)) + (15*b^3*c^2*x^5 + 25*a*b^2*c^2*x^3 + 4*a^2*b*c^2*x)*sqrt(b
*x^2 + a)*sqrt(c*x))/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 9.32, size = 46, normalized size = 0.14

$$\frac{a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(3/2),x)
[Out] a**(3/2)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-3/2, 7/4), (11/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{5/2}\,\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a + b*x^2)^(3/2),x)
[Out] int((c*x)^(5/2)*(a + b*x^2)^(3/2), x)
```

3.599 $\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=181

$$\frac{8a^2c\sqrt{cx}\,\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{4a^{11/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\right.}}}{77b^{5/4}\sqrt{a+}}$$

```
[Out] 2/11*(c*x)^(5/2)*(b*x^2+a)^(3/2)/c+12/77*a*(c*x)^(5/2)*(b*x^2+a)^(1/2)/c+8/
77*a^2*c*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b-4/77*a^(11/4)*c^(3/2)*(cos(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(
1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)
^(1/2)/b^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {285, 327, 335, 226}

$$-\frac{4a^{11/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{a+bx^2}}+\frac{8a^2c\sqrt{cx}\,\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (8*a^2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(77*b) + (12*a*(c*x)^(5/2)*Sqrt[a + b*x
^2])/(77*c) + (2*(c*x)^(5/2)*(a + b*x^2)^(3/2))/(11*c) - (4*a^(11/4)*c^(3/2
)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF
[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(77*b^(5/4)*Sqrt[a
+ b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{3/2}\left(a+bx^2\right)^{3/2}\,dx = \frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}+\frac{1}{11}(6a)\int (cx)^{3/2}\sqrt{a+bx^2}\,dx$$

$$= \frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}+\frac{1}{77}\left(12a^2\right)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{8a^2c\sqrt{cx}\,\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{\left(4a^3c^2\right)}{\quad}$$

$$= \frac{8a^2c\sqrt{cx}\,\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{\left(8a^3c\right)}{\quad}$$

$$= \frac{8a^2c\sqrt{cx}\,\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{4a^{11/4}c}{\quad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 89, normalized size = 0.49

$$\frac{2c\sqrt{cx}\,\sqrt{a+bx^2}\left((a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{11b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hyp
ergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.04, size = 150, normalized size = 0.83

method	result
default	$-\frac{2c\sqrt{cx}\left(-7b^4x^7+2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{-\right.}{77x\sqrt{b\,x^2+a}\,b^2}$
risch	$\frac{2(7b^2x^4+13ab\,x^2+4a^2)x\sqrt{b\,x^2+a}\,c^2}{77b\sqrt{cx}}-\frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{Ell}}{77b^2\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(\frac{2bc\,x^4\sqrt{bc\,x^3+acx}}{11}+\frac{26ac\,x^2\sqrt{bc\,x^3+acx}}{77}+\frac{8a^2c\sqrt{bc\,x^3+acx}}{77b}-\frac{4a^3c^2\sqrt{-ab}\,\sqrt{}}{}\right.}{cx\sqrt{b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(-7*b^4*x^7+2*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*(-a*b)^(1/2)*a^3-20*a*b^3*x^5-17*a^2*b^2*x^3-4*a^3*b*x)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 69, normalized size = 0.38

$$-\frac{2\left(4\sqrt{bc}\,a^3c\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\left(7b^3cx^4+13ab^2cx^2+4a^2bc\right)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{77b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*sqrt(b*c)*a^3*c*weierstrassPInverse(-4*a/b, 0, x) - (7*b^3*c*x^4 +
 13*a*b^2*c*x^2 + 4*a^2*b*c)*sqrt(b*x^2 + a)*sqrt(c*x))/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 2.80, size = 46, normalized size = 0.25

$$\frac{a^{\frac{3}{2}}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-3/2, 5/4), (9/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(3/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{3/2}\,{\left(b\,x^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(3/2)*(a + b*x^2)^(3/2), x)
```

3.600 $\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=297

$$\frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{8a^2\sqrt{cx}\sqrt{a+bx^2}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}-\frac{8a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{\ldots}\right.}}}{15b^{3/4}\sqrt{a\ldots}}$$

```
[Out] 2/9*(c*x)^(3/2)*(b*x^2+a)^(3/2)/c+4/15*a*(c*x)^(3/2)*(b*x^2+a)^(1/2)/c+8/15
*a^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(a^(1/2)+x*b^(1/2))-8/15*a^(9/4)*(
cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(
a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)+4/15*a^(9/4)*(cos(2*arc
tan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*
x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/
4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x
*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 335, 311, 226, 1210}

$$\frac{4a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}}-\frac{8a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}}+\frac{8a^2\sqrt{cx}\sqrt{a+bx^2}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a + b*x^2)^(3/2),x]
```

```
[Out] (4*a*(c*x)^(3/2)*Sqrt[a + b*x^2])/(15*c) + (8*a^2*Sqrt[c*x]*Sqrt[a + b*x^2]
)/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) + (2*(c*x)^(3/2)*(a + b*x^2)^(3/2))/(9
*c) - (8*a^(9/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/
2])/(15*b^(3/4)*Sqrt[a + b*x^2]) + (4*a^(9/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{cx}\left(a+bx^2\right)^{3/2}dx = \frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{1}{3}(2a)\int \sqrt{cx}\sqrt{a+bx^2}\,dx$$

$$= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{1}{15}\left(4a^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}dx$$

$$= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{\left(8a^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}dx,x,\right.}{15c}$$

$$= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{\left(8a^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}dx,\right.}{15\sqrt{b}}$$

$$= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{8a^2\sqrt{cx}\sqrt{a+bx^2}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}-\frac{8a^{9/4}\sqrt{c}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 57, normalized size = 0.19

$$\frac{2ax\sqrt{cx}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a*x*Sqrt[c*x]*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2
)/a)])/(3*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.04, size = 218, normalized size = 0.73

method	result

default	$\frac{2\sqrt{cx}\left(5b^3x^6+12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^3-6\sqrt{\right.}{45\sqrt{bx^2+}}$
risch	$\frac{2x^2(5bx^2+11a)\sqrt{bx^2+a}\,c}{45\sqrt{cx}}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}}{}\right.}{15b\sqrt{bc}}$
elliptic	$\frac{\sqrt{cx}\sqrt{cx(bx^2+a)}\left(\frac{2bx^3\sqrt{bcx^3+acx}}{9}+\frac{22ax\sqrt{bcx^3+acx}}{45}+\frac{4a^2c\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2(x}{}}}{}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b*(5*b^3*x^6+12*((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)
^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)*a^3-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+16*a*b^2*x^4+11*a^2*b*x^2)/x
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 63, normalized size = 0.21

$$-\frac{2\left(12\sqrt{bc}\,a^2\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)-\left(5\,b^2x^3+11\,abx\right)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{45\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45*(12*sqrt(b*c)*a^2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a
/b, 0, x)) - (5*b^2*x^3 + 11*a*b*x)*sqrt(b*x^2 + a)*sqrt(c*x))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 1.39, size = 46, normalized size = 0.15

$$\frac{a^{\frac{3}{2}}\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-3/2, 3/4), (7/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x}\ {\left(b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a + b*x^2)^(3/2),x)
[Out] int((c*x)^(1/2)*(a + b*x^2)^(3/2), x)
```

3.601 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=152

$$\frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c}+\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right.}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

```
[Out] 2/7*(b*x^2+a)^(3/2)*(c*x)^(1/2)/c+4/7*a*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c+4/7*a
^(7/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(
a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {285, 335, 226}

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}+\frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[c*x],x]
```

```
[Out] (4*a*Sqrt[c*x]*Sqrt[a + b*x^2])/(7*c) + (2*Sqrt[c*x]*(a + b*x^2)^(3/2))/(7*
c) + (4*a^(7/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(7*b^
(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx &= \frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c}+\frac{1}{7}(6a)\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx\\
&= \frac{4a\sqrt{cx}\,\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c}+\frac{1}{7}\left(4a^2\right)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx\\
&= \frac{4a\sqrt{cx}\,\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c}+\frac{\left(8a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{7c}\\
&= \frac{4a\sqrt{cx}\,\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\,(a+bx^2)^{3/2}}{7c}+\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(}{7\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.70, size = 55, normalized size = 0.36

$$\frac{2ax\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\,\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[c*x],x]
```

```
[Out] (2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/(Sq
rt[c*x]*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.04, size = 134, normalized size = 0.88

method	result
default	$\frac{\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a^2}{7}+\frac{2b^3x^5}{7}+\frac{8a\,b^2x^3}{7}}{\sqrt{b\,x^2+a}\,b\sqrt{cx}}$
risch	$\frac{2(b\,x^2+3a)x\sqrt{b\,x^2+a}}{7\sqrt{cx}}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\ldots}\right.}{7b\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx\,(b\,x^2+a)}\left(\frac{2b\,x^2\sqrt{bc\,x^3+acx}}{7c}+\frac{6a\sqrt{bc\,x^3+acx}}{7c}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a\ldots}}{b}\right)}{\sqrt{-ab}}}\ldots}{7b\sqrt{\ldots}}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7/(b*x^2+a)^(1/2)*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2+b^3*x^5
+4*a*b^2*x^3+3*a^2*b*x)/b/(c*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(c*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 55, normalized size = 0.36

$$\frac{2\left(4\sqrt{bc}\,a^2\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+(b^2x^2+3ab)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{7bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*(4*sqrt(b*c)*a^2*weierstrassPInverse(-4*a/b, 0, x) + (b^2*x^2 + 3*a*b)*
sqrt(b*x^2 + a)*sqrt(c*x))/(b*c)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.01, size = 46, normalized size = 0.30

$$\frac{a^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(1/2),x)
```

```
[Out] a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(c)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c*x)^(1/2), x)
```

3.602 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}} \, dx$

Optimal. Leaf size=296

$$\frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}+\frac{24a\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{5c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}}-\frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right.}}}{5c^{3/2}\sqrt{a+bx}}$$

```
[Out] -2*(b*x^2+a)^(3/2)/c/(c*x)^(1/2)+12/5*b*(c*x)^(3/2)*(b*x^2+a)^(1/2)/c^3+24/
5*a*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^2/(a^(1/2)+x*b^(1/2))-24/5*a^(5/4
)*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a
)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(3/2)/(b*x^2+a)^(1/2)+12/5*a^(5/4)*b^(1/4)
*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/c^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {283, 285, 335, 311, 226, 1210}

$$\frac{12a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}}-\frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}}+\frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}+\frac{24a\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{5c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(3/2),x]
```

```
[Out] (12*b*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*c^3) + (24*a*Sqrt[b]*Sqrt[c*x]*Sqrt[a
 + b*x^2])/(5*c^2*(Sqrt[a] + Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(c*Sqrt[c*
x]) - (24*a^(5/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1
/2])/(5*c^(3/2)*Sqrt[a + b*x^2]) + (12*a^(5/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*c^(3/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = -\frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}} + \frac{(6b)\int \sqrt{cx}\,\sqrt{a+bx^2}\,dx}{c^2}$$

$$= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3} - \frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}} + \frac{(12ab)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5c^2}$$

$$= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3} - \frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}} + \frac{(24ab)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5c^3}$$

$$= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3} - \frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}} + \frac{\left(24a^{3/2}\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right.}{5c^2}$$

$$= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3} + \frac{24a\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{5c^2\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}} - \frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 55, normalized size = 0.19

$$-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(cx)^(3/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])/(
(c*x)^(3/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.06, size = 208, normalized size = 0.70

method	result

default	$\frac{\frac{24\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^{2}}{5}-\frac{12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\cdots}{\cdots}}{\sqrt{b\,x^{2}+a}\,c\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-b\,x^{2}+5a\right)}{5c\sqrt{cx}}+\frac{12a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{-ab}\cdots}{\cdots}\cdots\right)}{5\sqrt{bc\,x^{3}\cdots}}$
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\,\left(-\frac{2\left(c\,x^{2}b+ac\right)a}{c^{2}\sqrt{x\left(c\,x^{2}b+ac\right)}}+\frac{2bx\sqrt{bc\,x^{3}+acx}}{5c^{2}}+\frac{12a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\cdots}{\cdots}\right)}{\sqrt{cx}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+b
^2*x^4-4*a*b*x^2-5*a^2)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 60, normalized size = 0.20

$$-\frac{2\left(12\sqrt{bc}\,ax\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)-\sqrt{bx^2+a}\,(bx^2-5a)\sqrt{cx}\right)}{5c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(12*sqrt(b*c)*a*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/
b, 0, x)) - sqrt(b*x^2 + a)*(b*x^2 - 5*a)*sqrt(c*x))/(c^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.03, size = 49, normalized size = 0.17

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(3/2),x)
```

```
[Out] a**(3/2)*gamma(-1/4)*hyper((-3/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(3/2),x)
[Out] int((a + b*x^2)^(3/2)/(c*x)^(3/2), x)
```

3.603 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}} \, dx$

Optimal. Leaf size=152

$$\frac{4b\sqrt{cx}\,\sqrt{a+bx^2}}{3c^3}-\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}+\frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}}$$

```
[Out] -2/3*(b*x^2+a)^(3/2)/c/(c*x)^(3/2)+4/3*b*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^3+4/
3*a^(3/4)*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arct
an(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*(
(b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {283, 285, 335, 226}

$$\frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}}+\frac{4b\sqrt{cx}\,\sqrt{a+bx^2}}{3c^3}-\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (4*b*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*c^3) - (2*(a + b*x^2)^(3/2))/(3*c*(c*x)^
(3/2)) + (4*a^(3/4)*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
 1/2])/(3*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx &= -\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}+\frac{(2b)\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx}{c^2}\\
&= \frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3}-\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}+\frac{(4ab)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{3c^2}\\
&= \frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3}-\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}+\frac{(8ab)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3c^3}\\
&= \frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3}-\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}+\frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(2}{3c^{5/2}\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 57, normalized size = 0.38

$$-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)])/(
3*(c*x)^(5/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.05, size = 125, normalized size = 0.82

method	result
default	$\frac{\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,ax}{3}+\frac{2b^2x^4}{3}-\frac{2a^2}{3}}{\sqrt{b\,x^2+a}\,x\,c^2\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-b\,x^2+a)}{3x\,c^2\sqrt{cx}}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\ldots}\right.}{3\sqrt{bc\,x^3+acx}\,c^2\sqrt{cx}\,\sqrt{b\,x^2+a}\ldots}$
elliptic	$\frac{\sqrt{cx\,(b\,x^2+a)}\left(-\frac{2a\sqrt{bc\,x^3+acx}}{3c^3x^2}+\frac{2b\sqrt{bc\,x^3+acx}}{3c^3}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\ldots}{3c^2\sqrt{b}\ldots}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)/x*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a*x+b^2*x
^4-a^2)/c^2/(c*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 53, normalized size = 0.35

$$\frac{2\left(4\sqrt{bc}\,ax^2\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^2+a}\,(bx^2-a)\sqrt{cx}\right)}{3\,c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(4*sqrt(b*c)*a*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^2 + a)*
(b*x^2 - a)*sqrt(c*x))/(c^3*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.16, size = 49, normalized size = 0.32

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(5/2),x)
```

```
[Out] a**(3/2)*gamma(-3/4)*hyper((-3/2, -3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{\left(c\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c*x)^(5/2), x)
```

3.604 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=297

$$-\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}+\frac{24b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}}-\frac{24\sqrt[4]{a}\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\Big(}{5c^{7/2}\sqrt{a+bx^2}}$$

```
[Out] -2/5*(b*x^2+a)^(3/2)/c/(c*x)^(5/2)-12/5*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)+2
4/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^4/(a^(1/2)+x*b^(1/2))-24/5*a^(1/4
)*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a
)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(7/2)/(b*x^2+a)^(1/2)+12/5*a^(1/4)*b^(5/4)
*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/c^(7/2)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {283, 335, 311, 226, 1210}

$$\frac{12\sqrt[4]{a}\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}}-\frac{24\sqrt[4]{a}\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}}+\frac{24b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}-\frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

```
[Out] (-12*b*Sqrt[a + b*x^2])/(5*c^3*Sqrt[c*x]) + (24*b^(3/2)*Sqrt[c*x]*Sqrt[a +
b*x^2])/(5*c^4*(Sqrt[a] + Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(5*c*(c*x)^(5
/2)) - (24*a^(1/4)*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
1/2])/(5*c^(7/2)*Sqrt[a + b*x^2]) + (12*a^(1/4)*b^(5/4)*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx = -\frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}} + \frac{(6b)\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx}{5c^2}$$

$$= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}} - \frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}} + \frac{(12b^2)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5c^4}$$

$$= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}} - \frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}} + \frac{(24b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5c^5}$$

$$= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}} - \frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}} + \frac{(24\sqrt{a}\,b^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5c^4} -$$

$$= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}} + \frac{24b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5c^4\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}} - \frac{24\sqrt[4]{a}\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 57, normalized size = 0.19

$$-\frac{2ax\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)])/
(5*(c*x)^(7/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.06, size = 216, normalized size = 0.73

method	result

default	$\frac{\frac{24\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^{2}}{5}-\frac{12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\cdots}{\cdots}}{x^{2}\sqrt{b\,x^{2}+a}\,c^{3}\sqrt{cx}\cdots}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,(7b\,x^{2}+a)}{5x^{2}c^{3}\sqrt{cx}}+\frac{12b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{E}\cdots}{\cdots}\cdots\right.}{5\sqrt{bc\,x^{3}+\cdots}}$
elliptic	$\frac{\sqrt{cx\,(b\,x^{2}+a)}\left(-\frac{2a\sqrt{bc\,x^{3}+acx}}{5c^{4}x^{3}}-\frac{14\left(c\,x^{2}b+ac\right)b}{5c^{4}\sqrt{x\,(c\,x^{2}b+ac)}}+\frac{12b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-\cdots}}{b}\right)\cdots}{\sqrt{-a\cdots}}}\cdots}{\cdots}\right.}{\sqrt{cx}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/x^2*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-6*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))*a*b*x^2-7*b^2*x^4-8*a*b*x^2-a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 60, normalized size = 0.20

$$-\frac{2\left(12\sqrt{bc}\,bx^3\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(7bx^2+a\right)\sqrt{bx^2+a}\sqrt{cx}\right)}{5c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(12*sqrt(b*c)*b*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*
a/b, 0, x)) + (7*b*x^2 + a)*sqrt(b*x^2 + a)*sqrt(c*x))/(c^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.44, size = 53, normalized size = 0.18

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(7/2),x)
```

```
[Out] a**(3/2)*gamma(-5/4)*hyper((-3/2, -5/4), (-1/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{{\left(c\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(7/2),x)
[Out] int((a + b*x^2)^(3/2)/(c*x)^(7/2), x)
```

3.605 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}}-\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}+\frac{4b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\,c^{9/2}\sqrt{a+bx^2}}$$

```
[Out] -2/7*(b*x^2+a)^(3/2)/c/(c*x)^(7/2)-4/7*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(3/2)+4/
7*b^(7/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a
)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/c^(9/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {283, 335, 226}

$$\frac{4b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\,c^{9/2}\sqrt{a+bx^2}}-\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}}-\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(9/2),x]
```

```
[Out] (-4*b*Sqrt[a + b*x^2])/(7*c^3*(c*x)^(3/2)) - (2*(a + b*x^2)^(3/2))/(7*c*(c*
x)^(7/2)) + (4*b^(7/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2]
)/(7*a^(1/4)*c^(9/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx &= -\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}+\frac{(6b)\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx}{7c^2}\\ &= -\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}}-\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}+\frac{(4b^2)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{7c^4}\\ &= -\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}}-\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}+\frac{(8b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{7c^5}\\ &= -\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}}-\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}+\frac{4b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\right.}{7\sqrt[4]{a}\,c^{9/2}\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.38

$$-\frac{2ax\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(9/2),x]
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)])/
(7*(c*x)^(9/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.08, size = 135, normalized size = 0.89

method	result
default	$\frac{\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^{3}}{7}-\frac{6b^{2}x^{4}}{7}-\frac{8abx^{2}}{7}\cdots}{\sqrt{b\,x^{2}+a}\,x^{3}c^{4}\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,(3b\,x^{2}+a)}{7x^{3}c^{4}\sqrt{cx}}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.}{7\sqrt{bc\,x^{3}+acx}\,c^{4}\sqrt{cx}\,\sqrt{b\,x^{2}+a}\cdots}$
elliptic	$\frac{\sqrt{cx\,(b\,x^{2}+a)}\left(-\frac{2a\sqrt{bc\,x^{3}+acx}}{7c^{5}x^{4}}-\frac{6b\sqrt{bc\,x^{3}+acx}}{7c^{5}x^{2}}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)\cdots}{\sqrt{-ab}}}\cdots}{7c^{4}\sqrt{b\cdots}}\cdots\right.}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/7/(b*x^2+a)^(1/2)/x^3*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*
((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elliptic
F(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b*x^3-3
*b^2*x^4-4*a*b*x^2-a^2)/c^4/(c*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 53, normalized size = 0.35

$$\frac{2\left(4\sqrt{bc}\,bx^{4}\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-(3\,bx^{2}+a)\sqrt{bx^{2}+a}\,\sqrt{cx}\right)}{7\,c^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/7*(4*sqrt(b*c)*b*x^4*weierstrassPInverse(-4*a/b, 0, x) - (3*b*x^2 + a)*sq
rt(b*x^2 + a)*sqrt(c*x))/(c^5*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 19.52, size = 53, normalized size = 0.35

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{4}, -\frac{3}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a} \right)}{2c^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(9/2),x)
```

```
[Out] a**(3/2)*gamma(-7/4)*hyper((-7/4, -3/2), (-3/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(9/2)*x**(7/2)*gamma(-3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{{\left(c\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(9/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c*x)^(9/2), x)
```

3.606 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}} \, dx$

Optimal. Leaf size=331

$$-\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}+\frac{8b^{5/2}\sqrt{cx}\,\sqrt{a+bx^2}}{15ac^6\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}-\frac{8b^{9/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\right.}}}{15a^{3/4}c^{11/2}}$$

```
[Out] -2/9*(b*x^2+a)^(3/2)/c/(c*x)^(9/2)-4/15*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(5/2)-8
/15*b^2*(b*x^2+a)^(1/2)/a/c^5/(c*x)^(1/2)+8/15*b^(5/2)*(c*x)^(1/2)*(b*x^2+a
)^(1/2)/a/c^6/(a^(1/2)+x*b^(1/2))-8/15*b^(9/4)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)
/c^(11/2)/(b*x^2+a)^(1/2)+4/15*b^(9/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))
*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*
(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/c^(11/2
)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {283, 331, 335, 311, 226, 1210}

$$\frac{4b^{9/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}-\frac{8b^{9/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}+\frac{8b^{5/2}\sqrt{cx}\,\sqrt{a+bx^2}}{15ac^6\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}-\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-4*b*Sqrt[a + b*x^2])/(15*c^3*(c*x)^(5/2)) - (8*b^2*Sqrt[a + b*x^2])/(15*a
*c^5*Sqrt[c*x]) + (8*b^(5/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(15*a*c^6*(Sqrt[a]
+ Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(9*c*(c*x)^(9/2)) - (8*b^(9/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*a^(3/4)*c^(11/2)*Sqrt[a
 + b*x^2]) + (4*b^(9/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2
])/(15*a^(3/4)*c^(11/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}} + \frac{(2b)\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx}{3c^2}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}} - \frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}} + \frac{(4b^2)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{15c^4}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}} - \frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}} - \frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}} + \frac{(4b^3)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{15ac^6}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}} - \frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}} - \frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}} + \frac{(8b^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{\right.}{15ac^7}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}} - \frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}} - \frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}} + \frac{(8b^{5/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x\right.}{15\sqrt{a}\,c^6}$$

$$= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}} - \frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}} + \frac{8b^{5/2}\sqrt{cx}\,\sqrt{a+bx^2}}{15ac^6\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}} - \frac{8b^{9/4}\left(\sqrt{a\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 57, normalized size = 0.17

$$-\frac{2ax\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)])/
(9*(c*x)^(11/2)*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.09, size = 234, normalized size = 0.71

method	result
default	$\frac{8\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^{2}x^{4}}{15} - \frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{}}{x^{4}\sqrt{b\,x^{2}+a}\;a\,c^{5}}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,(12b^{2}x^{4}+11ab\,x^{2}+5a^{2})}{45x^{4}a\,c^{5}\sqrt{cx}} + \frac{4b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\right.}{1}$
elliptic	$\frac{\sqrt{cx\,(b\,x^{2}+a)}\left(-\frac{2a\sqrt{bc\,x^{3}+acx}}{9c^{6}x^{5}}-\frac{22b\sqrt{bc\,x^{3}+acx}}{45c^{6}x^{3}}-\frac{8\left(c\,x^{2}b+ac\right)b^{2}}{15a\,c^{6}\sqrt{x\,(c\,x^{2}b+ac)}}+\frac{4b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-}}{b}\right)}{\sqrt{-}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/x^4*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*x^4-6*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^2*x^4-12*b^3*x^6-23*a*b^2*x^4-16*a^2*b*x^2-5*a^3)/(b*x^2+a)^(1/2)
/a/c^5/(c*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 78, normalized size = 0.24

$$-\frac{2\left(12\sqrt{bc}\,b^2x^5\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(12\,b^2x^4+11\,abx^2+5\,a^2\right)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{45\,ac^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*(12*sqrt(b*c)*b^2*x^5*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(
-4*a/b, 0, x)) + (12*b^2*x^4 + 11*a*b*x^2 + 5*a^2)*sqrt(b*x^2 + a)*sqrt(c*x
))/(a*c^6*x^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 51.69, size = 53, normalized size = 0.16

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{3}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{11}{2}}x^{\frac{9}{2}}\Gamma\left(-\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(11/2),x)
```

```
[Out] a**(3/2)*gamma(-9/4)*hyper((-9/4, -3/2), (-5/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(11/2)*x**(9/2)*gamma(-5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(11/2),x)
[Out] int((a + b*x^2)^(3/2)/(c*x)^(11/2), x)
```

3.607 $\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx$

Optimal. Leaf size=128

$$-\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}-\frac{3\sqrt[4]{6}\,ac^2\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] -3/5*6^(1/4)*a*c^2*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)
^(1/2)*(-2*x^2+3)^(1/2)/x^(1/2)/(-2*a*x^2+3*a)^(1/2)-2/15*c*(c*x)^(3/2)*(-2
*a*x^2+3*a)^(1/2)+2/9*(c*x)^(7/2)*(-2*a*x^2+3*a)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {285, 327, 326, 325, 324, 435}

$$-\frac{3\sqrt[4]{6}\,ac^2\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\,\sqrt{3a-2ax^2}}+\frac{2\sqrt{3a-2ax^2}\,(cx)^{7/2}}{9c}-\frac{2}{15}c\sqrt{3a-2ax^2}\,(cx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (-2*c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/15 + (2*(c*x)^(7/2)*Sqrt[3*a - 2*a*x
^2])/(9*c) - (3*6^(1/4)*a*c^2*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sq
rt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 324

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-b/a
)^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b/
a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

Rule 325

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b}
```

```
, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

Rule 326

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-b/a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx = \frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c} + \frac{1}{3}(2a)\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2} + \frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c} + \frac{1}{5}\left(3ac^2\right)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,d$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2} + \frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c} + \frac{\left(3ac^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax}}}{5\sqrt{x}}$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2} + \frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c} + \frac{\left(3ac^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)}{5\sqrt{x}\sqrt{3a-2}}$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2} + \frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c} - \frac{\left(3\sqrt[4]{2}\,3^{3/4}ac^2\sqrt{cx}\sqrt{1-}\right.}{}$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2} + \frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c} - \frac{3\sqrt[4]{6}\,ac^2\sqrt{cx}\sqrt{3-2x^2}\,E}{5\sqrt{x}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.95, size = 74, normalized size = 0.58

$$\frac{c(cx)^{3/2}\sqrt{a\left(3-2x^2\right)}\left(-\left(3-2x^2\right)^{3/2}+3\sqrt{3}\;{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)\right)}{9\sqrt{3-2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*(c*x)^(3/2)*Sqrt[a*(3 - 2*x^2)]*(-(3 - 2*x^2)^(3/2) + 3*Sqrt[3]*Hypergeo
metric2F1[-1/2, 3/4, 7/4, (2*x^2)/3]))/(9*Sqrt[3 - 2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(100) = 200.
time = 0.14, size = 237, normalized size = 1.85

method	result
risch	$-\frac{2x^2(5x^2-3)(2x^2-3)c^3a}{45\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\left(-\right.}{}$
elliptic	$-\frac{\sqrt{cx}\,\sqrt{-a(2x^2-3)}\,\sqrt{-cxa(2x^2-3)}\left(\frac{2c^2x^3\sqrt{-2ac\,x^3+3acx}}{9}-\frac{2c^2x\sqrt{-2ac\,x^3+3acx}}{15}+\frac{c^3a\sqrt{(}}{}\right.}{}$
default	$\frac{c^2\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(80x^6+18\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/180*c^2/x*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(80*x^6+18*((-2*x+2^(1/2)*3^(1
/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/
```

```
6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2)
)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)-9*((-2*x+2^(1/2)*3^
(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(
1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/
2))*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)-168*x^4+72*x^2)/(
2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 56, normalized size = 0.44

$$\frac{3}{5}\sqrt{2}\sqrt{-ac}\,c^2\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x)) + \frac{2}{45}\left(5\,c^2x^3 - 3\,c^2x\right)\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/5*sqrt(2)*sqrt(-a*c)*c^2*weierstrassZeta(6, 0, weierstrassPInverse(6, 0,
x)) + 2/45*(5*c^2*x^3 - 3*c^2*x)*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.11, size = 53, normalized size = 0.41

$$\frac{\sqrt{3}\,\sqrt{a}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), 2*
x**2*exp_polar(2*I*pi)/3)/(2*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{5/2}\,\sqrt{3\,a-2\,a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2), x)
```

3.608 $\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx$

Optimal. Leaf size=117

$$-\frac{2}{7}c\sqrt{cx}\,\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{7\sqrt{a\left(3-2x^2\right)}}$$

```
[Out] 1/7*6^(3/4)*a*c^(3/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*
(-2*x^2+3)^(1/2)/(a*(-2*x^2+3))^(1/2)+2/7*(c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2)/
c-2/7*c*(c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {285, 327, 335, 230, 227}

$$\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{7\sqrt{a\left(3-2x^2\right)}}+\frac{2\sqrt{3a-2ax^2}\,(cx)^{5/2}}{7c}-\frac{2}{7}c\sqrt{3a-2ax^2}\,\sqrt{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (-2*c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/7 + (2*(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]
)/(7*c) + (6^(3/4)*a*c^(3/2)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*
Sqrt[c*x])/Sqrt[c]], -1])/(7*Sqrt[a*(3 - 2*x^2)])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx &= \frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{1}{7}(6a)\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx \\
&= -\frac{2}{7}c\sqrt{cx}\,\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{1}{7}\left(3ac^2\right)\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}} \\
&= -\frac{2}{7}c\sqrt{cx}\,\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{1}{7}(6ac)\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax}{c^2}}}\right. \\
&= -\frac{2}{7}c\sqrt{cx}\,\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{\left(2\sqrt{3}\,ac\sqrt{3-2x^2}\right)\text{Subst}\left(\right.}{7\sqrt{a\,(3-}} \\
&= -\frac{2}{7}c\sqrt{cx}\,\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\right.}{7\sqrt{a\,(3-2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.37, size = 74, normalized size = 0.63

$$\frac{c\sqrt{cx}\,\sqrt{a\,(3-2x^2)}\,\left(-(3-2x^2)^{3/2}+3\sqrt{3}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};\frac{2x^2}{3}\right)\right)}{7\sqrt{3-2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)]*(-(3 - 2*x^2)^(3/2) + 3*Sqrt[3]*Hypergeome
tric2F1[-1/2, 1/4, 5/4, (2*x^2)/3]))/(7*Sqrt[3 - 2*x^2])
```

Maple [A]
time = 0.10, size = 133, normalized size = 1.14

method	result
default	$-\frac{c\sqrt{cx}\,\sqrt{-a\,(2x^2-3)}\,\left(-8x^5+\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-}\right.}{14x(2x^2-3)}$
risch	$-\frac{2(x^2-1)x(2x^2-3)c^2a}{7\sqrt{cx}\,\sqrt{-a\,(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\,\mathrm{Ell}}{126\sqrt{-2ac\,x^3+3acx}\,\sqrt{cx}}$
elliptic	$-\frac{\sqrt{cx}\,\sqrt{-a\,(2x^2-3)}\,\sqrt{-cxa\,(2x^2-3)}\,\left(\frac{2c\,x^2\sqrt{-2ac\,x^3+3acx}}{7}-\frac{2c\sqrt{-2ac\,x^3+3acx}}{7}+\frac{c^2a\sqrt{6}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*c*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(-8*x^5+((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1
```

```
/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2),1/2*2^(1/2))+20*x^3-12*x)/x/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 45, normalized size = 0.38

$$-\frac{3}{7}\sqrt{2}\sqrt{-ac}\,c\text{weierstrassPInverse}(6,0,x)+\frac{2}{7}\sqrt{-2\,ax^2+3\,a}\,(cx^2-c)\sqrt{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -3/7*sqrt(2)*sqrt(-a*c)*c*weierstrassPInverse(6, 0, x) + 2/7*sqrt(-2*a*x^2
+ 3*a)*(c*x^2 - c)*sqrt(c*x)
```

Sympy [A]
time = 1.39, size = 53, normalized size = 0.45

$$\frac{\sqrt{3}\sqrt{a}\,c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), 2*x
**2*exp_polar(2*I*pi)/3)/(2*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{3/2}\,\sqrt{3\,a-2\,a\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2),x)
[Out] int((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2), x)
```

3.609 $\int \sqrt{cx}\,\sqrt{3a-2ax^2}\,dx$

Optimal. Leaf size=99

$$\frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}-\frac{6\sqrt[4]{6}\,a\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] -6/5*6^(1/4)*a*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/
2)*(-2*x^2+3)^(1/2)/x^(1/2)/(-2*a*x^2+3*a)^(1/2)+2/5*(c*x)^(3/2)*(-2*a*x^2+
3*a)^(1/2)/c
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {285, 326, 325, 324, 435}

$$\frac{2\sqrt{3a-2ax^2}\,(cx)^{3/2}}{5c}-\frac{6\sqrt[4]{6}\,a\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\,\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/(5*c) - (6*6^(1/4)*a*Sqrt[c*x]*Sqrt[3 -
 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*Sqrt[x]*Sqrt[
3*a - 2*a*x^2])
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 324

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-b/a
)^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b/
a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

Rule 325

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b}
, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

Rule 326

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-b/a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx}\,\sqrt{3a-2ax^2}\,dx &= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}+\frac{1}{5}(6a)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}+\frac{\left(6a\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{5\sqrt{x}}\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}+\frac{\left(6a\sqrt{cx}\,\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{5\sqrt{x}\,\sqrt{3a-2ax^2}}\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}-\frac{\left(6\sqrt[4]{2}\,3^{3/4}a\sqrt{cx}\,\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx\right.}{5\sqrt{x}\,\sqrt{3a-2ax^2}}\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}-\frac{6\sqrt[4]{6}\,a\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}}\,x}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\,\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 7.33, size = 51, normalized size = 0.52

$$\frac{2x\sqrt{cx}\,\sqrt{a\,(3-2x^2)}\;{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)}{\sqrt{9-6x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, (2*x^2
)/3])/Sqrt[9 - 6*x^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(77) = 154.
time = 0.08, size = 229, normalized size = 2.31

method	result
risch	$-\frac{2x^2(2x^2-3)ac}{5\sqrt{cx}\,\sqrt{-a\,(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\,\left(-\sqrt{}\right.}{4}$

elliptic	$-\frac{\sqrt{cx}\,\sqrt{-a\left(2x^2-3\right)}\,\sqrt{-cxa\left(2x^2-3\right)}\left(\frac{2x\sqrt{-2ac\,x^3+3acx}}{5}+\frac{ac\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{cx}\,\sqrt{-a\left(2x^2-3\right)}\left(2\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{(}}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(2*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^
(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/
2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*x
+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)-((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^
(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/
2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*x
+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)+8*x^4-12*x^2)/x/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 39, normalized size = 0.39

$$\frac{2}{5}\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,x+\frac{6}{5}\sqrt{2}\,\sqrt{-ac}\,\mathrm{weierstrassZeta}(6,0,\mathrm{weierstrassPInverse}(6,0,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*x + 6/5*sqrt(2)*sqrt(-a*c)*weierstrassZe
ta(6, 0, weierstrassPInverse(6, 0, x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 53, normalized size = 0.54

$$\frac{\sqrt{3}\,\sqrt{a}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), 2*x*
*2*exp_polar(2*I*pi)/3)/(2*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x}\,\sqrt{3\,a-2\,a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2), x)
```

3.610 $\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{cx}\,\sqrt{3a-2ax^2}}{3c}+\frac{2\,2^{3/4}a\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\,\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\,\sqrt{c}\,\sqrt{a\left(3-2x^2\right)}}$$

```
[Out] 2/3*2^(3/4)*a*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+
3)^(1/2)*3^(3/4)/c^(1/2)/(a*(-2*x^2+3))^(1/2)+2/3*(c*x)^(1/2)*(-2*a*x^2+3*a
)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {285, 335, 230, 227}

$$\frac{2\,2^{3/4}a\sqrt{3-2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{\frac{2}{3}}\,\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\,\sqrt{c}\,\sqrt{a\left(3-2x^2\right)}}+\frac{2\sqrt{3a-2ax^2}\,\sqrt{cx}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*a - 2*a*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/(3*c) + (2*2^(3/4)*a*Sqrt[3 - 2*x^2]*Elli
pticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3^(1/4)*Sqrt[c]*Sqrt[a
*(3 - 2*x^2)])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\,\sqrt{3a-2ax^2}}{3c} + (2a)\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx$$

$$= \frac{2\sqrt{cx}\,\sqrt{3a-2ax^2}}{3c} + \frac{(4a)\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{2\sqrt{cx}\,\sqrt{3a-2ax^2}}{3c} + \frac{\left(4a\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{3}\,c\sqrt{a\left(3-2x^2\right)}}$$

$$= \frac{2\sqrt{cx}\,\sqrt{3a-2ax^2}}{3c} + \frac{2\,2^{3/4}a\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\,\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\,\sqrt{c}\,\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.81, size = 51, normalized size = 0.54

$$\frac{2x\sqrt{a\left(9-6x^2\right)}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};\frac{2x^2}{3}\right)}{\sqrt{cx}\,\sqrt{3-2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*a - 2*a*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, (2*x^2)/3])/(Sqr
t[c*x]*Sqrt[3 - 2*x^2])
```

Maple [A]
time = 0.08, size = 124, normalized size = 1.32

method	result
default	$-\frac{\sqrt{-a\left(2x^{2}-3\right)}\left(\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\right.}{3\sqrt{cx}\,\left(2x^{2}-3\right)}$
elliptic	$\frac{\sqrt{-cxa\left(2x^{2}-3\right)}\left(\frac{2\sqrt{-2ac\,x^{3}+3acx}}{3c}+\frac{a\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{27\sqrt{-2ac\,x^{3}}}\right.}{\sqrt{cx}\,\sqrt{-a\left(2x^{2}-3\right)}}$
risch	$-\frac{2x\left(2x^{2}-3\right)a}{3\sqrt{cx}\,\sqrt{-a\left(2x^{2}-3\right)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\ \mathrm{Ell}}{27\sqrt{-2ac\,x^{3}+3acx}\,\sqrt{cx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((
-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*Ell
ipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/
2*2^(1/2))-4*x^3+6*x)/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 40, normalized size = 0.43

$$-\frac{2\left(3\sqrt{2}\sqrt{-ac}\ \text{weierstrassPInverse}(6,0,x)-\sqrt{-2\,ax^2+3\,a}\ \sqrt{cx}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(2)*sqrt(-a*c)*weierstrassPInverse(6, 0, x) - sqrt(-2*a*x^2 + 3
*a)*sqrt(c*x))/c
```

Sympy [A]
time = 0.44, size = 53, normalized size = 0.56

$$\frac{\sqrt{3}\sqrt{a}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{2\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), 2*x**2*exp_po
lar(2*I*pi)/3)/(2*sqrt(c)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,a-2\,a\,x^2}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(1/2), x)
```

3.611 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}+\frac{4\sqrt[4]{6}\,a\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] 4*6^(1/4)*a*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*
(-2*x^2+3)^(1/2)/c^2/x^(1/2)/(-2*a*x^2+3*a)^(1/2)-2*(-2*a*x^2+3*a)^(1/2)/c/
(c*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {283, 326, 325, 324, 435}

$$\frac{4\sqrt[4]{6}\,a\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\,\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(c*Sqrt[c*x]) + (4*6^(1/4)*a*Sqrt[c*x]*Sqrt[3 - 2*
x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(c^2*Sqrt[x]*Sqrt[3
*a - 2*a*x^2])
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 324

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-b/a
)^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b/
a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

Rule 325

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b}
, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

Rule 326

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-b/a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}} - \frac{(4a)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{c^2}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}} - \frac{\left(4a\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{c^2\sqrt{x}}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}} - \frac{\left(4a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{c^2\sqrt{x}\sqrt{3a-2ax^2}}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}} + \frac{\left(4\sqrt[4]{2}\,3^{3/4}a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\cdots}}{\sqrt{2}\cdots}\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}} + \frac{4\sqrt[4]{6}\,a\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.01, size = 51, normalized size = 0.52

$$-\frac{2x\sqrt{a\left(9-6x^2\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\frac{2x^2}{3}\right)}{(cx)^{3/2}\sqrt{3-2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, (2*x^2)/3])/((
c*x)^(3/2)*Sqrt[3 - 2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(80) = 160.
time = 0.10, size = 225, normalized size = 2.30

method	result
risch	$\frac{2(2x^2-3)a}{c\sqrt{cx}\,\sqrt{-a\left(2x^2-3\right)}}-\frac{2\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\,\left(-\right.}{27}$

elliptic	$-\frac{\sqrt{-a(2x^2-3)}\,\sqrt{-cxa(2x^2-3)}\left(-\frac{2(-2cx^2a+3ac)}{c^2\sqrt{x(-2cx^2a+3ac)}}-\frac{2a\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{}}{}\right.}{}$
default	$-\frac{\sqrt{-a(2x^2-3)}\left(2\sqrt{(-2x+\sqrt{2}\,\sqrt{3})\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\,\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{(2x+}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-a*(2*x^2-3))^(1/2)*(2*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)
*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(
1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*x+2^(1/2)*3^(
1/2))*2^(1/2)*3^(1/2))^(1/2)-((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)
*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(
1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*x+2^(1/2)*3^(
1/2))*2^(1/2)*3^(1/2))^(1/2)+12*x^2-18)/c/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 46, normalized size = 0.47

$$-\frac{2\left(2\sqrt{2}\sqrt{-ac}\,x\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x))+\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\right)}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")

[Out] -2*(2*sqrt(2)*sqrt(-a*c)*x*weierstrassZeta(6, 0, weierstrassPInverse(6, 0, x)) + sqrt(-2*a*x^2 + 3*a)*sqrt(c*x))/(c^2*x)

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 56, normalized size = 0.57

$$\frac{\sqrt{3}\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(3/2),x)

[Out] sqrt(3)*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), 2*x**2*exp_polar(2*I*pi)/3)/(2*c**(3/2)*sqrt(x)*gamma(3/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,a-2\,a\,x^2}}{{\left(c\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(3/2),x)

[Out] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(3/2), x)

3.612 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}-\frac{4\,2^{3/4}a\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{3}\,c^{5/2}\sqrt{a(3-2x^2)}}$$

```
[Out] -4/9*2^(3/4)*a*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2
+3)^(1/2)*3^(3/4)/c^(5/2)/(a*(-2*x^2+3))^(1/2)-2/3*(-2*a*x^2+3*a)^(1/2)/c/(
c*x)^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {283, 335, 230, 227}

$$-\frac{4\,2^{3/4}a\sqrt{3-2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{3}\,c^{5/2}\sqrt{a(3-2x^2)}}-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(3*c*(c*x)^(3/2)) - (4*2^(3/4)*a*Sqrt[3 - 2*x^2]*E
llipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*3^(1/4)*c^(5/2)*S
qrt[a*(3 - 2*x^2)])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx &= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}-\frac{(4a)\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx}{3c^2}\\
&= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}-\frac{(8a)\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3c^3}\\
&= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}-\frac{\left(8a\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{3\sqrt{3}\,c^3\sqrt{a\left(3-2x^2\right)}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}-\frac{4\;2^{3/4}a\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{3}\,c^{5/2}\sqrt{a\left(3-2x^2\right)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.53

$$-\frac{2x\sqrt{a\left(3-2x^2\right)}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};\frac{2x^2}{3}\right)}{(cx)^{5/2}\sqrt{9-6x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*x*Sqrt[a*(3 - 2*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, (2*x^2)/3])/((
c*x)^(5/2)*Sqrt[9 - 6*x^2])
```

Maple [A]
time = 0.09, size = 129, normalized size = 1.34

method	result
default	$\frac{2\sqrt{-a\left(2x^{2}-3\right)}\,\left(\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\,\mathrm{Ell}}{9x\,c^{2}\sqrt{cx}\,\left(2x^{2}-3\right)}$
risch	$\frac{2\left(2x^{2}-3\right)a}{3x\,c^{2}\sqrt{cx}\,\sqrt{-a\left(2x^{2}-3\right)}}-\frac{2\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\,\mathrm{El}}{81\sqrt{-2ac\,x^{3}+3acx}\,c^{2}\sqrt{cx}}$
elliptic	$-\frac{\sqrt{-a\left(2x^{2}-3\right)}\,\sqrt{-cxa\left(2x^{2}-3\right)}\,\left(-\frac{2\sqrt{-2ac\,x^{3}+3acx}}{3c^{3}x^{2}}-\frac{2a\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6}}{}\right.}{\sqrt{cx}\,a\left(2x^{2}-3\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-
2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*Elli
pticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2
*2^(1/2))*x-6*x^2+9)/x/c^2/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 46, normalized size = 0.48

$$\frac{2\left(2\sqrt{2}\sqrt{-ac}\,x^2\text{weierstrassPInverse}(6,0,x)-\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\right)}{3\,c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
[Out] 2/3*(2*sqrt(2)*sqrt(-a*c)*x^2*weierstrassPInverse(6, 0, x) - sqrt(-2*a*x^2
+ 3*a)*sqrt(c*x))/(c^3*x^2)
```

Sympy [A]
time = 1.45, size = 49, normalized size = 0.51

$$\frac{\sqrt{2}\,i\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{3}{2x^2}\right)}{2c^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(5/2),x)
[Out] sqrt(2)*I*sqrt(a)*gamma(-1/4)*hyper((-1/2, 1/4), (5/4,), 3/(2*x**2))/(2*c**
(5/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,a-2\,a\,x^2}}{{\left(c\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(5/2),x)
[Out] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(5/2), x)
```

3.613 $\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}} dx$

Optimal. Leaf size=156

$$-\frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2}+\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}+\frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}$$

```
[Out] 2/7*c*(c*x)^(5/2)*(b*x^2+a)^(1/2)/b-10/21*a*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)
/b^2+5/21*a^(7/4)*c^(7/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(si
n(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {327, 335, 226}

$$\frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}-\frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2}+\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (-10*a*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(21*b^2) + (2*c*(c*x)^(5/2)*Sqrt[a +
b*x^2])/(7*b) + (5*a^(7/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(21*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} - \frac{(5ac^2)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx}{7b}$$
$$= -\frac{10ac^3\sqrt{cx}\,\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{(5a^2c^4)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{21b^2}$$
$$= -\frac{10ac^3\sqrt{cx}\,\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{(10a^2c^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{\right.}{21b^2}$$
$$= -\frac{10ac^3\sqrt{cx}\,\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx}{\left(\sqrt{a}+\sqrt{\right.}}}}{21b^{9/4}\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 87, normalized size = 0.56

$$\frac{2c^3\sqrt{cx}\left(-5a^2-2abx^2+3b^2x^4+5a^2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{21b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c^3*Sqrt[c*x]*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a
]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 141, normalized size = 0.90

method	result
default	$\frac{c^{3}\sqrt{cx}\,\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a^{2}+6b^{3}\right.}{21x\sqrt{b\,x^{2}+a}\,b^{3}}$
risch	$-\frac{2(-3b\,x^{2}+5a)x\sqrt{b\,x^{2}+a}\,c^{4}}{21b^{2}\sqrt{cx}}+\frac{5a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}}{21b^{3}\sqrt{bc\,x^{3}+acx}\,\sqrt{cx}\,\sqrt{b\,x^{2}}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^{2}+a)}\left(\frac{2c^{3}x^{2}\sqrt{bc\,x^{3}+acx}}{7b}-\frac{10c^{3}a\sqrt{bc\,x^{3}+acx}}{21b^{2}}+\frac{5c^{4}a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2}{}}}{}\right.}{cx\sqrt{b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*c^3/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b
)^(1/2)*a^2+6*b^3*x^5-4*a*b^2*x^3-10*a^2*b*x)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/sqrt(b*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 62, normalized size = 0.40

$$\frac{2\left(5\sqrt{bc}\,a^{2}c^{3}\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(3\,b^{2}c^{3}x^{2}-5\,abc^{3}\right)\sqrt{bx^{2}+a}\,\sqrt{cx}\right)}{21\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*sqrt(b*c)*a^2*c^3*weierstrassPInverse(-4*a/b, 0, x) + (3*b^2*c^3*x^
2 - 5*a*b*c^3)*sqrt(b*x^2 + a)*sqrt(c*x))/b^3
```

Sympy [C] Result contains complex when optimal does not.
time = 11.47, size = 44, normalized size = 0.28

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(a)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^{7/2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(7/2)/(a + b*x^2)^(1/2), x)
```

3.614 $\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=273

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac^2\sqrt{cx}\,\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{}}{\sqrt[4]{a}\,\sqrt{}}\right.\right.}{5b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] 2/5*c*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b-6/5*a*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b
^(3/2)/(a^(1/2)+x*b^(1/2))+6/5*a^(5/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)
/(b*x^2+a)^(1/2)-3/5*a^(5/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1
/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*E
llipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a
^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)
^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {327, 335, 311, 226, 1210}

$$-\frac{3a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}+\frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}-\frac{6ac^2\sqrt{cx}\,\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*b) - (6*a*c^2*Sqrt[c*x]*Sqrt[a + b*x^2
])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + (6*a^(5/4)*c^(5/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2]) - (3*a^(5/
4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(7/4)
*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{(3ac^2)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5b}$$

$$= \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{(6ac)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5b}$$

$$= \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{(6a^{3/2}c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5b^{3/2}} + \frac{(6a^{3/2}c^2)\,\text{Subst}\Big(}{}$$

$$= \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{6ac^2\sqrt{cx}\,\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right.}}}{5b^{7/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 69, normalized size = 0.25

$$\frac{2c(cx)^{3/2}\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{5b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*(c*x)^(3/2)*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2,
3/4, 7/4, -((b*x^2)/a)]))/(5*b*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 210, normalized size = 0.77

method	result
default	$-\frac{c^2\sqrt{cx}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2-3\sqrt{\frac{bx+}{\sqrt{}}}\right.}{5x\sqrt{b\,x^2+a}}$

risch	$\frac{2x^2\sqrt{b\,x^2+a}\,c^3}{5b\sqrt{cx}} - \frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{EllipticE}\cdots}{\cdots}\right.}{5b^2\sqrt{bc\,x^3+ac\cdots}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(\frac{2c^2x\sqrt{bc\,x^3+acx}}{5b} - \frac{3c^3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\cdots}}{\cdots}\right.}{cx\sqrt{b\,x^2\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a
^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-2*b^2*x^4-2*a*b*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

[Out] integrate((c*x)^(5/2)/sqrt(b*x^2 + a), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 54, normalized size = 0.20

$$\frac{2\left(\sqrt{bx^2+a}\,\sqrt{cx}\,bc^2x+3\,\sqrt{bc}\,ac^2\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] 2/5*(sqrt(b*x^2 + a)*sqrt(c*x)*b*c^2*x + 3*sqrt(b*c)*a*c^2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)))/b^2

Sympy [C] Result contains complex when optimal does not.
time = 3.50, size = 44, normalized size = 0.16

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(5/2)/(b*x**2+a)**(1/2),x)

[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(11/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((c*x)^(5/2)/sqrt(b*x^2 + a), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{5/2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(5/2)/(a + b*x^2)^(1/2),x)

[Out] int((c*x)^(5/2)/(a + b*x^2)^(1/2), x)

3.615 $\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=127

$$\frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

```
[Out] 2/3*c*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b-1/3*a^(3/4)*c^(3/2)*(cos(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/
a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2
))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2
)/b^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {327, 335, 226}

$$\frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*c^(3/2)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*S
qrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b} - \frac{(ac^2)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{3b}$$

$$= \frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b} - \frac{(2ac)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3b}$$

$$= \frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b} - \frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)\right.}{3b^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 69, normalized size = 0.54

$$\frac{2c\sqrt{cx}\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*Sqrt[c*x]*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/
2, 5/4, -((b*x^2)/a)]))/(3*b*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 125, normalized size = 0.98

method	result

default	$-\frac{c\sqrt{cx}\,\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a-2\right.}{3x\sqrt{b\,x^{2}+a}\,b^{2}}$
risch	$\frac{2x\sqrt{b\,x^{2}+a}\,c^{2}}{3b\sqrt{cx}}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-}}{b}\right.}{\sqrt{-a}}}\right.}{3b^{2}\sqrt{bc\,x^{3}+acx}\,\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(b\,x^{2}+a\right)}\left(\frac{2c\sqrt{bc\,x^{3}+acx}}{3b}-\frac{c^{2}a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{x}{\sqrt{-}}}}{3b^{2}\sqrt{bc\,x^{3}+acx}}\right.}{cx\sqrt{b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/
2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1
/2)*a-2*b^2*x^3-2*a*b*x)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(b*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 41, normalized size = 0.32

$$-\frac{2\left(\sqrt{bc}\,ac\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^{2}+a}\,\sqrt{cx}\,bc\right)}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(b*c)*a*c*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^2 + a)*sqr
t(c*x)*b*c)/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.90, size = 44, normalized size = 0.35

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(a)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(3/2)/(a + b*x^2)^(1/2), x)
```

3.616 $\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=236

$$\frac{2\sqrt{cx}\sqrt{a+bx^2}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt[4]{a}\sqrt{c}\ \cdots}{\cdots}$$

```
[Out] 2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(a^(1/2)+x*b^(1/2))-2*a^(1/4)*(cos(2*
arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)+a^(1/4)*(cos(2*arctan(b^(1/4)
*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2
)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {335, 311, 226, 1210}

$$\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}}+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[a + b*x^2],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) - (2*a^(1/4)*
Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(3/4)*Sqrt
[a + b*x^2]) + (a^(1/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c
])], 1/2])/(b^(3/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\left(2\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{b}} - \frac{\left(2\sqrt{a}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right.}{\sqrt{b}}$$

$$= \frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2\sqrt[4]{a}\,\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{c}}{\sqrt[4]{a}\,\sqrt{}}\right.\right.}{b^{3/4}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.01, size = 56, normalized size = 0.24

$$\frac{2x\sqrt{cx}\,\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c*x]/Sqrt[a + b*x^2],x]

[Out] (2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])/(3*Sqrt[a + b*x^2])

Maple [A]
time = 0.04, size = 132, normalized size = 0.56

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+2\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\right)\sqrt{cx}\;a\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\;\sqrt{2}\;\sqrt{\frac{-bx+}{\sqrt{}}}}{\sqrt{b\,x^2+a}\;bx}$
elliptic	$\frac{\sqrt{cx}\;\sqrt{cx\left(b\,x^2+a\right)}\;\sqrt{-ab}\;\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\;\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\;\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\mathrm{Elliptic}}{}\right.}{x\sqrt{b\,x^2+a}\;b\sqrt{bc\,x^3+acx}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+2*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))*(c*x)^(1/2)/(b*x^2+a)^(1/2)*a/b*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/x

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")

```
[Out] integrate(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 27, normalized size = 0.11

$$-\frac{2\,\sqrt{bc}\;\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x))/
b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.43, size = 44, normalized size = 0.19

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*sqrt(a)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.617 $\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

```
[Out] (cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b
^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(
1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {335, 226}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[
2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(1/4)*b^(1/4)*Sqr
t[c]*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.56

$$\frac{2x\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/(S
qrt[c*x]*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.04, size = 104, normalized size = 1.07

method	result
default	$\frac{\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{b\,x^2+a}\,b\sqrt{cx}}$
elliptic	$\frac{\sqrt{cx\,(b\,x^2+a)}\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{\sqrt{cx}\sqrt{b\,x^2+a}\,b\sqrt{bc\,x^3+acx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^2+a)^(1/2)*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1
/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))/b/(c*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 22, normalized size = 0.23

$$\frac{2\sqrt{bc}\,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0, x)/(b*c)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 44, normalized size = 0.45

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}\,\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*s
qrt(a)*sqrt(c)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/2)),x)
[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/2)), x)
```

3.618 $\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=268

$$-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}+\frac{2\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{ac^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}$$

```
[Out] -2*(b*x^2+a)^(1/2)/a/c/(c*x)^(1/2)+2*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/
c^2/(a^(1/2)+x*b^(1/2))-2*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*Elli
pticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1
/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/c^(3/2)/(b*x
^2+a)^(1/2)+b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*ar
ctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/c^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {331, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{ac^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x]) + (2*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2]
)/(a*c^2*(Sqrt[a] + Sqrt[b]*x)) - (2*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a
^(1/4)*Sqrt[c])], 1/2])/(a^(3/4)*c^(3/2)*Sqrt[a + b*x^2]) + (b^(1/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(3/4)*c^(3/2)*Sqrt[a + b
*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{b\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{ac^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{(2b)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{ac^3}$$

$$= -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{\left(2\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{a}\,c^2} - \frac{\left(2\sqrt{b}\right)\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{2\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{ac^2\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)}}}{a^{3/4}c^{3/2}\sqrt{a+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 54, normalized size = 0.20

$$-\frac{2x\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])/
((c*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 196, normalized size = 0.73

method	result
default	$\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}}{\sqrt{b\,x^2+a}\,c\sqrt{cx}\,a}$

risch	$-\frac{2\sqrt{b x^2+a}}{ac\sqrt{cx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(\right.}{-}}\right.}{b}\right.}{a\sqrt{bc x^3+acx}\,c\sqrt{}}$
elliptic	$\frac{\sqrt{cx\left(b x^2+a\right)}\left(-\frac{2\left(c x^2 b+ac\right)}{c^2 a\sqrt{x\left(c x^2 b+ac\right)}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}}{}\right.}{\sqrt{cx}\,\sqrt{b x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
lipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-2*b*x^2-2*a)
/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 51, normalized size = 0.19

$$-\frac{2\left(\sqrt{bc}\,x\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^2+a}\,\sqrt{cx}\right)}{ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(b*c)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x
)) + sqrt(b*x^2 + a)*sqrt(c*x))/(a*c^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.78, size = 48, normalized size = 0.18

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)
*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c\,x\right)}^{3/2}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(1/2)), x)
```

3.619 $\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=129

$$-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}$$

```
[Out] -2/3*(b*x^2+a)^(1/2)/a/c/(c*x)^(3/2)-1/3*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2
*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/
4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {331, 335, 226}

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/2)) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(5/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{b\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{3ac^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3ac^3}$$

$$= -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right.}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 56, normalized size = 0.43

$$-\frac{2x\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[a + b*x^2]),x]
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])/
(3*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 123, normalized size = 0.95

method	result

default	$-\frac{\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,x+2b\,x^{2}+2a}{3\sqrt{b\,x^{2}+a}\,xa\,c^{2}\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}}{3ax\,c^{2}\sqrt{cx}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{3a\sqrt{bc\,x^{3}+acx}\,c^{2}\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\left(-\frac{2\sqrt{bc\,x^{3}+acx}}{3c^{3}a\,x^{2}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{Ellipt}}{3a\,c^{2}\sqrt{bc\,x^{3}+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(b*x^2+a)^(1/2)/x*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x+2*b*x^2+
2*a)/a/c^2/(c*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 45, normalized size = 0.35

$$-\frac{2\left(\sqrt{bc}\,x^{2}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^{2}+a}\,\sqrt{cx}\right)}{3\,ac^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(b*c)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^2 + a)*sqr
t(c*x))/(a*c^3*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.99, size = 48, normalized size = 0.37

$$\frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a}\, c^{\frac{5}{2}} x^{\frac{3}{2}} \Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)
*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{5/2}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(a + b*x^2)^(1/2)), x)
```

3.620 $\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=306

$$-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}+\frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}}-\frac{6b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5a^2c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}$$

```
[Out] -2/5*(b*x^2+a)^(1/2)/a/c/(c*x)^(5/2)+6/5*b*(b*x^2+a)^(1/2)/a^2/c^3/(c*x)^(1
/2)-6/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/c^4/(a^(1/2)+x*b^(1/2))+6/5
*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)
/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/c^(7/2)/(b*x^2+a)^(1/2)-3/5*b^(5/4)*(
cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+
x*b^(1/2))^2)^(1/2)/a^(7/4)/c^(7/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {331, 335, 311, 226, 1210}

$$-\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}+\frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}-\frac{6b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5a^2c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(5*a*c*(c*x)^(5/2)) + (6*b*Sqrt[a + b*x^2])/(5*a^2*c^3
*Sqrt[c*x]) - (6*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*a^2*c^4*(Sqrt[a] + S
qrt[b]*x)) + (6*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2
])/(5*a^(7/4)*c^(7/2)*Sqrt[a + b*x^2]) - (3*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*a^(7/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} - \frac{(3b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{5ac^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{(3b^2)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5a^2c^4}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{(6b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5a^2c^5}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{(6b^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5a^{3/2}c^4} + \frac{(6\ldots}{\ldots}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{6b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5a^2c^4\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.18

$$-\frac{2x\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])
/(5*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.06, size = 219, normalized size = 0.72

method	result

default	$-\frac{6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^{2}-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\cdots}{5x^{2}\sqrt{b\,x^{2}+a}\,c^{3}\sqrt{\cdots}}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-3b\,x^{2}+a\right)}{5a^{2}x^{2}c^{3}\sqrt{cx}}-\frac{3b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{-ab}\cdots}{\cdots}\cdots\right)}{5a^{2}\sqrt{bc\,x\cdots}}$
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\,\left(-\frac{2\sqrt{bc\,x^{3}+acx}}{5c^{4}a\,x^{3}}+\frac{6\left(c\,x^{2}b+ac\right)b}{5a^{2}c^{4}\sqrt{x\left(c\,x^{2}b+ac\right)}}-\frac{3b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{\cdots}}{\cdots}\right)\cdots}{\sqrt{-\cdots}}}\cdots}{\cdots}\right)}{\sqrt{c\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-3*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))*a*b*x^2-6*b^2*x^4-4*a*b*x^2+2*a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/a^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 65, normalized size = 0.21

$$\frac{2\left(3\sqrt{bc}\,bx^3\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+(3\,bx^2-a)\sqrt{bx^2+a}\,\sqrt{cx}\right)}{5\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(b*c)*b*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/
b, 0, x)) + (3*b*x^2 - a)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*c^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.51, size = 51, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a
)*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{7/2}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(1/2)),x)
[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(1/2)), x)
```

3.621 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2}-\frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}}$$

```
[Out] -c*(c*x)^(5/2)/b/(b*x^2+a)^(1/2)+5/3*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^2-5/
6*a^(3/4)*c^(7/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arct
an(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*(
(b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {294, 327, 335, 226}

$$-\frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((c*(c*x)^(5/2))/(b*Sqrt[a + b*x^2])) + (5*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/
(3*b^2) - (5*a^(3/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(6*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx &= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{(5c^2)\int\frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2}-\frac{(5ac^4)\int\frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{6b^2}\\
&= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2}-\frac{(5ac^3)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3b^2}\\
&= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2}-\frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F}{6b^{9/4}\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 74, normalized size = 0.48

$$\frac{c^3\sqrt{cx}\left(5a+2bx^2-5a\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (c^3*Sqrt[c*x]*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b^2*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.09, size = 128, normalized size = 0.84

method	result
default	$-\frac{c^{3}\sqrt{cx}\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a-4\right.}{6x\sqrt{b\,x^{2}+a}\,b^{3}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(b\,x^{2}+a\right)}\left(\frac{c^{4}xa}{b^{2}\sqrt{\left(x^{2}+\frac{a}{b}\right)bcx}}+\frac{2c^{3}\sqrt{bc\,x^{3}+acx}}{3b^{2}}-\frac{5a\,c^{4}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\right.}{}}}{}\right.}{cx\sqrt{b\,x^{2}+a}}$
risch	$\frac{2x\sqrt{b\,x^{2}+a}\,c^{4}}{3b^{2}\sqrt{cx}}-\frac{a\left(\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-a}}}\right.}{b\sqrt{bc\,x^{3}+acx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c^3/x*(c*x)^(1/2)*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a-4*b^2*
x^3-10*a*b*x)/(b*x^2+a)^(1/2)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 86, normalized size = 0.56

$$-\frac{5\,(abc^3x^2+a^2c^3)\sqrt{bc}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(2\,b^2c^3x^2+5\,abc^3)\sqrt{bx^2+a}\,\sqrt{cx}}{3\,(b^4x^2+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(5*(a*b*c^3*x^2 + a^2*c^3)*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0, x)
 - (2*b^2*c^3*x^2 + 5*a*b*c^3)*sqrt(b*x^2 + a)*sqrt(c*x))/(b^4*x^2 + a*b^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.04, size = 44, normalized size = 0.29

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/2)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^{7/2}}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)/(a + b*x^2)^(3/2), x)
```

3.622 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{3c^2\sqrt{cx}\,\sqrt{a+bx^2}}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{3\sqrt[4]{a}\,c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\right)}{b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] -c*(c*x)^(3/2)/b/(b*x^2+a)^(1/2)+3*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/
(a^(1/2)+x*b^(1/2))-3*a^(1/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*
EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(
a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a
)^(1/2)+3/2*a^(1/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/
2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(
sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*
b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {294, 335, 311, 226, 1210}

$$\frac{3\sqrt[4]{a}\,c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{a}\,c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{a+bx^2}}+\frac{3c^2\sqrt{cx}\,\sqrt{a+bx^2}}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((c*(c*x)^(3/2))/(b*Sqrt[a + b*x^2])) + (3*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/
(b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) - (3*a^(1/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[
c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(7/4)*Sqrt[a + b*x^2]) + (3*a^(1/4)*c^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(7/4)*Sqrt[a
 + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}} + \frac{(3c^2)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{2b}$$

$$= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}} + \frac{(3c)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{b}$$

$$= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}} + \frac{\left(3\sqrt{a}\,c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{b^{3/2}} - \frac{\left(3\sqrt{a}\,c^2\right)\text{Subst}\left(\int -\right.}{b}$$

$$= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}} + \frac{3c^2\sqrt{cx}\,\sqrt{a+bx^2}}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{3\sqrt[4]{a}\,c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E(}{b^{7/4}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 60, normalized size = 0.23

$$-\frac{2c(cx)^{3/2}\left(-1+\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(3/2),x]
[Out] (-2*c*(c*x)^(3/2)*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4
, -((b*x^2)/a)]))/(b*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.07, size = 197, normalized size = 0.74

method	result
default	$\frac{c^2\sqrt{cx}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-3\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-a}}}\right.}{2x\sqrt{b\,x^2+a}\;b^2}$

elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(-\frac{c^3x^2}{b\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{3c^3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\cdots}}{\cdots}\right.}{cx\sqrt{b\,x^2\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c^2/x*(c*x)^(1/2)*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-3*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*a-2*b*x^2)/(b*x^2+a)^(1/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 76, normalized size = 0.29

$$-\frac{\sqrt{bx^2+a}\,\sqrt{cx}\,bc^2x+3\,(bc^2x^2+ac^2)\sqrt{bc}\;\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{b^3x^2+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*x^2 + a)*sqrt(c*x)*b*c^2*x + 3*(b*c^2*x^2 + a*c^2)*sqrt(b*c)*weier
strassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)))/(b^3*x^2 + a*b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.08, size = 44, normalized size = 0.17

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/2)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x\right)}^{5/2}}{{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(3/2), x)
```

3.623 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}} \, dx$

Optimal. Leaf size=125

$$-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}+\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{a+bx^2}}$$

```
[Out] -c*(c*x)^(1/2)/b/(b*x^2+a)^(1/2)+1/2*c^(3/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(
1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(
1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b
^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {294, 335, 226}

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((c*Sqrt[c*x])/(b*Sqrt[a + b*x^2])) + (c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])
/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}+\frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{2b}$$

$$= -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}+\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{b}$$

$$= -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}+\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 59, normalized size = 0.47

$$\frac{c\sqrt{cx}\left(-1+\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c*x]*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((
b*x^2)/a)]))/(b*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.04, size = 115, normalized size = 0.92

method	result
default	$\frac{c\sqrt{cx}\left(\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-2bx\right)}{2x\sqrt{b\,x^2+a}\,b^2}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(b\,x^2+a\right)}\left(-\frac{c^2x}{b\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{c^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{}{\sqrt{}}}}{2b^2\sqrt{bc\,x^3+acx}}\right.}{cx\sqrt{b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c/x*(c*x)^(1/2)*((-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2
^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*E
llipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-2*b*x)/(b*x^2
+a)^(1/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 61, normalized size = 0.49

$$-\frac{\sqrt{bx^2+a}\,\sqrt{cx}\,bc-(bcx^2+ac)\sqrt{bc}\,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)}{b^3x^2+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*x^2 + a)*sqrt(c*x)*b*c - (b*c*x^2 + a*c)*sqrt(b*c)*weierstrassPInv
erse(-4*a/b, 0, x))/(b^3*x^2 + a*b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.11, size = 44, normalized size = 0.35

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(3/2)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(3/2)/(a + b*x^2)^(3/2), x)
```

3.624 $\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=266

$$\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\sqrt{cx}\,\sqrt{a+bx^2}}{a\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] (c*x)^(3/2)/a/c/(b*x^2+a)^(1/2)-(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/b^(1/2)/(a^(1
/2)+x*b^(1/2))+(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1
/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^2+a)^(1/2)
-1/2*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c
*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^
2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {296, 335, 311, 226, 1210}

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}}+\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\sqrt{cx}\,\sqrt{a+bx^2}}{a\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(3/2),x]
```

```
[Out] (c*x)^(3/2)/(a*c*Sqrt[a + b*x^2]) - (Sqrt[c*x]*Sqrt[a + b*x^2])/(a*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x)) + (Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt
[c])], 1/2])/(a^(3/4)*b^(3/4)*Sqrt[a + b*x^2]) - (Sqrt[c]*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx = \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{ac}$$

$$= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{a}\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{c\right.}}{\sqrt{a}\sqrt{b}}$$

$$= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\right.}{a^{3/4}b^{3/4}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 59, normalized size = 0.22

$$\frac{2x\sqrt{cx}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(3/2),x]
[Out] (2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^
2)/a)])/(3*a*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 197, normalized size = 0.74

method	result
default	$-\frac{\sqrt{cx}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}\right.}{2\sqrt{b\,x^2+a}\,bxa}$

elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(\frac{c\,x^2}{a\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{c\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-a\cdots}}}\cdots}{\cdots}\right.}{cx\sqrt{b\,x^2+a\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x)^(1/2)*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))*a-2*b*x^2)/(b*x^2+a)^(1/2)/b/x/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 65, normalized size = 0.24

$$\frac{\sqrt{bx^2+a}\,\sqrt{cx}\,bx+(bx^2+a)\sqrt{bc}\ \mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{ab^2x^2+a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^2 + a)*sqrt(c*x)*b*x + (b*x^2 + a)*sqrt(b*c)*weierstrassZeta(-4*a
/b, 0, weierstrassPInverse(-4*a/b, 0, x)))/(a*b^2*x^2 + a^2*b)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 44, normalized size = 0.17

$$\frac{\sqrt{c}\, x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(3/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)/(a + b*x^2)^(3/2), x)
```

3.625 $\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}+\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}$$

```
[Out] (c*x)^(1/2)/a/c/(b*x^2+a)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/
4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*El
lipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^
(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/b^(1/4)/c^
(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {296, 335, 226}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c*x]/(a*c*Sqrt[a + b*x^2]) + ((Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(2*a^(5/4)*b^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{cx}}{ac\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{\sqrt{cx}}{ac\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{ac}$$

$$= \frac{\sqrt{cx}}{ac\sqrt{a+bx^2}} + \frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 59, normalized size = 0.47

$$\frac{x+x\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{a\sqrt{cx}\,\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(3/2)),x]
[Out] (x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/
(a*Sqrt[c*x]*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.05, size = 114, normalized size = 0.90

method	result

default	$\frac{\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+2bx}{2\sqrt{b\,x^{2}+a}\,ba\sqrt{cx}}$
elliptic	$\frac{\sqrt{cx\,(b\,x^{2}+a)}\left(\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bcx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{Ellip}}{2ab\sqrt{bc\,x^{3}+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+2*b*x)/(b*x^2+a)^(1/2)/b/a/(c
*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 58, normalized size = 0.46

$$\frac{(bx^2+a)\sqrt{bc}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^2+a}\,\sqrt{cx}\,b}{ab^2cx^2+a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b*x^2 + a)*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^2 + a)*
sqrt(c*x)*b)/(a*b^2*c*x^2 + a^2*b*c)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.84, size = 44, normalized size = 0.35

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(3/2)*sqrt(c)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(3/2)), x)
```

3.626 $\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=296

$$\frac{1}{ac\sqrt{cx}\,\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}}+\frac{3\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{a^2c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-}\right.}{a^{7/4}c^{3/2}\sqrt{a+bx^2}}$$

```
[Out] 1/a/c/(c*x)^(1/2)/(b*x^2+a)^(1/2)-3*(b*x^2+a)^(1/2)/a^2/c/(c*x)^(1/2)+3*b^(
1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/c^2/(a^(1/2)+x*b^(1/2))-3*b^(1/4)*(cos
(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)
/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b
^(1/2))^2)^(1/2)/a^(7/4)/c^(3/2)/(b*x^2+a)^(1/2)+3/2*b^(1/4)*(cos(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(
1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)
^(1/2)/a^(7/4)/c^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {296, 331, 335, 311, 226, 1210}

$$\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{7/4}c^{3/2}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{7/4}c^{3/2}\sqrt{a+bx^2}}+\frac{3\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{a^2c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}}+\frac{1}{ac\sqrt{cx}\,\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*c*Sqrt[c*x]*Sqrt[a + b*x^2]) - (3*Sqrt[a + b*x^2])/(a^2*c*Sqrt[c*x]) +
 (3*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2])/(a^2*c^2*(Sqrt[a] + Sqrt[b]*x)) - (3
*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(7/4)*c^(
3/2)*Sqrt[a + b*x^2]) + (3*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(2*a^(7/4)*c^(3/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/2}}\,dx = \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} + \frac{3\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{(3b)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{2a^2c^2}$$

$$= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{(3b)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{a^2c^3}$$

$$= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{\left(3\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{a^{3/2}c^2}$$

$$= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 57, normalized size = 0.19

$$-\frac{2x\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{a(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])/
(a*(c*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.08, size = 197, normalized size = 0.67

method	result

default	$\frac{6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{}}{2\sqrt{b\,x^2+a}\,c\sqrt{cx}\,a^2}$
elliptic	$\frac{\sqrt{cx\,(b\,x^2+a)}\left(-\frac{b\,x^2}{c\,a^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{2\left(c\,x^2b+ac\right)}{a^2c^2\sqrt{x\,(c\,x^2b+ac)}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\right.}{\sqrt{}}}}{}\right.}{\sqrt{c}}$

risch	$-\frac{2\sqrt{b\,x^2+a}}{a^2c\sqrt{cx}}+\frac{b^2\left(\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\ldots}\right.}{\ldots}\right.}{b^2\sqrt{bc\,x^3+acx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-6*b*x^
2-4*a)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 83, normalized size = 0.28

$$-\frac{3\,(bx^3+ax)\sqrt{bc}\ \mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(3\,bx^2+2\,a)\sqrt{bx^2+a}\ \sqrt{cx}}{a^2bc^2x^3+a^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] -(3*(b*x^3 + a*x)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(
-4*a/b, 0, x)) + (3*b*x^2 + 2*a)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*b*c^2*x^3
+ a^3*c^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.61, size = 48, normalized size = 0.16

$$\frac{\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/2),x)
[Out] gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{3/2}\,(b\,x^2+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x)
[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(3/2)), x)
```

3.627 $\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}}$$

```
[Out] 1/a/c/(c*x)^(3/2)/(b*x^2+a)^(1/2)-5/3*(b*x^2+a)^(1/2)/a^2/c/(c*x)^(3/2)-5/6
*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)
/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {296, 331, 335, 226}

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*c*(c*x)^(3/2)*Sqrt[a + b*x^2]) - (5*Sqrt[a + b*x^2])/(3*a^2*c*(c*x)^(3
/2)) - (5*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(6*
a^(9/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}+\frac{5\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx}{2a}\\ &= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}-\frac{(5b)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{6a^2c^2}\\ &= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}-\frac{(5b)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3a^2c^3}\\ &= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F}{6a^{9/4}c^{5/2}\sqrt{a+bx}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 59, normalized size = 0.38

$$-\frac{2x\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3a(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/
(3*a*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.08, size = 124, normalized size = 0.81

method	result
default	$-\frac{5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,x+10b\,x^2+4a}{6x\sqrt{b\,x^2+a}\,a^2c^2\sqrt{cx}}$
elliptic	$\frac{\sqrt{cx\,(b\,x^2+a)}\left(-\frac{bx}{c^2a^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{2\sqrt{bc\,x^3+acx}}{3a^2c^3x^2}-\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}}{6a^2c^2}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$
risch	$-\frac{2\sqrt{b\,x^2+a}}{3a^2x\,c^2\sqrt{cx}}-\frac{b\left(\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b\sqrt{bc\,x^3+acx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x+10*b*x^2+4*a)/(b*x^2+a
)^(1/2)/a^2/c^2/(c*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 79, normalized size = 0.51

$$-\frac{5\,(bx^4+ax^2)\sqrt{bc}\;\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(5\,bx^2+2\,a)\sqrt{bx^2+a}\,\sqrt{cx}}{3\,(a^2bc^3x^4+a^3c^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(5*(b*x^4 + a*x^2)*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0, x) + (5*b*
x^2 + 2*a)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*b*c^3*x^4 + a^3*c^3*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.47, size = 48, normalized size = 0.31

$$\frac{\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{5/2}\,{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(a + b*x^2)^(3/2)), x)
```

3.628 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=331

$$\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}}+\frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}}-\frac{21b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5a^3c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a}{\left(\sqrt{a}\right.}}}{5a^{11/4}}$$

```
[Out] 1/a/c/(c*x)^(5/2)/(b*x^2+a)^(1/2)-7/5*(b*x^2+a)^(1/2)/a^2/c/(c*x)^(5/2)+21/
5*b*(b*x^2+a)^(1/2)/a^3/c^3/(c*x)^(1/2)-21/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^
(1/2)/a^3/c^4/(a^(1/2)+x*b^(1/2))+21/5*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4
)/c^(7/2)/(b*x^2+a)^(1/2)-21/10*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a
^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))
)*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))
*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/c^(7/
2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {296, 331, 335, 311, 226, 1210}

$$-\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{10a^{11/4}c^{7/2}\sqrt{a+bx^2}}+\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{11/4}c^{7/2}\sqrt{a+bx^2}}-\frac{21b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5a^3c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}}-\frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*c*(c*x)^(5/2)*Sqrt[a + b*x^2]) - (7*Sqrt[a + b*x^2])/(5*a^2*c*(c*x)^(5
/2)) + (21*b*Sqrt[a + b*x^2])/(5*a^3*c^3*Sqrt[c*x]) - (21*b^(3/2)*Sqrt[c*x]
*Sqrt[a + b*x^2])/(5*a^3*c^4*(Sqrt[a] + Sqrt[b]*x)) + (21*b^(5/4)*(Sqrt[a]
+ Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(
b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*a^(11/4)*c^(7/2)*Sqrt[a + b
*x^2]) - (21*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/
(10*a^(11/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} + \frac{7\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} - \frac{(21b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{10a^2c^2}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{(21b^2)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{10a^3c^4}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{(21b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\cdots}}\cdots\right)}{5a^3c^5\cdots}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{\left(21b^{3/2}\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt{a\cdots}}\cdots\right)}{5a^{5/2}\cdots}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{21b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5a^3c^4\left(\sqrt{a}+\sqrt{b}\,x\right)} \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 59, normalized size = 0.18

$$-\frac{2x\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a(cx)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)])
/(5*a*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.08, size = 219, normalized size = 0.66

method	result
default	$-\frac{42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^{2}-21\sqrt{\frac{bx+\sqrt{-a}}{\sqrt{-ab}}}}{10x^{2}\sqrt{b\,x^{2}+a}\,c^{3}}$
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\left(\frac{b^{2}x^{2}}{c^{3}a^{3}\sqrt{\left(x^{2}+\frac{a}{b}\right)bcx}}-\frac{2\sqrt{bc\,x^{3}+acx}}{5a^{2}c^{4}x^{3}}+\frac{16\left(c\,x^{2}b+ac\right)b}{5a^{3}c^{4}\sqrt{x\left(c\,x^{2}b+ac\right)}}-\frac{21b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-}}{}\right)}{\sqrt{-}}}}{}\right)}{}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-8b\,x^{2}+a\right)}{5a^{3}x^{2}c^{3}\sqrt{cx}}-\frac{b^{2}\left(\frac{8\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}}{}\right)}{b\sqrt{bc}}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/x^2*(42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-21*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*a*b*x^2-42*b^2*x^4-28*a*b*x^2+4*a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/
a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 101, normalized size = 0.31

$$\frac{21\left(b^2x^5+abx^3\right)\sqrt{bc}\ \text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(21\,b^2x^4+14\,abx^2-2\,a^2\right)\sqrt{bx^2+a}\,\sqrt{cx}}{5\left(a^3bc^4x^5+a^4c^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(21*(b^2*x^5 + a*b*x^3)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, weierstras
sPInverse(-4*a/b, 0, x)) + (21*b^2*x^4 + 14*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a
)*sqrt(c*x))/(a^3*b*c^4*x^5 + a^4*c^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.71, size = 51, normalized size = 0.15

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},\frac{3}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/
2)*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{7/2}\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(3/2)), x)
```

3.629 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}}-\frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}}+\frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12\sqrt[4]{a}\,b^{9/4}\sqrt{a+bx^2}}$$

```
[Out] -1/3*c*(c*x)^(5/2)/b/(b*x^2+a)^(3/2)-5/6*c^3*(c*x)^(1/2)/b^2/(b*x^2+a)^(1/2
)+5/12*c^(7/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*
x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {294, 335, 226}

$$\frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12\sqrt[4]{a}\,b^{9/4}\sqrt{a+bx^2}}-\frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}}-\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(c*(c*x)^(5/2))/(b*(a + b*x^2)^(3/2)) - (5*c^3*Sqrt[c*x])/(6*b^2*Sqrt[
a + b*x^2]) + (5*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/
2])/(12*a^(1/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx &= -\frac{c(cx)^{5/2}}{3b\,(a+bx^2)^{3/2}}+\frac{(5c^2)\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{6b}\\
&= -\frac{c(cx)^{5/2}}{3b\,(a+bx^2)^{3/2}}-\frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}}+\frac{(5c^4)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{12b^2}\\
&= -\frac{c(cx)^{5/2}}{3b\,(a+bx^2)^{3/2}}-\frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}}+\frac{(5c^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{6b^2}\\
&= -\frac{c(cx)^{5/2}}{3b\,(a+bx^2)^{3/2}}-\frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}}+\frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-}\right.}{12\sqrt[4]{a}\,b^{9/4}\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 80, normalized size = 0.52

$$\frac{c^3\sqrt{cx}\left(-5a-7bx^2+5(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{6b^2\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (c^3*Sqrt[c*x]*(-5*a - 7*b*x^2 + 5*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.08, size = 219, normalized size = 1.41

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(\frac{a\,c^3\sqrt{bc\,x^3+acx}}{3b^4\left(x^2+\frac{a}{b}\right)^2}-\frac{7c^4x}{6b^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{5c^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(\right.}{}}}{}\right.}{cx\sqrt{b\,x^2+a}}$
default	$\frac{\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^2+5\sqrt{\frac{bx+}{}}\right.}{12x\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b*x^2+5*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*(-a*b)^(1/2)*a-14*b^2*x^3-10*a*b*x)*c^3/x*(c*x)^(1/2)/b^3/(b*x^2+
a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 108, normalized size = 0.70

$$\frac{5\,(b^2c^3x^4+2\,abc^3x^2+a^2c^3)\sqrt{bc}\,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(7\,b^2c^3x^2+5\,abc^3)\sqrt{bx^2+a}\,\sqrt{cx}}{6\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(5*(b^2*c^3*x^4 + 2*a*b*c^3*x^2 + a^2*c^3)*sqrt(b*c)*weierstrassPInvers
e(-4*a/b, 0, x) - (7*b^2*c^3*x^2 + 5*a*b*c^3)*sqrt(b*x^2 + a)*sqrt(c*x))/(b
^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.84, size = 44, normalized size = 0.28

$$\frac{c^{\frac{7}{2}} x^{\frac{9}{2}} \Gamma\left(\frac{9}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{9}{4}, \frac{5}{2} \\ \frac{13}{4} \end{matrix}\middle| \frac{b x^{2} e^{i \pi}}{a}\right)}{2 a^{\frac{5}{2}} \Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((9/4, 5/2), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/2)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/2}}{(b\,x^2+a)^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c*x)^(7/2)/(a + b*x^2)^(5/2), x)
```

3.630 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=304

$$-\frac{c(cx)^{3/2}}{3b\,(a+bx^2)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c^2\sqrt{cx}\,\sqrt{a+bx^2}}{2ab^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\Big(2\tan}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] -1/3*c*(c*x)^(3/2)/b/(b*x^2+a)^(3/2)+1/2*c*(c*x)^(3/2)/a/b/(b*x^2+a)^(1/2)-
1/2*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/b^(3/2)/(a^(1/2)+x*b^(1/2))+1/2*c^(5/
2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1
/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(7/4)/(b*x^2+a)^(1/2)-1/4*c^(5/2)*(cos(2*
arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1
/2))^2)^(1/2)/a^(3/4)/b^(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {294, 296, 335, 311, 226, 1210}

$$-\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}-\frac{c^2\sqrt{cx}\,\sqrt{a+bx^2}}{2ab^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c(cx)^{3/2}}{3b\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(c*(c*x)^(3/2))/(b*(a + b*x^2)^(3/2)) + (c*(c*x)^(3/2))/(2*a*b*Sqrt[a
+ b*x^2]) - (c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(2*a*b^(3/2)*(Sqrt[a] + Sqrt[b]
*x)) + (c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^
(3/4)*b^(7/4)*Sqrt[a + b*x^2]) - (c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b
*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1
/4)*Sqrt[c])], 1/2])/(4*a^(3/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{c(cx)^{3/2}}{3b\,(a+bx^2)^{3/2}} + \frac{c^2\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx}{2b}$$

$$= -\frac{c(cx)^{3/2}}{3b\,(a+bx^2)^{3/2}} + \frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}} - \frac{c^2\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{4ab}$$

$$= -\frac{c(cx)^{3/2}}{3b\,(a+bx^2)^{3/2}} + \frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}} - \frac{c\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2ab}$$

$$= -\frac{c(cx)^{3/2}}{3b\,(a+bx^2)^{3/2}} + \frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}} - \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2\sqrt{a}\,b^{3/2}} + \frac{c^2\text{Subst}}{}$$

$$= -\frac{c(cx)^{3/2}}{3b\,(a+bx^2)^{3/2}} + \frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}} - \frac{c^2\sqrt{cx}\,\sqrt{a+bx^2}}{2ab^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\ }}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 74, normalized size = 0.24

$$\frac{2c(cx)^{3/2}\left(-a+(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3ab\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*c*(c*x)^(3/2)*(-a + (a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/
4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a*b*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.08, size = 385, normalized size = 1.27

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(-\frac{c^2x\sqrt{bc\,x^3+acx}}{3b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{c^3x^2}{2ba\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{c^3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x\right.}{}}}{}\right.}{c}$
default	$-\frac{\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-3\sqrt{\frac{bx+\sqrt{-a}}{\sqrt{-ab}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
a*b*x^2+6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2
-6*b^2*x^4-2*a*b*x^2)*c^2/x*(c*x)^(1/2)/b^2/a/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")

[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 118, normalized size = 0.39

$$\frac{3\,(b^2c^2x^4+2\,abc^2x^2+a^2c^2)\sqrt{bc}\;\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+(3\,b^2c^2x^3+abc^2x)\sqrt{bx^2+a}\,\sqrt{cx}}{6\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")

[Out] 1/6*(3*(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (3*b^2*c^2*x^3 + a*b*c^2*x)*sqrt(b*x^2 + a)*sqrt(c*x))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)

Sympy [C] Result contains complex when optimal does not.
time = 4.06, size = 44, normalized size = 0.14

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(5/2)/(b*x**2+a)**(5/2),x)

[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 5/2), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(11/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")

[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(c\,x)}^{5/2}}{{(b\,x^2+a)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(5/2)/(a + b*x^2)^(5/2),x)

[Out] int((c*x)^(5/2)/(a + b*x^2)^(5/2), x)

3.631 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{c\sqrt{cx}}{3b\,(a+bx^2)^{3/2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}}+\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}$$

```
[Out] -1/3*c*(c*x)^(1/2)/b/(b*x^2+a)^(3/2)+1/6*c*(c*x)^(1/2)/a/b/(b*x^2+a)^(1/2)+
1/12*c^(3/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^
2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/b^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {294, 296, 335, 226}

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{3b\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(c*Sqrt[c*x])/(b*(a + b*x^2)^(3/2)) + (c*Sqrt[c*x])/(6*a*b*Sqrt[a + b*
x^2]) + (c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*
x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(12*
a^(5/4)*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}} + \frac{c^2\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx}{6b}$$

$$= -\frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}} + \frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} + \frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{12ab}$$

$$= -\frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}} + \frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{6ab}$$

$$= -\frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}} + \frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} + \frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan\right.}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 79, normalized size = 0.51

$$\frac{c\sqrt{cx}\left(-a+bx^2+(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{6ab(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(5/2),x]

[Out] (c*Sqrt[c*x]*(-a + b*x^2 + (a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F
1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*a*b*(a + b*x^2)^(3/2))

Maple [A]
time = 0.06, size = 218, normalized size = 1.40

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(bx^2+a\right)}\left(-\frac{c\sqrt{bc\,x^3+acx}}{3b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{c^2x}{6ba\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{c^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\cdots\right.}{\sqrt{\cdots}}}\cdots}{\cdots}\right.\cdots}{cx\sqrt{bx^2+a}\cdots}$
default	$\frac{\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^2+\sqrt{\frac{bx+\sqrt{-\cdots}}{\sqrt{-\cdots}}}\cdots\right.}{12xa\,b^2\left(b\,x^2\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(3/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/12*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b*x^2+((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*(-a*b)^(1/2)*a+2*b^2*x^3-2*a*b*x)*c/x*(c*x)^(1/2)/a/b^2/(b*x^2+a)^(3/
2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")

[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 98, normalized size = 0.63

$$\frac{(b^2cx^4 + 2\,abcx^2 + a^2c)\sqrt{bc}\ \text{weierstrassPInverse}\left(-\frac{4\,a}{b}, 0, x\right) + (b^2cx^2 - abc)\sqrt{bx^2 + a}\ \sqrt{cx}}{6\,(ab^4x^4 + 2\,a^2b^3x^2 + a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*sqrt(b*c)*weierstrassPInverse(-4*a/b
, 0, x) + (b^2*c*x^2 - a*b*c)*sqrt(b*x^2 + a)*sqrt(c*x))/(a*b^4*x^4 + 2*a^2
*b^3*x^2 + a^3*b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.10, size = 44, normalized size = 0.28

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{5}{4}, \frac{5}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/2), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(5/2)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c*x)^(3/2)/(a + b*x^2)^(5/2), x)
```

3.632 $\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=302

$$\frac{(cx)^{3/2}}{3ac\,(a+bx^2)^{3/2}}+\frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}}-\frac{\sqrt{cx}\,\sqrt{a+bx^2}}{2a^2\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\Big(2\tan}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] 1/3*(c*x)^(3/2)/a/c/(b*x^2+a)^(3/2)+1/2*(c*x)^(3/2)/a^2/c/(b*x^2+a)^(1/2)-1
/2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/b^(1/2)/(a^(1/2)+x*b^(1/2))+1/2*(cos(2*a
rctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/a^(7/4)/b^(3/4)/(b*x^2+a)^(1/2)-1/4*(cos(2*arctan(b^(1
/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)
/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/
2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/a^(7/4)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {296, 335, 311, 226, 1210}

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}}+\frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}}-\frac{\sqrt{cx}\,\sqrt{a+bx^2}}{2a^2\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{(cx)^{3/2}}{3ac\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(5/2),x]
```

```
[Out] (c*x)^(3/2)/(3*a*c*(a + b*x^2)^(3/2)) + (c*x)^(3/2)/(2*a^2*c*Sqrt[a + b*x^2
]) - (Sqrt[c*x]*Sqrt[a + b*x^2])/(2*a^2*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) + (S
qrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(7/4)*b^(
3/4)*Sqrt[a + b*x^2]) - (Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[
c])], 1/2])/(4*a^(7/4)*b^(3/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx = \frac{(cx)^{3/2}}{3ac(a+bx^2)^{3/2}} + \frac{\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx}{2a}$$

$$= \frac{(cx)^{3/2}}{3ac(a+bx^2)^{3/2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{4a^2}$$

$$= \frac{(cx)^{3/2}}{3ac(a+bx^2)^{3/2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2a^2c}$$

$$= \frac{(cx)^{3/2}}{3ac(a+bx^2)^{3/2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2a^{3/2}\sqrt{b}} + \frac{\text{Subst}\left(\int -\right.}{}$$

$$= \frac{(cx)^{3/2}}{3ac(a+bx^2)^{3/2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{2a^2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\cdots}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 59, normalized size = 0.20

$$\frac{2x\sqrt{cx}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, -((b*x^
2)/a)])/(3*a^2*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.06, size = 382, normalized size = 1.26

method	result

elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\,(b\,x^2+a)}\left(\frac{x\sqrt{bc\,x^3+acx}}{3a\,b^2\left(x^2+\frac{a}{b}\right)^2}+\frac{c\,x^2}{2a^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{c\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{\ldots}}{\ldots}\right)}{\sqrt{\ldots}}}\ldots}{\ldots}\right.}{cx\ldots}$
default	$-\frac{\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-3\sqrt{\frac{bx+\sqrt{-\ldots}}{\sqrt{-ab}}}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
a*b*x^2+6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2
-6*b^2*x^4-10*a*b*x^2)*(c*x)^(1/2)/b/a^2/x/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 103, normalized size = 0.34

$$\frac{3\,(b^2x^4+2\,abx^2+a^2)\sqrt{bc}\,\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(3\,b^2x^3+5\,abx)\sqrt{bx^2+a}\,\sqrt{cx}}{6\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")

[Out] 1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (3*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)

Sympy [C] Result contains complex when optimal does not.
time = 1.49, size = 44, normalized size = 0.15

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(1/2)/(b*x**2+a)**(5/2),x)

[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(7/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/2)/(a + b*x^2)^(5/2),x)

[Out] int((c*x)^(1/2)/(a + b*x^2)^(5/2), x)

3.633 $\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{\sqrt{cx}}{3ac\,(a+bx^2)^{3/2}}+\frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}}+\frac{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}$$

```
[Out] 1/3*(c*x)^(1/2)/a/c/(b*x^2+a)^(3/2)+5/6*(c*x)^(1/2)/a^2/c/(b*x^2+a)^(1/2)+5
/12*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*
x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(
1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/b^(1/4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {296, 335, 226}

$$\frac{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{3ac\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] Sqrt[c*x]/(3*a*c*(a + b*x^2)^(3/2)) + (5*Sqrt[c*x])/(6*a^2*c*Sqrt[a + b*x^2
]) + (5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(12*a^(9/4)*b
^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx &= \frac{\sqrt{cx}}{3ac\,(a+bx^2)^{3/2}}+\frac{5\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx}{6a}\\ &= \frac{\sqrt{cx}}{3ac\,(a+bx^2)^{3/2}}+\frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}}+\frac{5\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{12a^2}\\ &= \frac{\sqrt{cx}}{3ac\,(a+bx^2)^{3/2}}+\frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}}+\frac{5\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{6a^2c}\\ &= \frac{\sqrt{cx}}{3ac\,(a+bx^2)^{3/2}}+\frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}}+\frac{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\,\mathrm{ta}\right.}{12a^{9/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 79, normalized size = 0.50

$$\frac{7ax+5bx^3+5x(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{6a^2\sqrt{cx}\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] (7*a*x + 5*b*x^3 + 5*x*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/
4, 1/2, 5/4, -((b*x^2)/a)])/(6*a^2*Sqrt[c*x]*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.08, size = 216, normalized size = 1.38

method	result
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\,\left(\frac{\sqrt{bc\,x^{3}+acx}}{3ac\,b^{2}\left(x^{2}+\frac{a}{b}\right)^{2}}+\frac{5x}{6a^{2}\sqrt{\left(x^{2}+\frac{a}{b}\right)bcx}}+\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}}{12a^{2}b\sqrt{b}}\right.}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$
default	$\frac{5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^{2}+5\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}}{12\sqrt{cx}\,a^{2}b(b\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b*x^2+5*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*(-a*b)^(1/2)*a+10*b^2*x^3+14*a*b*x)/(c*x)^(1/2)/a^2/b/(b*x^2+a)^(
3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 97, normalized size = 0.62

$$\frac{5\,(b^2x^4+2\,abx^2+a^2)\sqrt{bc}\ \mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+(5\,b^2x^2+7\,ab)\sqrt{bx^2+a}\,\sqrt{cx}}{6\,(a^2b^3cx^4+2\,a^3b^2cx^2+a^4bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

[Out] 1/6*(5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0, x) + (5*b^2*x^2 + 7*a*b)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*b^3*c*x^4 + 2*a^3*b^2*c*x^2 + a^4*b*c)

Sympy [C] Result contains complex when optimal does not.
time = 2.39, size = 44, normalized size = 0.28

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/2),x)

[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*sqrt(c)*gamma(5/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(5/2)),x)

[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(5/2)), x)

3.634 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=333

$$\frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}}+\frac{7}{6a^2c\sqrt{cx}\,\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}}+\frac{7\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{2a^3c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{}}{}$$

```
[Out] 1/3/a/c/(b*x^2+a)^(3/2)/(c*x)^(1/2)+7/6/a^2/c/(c*x)^(1/2)/(b*x^2+a)^(1/2)-7
/2*(b*x^2+a)^(1/2)/a^3/c/(c*x)^(1/2)+7/2*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2
)/a^3/c^2/(a^(1/2)+x*b^(1/2))-7/2*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)
/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2
)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2
))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/c^(
3/2)/(b*x^2+a)^(1/2)+7/4*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*Ellip
ticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/
2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/c^(3/2)/(b*x
^2+a)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {296, 331, 335, 311, 226, 1210}

$$\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{11/4}c^{3/2}\sqrt{a+bx^2}}-\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{11/4}c^{3/2}\sqrt{a+bx^2}}+\frac{7\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{2a^3c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}}+\frac{7}{6a^2c\sqrt{cx}\,\sqrt{a+bx^2}}+\frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*c*Sqrt[c*x]*(a + b*x^2)^(3/2)) + 7/(6*a^2*c*Sqrt[c*x]*Sqrt[a + b*x^2
]) - (7*Sqrt[a + b*x^2])/(2*a^3*c*Sqrt[c*x]) + (7*Sqrt[b]*Sqrt[c*x]*Sqrt[a
+ b*x^2])/(2*a^3*c^2*(Sqrt[a] + Sqrt[b]*x)) - (7*b^(1/4)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*S
qrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(11/4)*c^(3/2)*Sqrt[a + b*x^2]) +
(7*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(4*a^(11/4
)*c^(3/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}\,dx = \frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}} + \frac{7\int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/2}}\,dx}{6a}$$

$$= \frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\,\sqrt{a+bx^2}} + \frac{7\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{4a^2}$$

$$= \frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\,\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{(7b)\int \frac{\sqrt{cx}}{\sqrt{a+bx}}}{4a^3c^2}$$

$$= \frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\,\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{(7b)\text{Subst}\left(\int -\right.}{}$$

$$= \frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\,\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{\left(7\sqrt{b}\right)\text{Subst}}{}$$

$$= \frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\,\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{7\sqrt{b}\,\sqrt{cx}\,\sqrt{a-}}{2a^3c^2\left(\sqrt{a}+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.17

$$-\frac{2x\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{a^2(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 5/2, 3/4, -((b*x^2)/a)])/
(a^2*(c*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.11, size = 384, normalized size = 1.15

method	result
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\left(-\frac{x\sqrt{bc\,x^{3}+acx}}{3a^{2}c^{2}b\left(x^{2}+\frac{a}{b}\right)^{2}}-\frac{3b\,x^{2}}{2c\,a^{3}\sqrt{\left(x^{2}+\frac{a}{b}\right)bcx}}-\frac{2\left(c\,x^{2}b+ac\right)}{a^{3}c^{2}\sqrt{x\left(c\,x^{2}b+ac\right)}}+\frac{7\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}}{}\right.}{}$
default	$\frac{42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^{2}-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}}{}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}}{a^{3}c\sqrt{cx}}+\frac{b\left(\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\right.\right.}{}\right.}{b\sqrt{bc\,x^{3}+acx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-21*((b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^
(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
*a*b*x^2+42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-21*((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
a^2-42*b^2*x^4-70*a*b*x^2-24*a^2)/a^3/c/(c*x)^(1/2)/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 119, normalized size = 0.36

$$-\frac{21\,(b^2x^5+2\,abx^3+a^2x)\sqrt{bc}\,\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+(21\,b^2x^4+35\,abx^2+12\,a^2)\sqrt{bx^2+a}\,\sqrt{cx}}{6\,(a^3b^2c^2x^5+2\,a^4bc^2x^3+a^5c^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(21*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0,
 weierstrassPInverse(-4*a/b, 0, x)) + (21*b^2*x^4 + 35*a*b*x^2 + 12*a^2)*sq
rt(b*x^2 + a)*sqrt(c*x))/(a^3*b^2*c^2*x^5 + 2*a^4*b*c^2*x^3 + a^5*c^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.46, size = 48, normalized size = 0.14

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(5/2)), x)
```

3.635 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(\right.}{4a^{13/4}c^{5/2}\sqrt{a+bx^2}}$$

```
[Out] 1/3/a/c/(c*x)^(3/2)/(b*x^2+a)^(3/2)+3/2/a^2/c/(c*x)^(3/2)/(b*x^2+a)^(1/2)-5
/2*(b*x^2+a)^(1/2)/a^3/c/(c*x)^(3/2)-5/4*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2
*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(13
/4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {296, 331, 335, 226}

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{13/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}+\frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*c*(c*x)^(3/2)*(a + b*x^2)^(3/2)) + 3/(2*a^2*c*(c*x)^(3/2)*Sqrt[a + b
*x^2]) - (5*Sqrt[a + b*x^2])/(2*a^3*c*(c*x)^(3/2)) - (5*b^(3/4)*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(4*a^(13/4)*c^(5/2)*Sqrt[a + b*x
^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/2}}\,dx = \frac{1}{3ac(cx)^{3/2}(a+bx^2)^{3/2}} + \frac{3\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}}\,dx}{2a}$$

$$= \frac{1}{3ac(cx)^{3/2}(a+bx^2)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} + \frac{15\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx}{4a^2}$$

$$= \frac{1}{3ac(cx)^{3/2}(a+bx^2)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}} - \frac{(5b)\int \frac{}{\sqrt{cx}\,\sqrt{\cdots}}}{4a^3\cdots}$$

$$= \frac{1}{3ac(cx)^{3/2}(a+bx^2)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}} - \frac{(5b)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{1}{3ac(cx)^{3/2}(a+bx^2)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}} - \frac{5b^{3/4}\left(\sqrt{a}+\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 59, normalized size = 0.32

$$-\frac{2x\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(-\frac{3}{4},\frac{5}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^2(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 5/2, 1/4, -((b*x^2)/a)])/
(3*a^2*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.12, size = 227, normalized size = 1.23

method	result
elliptic	$\frac{\sqrt{cx\,(b\,x^2+a)}\left(-\frac{\sqrt{bc\,x^3+acx}}{3a^2c^3b\left(x^2+\frac{a}{b}\right)^2}-\frac{11bx}{6c^2a^3\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{2\sqrt{bc\,x^3+acx}}{3a^3c^3x^2}-\frac{5\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}}{}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$
default	$-\frac{15\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^3+15\sqrt{\frac{b}{}}}{12x\,c^2\sqrt{C}}$
risch	$-\frac{2\sqrt{b\,x^2+a}}{3a^3x\,c^2\sqrt{cx}}-\frac{b\left(\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right)}{\sqrt{-ab}}}\right.}{b\sqrt{bc\,x^3+acx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(15*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1
```

```
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b*x^3+15*((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*(-a*b)^(1/2)*a*x+30*b^2*x^4+42*a*b*x^2+8*a^2)/x/c^2/(c*x)^(1/2
)/a^3/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 115, normalized size = 0.62

$$-\frac{15\,(b^2x^6+2\,abx^4+a^2x^2)\sqrt{bc}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(15\,b^2x^4+21\,abx^2+4\,a^2)\sqrt{bx^2+a}\ \sqrt{cx}}{6\,(a^3b^2c^3x^6+2\,a^4bc^3x^4+a^5c^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(15*(b^2*x^6 + 2*a*b*x^4 + a^2*x^2)*sqrt(b*c)*weierstrassPInverse(-4*a
/b, 0, x) + (15*b^2*x^4 + 21*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a)*sqrt(c*x))/(a
^3*b^2*c^3*x^6 + 2*a^4*b*c^3*x^4 + a^5*c^3*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.64, size = 48, normalized size = 0.26

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{5/2}\,{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(a + b*x^2)^(5/2)), x)
```

3.636 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=362

$$\frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}+\frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}}-\frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}}+\frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}}-\frac{77b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{10a^4c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{77b^{5/4}\cdots}{\cdots}$$

[Out] 1/3/a/c/(c*x)^(5/2)/(b*x^2+a)^(3/2)+11/6/a^2/c/(c*x)^(5/2)/(b*x^2+a)^(1/2)-77/30*(b*x^2+a)^(1/2)/a^3/c/(c*x)^(5/2)+77/10*b*(b*x^2+a)^(1/2)/a^4/c^3/(c*x)^(1/2)-77/10*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^4/c^4/(a^(1/2)+x*b^(1/2))+77/10*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(15/4)/c^(7/2)/(b*x^2+a)^(1/2)-77/20*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(15/4)/c^(7/2)/(b*x^2+a)^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {296, 331, 335, 311, 226, 1210}

$$-\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{20a^{15/4}c^{7/2}\sqrt{a+bx^2}}+\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{10a^{15/4}c^{7/2}\sqrt{a+bx^2}}-\frac{77b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{10a^4c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}}-\frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}}+\frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}}+\frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x]

[Out] 1/(3*a*c*(c*x)^(5/2)*(a + b*x^2)^(3/2)) + 11/(6*a^2*c*(c*x)^(5/2)*Sqrt[a + b*x^2]) - (77*Sqrt[a + b*x^2])/(30*a^3*c*(c*x)^(5/2)) + (77*b*Sqrt[a + b*x^2])/(10*a^4*c^3*Sqrt[c*x]) - (77*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(10*a^4*c^4*(Sqrt[a] + Sqrt[b]*x)) + (77*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(10*a^(15/4)*c^(7/2)*Sqrt[a + b*x^2]) - (77*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(20*a^(15/4)*c^(7/2)*Sqrt[a + b*x^2])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}}\,dx = \frac{1}{3ac(cx)^{5/2}(a+bx^2)^{3/2}} + \frac{11\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}\,dx}{6a}$$

$$= \frac{1}{3ac(cx)^{5/2}(a+bx^2)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} + \frac{77\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx}{12a^2}$$

$$= \frac{1}{3ac(cx)^{5/2}(a+bx^2)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} - \frac{(77b)\int \frac{}{(cx)^{3/}}}{20}$$

$$= \frac{1}{3ac(cx)^{5/2}(a+bx^2)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}}$$

$$= \frac{1}{3ac(cx)^{5/2}(a+bx^2)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}}$$

$$= \frac{1}{3ac(cx)^{5/2}(a+bx^2)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}}$$

$$= \frac{1}{3ac(cx)^{5/2}(a+bx^2)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 59, normalized size = 0.16

$$-\frac{2x\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^2(cx)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 5/2, -1/4, -((b*x^2)/a)])
/(5*a^2*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A]

time = 0.10, size = 410, normalized size = 1.13

method	result
elliptic	$\frac{\sqrt{cx\,(b\,x^2+a)}\left(\frac{x\sqrt{bc\,x^3+acx}}{3a^3c^4\left(x^2+\frac{a}{b}\right)^2}+\frac{5b^2x^2}{2c^3a^4\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{2\sqrt{bc\,x^3+acx}}{5a^3c^4x^3}+\frac{26\left(c\,x^2b+ac\right)b}{5a^4c^4\sqrt{x\,(c\,x^2b+ac)}}- \cdots\right.}{\cdots}$
default	$-\frac{462\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^2x^4-231\sqrt{\frac{bx+\sqrt{\cdots}}{\sqrt{-\cdots}}}\cdots}{\cdots}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,\left(-13b\,x^2+a\right)}{5a^4x^2c^3\sqrt{cx}}-\frac{b^2\left(\frac{13\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-\cdots}}{\cdots}\cdots\right)}{b\sqrt{\cdots}}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(462*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*x^4-231*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^2*x^4+462*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*x^2-231*((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))*a^2*b*x^2-462*b^3*x^6-770*a*b^2*x^4-264*a^2*b*x^2+24*a^3)/x^
2/a^4/c^3/(c*x)^(1/2)/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 137, normalized size = 0.38

$$\frac{231\left(b^3x^7+2\,ab^2x^5+a^2bx^3\right)\sqrt{bc}\ \mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(231\,b^3x^6+385\,ab^2x^4+132\,a^2bx^2-12\,a^3\right)\sqrt{bx^2+a}\,\sqrt{cx}}{30\left(a^4b^2c^4x^7+2\,a^5bc^4x^5+a^6c^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(231*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(b*c)*weierstrassZeta(-4*
a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (231*b^3*x^6 + 385*a*b^2*x^4 +
 132*a^2*b*x^2 - 12*a^3)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^4*b^2*c^4*x^7 + 2*a^
5*b*c^4*x^5 + a^6*c^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 27.71, size = 51, normalized size = 0.14

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},\frac{5}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/2),x)

[Out] gamma(-5/4)*hyper((-5/4, 5/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*c**(7/2)*x**(5/2)*gamma(-1/4))

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(7/2)), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x)

[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(5/2)), x)

3.637 $\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}-\frac{9\sqrt[4]{3}\,c^2\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\;2^{3/4}\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] -9/10*3^(1/4)*c^2*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^
(1/2)*(-2*x^2+3)^(1/2)*2^(1/4)/x^(1/2)/(-2*a*x^2+3*a)^(1/2)-1/5*c*(c*x)^(3/
2)*(-2*a*x^2+3*a)^(1/2)/a
```

Rubi [A]
time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {327, 326, 325, 324, 435}

$$-\frac{9\sqrt[4]{3}\,c^2\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\;2^{3/4}\sqrt{x}\,\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}\,(cx)^{3/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] -1/5*(c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/a - (9*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[
3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*2^(3/4)*Sq
rt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 324

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-b/a
)^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b/
a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

Rule 325

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b}
, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

Rule 326

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
```

```
-b/a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx &= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}+\frac{1}{10}\left(9c^2\right)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx \\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}+\frac{\left(9c^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{10\sqrt{x}} \\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}+\frac{\left(9c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{10\sqrt{x}\sqrt{3a-2ax^2}} \\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}-\frac{\left(9\left(\frac{3}{2}\right)^{3/4}c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x,\right.}{5\sqrt{x}\sqrt{3a-2ax^2}} \\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}-\frac{9\sqrt[4]{3}\,c^2\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\,2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 61, normalized size = 0.57

$$\frac{c(cx)^{3/2}\left(-3+2x^2+\sqrt{9-6x^2}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)\right)}{5\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*(c*x)^(3/2)*(-3 + 2*x^2 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/2, 3/4, 7/
4, (2*x^2)/3]))/(5*Sqrt[a*(3 - 2*x^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(83) = 166$.
time = 0.09, size = 235, normalized size = 2.20

method	result
elliptic	$\frac{\sqrt{cx}\ \sqrt{-cxa\left(2x^2-3\right)}\left(-\frac{c^2x\sqrt{-2ac\,x^3+3acx}}{5a}+\frac{c^3\sqrt{6}\ \sqrt{3}\ \sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\ \sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right.}}{}\right.}{cx}$

risch	$\frac{x^2(2x^2-3)c^3}{5\sqrt{cx}\sqrt{-a(2x^2-3)}} + \frac{\sqrt{6}\sqrt{3}\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\sqrt{-3x\sqrt{6}}\left(-\sqrt{\ldots}\right.}{60\ldots}$
default	$\frac{c^2\sqrt{cx}\sqrt{-a(2x^2-3)}\left(6\sqrt{(-2x+\sqrt{2}\sqrt{3})\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-x\sqrt{2}\sqrt{3}}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\sqrt{2}\sqrt{\ldots}}{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*c^2/x*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)/a*(6*((-2*x+2^(1/2)*3^(1/2))*2^
(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/
2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/
2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)-3*((-2*x+2^(1/2)*3^(1/2))*
2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(
1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)-16*x^4+24*x^2)/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 50, normalized size = 0.47

$$-\frac{2\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,c^2x-9\,\sqrt{2}\,\sqrt{-ac}\,c^2\mathrm{weierstrassZeta}(6,0,\mathrm{weierstrassPInverse}(6,0,x))}{10\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

[Out] -1/10*(2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x - 9*sqrt(2)*sqrt(-a*c)*c^2*weierstrassZeta(6, 0, weierstrassPInverse(6, 0, x)))/a

Sympy [C] Result contains complex when optimal does not.
time = 3.48, size = 51, normalized size = 0.48

$$\frac{\sqrt{3}\, c^{\frac{5}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{2x^2 e^{2i\pi}}{3}\right)}{6\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(5/2)/(-2*a*x**2+3*a)**(1/2),x)

[Out] sqrt(3)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), 2*x**2*exp_polar(2*I*pi)/3)/(6*sqrt(a)*gamma(11/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")

[Out] integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(5/2)/(3*a - 2*a*x^2)^(1/2),x)

[Out] int((c*x)^(5/2)/(3*a - 2*a*x^2)^(1/2), x)

3.638 $\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=88

$$-\frac{c\sqrt{cx}\,\sqrt{3a-2ax^2}}{3a}+\frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\,\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{6}\,\sqrt{a\left(3-2x^2\right)}}$$

```
[Out] 1/6*c^(3/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3)
^(1/2)*6^(3/4)/(a*(-2*x^2+3))^(1/2)-1/3*c*(c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2)/
a
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {327, 335, 230, 227}

$$\frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{\frac{2}{3}}\,\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{6}\,\sqrt{a\left(3-2x^2\right)}}-\frac{c\sqrt{3a-2ax^2}\,\sqrt{cx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] -1/3*(c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/a + (c^(3/2)*Sqrt[3 - 2*x^2]*Ellipti
cF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(6^(1/4)*Sqrt[a*(3 - 2*x^2
)])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx &= -\frac{c\sqrt{cx}\,\sqrt{3a-2ax^2}}{3a}+\frac{1}{2}c^2\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx\\
&= -\frac{c\sqrt{cx}\,\sqrt{3a-2ax^2}}{3a}+c\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)\\
&= -\frac{c\sqrt{cx}\,\sqrt{3a-2ax^2}}{3a}+\frac{\left(c\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{\sqrt{3}\,\sqrt{a\left(3-2x^2\right)}}\\
&= -\frac{c\sqrt{cx}\,\sqrt{3a-2ax^2}}{3a}+\frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\,\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{6}\,\sqrt{a\left(3-2x^2\right)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 61, normalized size = 0.69

$$\frac{c\sqrt{cx}\left(-3+2x^2+\sqrt{9-6x^2}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)\right)}{3\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*Sqrt[c*x]*(-3 + 2*x^2 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4,
 (2*x^2)/3]))/(3*Sqrt[a*(3 - 2*x^2)])
```

Maple [A]
time = 0.09, size = 131, normalized size = 1.49

method	result
default	$-\frac{c\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(\sqrt{(2x+\sqrt{2}\,\sqrt{3})\sqrt{2}\,\sqrt{3}}\,\sqrt{(-2x+\sqrt{2}\,\sqrt{3})\sqrt{2}\,\sqrt{3}}\,\sqrt{-x\sqrt{2}}\right.}{12xa(2x^2-3)}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa(2x^2-3)}\left(-\frac{c\sqrt{-2acx^3+3acx}}{3a}+\frac{c^2\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right.}}{108\sqrt{-}}\right.}{cx\sqrt{-a(2x^2-3)}}$
risch	$\frac{x(2x^2-3)c^2}{3\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\ \mathrm{Ellipt}}{108\sqrt{-2acx^3+3acx}\,\sqrt{cx}\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*c*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^
(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(
1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^
(1/2))^(1/2),1/2*2^(1/2))+8*x^3-12*x)/x/a/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 42, normalized size = 0.48

$$-\frac{3\sqrt{2}\sqrt{-ac}\,c\mathrm{weierstrassPInverse}(6,0,x)+2\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,c}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(2)*sqrt(-a*c)*c*weierstrassPInverse(6, 0, x) + 2*sqrt(-2*a*x^2
 + 3*a)*sqrt(c*x)*c)/a
```

Sympy [A]
time = 0.89, size = 51, normalized size = 0.58

$$\frac{\sqrt{3}\,c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\,\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), 2*x**2*exp_p
olar(2*I*pi)/3)/(6*sqrt(a)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^{3/2}}{\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(3/2)/(3*a - 2*a*x^2)^(1/2), x)
```

3.639 $\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt[4]{6}\,\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] -6^(1/4)*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*(-2
*x^2+3)^(1/2)/x^(1/2)/(-2*a*x^2+3*a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {326, 325, 324, 435}

$$-\frac{\sqrt[4]{6}\,\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\,\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] -((6^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/S
qrt[6]], 2])/(Sqrt[x]*Sqrt[3*a - 2*a*x^2]))
```

Rule 324

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-b/a
)^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b/
a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

Rule 325

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b}
, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

Rule 326

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
```

```
-b/a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx &= \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{\sqrt{x}} \\
&= \frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{\sqrt{x}\sqrt{3a-2ax^2}} \\
&= -\frac{\left(\sqrt[4]{2}\,3^{3/4}\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{\frac{2}{3}}\,x}}{\sqrt{2}}\right)}{\sqrt{x}\sqrt{3a-2ax^2}} \\
&= -\frac{\sqrt[4]{6}\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 53, normalized size = 0.79

$$\frac{2x\sqrt{cx}\sqrt{3-2x^2}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)}{3\sqrt{a\,(9-6x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[3*a - 2*a*x^2],x]
[Out] (2*x*Sqrt[c*x]*Sqrt[3 - 2*x^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (2*x^2)/3])
/(3*Sqrt[a*(9 - 6*x^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(53) = 106$.
time = 0.06, size = 165, normalized size = 2.46

method	result
elliptic	$\dfrac{\sqrt{cx}\,\sqrt{-cxa\,(2x^2-3)}\,\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\,\Big(-\sqrt{6}}{54x\sqrt{-a\,(2x^2-3)}\,\sqrt{-}}$
default	$\dfrac{\sqrt{cx}\,\sqrt{-a\,(2x^2-3)}\,\sqrt{2}\,\sqrt{(2x+\sqrt{2}\,\sqrt{3})\,\sqrt{2}\,\sqrt{3}}\,\sqrt{(-2x+\sqrt{2}\,\sqrt{3})\,\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x
*2^(1/2)*3^(1/2))^(1/2)*(2*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1
/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-EllipticF(1/6*3^(1/2)*2^(1/2)*((2*
x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2)))/x/a/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 20, normalized size = 0.30

$$\frac{\sqrt{2}\,\sqrt{-ac}\,\mathrm{weierstrassZeta}(6,0,\mathrm{weierstrassPInverse}(6,0,x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(-a*c)*weierstrassZeta(6, 0, weierstrassPInverse(6, 0, x))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.42, size = 51, normalized size = 0.76

$$\frac{\sqrt{3}\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), 2*x**2*exp_po
lar(2*I*pi)/3)/(6*sqrt(a)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(3*a - 2*a*x^2)^(1/2), x)
```

3.640 $\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a(3-2x^2)}}$$

```
[Out] 1/3*2^(3/4)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3)
^(1/2)*3^(3/4)/c^(1/2)/(a*(-2*x^2+3))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {335, 230, 227}

$$\frac{2^{3/4}\sqrt{3-2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]],
 -1])/(3^(1/4)*Sqrt[c]*Sqrt[a*(3 - 2*x^2)])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx = \frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\left(2\sqrt{3-2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{3}\,c\sqrt{a\,(3-2x^2)}}$$

$$= \frac{2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\,\sqrt{c}\,\sqrt{a\,(3-2x^2)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 56, normalized size = 0.89

$$\frac{2x\sqrt{3-2x^2}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)}{\sqrt{3}\,\sqrt{cx}\,\sqrt{a\,(3-2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2)/3])/(Sqrt[3]*
Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(50) = 100.
time = 0.06, size = 117, normalized size = 1.86

method	result

elliptic	$\frac{\sqrt{-cxa\left(2x^{2}-3\right)}\,\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}}{}\right.}{54\sqrt{cx}\,\sqrt{-a\left(2x^{2}-3\right)}\,\sqrt{-2ac\,x^{3}+3acx}}$
default	$-\frac{\sqrt{-a\left(2x^{2}-3\right)}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\,\mathrm{E}}{6\sqrt{cx}\,a\left(2x^{2}-3\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-a*(2*x^2-3))^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-
2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*Elli
pticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2
*2^(1/2))/(c*x)^(1/2)/a/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 21, normalized size = 0.33

$$-\frac{\sqrt{2}\,\sqrt{-ac}\ \mathrm{weierstrassPInverse}(6,0,x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*sqrt(-a*c)*weierstrassPInverse(6, 0, x)/(a*c)
```

Sympy [A]
time = 0.50, size = 51, normalized size = 0.81

$$\frac{\sqrt{3}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{2x^{2}e^{2i\pi}}{3}\right)}{6\sqrt{a}\,\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**2*exp_polar(2*I*p
i)/3)/(6*sqrt(a)*sqrt(c)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x}\,\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2)), x)
```

3.641 $\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}+\frac{2\sqrt[4]{2}\,\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] 2/3*2^(1/4)*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*
(-2*x^2+3)^(1/2)*3^(1/4)/c^2/x^(1/2)/(-2*a*x^2+3*a)^(1/2)-2/3*(-2*a*x^2+3*a
)^(1/2)/a/c/(c*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {331, 326, 325, 324, 435}

$$\frac{2\sqrt[4]{2}\,\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\,\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(3*a*c*Sqrt[c*x]) + (2*2^(1/4)*Sqrt[c*x]*Sqrt[3 -
2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(3^(3/4)*c^2*Sqrt
[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 324

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-b/a
)^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b/
a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

Rule 325

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b}
, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

Rule 326

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
```

```
-b/a, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx &= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}-\frac{2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{3c^2} \\
&= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}-\frac{\left(2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{3c^2\sqrt{x}} \\
&= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}-\frac{\left(2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{3c^2\sqrt{x}\sqrt{3a-2ax^2}} \\
&= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}+\frac{\left(2\sqrt[4]{\frac{2}{3}}\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\cdots}}{\cdots}\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}} \\
&= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}+\frac{2\sqrt[4]{2}\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 51, normalized size = 0.48

$$-\frac{2x\sqrt{3-2x^2}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\frac{2x^2}{3}\right)}{(cx)^{3/2}\sqrt{a\left(9-6x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[-1/4, 1/2, 3/4, (2*x^2)/3])/((c*x)^
(3/2)*Sqrt[a*(9 - 6*x^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(85) = 170$.
time = 0.08, size = 228, normalized size = 2.13

method	result
risch	$\frac{\frac{4x^2}{3}-2}{c\sqrt{cx}\,\sqrt{-a\left(2x^2-3\right)}}-\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3x\sqrt{6}}\,\left(-\sqrt{\right.}}{81\sqrt{}}$

elliptic	$\frac{\sqrt{-cxa\left(2x^2-3\right)}\left(-\frac{2\left(-2c\,x^2a+3ac\right)}{3a\,c^2\sqrt{x\left(-2c\,x^2a+3ac\right)}}-\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\ldots}}{\ldots}\right.}{\sqrt{cx}\ldots}$
default	$-\frac{\sqrt{-a\left(2x^2-3\right)}\left(2\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{(2x+\ldots}}{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*(-a*(2*x^2-3))^(1/2)*(2*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^
(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*x+2^(1/2)*3^
(1/2))*2^(1/2)*3^(1/2))^(1/2)-((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^
(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*x+2^(1/2)*3^
(1/2))*2^(1/2)*3^(1/2))^(1/2)+24*x^2-36)/c/(c*x)^(1/2)/a/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 48, normalized size = 0.45

$$-\frac{2\left(\sqrt{2}\sqrt{-ac}\,x\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x))+\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\right)}{3\,ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*sqrt(-a*c)*x*weierstrassZeta(6, 0, weierstrassPInverse(6, 0,
x)) + sqrt(-2*a*x^2 + 3*a)*sqrt(c*x))/(a*c^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.76, size = 54, normalized size = 0.50

$$\frac{\sqrt{3}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\,c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(6*sqrt(a)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{3/2}\,\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2)), x)
```

3.642 $\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}+\frac{2\,2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{9\sqrt[4]{3}\,c^{5/2}\sqrt{a\left(3-2x^2\right)}}$$

```
[Out] 2/27*2^(3/4)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3
)^(1/2)*3^(3/4)/c^(5/2)/(a*(-2*x^2+3))^(1/2)-2/9*(-2*a*x^2+3*a)^(1/2)/a/c/(
c*x)^(3/2)
```

Rubi [A]

time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {331, 335, 230, 227}

$$\frac{2\,2^{3/4}\sqrt{3-2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{9\sqrt[4]{3}\,c^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(9*a*c*(c*x)^(3/2)) + (2*2^(3/4)*Sqrt[3 - 2*x^2]*E
llipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(9*3^(1/4)*c^(5/2)*S
qrt[a*(3 - 2*x^2)])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx &= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{9c^2}\\
&= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}+\frac{4\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{9c^3}\\
&= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}+\frac{\left(4\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{9\sqrt{3}\,c^3\sqrt{a\left(3-2x^2\right)}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}+\frac{2\,2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{9\sqrt[4]{3}\,c^{5/2}\sqrt{a\left(3-2x^2\right)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 53, normalized size = 0.54

$$-\frac{2x\sqrt{3-2x^2}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};\frac{2x^2}{3}\right)}{3(cx)^{5/2}\sqrt{a\left(9-6x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[-3/4, 1/2, 1/4, (2*x^2)/3])/(3*(c*x
)^(5/2)*Sqrt[a*(9 - 6*x^2)])
```

Maple [A]
time = 0.08, size = 132, normalized size = 1.35

method	result
default	$-\frac{\sqrt{-a(2x^2-3)}\left(\sqrt{(2x+\sqrt{2}\sqrt{3})\sqrt{2}\sqrt{3}}\sqrt{(-2x+\sqrt{2}\sqrt{3})\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{2}\sqrt{3}}\,\mathrm{E}\right.}{27xac^2\sqrt{cx}\,(2x^2-3)}$
elliptic	$\frac{\sqrt{-cxa(2x^2-3)}\left(-\frac{2\sqrt{-2acx^3+3acx}}{9ac^3x^2}+\frac{\sqrt{6}\sqrt{3}\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{243c^2\sqrt{-2acx^3}}\right.}{\sqrt{cx}\sqrt{-a(2x^2-3)}}$
risch	$\frac{\frac{4x^2}{9}-\frac{2}{3}}{xc^2\sqrt{cx}\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\sqrt{3}\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\sqrt{-3x\sqrt{6}}\,\mathrm{Ellip}}{243\sqrt{-2acx^3+3acx}\,c^2\sqrt{cx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(
(-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*El
lipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1
/2*2^(1/2))*x+12*x^2-18)/x/a/c^2/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 47, normalized size = 0.48

$$-\frac{2\left(\sqrt{2}\,\sqrt{-ac}\,x^2\text{weierstrassPInverse}(6,0,x)+\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\right)}{9\,ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*(sqrt(2)*sqrt(-a*c)*x^2*weierstrassPInverse(6, 0, x) + sqrt(-2*a*x^2 +
 3*a)*sqrt(c*x))/(a*c^3*x^2)
```

Sympy [A]
time = 1.99, size = 54, normalized size = 0.55

$$\frac{\sqrt{3}\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\,c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(6*sqrt(a)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{5/2}\,\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2)), x)
```

3.643 $\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}+\frac{3\sqrt[4]{3}\,c^2\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{2\,2^{3/4}a\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] 1/2*c*(c*x)^(3/2)/a/(-2*a*x^2+3*a)^(1/2)+3/4*3^(1/4)*c^2*EllipticE(1/6*(3-x
*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*(-2*x^2+3)^(1/2)*2^(1/4)/a/x^(
1/2)/(-2*a*x^2+3*a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {294, 326, 325, 324, 435}

$$\frac{3\sqrt[4]{3}\,c^2\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{2\,2^{3/4}a\sqrt{x}\,\sqrt{3a-2ax^2}}+\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*(c*x)^(3/2))/(2*a*Sqrt[3*a - 2*a*x^2]) + (3*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[3
 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(2*2^(3/4)*a*S
qrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 324

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-b/a
)^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b/
a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

Rule 325

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b}
, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

Rule 326

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-b/a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{(3c^2)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{4a}$$

$$= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{\left(3c^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{4a\sqrt{x}}$$

$$= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{\left(3c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{4a\sqrt{x}\sqrt{3a-2ax^2}}$$

$$= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} + \frac{\left(3\left(\frac{3}{2}\right)^{3/4}c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\cdots}}{\sqrt{\cdots}}\cdots\right)}{2a\sqrt{x}\sqrt{3a-2ax^2}}$$

$$= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} + \frac{3\sqrt[4]{3}\,c^2\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{2\,2^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 59, normalized size = 0.54

$$\frac{c(cx)^{3/2}\left(-3+\sqrt{9-6x^2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\frac{2x^2}{3}\right)\right)}{3a\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*(c*x)^(3/2)*(-3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[3/4, 3/2, 7/4, (2*x^
2)/3]))/(3*a*Sqrt[a*(3 - 2*x^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(86) = 172$.
time = 0.09, size = 230, normalized size = 2.09

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa\left(2x^2-3\right)}\left(\frac{c^3x^2}{2a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}-\frac{c^3\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right.}}{}\right.}{cx}$
default	$-\frac{c^2\sqrt{cx}\,\sqrt{-a\left(2x^2-3\right)}\left(2\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*c^2/x*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(2*((-2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2
)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2
)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)-((-2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2
)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2
)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)+8*x^2)/a^2/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 74, normalized size = 0.67

$$-\frac{2\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,c^2x+3\,\sqrt{2}\,(2\,c^2x^2-3\,c^2)\sqrt{-ac}\,\mathrm{weierstrassZeta}(6,0,\mathrm{weierstrassPInverse}(6,0,x))}{4\,(2\,a^2x^2-3\,a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x + 3*sqrt(2)*(2*c^2*x^2 - 3*c^2
)*sqrt(-a*c)*weierstrassZeta(6, 0, weierstrassPInverse(6, 0, x)))/(2*a^2*x^
2 - 3*a^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.03, size = 51, normalized size = 0.46

$$\frac{\sqrt{3}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), 2*x**2*exp_
polar(2*I*pi)/3)/(18*a**(3/2)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{\left(3\,a-2\,a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x)
[Out] int((c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2), x)
```

3.644 $\int \frac{(cx)^{3/2}}{\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{2\sqrt[4]{6}\,a\sqrt{a\left(3-2x^2\right)}}$$

```
[Out] -1/12*c^(3/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+
3)^(1/2)*6^(3/4)/a/(a*(-2*x^2+3))^(1/2)+1/2*c*(c*x)^(1/2)/a/(-2*a*x^2+3*a)^
(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {294, 335, 230, 227}

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{2\sqrt[4]{6}\,a\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c*x])/(2*a*Sqrt[3*a - 2*a*x^2]) - (c^(3/2)*Sqrt[3 - 2*x^2]*Elliptic
F[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(2*6^(1/4)*a*Sqrt[a*(3 - 2*
x^2)])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx &= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c^2\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx}{4a}\\
&= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2a}\\
&= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{\left(c\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{2\sqrt{3}\,a\sqrt{a\,(3-2x^2)}}\\
&= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{2\sqrt[4]{6}\,a\sqrt{a\,(3-2x^2)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 59, normalized size = 0.63

$$-\frac{c\sqrt{cx}\left(-3+\sqrt{9-6x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)\right)}{6a\sqrt{a\,(3-2x^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x]

[Out] -1/6*(c*Sqrt[c*x]*(-3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2)/3]))/(a*Sqrt[a*(3 - 2*x^2)])

Maple [A]
time = 0.09, size = 126, normalized size = 1.34

method	result
default	$\frac{c\sqrt{cx}\,\sqrt{-a\left(2x^2-3\right)}\left(\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-x\sqrt{2}\cdots}\right.}{24x\,a^2\left(2x^2-3\right)}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa\left(2x^2-3\right)}\left(\frac{c^2x}{2a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}-\frac{c^2\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\cdots\right.}}{216a\sqrt{\cdots}}\right.}{cx\sqrt{-a\left(2x^2-3\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/24*c*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-12*x)/x/a^2/(2*x^2-3)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x)^(3/2)/(-2*a*x^2 + 3*a)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 63, normalized size = 0.67

$$\frac{\sqrt{2}\,(2\,cx^2-3\,c)\sqrt{-ac}\ \text{weierstrassPInverse}(6,0,x)-2\,\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,c}{4\,(2\,a^2x^2-3\,a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(2*c*x^2 - 3*c)*sqrt(-a*c)*weierstrassPInverse(6, 0, x) - 2*sq
rt(-2*a*x^2 + 3*a)*sqrt(c*x)*c)/(2*a^2*x^2 - 3*a^2)
```

Sympy [A]
time = 1.02, size = 51, normalized size = 0.54

$$\frac{\sqrt{3}\,c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), 2*x**2*exp_p
olar(2*I*pi)/3)/(18*a**(3/2)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(c\,x)}^{3/2}}{{\left(3\,a-2\,a\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2), x)
```

3.645 $\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] 1/3*(c*x)^(3/2)/a/c/(-2*a*x^2+3*a)^(1/2)+1/6*EllipticE(1/6*(3-x*6^(1/2))^(1
/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*(-2*x^2+3)^(1/2)*6^(1/4)/a/x^(1/2)/(-2*a*x
^2+3*a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {296, 326, 325, 324, 435}

$$\frac{\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\,\sqrt{3a-2ax^2}}+\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*x)^(3/2)/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[c*x]*Sqrt[3 - 2*x^2]*Ellipt
icE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(6^(3/4)*a*Sqrt[x]*Sqrt[3*a -
2*a*x^2])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 324

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-b/a
)^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b/
a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

Rule 325

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b}
, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

Rule 326

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-b/a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx &= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} - \frac{\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{6a} \\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} - \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{6a\sqrt{x}} \\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} - \frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{6a\sqrt{x}\sqrt{3a-2ax^2}} \\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} + \frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{\frac{2}{3}}\,x}}{\sqrt{2}}\right)}{2^{3/4}\sqrt[4]{3}\,a\sqrt{x}\sqrt{3a-2ax^2}} \\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} + \frac{\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|\,2\right)}{6^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.57, size = 58, normalized size = 0.57

$$\frac{2x\sqrt{cx}\,(3-2x^2)^{3/2}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\frac{2x^2}{3}\right)}{9\sqrt{3}\,(a(3-2x^2))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[c*x]*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[3/4, 3/2, 7/4, (2*x^2)/3
])/(9*Sqrt[3]*(a*(3 - 2*x^2))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(82) = 164.
time = 0.07, size = 227, normalized size = 2.25

method	result
elliptic	$\dfrac{\sqrt{cx}\,\sqrt{-cxa\,(2x^2-3)}\left(\dfrac{c\,x^2}{3a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}-\dfrac{c\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right.}}{}\right.}{}$
default	$-\dfrac{\sqrt{cx}\,\sqrt{-a\,(2x^2-3)}\left(2\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\;\mathrm{EllipticE}\left(\dfrac{\sqrt{3}\,\sqrt{2}\,\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/72*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(2*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3
^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1
/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*
x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)-((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3
^(1/2))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1
/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*
x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)+24*x^2)/a^2/x/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 63, normalized size = 0.62

$$-\frac{\sqrt{2}\,\sqrt{-ac}\,(2\,x^2-3)\mathrm{weierstrassZeta}(6,0,\mathrm{weierstrassPInverse}(6,0,x))+2\,\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,x}{6\,(2\,a^2x^2-3\,a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(2)*sqrt(-a*c)*(2*x^2 - 3)*weierstrassZeta(6, 0, weierstrassPInve
rse(6, 0, x)) + 2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*x)/(2*a^2*x^2 - 3*a^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.59, size = 51, normalized size = 0.50

$$\frac{\sqrt{3}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), 2*x**2*exp_po
lar(2*I*pi)/3)/(18*a**(3/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{{\left(3\,a-2\,a\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(3*a - 2*a*x^2)^(3/2),x)
[Out] int((c*x)^(1/2)/(3*a - 2*a*x^2)^(3/2), x)
```

3.646 $\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{6}\,a\sqrt{c}\,\sqrt{a(3-2x^2)}}$$

```
[Out] 1/18*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3)^(1/2)*
6^(3/4)/a/c^(1/2)/(a*(-2*x^2+3))^(1/2)+1/3*(c*x)^(1/2)/a/c/(-2*a*x^2+3*a)^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {296, 335, 230, 227}

$$\frac{\sqrt{3-2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{6}\,a\sqrt{c}\,\sqrt{a(3-2x^2)}}+\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c*x]/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[3 - 2*x^2]*EllipticF[ArcSin[(
(2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*6^(1/4)*a*Sqrt[c]*Sqrt[a*(3 - 2*x^
2)])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx &= \frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{6a}\\
&= \frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3ac}\\
&= \frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{3\sqrt{3}\,ac\sqrt{a\,(3-2x^2)}}\\
&= \frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{6}\,a\sqrt{c}\,\sqrt{a\,(3-2x^2)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.84, size = 59, normalized size = 0.61

$$
\frac{x\left(3+\sqrt{9-6x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)\right)}{9a\sqrt{cx}\,\sqrt{a\,(3-2x^2)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] (x*(3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2)/3]))/(9*a*
Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)])
```

Maple [A]
time = 0.07, size = 122, normalized size = 1.27

method	result
default	$-\frac{\sqrt{-a\left(2x^2-3\right)}\left(\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\,\mathrm{E}\ldots\right.}{36a^2\sqrt{cx}\,\left(2x^2-3\right)}$
elliptic	$\frac{\sqrt{-cxa\left(2x^2-3\right)}\left(\frac{x}{3a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\ldots}{324a\sqrt{-2ac\,x^3\ldots}}\right.}{\sqrt{cx}\,\sqrt{-a\left(2x^2-3\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(-a*(2*x^2-3))^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(
(-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*El
lipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1
/2*2^(1/2))+12*x)/a^2/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 61, normalized size = 0.64

$$-\frac{\sqrt{2}\,\sqrt{-ac}\,(2\,x^2-3)\mathrm{weierstrassPInverse}(6,0,x)+2\,\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{6\,(2\,a^2cx^2-3\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(2)*sqrt(-a*c)*(2*x^2 - 3)*weierstrassPInverse(6, 0, x) + 2*sqrt(
-2*a*x^2 + 3*a)*sqrt(c*x))/(2*a^2*c*x^2 - 3*a^2*c)
```

Sympy [A]
time = 0.83, size = 51, normalized size = 0.53

$$\frac{\sqrt{3}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 2*x**2*exp_polar(2*I*p
i)/3)/(18*a**(3/2)*sqrt(c)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,{\left(3\,a-2\,a\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(3/2)), x)
```

3.647 $\int \frac{1}{(cx)^{3/2}(3a-2ax^2)^{3/2}}\,dx$

Optimal. Leaf size=140

$$\frac{1}{3ac\sqrt{cx}\,\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\sqrt[4]{2}\,\sqrt{cx}\,\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}ac^2\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] 1/3/a/c/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2)+1/3*2^(1/4)*EllipticE(1/6*(3-x*6^(
1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*(-2*x^2+3)^(1/2)*3^(1/4)/a/c^2/x^(
1/2)/(-2*a*x^2+3*a)^(1/2)-1/3*(-2*a*x^2+3*a)^(1/2)/a^2/c/(c*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {296, 331, 326, 325, 324, 435}

$$-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\sqrt[4]{2}\,\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}ac^2\sqrt{x}\,\sqrt{3a-2ax^2}}+\frac{1}{3ac\sqrt{3a-2ax^2}\,\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] 1/(3*a*c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]) - Sqrt[3*a - 2*a*x^2]/(3*a^2*c*Sqrt
[c*x]) + (2^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[
6]*x]/Sqrt[6]], 2])/(3^(3/4)*a*c^2*Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 324

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-b/a
)^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b/
a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

Rule 325

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b}
, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

Rule 326

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-b/a, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}(3a-2ax^2)^{3/2}}\,dx = \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}} + \frac{\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx}{2a}$$

$$= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}} - \frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}} - \frac{\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{3ac^2}$$

$$= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}} - \frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}} - \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{3ac^2\sqrt{x}}$$

$$= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}} - \frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}} - \frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}}{3ac^2\sqrt{x}\sqrt{3a-2ax^2}}$$

$$= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}} - \frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}} + \frac{\left(\sqrt[4]{\frac{2}{3}}\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\Bigg(\int}{ac^2\sqrt{x}\sqrt{3}}$$

$$= \frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}} - \frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}} + \frac{\sqrt[4]{2}\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3}}{}\right.\right.}{3^{3/4}ac^2\sqrt{x}\sqrt{3a-2a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 58, normalized size = 0.41

$$-\frac{2x(3-2x^2)^{3/2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};\frac{2x^2}{3}\right)}{3\sqrt{3}\,(cx)^{3/2}\,(a\,(3-2x^2))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] (-2*x*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[-1/4, 3/2, 3/4, (2*x^2)/3])/(3*Sq
rt[3]*(c*x)^(3/2)*(a*(3 - 2*x^2))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(113) = 226.
time = 0.08, size = 228, normalized size = 1.63

method	result
elliptic	$\frac{\sqrt{-cxa\,(2x^2-3)}\left(\frac{2x^2}{9ac\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}-\frac{2\left(-2c\,x^2a+3ac\right)}{9a^2c^2\sqrt{x\left(-2c\,x^2a+3ac\right)}}-\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)}}{}\right.}{}$
default	$-\frac{\sqrt{-a\,(2x^2-3)}\left(2\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\,\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{(2x}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(-a*(2*x^2-3))^(1/2)*(2*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^
(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*x+2^(1/2)*3^
(1/2))*2^(1/2)*3^(1/2))^(1/2)-((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^
(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*((2*x+2^(1/2)*3^
(1/2))*2^(1/2)*3^(1/2))^(1/2)+24*x^2-24)/a^2/c/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 76, normalized size = 0.54

$$-\frac{\sqrt{2}\,(2\,x^3-3\,x)\sqrt{-ac}\,\mathrm{weierstrassZeta}(6,0,\mathrm{weierstrassPInverse}(6,0,x))+2\,\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,(x^2-1)}{3\,(2\,a^2c^2x^3-3\,a^2c^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
[Out] -1/3*(sqrt(2)*(2*x^3 - 3*x)*sqrt(-a*c)*weierstrassZeta(6, 0, weierstrassPIn
verse(6, 0, x)) + 2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*(x^2 - 1))/(2*a^2*c^2*x^
3 - 3*a^2*c^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.61, size = 54, normalized size = 0.39

$$\frac{\sqrt{3}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-2*a*x**2+3*a)**(3/2),x)
[Out] sqrt(3)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(18*a**(3/2)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{3/2}\,(3\,a-2\,a\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x)
[Out] int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)), x)
```

3.648 $\int \frac{1}{(cx)^{5/2}(3a-2ax^2)^{3/2}}\,dx$

Optimal. Leaf size=132

$$\frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{5\ 2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{27\sqrt[4]{3}\,ac^{5/2}\sqrt{a\left(3-2x^2\right)}}$$

```
[Out] 5/81*2^(3/4)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3
)^(1/2)*3^(3/4)/a/c^(5/2)/(a*(-2*x^2+3))^(1/2)+1/3/a/c/(c*x)^(3/2)/(-2*a*x^
2+3*a)^(1/2)-5/27*(-2*a*x^2+3*a)^(1/2)/a^2/c/(c*x)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {296, 331, 335, 230, 227}

$$-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{5\ 2^{3/4}\sqrt{3-2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{27\sqrt[4]{3}\,ac^{5/2}\sqrt{a\left(3-2x^2\right)}}+\frac{1}{3ac\sqrt{3a-2ax^2}\,(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] 1/(3*a*c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]) - (5*Sqrt[3*a - 2*a*x^2])/(27*a^2
*c*(c*x)^(3/2)) + (5*2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*
Sqrt[c*x])/Sqrt[c]], -1])/(27*3^(1/4)*a*c^(5/2)*Sqrt[a*(3 - 2*x^2)])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(3a-2ax^2\right)^{3/2}}\,dx = \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}} + \frac{5\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx}{6a}$$

$$= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}} - \frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}} + \frac{5\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{27ac^2}$$

$$= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}} - \frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}} + \frac{10\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x \ldots\right.}{27ac^3}$$

$$= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}} - \frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}} + \frac{\left(10\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-\ldots}}\right.}{27\sqrt{3}\,ac^3\sqrt{a\left(3-\ldots\right.}}$$

$$= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}} - \frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}} + \frac{5\,2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\ldots}}{\ldots}\right.\right.}{27\sqrt[4]{3}\,ac^{5/2}\sqrt{a\left(3-\ldots\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 58, normalized size = 0.44

$$-\frac{2x\left(3-2x^2\right)^{3/2}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};\frac{2x^2}{3}\right)}{9\sqrt{3}\,(cx)^{5/2}\left(a\left(3-2x^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] (-2*x*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[-3/4, 3/2, 1/4, (2*x^2)/3])/(9*Sq
rt[3]*(c*x)^(5/2)*(a*(3 - 2*x^2))^(3/2))
```

Maple [A]
time = 0.07, size = 133, normalized size = 1.01

method	result

default	$-\frac{\sqrt{-a\left(2x^{2}-3\right)}\left(5\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-x\sqrt{2}\,\sqrt{3}}\right.}{162x\,a^{2}c^{2}\sqrt{cx}\,\left(2x^{2}-3\right)}$
elliptic	$\frac{\sqrt{-cxa\left(2x^{2}-3\right)}\left(\frac{2x}{9a\,c^{2}\sqrt{-2\left(x^{2}-\frac{3}{2}\right)acx}}-\frac{2\sqrt{-2ac\,x^{3}+3acx}}{27a^{2}c^{3}x^{2}}+\frac{5\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{}\right.}{\sqrt{cx}\,\sqrt{-a\left(2x^{2}-3\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/162*(-a*(2*x^2-3))^(1/2)*(5*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)
*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
),1/2*2^(1/2))*x+60*x^2-36)/x/a^2/c^2/(c*x)^(1/2)/(2*x^2-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 80, normalized size = 0.61

$$-\frac{5\sqrt{2}\left(2\,x^{4}-3\,x^{2}\right)\sqrt{-ac}\ \mathrm{weierstrassPInverse}(6,0,x)+2\sqrt{-2\,ax^{2}+3\,a}\,\sqrt{cx}\left(5\,x^{2}-3\right)}{27\left(2\,a^{2}c^{3}x^{4}-3\,a^{2}c^{3}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/27*(5*sqrt(2)*(2*x^4 - 3*x^2)*sqrt(-a*c)*weierstrassPInverse(6, 0, x) +
2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*(5*x^2 - 3))/(2*a^2*c^3*x^4 - 3*a^2*c^3*x^
2)
```

Sympy [A]
time = 4.69, size = 54, normalized size = 0.41

$$\frac{\sqrt{3}\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-2*a*x**2+3*a)**(3/2),x)
[Out] sqrt(3)*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(18*a**(3/2)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{5/2}\,{\left(3\,a-2\,a\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x)
[Out] int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)), x)
```

3.649 $\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2F\left(\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)\middle|-1\right)}{\sqrt{a}}$$

[Out] 2*EllipticF(a^(1/2)*x^(1/2),I)/a^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {335, 227}

$$\frac{2F\left(\text{ArcSin}\left(\sqrt{a}\sqrt{x}\right)\middle|-1\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[1 - a^2*x^2]),x]

[Out] (2*EllipticF[ArcSin[Sqrt[a]*Sqrt[x]], -1])/Sqrt[a]

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^4}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2F\left(\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)\middle|-1\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 24, normalized size = 1.14

$$2\sqrt{x}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] 2*Sqrt[x]*Hypergeometric2F1[1/4, 1/2, 5/4, a^2*x^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(15) = 30$.
time = 0.12, size = 66, normalized size = 3.14

method	result	size
meijerg	$2\sqrt{x}\ \text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],a^2x^2\right)$	19
default	$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax+1}\,\sqrt{-2ax+2}\,\sqrt{-ax}\ \text{EllipticF}\left(\sqrt{ax+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{x}\ a(a^2x^2-1)}$	66
elliptic	$\frac{\sqrt{-x\left(a^2x^2-1\right)}\,\sqrt{a\left(x+\frac{1}{a}\right)}\,\sqrt{-2a\left(x-\frac{1}{a}\right)}\,\sqrt{-ax}\ \text{EllipticF}\left(\sqrt{a\left(x+\frac{1}{a}\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{x}\,\sqrt{-a^2x^2+1}\ a\sqrt{-a^2x^3+x}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x^(1/2)*(-a^2*x^2+1)^(1/2)*(a*x+1)^(1/2)*(-2*a*x+2)^(1/2)*(-a*x)^(1/2)*E
llipticF((a*x+1)^(1/2),1/2*2^(1/2))/a/(a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.
time = 0.39, size = 36, normalized size = 1.71

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a**2*x**2*exp_polar(2*I*pi))/(
2*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

3.650 $\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx$

Optimal. Leaf size=67

$$\frac{\left(1+\sqrt{a}\,x\right)\sqrt{\frac{1+ax^2}{\left(1+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{a}\,\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt{1+ax^2}}$$

```
[Out] (cos(2*arctan(a^(1/4)*x^(1/2)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/2)))*Ell
ipticF(sin(2*arctan(a^(1/4)*x^(1/2))),1/2*2^(1/2))*(1+x*a^(1/2))*((a*x^2+1)
/(1+x*a^(1/2))^2)^(1/2)/a^(1/4)/(a*x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {335, 226}

$$\frac{\left(\sqrt{a}\,x+1\right)\sqrt{\frac{ax^2+1}{\left(\sqrt{a}\,x+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{a}\,\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt{ax^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 + a*x^2]),x]
```

```
[Out] ((1 + Sqrt[a]*x)*Sqrt[(1 + a*x^2)/(1 + Sqrt[a]*x)^2]*EllipticF[2*ArcTan[a^(
1/4)*Sqrt[x]], 1/2])/(a^(1/4)*Sqrt[1 + a*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,\sqrt{1+ax^2}}\,dx = 2\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^4}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\left(1+\sqrt{a}\,x\right)\sqrt{\frac{1+ax^2}{\left(1+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{a}\,\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt{1+ax^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 23, normalized size = 0.34

$$2\sqrt{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 + a*x^2]),x]
```

```
[Out] 2*Sqrt[x]*Hypergeometric2F1[1/4, 1/2, 5/4, -(a*x^2)]
```

Maple [A]
time = 0.08, size = 73, normalized size = 1.09

method	result
meijerg	$2\sqrt{x}\,\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-a\,x^2\right)$
default	$-\frac{\sqrt{-x\sqrt{-a}+1}\,\sqrt{2}\,\sqrt{x\sqrt{-a}+1}\,\sqrt{x\sqrt{-a}}\,\operatorname{EllipticF}\left(\sqrt{-x\sqrt{-a}+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{x}\,\sqrt{a\,x^2+1}\,\sqrt{-a}}$
elliptic	$-\frac{\sqrt{x\left(a\,x^2+1\right)}\,\sqrt{-\left(x-\frac{1}{\sqrt{-a}}\right)\sqrt{-a}}\,\sqrt{2}\,\sqrt{\left(x+\frac{1}{\sqrt{-a}}\right)\sqrt{-a}}\,\sqrt{x\sqrt{-a}}\,\operatorname{EllipticF}\left(\sqrt{-\right.}\right.}{\sqrt{x}\,\sqrt{a\,x^2+1}\,\sqrt{-a}\,\sqrt{a\,x^3+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(a*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x^(1/2)/(a*x^2+1)^(1/2)*(-x*(-a)^(1/2)+1)^(1/2)*2^(1/2)*(x*(-a)^(1/2)+1)
^(1/2)*(x*(-a)^(1/2))^(1/2)*EllipticF((-x*(-a)^(1/2)+1)^(1/2),1/2*2^(1/2))/
(-a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 13, normalized size = 0.19

$$\frac{2\,\mathrm{weierstrassPInverse}\left(-\frac{4}{a},0,x\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*weierstrassPInverse(-4/a, 0, x)/sqrt(a)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.36, size = 32, normalized size = 0.48

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|ax^2e^{i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(a*x**2+1)**(1/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a*x**2*exp_polar(I*pi))/(2*gam
ma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x}\,\sqrt{a\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x^2 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x^2 + 1)^(1/2)), x)
```

3.651 $\int x^m(a+bx^2)^{3/2}\,dx$

Optimal. Leaf size=50

$$\frac{x^{1+m}(a+bx^2)^{5/2}\ {}_2F_1\left(1,\frac{6+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

```
[Out] x^(1+m)*(b*x^2+a)^(5/2)*hypergeom([1, 3+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m
)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{ax^{m+1}\sqrt{a+bx^2}\ {}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m(a+bx^2)^{3/2}\,dx = \frac{\left(a\sqrt{a+bx^2}\right)\int x^m\left(1+\frac{bx^2}{a}\right)^{3/2}dx}{\sqrt{1+\frac{bx^2}{a}}}$$
$$= \frac{ax^{1+m}\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}$$

Mathematica [A]
time = 0.19, size = 66, normalized size = 1.32

$$\frac{ax^{1+m}\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^(3/2),x]
[Out] (a*x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, 1 + (1 + m)
/2, -((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^m\left(b\,x^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^(3/2),x)
[Out] int(x^m*(b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/2)*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.45, size = 54, normalized size = 1.08

$$\frac{a^{\frac{3}{2}} x x^{m} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{b x^{2} e^{i \pi}}{a}\right)}{2 \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**(3/2),x)
[Out] a**(3/2)*x*x**m*gamma(m/2 + 1/2)*hyper((-3/2, m/2 + 1/2), (m/2 + 3/2,), b*x
**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(b\,x^2+a\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^(3/2),x)
[Out] int(x^m*(a + b*x^2)^(3/2), x)
```

3.652 $\int x^m \sqrt{a+bx^2}\, dx$

Optimal. Leaf size=50

$$\frac{x^{1+m}(a+bx^2)^{3/2}\ {}_2F_1\left(1,\frac{4+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

```
[Out] x^(1+m)*(b*x^2+a)^(3/2)*hypergeom([1, 2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m
)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{x^{m+1}\sqrt{a+bx^2}\ {}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}\int x^m\sqrt{1+\frac{bx^2}{a}}\,dx}{\sqrt{1+\frac{bx^2}{a}}}$$

$$=\frac{x^{1+m}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}$$

Mathematica [A]
time = 0.14, size = 65, normalized size = 1.30

$$\frac{x^{1+m}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[a + b*x^2],x]
[Out] (x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, 1 + (1 + m)/2
, -((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^m\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^(1/2),x)
[Out] int(x^m*(b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 54, normalized size = 1.08

$$\frac{\sqrt{a}\, x x^{m} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{b x^{2} e^{i \pi}}{a}\right)}{2 \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^m*(a + b*x^2)^(1/2), x)
```

3.653 $\int \frac{x^m}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{1+m}\sqrt{a+bx^2}\;{}_2F_1\left(1,\frac{2+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

```
[Out] x^(1+m)*hypergeom([1, 1+1/2*m],[3/2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(1+m
)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}} = \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.18, size = 65, normalized size = 1.30

$$\frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 53, normalized size = 1.06

$$\frac{x x^{m} \Gamma\left(\frac{m}{2} + \frac{1}{2}\right) {}_{2}F_{1}\left(\left.\begin{matrix} \frac{1}{2}, \frac{m}{2} + \frac{1}{2} \\ \frac{m}{2} + \frac{3}{2} \end{matrix}\right| \frac{b x^{2} e^{i \pi}}{a}\right)}{2 \sqrt{a} \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{b\,x^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^m/(a + b*x^2)^(1/2), x)
```

3.654 $\int \frac{x^m}{(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=48

$$\frac{x^{1+m}\,{}_2F_1\left(1,\frac{m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)\sqrt{a+bx^2}}$$

```
[Out] x^(1+m)*hypergeom([1, 1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.38, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/(a*(1 + m)*Sqrt[a + b*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2}}$$
$$= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)\sqrt{a+bx^2}}$$

Mathematica [**A**]
time = 0.20, size = 68, normalized size = 1.42

$$\frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^(3/2),x]
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/(a*(1 + m)*Sqrt[a + b*x^2])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(3/2),x)
[Out] int(x^m/(b*x^2+a)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m/(b*x^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.67, size = 53, normalized size = 1.10

$$\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(3/2),x)
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((3/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate(x^m/(b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^(3/2),x)
[Out] int(x^m/(a + b*x^2)^(3/2), x)
```

3.655 $\int \frac{x^m}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{1+m}\,{}_2F_1\left(1,\frac{1}{2}(-2+m);\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)\left(a+bx^2\right)^{3/2}}$$

```
[Out] x^(1+m)*hypergeom([1, -1+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m)/(b*x^2+a)^(3/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{a^2\sqrt{a+bx^2}}$$
$$= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.29, size = 68, normalized size = 1.36

$$\frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^(5/2),x]
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(5/2),x)
[Out] int(x^m/(b*x^2+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x^m/(b*x^2 + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x
)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.51, size = 53, normalized size = 1.06

$$\frac{x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} \frac{5}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(5/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((5/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*a**(5/2)*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^m/(a + b*x^2)^(5/2), x)
```

3.656 $\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{3+m}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{4+m}{2};\frac{5+m}{2};-\frac{bx^2}{a}\right)}{a(3+m)}$$

```
[Out] x^(3+m)*hypergeom([1, 2+1/2*m],[5/2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(3+m
)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$\frac{x^{m+3}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};-\frac{bx^2}{a}\right)}{(m+3)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(3 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2,
 -((b*x^2)/a)])/((3 + m)*Sqrt[a + b*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{2+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}$$
$$= \frac{x^{3+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};-\frac{bx^2}{a}\right)}{(3+m)\sqrt{a+bx^2}}$$

Mathematica [**A**]
time = 0.17, size = 65, normalized size = 1.30

$$\frac{x^{3+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};1+\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(3+m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)/Sqrt[a + b*x^2],x]
[Out] (x^(3 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (3 + m)/2, 1 + (3 + m
)/2, -((b*x^2)/a)])/((3 + m)*Sqrt[a + b*x^2])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{2+m}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)/(b*x^2+a)^(1/2),x)
[Out] int(x^(2+m)/(b*x^2+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(m + 2)/sqrt(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m + 2)/sqrt(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.29, size = 54, normalized size = 1.08

$$\frac{x^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\\frac{m}{2}+\frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**3*x**m*gamma(m/2 + 3/2)*hyper((1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp
_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 5/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^(m + 2)/(a + b*x^2)^(1/2), x)
```

3.657 $\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{2+m}\sqrt{a+bx^2}\ {}_2F_1\left(1,\frac{3+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{a(2+m)}$$

```
[Out] x^(2+m)*hypergeom([1, 3/2+1/2*m],[2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(2+m
)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)])/((2 + m)*Sqrt[a + b*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}$$
$$= \frac{x^{2+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{(2+m)\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.18, size = 65, normalized size = 1.30

$$\frac{x^{2+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};1+\frac{2+m}{2};-\frac{bx^2}{a}\right)}{(2+m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)/Sqrt[a + b*x^2],x]
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, 1 + (2 + m
)/2, -((b*x^2)/a)])/((2 + m)*Sqrt[a + b*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{1+m}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)/(b*x^2+a)^(1/2),x)
[Out] int(x^(1+m)/(b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(m + 1)/sqrt(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m + 1)/sqrt(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.87, size = 48, normalized size = 0.96

$$\frac{x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar
(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+1}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^(m + 1)/(a + b*x^2)^(1/2), x)
```

3.658 $\int \frac{x^m}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{1+m}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

```
[Out] x^(1+m)*hypergeom([1, 1+1/2*m],[3/2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(1+m
)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}} = \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 1.30

$$\frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^2],x]
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(1/2),x)
[Out] int(x^m/(b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^m/sqrt(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 53, normalized size = 1.06

$$\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(1/2),x)
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^(1/2),x)
[Out] int(x^m/(a + b*x^2)^(1/2), x)
```

3.659 $\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=46

$$\frac{x^m\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{am}$$

```
[Out] x^m*hypergeom([1, 1/2+1/2*m],[1+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/m
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b*x^2)/a
)])/(m*Sqrt[a + b*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{-1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}$$
$$= \frac{x^m\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.17, size = 57, normalized size = 1.24

$$\frac{x^m\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{m}{2};1+\frac{m}{2};-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)/Sqrt[a + b*x^2],x]
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, 1 + m/2, -((b*x^2)/a)]
)/(m*Sqrt[a + b*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+m}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)/(b*x^2+a)^(1/2),x)
[Out] int(x^(-1+m)/(b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(m - 1)/sqrt(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m - 1)/sqrt(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.46, size = 41, normalized size = 0.89

$$\frac{x^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(2*
sqrt(a)*gamma(m/2 + 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-1}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^(m - 1)/(a + b*x^2)^(1/2), x)
```

3.660 $\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{x^{-1+m}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m}{2};\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a(1-m)}$$

```
[Out] -x^(-1+m)*hypergeom([1, 1/2*m],[1/2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(1-m
)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$-\frac{x^{m-1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{(1-m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] -((x^(-1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m
)/2, -((b*x^2)/a)])/((1 - m)*Sqrt[a + b*x^2]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{-2+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}$$
$$= -\frac{x^{-1+m}\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{bx^2}{a}\right)}{(1-m)\sqrt{a+bx^2}}$$

Mathematica [**A**]
time = 0.17, size = 65, normalized size = 1.27

$$\frac{x^{-1+m}\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+m);1+\frac{1}{2}(-1+m);-\frac{bx^2}{a}\right)}{(-1+m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(-1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (-1 + m)/2, 1 + (-1
+ m)/2, -((b*x^2)/a)])/((-1 + m)*Sqrt[a + b*x^2])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{-2+m}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^(-2+m)/(b*x^2+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^(m - 2)/sqrt(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.14, size = 53, normalized size = 1.04

$$\frac{x^m\Gamma\left(\frac{m}{2}-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}-\frac{1}{2}\\ \frac{m}{2}+\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,x\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)/(b*x**2+a)**(1/2),x)
[Out] x**m*gamma(m/2 - 1/2)*hyper((1/2, m/2 - 1/2), (m/2 + 1/2,), b*x**2*exp_pola
r(I*pi)/a)/(2*sqrt(a)*x*gamma(m/2 + 1/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(x^(m - 2)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)/(a + b*x^2)^(1/2),x)
[Out] int(x^(m - 2)/(a + b*x^2)^(1/2), x)
```

3.661 $\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=17

$$x^{2+m}\sqrt{a+bx^2}$$

```
[Out] x^(2+m)*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {460}

$$x^{m+2}\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] x^(2 + m)*Sqrt[a + b*x^2]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx = x^{2+m}\sqrt{a+bx^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.34, size = 97, normalized size = 5.71

$$\frac{x^{2+m}\sqrt{a+bx^2}\left((3+m)\,{}_2F_1\left(-\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)-{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)\right)}{(2+m)\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[a + b*x^2]*((3 + m)*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/
2, -((b*x^2)/a)] - Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -((b*x^2)/a)]))
/((2 + m)*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.94

method	result	size
gosper	$x^{2+m}\sqrt{b\,x^{2}+a}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(2+m)*(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.32, size = 16, normalized size = 0.94

$$\sqrt{bx^2+a}\,x^2x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] sqrt(b*x^2 + a)*x^2*x^m
```

Fricas [A]
time = 2.58, size = 16, normalized size = 0.94

$$\sqrt{bx^2+a}\,xx^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(b*x^2 + a)*x*x^(m + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.74, size = 202, normalized size = 11.88

$$\frac{\sqrt{a}\,mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{\sqrt{a}\,x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\Gamma\left(\frac{m}{2}+2\right)}+\frac{bmx^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+3\right)}+\frac{3bx^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*(a*(2+m)+b*(3+m)*x**2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*m*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2
*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + sqrt(a)*x**2*x**m*gamma(m/2 + 1)*h
yper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/gamma(m/2 + 2) +
 b*m*x**4*x**m*gamma(m/2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_
polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3)) + 3*b*x**4*x**m*gamma(m/2 + 2)*hy
per((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(
m/2 + 3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*(m + 3)*x^2 + a*(m + 2))*x^(m + 1)/sqrt(b*x^2 + a), x)
```

Mupad [B]
time = 5.19, size = 24, normalized size = 1.41

$$\frac{x^{m+1}\,\left(b\,x^3+a\,x\right)}{\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m + 1)*(a*(m + 2) + b*x^2*(m + 3)))/(a + b*x^2)^(1/2),x)
```

```
[Out] (x^(m + 1)*(a*x + b*x^3))/(a + b*x^2)^(1/2)
```

3.662 $\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right) dx$

Optimal. Leaf size=17

$$x^{2+m}\sqrt{a+bx^2}$$

```
[Out] x^(2+m)*(b*x^2+a)^(1/2)
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 7.47, number of steps used = 5, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {372, 371}

$$\frac{ax^{m+2}\sqrt{\frac{bx^2}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}+\frac{b(m+3)x^{m+4}\sqrt{\frac{bx^2}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sqrt[a + b*x^2],x]
```

```
[Out] (a*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/Sqrt[a + b*x^2] + (b*(3 + m)*x^(4 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, -((b*x^2)/a)])/((4 + m)*Sqrt[a + b*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)dx = (a(2+m))\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx+(b(3+m))\int \frac{x^{3+m}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{\left(a(2+m)\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}} + \frac{\left(b(3+m)\sqrt{1+\frac{b}{}}\right.}{\sqrt{a}}$$

$$= \frac{ax^{2+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}} + \frac{b(3+m)x^{4+m}\sqrt{1}}{(4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 97, normalized size = 5.71

$$\frac{x^{2+m}\sqrt{a+bx^2}\left((3+m)\,{}_2F_1\left(-\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)-{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)\right)}{(2+m)\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sqr
t[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[a + b*x^2]*((3 + m)*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/
2, -((b*x^2)/a)] - Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -((b*x^2)/a)]))
/((2 + m)*Sqrt[1 + (b*x^2)/a])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a(2+m)\,x^{1+m}}{\sqrt{b\,x^2+a}}+\frac{b(3+m)\,x^{3+m}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x)
```

Maxima [A]
time = 0.33, size = 16, normalized size = 0.94

$$\sqrt{bx^2+a}\,x^2x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*x^2*x^m
```

Fricas [A]
time = 1.26, size = 18, normalized size = 1.06

$$\frac{\sqrt{bx^2+a}\,x^{m+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x^(m + 3)/x
```

Sympy [C] Result contains complex when optimal does not.
time = 2.66, size = 105, normalized size = 6.18

$$\frac{\sqrt{a}\,x^2x^m(m+2)\,\Gamma\left(\frac{m}{2}+1\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{bx^4x^m(m+3)\,\Gamma\left(\frac{m}{2}+2\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x**(1+m)/(b*x**2+a)**(1/2)+b*(3+m)*x**(3+m)/(b*x**2+a)**(
1/2),x)
```

```
[Out] sqrt(a)*x**2*x**m*(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,),
b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + b*x**4*x**m*(m + 3)*gamma(m/
2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*gamma(m/2 + 3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(b*(m + 3)*x^(m + 3)/sqrt(b*x^2 + a) + a*(m + 2)*x^(m + 1)/sqrt(b*
x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{a\,x^{m+1}\,\left(m+2\right)}{\sqrt{b\,x^2+a}}+\frac{b\,x^{m+3}\,\left(m+3\right)}{\sqrt{b\,x^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(m + 1)*(m + 2))/(a + b*x^2)^(1/2) + (b*x^(m + 3)*(m + 3))/(a + b*
x^2)^(1/2),x)
[Out] int((a*x^(m + 1)*(m + 2))/(a + b*x^2)^(1/2) + (b*x^(m + 3)*(m + 3))/(a + b*
x^2)^(1/2), x)
```

3.663 $\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

```
[Out] x^m/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {460}

$$\frac{x^m}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] x^m/Sqrt[a + b*x^2]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx = \frac{x^m}{\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.41, size = 131, normalized size = 8.73

$$\frac{x^m\sqrt{a+bx^2}\left(a(2+m)\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2};1+\frac{m}{2};-\frac{bx^2}{a}\right)-bx^2\left(m\,{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)+{}_2F_1\left(\frac{3}{2},1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)\right)\right)}{a^2(2+m)\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (x^m*Sqrt[a + b*x^2]*(a*(2 + m)*Hypergeometric2F1[-1/2, m/2, 1 + m/2, -((b*
x^2)/a)] - b*x^2*(m*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -((b*x^2)/a)]
+ Hypergeometric2F1[3/2, 1 + m/2, 2 + m/2, -((b*x^2)/a)])))/(a^2*(2 + m)*Sq
rt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.93

method	result	size
gosper	$\frac{x^{m}}{\sqrt{b\,x^{2}+a}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^m/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.33, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] x^m/sqrt(b*x^2 + a)
```

Fricas [A]
time = 1.63, size = 16, normalized size = 1.07

$$\frac{xx^{m-1}}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] x*x^(m - 1)/sqrt(b*x^2 + a)
```

Sympy [C] Result contains complex when optimal does not.
time = 23.52, size = 97, normalized size = 6.47

$$\frac{mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}\\\frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+1\right)}+\frac{bx^2x^m(m-1)\,\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*(a*m+b*(-1+m)*x**2)/(b*x**2+a)**(3/2),x)
```

```
[Out] m*x**m*gamma(m/2)*hyper((3/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*gamma(m/2 + 1)) + b*x**2*x**m*(m - 1)*gamma(m/2 + 1)*hyper((3/2,
m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*(m - 1)*x^2 + a*m)*x^(m - 1)/(b*x^2 + a)^(3/2), x)
```

Mupad [B]
time = 5.43, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(a*m + b*x^2*(m - 1)))/(a + b*x^2)^(3/2),x)
```

```
[Out] x^m/(a + b*x^2)^(1/2)
```

3.664 $\int \left(-\frac{bx^{1+m}}{(a+bx^2)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right) dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

[Out] x^m/(b*x^2+a)^(1/2)

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, antiderivative size = 123, normalized size of antiderivative = 8.20, number of steps used = 5, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {372, 371}

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}-\frac{bx^{m+2}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x^2],x]

[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b*x^2)/a)])/Sqrt[a + b*x^2] - (b*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*(2 + m)*Sqrt[a + b*x^2])

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(-\frac{bx^{1+m}}{(a+bx^2)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)dx=-\left(b\int \frac{x^{1+m}}{(a+bx^2)^{3/2}}dx\right)+m\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}dx$$

$$=-\frac{\left(b\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{1+m}}{\left(1+\frac{bx^2}{a}\right)^{3/2}}dx}{a\sqrt{a+bx^2}}+\frac{\left(m\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{-1+m}}{\sqrt{1+\frac{bx^2}{a}}}$$

$$=\frac{x^m\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}-\frac{bx^{2+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{2},\right.}{a(2+m)\sqrt{a+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 131, normalized size = 8.73

$$\frac{x^m\sqrt{a+bx^2}\left(a(2+m)\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2};1+\frac{m}{2};-\frac{bx^2}{a}\right)-bx^2\left(m\,{}_2F_1\left(\frac{1}{2},1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)+{}_2F_1\left(\frac{3}{2},1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)\right)\right)}{a^2(2+m)\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x^
2],x]
```

```
[Out] (x^m*Sqrt[a + b*x^2]*(a*(2 + m)*Hypergeometric2F1[-1/2, m/2, 1 + m/2, -((b*
x^2)/a)] - b*x^2*(m*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -((b*x^2)/a)]
+ Hypergeometric2F1[3/2, 1 + m/2, 2 + m/2, -((b*x^2)/a)])))/(a^2*(2 + m)*Sq
rt[1 + (b*x^2)/a])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int -\frac{b\,x^{1+m}}{(b\,x^2+a)^{\frac{3}{2}}}+\frac{m\,x^{-1+m}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x)
```

Maxima [A]
time = 0.33, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="maxima")
```

```
[Out] x^m/sqrt(b*x^2 + a)
```

Fricas [A]
time = 1.54, size = 26, normalized size = 1.73

$$\frac{\sqrt{bx^2+a}\,x^{m+1}}{bx^3+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x^(m + 1)/(b*x^3 + a*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.88, size = 94, normalized size = 6.27

$$\frac{mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+1\right)}-\frac{bx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x**(1+m)/(b*x**2+a)**(3/2)+m*x**(-1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] m*x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*gamma(m/2 + 1)) - b*x**2*x**m*gamma(m/2 + 1)*hyper((3/2, m/2 + 1)
, (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(m*x^(m - 1)/sqrt(b*x^2 + a) - b*x^(m + 1)/(b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$-\int \frac{b\,x^{m+1}}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{m\,x^{m-1}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^(m - 1))/(a + b*x^2)^(1/2) - (b*x^(m + 1))/(a + b*x^2)^(3/2),x)
[Out] -int((b*x^(m + 1))/(a + b*x^2)^(3/2) - (m*x^(m - 1))/(a + b*x^2)^(1/2), x)
```

3.665 $\int x^7 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3(a+bx^2)^{4/3}}{8b^4}+\frac{9a^2(a+bx^2)^{7/3}}{14b^4}-\frac{9a(a+bx^2)^{10/3}}{20b^4}+\frac{3(a+bx^2)^{13/3}}{26b^4}$$

```
[Out] -3/8*a^3*(b*x^2+a)^(4/3)/b^4+9/14*a^2*(b*x^2+a)^(7/3)/b^4-9/20*a*(b*x^2+a)^
(10/3)/b^4+3/26*(b*x^2+a)^(13/3)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {272, 45}

$$-\frac{3a^3(a+bx^2)^{4/3}}{8b^4}+\frac{9a^2(a+bx^2)^{7/3}}{14b^4}+\frac{3(a+bx^2)^{13/3}}{26b^4}-\frac{9a(a+bx^2)^{10/3}}{20b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(4/3))/(8*b^4) + (9*a^2*(a + b*x^2)^(7/3))/(14*b^4) - (
9*a*(a + b*x^2)^(10/3))/(20*b^4) + (3*(a + b*x^2)^(13/3))/(26*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3 \sqrt[3]{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3\sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right) dx, x, x^2\right)\\ &= -\frac{3a^3(a+bx^2)^{4/3}}{8b^4}+\frac{9a^2(a+bx^2)^{7/3}}{14b^4}-\frac{9a(a+bx^2)^{10/3}}{20b^4}+\frac{3(a+bx^2)^{13/3}}{26b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.62

$$\frac{3(a+bx^2)^{4/3}\left(-81a^3+108a^2bx^2-126ab^2x^4+140b^3x^6\right)}{3640b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-81*a^3 + 108*a^2*b*x^2 - 126*a*b^2*x^4 + 140*b^3*x^6
))/(3640*b^4)
```

Maple [A]
time = 0.06, size = 47, normalized size = 0.59

method	result	size
gosper	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-140b^{3}x^{6}+126a\,b^{2}x^{4}-108a^{2}b\,x^{2}+81a^{3}\right)}{3640b^{4}}$	47
trager	$-\frac{3\left(-140b^{4}x^{8}-14a\,b^{3}x^{6}+18a^{2}b^{2}x^{4}-27a^{3}b\,x^{2}+81a^{4}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{3640b^{4}}$	58
risch	$-\frac{3\left(-140b^{4}x^{8}-14a\,b^{3}x^{6}+18a^{2}b^{2}x^{4}-27a^{3}b\,x^{2}+81a^{4}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{3640b^{4}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/3640*(b*x^2+a)^(4/3)*(-140*b^3*x^6+126*a*b^2*x^4-108*a^2*b*x^2+81*a^3)/b
^4
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{13}{3}}}{26\,b^4}-\frac{9\,(bx^2+a)^{\frac{10}{3}}\,a}{20\,b^4}+\frac{9\,(bx^2+a)^{\frac{7}{3}}\,a^2}{14\,b^4}-\frac{3\,(bx^2+a)^{\frac{4}{3}}\,a^3}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/26*(b*x^2 + a)^(13/3)/b^4 - 9/20*(b*x^2 + a)^(10/3)*a/b^4 + 9/14*(b*x^2 +
 a)^(7/3)*a^2/b^4 - 3/8*(b*x^2 + a)^(4/3)*a^3/b^4
```

Fricas [A]
time = 1.03, size = 57, normalized size = 0.71

$$\frac{3\,(140\,b^4x^8+14\,ab^3x^6-18\,a^2b^2x^4+27\,a^3bx^2-81\,a^4)(bx^2+a)^{\frac{1}{3}}}{3640\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/3640*(140*b^4*x^8 + 14*a*b^3*x^6 - 18*a^2*b^2*x^4 + 27*a^3*b*x^2 - 81*a^4
)*(b*x^2 + a)^(1/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1795 vs. $2(75) = 150$.
time = 1.35, size = 1795, normalized size = 22.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(1/3),x)
```

```
[Out] -243*a**(73/3)*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x*
*2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8
+ 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 243*a**(73/3)/(3640*a*
*20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7
*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x
**12) - 1377*a**(70/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 2184
0*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a
**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 1458*a*
*(70/3)*b*x**2/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*
x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**
10 + 3640*a**14*b**10*x**12) - 3213*a**(67/3)*b**2*x**4*(1 + b*x**2/a)**(1/
3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800
*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a*
*14*b**10*x**12) + 3645*a**(67/3)*b**2*x**4/(3640*a**20*b**4 + 21840*a**19*
b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**
8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) - 3927*a**(64/3)*
b**3*x**6*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 +
54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 218
40*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 4860*a**(64/3)*b**3*x**6/(3
640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**1
7*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b
**10*x**12) - 2163*a**(61/3)*b**4*x**8*(1 + b*x**2/a)**(1/3)/(3640*a**20*b*
*4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6
+ 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12)
+ 3645*a**(61/3)*b**4*x**8/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600
*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a*
*15*b**9*x**10 + 3640*a**14*b**10*x**12) + 1827*a**(58/3)*b**5*x**10*(1 + b
*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6
*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x*
*10 + 3640*a**14*b**10*x**12) + 1458*a**(58/3)*b**5*x**10/(3640*a**20*b**4
```

```
+ 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 5
4600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 6
573*a**(55/3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**
19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*
b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 243*a**(55/3
)*b**6*x**12/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x*
*4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10
 + 3640*a**14*b**10*x**12) + 8787*a**(52/3)*b**7*x**14*(1 + b*x**2/a)**(1/3
)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*
a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**
14*b**10*x**12) + 6498*a**(49/3)*b**8*x**16*(1 + b*x**2/a)**(1/3)/(3640*a**
20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*
x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x*
*12) + 2562*a**(46/3)*b**9*x**18*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 2
1840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 5460
0*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 420*
a**(43/3)*b**10*x**20*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*
b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**
8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12)
```

Giac [A]
time = 1.30, size = 57, normalized size = 0.71

$$\frac{3\left(140\left(bx^2+a\right)^{\frac{13}{3}}-546\left(bx^2+a\right)^{\frac{10}{3}}a+780\left(bx^2+a\right)^{\frac{7}{3}}a^2-455\left(bx^2+a\right)^{\frac{4}{3}}a^3\right)}{3640\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/3640*(140*(b*x^2 + a)^(13/3) - 546*(b*x^2 + a)^(10/3)*a + 780*(b*x^2 + a)
^(7/3)*a^2 - 455*(b*x^2 + a)^(4/3)*a^3)/b^4
```

Mupad [B]
time = 5.25, size = 55, normalized size = 0.69

$$\left(b\,x^2+a\right)^{1/3}\left(\frac{3\,x^8}{26}-\frac{243\,a^4}{3640\,b^4}+\frac{3\,a\,x^6}{260\,b}-\frac{27\,a^2\,x^4}{1820\,b^2}+\frac{81\,a^3\,x^2}{3640\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(1/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((3*x^8)/26 - (243*a^4)/(3640*b^4) + (3*a*x^6)/(260*b) -
(27*a^2*x^4)/(1820*b^2) + (81*a^3*x^2)/(3640*b^3))
```

3.666 $\int x^5 \sqrt[3]{a+bx^2}\, dx$

Optimal. Leaf size=59

$$\frac{3a^2(a+bx^2)^{4/3}}{8b^3}-\frac{3a(a+bx^2)^{7/3}}{7b^3}+\frac{3(a+bx^2)^{10/3}}{20b^3}$$

```
[Out] 3/8*a^2*(b*x^2+a)^(4/3)/b^3-3/7*a*(b*x^2+a)^(7/3)/b^3+3/20*(b*x^2+a)^(10/3)
/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{3a^2(a+bx^2)^{4/3}}{8b^3}+\frac{3(a+bx^2)^{10/3}}{20b^3}-\frac{3a(a+bx^2)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(4/3))/(8*b^3) - (3*a*(a + b*x^2)^(7/3))/(7*b^3) + (3*(a
 + b*x^2)^(10/3))/(20*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt[3]{a+bx^2}\, dx &= \frac{1}{2}\text{Subst}\left(\int x^2 \sqrt[3]{a+bx}\, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2 \sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right) dx, x, x^2\right)\\
&= \frac{3a^2(a+bx^2)^{4/3}}{8b^3}-\frac{3a(a+bx^2)^{7/3}}{7b^3}+\frac{3(a+bx^2)^{10/3}}{20b^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{3(a+bx^2)^{4/3}(9a^2-12abx^2+14b^2x^4)}{280b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(4/3)*(9*a^2 - 12*a*b*x^2 + 14*b^2*x^4))/(280*b^3)
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.61

method	result	size
gosper	$\frac{3(b x^2+a)^{\frac{4}{3}}(14b^2x^4-12ab x^2+9a^2)}{280b^3}$	36
trager	$\frac{3(14b^3x^6+2a b^2x^4-3a^2b x^2+9a^3)(b x^2+a)^{\frac{1}{3}}}{280b^3}$	47
risch	$\frac{3(14b^3x^6+2a b^2x^4-3a^2b x^2+9a^3)(b x^2+a)^{\frac{1}{3}}}{280b^3}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/280*(b*x^2+a)^(4/3)*(14*b^2*x^4-12*a*b*x^2+9*a^2)/b^3
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.80

$$\frac{3(bx^2+a)^{\frac{10}{3}}}{20b^3}-\frac{3(bx^2+a)^{\frac{7}{3}}a}{7b^3}+\frac{3(bx^2+a)^{\frac{4}{3}}a^2}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 3/20*(b*x^2 + a)^(10/3)/b^3 - 3/7*(b*x^2 + a)^(7/3)*a/b^3 + 3/8*(b*x^2 + a)
^(4/3)*a^2/b^3
```

Fricas [A]
time = 1.33, size = 46, normalized size = 0.78

$$\frac{3(14b^3x^6+2ab^2x^4-3a^2bx^2+9a^3)(bx^2+a)^{\frac{1}{3}}}{280b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/280*(14*b^3*x^6 + 2*a*b^2*x^4 - 3*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(1/3)/b^
3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(54) = 108$.
time = 0.90, size = 700, normalized size = 11.86

$$\frac{27a^{\frac{34}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}-\frac{27a^{\frac{34}{3}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}+\frac{72a^{\frac{31}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}-\frac{81a^{\frac{31}{3}}bx^2}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}+\frac{60a^{\frac{28}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}-\frac{81a^{\frac{28}{3}}b^2x^4}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}+\frac{60a^{\frac{25}{3}}b^3x^6\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}-\frac{27a^{\frac{25}{3}}b^3x^6}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}+\frac{135a^{\frac{22}{3}}b^4x^8\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}+\frac{132a^{\frac{19}{3}}b^5x^{10}\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}+\frac{42a^{\frac{16}{3}}b^6x^{12}\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(34/3)*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 84
0*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(34/3)/(280*a**8*b**3 + 840*
a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 72*a**(31/3)*b*
x**2*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b
**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(31/3)*b*x**2/(280*a**8*b**3 + 840*a
**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60*a**(28/3)*b**
2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6
*b**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(28/3)*b**2*x**4/(280*a**8*b**3 +
840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60*a**(25/3
)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840
*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(25/3)*b**3*x**6/(280*a**8*b*
*3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 135*a*
*(22/3)*b**4*x**8*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2
 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 132*a**(19/3)*b**5*x**10*(1 +
 b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4
+ 280*a**5*b**6*x**6) + 42*a**(16/3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(280*
a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6)
```

Giac [A]
time = 0.82, size = 43, normalized size = 0.73

$$\frac{3\left(14\,(bx^2+a)^{\frac{10}{3}}-40\,(bx^2+a)^{\frac{7}{3}}a+35\,(bx^2+a)^{\frac{4}{3}}a^2\right)}{280\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/280*(14*(b*x^2 + a)^(10/3) - 40*(b*x^2 + a)^(7/3)*a + 35*(b*x^2 + a)^(4/3
)*a^2)/b^3
```

Mupad [B]
time = 4.80, size = 44, normalized size = 0.75

$$\left(b\,x^2+a\right)^{1/3}\,\left(\frac{3\,x^6}{20}+\frac{27\,a^3}{280\,b^3}+\frac{3\,a\,x^4}{140\,b}-\frac{9\,a^2\,x^2}{280\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(1/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((3*x^6)/20 + (27*a^3)/(280*b^3) + (3*a*x^4)/(140*b) - (9
*a^2*x^2)/(280*b^2))
```

3.667 $\int x^3 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=38

$$-\frac{3a(a+bx^2)^{4/3}}{8b^2}+\frac{3(a+bx^2)^{7/3}}{14b^2}$$

```
[Out] -3/8*a*(b*x^2+a)^(4/3)/b^2+3/14*(b*x^2+a)^(7/3)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{3(a+bx^2)^{7/3}}{14b^2}-\frac{3a(a+bx^2)^{4/3}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(4/3))/(8*b^2) + (3*(a + b*x^2)^(7/3))/(14*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt[3]{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a\sqrt[3]{a+bx}}{b}+\frac{(a+bx)^{4/3}}{b}\right) dx, x, x^2\right)\\
&= -\frac{3a(a+bx^2)^{4/3}}{8b^2}+\frac{3(a+bx^2)^{7/3}}{14b^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx^2}\,(-3a^2+abx^2+4b^2x^4)}{56b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(-3*a^2 + a*b*x^2 + 4*b^2*x^4))/(56*b^2)
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.66

method	result	size
gosper	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-4b\,x^{2}+3a\right)}{56b^{2}}$	25
trager	$-\frac{3\left(-4b^{2}x^{4}-ab\,x^{2}+3a^{2}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{56b^{2}}$	36
risch	$-\frac{3\left(-4b^{2}x^{4}-ab\,x^{2}+3a^{2}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{56b^{2}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/56*(b*x^2+a)^(4/3)*(-4*b*x^2+3*a)/b^2
```

Maxima [A]
time = 0.33, size = 30, normalized size = 0.79

$$\frac{3\,(bx^2+a)^{\frac{7}{3}}}{14\,b^2}-\frac{3\,(bx^2+a)^{\frac{4}{3}}a}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b^2 - 3/8*(b*x^2 + a)^(4/3)*a/b^2
```

Fricas [A]
time = 1.95, size = 34, normalized size = 0.89

$$\frac{3\,(4\,b^2x^4+abx^2-3\,a^2)(bx^2+a)^{\frac{1}{3}}}{56\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/56*(4*b^2*x^4 + a*b*x^2 - 3*a^2)*(b*x^2 + a)^(1/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(34) = 68$.
time = 0.59, size = 223, normalized size = 5.87

$$-\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}+\frac{9a^{\frac{13}{3}}}{56a^2b^2+56ab^3x^2}-\frac{6a^{\frac{10}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}+\frac{9a^{\frac{10}{3}}bx^2}{56a^2b^2+56ab^3x^2}+\frac{15a^{\frac{7}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}+\frac{12a^{\frac{4}{3}}b^3x^6\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/3),x)
```

```
[Out] -9*a**(13/3)*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**(
13/3)/(56*a**2*b**2 + 56*a*b**3*x**2) - 6*a**(10/3)*b*x**2*(1 + b*x**2/a)**
(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**(10/3)*b*x**2/(56*a**2*b**2 +
56*a*b**3*x**2) + 15*a**(7/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2
 + 56*a*b**3*x**2) + 12*a**(4/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(56*a**2*b
**2 + 56*a*b**3*x**2)
```

Giac [A]
time = 1.13, size = 29, normalized size = 0.76

$$\frac{3\left(4\left(bx^2+a\right)^{\frac{7}{3}}-7\left(bx^2+a\right)^{\frac{4}{3}}a\right)}{56\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/56*(4*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(4/3)*a)/b^2
```

Mupad [B]
time = 4.72, size = 33, normalized size = 0.87

$$\left(b\,x^2+a\right)^{1/3}\,\left(\frac{3\,x^4}{14}-\frac{9\,a^2}{56\,b^2}+\frac{3\,a\,x^2}{56\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(1/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((3*x^4)/14 - (9*a^2)/(56*b^2) + (3*a*x^2)/(56*b))
```

3.668 $\int x\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=18

$$\frac{3(a+bx^2)^{4/3}}{8b}$$

[Out] 3/8*(b*x^2+a)^(4/3)/b

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{3(a+bx^2)^{4/3}}{8b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^(1/3),x]

[Out] (3*(a + b*x^2)^(4/3))/(8*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt[3]{a+bx^2}\,dx = \frac{3(a+bx^2)^{4/3}}{8b}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{3(a+bx^2)^{4/3}}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^(1/3),x]

[Out] (3*(a + b*x^2)^(4/3))/(8*b)

Maple [A]
time = 0.03, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{3(b\,x^2+a)^{\frac{4}{3}}}{8b}$	15
derivativedivides	$\frac{3(b\,x^2+a)^{\frac{4}{3}}}{8b}$	15
default	$\frac{3(b\,x^2+a)^{\frac{4}{3}}}{8b}$	15
trager	$\frac{3(b\,x^2+a)^{\frac{4}{3}}}{8b}$	15
risch	$\frac{3(b\,x^2+a)^{\frac{4}{3}}}{8b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*(b*x^2+a)^(4/3)/b
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.78

$$\frac{3\,{\left(b x^{2} + a\right)}^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

Fricas [A]
time = 1.35, size = 14, normalized size = 0.78

$$\frac{3\,{\left(b x^{2} + a\right)}^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.
time = 0.07, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3a\sqrt[3]{a + bx^2}}{8b} + \frac{3x^2\sqrt[3]{a + bx^2}}{8} & \text{for}\: b \neq 0 \\ \frac{\sqrt[3]{a}\,x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/3),x)
```

```
[Out] Piecewise((3*a*(a + b*x**2)**(1/3)/(8*b) + 3*x**2*(a + b*x**2)**(1/3)/8, Ne
(b, 0)), (a**(1/3)*x**2/2, True))
```

Giac [A]
time = 1.11, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

Mupad [B]
time = 4.66, size = 14, normalized size = 0.78

$$\frac{3\,(b\,x^2+a)^{4/3}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(1/3),x)
```

```
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

3.669 $\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx$

Optimal. Leaf size=101

$$\frac{3}{2}\sqrt[3]{a+bx^2}-\frac{1}{2}\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)+\frac{3}{4}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)$$

```
[Out] 3/2*(b*x^2+a)^(1/3)-1/2*a^(1/3)*ln(x)+3/4*a^(1/3)*ln(a^(1/3)-(b*x^2+a)^(1/3
))-1/2*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 52, 59, 631, 210, 31}

$$-\frac{1}{2}\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)+\frac{3}{2}\sqrt[3]{a+bx^2}+\frac{3}{4}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/2 - (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^2\right)\\ &= \frac{3}{2}\sqrt[3]{a+bx^2}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\ &= \frac{3}{2}\sqrt[3]{a+bx^2}-\frac{1}{2}\sqrt[3]{a}\,\log(x)-\frac{1}{4}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)-\frac{1}{4}\left(3a^{2/}\right.\\ &= \frac{3}{2}\sqrt[3]{a+bx^2}-\frac{1}{2}\sqrt[3]{a}\,\log(x)+\frac{3}{4}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{}{-3}\right.\\ &= \frac{3}{2}\sqrt[3]{a+bx^2}-\frac{1}{2}\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)+\frac{3}{4}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\right.\end{aligned}$$

Mathematica [A]
time = 0.10, size = 126, normalized size = 1.25

$$\frac{1}{4}\left(6\sqrt[3]{a+bx^2}-2\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{a}\,\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-\sqrt[3]{a}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x,x]
```

```
[Out] (6*(a + b*x^2)^(1/3) - 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/
a^(1/3))/Sqrt[3]] + 2*a^(1/3)*Log[-a^(1/3) + (a + b*x^2)^(1/3)] - a^(1/3)*L
og[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{2}+a)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x,x)
```

Maxima [A]
time = 0.51, size = 97, normalized size = 0.96

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^{2}+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{1}{3}}\log\left((bx^{2}+a)^{\frac{2}{3}}+(bx^{2}+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx^{2}+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\,(bx^{2}+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a)^(1/3
)
```

Fricas [A]
time = 1.89, size = 102, normalized size = 1.01

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,(bx^{2}+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{4}\,a^{\frac{1}{3}}\log\left((bx^{2}+a)^{\frac{2}{3}}+(bx^{2}+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx^{2}+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\,(bx^{2}+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqrt
(3)*a)/a) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a)^
(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 46, normalized size = 0.46

$$-\frac{\sqrt[3]{b}\,x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x,x)
```

```
[Out] -b**(1/3)*x**(2/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(2/3))
```

Giac [A]
time = 1.63, size = 98, normalized size = 0.97

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{1}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{1}{3}}\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{2}\,(bx^2+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(1/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/2*(b*x^2 + a)
^(1/3)
```

Mupad [B]
time = 4.74, size = 115, normalized size = 1.14

$$\frac{a^{1/3}\ln\left(\frac{9\,a\,(b\,x^2+a)^{1/3}}{4}-\frac{9\,a^{4/3}}{4}\right)}{2}+\frac{3\,(b\,x^2+a)^{1/3}}{2}-\frac{a^{1/3}\ln\left(\frac{9\,a^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2}+\frac{9\,a\,(b\,x^2+a)^{1/3}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2}+a^{1/3}\ln\left(\frac{9\,a\,(b\,x^2+a)^{1/3}}{2}-9\,a^{4/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x,x)
```

```
[Out] (a^(1/3)*log((9*a*(a + b*x^2)^(1/3))/4 - (9*a^(4/3))/4))/2 + (3*(a + b*x^2)
^(1/3))/2 - (a^(1/3)*log((9*a^(4/3)*((3^(1/2)*1i)/2 + 1/2))/2 + (9*a*(a + b
*x^2)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/2 + a^(1/3)*log((9*a*(a + b*x^2)^(1
/3))/2 - 9*a^(4/3)*((3^(1/2)*1i)/4 - 1/4))*((3^(1/2)*1i)/4 - 1/4)
```

3.670 $\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt[3]{a+bx^2}}{2x^2}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}$$

```
[Out] -1/2*(b*x^2+a)^(1/3)/x^2-1/6*b*ln(x)/a^(2/3)+1/4*b*ln(a^(1/3)-(b*x^2+a)^(1/
3))/a^(2/3)-1/6*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a
^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 43, 59, 631, 210, 31}

$$-\frac{b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}-\frac{\sqrt[3]{a+bx^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x^2)^(1/3)/x^2 - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(2*Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(4*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[3]{a+bx^2}}{2x^2} + \frac{1}{6}b\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b\log(x)}{6a^{2/3}} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+\cdots}\right.}{4\sqrt[3]{\cdots}}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}} + \frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a\cdots}}{\sqrt[3]{\cdots}}\right.}{2a^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}\,a^{2/3}} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}$$

Mathematica [A]

time = 0.15, size = 135, normalized size = 1.26

$$-\frac{6a^{2/3}\sqrt[3]{a+bx^2}+2\sqrt{3}\,bx^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)+bx^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{12a^{2/3}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^3,x]
```

```
[Out] -1/12*(6*a^(2/3)*(a + b*x^2)^(1/3) + 2*Sqrt[3]*b*x^2*ArcTan[(1 + (2*(a + b*
x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x^2*Log[-a^(1/3) + (a + b*x^2)^(1/3)] +
 b*x^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(a^(2/
3)*x^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^3,x)
```

Maxima [A]
time = 0.51, size = 103, normalized size = 0.96

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{6\,a^{\frac{2}{3}}}-\frac{b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{12\,a^{\frac{2}{3}}}+\frac{b\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6\,a^{\frac{2}{3}}}-\frac{(bx^2+a)^{\frac{1}{3}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(2/3) - 1/12*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(2/3) + 1/6*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/2*(b*x^2 +
a)^(1/3)/x^2
```

Fricas [A]
time = 1.42, size = 155, normalized size = 1.45

$$-\frac{2\sqrt{3}\,(a^2)^{\frac{1}{6}}abx^2\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}\,(a^2)^{\frac{1}{3}}a+2\sqrt{3}\,(bx^2+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{3\,a^2}\right)+(a^2)^{\frac{2}{3}}bx^2\log\left((bx^2+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx^2+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)-2\,(a^2)^{\frac{2}{3}}bx^2\log\left((bx^2+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)+6\,(bx^2+a)^{\frac{1}{3}}a^2}{12\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(3)*(a^2)^(1/6)*a*b*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^
(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*b*x^2
*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) -
 2*(a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^2 + a)
^(1/3)*a^2)/(a^2*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.58, size = 42, normalized size = 0.39

$$-\frac{\sqrt[3]{b}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**3,x)
```

```
[Out] -b**(1/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(4/3)*gamma(5/3))
```

Giac [A]
time = 1.39, size = 115, normalized size = 1.07

$$-\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2b^2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{6\left(bx^2+a\right)^{\frac{1}{3}}b}{x^2}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(2/3) + b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(2/3) - 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*
x^2 + a)^(1/3)*b/x^2)/b
```

Mupad [B]
time = 4.88, size = 125, normalized size = 1.17

$$\frac{b\ln\left(\frac{3b\left(bx^2+a\right)^{1/3}}{2}-\frac{3a^{1/3}b}{2}\right)}{6a^{2/3}}-\frac{\left(bx^2+a\right)^{1/3}}{2x^2}-\frac{\ln\left(\frac{3a^{1/3}\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4}+\frac{3b\left(bx^2+a\right)^{1/3}}{2}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{12a^{2/3}}-\frac{\ln\left(\frac{3a^{1/3}\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{4}+\frac{3b\left(bx^2+a\right)^{1/3}}{2}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{12a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^3,x)
```

```
[Out] (b*log((3*b*(a + b*x^2)^(1/3))/2 - (3*a^(1/3)*b)/2))/(6*a^(2/3)) - (a + b*x
^2)^(1/3)/(2*x^2) - (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/4 + (3*b*(a + b*x^2
)^(1/3))/2)*(b - 3^(1/2)*b*1i))/(12*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1/2)
*b*1i))/4 + (3*b*(a + b*x^2)^(1/3))/2)*(b + 3^(1/2)*b*1i))/(12*a^(2/3))
```

3.671 $\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx$

Optimal. Leaf size=135

$$-\frac{\sqrt[3]{a+bx^2}}{4x^4}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}+\frac{b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}$$

```
[Out] -1/4*(b*x^2+a)^(1/3)/x^4-1/12*b*(b*x^2+a)^(1/3)/a/x^2+1/18*b^2*ln(x)/a^(5/3
)-1/12*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(5/3)+1/18*b^2*arctan(1/3*(a^(1/3)
+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {272, 43, 44, 59, 631, 210, 31}

$$\frac{b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}-\frac{\sqrt[3]{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^5,x]
```

```
[Out] -1/4*(a + b*x^2)^(1/3)/x^4 - (b*(a + b*x^2)^(1/3))/(12*a*x^2) + (b^2*ArcTan
[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(5/3)) +
(b^2*Log[x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(5
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} + \frac{1}{12}b\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} - \frac{b^2\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)}{18a}$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2\log(x)}{18a^{5/3}} + \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{12a^{5/3}} + \frac{b^2\text{Sub}}{}$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2\log(x)}{18a^{5/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}} - \frac{b^2\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{}$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{5/3}} + \frac{b^2\log(x)}{18a^{5/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{\right.}}{12a^{5/3}}$$

Mathematica [A]
time = 0.16, size = 158, normalized size = 1.17

$$\frac{(-3a-bx^2)\sqrt[3]{a+bx^2}}{12ax^4} + \frac{b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{5/3}} - \frac{b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)}{18a^{5/3}} + \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{36a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^5,x]
```

```
[Out] ((-3*a - b*x^2)*(a + b*x^2)^(1/3))/(12*a*x^4) + (b^2*ArcTan[1/Sqrt[3] + (2*
(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(5/3)) - (b^2*Log[-a^(1
/3) + (a + b*x^2)^(1/3)])/(18*a^(5/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a + b*
x^2)^(1/3) + (a + b*x^2)^(2/3)])/(36*a^(5/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^5,x)
```

Maxima [A]
time = 0.52, size = 155, normalized size = 1.15

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{5}{3}}}+\frac{b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36\,a^{\frac{5}{3}}}-\frac{b^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,a^{\frac{5}{3}}}-\frac{(bx^2+a)^{\frac{4}{3}}b^2+2\,(bx^2+a)^{\frac{1}{3}}ab^2}{12\,\left((bx^2+a)^2a-2\,(bx^2+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(5/3) + 1/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(5/3) - 1/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(5/3) - 1/12*(
(b*x^2 + a)^(4/3)*b^2 + 2*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2*a - 2*(b*
x^2 + a)*a^2 + a^3)
```

Fricas [A]
time = 1.56, size = 199, normalized size = 1.47

$$\frac{2\,\sqrt{3}\,ab^2x^4\sqrt{-(-a^2)^{\frac{1}{3}}}\,\arctan\left(-\frac{\left(\sqrt{3}\,(-a^2)^{\frac{1}{3}}a-2\,\sqrt{3}\,(bx^2+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}\right)\sqrt{-(-a^2)^{\frac{1}{3}}}}{3\,a^2}\right)+(-a^2)^{\frac{2}{3}}\,b^2x^4\log\left((bx^2+a)^{\frac{2}{3}}a-(-a^2)^{\frac{1}{3}}\,a+(bx^2+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}\right)-2\,(-a^2)^{\frac{2}{3}}\,b^2x^4\log\left((bx^2+a)^{\frac{1}{3}}a-(-a^2)^{\frac{2}{3}}\right)-3\,(a^2bx^2+3\,a^3)(bx^2+a)^{\frac{1}{3}}}{36\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="fricas")
```

```
[Out] 1/36*(2*sqrt(3)*a*b^2*x^4*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2)
 + (-a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 +
 a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a -
(-a^2)^(2/3)) - 3*(a^2*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^3*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 42, normalized size = 0.31

$$-\frac{\sqrt[3]{b}\,\Gamma\left(\frac{5}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{10}{3}}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**5,x)
```

```
[Out] -b**(1/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(10/3)*gamma(8/3))
```

Giac [A]
time = 1.56, size = 140, normalized size = 1.04

$$\frac{\frac{2\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\,b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left(\left(bx^2+a\right)^{\frac{4}{3}}b^3+2\left(bx^2+a\right)^{\frac{1}{3}}ab^3\right)}{ab^2x^4}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="giac")
```

```
[Out] 1/36*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(5/3) + b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3))/a^(5/3) - 2*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*
x^2 + a)^(4/3)*b^3 + 2*(b*x^2 + a)^(1/3)*a*b^3)/(a*b^2*x^4))/b
```

Mupad [B]
time = 5.11, size = 217, normalized size = 1.61

$$\frac{b^2\ln\left(\frac{b^2}{2\,(-a)^{2/3}}-\frac{b^2\,(b\,x^2+a)^{1/3}}{2\,a}\right)}{18\,(-a)^{5/3}}-\frac{\ln\left(\frac{b^2+\sqrt{3}\,b^2\,1\mathrm{i}}{4\,(-a)^{2/3}}+\frac{b^2\,(b\,x^2+a)^{1/3}}{2\,a}\right)\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{36\,(-a)^{5/3}}-\frac{\frac{b^2\,(b\,x^2+a)^{1/3}}{3}+\frac{b^2\,(b\,x^2+a)^{4/3}}{6\,a}}{2\,(b\,x^2+a)^2-4\,a\,(b\,x^2+a)+2\,a^2}+\frac{b^2\ln\left(\frac{b^2\,(b\,x^2+a)^{1/3}}{2\,a}-\frac{b^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,(-a)^{2/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18\,(-a)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^5,x)
```

```
[Out] (b^2*log(b^2/(2*(-a)^(2/3)) - (b^2*(a + b*x^2)^(1/3))/(2*a)))/(18*(-a)^(5/3
)) - (log((3^(1/2)*b^2*1i + b^2)/(4*(-a)^(2/3)) + (b^2*(a + b*x^2)^(1/3))/(
2*a))*(3^(1/2)*b^2*1i + b^2))/(36*(-a)^(5/3)) - ((b^2*(a + b*x^2)^(1/3))/3
+ (b^2*(a + b*x^2)^(4/3))/(6*a))/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2
) + (b^2*log((b^2*(a + b*x^2)^(1/3))/(2*a) - (b^2*((3^(1/2)*1i)/2 - 1/2))/(
2*(-a)^(2/3)))*((3^(1/2)*1i)/2 - 1/2))/(18*(-a)^(5/3))
```

3.672 $\int x^4\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=314

$$-\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}+\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\cdots}{\left(\left(1-\cdots\right.\right.}}\cdots}{935b^3x\cdots}$$

```
[Out] -54/935*a^2*x*(b*x^2+a)^(1/3)/b^2+6/187*a*x^3*(b*x^2+a)^(1/3)/b+3/17*x^5*(b
*x^2+a)^(1/3)-54/935*3^(3/4)*a^3*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x
^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I
-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^3/x/(-a^(1/
3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)
```

Rubi [A]
time = 0.20, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {285, 327, 242, 225}

$$-\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{935b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}+\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(1/3),x]
```

```
[Out] (-54*a^2*x*(a + b*x^2)^(1/3))/(935*b^2) + (6*a*x^3*(a + b*x^2)^(1/3))/(187*
b) + (3*x^5*(a + b*x^2)^(1/3))/17 - (54*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
+ b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
```

```
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\sqrt[3]{a+bx^2}\,dx = \frac{3}{17}x^5\sqrt[3]{a+bx^2} + \frac{1}{17}(2a)\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx$$
$$= \frac{6ax^3\sqrt[3]{a+bx^2}}{187b} + \frac{3}{17}x^5\sqrt[3]{a+bx^2} - \frac{(18a^2)\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx}{187b}$$
$$= -\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2} + \frac{6ax^3\sqrt[3]{a+bx^2}}{187b} + \frac{3}{17}x^5\sqrt[3]{a+bx^2} + \frac{(54a^3)\int \frac{1}{(a+bx^2)^{2/3}}\,dx}{935b^2}$$
$$= -\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2} + \frac{6ax^3\sqrt[3]{a+bx^2}}{187b} + \frac{3}{17}x^5\sqrt[3]{a+bx^2} + \frac{\left(81a^3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-}}\right.}{935}$$
$$= -\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2} + \frac{6ax^3\sqrt[3]{a+bx^2}}{187b} + \frac{3}{17}x^5\sqrt[3]{a+bx^2} - \frac{54\ 3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.86, size = 94, normalized size = 0.30

$$\frac{3x\sqrt[3]{a+bx^2}\left(\sqrt[3]{1+\frac{bx^2}{a}}\,(-9a^2+2abx^2+11b^2x^4)+9a^2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{187b^2\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-9*a^2 + 2*a*b*x^2 + 11*b^2*
x^4) + 9*a^2*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)]))/(187*b^2*(1
+ (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^4\left(b\,x^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/3),x)
[Out] int(x^4*(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3)*x^4, x)
```

Sympy [A]
time = 0.44, size = 29, normalized size = 0.09

$$\frac{\sqrt[3]{a}\, x^5 {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/3),x)
[Out] a**(1/3)*x**5*hyper((-1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(b\,x^2+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(1/3),x)
```

```
[Out] int(x^4*(a + b*x^2)^(1/3), x)
```

3.673 $\int x^2\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=290

$$\frac{6ax\sqrt[3]{a+bx^2}}{55b}+\frac{3}{11}x^3\sqrt[3]{a+bx^2}+\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\cdots}}}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a}\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right.}}}$$

```
[Out] 6/55*a*x*(b*x^2+a)^(1/3)/b+3/11*x^3*(b*x^2+a)^(1/3)+6/55*3^(3/4)*a^2*(a^(1/
3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)
^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/
2*6^(1/2)-1/2*2^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a
)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 242, 225}

$$\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{6ax\sqrt[3]{a+bx^2}}{55b}+\frac{3}{11}x^3\sqrt[3]{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(1/3),x]
```

```
[Out] (6*a*x*(a + b*x^2)^(1/3))/(55*b) + (3*x^3*(a + b*x^2)^(1/3))/11 + (6*3^(3/4
)*Sqrt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/
3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^2*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
```

```
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\sqrt[3]{a+bx^2}\,dx &= \frac{3}{11}x^3\sqrt[3]{a+bx^2}+\frac{1}{11}(2a)\int\frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= \frac{6ax\sqrt[3]{a+bx^2}}{55b}+\frac{3}{11}x^3\sqrt[3]{a+bx^2}-\frac{\left(6a^2\right)\int\frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{55b}\\
&= \frac{6ax\sqrt[3]{a+bx^2}}{55b}+\frac{3}{11}x^3\sqrt[3]{a+bx^2}-\frac{\left(9a^2\sqrt{bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right.}{55b^2x}\\
&= \frac{6ax\sqrt[3]{a+bx^2}}{55b}+\frac{3}{11}x^3\sqrt[3]{a+bx^2}+\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+}{\left(\left(1\right.\right.}}}{55b^2x}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.56, size = 62, normalized size = 0.21

$$\frac{3x\sqrt[3]{a+bx^2}\left(a+bx^2-\frac{a\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*(a + b*x^2 - (a*Hypergeometric2F1[-1/3, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(11*b)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(b\,x^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/3),x)
```

```
[Out] int(x^2*(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*x^2, x)
```

Sympy [A]
time = 0.40, size = 29, normalized size = 0.10

$$\frac{\sqrt[3]{a}\, x^3 {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*x**3*hyper((-1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(b\,x^2+a\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/3),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/3), x)
```

3.674 $\int \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=266

$$\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1\right.}{\left(1\right.}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] 3/5*x*(b*x^2+a)^(1/3)-2/5*3^(3/4)*a*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(
b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),
2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b/x/(-a^(1
/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)
```

Rubi [A]

time = 0.09, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 242, 225}

$$\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a +
b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rubi steps

$$\int \sqrt[3]{a+bx^2}\,dx = \frac{3}{5}x\sqrt[3]{a+bx^2}+\frac{1}{5}(2a)\int \frac{1}{(a+bx^2)^{2/3}}\,dx$$

$$= \frac{3}{5}x\sqrt[3]{a+bx^2}+\frac{\left(3a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{5bx}$$

$$= \frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b\cdots}\right.}}\cdots}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.54, size = 46, normalized size = 0.17

$$\frac{x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3),x]
[Out] (x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3),x)
[Out] int((b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3), x)
```

Sympy [A]
time = 0.38, size = 26, normalized size = 0.10

$$\sqrt[3]{a}\,x\,{}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3),x)
[Out] a**(1/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3), x)
```

Mupad [B]
time = 4.59, size = 37, normalized size = 0.14

$$\frac{x\,{\left(b\,x^2+a\right)}^{1/3}\,{{}}_2{\mathrm{F}}_1\left(-\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3),x)
```

```
[Out] (x*(a + b*x^2)^(1/3)*hypergeom([-1/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(1/3)
```

3.675 $\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=260

$$-\frac{\sqrt[3]{a+bx^2}}{x}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] -(b*x^2+a)^(1/3)/x-2/3*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3
)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2)
)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/x/(-a^(1/3)*(a^
(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {283, 242, 225}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x^2)^(1/3)/x) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
```

```
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx &= -\frac{\sqrt[3]{a+bx^2}}{x}+\frac{1}{3}(2b)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx\\ &= -\frac{\sqrt[3]{a+bx^2}}{x}+\frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{x}\\ &= -\frac{\sqrt[3]{a+bx^2}}{x}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\cdots}{\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a\cdots}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right.}}\cdots}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.79, size = 49, normalized size = 0.19

$$-\frac{\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(1/3)*Hypergeometric2F1[-1/2, -1/3, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(1/3)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \,x^{2}+a \right)^{\frac{1}{3}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^2,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/x^2, x)
```

Sympy [A]
time = 0.42, size = 29, normalized size = 0.11

$$-\frac{\sqrt[3]{a}\, {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{1}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**2,x)
```

```
[Out] -a**(1/3)*hyper((-1/2, -1/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/x^2, x)
```

Mupad [B]
time = 4.78, size = 40, normalized size = 0.15

$$-\frac{3\left(b\,x^2+a\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{6};\ \frac{7}{6};\ -\frac{a}{b\,x^2}\right)}{x\left(\frac{a}{b\,x^2}+1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^2,x)
[Out] -(3*(a + b*x^2)^(1/3)*hypergeom([-1/3, 1/6], 7/6, -a/(b*x^2)))/(x*(a/(b*x^2
) + 1)^(1/3))
```

3.676 $\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}+\frac{2\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\Big(\cdots}{9\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\cdots}\right)}}}$$

```
[Out] -1/3*(b*x^2+a)^(1/3)/x^3-2/9*b*(b*x^2+a)^(1/3)/a/x+2/27*b*(a^(1/3)-(b*x^2+a
)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x
^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1
/2*2^(1/2))*3^(3/4)/a/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 331, 242, 225}

$$\frac{2\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}-\frac{\sqrt[3]{a+bx^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^4,x]
```

```
[Out] -1/3*(a + b*x^2)^(1/3)/x^3 - (2*b*(a + b*x^2)^(1/3))/(9*a*x) + (2*Sqrt[2 -
Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2
)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*Sqrt[-((a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[3]{a+bx^2}}{3x^3}+\frac{1}{9}(2b)\int \frac{1}{x^2\,(a+bx^2)^{2/3}}\,dx$$
$$= -\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}-\frac{(2b^2)\int \frac{1}{(a+bx^2)^{2/3}}\,dx}{27a}$$
$$= -\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}-\frac{\left(b\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{9ax}$$
$$= -\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}+\frac{2\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}{9\sqrt[4]{3}\,ax\sqrt{-\frac{}{\left(\left(1\right.\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.18

$$-\frac{\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^4,x]
```

```
[Out] -1/3*((a + b*x^2)^(1/3)*Hypergeometric2F1[-3/2, -1/3, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^4,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3)/x^4, x)
```

Sympy [A]
time = 0.46, size = 34, normalized size = 0.12

$$-\frac{\sqrt[3]{a}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},-\frac{1}{3}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**4,x)
[Out] -a**(1/3)*hyper((-3/2, -1/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^4,x)
[Out] int((a + b*x^2)^(1/3)/x^4, x)
```

3.677 $\int x^7(a+bx^2)^{2/3}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3(a+bx^2)^{5/3}}{10b^4}+\frac{9a^2(a+bx^2)^{8/3}}{16b^4}-\frac{9a(a+bx^2)^{11/3}}{22b^4}+\frac{3(a+bx^2)^{14/3}}{28b^4}$$

```
[Out] -3/10*a^3*(b*x^2+a)^(5/3)/b^4+9/16*a^2*(b*x^2+a)^(8/3)/b^4-9/22*a*(b*x^2+a)
^(11/3)/b^4+3/28*(b*x^2+a)^(14/3)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {272, 45}

$$-\frac{3a^3(a+bx^2)^{5/3}}{10b^4}+\frac{9a^2(a+bx^2)^{8/3}}{16b^4}+\frac{3(a+bx^2)^{14/3}}{28b^4}-\frac{9a(a+bx^2)^{11/3}}{22b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(5/3))/(10*b^4) + (9*a^2*(a + b*x^2)^(8/3))/(16*b^4) -
(9*a*(a + b*x^2)^(11/3))/(22*b^4) + (3*(a + b*x^2)^(14/3))/(28*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{2/3}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{2/3}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right)\,dx,\right.\\&=-\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.62

$$\frac{3(a+bx^2)^{5/3}(-81a^3+135a^2bx^2-180ab^2x^4+220b^3x^6)}{6160b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3)*(-81*a^3 + 135*a^2*b*x^2 - 180*a*b^2*x^4 + 220*b^3*x^6
))/(6160*b^4)
```

Maple [A]
time = 0.05, size = 47, normalized size = 0.59

method	result	size
gosper	$-\frac{3(b\,x^2+a)^{\frac{5}{3}}(-220b^3x^6+180a\,b^2x^4-135a^2b\,x^2+81a^3)}{6160b^4}$	47
trager	$-\frac{3(-220b^4x^8-40a\,b^3x^6+45a^2b^2x^4-54a^3b\,x^2+81a^4)(b\,x^2+a)^{\frac{2}{3}}}{6160b^4}$	58
risch	$-\frac{3(-220b^4x^8-40a\,b^3x^6+45a^2b^2x^4-54a^3b\,x^2+81a^4)(b\,x^2+a)^{\frac{2}{3}}}{6160b^4}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/6160*(b*x^2+a)^(5/3)*(-220*b^3*x^6+180*a*b^2*x^4-135*a^2*b*x^2+81*a^3)/b
^4
```

Maxima [A]
time = 0.33, size = 64, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{14}{3}}}{28\,b^4}-\frac{9\,(bx^2+a)^{\frac{11}{3}}\,a}{22\,b^4}+\frac{9\,(bx^2+a)^{\frac{8}{3}}\,a^2}{16\,b^4}-\frac{3\,(bx^2+a)^{\frac{5}{3}}\,a^3}{10\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/28*(b*x^2 + a)^(14/3)/b^4 - 9/22*(b*x^2 + a)^(11/3)*a/b^4 + 9/16*(b*x^2 +
 a)^(8/3)*a^2/b^4 - 3/10*(b*x^2 + a)^(5/3)*a^3/b^4
```

Fricas [A]
time = 0.97, size = 57, normalized size = 0.71

$$\frac{3\,(220\,b^4x^8+40\,ab^3x^6-45\,a^2b^2x^4+54\,a^3bx^2-81\,a^4)(bx^2+a)^{\frac{2}{3}}}{6160\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="fricas")

[Out] 3/6160*(220*b^4*x^8 + 40*a*b^3*x^6 - 45*a^2*b^2*x^4 + 54*a^3*b*x^2 - 81*a^4)*(b*x^2 + a)^(2/3)/b^4

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1795 vs. $2(75) = 150$.
time = 1.45, size = 1795, normalized size = 22.44

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(b*x**2+a)**(2/3),x)

[Out] -243*a**(74/3)*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 243*a**(74/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 1296*a**(71/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 1458*a**(71/3)*b*x**2/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 2808*a**(68/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 3645*a**(68/3)*b**2*x**4/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 3120*a**(65/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4860*a**(65/3)*b**3*x**6/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 1050*a**(62/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 3645*a**(62/3)*b**4*x**8/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4032*a**(59/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 1458*a**(59/3)*b**5*x**10/(6160*

```
a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b
**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**1
0*x**12) + 11004*a**(56/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**
4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6
+ 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12)
+ 243*a**(56/3)*b**6*x**12/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400
*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a
**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 14352*a**(53/3)*b**7*x**14*(1 +
 b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b*
*6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9
*x**10 + 6160*a**14*b**10*x**12) + 10485*a**(50/3)*b**8*x**16*(1 + b*x**2/a
)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 +
 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 +
6160*a**14*b**10*x**12) + 4080*a**(47/3)*b**9*x**18*(1 + b*x**2/a)**(2/3)/(
6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a*
*17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14
*b**10*x**12) + 660*a**(44/3)*b**10*x**20*(1 + b*x**2/a)**(2/3)/(6160*a**20
*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x
**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**
12)
```

Giac [A]
time = 1.72, size = 57, normalized size = 0.71

$$\frac{3\left(220\left(bx^2+a\right)^{\frac{14}{3}}-840\left(bx^2+a\right)^{\frac{11}{3}}a+1155\left(bx^2+a\right)^{\frac{8}{3}}a^2-616\left(bx^2+a\right)^{\frac{5}{3}}a^3\right)}{6160\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/6160*(220*(b*x^2 + a)^(14/3) - 840*(b*x^2 + a)^(11/3)*a + 1155*(b*x^2 + a
)^(8/3)*a^2 - 616*(b*x^2 + a)^(5/3)*a^3)/b^4
```

Mupad [B]
time = 4.63, size = 55, normalized size = 0.69

$$\left(b\,x^2+a\right)^{2/3}\,\left(\frac{3\,x^8}{28}-\frac{243\,a^4}{6160\,b^4}+\frac{3\,a\,x^6}{154\,b}-\frac{27\,a^2\,x^4}{1232\,b^2}+\frac{81\,a^3\,x^2}{3080\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(2/3),x)
```

```
[Out] (a + b*x^2)^(2/3)*((3*x^8)/28 - (243*a^4)/(6160*b^4) + (3*a*x^6)/(154*b) -
(27*a^2*x^4)/(1232*b^2) + (81*a^3*x^2)/(3080*b^3))
```

3.678 $\int x^5(a+bx^2)^{2/3}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2(a+bx^2)^{5/3}}{10b^3}-\frac{3a(a+bx^2)^{8/3}}{8b^3}+\frac{3(a+bx^2)^{11/3}}{22b^3}$$

```
[Out] 3/10*a^2*(b*x^2+a)^(5/3)/b^3-3/8*a*(b*x^2+a)^(8/3)/b^3+3/22*(b*x^2+a)^(11/3
)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {272, 45}

$$\frac{3a^2(a+bx^2)^{5/3}}{10b^3}+\frac{3(a+bx^2)^{11/3}}{22b^3}-\frac{3a(a+bx^2)^{8/3}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(5/3))/(10*b^3) - (3*a*(a + b*x^2)^(8/3))/(8*b^3) + (3*(
a + b*x^2)^(11/3))/(22*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{2/3}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{2/3}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right)\,dx,x,x^2\right)\\&=\frac{3a^2(a+bx^2)^{5/3}}{10b^3}-\frac{3a(a+bx^2)^{8/3}}{8b^3}+\frac{3(a+bx^2)^{11/3}}{22b^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{3(a+bx^2)^{5/3}(9a^2-15abx^2+20b^2x^4)}{440b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(a + b*x^2)^(2/3),x]

[Out] (3*(a + b*x^2)^(5/3)*(9*a^2 - 15*a*b*x^2 + 20*b^2*x^4))/(440*b^3)

Maple [A]
time = 0.04, size = 36, normalized size = 0.61

method	result	size
gosper	$\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(20b^{2}x^{4}-15ab\,x^{2}+9a^{2}\right)}{440b^{3}}$	36
trager	$\frac{3\left(20b^{3}x^{6}+5a\,b^{2}x^{4}-6a^{2}b\,x^{2}+9a^{3}\right)\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{440b^{3}}$	47
risch	$\frac{3\left(20b^{3}x^{6}+5a\,b^{2}x^{4}-6a^{2}b\,x^{2}+9a^{3}\right)\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{440b^{3}}$	47

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)

[Out] 3/440*(b*x^2+a)^(5/3)*(20*b^2*x^4-15*a*b*x^2+9*a^2)/b^3

Maxima [A]
time = 0.28, size = 47, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{11}{3}}}{22\,b^3}-\frac{3\,(bx^2+a)^{\frac{8}{3}}a}{8\,b^3}+\frac{3\,(bx^2+a)^{\frac{5}{3}}a^2}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="maxima")

[Out] 3/22*(b*x^2 + a)^(11/3)/b^3 - 3/8*(b*x^2 + a)^(8/3)*a/b^3 + 3/10*(b*x^2 + a
)^(5/3)*a^2/b^3

Fricas [A]
time = 0.81, size = 46, normalized size = 0.78

$$\frac{3\,(20\,b^3x^6+5\,ab^2x^4-6\,a^2bx^2+9\,a^3)(bx^2+a)^{\frac{2}{3}}}{440\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/440*(20*b^3*x^6 + 5*a*b^2*x^4 - 6*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(2/3)/b^
3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(54) = 108$.
time = 1.01, size = 700, normalized size = 11.86

$$\frac{27a^{\frac{35}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}-\frac{27a^{\frac{35}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}+\frac{63a^{\frac{32}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}-\frac{81a^{\frac{32}{3}}bx^2}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}+\frac{42a^{\frac{29}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}-\frac{81a^{\frac{29}{3}}b^2x^4}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}+\frac{78a^{\frac{26}{3}}b^3x^6\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}-\frac{27a^{\frac{26}{3}}b^3x^6}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}+\frac{207a^{\frac{23}{3}}b^4x^8\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}+\frac{195a^{\frac{20}{3}}b^5x^{10}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}+\frac{60a^{\frac{17}{3}}b^6x^{12}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(2/3),x)
```

```
[Out] 27*a**(35/3)*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1
320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(35/3)/(440*a**8*b**3 + 13
20*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 63*a**(32/3
)*b*x**2*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*
a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(32/3)*b*x**2/(440*a**8*b**3 +
 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 42*a**(2
9/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 +
 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(29/3)*b**2*x**4/(440*a*
*8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) +
 78*a**(26/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b*
*4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(26/3)*b**3*x**
6/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**
6*x**6) + 207*a**(23/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 13
20*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 195*a**(20/
3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 +
1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 60*a**(17/3)*b**6*x**12*(1 + b*
x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 +
 440*a**5*b**6*x**6)
```

Giac [A]
time = 1.74, size = 43, normalized size = 0.73

$$\frac{3\left(20\left(bx^2+a\right)^{\frac{11}{3}}-55\left(bx^2+a\right)^{\frac{8}{3}}a+44\left(bx^2+a\right)^{\frac{5}{3}}a^2\right)}{440\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/440*(20*(b*x^2 + a)^(11/3) - 55*(b*x^2 + a)^(8/3)*a + 44*(b*x^2 + a)^(5/3
)*a^2)/b^3
```

Mupad [B]
time = 4.66, size = 44, normalized size = 0.75

$$\left(b\,x^2+a\right)^{2/3}\,\left(\frac{3\,x^6}{22}+\frac{27\,a^3}{440\,b^3}+\frac{3\,a\,x^4}{88\,b}-\frac{9\,a^2\,x^2}{220\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(2/3),x)
```

```
[Out] (a + b*x^2)^(2/3)*((3*x^6)/22 + (27*a^3)/(440*b^3) + (3*a*x^4)/(88*b) - (9*
a^2*x^2)/(220*b^2))
```

3.679 $\int x^3(a+bx^2)^{2/3}\,dx$

Optimal. Leaf size=38

$$-\frac{3a(a+bx^2)^{5/3}}{10b^2}+\frac{3(a+bx^2)^{8/3}}{16b^2}$$

```
[Out] -3/10*a*(b*x^2+a)^(5/3)/b^2+3/16*(b*x^2+a)^(8/3)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{3(a+bx^2)^{8/3}}{16b^2}-\frac{3a(a+bx^2)^{5/3}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(5/3))/(10*b^2) + (3*(a + b*x^2)^(8/3))/(16*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{2/3}\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{2/3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right)\,dx,x,x^2\right)\\ &= -\frac{3a(a+bx^2)^{5/3}}{10b^2}+\frac{3(a+bx^2)^{8/3}}{16b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.03

$$\frac{3(a+bx^2)^{2/3}\left(-3a^2+2abx^2+5b^2x^4\right)}{80b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(2/3)*(-3*a^2 + 2*a*b*x^2 + 5*b^2*x^4))/(80*b^2)
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.66

method	result	size
gosper	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(-5b\,x^{2}+3a\right)}{80b^{2}}$	25
trager	$-\frac{3\left(-5b^{2}x^{4}-2ab\,x^{2}+3a^{2}\right)\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{80b^{2}}$	36
risch	$-\frac{3\left(-5b^{2}x^{4}-2ab\,x^{2}+3a^{2}\right)\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{80b^{2}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
[Out] -3/80*(b*x^2+a)^(5/3)*(-5*b*x^2+3*a)/b^2
```

Maxima [A]
time = 0.28, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{8}{3}}}{16\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{5}{3}}a}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] 3/16*(b*x^2 + a)^(8/3)/b^2 - 3/10*(b*x^2 + a)^(5/3)*a/b^2
```

Fricas [A]
time = 1.06, size = 35, normalized size = 0.92

$$\frac{3\left(5\,b^2x^4+2\,abx^2-3\,a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{80\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/80*(5*b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3)/b^2
```

Sympy [A]
time = 0.15, size = 66, normalized size = 1.74

$$\begin{cases} -\frac{9a^2(a+bx^2)^{\frac{2}{3}}}{80b^2} + \frac{3ax^2(a+bx^2)^{\frac{2}{3}}}{40b} + \frac{3x^4(a+bx^2)^{\frac{2}{3}}}{16} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((-9*a**2*(a + b*x**2)**(2/3)/(80*b**2) + 3*a*x**2*(a + b*x**2)**(
2/3)/(40*b) + 3*x**4*(a + b*x**2)**(2/3)/16, Ne(b, 0)), (a**(2/3)*x**4/4, T
rue))
```

Giac [A]
time = 2.48, size = 29, normalized size = 0.76

$$\frac{3\left(5\,(bx^2+a)^{\frac{8}{3}} - 8\,(bx^2+a)^{\frac{5}{3}}a\right)}{80\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/80*(5*(b*x^2 + a)^(8/3) - 8*(b*x^2 + a)^(5/3)*a)/b^2
```

Mupad [B]
time = 4.71, size = 33, normalized size = 0.87

$$\left(b\,x^2+a\right)^{2/3}\,\left(\frac{3\,x^4}{16}-\frac{9\,a^2}{80\,b^2}+\frac{3\,a\,x^2}{40\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(2/3),x)
```

```
[Out] (a + b*x^2)^(2/3)*((3*x^4)/16 - (9*a^2)/(80*b^2) + (3*a*x^2)/(40*b))
```

3.680 $\int x(a+bx^2)^{2/3}\,dx$

Optimal. Leaf size=18

$$\frac{3(a+bx^2)^{5/3}}{10b}$$

[Out] 3/10*(b*x^2+a)^(5/3)/b

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\frac{3(a+bx^2)^{5/3}}{10b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^(2/3),x]

[Out] (3*(a + b*x^2)^(5/3))/(10*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{2/3}\,dx=\frac{3(a+bx^2)^{5/3}}{10b}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{3(a+bx^2)^{5/3}}{10b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^(2/3),x]

[Out] (3*(a + b*x^2)^(5/3))/(10*b)

Maple [A]
time = 0.03, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{3(b\,x^{2}+a)^{\frac{5}{3}}}{10b}$	15
derivativedivides	$\frac{3(b\,x^{2}+a)^{\frac{5}{3}}}{10b}$	15
default	$\frac{3(b\,x^{2}+a)^{\frac{5}{3}}}{10b}$	15
trager	$\frac{3(b\,x^{2}+a)^{\frac{5}{3}}}{10b}$	15
risch	$\frac{3(b\,x^{2}+a)^{\frac{5}{3}}}{10b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/10*(b*x^2+a)^(5/3)/b
```

Maxima [A]
time = 0.34, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

Fricas [A]
time = 1.07, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.
time = 0.09, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3a(a+bx^2)^{\frac{2}{3}}}{10b} + \frac{3x^2(a+bx^2)^{\frac{2}{3}}}{10} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((3*a*(a + b*x**2)**(2/3)/(10*b) + 3*x**2*(a + b*x**2)**(2/3)/10,
Ne(b, 0)), (a**(2/3)*x**2/2, True))
```

Giac [A]
time = 1.92, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

Mupad [B]
time = 4.58, size = 14, normalized size = 0.78

$$\frac{3\,{\left(b\,x^2+a\right)}^{5/3}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(2/3),x)
```

```
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

3.681 $\int \frac{(a+bx^2)^{2/3}}{x} dx$

Optimal. Leaf size=101

$$\frac{3}{4}\left(a+bx^2\right)^{2/3}+\frac{1}{2}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)$$

```
[Out] 3/4*(b*x^2+a)^(2/3)-1/2*a^(2/3)*ln(x)+3/4*a^(2/3)*ln(a^(1/3)-(b*x^2+a)^(1/3
))+1/2*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1
/2)
```

Rubi [A]

time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 52, 57, 631, 210, 31}

$$\frac{1}{2}\sqrt{3}\,a^{2/3}\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)+\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}\left(a+bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(2/3))/4 + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{2/3}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3}-\frac{1}{2}a^{2/3}\log(x)-\frac{1}{4}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)+\frac{1}{4}(3\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3}-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int\right.\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3}+\frac{1}{2}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}\right.
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 126, normalized size = 1.25

$$\frac{1}{4}\left(3\left(a+bx^2\right)^{2/3}+2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2a^{2/3}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(2/3) + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/
a^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[-a^(1/3) + (a + b*x^2)^(1/3)] - a^(2/3)*L
og[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x,x)
```

Maxima [A]
time = 0.49, size = 97, normalized size = 0.96

$$\frac{1}{2}\,\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^{2}+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{2}{3}}\log\left((bx^{2}+a)^{\frac{2}{3}}+(bx^{2}+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{2}{3}}\log\left((bx^{2}+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx^{2}+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - 1/4*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 1/2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/4*(b*x^2 + a)^(2/3)
```

Fricas [A]
time = 0.98, size = 122, normalized size = 1.21

$$\frac{1}{2}\,\sqrt{3}\,\left(a^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\,(bx^{2}+a)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{1}{3}}}{3\,a}\right)-\frac{1}{4}\,\left(a^{2}\right)^{\frac{1}{3}}\log\left((bx^{2}+a)^{\frac{2}{3}}a+\left(a^{2}\right)^{\frac{1}{3}}a+(bx^{2}+a)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{2}{3}}\right)+\frac{1}{2}\,\left(a^{2}\right)^{\frac{1}{3}}\log\left((bx^{2}+a)^{\frac{1}{3}}a-\left(a^{2}\right)^{\frac{2}{3}}\right)+\frac{3}{4}\,(bx^{2}+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)
*(a^2)^(1/3))/a) - 1/4*(a^2)^(1/3)*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a
+ (b*x^2 + a)^(1/3)*(a^2)^(2/3)) + 1/2*(a^2)^(1/3)*log((b*x^2 + a)^(1/3)*a
- (a^2)^(2/3)) + 3/4*(b*x^2 + a)^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.56, size = 46, normalized size = 0.46

$$-\frac{b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x,x)
```

```
[Out] -b**(2/3)*x**(4/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(1/3))
```

Giac [A]
time = 3.12, size = 98, normalized size = 0.97

$$\frac{1}{2}\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{2}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{4}\left(bx^2+a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - 1/4*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 1/2*a^(2/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/4*(b*x^2 + a)^
(2/3)
```

Mupad [B]
time = 4.67, size = 125, normalized size = 1.24

$$\frac{3\,{\left(b\,x^2+a\right)}^{2/3}}{4}+\frac{a^{2/3}\,\ln\left(\frac{9\,a^2\,{\left(b\,x^2+a\right)}^{1/3}}{4}-\frac{9\,a^{7/3}}{4}\right)}{2}-\frac{a^{2/3}\,\ln\left(\frac{9\,a^2\,{\left(b\,x^2+a\right)}^{1/3}}{4}-\frac{9\,a^{7/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{4}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+a^{2/3}\,\ln\left(\frac{9\,a^2\,{\left(b\,x^2+a\right)}^{1/3}}{4}-9\,a^{7/3}\,{\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}^2\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x,x)
```

```
[Out] (3*(a + b*x^2)^(2/3))/4 + (a^(2/3)*log((9*a^2*(a + b*x^2)^(1/3))/4 - (9*a^(
7/3))/4))/2 - (a^(2/3)*log((9*a^2*(a + b*x^2)^(1/3))/4 - (9*a^(7/3)*((3^(1/
2)*1i)/2 + 1/2)^2)/4)*((3^(1/2)*1i)/2 + 1/2))/2 + a^(2/3)*log((9*a^2*(a + b
*x^2)^(1/3))/4 - 9*a^(7/3)*((3^(1/2)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4)
```

3.682 $\int \frac{(a+bx^2)^{2/3}}{x^3} \, dx$

Optimal. Leaf size=104

$$-\frac{(a+bx^2)^{2/3}}{2x^2}+\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}$$

```
[Out] -1/2*(b*x^2+a)^(2/3)/x^2-1/3*b*ln(x)/a^(1/3)+1/2*b*ln(a^(1/3)-(b*x^2+a)^(1/
3))/a^(1/3)+1/3*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a
^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {272, 43, 57, 631, 210, 31}

$$\frac{b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{(a+bx^2)^{2/3}}{2x^2}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x^2)^(2/3)/x^2 + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3)
 - (a + b*x^2)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{2/3}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2}+\frac{1}{3}b\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{1}{2}b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a+bx^2}\right)-\frac{b\text{Sub}
\end{aligned}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{}}{}\right.}{\sqrt[3]{a}}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2}+\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}$$

Mathematica [A]

time = 0.14, size = 136, normalized size = 1.31

$$-\frac{(a+bx^2)^{2/3}}{2x^2}+\frac{b\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}+\frac{b\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)}{3\sqrt[3]{a}}-\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{6\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x^2)^(2/3)/x^2 + (b*ArcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)) + (b*Log[-a^(1/3) + (a + b*x^2)^(1/3)])
/(3*a^(1/3)) - (b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/
3)])/(6*a^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^3,x)
```

Maxima [A]
time = 0.50, size = 103, normalized size = 0.99

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}}-\frac{b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}}+\frac{b\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}-\frac{(bx^2+a)^{\frac{2}{3}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(1/3) - 1/6*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(1/3) + 1/3*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(1/3) - 1/2*(b*x^2 + a)
^(2/3)/x^2
```

Fricas [A]
time = 0.98, size = 290, normalized size = 2.79

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-a^{\frac{2}{3}}bx^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}bx^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(bx^2+a)^{\frac{2}{3}}a}{6\,ax^2},\ \frac{6\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}bx^2\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}bx^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}bx^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(bx^2+a)^{\frac{2}{3}}a}{6\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x^2*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(2*(b
*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*b*x^2*log((b*x^2 + a)^(2
/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x^2*log((b*x^2 + a
)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*a)/(a*x^2), 1/6*(6*sqrt(1/3)*a^(2/
3)*b*x^2*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3
)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^
(2/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*a)/(a*x^
2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 42, normalized size = 0.40

$$-\frac{b^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**3,x)
```

```
[Out] -b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(2/3)*gamma(4/3))
```

Giac [A]
time = 1.94, size = 116, normalized size = 1.12

$$\frac{\frac{2\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{b^{2}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\,b^{2}\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}-\frac{3\left(bx^{2}+a\right)^{\frac{2}{3}}b}{x^{2}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(1/3) - b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(1/3) + 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x^
2 + a)^(2/3)*b/x^2)/b
```

Mupad [B]
time = 4.89, size = 136, normalized size = 1.31

$$\frac{b\,\ln\left(a^{1/3}\,b^2-b^2\,{\left(b\,x^2+a\right)}^{1/3}\right)}{3\,a^{1/3}}-\frac{{\left(b\,x^2+a\right)}^{2/3}}{2\,x^2}-\frac{\ln\left(\frac{a^{1/3}\,{\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2}{4}-b^2\,{\left(b\,x^2+a\right)}^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{6\,a^{1/3}}-\frac{\ln\left(\frac{a^{1/3}\,{\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2}{4}-b^2\,{\left(b\,x^2+a\right)}^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{6\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^3,x)
```

```
[Out] (b*log(a^(1/3)*b^2 - b^2*(a + b*x^2)^(1/3)))/(3*a^(1/3)) - (a + b*x^2)^(2/3
)/(2*x^2) - (log((a^(1/3)*(b - 3^(1/2)*b*1i)^2)/4 - b^2*(a + b*x^2)^(1/3))*
(b - 3^(1/2)*b*1i))/(6*a^(1/3)) - (log((a^(1/3)*(b + 3^(1/2)*b*1i)^2)/4 - b
^2*(a + b*x^2)^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(1/3))
```

3.683 $\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx$

Optimal. Leaf size=135

$$-\frac{(a+bx^2)^{2/3}}{4x^4}-\frac{b(a+bx^2)^{2/3}}{6ax^2}-\frac{b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}$$

```
[Out] -1/4*(b*x^2+a)^(2/3)/x^4-1/6*b*(b*x^2+a)^(2/3)/a/x^2+1/18*b^2*ln(x)/a^(4/3)
-1/12*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(4/3)-1/18*b^2*arctan(1/3*(a^(1/3)+
2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {272, 43, 44, 57, 631, 210, 31}

$$-\frac{b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b(a+bx^2)^{2/3}}{6ax^2}-\frac{(a+bx^2)^{2/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^5,x]
```

```
[Out] -1/4*(a + b*x^2)^(2/3)/x^4 - (b*(a + b*x^2)^(2/3))/(6*a*x^2) - (b^2*ArcTan[
(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(4/3)) + (
b^2*Log[x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(4/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^{2/3}}{4x^4} + \frac{1}{6}b\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^{2/3}}{4x^4} - \frac{b(a+bx^2)^{2/3}}{6ax^2} - \frac{b^2\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{18a}$$
$$= -\frac{(a+bx^2)^{2/3}}{4x^4} - \frac{b(a+bx^2)^{2/3}}{6ax^2} + \frac{b^2\log(x)}{18a^{4/3}} + \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{12a^{4/3}} -$$
$$= -\frac{(a+bx^2)^{2/3}}{4x^4} - \frac{b(a+bx^2)^{2/3}}{6ax^2} + \frac{b^2\log(x)}{18a^{4/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}} + \frac{b^2\text{Subst}\left(\int\right.}{}$$
$$= -\frac{(a+bx^2)^{2/3}}{4x^4} - \frac{b(a+bx^2)^{2/3}}{6ax^2} - \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{4/3}} + \frac{b^2\log(x)}{18a^{4/3}} - \frac{b^2\log\left(\sqrt[3]{a}\right.}{}$$

Mathematica [A]
time = 0.19, size = 158, normalized size = 1.17

$$\frac{(-3a-2bx^2)(a+bx^2)^{2/3}}{12ax^4} - \frac{b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{4/3}} - \frac{b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)}{18a^{4/3}} + \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{36a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^5,x]
```

```
[Out] ((-3*a - 2*b*x^2)*(a + b*x^2)^(2/3))/(12*a*x^4) - (b^2*ArcTan[1/Sqrt[3] + (
2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(4/3)) - (b^2*Log[-a^
(1/3) + (a + b*x^2)^(1/3)])/(18*a^(4/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a +
b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(36*a^(4/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^5,x)
```

Maxima [A]
time = 0.51, size = 155, normalized size = 1.15

$$-\frac{\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^{2}+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{4}{3}}}+\frac{b^{2}\log\left((bx^{2}+a)^{\frac{2}{3}}+(bx^{2}+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36\,a^{\frac{4}{3}}}-\frac{b^{2}\log\left((bx^{2}+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,a^{\frac{4}{3}}}-\frac{2\,(bx^{2}+a)^{\frac{5}{3}}b^{2}+(bx^{2}+a)^{\frac{2}{3}}ab^{2}}{12\left((bx^{2}+a)^{2}a-2\,(bx^{2}+a)a^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) + 1/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(4/3) - 1/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(4/3) - 1/12*
(2*(b*x^2 + a)^(5/3)*b^2 + (b*x^2 + a)^(2/3)*a*b^2)/((b*x^2 + a)^2*a - 2*(b
*x^2 + a)*a^2 + a^3)
```

Fricas [A]
time = 0.97, size = 380, normalized size = 2.81

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab^{2}x^{4}\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx^{2}-3\sqrt{\frac{1}{3}}\left(2\,(bx^{2}+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx^{2}+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx^{2}+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x^{2}}\right)+(-a)^{\frac{2}{3}}b^{2}x^{4}\log\left((bx^{2}+a)^{\frac{2}{3}}-(bx^{2}+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}b^{2}x^{4}\log\left((bx^{2}+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\,(2\,abx^{2}+3\,a^{2})(bx^{2}+a)^{\frac{2}{3}}}{36\,a^{2}x^{4}},\ -\frac{6\sqrt{\frac{1}{3}}\,ab^{2}x^{4}\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx^{2}+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}b^{2}x^{4}\log\left((bx^{2}+a)^{\frac{2}{3}}-(bx^{2}+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2\,(-a)^{\frac{2}{3}}b^{2}x^{4}\log\left((bx^{2}+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\,(2\,abx^{2}+3\,a^{2})(bx^{2}+a)^{\frac{2}{3}}}{36\,a^{2}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="fricas")
```

```
[Out] [1/36*(3*sqrt(1/3)*a*b^2*x^4*sqrt((-a)^(1/3)/a)*log((2*b*x^2 - 3*sqrt(1/3)*
(2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt(
(-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + (-a)^(2/3)*b^2
*x^4*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2
*(-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) - 3*(2*a*b*x^2 + 3*
a^2)*(b*x^2 + a)^(2/3))/(a^2*x^4), -1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-(-a)^
(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/
3)/a)) - (-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^
(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) + (-a)^(1/
3)) + 3*(2*a*b*x^2 + 3*a^2)*(b*x^2 + a)^(2/3))/(a^2*x^4)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.10, size = 42, normalized size = 0.31

$$-\frac{b^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**5,x)
```

```
[Out] -b**(2/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(8/3)*gamma(7/3))
```

Giac [A]
time = 2.90, size = 141, normalized size = 1.04

$$-\frac{\frac{2\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\,b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2\left(bx^2+a\right)^{\frac{5}{3}}b^3+\left(bx^2+a\right)^{\frac{2}{3}}ab^3\right)}{ab^2x^4}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="giac")
```

```
[Out] -1/36*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(4/3) - b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(4/3) + 2*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*
(b*x^2 + a)^(5/3)*b^3 + (b*x^2 + a)^(2/3)*a*b^3)/(a*b^2*x^4))/b
```

Mupad [B]
time = 5.16, size = 212, normalized size = 1.57

$$\frac{(-1)^{1/3}\,b^2\ln\left(\left(b\,x^2+a\right)^{1/3}-(-1)^{2/3}\,a^{1/3}\right)}{18\,a^{4/3}}-\frac{\frac{b^2\,\left(b\,x^2+a\right)^{2/3}}{6}+\frac{b^2\,\left(b\,x^2+a\right)^{5/3}}{3\,a}}{2\,\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2}+\frac{(-1)^{1/3}\,b^2\ln\left(\frac{b^4\,\left(b\,x^2+a\right)^{1/3}}{36\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{36\,a^{5/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18\,a^{4/3}}-\frac{(-1)^{1/3}\,b^2\ln\left(\frac{b^4\,\left(b\,x^2+a\right)^{1/3}}{36\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{36\,a^{5/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^5,x)
```

```
[Out] ((-1)^(1/3)*b^2*log((a + b*x^2)^(1/3) - (-1)^(2/3)*a^(1/3)))/(18*a^(4/3)) -
 ((b^2*(a + b*x^2)^(2/3))/6 + (b^2*(a + b*x^2)^(5/3))/(3*a))/(2*(a + b*x^2)
^2 - 4*a*(a + b*x^2) + 2*a^2) + ((-1)^(1/3)*b^2*log((b^4*(a + b*x^2)^(1/3))
/(36*a^2) - ((-1)^(2/3)*b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(36*a^(5/3)))*((3^(1/
2)*1i)/2 - 1/2))/(18*a^(4/3)) - ((-1)^(1/3)*b^2*log((b^4*(a + b*x^2)^(1/3))
/(36*a^2) - ((-1)^(2/3)*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(36*a^(5/3)))*((3^(1/
2)*1i)/2 + 1/2))/(18*a^(4/3))
```

3.684 $\int x^4 (a+bx^2)^{2/3}\, dx$

Optimal. Leaf size=601

$$-\frac{108a^2x(a+bx^2)^{2/3}}{1729b^2}+\frac{12ax^3(a+bx^2)^{2/3}}{247b}+\frac{3}{19}x^5(a+bx^2)^{2/3}-\frac{324a^3x}{1729b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{162\sqrt{}}{}$$

```
[Out] -108/1729*a^2*x*(b*x^2+a)^(2/3)/b^2+12/247*a*x^3*(b*x^2+a)^(2/3)/b+3/19*x^5
*(b*x^2+a)^(2/3)-324/1729*a^3*x/b^2/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-
108/1729*3^(3/4)*a^(10/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(
1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1
/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/
3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+162/1729*3^(1/4)*a^(10/
3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)
))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*
(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(
-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {285, 327, 241, 310, 225, 1893}

$$-\frac{108\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{1729b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{1729b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{324a^3x}{1729b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{108a^2x(a+bx^2)^{2/3}}{1729b^2}+\frac{3}{19}x^5(a+bx^2)^{2/3}+\frac{12ax^3(a+bx^2)^{2/3}}{247b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(2/3),x]
```

```
[Out] (-108*a^2*x*(a + b*x^2)^(2/3))/(1729*b^2) + (12*a*x^3*(a + b*x^2)^(2/3))/(2
47*b) + (3*x^5*(a + b*x^2)^(2/3))/19 - (324*a^3*x)/(1729*b^2*((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))) + (162*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a
^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
+ b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) -
 (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (10
```

```
8*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^
(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b^3
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{2/3}\,dx = \frac{3}{19}x^5\left(a+bx^2\right)^{2/3}+\frac{1}{19}(4a)\int\frac{x^4}{\sqrt[3]{a+bx^2}}\,dx$$

$$=\frac{12ax^3(a+bx^2)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}-\frac{(36a^2)\int\frac{x^2}{\sqrt[3]{a+bx^2}}\,dx}{247b}$$

$$=-\frac{108a^2x(a+bx^2)^{2/3}}{1729b^2}+\frac{12ax^3(a+bx^2)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}+\frac{(108a^3)\int\frac{1}{\sqrt[3]{a+bx}}}{1729b^2}$$

$$=-\frac{108a^2x(a+bx^2)^{2/3}}{1729b^2}+\frac{12ax^3(a+bx^2)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}+\frac{\left(162a^3\sqrt{bx^2}\right)\text{Sub}}{}$$

$$=-\frac{108a^2x(a+bx^2)^{2/3}}{1729b^2}+\frac{12ax^3(a+bx^2)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}-\frac{\left(162a^3\sqrt{bx^2}\right)\text{Sub}}{}$$

$$=-\frac{108a^2x(a+bx^2)^{2/3}}{1729b^2}+\frac{12ax^3(a+bx^2)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}-\frac{32}{1729b^2\left(\left(1-\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.87, size = 94, normalized size = 0.16

$$\frac{3x(a+bx^2)^{2/3}\left(\left(1+\frac{bx^2}{a}\right)^{2/3}\left(-9a^2+4abx^2+13b^2x^4\right)+9a^2\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{247b^2\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3)*((1 + (b*x^2)/a)^(2/3)*(-9*a^2 + 4*a*b*x^2 + 13*b^2*
x^4) + 9*a^2*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)]))/(247*b^2*(1
+ (b*x^2)/a)^(2/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{4} \left(b\,x^{2}+a\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(2/3),x)
```

```
[Out] int(x^4*(b*x^2+a)^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)*x^4, x)
```

Sympy [**A**]
time = 0.48, size = 29, normalized size = 0.05

$$\frac{a^{\frac{2}{3}} x^{5} {{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(2/3),x)
```

```
[Out] a**(2/3)*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(b\,x^2+a\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(2/3),x)
```

```
[Out] int(x^4*(a + b*x^2)^(2/3), x)
```

3.685 $\int x^2(a+bx^2)^{2/3}\,dx$

Optimal. Leaf size=577

$$\frac{12ax(a+bx^2)^{2/3}}{91b}+\frac{3}{13}x^3(a+bx^2)^{2/3}+\frac{36a^2x}{91b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{18\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}-\right.}{}$$

```
[Out] 12/91*a*x*(b*x^2+a)^(2/3)/b+3/13*x^3*(b*x^2+a)^(2/3)+36/91*a^2*x/b/(-(b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+12/91*3^(3/4)*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1/
3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(
b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^2/x/(-a^(
1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)-18/91*3^(1/4)*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(
1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1
/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^2/x/(-a^(1/3)*(a^(
1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {285, 327, 241, 310, 225, 1893}

$$\frac{12\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{18\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{36a^2x}{91b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{12ax(a+bx^2)^{2/3}}{91b}+\frac{3}{13}x^3(a+bx^2)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(2/3),x]
```

```
[Out] (12*a*x*(a + b*x^2)^(2/3))/(91*b) + (3*x^3*(a + b*x^2)^(2/3))/13 + (36*a^2*
x)/(91*b*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (18*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b^2*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))^2)]) + (12*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3]
```

```
)*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(91*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^{2/3}dx=\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}+\frac{1}{13}(4a)\int\frac{x^2}{\sqrt[3]{a+bx^2}}dx$$

$$=\frac{12ax(a+bx^2)^{2/3}}{91b}+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}-\frac{(12a^2)\int\frac{1}{\sqrt[3]{a+bx^2}}dx}{91b}$$

$$=\frac{12ax(a+bx^2)^{2/3}}{91b}+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}-\frac{\left(18a^2\sqrt{bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}dx,x,$$

$$=\frac{12ax(a+bx^2)^{2/3}}{91b}+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}+\frac{\left(18a^2\sqrt{bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}dx$$

$$18\sqrt{}$$

$$=\frac{12ax(a+bx^2)^{2/3}}{91b}+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}+\frac{36a^2x}{91b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.39, size = 62, normalized size = 0.11

$$\frac{3x(a+bx^2)^{2/3}\left(a+bx^2-\frac{a\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\right)}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3)*(a + b*x^2 - (a*Hypergeometric2F1[-2/3, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(2/3)))/(13*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{2} \left(b\,x^{2}+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(2/3),x)
[Out] int(x^2*(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(2/3)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)*x^2, x)
```

Sympy [A]
time = 0.44, size = 29, normalized size = 0.05

$$\frac{a^{\frac{2}{3}} x^{3} {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(2/3),x)
[Out] a**(2/3)*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(2/3)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(b\,x^2+a\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(2/3),x)
[Out] int(x^2*(a + b*x^2)^(2/3), x)
```

3.686 $\int (a+bx^2)^{2/3}\,dx$

Optimal. Leaf size=550

$$\frac{3}{7}x\left(a+bx^2\right)^{2/3}-\frac{12ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{6\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{\cdots}\right.\right.}}}{7bx\sqrt{-\frac{\cdots}{\left(\cdots\right.}}}$$

```
[Out] 3/7*x*(b*x^2+a)^(2/3)-12/7*a*x/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-4/7*3
^(3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3
)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2
)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+6/7*3^(1/4)*a^(4/3)*(a^(1/3)-(b*x^2+
a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*
x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+
1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {201, 241, 310, 225, 1893}

$$-\frac{4\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{12ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3))/7 - (12*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))) + (6*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (4*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSi
```

```
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*
x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{2/3}\,dx = \frac{3}{7}x\left(a+bx^2\right)^{2/3}+\frac{1}{7}(4a)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$$

$$= \frac{3}{7}x\left(a+bx^2\right)^{2/3}+\frac{\left(6a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{7bx}$$

$$= \frac{3}{7}x\left(a+bx^2\right)^{2/3}-\frac{\left(6a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{7bx}+\frac{\left(6\sqrt{2\left(2\right.}\right.}{}$$

$$= \frac{3}{7}x\left(a+bx^2\right)^{2/3}-\frac{12ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{\right.}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.27, size = 46, normalized size = 0.08

$$\frac{x(a+bx^2)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3),x]
```

```
[Out] (x*(a + b*x^2)^(2/3)*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(2/3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3),x)
```

```
[Out] int((b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3), x)
```

Sympy [A]
time = 0.41, size = 26, normalized size = 0.05

$$a^{\frac{2}{3}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3),x)
[Out] a**(2/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(2/3), x)
```

Mupad [B]
time = 5.17, size = 37, normalized size = 0.07

$$\frac{x\,{\left(b\,x^2+a\right)}^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3),x)
```

```
[Out] (x*(a + b*x^2)^(2/3)*hypergeom([-2/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(2/3)
```

3.687 $\int \frac{(a+bx^2)^{2/3}}{x^2} \, dx$

Optimal. Leaf size=538

$$-\frac{(a+bx^2)^{2/3}}{x}-\frac{4bx}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}+\frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a}}{\left(\left(1-\sqrt{3}\right)\right.}}}{x\sqrt{-\frac{\sqrt[3]{}}{\left(\left(1-\right.\right.}}}$$

```
[Out] -(b*x^2+a)^(2/3)/x-4*b*x/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-4/3*a^(1/3)
*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))
/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^
(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
))^2)^(1/2)*3^(3/4)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+2*3^(1/4)*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*
EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*
(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/
3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2
))/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 538, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {283, 241, 310, 225, 1893}

$$-\frac{4\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{4bx}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}-\frac{(a+bx^2)^{2/3}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x^2)^(2/3)/x) - (4*b*x)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
) + (2*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
 + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2)]) - (4*Sqrt[2]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1
/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - S
```

```
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx = -\frac{(a+bx^2)^{2/3}}{x} + \frac{1}{3}(4b)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$$

$$= -\frac{(a+bx^2)^{2/3}}{x} + \frac{\left(2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{x}$$

$$= -\frac{(a+bx^2)^{2/3}}{x} - \frac{\left(2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{x} + \frac{\left(2\sqrt{2\left(2\right.}\right.}{}$$

$$= -\frac{(a+bx^2)^{2/3}}{x} - \frac{4bx}{\left(1-\sqrt{3}\right)\sqrt[3]{a} - \sqrt[3]{a+bx^2}} + \frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.77, size = 49, normalized size = 0.09

$$-\frac{(a+bx^2)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(2/3)*Hypergeometric2F1[-2/3, -1/2, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(2/3)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^2,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(2/3)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)/x^2, x)
```

Sympy [A]
time = 0.43, size = 29, normalized size = 0.05

$$-\frac{a^{\frac{2}{3}}{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**2,x)
[Out] -a**(2/3)*hyper((-2/3, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(2/3)/x^2, x)
```

Mupad [B]
time = 5.47, size = 40, normalized size = 0.07

$$\frac{3\,{\left(b\,x^2+a\right)}^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},-\frac{1}{6};\ \frac{5}{6};\ -\frac{a}{b\,x^2}\right)}{x\,{\left(\frac{a}{b\,x^2}+1\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^2,x)
```

```
[Out] (3*(a + b*x^2)^(2/3)*hypergeom([-2/3, -1/6], 5/6, -a/(b*x^2)))/(x*(a/(b*x^2
) + 1)^(2/3))
```

3.688 $\int \frac{(a+bx^2)^{2/3}}{x^4} dx$

Optimal. Leaf size=575

$$-\frac{(a+bx^2)^{2/3}}{3x^3}-\frac{4b(a+bx^2)^{2/3}}{9ax}-\frac{4b^2x}{9a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{2\sqrt{2+\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a}{\ }}}{}$$

```
[Out] -1/3*(b*x^2+a)^(2/3)/x^3-4/9*b*(b*x^2+a)^(2/3)/a/x-4/9*b^2*x/a/(-(b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))-4/27*b*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*
x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*
I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+2/9
*b*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)
))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*
(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2
+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {283, 331, 241, 310, 225, 1893}

$$-\frac{4\sqrt{2}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt{2+\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{3\ 3^{3/4}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{4b^2x}{9a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{4b(a+bx^2)^{2/3}}{9ax}-\frac{(a+bx^2)^{2/3}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^4,x]
```

```
[Out] -1/3*(a + b*x^2)^(2/3)/x^3 - (4*b*(a + b*x^2)^(2/3))/(9*a*x) - (4*b^2*x)/(9
*a*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (2*Sqrt[2 + Sqrt[3]]*b*(a
^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
+ b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2
)]) - (4*Sqrt[2]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
```

```
 + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^(2/3
)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx = -\frac{(a+bx^2)^{2/3}}{3x^3}+\frac{1}{9}(4b)\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx$$

$$= -\frac{(a+bx^2)^{2/3}}{3x^3}-\frac{4b(a+bx^2)^{2/3}}{9ax}+\frac{(4b^2)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{27a}$$

$$= -\frac{(a+bx^2)^{2/3}}{3x^3}-\frac{4b(a+bx^2)^{2/3}}{9ax}+\frac{\left(2b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{9ax}$$

$$= -\frac{(a+bx^2)^{2/3}}{3x^3}-\frac{4b(a+bx^2)^{2/3}}{9ax}-\frac{\left(2b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right.}{9ax}$$

$$= -\frac{(a+bx^2)^{2/3}}{3x^3}-\frac{4b(a+bx^2)^{2/3}}{9ax}-\frac{4b^2x}{9a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{2\sqrt{2+\sqrt{3}}\,b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.09

$$-\frac{(a+bx^2)^{2/3}\,{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^4,x]
```

```
[Out] -1/3*((a + b*x^2)^(2/3)*Hypergeometric2F1[-3/2, -2/3, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a \right)^{\frac{2}{3}}}{x^{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^4,x)
[Out] int((b*x^2+a)^(2/3)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(2/3)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)/x^4, x)
```

Sympy [A]
time = 0.47, size = 34, normalized size = 0.06

$$-\frac{a^{\frac{2}{3}}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, -\frac{2}{3} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**4,x)
[Out] -a**(2/3)*hyper((-3/2, -2/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{2/3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^4,x)
```

```
[Out] int((a + b*x^2)^(2/3)/x^4, x)
```

3.689 $\int x^7 (a+bx^2)^{4/3}\, dx$

Optimal. Leaf size=80

$$-\frac{3a^3(a+bx^2)^{7/3}}{14b^4}+\frac{9a^2(a+bx^2)^{10/3}}{20b^4}-\frac{9a(a+bx^2)^{13/3}}{26b^4}+\frac{3(a+bx^2)^{16/3}}{32b^4}$$

```
[Out] -3/14*a^3*(b*x^2+a)^(7/3)/b^4+9/20*a^2*(b*x^2+a)^(10/3)/b^4-9/26*a*(b*x^2+a
)^(13/3)/b^4+3/32*(b*x^2+a)^(16/3)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$-\frac{3a^3(a+bx^2)^{7/3}}{14b^4}+\frac{9a^2(a+bx^2)^{10/3}}{20b^4}+\frac{3(a+bx^2)^{16/3}}{32b^4}-\frac{9a(a+bx^2)^{13/3}}{26b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(7/3))/(14*b^4) + (9*a^2*(a + b*x^2)^(10/3))/(20*b^4) -
 (9*a*(a + b*x^2)^(13/3))/(26*b^4) + (3*(a + b*x^2)^(16/3))/(32*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{4/3}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{4/3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}\right)\,a\right.\\ &= -\frac{3a^3(a+bx^2)^{7/3}}{14b^4}+\frac{9a^2(a+bx^2)^{10/3}}{20b^4}-\frac{9a(a+bx^2)^{13/3}}{26b^4}+\frac{3(a+bx^2)^{16/3}}{32b^4}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 0.62

$$\frac{3(a+bx^2)^{7/3}\left(-81a^3+189a^2bx^2-315ab^2x^4+455b^3x^6\right)}{14560b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3)*(-81*a^3 + 189*a^2*b*x^2 - 315*a*b^2*x^4 + 455*b^3*x^6
))/(14560*b^4)
```

Maple [A]
time = 0.05, size = 47, normalized size = 0.59

method	result	size
gosper	$-\frac{3(b\,x^2+a)^{\frac{7}{3}}\left(-455b^3x^6+315a\,b^2x^4-189a^2b\,x^2+81a^3\right)}{14560b^4}$	47
trager	$-\frac{3\left(-455b^5x^{10}-595a\,b^4x^8-14a^2b^3x^6+18a^3b^2x^4-27a^4b\,x^2+81a^5\right)(b\,x^2+a)^{\frac{1}{3}}}{14560b^4}$	69
risch	$-\frac{3\left(-455b^5x^{10}-595a\,b^4x^8-14a^2b^3x^6+18a^3b^2x^4-27a^4b\,x^2+81a^5\right)(b\,x^2+a)^{\frac{1}{3}}}{14560b^4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/14560*(b*x^2+a)^(7/3)*(-455*b^3*x^6+315*a*b^2*x^4-189*a^2*b*x^2+81*a^3)/
b^4
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{16}{3}}}{32\,b^4}-\frac{9\,(bx^2+a)^{\frac{13}{3}}a}{26\,b^4}+\frac{9\,(bx^2+a)^{\frac{10}{3}}a^2}{20\,b^4}-\frac{3\,(bx^2+a)^{\frac{7}{3}}a^3}{14\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/32*(b*x^2 + a)^(16/3)/b^4 - 9/26*(b*x^2 + a)^(13/3)*a/b^4 + 9/20*(b*x^2 +
 a)^(10/3)*a^2/b^4 - 3/14*(b*x^2 + a)^(7/3)*a^3/b^4
```

Fricas [A]
time = 0.85, size = 68, normalized size = 0.85

$$\frac{3\,(455\,b^5x^{10}+595\,ab^4x^8+14\,a^2b^3x^6-18\,a^3b^2x^4+27\,a^4bx^2-81\,a^5)(bx^2+a)^{\frac{1}{3}}}{14560\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/14560*(455*b^5*x^10 + 595*a*b^4*x^8 + 14*a^2*b^3*x^6 - 18*a^3*b^2*x^4 + 2
7*a^4*b*x^2 - 81*a^5)*(b*x^2 + a)^(1/3)/b^4
```

Sympy [A]
time = 0.50, size = 136, normalized size = 1.70

$$\begin{cases} -\frac{243a^5\sqrt[3]{a+bx^2}}{14560b^4} + \frac{81a^4x^2\sqrt[3]{a+bx^2}}{14560b^3} - \frac{27a^3x^4\sqrt[3]{a+bx^2}}{7280b^2} + \frac{3a^2x^6\sqrt[3]{a+bx^2}}{1040b} + \frac{51ax^8\sqrt[3]{a+bx^2}}{416} + \frac{3bx^{10}\sqrt[3]{a+bx^2}}{32} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-243*a**5*(a + b*x**2)**(1/3)/(14560*b**4) + 81*a**4*x**2*(a + b
*x**2)**(1/3)/(14560*b**3) - 27*a**3*x**4*(a + b*x**2)**(1/3)/(7280*b**2) +
 3*a**2*x**6*(a + b*x**2)**(1/3)/(1040*b) + 51*a*x**8*(a + b*x**2)**(1/3)/4
16 + 3*b*x**10*(a + b*x**2)**(1/3)/32, Ne(b, 0)), (a**(4/3)*x**8/8, True))
```

Giac [A]
time = 1.16, size = 57, normalized size = 0.71

$$\frac{3\left(455\left(bx^2+a\right)^{\frac{16}{3}} - 1680\left(bx^2+a\right)^{\frac{13}{3}}a + 2184\left(bx^2+a\right)^{\frac{10}{3}}a^2 - 1040\left(bx^2+a\right)^{\frac{7}{3}}a^3\right)}{14560\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/14560*(455*(b*x^2 + a)^(16/3) - 1680*(b*x^2 + a)^(13/3)*a + 2184*(b*x^2 +
 a)^(10/3)*a^2 - 1040*(b*x^2 + a)^(7/3)*a^3)/b^4
```

Mupad [B]
time = 5.19, size = 64, normalized size = 0.80

$$\left(b\,x^2+a\right)^{1/3}\left(\frac{51\,a\,x^8}{416}+\frac{3\,b\,x^{10}}{32}-\frac{243\,a^5}{14560\,b^4}+\frac{3\,a^2\,x^6}{1040\,b}-\frac{27\,a^3\,x^4}{7280\,b^2}+\frac{81\,a^4\,x^2}{14560\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(4/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((51*a*x^8)/416 + (3*b*x^10)/32 - (243*a^5)/(14560*b^4) +
 (3*a^2*x^6)/(1040*b) - (27*a^3*x^4)/(7280*b^2) + (81*a^4*x^2)/(14560*b^3))
```

3.690 $\int x^5(a+bx^2)^{4/3}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2(a+bx^2)^{7/3}}{14b^3}-\frac{3a(a+bx^2)^{10/3}}{10b^3}+\frac{3(a+bx^2)^{13/3}}{26b^3}$$

```
[Out] 3/14*a^2*(b*x^2+a)^(7/3)/b^3-3/10*a*(b*x^2+a)^(10/3)/b^3+3/26*(b*x^2+a)^(13
/3)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {272, 45}

$$\frac{3a^2(a+bx^2)^{7/3}}{14b^3}+\frac{3(a+bx^2)^{13/3}}{26b^3}-\frac{3a(a+bx^2)^{10/3}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(7/3))/(14*b^3) - (3*a*(a + b*x^2)^(10/3))/(10*b^3) + (3
*(a + b*x^2)^(13/3))/(26*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{4/3}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{4/3}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{4/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}\right)dx,x,x^2\right)\\&=\frac{3a^2(a+bx^2)^{7/3}}{14b^3}-\frac{3a(a+bx^2)^{10/3}}{10b^3}+\frac{3(a+bx^2)^{13/3}}{26b^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{3(a+bx^2)^{7/3}(9a^2-21abx^2+35b^2x^4)}{910b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(4/3),x]
[Out] (3*(a + b*x^2)^(7/3)*(9*a^2 - 21*a*b*x^2 + 35*b^2*x^4))/(910*b^3)
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.61

method	result	size
gosper	$\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(35b^{2}x^{4}-21ab\,x^{2}+9a^{2}\right)}{910b^{3}}$	36
trager	$\frac{3\left(35b^{4}x^{8}+49a\,b^{3}x^{6}+2a^{2}b^{2}x^{4}-3a^{3}b\,x^{2}+9a^{4}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{910b^{3}}$	58
risch	$\frac{3\left(35b^{4}x^{8}+49a\,b^{3}x^{6}+2a^{2}b^{2}x^{4}-3a^{3}b\,x^{2}+9a^{4}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{910b^{3}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/910*(b*x^2+a)^(7/3)*(35*b^2*x^4-21*a*b*x^2+9*a^2)/b^3
```

Maxima [A]
time = 0.31, size = 47, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{13}{3}}}{26\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{10}{3}}a}{10\,b^3}+\frac{3\left(bx^2+a\right)^{\frac{7}{3}}a^2}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] 3/26*(b*x^2 + a)^(13/3)/b^3 - 3/10*(b*x^2 + a)^(10/3)*a/b^3 + 3/14*(b*x^2 +
 a)^(7/3)*a^2/b^3
```

Fricas [A]
time = 0.90, size = 57, normalized size = 0.97

$$\frac{3\left(35\,b^4x^8+49\,ab^3x^6+2\,a^2b^2x^4-3\,a^3bx^2+9\,a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}}{910\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/910*(35*b^4*x^8 + 49*a*b^3*x^6 + 2*a^2*b^2*x^4 - 3*a^3*b*x^2 + 9*a^4)*(b*
x^2 + a)^(1/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.
time = 0.37, size = 112, normalized size = 1.90

$$\begin{cases} \frac{27a^4\sqrt[3]{a+bx^2}}{910b^3} - \frac{9a^3x^2\sqrt[3]{a+bx^2}}{910b^2} + \frac{3a^2x^4\sqrt[3]{a+bx^2}}{455b} + \frac{21ax^6\sqrt[3]{a+bx^2}}{130} + \frac{3bx^8\sqrt[3]{a+bx^2}}{26} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((27*a**4*(a + b*x**2)**(1/3)/(910*b**3) - 9*a**3*x**2*(a + b*x**2
)**(1/3)/(910*b**2) + 3*a**2*x**4*(a + b*x**2)**(1/3)/(455*b) + 21*a*x**6*(
a + b*x**2)**(1/3)/130 + 3*b*x**8*(a + b*x**2)**(1/3)/26, Ne(b, 0)), (a**(4
/3)*x**6/6, True))
```

Giac [A]
time = 1.74, size = 43, normalized size = 0.73

$$\frac{3\left(35\left(bx^2+a\right)^{\frac{13}{3}} - 91\left(bx^2+a\right)^{\frac{10}{3}}a + 65\left(bx^2+a\right)^{\frac{7}{3}}a^2\right)}{910\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/910*(35*(b*x^2 + a)^(13/3) - 91*(b*x^2 + a)^(10/3)*a + 65*(b*x^2 + a)^(7/
3)*a^2)/b^3
```

Mupad [B]
time = 5.10, size = 53, normalized size = 0.90

$${\left(b\,x^2+a\right)}^{1/3}\,\left(\frac{21\,a\,x^6}{130}+\frac{3\,b\,x^8}{26}+\frac{27\,a^4}{910\,b^3}+\frac{3\,a^2\,x^4}{455\,b}-\frac{9\,a^3\,x^2}{910\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(4/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((21*a*x^6)/130 + (3*b*x^8)/26 + (27*a^4)/(910*b^3) + (3*
a^2*x^4)/(455*b) - (9*a^3*x^2)/(910*b^2))
```

3.691 $\int x^3(a+bx^2)^{4/3}\,dx$

Optimal. Leaf size=38

$$-\frac{3a(a+bx^2)^{7/3}}{14b^2}+\frac{3(a+bx^2)^{10/3}}{20b^2}$$

```
[Out] -3/14*a*(b*x^2+a)^(7/3)/b^2+3/20*(b*x^2+a)^(10/3)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {272, 45}

$$\frac{3(a+bx^2)^{10/3}}{20b^2}-\frac{3a(a+bx^2)^{7/3}}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(7/3))/(14*b^2) + (3*(a + b*x^2)^(10/3))/(20*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{4/3}\,dx&=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^{4/3}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^{4/3}}{b}+\frac{(a+bx)^{7/3}}{b}\right)\,dx,x,x^2\right)\\&=-\frac{3a(a+bx^2)^{7/3}}{14b^2}+\frac{3(a+bx^2)^{10/3}}{20b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.74

$$\frac{3(a+bx^2)^{7/3}\,(-3a+7bx^2)}{140b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(4/3),x]
[Out] (3*(a + b*x^2)^(7/3)*(-3*a + 7*b*x^2))/(140*b^2)
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.66

method	result	size
gosper	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(-7b\,x^{2}+3a\right)}{140b^{2}}$	25
trager	$-\frac{3\left(-7b^{3}x^{6}-11a\,b^{2}x^{4}-a^{2}b\,x^{2}+3a^{3}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{140b^{2}}$	47
risch	$-\frac{3\left(-7b^{3}x^{6}-11a\,b^{2}x^{4}-a^{2}b\,x^{2}+3a^{3}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{140b^{2}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
[Out] -3/140*(b*x^2+a)^(7/3)*(-7*b*x^2+3*a)/b^2
```

Maxima [A]
time = 0.28, size = 30, normalized size = 0.79

$$\frac{3\,(bx^2+a)^{\frac{10}{3}}}{20\,b^2}-\frac{3\,(bx^2+a)^{\frac{7}{3}}a}{14\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] 3/20*(b*x^2 + a)^(10/3)/b^2 - 3/14*(b*x^2 + a)^(7/3)*a/b^2
```

Fricas [A]
time = 1.10, size = 45, normalized size = 1.18

$$\frac{3\,(7\,b^3x^6+11\,ab^2x^4+a^2bx^2-3\,a^3)(bx^2+a)^{\frac{1}{3}}}{140\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/140*(7*b^3*x^6 + 11*a*b^2*x^4 + a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(34) = 68$.
time = 0.28, size = 88, normalized size = 2.32

$$\begin{cases} -\frac{9a^3\sqrt[3]{a+bx^2}}{140b^2} + \frac{3a^2x^2\sqrt[3]{a+bx^2}}{140b} + \frac{33ax^4\sqrt[3]{a+bx^2}}{140} + \frac{3bx^6\sqrt[3]{a+bx^2}}{20} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-9*a**3*(a + b*x**2)**(1/3)/(140*b**2) + 3*a**2*x**2*(a + b*x**2
)**(1/3)/(140*b) + 33*a*x**4*(a + b*x**2)**(1/3)/140 + 3*b*x**6*(a + b*x**2
)**(1/3)/20, Ne(b, 0)), (a**(4/3)*x**4/4, True))
```

Giac [A]
time = 2.36, size = 29, normalized size = 0.76

$$\frac{3\left(7\,(bx^2+a)^{\frac{10}{3}} - 10\,(bx^2+a)^{\frac{7}{3}}a\right)}{140\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/140*(7*(b*x^2 + a)^(10/3) - 10*(b*x^2 + a)^(7/3)*a)/b^2
```

Mupad [B]
time = 5.05, size = 42, normalized size = 1.11

$$\left(b\,x^2+a\right)^{1/3}\,\left(\frac{33\,a\,x^4}{140}+\frac{3\,b\,x^6}{20}-\frac{9\,a^3}{140\,b^2}+\frac{3\,a^2\,x^2}{140\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(4/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((33*a*x^4)/140 + (3*b*x^6)/20 - (9*a^3)/(140*b^2) + (3*a
^2*x^2)/(140*b))
```

3.692 $\int x(a+bx^2)^{4/3}\,dx$

Optimal. Leaf size=18

$$\frac{3(a+bx^2)^{7/3}}{14b}$$

```
[Out] 3/14*(b*x^2+a)^(7/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\frac{3(a+bx^2)^{7/3}}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{4/3}\,dx=\frac{3(a+bx^2)^{7/3}}{14b}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{3(a+bx^2)^{7/3}}{14b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

Maple [A]
time = 0.04, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{3(bx^2+a)^{\frac{7}{3}}}{14b}$	15
derivativedivides	$\frac{3(bx^2+a)^{\frac{7}{3}}}{14b}$	15
default	$\frac{3(bx^2+a)^{\frac{7}{3}}}{14b}$	15
trager	$\frac{3(b^2x^4+2abx^2+a^2)(bx^2+a)^{\frac{1}{3}}}{14b}$	33
risch	$\frac{3(b^2x^4+2abx^2+a^2)(bx^2+a)^{\frac{1}{3}}}{14b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/14*(b*x^2+a)^(7/3)/b
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{7}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] 3/14*(b*x^2 + a)^(7/3)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.
time = 0.95, size = 32, normalized size = 1.78

$$\frac{3\,(b^2x^4+2\,abx^2+a^2)(bx^2+a)^{\frac{1}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] 3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(14) = 28$.

time = 0.19, size = 65, normalized size = 3.61

$$\begin{cases} \frac{3a^2\sqrt[3]{a+bx^2}}{14b} + \frac{3ax^2\sqrt[3]{a+bx^2}}{7} + \frac{3bx^4\sqrt[3]{a+bx^2}}{14} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((3*a**2*(a + b*x**2)**(1/3)/(14*b) + 3*a*x**2*(a + b*x**2)**(1/3)
/7 + 3*b*x**4*(a + b*x**2)**(1/3)/14, Ne(b, 0)), (a**(4/3)*x**2/2, True))
```

Giac [A]
time = 1.92, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b
```

Mupad [B]
time = 4.99, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^2+a\right)^{7/3}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(4/3),x)
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

3.693 $\int \frac{(a+bx^2)^{4/3}}{x} dx$

Optimal. Leaf size=117

$$\frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a}\right.$$

```
[Out] 3/2*a*(b*x^2+a)^(1/3)+3/8*(b*x^2+a)^(4/3)-1/2*a^(4/3)*ln(x)+3/4*a^(4/3)*ln(
a^(1/3)-(b*x^2+a)^(1/3))-1/2*a^(4/3)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))
/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 52, 59, 631, 210, 31}

$$-\frac{1}{2}\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)+\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x,x]
```

```
[Out] (3*a*(a + b*x^2)^(1/3))/2 + (3*(a + b*x^2)^(4/3))/8 - (Sqrt[3]*a^(4/3)*ArcT
an[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(4/3)*Log[x])
/2 + (3*a^(4/3)*Log[a^(1/3) - (a + b*x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{4/3}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{8}(a+bx^2)^{4/3}+\frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}(a+bx^2)^{4/3}+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}(a+bx^2)^{4/3}-\frac{1}{2}a^{4/3}\log(x)-\frac{1}{4}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a}\right.\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}(a+bx^2)^{4/3}-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\left(3a^{4/}\right.\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}(a+bx^2)^{4/3}-\frac{1}{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{4/3}\log(x)+
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 135, normalized size = 1.15

$$\frac{1}{8}\left(3\sqrt[3]{a+bx^2}\,(5a+bx^2)-4\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4a^{4/3}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-2a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(5*a + b*x^2) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1 + (2*(a + 
b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[-a^(1/3) + (a + b*x^2)^(1/3
)] - 2*a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]
)/8
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x,x)
```

Maxima [A]
time = 0.50, size = 109, normalized size = 0.93

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{4}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{8}\,(bx^2+a)^{\frac{4}{3}}+\frac{3}{2}\,(bx^2+a)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 + a)^(4/3
) + 3/2*(b*x^2 + a)^(1/3)*a
```

Fricas [A]
time = 0.80, size = 111, normalized size = 0.95

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{4}\,a^{\frac{4}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{8}\,(bx^2+5\,a)\,(bx^2+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqrt
(3)*a)/a) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 + 5*a
)*(b*x^2 + a)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.82, size = 49, normalized size = 0.42

$$-\frac{b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{4}{3}\\ -\frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x,x)
```

```
[Out] -b**(4/3)*x**(8/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), a*exp_polar(I*p
i)/(b*x**2))/(2*gamma(-1/3))
```

Giac [A]
time = 2.43, size = 110, normalized size = 0.94

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{4}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{8}\left(bx^2+a\right)^{\frac{4}{3}}+\frac{3}{2}\left(bx^2+a\right)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(4/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/8*(b*x^2 + a)
^(4/3) + 3/2*(b*x^2 + a)^(1/3)*a
```

Mupad [B]
time = 5.00, size = 133, normalized size = 1.14

$$\frac{3\,a\,{\left(b\,x^2+a\right)}^{1/3}}{2}+\frac{3\,{\left(b\,x^2+a\right)}^{4/3}}{8}+\frac{a^{4/3}\,\ln\left(\frac{9\,a^2\,{\left(b\,x^2+a\right)}^{1/3}}{2}-\frac{9\,a^{7/3}}{2}\right)}{2}-\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+\frac{9\,a^2\,{\left(b\,x^2+a\right)}^{1/3}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+a^{4/3}\,\ln\left(9\,a^{7/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)-\frac{9\,a^2\,{\left(b\,x^2+a\right)}^{1/3}}{2}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x,x)
```

```
[Out] (3*a*(a + b*x^2)^(1/3))/2 + (3*(a + b*x^2)^(4/3))/8 + (a^(4/3)*log((9*a^2*(
a + b*x^2)^(1/3))/2 - (9*a^(7/3))/2))/2 - (a^(4/3)*log((9*a^(7/3)*((3^(1/2)
*1i)/2 + 1/2))/2 + (9*a^2*(a + b*x^2)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/2 +
 a^(4/3)*log(9*a^(7/3)*((3^(1/2)*1i)/4 - 1/4) - (9*a^2*(a + b*x^2)^(1/3))/2
)*((3^(1/2)*1i)/4 - 1/4)
```

3.694 $\int \frac{(a+bx^2)^{4/3}}{x^3} dx$

Optimal. Leaf size=116

$$2b\sqrt[3]{a+bx^2}-\frac{(a+bx^2)^{4/3}}{2x^2}-\frac{2\sqrt[3]{a}\,b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.$$

```
[Out] 2*b*(b*x^2+a)^(1/3)-1/2*(b*x^2+a)^(4/3)/x^2-2/3*a^(1/3)*b*ln(x)+a^(1/3)*b*l
n(a^(1/3)-(b*x^2+a)^(1/3))-2/3*a^(1/3)*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1
/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {272, 43, 52, 59, 631, 210, 31}

$$-\frac{2\sqrt[3]{a}\,b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{(a+bx^2)^{4/3}}{2x^2}+2b\sqrt[3]{a+bx^2}+\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{2}{3}\sqrt[3]{a}\,b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^3,x]
```

```
[Out] 2*b*(a + b*x^2)^(1/3) - (a + b*x^2)^(4/3)/(2*x^2) - (2*a^(1/3)*b*ArcTan[(a^
(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log
[x])/3 + a^(1/3)*b*Log[a^(1/3) - (a + b*x^2)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{(a+bx^2)^{4/3}}{2x^2}+\frac{1}{3}(2b)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx,x,x^2\right)\\
&= 2b\sqrt[3]{a+bx^2}-\frac{(a+bx^2)^{4/3}}{2x^2}+\frac{1}{3}(2ab)\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= 2b\sqrt[3]{a+bx^2}-\frac{(a+bx^2)^{4/3}}{2x^2}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)-\left(\sqrt[3]{a}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+\right.\\
&= 2b\sqrt[3]{a+bx^2}-\frac{(a+bx^2)^{4/3}}{2x^2}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\left(2\sqrt[3]{a}\right.\\
&= 2b\sqrt[3]{a+bx^2}-\frac{(a+bx^2)^{4/3}}{2x^2}-\frac{2\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+\sqrt[3]{a}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 140, normalized size = 1.21

$$\frac{1}{6}\left(-\frac{3(a-3bx^2)\sqrt[3]{a+bx^2}}{x^2}-4\sqrt{3}\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{a}\,b\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-2\sqrt[3]{a}\,b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^3,x]
```

```
[Out] ((-3*(a - 3*b*x^2)*(a + b*x^2)^(1/3))/x^2 - 4*Sqrt[3]*a^(1/3)*b*ArcTan[(1 +
 (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*b*Log[-a^(1/3) + (a +
b*x^2)^(1/3)] - 2*a^(1/3)*b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a +
b*x^2)^(2/3)])/6
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^3,x)
```

Maxima [A]
time = 0.52, size = 116, normalized size = 1.00

$$-\frac{2}{3}\sqrt{3}\,a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{3}\,a^{\frac{1}{3}}b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{2}{3}\,a^{\frac{1}{3}}b\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\,(bx^2+a)^{\frac{1}{3}}b-\frac{(bx^2+a)^{\frac{1}{3}}a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*a^(1/3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a
^(1/3)) - 1/3*a^(1/3)*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 2/3*a^(1/3)*b*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a
)^(1/3)*b - 1/2*(b*x^2 + a)^(1/3)*a/x^2
```

Fricas [A]
time = 0.74, size = 129, normalized size = 1.11

$$-\frac{4\,\sqrt{3}\,a^{\frac{1}{3}}bx^2\arctan\left(\frac{2\,\sqrt{3}\,(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+2\,a^{\frac{1}{3}}bx^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}bx^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(3\,bx^2-a)(bx^2+a)^{\frac{1}{3}}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*a^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/
3) + sqrt(3)*a)/a) + 2*a^(1/3)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1
/3)*a^(1/3) + a^(2/3)) - 4*a^(1/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)) -
 3*(3*b*x^2 - a)*(b*x^2 + a)^(1/3))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.89, size = 46, normalized size = 0.40

$$-\frac{b^{\frac{4}{3}}x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**3,x)
```

```
[Out] -b**(4/3)*x**(2/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(2/3))
```

Giac [A]
time = 1.44, size = 131, normalized size = 1.13

$$-\frac{4\,\sqrt{3}\,a^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+2\,a^{\frac{1}{3}}b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}b^2\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)-9\,(bx^2+a)^{\frac{1}{3}}b^2+\frac{3\,(bx^2+a)^{\frac{1}{3}}ab}{x^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/6*(4*sqrt(3)*a^(1/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/
3))/a^(1/3)) + 2*a^(1/3)*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1
/3) + a^(2/3)) - 4*a^(1/3)*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) - 9*(b
*x^2 + a)^(1/3)*b^2 + 3*(b*x^2 + a)^(1/3)*a*b/x^2)/b
```

Mupad [B]
time = 5.42, size = 141, normalized size = 1.22

$$\frac{3\,b\,{\left(b\,x^2+a\right)}^{1/3}}{2}-\frac{a\,{\left(b\,x^2+a\right)}^{1/3}}{2\,x^2}+\frac{2\,a^{1/3}\,b\,\ln\left(6\,a^{4/3}\,b-6\,a\,b\,{\left(b\,x^2+a\right)}^{1/3}\right)}{3}+\frac{a^{1/3}\,b\,\ln\left(6\,a\,b\,{\left(b\,x^2+a\right)}^{1/3}-3\,a^{4/3}\,b\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{a^{1/3}\,b\,\ln\left(3\,a^{4/3}\,b\,\left(1+\sqrt{3}\,1\mathrm{i}\right)+6\,a\,b\,{\left(b\,x^2+a\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^3,x)
```

```
[Out] (3*b*(a + b*x^2)^(1/3))/2 - (a*(a + b*x^2)^(1/3))/(2*x^2) + (2*a^(1/3)*b*lo
g(6*a^(4/3)*b - 6*a*b*(a + b*x^2)^(1/3)))/3 + (a^(1/3)*b*log(6*a*b*(a + b*x
^2)^(1/3) - 3*a^(4/3)*b*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (a^(1/3)*b*
log(3*a^(4/3)*b*(3^(1/2)*1i + 1) + 6*a*b*(a + b*x^2)^(1/3))*(3^(1/2)*1i + 1
))/3
```

3.695 $\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx$

Optimal. Leaf size=132

$$-\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{(a+bx^2)^{4/3}}{4x^4}-\frac{b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}}$$

```
[Out] -1/3*b*(b*x^2+a)^(1/3)/x^2-1/4*(b*x^2+a)^(4/3)/x^4-1/9*b^2*ln(x)/a^(2/3)+1/
6*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(2/3)-1/9*b^2*arctan(1/3*(a^(1/3)+2*(b*
x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 43, 59, 631, 210, 31}

$$-\frac{b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{(a+bx^2)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^5,x]
```

```
[Out] -1/3*(b*(a + b*x^2)^(1/3))/x^2 - (a + b*x^2)^(4/3)/(4*x^4) - (b^2*ArcTan[(a
^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^
2*Log[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^{4/3}}{4x^4}+\frac{1}{3}b\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{(a+bx^2)^{4/3}}{4x^4}+\frac{1}{9}b^2\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{(a+bx^2)^{4/3}}{4x^4}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{6a^{2/3}}-\ldots\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{(a+bx^2)^{4/3}}{4x^4}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}}+\frac{b^2\text{Subst}\left(\int \ldots\right.}{\ldots}\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{(a+bx^2)^{4/3}}{4x^4}-\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}\ldots\right.}{6\ldots}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 148, normalized size = 1.12

$$\frac{1}{36}\left(-\frac{3\sqrt[3]{a+bx^2}\,(3a+7bx^2)}{x^4}-\frac{4\sqrt{3}\,b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{4b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)}{a^{2/3}}-\frac{2b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{a^{2/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^5,x]
```

```
[Out] ((-3*(a + b*x^2)^(1/3)*(3*a + 7*b*x^2))/x^4 - (4*Sqrt[3]*b^2*ArcTan[(1 + (2
*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*b^2*Log[-a^(1/3) + (a +
 b*x^2)^(1/3)])/a^(2/3) - (2*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3)])/a^(2/3))/36
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^5,x)
```

Maxima [A]
time = 0.53, size = 152, normalized size = 1.15

$$-\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}}-\frac{b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{2}{3}}}+\frac{b^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{2}{3}}}-\frac{7\,(bx^2+a)^{\frac{4}{3}}b^2-4\,(bx^2+a)^{\frac{1}{3}}ab^2}{12\left((bx^2+a)^2-2\,(bx^2+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(2/3) - 1/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(2/3) + 1/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/12*(7
*(b*x^2 + a)^(4/3)*b^2 - 4*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2 - 2*(b*x
^2 + a)*a + a^2)
```

Fricas [A]
time = 0.99, size = 174, normalized size = 1.32

$$-\frac{4\sqrt{3}\,(a^2)^{\frac{1}{6}}ab^2x^4\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}\,(a^2)^{\frac{1}{3}}a+2\sqrt{3}\,(bx^2+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{3\,a^2}\right)+2\,(a^2)^{\frac{2}{3}}b^2x^4\log\left((bx^2+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx^2+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)-4\,(a^2)^{\frac{2}{3}}b^2x^4\log\left((bx^2+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)+3\,(7\,a^2bx^2+3\,a^3)(bx^2+a)^{\frac{1}{3}}}{36\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="fricas")
```

```
[Out] -1/36*(4*sqrt(3)*(a^2)^(1/6)*a*b^2*x^4*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2
)^(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 2*(a^2)^(2/3)*b
^2*x^4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2
/3)) - 4*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*
a^2*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^2*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.96, size = 42, normalized size = 0.32

$$-\frac{b^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**5,x)
```

```
[Out] -b**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(4/3)*gamma(5/3))
```

Giac [A]
time = 1.74, size = 139, normalized size = 1.05

$$-\frac{\frac{4\sqrt{3}\,b^{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2\,b^{3}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{4\,b^{3}\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{3\left(7\left(bx^{2}+a\right)^{\frac{4}{3}}b^{3}-4\left(bx^{2}+a\right)^{\frac{1}{3}}ab^{3}\right)}{b^{2}x^{4}}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="giac")
```

```
[Out] -1/36*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(2/3) + 2*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(2/3) - 4*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(
7*(b*x^2 + a)^(4/3)*b^3 - 4*(b*x^2 + a)^(1/3)*a*b^3)/(b^2*x^4))/b
```

Mupad [B]
time = 5.49, size = 191, normalized size = 1.45

$$\frac{b^{2}\,\ln\left(b^{2}\,{\left(b\,x^{2}+a\right)}^{1/3}-a^{1/3}\,b^{2}\right)}{9\,a^{2/3}}-\frac{\ln\left(\frac{a^{1/3}\,\left(b^{2}+\sqrt{3}\,b^{2}\,1{}\mathrm{i}\right)}{2}+b^{2}\,{\left(b\,x^{2}+a\right)}^{1/3}\right)\,\left(b^{2}+\sqrt{3}\,b^{2}\,1{}\mathrm{i}\right)}{18\,a^{2/3}}-\frac{\frac{7\,b^{2}\,{\left(b\,x^{2}+a\right)}^{4/3}}{6}-\frac{2\,a\,b^{2}\,{\left(b\,x^{2}+a\right)}^{1/3}}{3}}{2\,{\left(b\,x^{2}+a\right)}^{2}-4\,a\,\left(b\,x^{2}+a\right)+2\,a^{2}}+\frac{b^{2}\,\ln\left(b^{2}\,{\left(b\,x^{2}+a\right)}^{1/3}-a^{1/3}\,b^{2}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^5,x)
```

```
[Out] (b^2*log(b^2*(a + b*x^2)^(1/3) - a^(1/3)*b^2))/(9*a^(2/3)) - (log((a^(1/3)*
(3^(1/2)*b^2*1i + b^2))/2 + b^2*(a + b*x^2)^(1/3))*(3^(1/2)*b^2*1i + b^2))/
(18*a^(2/3)) - ((7*b^2*(a + b*x^2)^(4/3))/6 - (2*a*b^2*(a + b*x^2)^(1/3))/3
)/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2) + (b^2*log(b^2*(a + b*x^2)^(1
/3) - a^(1/3)*b^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(2/3
))
```

3.696 $\int x^4(a+bx^2)^{4/3}\,dx$

Optimal. Leaf size=335

$$-\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}+\frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}-\frac{432\ 3^{3/4}\sqrt{2-\sqrt{3}}\ a^4\left(\sqrt[3]{a}\right.}{}$$

```
[Out] -432/21505*a^3*x*(b*x^2+a)^(1/3)/b^2+48/4301*a^2*x^3*(b*x^2+a)^(1/3)/b+24/3
91*a*x^5*(b*x^2+a)^(1/3)+3/23*x^5*(b*x^2+a)^(4/3)-432/21505*3^(3/4)*a^4*(a^
(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2
+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)-1/2*2^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 242, 225}

$$-\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}+\frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}-\frac{432\ 3^{3/4}\sqrt{2-\sqrt{3}}\ a^4\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{21505b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(4/3),x]
```

```
[Out] (-432*a^3*x*(a + b*x^2)^(1/3))/(21505*b^2) + (48*a^2*x^3*(a + b*x^2)^(1/3))
/(4301*b) + (24*a*x^5*(a + b*x^2)^(1/3))/391 + (3*x^5*(a + b*x^2)^(4/3))/23
 - (432*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(21505*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4(a+bx^2)^{4/3}\,dx = \frac{3}{23}x^5(a+bx^2)^{4/3}+\frac{1}{23}(8a)\int x^4\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5(a+bx^2)^{4/3}+\frac{1}{391}(16a^2)\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx$$

$$= \frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5(a+bx^2)^{4/3}-\frac{(144a^3)\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx}{4301b}$$

$$= -\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}+\frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5(a+bx^2)^{4/3}+$$

$$= -\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}+\frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5(a+bx^2)^{4/3}+$$

$$= -\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}+\frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5(a+bx^2)^{4/3}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.71, size = 79, normalized size = 0.24

$$\frac{3x\sqrt[3]{a+bx^2}\left(-\left((9a-17bx^2)\left(a+bx^2\right)^2\right)+\frac{9a^3\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}\right)}{391b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*(-((9*a - 17*b*x^2)*(a + b*x^2)^2) + (9*a^3*Hypergeo
metric2F1[-4/3, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(391*b^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^4\left(b\,x^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(4/3),x)
```

```
[Out] int(x^4*(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^6 + a*x^4)*(b*x^2 + a)^(1/3), x)
```

Sympy [A]
time = 0.59, size = 29, normalized size = 0.09

$$\frac{a^{\frac{4}{3}} x^5 {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*x**5*hyper((-4/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(b\,x^2+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(4/3),x)
[Out] int(x^4*(a + b*x^2)^(4/3), x)
```

3.697 $\int x^2(a+bx^2)^{4/3}\,dx$

Optimal. Leaf size=311

$$\frac{48a^2x\sqrt[3]{a+bx^2}}{935b}+\frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}+\frac{48\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\cdots}{\left(\left(1\cdots\right.\right.}}}{935b^2x\cdots}$$

[Out] 48/935*a^2*x*(b*x^2+a)^(1/3)/b+24/187*a*x^3*(b*x^2+a)^(1/3)+3/17*x^3*(b*x^2+a)^(4/3)+48/935*3^(3/4)*a^3*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.14, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {285, 327, 242, 225}

$$\frac{48a^2x\sqrt[3]{a+bx^2}}{935b}+\frac{48\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\operatorname{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{935b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^2)^(4/3),x]

[Out] (48*a^2*x*(a + b*x^2)^(1/3))/(935*b) + (24*a*x^3*(a + b*x^2)^(1/3))/187 + (3*x^3*(a + b*x^2)^(4/3))/17 + (48*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])

Rule 225

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])

```
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2(a+bx^2)^{4/3}\,dx = \frac{3}{17}x^3(a+bx^2)^{4/3} + \frac{1}{17}(8a)\int x^2\sqrt[3]{a+bx^2}\,dx$$
$$= \frac{24}{187}ax^3\sqrt[3]{a+bx^2} + \frac{3}{17}x^3(a+bx^2)^{4/3} + \frac{1}{187}(16a^2)\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx$$
$$= \frac{48a^2x\sqrt[3]{a+bx^2}}{935b} + \frac{24}{187}ax^3\sqrt[3]{a+bx^2} + \frac{3}{17}x^3(a+bx^2)^{4/3} - \frac{(48a^3)\int \frac{1}{(a+bx^2)^{2/3}}\,dx}{935b}$$
$$= \frac{48a^2x\sqrt[3]{a+bx^2}}{935b} + \frac{24}{187}ax^3\sqrt[3]{a+bx^2} + \frac{3}{17}x^3(a+bx^2)^{4/3} - \frac{\left(72a^3\sqrt{bx^2}\right)\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{48a^2x\sqrt[3]{a+bx^2}}{935b} + \frac{24}{187}ax^3\sqrt[3]{a+bx^2} + \frac{3}{17}x^3(a+bx^2)^{4/3} + \frac{48\ 3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.27, size = 67, normalized size = 0.22

$$\frac{3x\sqrt[3]{a+bx^2}\left((a+bx^2)^2 - \frac{a^2\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*((a + b*x^2)^2 - (a^2*Hypergeometric2F1[-4/3, 1/2, 3
/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(17*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2\left(b\,x^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(4/3),x)
```

```
[Out] int(x^2*(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^4 + a*x^2)*(b*x^2 + a)^(1/3), x)
```

Sympy [A]
time = 0.54, size = 29, normalized size = 0.09

$$\frac{a^{\frac{4}{3}} x^{3} {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(4/3),x)
[Out] a**(4/3)*x**3*hyper((-4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(b\,x^2+a\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(4/3),x)
[Out] int(x^2*(a + b*x^2)^(4/3), x)
```

3.698 $\int (a+bx^2)^{4/3}\,dx$

Optimal. Leaf size=285

$$\frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}-\frac{16\ 3^{3/4}\sqrt{2-\sqrt{3}}\ a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\ \sqrt[3]{a+bx^2}+(a}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}\right.}}}{55bx\sqrt{-\frac{\sqrt[3]{a}\ \left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}$$

```
[Out] 24/55*a*x*(b*x^2+a)^(1/3)+3/11*x*(b*x^2+a)^(4/3)-16/55*3^(3/4)*a^2*(a^(1/3)
-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(
1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*
6^(1/2)-1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 242, 225}

$$-\frac{16\ 3^{3/4}\sqrt{2-\sqrt{3}}\ a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\ \sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{55bx\sqrt{-\frac{\sqrt[3]{a}\ \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3),x]
```

```
[Out] (24*a*x*(a + b*x^2)^(1/3))/55 + (3*x*(a + b*x^2)^(4/3))/11 - (16*3^(3/4)*Sq
rt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(
a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1
- Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rubi steps

$$\int \left(a+bx^2\right)^{4/3}\,dx = \frac{3}{11}x\left(a+bx^2\right)^{4/3}+\frac{1}{11}(8a)\int \sqrt[3]{a+bx^2}\,dx$$

$$= \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}+\frac{1}{55}\left(16a^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}+\frac{\left(24a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+b\right.}{55bx}$$

$$= \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}-\frac{16\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}}{((}}}{55b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.78, size = 47, normalized size = 0.16

$$\frac{ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3),x]
```

```
[Out] (a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/2, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^(1/3)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^{2}+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+a)^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3), x)
```

Sympy [**A**]
time = 0.52, size = 26, normalized size = 0.09

$$a^{\frac{4}{3}}x\,{}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*x*hyper((-4/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3), x)
```

Mupad [B]
time = 5.16, size = 37, normalized size = 0.13

$$\frac{x\,{\left(b\,x^2+a\right)}^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3),x)
```

```
[Out] (x*(a + b*x^2)^(4/3)*hypergeom([-4/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(4/3)
```

3.699 $\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx$

Optimal. Leaf size=280

$$\frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{(a+bx^2)^{4/3}}{x}-\frac{16\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \cdots}{5\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+\cdots}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a\cdots}\right.}}}$$

```
[Out] 8/5*b*x*(b*x^2+a)^(1/3)-(b*x^2+a)^(4/3)/x-16/15*a*(a^(1/3)-(b*x^2+a)^(1/3))
*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2
/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/
2))*3^(3/4)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 201, 242, 225}

$$-\frac{16\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{(a+bx^2)^{4/3}}{x}+\frac{8}{5}bx\sqrt[3]{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^2,x]
```

```
[Out] (8*b*x*(a + b*x^2)^(1/3))/5 - (a + b*x^2)^(4/3)/x - (16*Sqrt[2 - Sqrt[3]]*a
*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx &= -\frac{(a+bx^2)^{4/3}}{x}+\frac{1}{3}(8b)\int \sqrt[3]{a+bx^2}\,dx\\
&= \frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{(a+bx^2)^{4/3}}{x}+\frac{1}{15}(16ab)\int \frac{1}{(a+bx^2)^{2/3}}\,dx\\
&= \frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{(a+bx^2)^{4/3}}{x}+\frac{\left(8a\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{5x}\\
&= \frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{(a+bx^2)^{4/3}}{x}-\frac{16\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a-\cdots}}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{5\sqrt[4]{3}\,x\sqrt{-\cdots}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.33, size = 50, normalized size = 0.18

$$-\frac{a\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^2,x]
```

```
[Out] -((a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/2, 1/2, -((b*x^2)/a)])/(x
*(1 + (b*x^2)/a)^(1/3)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^2,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)/x^2, x)
```

Sympy [A]
time = 0.54, size = 29, normalized size = 0.10

$$-\frac{a^{\frac{4}{3}}{}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**2,x)
[Out] -a**(4/3)*hyper((-4/3, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/x^2, x)
```

Mupad [B]
time = 5.61, size = 40, normalized size = 0.14

$$\frac{3\,{\left(b\,x^2+a\right)}^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},-\frac{5}{6};\ \frac{1}{6};\ -\frac{a}{b\,x^2}\right)}{5\,x\,{\left(\frac{a}{b\,x^2}+1\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^2,x)
[Out] (3*(a + b*x^2)^(4/3)*hypergeom([-4/3, -5/6], 1/6, -a/(b*x^2)))/(5*x*(a/(b*x
^2) + 1)^(4/3))
```

3.700 $\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx$

Optimal. Leaf size=284

$$-\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{(a+bx^2)^{4/3}}{3x^3}-\frac{16\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}{9\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\right.}}}$$

```
[Out] -8/9*b*(b*x^2+a)^(1/3)/x-1/3*(b*x^2+a)^(4/3)/x^3-16/27*b*(a^(1/3)-(b*x^2+a)
^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^
2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/
2*2^(1/2))*3^(3/4)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {283, 242, 225}

$$-\frac{16\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{(a+bx^2)^{4/3}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^4,x]
```

```
[Out] (-8*b*(a + b*x^2)^(1/3))/(9*x) - (a + b*x^2)^(4/3)/(3*x^3) - (16*Sqrt[2 - S
qrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)
^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*
EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*x*Sqrt[-((a^(1/
3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx &= -\frac{(a+bx^2)^{4/3}}{3x^3}+\frac{1}{9}(8b)\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx\\
&= -\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{(a+bx^2)^{4/3}}{3x^3}+\frac{1}{27}\left(16b^2\right)\int \frac{1}{(a+bx^2)^{2/3}}\,dx\\
&= -\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{(a+bx^2)^{4/3}}{3x^3}+\frac{\left(8b\sqrt{bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{9x}\\
&= -\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{(a+bx^2)^{4/3}}{3x^3}-\frac{16\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a \cdots}}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{9\sqrt[4]{3}\,x\sqrt{-\cdots}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 52, normalized size = 0.18

$$-\frac{a\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^4,x]
```

```
[Out] -1/3*(a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-3/2, -4/3, -1/2, -((b*x^2)/a)]
)/(x^3*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^4,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)/x^4, x)
```

Sympy [A]
time = 0.53, size = 34, normalized size = 0.12

$$-\frac{a^{\frac{4}{3}}{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, -\frac{4}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a} \right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**4,x)
[Out] -a**(4/3)*hyper((-3/2, -4/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^4,x)
[Out] int((a + b*x^2)^(4/3)/x^4, x)
```

3.701 $\int x(-1+x^2)^{7/3}\,dx$

Optimal. Leaf size=13

$$\frac{3}{20}\left(-1+x^2\right)^{10/3}$$

[Out] 3/20*(x^2-1)^(10/3)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

Antiderivative was successfully verified.

[In] Int[x*(-1 + x^2)^(7/3),x]

[Out] (3*(-1 + x^2)^(10/3))/20

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(-1+x^2\right)^{7/3}\,dx=\frac{3}{20}\left(-1+x^2\right)^{10/3}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{20}\left(-1+x^2\right)^{10/3}$$

Antiderivative was successfully verified.

[In] Integrate[x*(-1 + x^2)^(7/3),x]

[Out] (3*(-1 + x^2)^(10/3))/20

Maple [A]
time = 0.10, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$\frac{3(x^2-1)^{\frac{10}{3}}}{20}$	10
default	$\frac{3(x^2-1)^{\frac{10}{3}}}{20}$	10
gosper	$\frac{3(x-1)(x+1)(x^2-1)^{\frac{7}{3}}}{20}$	16
risch	$\frac{3(x^2-1)^{\frac{1}{3}}(x^6-3x^4+3x^2-1)}{20}$	25
trager	$\left(\frac{3}{20}x^6-\frac{9}{20}x^4+\frac{9}{20}x^2-\frac{3}{20}\right)(x^2-1)^{\frac{1}{3}}$	26
meijerg	$\frac{\mathrm{signum}(x^2-1)^{\frac{7}{3}}x^2\,\mathrm{hypergeom}\left(\left[-\frac{7}{3},1\right],[2],x^2\right)}{2(-\mathrm{signum}(x^2-1))^{\frac{7}{3}}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^(7/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/20*(x^2-1)^(10/3)
```

Maxima [A]
time = 0.29, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^2-1\right)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="maxima")
```

```
[Out] 3/20*(x^2 - 1)^(10/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.
time = 1.03, size = 24, normalized size = 1.85

$$\frac{3}{20}\left(x^6-3\,x^4+3\,x^2-1\right)\left(x^2-1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="fricas")
```

```
[Out] 3/20*(x^6 - 3*x^4 + 3*x^2 - 1)*(x^2 - 1)^(1/3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(10) = 20$.
time = 0.36, size = 56, normalized size = 4.31

$$\frac{3x^6\sqrt[3]{x^2-1}}{20}-\frac{9x^4\sqrt[3]{x^2-1}}{20}+\frac{9x^2\sqrt[3]{x^2-1}}{20}-\frac{3\sqrt[3]{x^2-1}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**(7/3),x)
```

```
[Out] 3*x**6*(x**2 - 1)**(1/3)/20 - 9*x**4*(x**2 - 1)**(1/3)/20 + 9*x**2*(x**2 -
1)**(1/3)/20 - 3*(x**2 - 1)**(1/3)/20
```

Giac [A]
time = 1.60, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^{2}-1\right)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="giac")
```

```
[Out] 3/20*(x^2 - 1)^(10/3)
```

Mupad [B]
time = 5.13, size = 25, normalized size = 1.92

$$\left(x^{2}-1\right)^{1/3}\left(\frac{3\,x^{6}}{20}-\frac{9\,x^{4}}{20}+\frac{9\,x^{2}}{20}-\frac{3}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^(7/3),x)
```

```
[Out] (x^2 - 1)^(1/3)*((9*x^2)/20 - (9*x^4)/20 + (3*x^6)/20 - 3/20)
```

3.702 $\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3(a+bx^2)^{2/3}}{4b^4}+\frac{9a^2(a+bx^2)^{5/3}}{10b^4}-\frac{9a(a+bx^2)^{8/3}}{16b^4}+\frac{3(a+bx^2)^{11/3}}{22b^4}$$

```
[Out] -3/4*a^3*(b*x^2+a)^(2/3)/b^4+9/10*a^2*(b*x^2+a)^(5/3)/b^4-9/16*a*(b*x^2+a)^
(8/3)/b^4+3/22*(b*x^2+a)^(11/3)/b^4
```

Rubi [A]

time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$-\frac{3a^3(a+bx^2)^{2/3}}{4b^4}+\frac{9a^2(a+bx^2)^{5/3}}{10b^4}+\frac{3(a+bx^2)^{11/3}}{22b^4}-\frac{9a(a+bx^2)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(2/3))/(4*b^4) + (9*a^2*(a + b*x^2)^(5/3))/(10*b^4) - (
9*a*(a + b*x^2)^(8/3))/(16*b^4) + (3*(a + b*x^2)^(11/3))/(22*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)\,dx,x,x^2\right)\\
&= -\frac{3a^3(a+bx^2)^{2/3}}{4b^4}+\frac{9a^2(a+bx^2)^{5/3}}{10b^4}-\frac{9a(a+bx^2)^{8/3}}{16b^4}+\frac{3(a+bx^2)^{11/3}}{22b^4}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.62

$$\frac{3(a+bx^2)^{2/3}(-81a^3+54a^2bx^2-45ab^2x^4+40b^3x^6)}{880b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3)*(-81*a^3 + 54*a^2*b*x^2 - 45*a*b^2*x^4 + 40*b^3*x^6))/
(880*b^4)
```

Maple [A]
time = 0.04, size = 47, normalized size = 0.59

method	result	size
gosper	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{2}{3}}\left(-40b^{3}x^{6}+45a\,b^{2}x^{4}-54a^{2}b\,x^{2}+81a^{3}\right)}{880b^{4}}$	47
trager	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{2}{3}}\left(-40b^{3}x^{6}+45a\,b^{2}x^{4}-54a^{2}b\,x^{2}+81a^{3}\right)}{880b^{4}}$	47
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{2}{3}}\left(-40b^{3}x^{6}+45a\,b^{2}x^{4}-54a^{2}b\,x^{2}+81a^{3}\right)}{880b^{4}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/880*(b*x^2+a)^(2/3)*(-40*b^3*x^6+45*a*b^2*x^4-54*a^2*b*x^2+81*a^3)/b^4
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{11}{3}}}{22\,b^4}-\frac{9\,(bx^2+a)^{\frac{8}{3}}a}{16\,b^4}+\frac{9\,(bx^2+a)^{\frac{5}{3}}a^2}{10\,b^4}-\frac{3\,(bx^2+a)^{\frac{2}{3}}a^3}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/22*(b*x^2 + a)^(11/3)/b^4 - 9/16*(b*x^2 + a)^(8/3)*a/b^4 + 9/10*(b*x^2 +
a)^(5/3)*a^2/b^4 - 3/4*(b*x^2 + a)^(2/3)*a^3/b^4
```

Fricas [A]
time = 1.40, size = 46, normalized size = 0.58

$$\frac{3\,(40\,b^3x^6-45\,ab^2x^4+54\,a^2bx^2-81\,a^3)(bx^2+a)^{\frac{2}{3}}}{880\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="fricas")

[Out] 3/880*(40*b^3*x^6 - 45*a*b^2*x^4 + 54*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(2/3)/b^4

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(75) = 150$.
time = 1.31, size = 1690, normalized size = 21.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(b*x**2+a)**(1/3),x)

[Out] -243*a**(71/3)*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 243*a**(71/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 1296*a**(68/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1458*a**(68/3)*b*x**2/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 2808*a**(65/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 3645*a**(65/3)*b**2*x**4/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 3120*a**(62/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 4860*a**(62/3)*b**3*x**6/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 1710*a**(59/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 3645*a**(59/3)*b**4*x**8/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 72*a**(56/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1458*a**(56/3)*b**5*x**10/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1104*a**(53/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(88

```
0*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b
**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*
x**12) + 243*a**(53/3)*b**6*x**12/(880*a**20*b**4 + 5280*a**19*b**5*x**2 +
13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 528
0*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1152*a**(50/3)*b**7*x**14*(1
+ b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**
6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x*
*10 + 880*a**14*b**10*x**12) + 585*a**(47/3)*b**8*x**16*(1 + b*x**2/a)**(2/
3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a
**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*
b**10*x**12) + 120*a**(44/3)*b**9*x**18*(1 + b*x**2/a)**(2/3)/(880*a**20*b*
*4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 +
 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12)
```

Giac [A]
time = 1.12, size = 61, normalized size = 0.76

$$-\frac{3\,(bx^2+a)^{\frac{2}{3}}a^3}{4\,b^4}+\frac{3\left(40\,(bx^2+a)^{\frac{11}{3}}-165\,(bx^2+a)^{\frac{8}{3}}a+264\,(bx^2+a)^{\frac{5}{3}}a^2\right)}{880\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] -3/4*(b*x^2 + a)^(2/3)*a^3/b^4 + 3/880*(40*(b*x^2 + a)^(11/3) - 165*(b*x^2
+ a)^(8/3)*a + 264*(b*x^2 + a)^(5/3)*a^2)/b^4
```

Mupad [B]
time = 5.32, size = 48, normalized size = 0.60

$$-{\left(b\,x^2+a\right)}^{2/3}\,\left(\frac{243\,a^3}{880\,b^4}-\frac{3\,x^6}{22\,b}+\frac{27\,a\,x^4}{176\,b^2}-\frac{81\,a^2\,x^2}{440\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(1/3),x)
```

```
[Out] -(a + b*x^2)^(2/3)*((243*a^3)/(880*b^4) - (3*x^6)/(22*b) + (27*a*x^4)/(176*
b^2) - (81*a^2*x^2)/(440*b^3))
```

3.703 $\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2(a+bx^2)^{2/3}}{4b^3}-\frac{3a(a+bx^2)^{5/3}}{5b^3}+\frac{3(a+bx^2)^{8/3}}{16b^3}$$

```
[Out] 3/4*a^2*(b*x^2+a)^(2/3)/b^3-3/5*a*(b*x^2+a)^(5/3)/b^3+3/16*(b*x^2+a)^(8/3)/
b^3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{3a^2(a+bx^2)^{2/3}}{4b^3}+\frac{3(a+bx^2)^{8/3}}{16b^3}-\frac{3a(a+bx^2)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(2/3))/(4*b^3) - (3*a*(a + b*x^2)^(5/3))/(5*b^3) + (3*(a
 + b*x^2)^(8/3))/(16*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}}-\frac{2a(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{5/3}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{3a^2(a+bx^2)^{2/3}}{4b^3}-\frac{3a(a+bx^2)^{5/3}}{5b^3}+\frac{3(a+bx^2)^{8/3}}{16b^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{3(a+bx^2)^{2/3}\left(9a^2-6abx^2+5b^2x^4\right)}{80b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3)*(9*a^2 - 6*a*b*x^2 + 5*b^2*x^4))/(80*b^3)
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.61

method	result	size
gosper	$\frac{3\left(b\,x^2+a\right)^{\frac{2}{3}}\left(5b^2x^4-6ab\,x^2+9a^2\right)}{80b^3}$	36
trager	$\frac{3\left(b\,x^2+a\right)^{\frac{2}{3}}\left(5b^2x^4-6ab\,x^2+9a^2\right)}{80b^3}$	36
risch	$\frac{3\left(b\,x^2+a\right)^{\frac{2}{3}}\left(5b^2x^4-6ab\,x^2+9a^2\right)}{80b^3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/80*(b*x^2+a)^(2/3)*(5*b^2*x^4-6*a*b*x^2+9*a^2)/b^3
```

Maxima [A]
time = 0.33, size = 47, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{8}{3}}}{16\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{5}{3}}a}{5\,b^3}+\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 3/16*(b*x^2 + a)^(8/3)/b^3 - 3/5*(b*x^2 + a)^(5/3)*a/b^3 + 3/4*(b*x^2 + a)^
(2/3)*a^2/b^3
```

Fricas [A]
time = 1.00, size = 35, normalized size = 0.59

$$\frac{3\left(5\,b^2x^4-6\,abx^2+9\,a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{80\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/80*(5*b^2*x^4 - 6*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(2/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(54) = 108$.
time = 0.86, size = 631, normalized size = 10.69

$$\frac{27a^{\frac{32}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}-\frac{27a^{\frac{32}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{63a^{\frac{29}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}-\frac{81a^{\frac{29}{3}}bx^2}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{42a^{\frac{26}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}-\frac{81a^{\frac{26}{3}}b^2x^4}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{18a^{\frac{23}{3}}b^3x^6\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}-\frac{27a^{\frac{23}{3}}b^3x^6}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{27a^{\frac{20}{3}}b^4x^8\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{15a^{\frac{17}{3}}b^5x^{10}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(32/3)*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240
*a**6*b**5*x**4 + 80*a**5*b**6*x**6) - 27*a**(32/3)/(80*a**8*b**3 + 240*a**
7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 63*a**(29/3)*b*x**2
*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x
**4 + 80*a**5*b**6*x**6) - 81*a**(29/3)*b*x**2/(80*a**8*b**3 + 240*a**7*b**
4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 42*a**(26/3)*b**2*x**4*(
1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**
4 + 80*a**5*b**6*x**6) - 81*a**(26/3)*b**2*x**4/(80*a**8*b**3 + 240*a**7*b*
*4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 18*a**(23/3)*b**3*x**6*
(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x*
*4 + 80*a**5*b**6*x**6) - 27*a**(23/3)*b**3*x**6/(80*a**8*b**3 + 240*a**7*b
**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 27*a**(20/3)*b**4*x**8
*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x
**4 + 80*a**5*b**6*x**6) + 15*a**(17/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(8
0*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6)
```

Giac [A]
time = 1.02, size = 47, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{2}{3}}a^2}{4\,b^3}+\frac{3\left(5\,(bx^2+a)^{\frac{8}{3}}-16\,(bx^2+a)^{\frac{5}{3}}a\right)}{80\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(b*x^2 + a)^(2/3)*a^2/b^3 + 3/80*(5*(b*x^2 + a)^(8/3) - 16*(b*x^2 + a)^
(5/3)*a)/b^3
```

Mupad [B]
time = 5.21, size = 36, normalized size = 0.61

$${\left(b\,x^2+a\right)}^{2/3}\,\left(\frac{27\,a^2}{80\,b^3}+\frac{3\,x^4}{16\,b}-\frac{9\,a\,x^2}{40\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(1/3),x)
[Out] (a + b*x^2)^(2/3)*((27*a^2)/(80*b^3) + (3*x^4)/(16*b) - (9*a*x^2)/(40*b^2))
```

3.704 $\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=38

$$-\frac{3a(a+bx^2)^{2/3}}{4b^2}+\frac{3(a+bx^2)^{5/3}}{10b^2}$$

```
[Out] -3/4*a*(b*x^2+a)^(2/3)/b^2+3/10*(b*x^2+a)^(5/3)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{3(a+bx^2)^{5/3}}{10b^2}-\frac{3a(a+bx^2)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(2/3))/(4*b^2) + (3*(a + b*x^2)^(5/3))/(10*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b}\right)\,dx,x,x^2\right)\\ &= -\frac{3a(a+bx^2)^{2/3}}{4b^2}+\frac{3(a+bx^2)^{5/3}}{10b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.74

$$\frac{3(a+bx^2)^{2/3}(-3a+2bx^2)}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3)*(-3*a + 2*b*x^2))/(20*b^2)
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.66

method	result	size
gosper	$-\frac{3(b\,x^2+a)^{\frac{2}{3}}(-2b\,x^2+3a)}{20b^2}$	25
trager	$-\frac{3(b\,x^2+a)^{\frac{2}{3}}(-2b\,x^2+3a)}{20b^2}$	25
risch	$-\frac{3(b\,x^2+a)^{\frac{2}{3}}(-2b\,x^2+3a)}{20b^2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/20*(b*x^2+a)^(2/3)*(-2*b*x^2+3*a)/b^2
```

Maxima [A]
time = 0.27, size = 30, normalized size = 0.79

$$\frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b^2}-\frac{3\,(bx^2+a)^{\frac{2}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b^2 - 3/4*(b*x^2 + a)^(2/3)*a/b^2
```

Fricas [A]
time = 1.03, size = 24, normalized size = 0.63

$$\frac{3\,(2\,bx^2-3\,a)(bx^2+a)^{\frac{2}{3}}}{20\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/20*(2*b*x^2 - 3*a)*(b*x^2 + a)^(2/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(34) = 68$.
time = 0.55, size = 178, normalized size = 4.68

$$-\frac{9a^{\frac{11}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}+\frac{9a^{\frac{11}{3}}}{20a^2b^2+20ab^3x^2}-\frac{3a^{\frac{8}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}+\frac{9a^{\frac{8}{3}}bx^2}{20a^2b^2+20ab^3x^2}+\frac{6a^{\frac{5}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/3),x)
```

```
[Out] -9*a**(11/3)*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**(
11/3)/(20*a**2*b**2 + 20*a*b**3*x**2) - 3*a**(8/3)*b*x**2*(1 + b*x**2/a)**(
2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**(8/3)*b*x**2/(20*a**2*b**2 + 20
*a*b**3*x**2) + 6*a**(5/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 +
20*a*b**3*x**2)
```

Giac [A]
time = 1.06, size = 30, normalized size = 0.79

$$\frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b^2}-\frac{3\,(bx^2+a)^{\frac{2}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b^2 - 3/4*(b*x^2 + a)^(2/3)*a/b^2
```

Mupad [B]
time = 5.01, size = 24, normalized size = 0.63

$$-\frac{3\,{\left(b\,x^2+a\right)}^{2/3}\,\left(3\,a-2\,b\,x^2\right)}{20\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(1/3),x)
```

```
[Out] -(3*(a + b*x^2)^(2/3)*(3*a - 2*b*x^2))/(20*b^2)
```

3.705 $\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=18

$$\frac{3(a+bx^2)^{2/3}}{4b}$$

```
[Out] 3/4*(b*x^2+a)^(2/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\frac{3(a+bx^2)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx = \frac{3(a+bx^2)^{2/3}}{4b}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{3(a+bx^2)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

Maple [A]
time = 0.03, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
derivativedivides	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
default	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
trager	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
risch	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/4*(b*x^2+a)^(2/3)/b
```

Maxima [A]
time = 0.31, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

Fricas [A]
time = 1.21, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

Sympy [A]
time = 0.18, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3(a+bx^2)^{\frac{2}{3}}}{4b} & \text{for}\ b \neq 0 \\ \frac{x^2}{2\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/3),x)
```

```
[Out] Piecewise((3*(a + b*x**2)**(2/3)/(4*b), Ne(b, 0)), (x**2/(2*a**(1/3)), True
))
```

Giac [A]
time = 1.35, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

Mupad [B]
time = 4.67, size = 14, normalized size = 0.78

$$\frac{3\,{\left(b\,x^2+a\right)}^{2/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(1/3),x)
```

```
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

3.706 $\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}$$

```
[Out] -1/2*ln(x)/a^(1/3)+3/4*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(1/3)+1/2*arctan(1/3*(
a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
```

Rubi [A]

time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 57, 631, 210, 31}

$$\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^(1
/3)) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(1/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right) - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x\right.}{4\sqrt[3]{a}}\\
&= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}\\
&= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 103, normalized size = 1.20

$$
\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{4\sqrt[3]{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[-a^(
1/3) + (a + b*x^2)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a +
b*x^2)^(2/3)])/(4*a^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(b\,x^{2}+a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/3),x)
[Out] int(1/x/(b*x^2+a)^(1/3),x)
```

Maxima [A]
time = 0.51, size = 86, normalized size = 1.00

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{1}{3}}}+\frac{\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(1/3)
```

Fricas [A]
time = 0.73, size = 235, normalized size = 2.73

$$\left[\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\,\log\left(\frac{2\,bx^{2}+\sqrt{3}\,\left(2\,\left(bx^{2}+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^{2}+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^{2}}\right)-a^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{4\,a},\ \frac{2\,\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{4\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] [1/4*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(b*x^2 + a)^(2/3
)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)
^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1
/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a, 1/4
*(2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
 + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 41, normalized size = 0.48

$$-\frac{\Gamma\left(\frac{1}{3}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^{2}}\right)}{2\sqrt[3]{b}\,x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1/
3)*x**(2/3)*gamma(4/3))
```

Giac [A]
time = 1.61, size = 87, normalized size = 1.01

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{1}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3)
```

Mupad [B]
time = 4.83, size = 106, normalized size = 1.23

$$\frac{\ln\left(\frac{9\left(bx^2+a\right)^{1/3}}{4}-\frac{9a^{1/3}}{4}\right)}{2a^{1/3}}+\frac{\ln\left(\frac{9\left(bx^2+a\right)^{1/3}}{4}-\frac{9a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4a^{1/3}}-\frac{\ln\left(\frac{9\left(bx^2+a\right)^{1/3}}{4}-\frac{9a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/3)),x)
```

```
[Out] log((9*(a + b*x^2)^(1/3))/4 - (9*a^(1/3))/4)/(2*a^(1/3)) + (log((9*(a + b*x
^2)^(1/3))/4 - (9*a^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/(4*a^(1
/3)) - (log((9*(a + b*x^2)^(1/3))/4 - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3
^(1/2)*1i + 1))/(4*a^(1/3))
```

3.707 $\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{(a+bx^2)^{2/3}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}$$

```
[Out] -1/2*(b*x^2+a)^(2/3)/a/x^2+1/6*b*ln(x)/a^(4/3)-1/4*b*ln(a^(1/3)-(b*x^2+a)^(
1/3))/a^(4/3)-1/6*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))
/a^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {272, 44, 57, 631, 210, 31}

$$-\frac{b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{(a+bx^2)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(1/3)),x]
```

```
[Out] -1/2*(a + b*x^2)^(2/3)/(a*x^2) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/
(Sqrt[3]*a^(1/3))])/(2*Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a
^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)^{2/3}}{2ax^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{(a+bx^2)^{2/3}}{2ax^2} + \frac{b\log(x)}{6a^{4/3}} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{4/3}} - \frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{}}\right.}{}$$

$$= -\frac{(a+bx^2)^{2/3}}{2ax^2} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}} + \frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2}{}\right.}{2a^{4/3}}$$

$$= -\frac{(a+bx^2)^{2/3}}{2ax^2} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}\,a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}$$

Mathematica [A]

time = 0.12, size = 136, normalized size = 1.24

$$-\frac{6\sqrt[3]{a}\,(a+bx^2)^{2/3}+2\sqrt{3}\,bx^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2bx^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-bx^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{12a^{4/3}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(1/3)),x]
```

```
[Out] -1/12*(6*a^(1/3)*(a + b*x^2)^(2/3) + 2*Sqrt[3]*b*x^2*ArcTan[(1 + (2*(a + b*
x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 2*b*x^2*Log[-a^(1/3) + (a + b*x^2)^(1/3)] -
 b*x^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(a^(4/
3)*x^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\,(b\,x^2+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(1/3),x)
```

Maxima [A]
time = 0.56, size = 118, normalized size = 1.07

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{6\,a^{\frac{4}{3}}}-\frac{(bx^2+a)^{\frac{2}{3}}b}{2\,((bx^2+a)a-a^2)}+\frac{b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{12\,a^{\frac{4}{3}}}-\frac{b\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6\,a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(4/3) - 1/2*(b*x^2 + a)^(2/3)*b/((b*x^2 + a)*a - a^2) + 1/12*b*log((b*x^2
 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/6*b*log((b*x
^2 + a)^(1/3) - a^(1/3))/a^(4/3)
```

Fricas [A]
time = 0.79, size = 344, normalized size = 3.13

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx^2-3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx^2+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x^2}\right)+(-a)^{\frac{2}{3}}\,bx^2\log\left((bx^2+a)^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\,bx^2\log\left((bx^2+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-6\,(bx^2+a)^{\frac{2}{3}}a}{12\,a^2x^2},-\frac{6\sqrt{\frac{1}{3}}\,abx^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}\,bx^2\log\left((bx^2+a)^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2\,(-a)^{\frac{2}{3}}\,bx^2\log\left((bx^2+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+6\,(bx^2+a)^{\frac{2}{3}}a}{12\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(1/3)*a*b*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x^2 - 3*sqrt(1/3)*(2
*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-
a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + (-a)^(2/3)*b*x^2
*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a
)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^2 + a)^(2/3)*a)/
(a^2*x^2), -1/12*(6*sqrt(1/3)*a*b*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*
(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x^2*
log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)
^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) + 6*(b*x^2 + a)^(2/3)*a)/(
a^2*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.66, size = 41, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}\,x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1/
3)*x**(8/3)*gamma(7/3))
```

Giac [A]
time = 1.63, size = 119, normalized size = 1.08

$$-\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2b^2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{6\left(bx^2+a\right)^{\frac{2}{3}}b}{ax^2}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/12*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(4/3) - b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(4/3) + 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*
x^2 + a)^(2/3)*b/(a*x^2))/b
```

Mupad [B]
time = 5.01, size = 138, normalized size = 1.25

$$-\frac{b\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{6\,a^{4/3}}-\frac{\left(b\,x^2+a\right)^{2/3}}{2\,a\,x^2}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)^2}{16\,a^{5/3}}-\frac{b^2\left(b\,x^2+a\right)^{1/3}}{4\,a^2}\right)\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{12\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)^2}{16\,a^{5/3}}-\frac{b^2\left(b\,x^2+a\right)^{1/3}}{4\,a^2}\right)\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{12\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)^2/(16*a^(5/3)) - (b^2*(a + b*x^2)^(1/3))/(4*a^2))*(
b - 3^(1/2)*b*1i))/(12*a^(4/3)) - (a + b*x^2)^(2/3)/(2*a*x^2) - (b*log((a +
 b*x^2)^(1/3) - a^(1/3)))/(6*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(16*a^(5/
3)) - (b^2*(a + b*x^2)^(1/3))/(4*a^2))*(b + 3^(1/2)*b*1i))/(12*a^(4/3))
```

3.708 $\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{(a+bx^2)^{2/3}}{4ax^4}+\frac{b(a+bx^2)^{2/3}}{3a^2x^2}+\frac{b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}$$

```
[Out] -1/4*(b*x^2+a)^(2/3)/a/x^4+1/3*b*(b*x^2+a)^(2/3)/a^2/x^2-1/9*b^2*ln(x)/a^(7
/3)+1/6*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(7/3)+1/9*b^2*arctan(1/3*(a^(1/3)
+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {272, 44, 57, 631, 210, 31}

$$\frac{b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b(a+bx^2)^{2/3}}{3a^2x^2}-\frac{(a+bx^2)^{2/3}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(1/3)),x]
```

```
[Out] -1/4*(a + b*x^2)^(2/3)/(a*x^4) + (b*(a + b*x^2)^(2/3))/(3*a^2*x^2) + (b^2*A
rcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3
)) - (b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a
^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^{2/3}}{4ax^4} - \frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{3a}$$
$$= -\frac{(a+bx^2)^{2/3}}{4ax^4} + \frac{b(a+bx^2)^{2/3}}{3a^2x^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{9a^2}$$
$$= -\frac{(a+bx^2)^{2/3}}{4ax^4} + \frac{b(a+bx^2)^{2/3}}{3a^2x^2} - \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{6a^{7/3}} +$$
$$= -\frac{(a+bx^2)^{2/3}}{4ax^4} + \frac{b(a+bx^2)^{2/3}}{3a^2x^2} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}} - \frac{b^2\text{Subst}\left(\right.}{}$$
$$= -\frac{(a+bx^2)^{2/3}}{4ax^4} + \frac{b(a+bx^2)^{2/3}}{3a^2x^2} + \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{\right.}{}$$

Mathematica [A]
time = 0.14, size = 143, normalized size = 1.04

$$\frac{\frac{3\sqrt[3]{a}\,(a+bx^2)^{2/3}(-3a+4bx^2)}{x^4} + 4\sqrt{3}\,b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right) - 2b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{36a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(1/3)),x]
```

```
[Out] ((3*a^(1/3)*(a + b*x^2)^(2/3)*(-3*a + 4*b*x^2))/x^4 + 4*Sqrt[3]*b^2*ArcTan[
(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 4*b^2*Log[-a^(1/3) + (a + b*
x^2)^(1/3)] - 2*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(
2/3)])/(36*a^(7/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(b\,x^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/3),x)
[Out] int(1/x^5/(b*x^2+a)^(1/3),x)
```

Maxima [A]
time = 0.56, size = 158, normalized size = 1.14

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}}-\frac{b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{7}{3}}}+\frac{b^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}}+\frac{4\,(bx^2+a)^{\frac{5}{3}}b^2-7\,(bx^2+a)^{\frac{2}{3}}ab^2}{12\left((bx^2+a)^2a^2-2\,(bx^2+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(7/3) - 1/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(7/3) + 1/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(7/3) + 1/12*(4*
(b*x^2 + a)^(5/3)*b^2 - 7*(b*x^2 + a)^(2/3)*a*b^2)/((b*x^2 + a)^2*a^2 - 2*(
b*x^2 + a)*a^3 + a^4)
```

Fricas [A]
time = 1.17, size = 326, normalized size = 2.36

$$\left[\frac{6\sqrt{\frac{1}{3}}\,ab^2x^4\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^2+3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^2}\right)-2\,a^{\frac{2}{3}}b^2x^4\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4\,a^{\frac{2}{3}}b^2x^4\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(4abx^2-3a^2)(bx^2+a)^{\frac{2}{3}}}{36\,a^3x^4},\ \frac{12\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}b^2x^4\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-2\,a^{\frac{2}{3}}b^2x^4\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4\,a^{\frac{2}{3}}b^2x^4\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(4abx^2-3a^2)(bx^2+a)^{\frac{2}{3}}}{36\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] [1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(2
*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)
) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 2*a^(2/3)*b^2*x^4*log((b*x^2
+ a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*log((
b*x^2 + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))/(a^3
*x^4), 1/36*(12*sqrt(1/3)*a^(2/3)*b^2*x^4*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(
1/3) + a^(1/3))/a^(1/3)) - 2*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) + (b*x^2
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) -
a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))/(a^3*x^4)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.47, size = 41, normalized size = 0.30

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}\,x^{\frac{14}{3}}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(7/3)*hyper((1/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1
/3)*x**(14/3)*gamma(10/3))
```

Giac [A]
time = 1.53, size = 142, normalized size = 1.03

$$\frac{\frac{4\sqrt{3}\,b^{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2\,b^{3}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{4\,b^{3}\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}}+\frac{3\left(4\left(bx^{2}+a\right)^{\frac{5}{3}}b^{3}-7\left(bx^{2}+a\right)^{\frac{2}{3}}ab^{3}\right)}{a^{2}b^{2}x^{4}}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/36*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(7/3) - 2*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(7/3) + 4*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4
*(b*x^2 + a)^(5/3)*b^3 - 7*(b*x^2 + a)^(2/3)*a*b^3)/(a^2*b^2*x^4))/b
```

Mupad [B]
time = 5.06, size = 201, normalized size = 1.46

$$\frac{b^{2}\ln\left(\left(b\,x^{2}+a\right)^{1/3}-a^{1/3}\right)}{9\,a^{7/3}}-\frac{\ln\left(\frac{b^{4}\left(b\,x^{2}+a\right)^{1/3}}{9\,a^{4}}-\frac{\left(b^{2}+\sqrt{3}\,b^{2}\,1{}\mathrm{i}\right)^{2}}{36\,a^{11/3}}\right)\left(b^{2}+\sqrt{3}\,b^{2}\,1{}\mathrm{i}\right)}{18\,a^{7/3}}-\frac{\frac{7\,b^{2}\left(b\,x^{2}+a\right)^{2/3}}{6\,a}-\frac{2\,b^{2}\left(b\,x^{2}+a\right)^{5/3}}{3\,a^{2}}}{2\left(b\,x^{2}+a\right)^{2}-4\,a\left(b\,x^{2}+a\right)+2\,a^{2}}+\frac{b^{2}\ln\left(\frac{b^{4}\left(b\,x^{2}+a\right)^{1/3}}{9\,a^{4}}-\frac{b^{4}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^{2}}{9\,a^{11/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(1/3)),x)
```

```
[Out] (b^2*log((a + b*x^2)^(1/3) - a^(1/3)))/(9*a^(7/3)) - (log((b^4*(a + b*x^2)^
(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(36*a^(11/3)))*(3^(1/2)*b^2*1i +
b^2))/(18*a^(7/3)) - ((7*b^2*(a + b*x^2)^(2/3))/(6*a) - (2*b^2*(a + b*x^2)^
(5/3))/(3*a^2))/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2) + (b^2*log((b^4
*(a + b*x^2)^(1/3))/(9*a^4) - (b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(9*a^(11/3)))*
((3^(1/2)*1i)/2 - 1/2))/(9*a^(7/3))
```

3.709 $\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=580

$$-\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2}+\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{81a^2x}{91b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{81\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}-\right.}{}$$

```
[Out] -27/91*a*x*(b*x^2+a)^(2/3)/b^2+3/13*x^3*(b*x^2+a)^(2/3)/b-81/91*a^2*x/b^2/(
-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-27/91*3^(3/4)*a^(7/3)*(a^(1/3)-(b*x^2
+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^
(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^3/
x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)+81/182*3^(1/4)*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*
x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*
I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^3/x/(-a^(1
/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)
```

Rubi [A]
time = 0.27, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {327, 241, 310, 225, 1893}

$$-\frac{27\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{91b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{182b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{81a^2x}{91b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2}+\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(1/3),x]
```

```
[Out] (-27*a*x*(a + b*x^2)^(2/3))/(91*b^2) + (3*x^3*(a + b*x^2)^(2/3))/(13*b) - (
81*a^2*x)/(91*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (81*3^(1/4
)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(182*b^3
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
```

```
(a + b*x^2)^(1/3))^2)]) - (27*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a + b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(91*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
```

```
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = \frac{3x^3(a+bx^2)^{2/3}}{13b} - \frac{(9a)\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx}{13b}$$

$$= -\frac{27ax(a+bx^2)^{2/3}}{91b^2} + \frac{3x^3(a+bx^2)^{2/3}}{13b} + \frac{(27a^2)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{91b^2}$$

$$= -\frac{27ax(a+bx^2)^{2/3}}{91b^2} + \frac{3x^3(a+bx^2)^{2/3}}{13b} + \frac{\left(81a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+\right.}{182b^3x}$$

$$= -\frac{27ax(a+bx^2)^{2/3}}{91b^2} + \frac{3x^3(a+bx^2)^{2/3}}{13b} - \frac{\left(81a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{\right.}{182b^3x}$$

$$= -\frac{27ax(a+bx^2)^{2/3}}{91b^2} + \frac{3x^3(a+bx^2)^{2/3}}{13b} - \frac{81a^2x}{91b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{81\sqrt[4]{3}\ \sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.47, size = 79, normalized size = 0.14

$$\frac{3\left(-9a^2x-2abx^3+7b^2x^5+9a^2x\sqrt[3]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{91b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(1/3),x]
[Out] (3*(-9*a^2*x - 2*a*b*x^3 + 7*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(1/3)*Hyperg
eometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)]))/(91*b^2*(a + b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/3),x)
[Out] int(x^4/(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral(x^4/(b*x^2 + a)^(1/3), x)
```

Sympy [A]
time = 0.41, size = 27, normalized size = 0.05

$$\frac{x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/3),x)
[Out] x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(b\,x^2+a)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/3),x)
[Out] int(x^4/(a + b*x^2)^(1/3), x)
```

3.710 $\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=556

$$\frac{3x(a+bx^2)^{2/3}}{7b}+\frac{9ax}{7b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{\ }}{\left(\left(1-\right.\right.}}}{14b^2x\sqrt{\ }}$$

```
[Out] 3/7*x*(b*x^2+a)^(2/3)/b+9/7*a*x/b/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+3/
7*3^(3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(
1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(
1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-9/14*3^(1/4)*a^(4/3)*(a^(1/3)-(
b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a
)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/
3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {327, 241, 310, 225, 1893}

$$\frac{3\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{7b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{14b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{9ax}{7b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{3x(a+bx^2)^{2/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3))/(7*b) + (9*a*x)/(7*b*((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))) - (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(14*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (3*Sqrt[2]*3^(3/4)*a^(4/
3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
```

```
+ (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx = \frac{3x(a+bx^2)^{2/3}}{7b} - \frac{(3a)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{7b}$$

$$= \frac{3x(a+bx^2)^{2/3}}{7b} - \frac{\left(9a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{14b^2x}$$

$$= \frac{3x(a+bx^2)^{2/3}}{7b} + \frac{\left(9a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{14b^2x} - \frac{\left(9\sqrt{\frac{1}{2}\left(2 \cdots\right.}\right.}{\cdots}$$

$$= \frac{3x(a+bx^2)^{2/3}}{7b} + \frac{9ax}{7b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.09, size = 62, normalized size = 0.11

$$\frac{3x\left(a+bx^2-a\sqrt[3]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{7b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2,
-((b*x^2)/a)]))/(7*b*(a + b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/3),x)
```

[Out] int(x^2/(b*x^2+a)^(1/3),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="maxima")

[Out] integrate(x^2/(b*x^2 + a)^(1/3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="fricas")

[Out] integral(x^2/(b*x^2 + a)^(1/3), x)

Sympy [A]
time = 0.38, size = 27, normalized size = 0.05

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**2+a)**(1/3),x)

[Out] x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="giac")

[Out] integrate(x^2/(b*x^2 + a)^(1/3), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/3),x)
```

```
[Out] int(x^2/(a + b*x^2)^(1/3), x)
```

3.711 $\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=529

$$-\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}+\frac{3\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+b\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx\cdots}\right.}}}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a\cdots}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right.}}}$$

[Out] -3*x/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-3^(3/4)*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3/2*3^(1/4)*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.18, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {241, 310, 225, 1893}

$$-\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-1/3),x]

[Out] (-3*x)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^

```
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2bx}$$

$$= -\frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2bx} + \frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt[3]{a}\sqrt{bx^2}\right.}{}$$

$$= -\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\cdots}{\left(\left(1-\cdots\right.\right.}}}{2bx\sqrt{-\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.08, size = 46, normalized size = 0.09

$$\frac{x\sqrt[3]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1/3),x]
[Out] (x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(1/3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/3),x)
[Out] int(1/(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(-1/3), x)
```

Sympy [A]
time = 0.37, size = 24, normalized size = 0.05

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/3),x)
[Out] x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-1/3), x)
```

Mupad [B]
time = 4.63, size = 37, normalized size = 0.07

$$\frac{x\,\left(\frac{b\,x^2}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/3),x)
[Out] (x*((b*x^2)/a + 1)^(1/3)*hypergeom([1/3, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(1/3)
```

3.712 $\int \frac{1}{x^2 \sqrt[3]{a+bx^2}} \, dx$

Optimal. Leaf size=546

$$-\frac{\left(a+bx^2\right)^{2/3}}{ax}-\frac{bx}{a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}{2a^{2/3}x\sqrt{-\frac{\cdots}{\left(\left(\cdots\right.\right.}}}$$

```
[Out] -(b*x^2+a)^(2/3)/a/x-b*x/a/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-1/3*(a^(1
/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*
(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*3^(3/4)/a^(2/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+1/2*3^(1/4)*(a^(1/3)-(b*x^2+a)^(1/3))*Elli
pticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/
(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/a
^(2/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {331, 241, 310, 225, 1893}

$$-\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{2a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{bx}{a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{\left(a+bx^2\right)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(1/3)),x]
```

```
[Out] -((a + b*x^2)^(2/3)/(a*x)) - (b*x)/(a*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^
(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(2*a^(2/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (Sqrt[2]*(a^(1/3) - (a + b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
```

```
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(3^(1/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{2/3}}{ax} + \frac{b\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{3a}$$

$$= -\frac{(a+bx^2)^{2/3}}{ax} + \frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2ax}$$

$$= -\frac{(a+bx^2)^{2/3}}{ax} - \frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2ax} + \frac{\left(\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right.}\right.}{}$$

$$= -\frac{(a+bx^2)^{2/3}}{ax} - \frac{bx}{a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.33, size = 49, normalized size = 0.09

$$-\frac{\sqrt[3]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 1/3, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(1/3)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(b\,x^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)/(b*x^4 + a*x^2), x)
```

Sympy [A]
time = 0.41, size = 27, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{\sqrt[3]{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/3),x)
[Out] -hyper((-1/2, 1/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/3)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^2), x)
```

Mupad [B]
time = 4.83, size = 40, normalized size = 0.07

$$-\frac{3\,{\left(\frac{a}{b\,x^2}+1\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{5}{6};\ \frac{11}{6};\ -\frac{a}{b\,x^2}\right)}{5\,x\,{\left(b\,x^2+a\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/3)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(1/3)*hypergeom([1/3, 5/6], 11/6, -a/(b*x^2)))/(5*x*(a
+ b*x^2)^(1/3))
```

3.713 $\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=578

$$-\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}+\frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x}+\frac{5b^2x}{9a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{5\sqrt{2+\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{}$$

[Out] -1/3*(b*x^2+a)^(2/3)/a/x^3+5/9*b*(b*x^2+a)^(2/3)/a^2/x+5/9*b^2*x/a^2/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+5/27*b*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-5/18*b*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.26, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {331, 241, 310, 225, 1893}

$$\frac{5\sqrt{2}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{5\sqrt{2+\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{6\ 3^{3/4}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{5b^2x}{9a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x}-\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^2)^(1/3)),x]

[Out] -1/3*(a + b*x^2)^(2/3)/(a*x^3) + (5*b*(a + b*x^2)^(2/3))/(9*a^2*x) + (5*b^2*x)/(9*a^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (5*Sqrt[2 + Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*3^(3/4)*a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (5*Sqrt[2]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^

```
(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4
)*a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx = -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} - \frac{(5b)\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx}{9a}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} + \frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x} - \frac{\left(5b^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{27a^2}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} + \frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x} - \frac{\left(5b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right.}{18a^2x}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} + \frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x} + \frac{\left(5b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+b\right.}{18a^2x}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3} + \frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x} + \frac{5b^2x}{9a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{5\sqrt{2+\sqrt{3}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.09

$$-\frac{\sqrt[3]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(1/3)),x]
[Out] -1/3*((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-3/2, 1/3, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(b\,x^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/3),x)
[Out] int(1/x^4/(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)/(b*x^6 + a*x^4), x)
```

Sympy [A]
time = 0.46, size = 32, normalized size = 0.06

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{1}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/3),x)
[Out] -hyper((-3/2, 1/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/3)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^(1/3)), x)
```

3.714 $\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{9a^2(a+bx^2)^{4/3}}{8b^4}-\frac{9a(a+bx^2)^{7/3}}{14b^4}+\frac{3(a+bx^2)^{10/3}}{20b^4}$$

```
[Out] -3/2*a^3*(b*x^2+a)^(1/3)/b^4+9/8*a^2*(b*x^2+a)^(4/3)/b^4-9/14*a*(b*x^2+a)^(
7/3)/b^4+3/20*(b*x^2+a)^(10/3)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {272, 45}

$$-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{9a^2(a+bx^2)^{4/3}}{8b^4}+\frac{3(a+bx^2)^{10/3}}{20b^4}-\frac{9a(a+bx^2)^{7/3}}{14b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(1/3))/(2*b^4) + (9*a^2*(a + b*x^2)^(4/3))/(8*b^4) - (9
*a*(a + b*x^2)^(7/3))/(14*b^4) + (3*(a + b*x^2)^(10/3))/(20*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx,x,x^2\right)\\
&= -\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{9a^2(a+bx^2)^{4/3}}{8b^4}-\frac{9a(a+bx^2)^{7/3}}{14b^4}+\frac{3(a+bx^2)^{10/3}}{20b^4}
\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\sqrt[3]{a+bx^2}\,(-81a^3+27a^2bx^2-18ab^2x^4+14b^3x^6)}{280b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(-81*a^3 + 27*a^2*b*x^2 - 18*a*b^2*x^4 + 14*b^3*x^6))/
(280*b^4)
```

Maple [**A**]
time = 0.04, size = 47, normalized size = 0.59

method	result	size
gosper	$-\frac{3(b\,x^2+a)^{\frac{1}{3}}(-14b^3x^6+18a\,b^2x^4-27a^2b\,x^2+81a^3)}{280b^4}$	47
trager	$-\frac{3(b\,x^2+a)^{\frac{1}{3}}(-14b^3x^6+18a\,b^2x^4-27a^2b\,x^2+81a^3)}{280b^4}$	47
risch	$-\frac{3(b\,x^2+a)^{\frac{1}{3}}(-14b^3x^6+18a\,b^2x^4-27a^2b\,x^2+81a^3)}{280b^4}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/280*(b*x^2+a)^(1/3)*(-14*b^3*x^6+18*a*b^2*x^4-27*a^2*b*x^2+81*a^3)/b^4
```

Maxima [**A**]
time = 0.28, size = 64, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{10}{3}}}{20\,b^4}-\frac{9\,(bx^2+a)^{\frac{7}{3}}a}{14\,b^4}+\frac{9\,(bx^2+a)^{\frac{4}{3}}a^2}{8\,b^4}-\frac{3\,(bx^2+a)^{\frac{1}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^4 - 9/14*(b*x^2 + a)^(7/3)*a/b^4 + 9/8*(b*x^2 + a
)^(4/3)*a^2/b^4 - 3/2*(b*x^2 + a)^(1/3)*a^3/b^4
```

Fricas [**A**]
time = 1.14, size = 46, normalized size = 0.58

$$\frac{3\,(14\,b^3x^6-18\,ab^2x^4+27\,a^2bx^2-81\,a^3)(bx^2+a)^{\frac{1}{3}}}{280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/280*(14*b^3*x^6 - 18*a*b^2*x^4 + 27*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(1/3)
/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(75) = 150$.
time = 1.31, size = 1690, normalized size = 21.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(2/3),x)
```

```
[Out] -243*a**(70/3)*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2
 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 168
0*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(70/3)/(280*a**20*b**4
 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 420
0*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 1377*a
**(67/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**
2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 16
80*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 1458*a**(67/3)*b*x**2/(280*a
**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x
**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12)
 - 3213*a**(64/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a*
*19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b*
*8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 3645*a**(64/3)*b
**2*x**4/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 56
00*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**
14*b**10*x**12) - 3927*a**(61/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(280*a**20
*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6
+ 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 4
860*a**(61/3)*b**3*x**6/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18
*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*
x**10 + 280*a**14*b**10*x**12) - 2583*a**(58/3)*b**4*x**8*(1 + b*x**2/a)**(
1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a
**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b
**10*x**12) + 3645*a**(58/3)*b**4*x**8/(280*a**20*b**4 + 1680*a**19*b**5*x*
*2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1
680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 693*a**(55/3)*b**5*x**10*(1
 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**
6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**1
0 + 280*a**14*b**10*x**12) + 1458*a**(55/3)*b**5*x**10/(280*a**20*b**4 + 16
80*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**
16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 273*a**(52/
3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2
```

```
+ 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680
*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(52/3)*b**6*x**12/(280*
a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*
x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12
) + 387*a**(49/3)*b**7*x**14*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a
**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b
**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 198*a**(46/3)*b
**8*x**16*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 42
00*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**
15*b**9*x**10 + 280*a**14*b**10*x**12) + 42*a**(43/3)*b**9*x**18*(1 + b*x**
2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 +
 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*
a**14*b**10*x**12)
```

Giac [A]

time = 0.96, size = 61, normalized size = 0.76

$$-\frac{3\,(bx^2+a)^{\frac{1}{3}}a^3}{2\,b^4}+\frac{3\left(14\,(bx^2+a)^{\frac{10}{3}}-60\,(bx^2+a)^{\frac{7}{3}}a+105\,(bx^2+a)^{\frac{4}{3}}a^2\right)}{280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -3/2*(b*x^2 + a)^(1/3)*a^3/b^4 + 3/280*(14*(b*x^2 + a)^(10/3) - 60*(b*x^2 +
 a)^(7/3)*a + 105*(b*x^2 + a)^(4/3)*a^2)/b^4
```

Mupad [B]

time = 4.75, size = 48, normalized size = 0.60

$$-{\left(b\,x^2+a\right)}^{1/3}\,\left(\frac{243\,a^3}{280\,b^4}-\frac{3\,x^6}{20\,b}+\frac{27\,a\,x^4}{140\,b^2}-\frac{81\,a^2\,x^2}{280\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(2/3),x)
```

```
[Out] -(a + b*x^2)^(1/3)*((243*a^3)/(280*b^4) - (3*x^6)/(20*b) + (27*a*x^4)/(140*
b^2) - (81*a^2*x^2)/(280*b^3))
```

3.715 $\int \frac{x^5}{(a+bx^2)^{2/3}} \, dx$

Optimal. Leaf size=59

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}-\frac{3a(a+bx^2)^{4/3}}{4b^3}+\frac{3(a+bx^2)^{7/3}}{14b^3}$$

```
[Out] 3/2*a^2*(b*x^2+a)^(1/3)/b^3-3/4*a*(b*x^2+a)^(4/3)/b^3+3/14*(b*x^2+a)^(7/3)/
b^3
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}+\frac{3(a+bx^2)^{7/3}}{14b^3}-\frac{3a(a+bx^2)^{4/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(1/3))/(2*b^3) - (3*a*(a + b*x^2)^(4/3))/(4*b^3) + (3*(a
 + b*x^2)^(7/3))/(14*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{2/3}}-\frac{2a\sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)\,dx,x,x^2\right)\\ &= \frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}-\frac{3a(a+bx^2)^{4/3}}{4b^3}+\frac{3(a+bx^2)^{7/3}}{14b^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx^2}\,(9a^2-3abx^2+2b^2x^4)}{28b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(9*a^2 - 3*a*b*x^2 + 2*b^2*x^4))/(28*b^3)
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.61

method	result	size
gosper	$\frac{3(b\,x^2+a)^{\frac{1}{3}}(2b^2x^4-3ab\,x^2+9a^2)}{28b^3}$	36
trager	$\frac{3(b\,x^2+a)^{\frac{1}{3}}(2b^2x^4-3ab\,x^2+9a^2)}{28b^3}$	36
risch	$\frac{3(b\,x^2+a)^{\frac{1}{3}}(2b^2x^4-3ab\,x^2+9a^2)}{28b^3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/28*(b*x^2+a)^(1/3)*(2*b^2*x^4-3*a*b*x^2+9*a^2)/b^3
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{7}{3}}}{14\,b^3}-\frac{3\,(bx^2+a)^{\frac{4}{3}}a}{4\,b^3}+\frac{3\,(bx^2+a)^{\frac{1}{3}}a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b^3 - 3/4*(b*x^2 + a)^(4/3)*a/b^3 + 3/2*(b*x^2 + a)^
(1/3)*a^2/b^3
```

Fricas [A]
time = 1.26, size = 35, normalized size = 0.59

$$\frac{3\,(2\,b^2x^4-3\,abx^2+9\,a^2)(bx^2+a)^{\frac{1}{3}}}{28\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/28*(2*b^2*x^4 - 3*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(1/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(54) = 108$.
time = 0.87, size = 631, normalized size = 10.69

$$\frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} - \frac{27a^{\frac{31}{3}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{72a^{\frac{28}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} - \frac{81a^{\frac{28}{3}}bx^2}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{60a^{\frac{25}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} - \frac{81a^{\frac{25}{3}}b^2x^4}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{18a^{\frac{22}{3}}b^3x^6\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} - \frac{27a^{\frac{22}{3}}b^3x^6}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{9a^{\frac{19}{3}}b^4x^8\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{6a^{\frac{16}{3}}b^5x^{10}\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(2/3),x)
```

```
[Out] 27*a**(31/3)*(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a
**6*b**5*x**4 + 28*a**5*b**6*x**6) - 27*a**(31/3)/(28*a**8*b**3 + 84*a**7*b
**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 72*a**(28/3)*b*x**2*(1
+ b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 +
28*a**5*b**6*x**6) - 81*a**(28/3)*b*x**2/(28*a**8*b**3 + 84*a**7*b**4*x**2
+ 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 60*a**(25/3)*b**2*x**4*(1 + b*x*
*2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**
5*b**6*x**6) - 81*a**(25/3)*b**2*x**4/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 8
4*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 18*a**(22/3)*b**3*x**6*(1 + b*x**2/
a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b
**6*x**6) - 27*a**(22/3)*b**3*x**6/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a
**6*b**5*x**4 + 28*a**5*b**6*x**6) + 9*a**(19/3)*b**4*x**8*(1 + b*x**2/a)**
(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*
x**6) + 6*a**(16/3)*b**5*x**10*(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**
7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6)
```

Giac [A]
time = 1.15, size = 47, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a^2}{2\,b^3} + \frac{3\left(2\left(bx^2+a\right)^{\frac{7}{3}} - 7\left(bx^2+a\right)^{\frac{4}{3}}a\right)}{28\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)*a^2/b^3 + 3/28*(2*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(
4/3)*a)/b^3
```

Mupad [B]
time = 4.77, size = 36, normalized size = 0.61

$$\left(b\,x^2+a\right)^{1/3}\,\left(\frac{27\,a^2}{28\,b^3}+\frac{3\,x^4}{14\,b}-\frac{9\,a\,x^2}{28\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(2/3),x)
[Out] (a + b*x^2)^(1/3)*((27*a^2)/(28*b^3) + (3*x^4)/(14*b) - (9*a*x^2)/(28*b^2))
```

3.716 $\int \frac{x^3}{(a+bx^2)^{2/3}} dx$

Optimal. Leaf size=38

$$-\frac{3a\sqrt[3]{a+bx^2}}{2b^2}+\frac{3(a+bx^2)^{4/3}}{8b^2}$$

```
[Out] -3/2*a*(b*x^2+a)^(1/3)/b^2+3/8*(b*x^2+a)^(4/3)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{3(a+bx^2)^{4/3}}{8b^2}-\frac{3a\sqrt[3]{a+bx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(1/3))/(2*b^2) + (3*(a + b*x^2)^(4/3))/(8*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx,x,x^2\right)\\
&= -\frac{3a\sqrt[3]{a+bx^2}}{2b^2}+\frac{3(a+bx^2)^{4/3}}{8b^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.71

$$\frac{3(-3a+bx^2)\sqrt[3]{a+bx^2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(-3*a + b*x^2)*(a + b*x^2)^(1/3))/(8*b^2)
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.66

method	result	size
gosper	$-\frac{3(b\,x^2+a)^{\frac{1}{3}}(-b\,x^2+3a)}{8b^2}$	25
trager	$-\frac{3(b\,x^2+a)^{\frac{1}{3}}(-b\,x^2+3a)}{8b^2}$	25
risch	$-\frac{3(b\,x^2+a)^{\frac{1}{3}}(-b\,x^2+3a)}{8b^2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*(b*x^2+a)^(1/3)*(-b*x^2+3*a)/b^2
```

Maxima [A]
time = 0.29, size = 30, normalized size = 0.79

$$\frac{3\,(bx^2+a)^{\frac{4}{3}}}{8\,b^2}-\frac{3\,(bx^2+a)^{\frac{1}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b^2 - 3/2*(b*x^2 + a)^(1/3)*a/b^2
```

Fricas [A]
time = 1.15, size = 23, normalized size = 0.61

$$\frac{3\,(bx^2+a)^{\frac{1}{3}}(bx^2-3\,a)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/8*(b*x^2 + a)^(1/3)*(b*x^2 - 3*a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(34) = 68$.
time = 0.56, size = 178, normalized size = 4.68

$$-\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9a^{\frac{10}{3}}}{8a^2b^2+8ab^3x^2}-\frac{6a^{\frac{7}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9a^{\frac{7}{3}}bx^2}{8a^2b^2+8ab^3x^2}+\frac{3a^{\frac{4}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(2/3),x)
```

```
[Out] -9*a**(10/3)*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(10
/3)/(8*a**2*b**2 + 8*a*b**3*x**2) - 6*a**(7/3)*b*x**2*(1 + b*x**2/a)**(1/3)
/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(7/3)*b*x**2/(8*a**2*b**2 + 8*a*b**3*
x**2) + 3*a**(4/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*
x**2)
```

Giac [A]
time = 0.79, size = 30, normalized size = 0.79

$$\frac{3\,(bx^2+a)^{\frac{4}{3}}}{8\,b^2}-\frac{3\,(bx^2+a)^{\frac{1}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b^2 - 3/2*(b*x^2 + a)^(1/3)*a/b^2
```

Mupad [B]
time = 4.79, size = 24, normalized size = 0.63

$$-\frac{3\,(b\,x^2+a)^{1/3}\,(3\,a-b\,x^2)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(2/3),x)
```

```
[Out] -(3*(a + b*x^2)^(1/3)*(3*a - b*x^2))/(8*b^2)
```

3.717 $\int \frac{x}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=18

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

```
[Out] 3/2*(b*x^2+a)^(1/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx^2}}{2b}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

Maple [A]
time = 0.04, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{3(b x^2+a)^{\frac{1}{3}}}{2b}$	15
derivativedivides	$\frac{3(b x^2+a)^{\frac{1}{3}}}{2b}$	15
default	$\frac{3(b x^2+a)^{\frac{1}{3}}}{2b}$	15
trager	$\frac{3(b x^2+a)^{\frac{1}{3}}}{2b}$	15
risch	$\frac{3(b x^2+a)^{\frac{1}{3}}}{2b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/2*(b*x^2+a)^(1/3)/b
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

Fricas [A]
time = 0.66, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

Sympy [A]
time = 0.19, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3\sqrt[3]{a+bx^2}}{2b} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((3*(a + b*x**2)**(1/3)/(2*b), Ne(b, 0)), (x**2/(2*a**(2/3)), True
))
```

Giac [A]
time = 0.78, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

Mupad [B]
time = 4.69, size = 14, normalized size = 0.78

$$\frac{3\,{\left(b\,x^2+a\right)}^{1/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(2/3),x)
```

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

3.718 $\int \frac{1}{x(a+bx^2)^{2/3}} dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}$$

```
[Out] -1/2*ln(x)/a^(2/3)+3/4*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(2/3)-1/2*arctan(1/3*(
a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 59, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/a^
(2/3) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(2/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)$$

$$= -\frac{\log(x)}{2a^{2/3}} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{3\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}$$

Mathematica [A]
time = 0.06, size = 101, normalized size = 1.17

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right) + \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{4a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/4*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log
[-a^(1/3) + (a + b*x^2)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3)])/a^(2/3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(b\,x^{2}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x/(b*x^2+a)^(2/3),x)
```

Maxima [A]
time = 0.53, size = 86, normalized size = 1.00

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,(bx^{2}+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{2}{3}}}-\frac{\log\left((bx^{2}+a)^{\frac{2}{3}}+(bx^{2}+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{2}{3}}}+\frac{\log\left((bx^{2}+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(61) = 122.
time = 0.77, size = 123, normalized size = 1.43

$$-\frac{2\,\sqrt{3}\,(a^{2})^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}\,(a^{2})^{\frac{1}{6}}\left((a^{2})^{\frac{1}{3}}a+2\,(bx^{2}+a)^{\frac{1}{3}}(a^{2})^{\frac{2}{3}}\right)}{3\,a^{2}}\right)+(a^{2})^{\frac{2}{3}}\log\left((bx^{2}+a)^{\frac{2}{3}}a+(a^{2})^{\frac{1}{3}}a+(bx^{2}+a)^{\frac{1}{3}}(a^{2})^{\frac{2}{3}}\right)-2\,(a^{2})^{\frac{2}{3}}\log\left((bx^{2}+a)^{\frac{1}{3}}a-(a^{2})^{\frac{2}{3}}\right)}{4\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*a
 + 2*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*log((b*x^2 + a)^(2/3
)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) - 2*(a^2)^(2/3)*log((b
*x^2 + a)^(1/3)*a - (a^2)^(2/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 41, normalized size = 0.48

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^{2}}\right)}{2b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*x**(4/3)*gamma(5/3))
```

Giac [A]

time = 1.33, size = 87, normalized size = 1.01

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{2}{3}}}-\frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{2}{3}}}+\frac{\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3)
```

Mupad [B]

time = 4.84, size = 102, normalized size = 1.19

$$\frac{\ln\left(\frac{9\,(b\,x^2+a)^{1/3}}{2}-\frac{9\,a^{1/3}}{2}\right)}{2\,a^{2/3}}+\frac{\ln\left(\frac{9\,a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}-\frac{9\,(b\,x^2+a)^{1/3}}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{2/3}}-\frac{\ln\left(\frac{9\,a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}+\frac{9\,(b\,x^2+a)^{1/3}}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(2/3)),x)
```

```
[Out] log((9*(a + b*x^2)^(1/3))/2 - (9*a^(1/3))/2)/(2*a^(2/3)) + (log((9*a^(1/3)*
(3^(1/2)*1i - 1))/4 - (9*(a + b*x^2)^(1/3))/2)*(3^(1/2)*1i - 1))/(4*a^(2/3)
) - (log((9*a^(1/3)*(3^(1/2)*1i + 1))/4 + (9*(a + b*x^2)^(1/3))/2)*(3^(1/2)
*1i + 1))/(4*a^(2/3))
```

3.719 $\int \frac{1}{x^3 (a+bx^2)^{2/3}} \, dx$

Optimal. Leaf size=107

$$-\frac{\sqrt[3]{a+bx^2}}{2ax^2}+\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}$$

```
[Out] -1/2*(b*x^2+a)^(1/3)/a/x^2+1/3*b*ln(x)/a^(5/3)-1/2*b*ln(a^(1/3)-(b*x^2+a)^(
1/3))/a^(5/3)+1/3*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))
/a^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 44, 59, 631, 210, 31}

$$\frac{b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/2*(a + b*x^2)^(1/3)/(a*x^2) + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/
(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(
1/3) - (a + b*x^2)^(1/3)])/(2*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2}-\frac{b\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)}{3a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2}+\frac{b\log(x)}{3a^{5/3}}+\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}+\frac{b\text{Subst}\left(\int \frac{}{a^{2/3}+\sqrt[3]{}}\right.}{}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}-\frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\right.}{a^{5/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2}+\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}\end{aligned}$$

Mathematica [A]

time = 0.10, size = 135, normalized size = 1.26

$$\frac{-3a^{2/3}\sqrt[3]{a+bx^2}+2\sqrt{3}\,bx^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)+bx^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{6a^{5/3}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*a^(2/3)*(a + b*x^2)^(1/3) + 2*Sqrt[3]*b*x^2*ArcTan[(1 + (2*(a + b*x^2)^
(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x^2*Log[-a^(1/3) + (a + b*x^2)^(1/3)] + b*x^
2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(6*a^(5/3)*
x^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(2/3),x)
```

Maxima [A]
time = 0.52, size = 118, normalized size = 1.10

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{5}{3}}}-\frac{\left(bx^2+a\right)^{\frac{1}{3}}b}{2\left(\left(bx^2+a\right)a-a^2\right)}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{5}{3}}}-\frac{b\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(5/3) - 1/2*(b*x^2 + a)^(1/3)*b/((b*x^2 + a)*a - a^2) + 1/6*b*log((b*x^2 +
 a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 1/3*b*log((b*x^2
 + a)^(1/3) - a^(1/3))/a^(5/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(81) = 162$.
time = 0.66, size = 182, normalized size = 1.70

$$\frac{2\sqrt{3}\,abx^2\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3\,a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)-3\left(bx^2+a\right)^{\frac{1}{3}}a^2}{6\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*b*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3
)*a - 2*sqrt(3)*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) +
(-a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 + a)^(
1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (-a^2)^
(2/3)) - 3*(b*x^2 + a)^(1/3)*a^2)/(a^3*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.68, size = 41, normalized size = 0.38

$$-\frac{\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2b^{\frac{2}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(5/3)*hyper((2/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*x**(10/3)*gamma(8/3))
```

Giac [A]
time = 1.19, size = 118, normalized size = 1.10

$$\frac{\frac{2\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^{2}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\,b^{2}\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left(bx^{2}+a\right)^{\frac{1}{3}}b}{ax^{2}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(5/3) + b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(5/3) - 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x^
2 + a)^(1/3)*b/(a*x^2))/b
```

Mupad [B]
time = 5.05, size = 130, normalized size = 1.21

$$\frac{\ln\left(\frac{3\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{2\,a^{2/3}}+\frac{3\,b\,\left(b\,x^2+a\right)^{1/3}}{a}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{6\,a^{5/3}}+\frac{\ln\left(\frac{3\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{2\,a^{2/3}}+\frac{3\,b\,\left(b\,x^2+a\right)^{1/3}}{a}\right)\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{6\,a^{5/3}}-\frac{b\,\ln\left({\left(b\,x^2+a\right)}^{1/3}-a^{1/3}\right)}{3\,a^{5/3}}-\frac{{\left(b\,x^2+a\right)}^{1/3}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(2/3)),x)
```

```
[Out] (log((3*(b - 3^(1/2)*b*1i))/(2*a^(2/3)) + (3*b*(a + b*x^2)^(1/3))/a)*(b - 3
^(1/2)*b*1i))/(6*a^(5/3)) + (log((3*(b + 3^(1/2)*b*1i))/(2*a^(2/3)) + (3*b*
(a + b*x^2)^(1/3))/a)*(b + 3^(1/2)*b*1i))/(6*a^(5/3)) - (b*log((a + b*x^2)^
(1/3) - a^(1/3)))/(3*a^(5/3)) - (a + b*x^2)^(1/3)/(2*a*x^2)
```

3.720 $\int \frac{1}{x^5 (a+bx^2)^{2/3}} \, dx$

Optimal. Leaf size=138

$$-\frac{\sqrt[3]{a+bx^2}}{4ax^4}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{5b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}$$

```
[Out] -1/4*(b*x^2+a)^(1/3)/a/x^4+5/12*b*(b*x^2+a)^(1/3)/a^2/x^2-5/18*b^2*ln(x)/a^
(8/3)+5/12*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(8/3)-5/18*b^2*arctan(1/3*(a^(
1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {272, 44, 59, 631, 210, 31}

$$-\frac{5b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{\sqrt[3]{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/4*(a + b*x^2)^(1/3)/(a*x^4) + (5*b*(a + b*x^2)^(1/3))/(12*a^2*x^2) - (5*
b^2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a
^(8/3)) - (5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x^2)^(1
/3)])/(12*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}-\frac{(5b)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx,x,x^2\right)}{12a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}+\frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)}{18a^2}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{5b^2\log(x)}{18a^{8/3}}-\frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}+\frac{\left(5b^2\right)\text{Sub}}{}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{5b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 143, normalized size = 1.04

$$\frac{\frac{3a^{2/3}\sqrt[3]{a+bx^2}(-3a+5bx^2)}{x^4}-10\sqrt{3}b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-5b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{36a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(2/3)),x]
```

```
[Out] ((3*a^(2/3)*(a + b*x^2)^(1/3)*(-3*a + 5*b*x^2))/x^4 - 10*Sqrt[3]*b^2*ArcTan
[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 10*b^2*Log[-a^(1/3) + (a +
b*x^2)^(1/3)] - 5*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)
^(2/3)])/(36*a^(8/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(2/3),x)
```

Maxima [A]
time = 0.51, size = 158, normalized size = 1.14

$$-\frac{5\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{8}{3}}}-\frac{5\,b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36\,a^{\frac{8}{3}}}+\frac{5\,b^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,a^{\frac{8}{3}}}+\frac{5\left(bx^2+a\right)^{\frac{4}{3}}b^2-8\left(bx^2+a\right)^{\frac{1}{3}}ab^2}{12\left(\left(bx^2+a\right)^2a^2-2\left(bx^2+a\right)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -5/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(8/3) - 5/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(8/3) + 5/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(8/3) + 1/12*
(5*(b*x^2 + a)^(4/3)*b^2 - 8*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2*a^2 -
2*(b*x^2 + a)*a^3 + a^4)
```

Fricas [A]
time = 0.93, size = 174, normalized size = 1.26

$$-\frac{10\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^4\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3\,a^2}\right)+5\left(a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-10\left(a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)-3\left(5\,a^2bx^2-3\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{36\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/36*(10*sqrt(3)*(a^2)^(1/6)*a*b^2*x^4*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^
2)^(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 5*(a^2)^(2/3)*
b^2*x^4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(
2/3)) - 10*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) - 3*(
5*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3))/(a^4*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.53, size = 41, normalized size = 0.30

$$-\frac{\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2b^{\frac{2}{3}}x^{\frac{16}{3}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(8/3)*hyper((2/3, 8/3), (11/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2
/3)*x**(16/3)*gamma(11/3))
```

Giac [A]
time = 1.15, size = 142, normalized size = 1.03

$$-\frac{\frac{10\sqrt{3}\,b^{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}}+\frac{5\,b^{3}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}}-\frac{10\,b^{3}\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}}-\frac{3\left(5\left(bx^{2}+a\right)^{\frac{4}{3}}b^{3}-8\left(bx^{2}+a\right)^{\frac{1}{3}}ab^{3}\right)}{a^{2}b^{2}x^{4}}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/36*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(8/3) + 5*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
a^(2/3))/a^(8/3) - 10*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(8/3) - 3
*(5*(b*x^2 + a)^(4/3)*b^3 - 8*(b*x^2 + a)^(1/3)*a*b^3)/(a^2*b^2*x^4))/b
```

Mupad [B]
time = 5.14, size = 193, normalized size = 1.40

$$\frac{5\,b^{2}\,\ln\left({\left(b\,x^{2}+a\right)}^{1/3}-a^{1/3}\right)}{18\,a^{8/3}}-\frac{\frac{4\,b^{2}\,{\left(b\,x^{2}+a\right)}^{1/3}}{3\,a}-\frac{5\,b^{2}\,{\left(b\,x^{2}+a\right)}^{4/3}}{6\,a^{2}}}{2\,{\left(b\,x^{2}+a\right)}^{2}-4\,a\,\left(b\,x^{2}+a\right)+2\,a^{2}}+\frac{5\,b^{2}\,\ln\left(\frac{5\,b^{2}\,{\left(b\,x^{2}+a\right)}^{1/3}}{2\,a^{2}}-\frac{5\,b^{2}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{18\,a^{8/3}}-\frac{5\,b^{2}\,\ln\left(\frac{5\,b^{2}\,{\left(b\,x^{2}+a\right)}^{1/3}}{2\,a^{2}}+\frac{5\,b^{2}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{18\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(2/3)),x)
```

```
[Out] (5*b^2*log((a + b*x^2)^(1/3) - a^(1/3)))/(18*a^(8/3)) - ((4*b^2*(a + b*x^2)
^(1/3))/(3*a) - (5*b^2*(a + b*x^2)^(4/3))/(6*a^2))/(2*(a + b*x^2)^2 - 4*a*(
a + b*x^2) + 2*a^2) + (5*b^2*log((5*b^2*(a + b*x^2)^(1/3))/(2*a^2) - (5*b^2
*((3^(1/2)*1i)/2 - 1/2))/(2*a^(5/3)))*((3^(1/2)*1i)/2 - 1/2))/(18*a^(8/3))
- (5*b^2*log((5*b^2*(a + b*x^2)^(1/3))/(2*a^2) + (5*b^2*((3^(1/2)*1i)/2 + 1
/2))/(2*a^(5/3)))*((3^(1/2)*1i)/2 + 1/2))/(18*a^(8/3))
```

3.721 $\int \frac{x^4}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=293

$$-\frac{27ax\sqrt[3]{a+bx^2}}{55b^2}+\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\cdots}}}}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}$$

```
[Out] -27/55*a*x*(b*x^2+a)^(1/3)/b^2+3/11*x^3*(b*x^2+a)^(1/3)/b-27/55*3^(3/4)*a^2
*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))
/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b
*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 242, 225}

$$-\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{27ax\sqrt[3]{a+bx^2}}{55b^2}+\frac{3x^3\sqrt[3]{a+bx^2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(2/3),x]
```

```
[Out] (-27*a*x*(a + b*x^2)^(1/3))/(55*b^2) + (3*x^3*(a + b*x^2)^(1/3))/(11*b) - (
27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55
*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx = \frac{3x^3\sqrt[3]{a+bx^2}}{11b} - \frac{(9a)\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx}{11b}$$

$$= -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2} + \frac{3x^3\sqrt[3]{a+bx^2}}{11b} + \frac{(27a^2)\int \frac{1}{(a+bx^2)^{2/3}}\,dx}{55b^2}$$

$$= -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2} + \frac{3x^3\sqrt[3]{a+bx^2}}{11b} + \frac{\left(81a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+\right.}{110b^3x}$$

$$= -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2} + \frac{3x^3\sqrt[3]{a+bx^2}}{11b} - \frac{27\ 3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/}}{(}}}{55b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 5.51, size = 79, normalized size = 0.27

$$\frac{3\left(-9a^2x-4abx^3+5b^2x^5+9a^2x\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{55b^2\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(2/3),x]
[Out] (3*(-9*a^2*x - 4*a*b*x^3 + 5*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(2/3)*Hyperg
eometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)]))/(55*b^2*(a + b*x^2)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(2/3),x)
[Out] int(x^4/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral(x^4/(b*x^2 + a)^(2/3), x)
```

Sympy [A]
time = 0.42, size = 27, normalized size = 0.09

$$\frac{x^5{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(2/3),x)
```

```
[Out] x**5*hyper((2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(2/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(2/3),x)
```

```
[Out] int(x^4/(a + b*x^2)^(2/3), x)
```

3.722 $\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=269

$$\frac{3x\sqrt[3]{a+bx^2}}{5b}+\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\left(1\cdots\right.\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] 3/5*x*(b*x^2+a)^(1/3)/b+3/5*3^(3/4)*a*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((
-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^2/x/(-
a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 242, 225}

$$\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3x\sqrt[3]{a+bx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3))/(5*b) + (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (
a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
```

```
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx = \frac{3x\sqrt[3]{a+bx^2}}{5b} - \frac{(3a)\int \frac{1}{(a+bx^2)^{2/3}}\,dx}{5b}$$

$$= \frac{3x\sqrt[3]{a+bx^2}}{5b} - \frac{\left(9a\sqrt{bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{10b^2x}$$

$$= \frac{3x\sqrt[3]{a+bx^2}}{5b} + \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a \cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+ \cdots}}}}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}- \cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a} \cdots\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.10, size = 62, normalized size = 0.23

$$\frac{3x\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{5b\,(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)]))/(5*b*(a + b*x^2)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(2/3),x)
```

```
[Out] int(x^2/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(2/3), x)
```

Sympy [A]
time = 0.39, size = 27, normalized size = 0.10

$$\frac{x^3\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(2/3),x)
```

```
[Out] x**3*hyper((2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(b\,x^2+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(2/3),x)
```

```
[Out] int(x^2/(a + b*x^2)^(2/3), x)
```

3.723 $\int \frac{1}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=246

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\cdots}}\right.\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] -3^(3/4)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3
^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^
(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3
))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {242, 225}

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-2/3),x]
```

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)]))
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
```

```
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2bx}$$

$$= -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\Big(\sin}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.01, size = 46, normalized size = 0.19

$$\frac{x\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)}{(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-2/3),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(2/3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-2/3), x)
```

Sympy [A]
time = 0.38, size = 24, normalized size = 0.10

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(2/3),x)
```

```
[Out] x*hyper((1/2, 2/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(2/3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-2/3), x)
```

Mupad [B]
time = 5.37, size = 37, normalized size = 0.15

$$\frac{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{2/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(2/3),x)
[Out] (x*((b*x^2)/a + 1)^(2/3)*hypergeom([1/2, 2/3], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(2/3)
```

3.724 $\int \frac{1}{x^2(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=265

$$-\frac{\sqrt[3]{a+bx^2}}{ax}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] -(b*x^2+a)^(1/3)/a/x+1/3*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1
/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/
2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a/x/(-a^(1/3)
*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 242, 225}

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(2/3)),x]
```

```
[Out] -((a + b*x^2)^(1/3)/(a*x)) + (Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(3^(1/4)*a*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
```

```
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 \left(a+bx^2\right)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx^2}}{ax}-\frac{b\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{3a}\\ &= -\frac{\sqrt[3]{a+bx^2}}{ax}-\frac{\sqrt{bx^2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{2ax}\\ &= -\frac{\sqrt[3]{a+bx^2}}{ax}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}}}{\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.55, size = 49, normalized size = 0.18

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(2/3)),x]
[Out] -(((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(2/3)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(2/3),x)
[Out] int(1/x^2/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3)/(b*x^4 + a*x^2), x)
```

Sympy [A]
time = 0.44, size = 27, normalized size = 0.10

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{1}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{a^{\frac{2}{3}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(2/3),x)
```

```
[Out] -hyper((-1/2, 2/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(2/3)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^2), x)
```

Mupad [B]
time = 5.46, size = 40, normalized size = 0.15

$$-\frac{3\left(\frac{a}{b\,x^2}+1\right)^{2/3}{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{7}{6};\ \frac{13}{6};\ -\frac{a}{b\,x^2}\right)}{7\,x\,\left(b\,x^2+a\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(2/3)),x)
[Out] -(3*(a/(b*x^2) + 1)^(2/3)*hypergeom([2/3, 7/6], 13/6, -a/(b*x^2)))/(7*x*(a
+ b*x^2)^(2/3))
```

3.725 $\int \frac{1}{x^4\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=293

$$-\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}-\frac{7\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\Bigl(\cdots}{9\sqrt[4]{3}\,a^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx\cdots}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\cdots}\right.}}}$$

```
[Out] -1/3*(b*x^2+a)^(1/3)/a/x^3+7/9*b*(b*x^2+a)^(1/3)/a^2/x-7/27*b*(a^(1/3)-(b*x
^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+
(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/
2)-1/2*2^(1/2))*3^(3/4)/a^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 242, 225}

$$\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}-\frac{7\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,a^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/3*(a + b*x^2)^(1/3)/(a*x^3) + (7*b*(a + b*x^2)^(1/3))/(9*a^2*x) - (7*Sqr
t[2 - Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a +
 b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^2*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^2}}{3ax^3}-\frac{(7b)\int \frac{1}{x^2(a+bx^2)^{2/3}}\,dx}{9a}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}+\frac{\left(7b^2\right)\int \frac{1}{(a+bx^2)^{2/3}}\,dx}{27a^2}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}+\frac{\left(7b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{18a^2x}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}-\frac{7\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a}}{\left(\left(1-\sqrt{3}\right)\right.}}}{9\sqrt[4]{3}\,a^2x\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 51, normalized size = 0.17

$$-\frac{\left(1+\frac{b x^{2}}{a}\right)^{2/3} {}_{2}F_{1}\left(-\frac{3}{2},\frac{2}{3};-\frac{1}{2};-\frac{b x^{2}}{a}\right)}{3 x^{3} \left(a+b x^{2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-3/2, 2/3, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(b\,x^{2}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/(b*x^6 + a*x^4), x)
```

Sympy [A]
time = 0.49, size = 32, normalized size = 0.11

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{2}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{b x^{2} e^{i\pi}}{a}\right)}{3 a^{\frac{2}{3}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(2/3),x)
[Out] -hyper((-3/2, 2/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(2/3)),x)
[Out] int(1/(x^4*(a + b*x^2)^(2/3)), x)
```

3.726 $\int \frac{x^7}{(a+bx^2)^{4/3}} \, dx$

Optimal. Leaf size=80

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2(a+bx^2)^{2/3}}{4b^4}-\frac{9a(a+bx^2)^{5/3}}{10b^4}+\frac{3(a+bx^2)^{8/3}}{16b^4}$$

```
[Out] 3/2*a^3/b^4/(b*x^2+a)^(1/3)+9/4*a^2*(b*x^2+a)^(2/3)/b^4-9/10*a*(b*x^2+a)^(5
/3)/b^4+3/16*(b*x^2+a)^(8/3)/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2(a+bx^2)^{2/3}}{4b^4}-\frac{9a(a+bx^2)^{5/3}}{10b^4}+\frac{3(a+bx^2)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a^3)/(2*b^4*(a + b*x^2)^(1/3)) + (9*a^2*(a + b*x^2)^(2/3))/(4*b^4) - (9*
a*(a + b*x^2)^(5/3))/(10*b^4) + (3*(a + b*x^2)^(8/3))/(16*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a+bx^2)^{4/3}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{4/3}} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{4/3}}+\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{(a+bx)^{5/3}}{b^3}\right) dx, x, x^2\right) \\
&= \frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2(a+bx^2)^{2/3}}{4b^4}-\frac{9a(a+bx^2)^{5/3}}{10b^4}+\frac{3(a+bx^2)^{8/3}}{16b^4}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.62

$$\frac{3(81a^3+27a^2bx^2-9ab^2x^4+5b^3x^6)}{80b^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(4/3),x]
[Out] (3*(81*a^3 + 27*a^2*b*x^2 - 9*a*b^2*x^4 + 5*b^3*x^6))/(80*b^4*(a + b*x^2)^(
1/3))
```

Maple [A]
time = 0.06, size = 47, normalized size = 0.59

method	result	size
gosper	$\frac{\frac{3}{16}b^3x^6-\frac{27}{80}a\,b^2x^4+\frac{81}{80}a^2b\,x^2+\frac{243}{80}a^3}{\left(b\,x^2+a\right)^{\frac{1}{3}}b^4}$	47
trager	$\frac{\frac{3}{16}b^3x^6-\frac{27}{80}a\,b^2x^4+\frac{81}{80}a^2b\,x^2+\frac{243}{80}a^3}{\left(b\,x^2+a\right)^{\frac{1}{3}}b^4}$	47
risch	$\frac{3\left(5b^2x^4-14ab\,x^2+41a^2\right)\left(b\,x^2+a\right)^{\frac{2}{3}}}{80b^4}+\frac{3a^3}{2b^4\left(b\,x^2+a\right)^{\frac{1}{3}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/80/(b*x^2+a)^(1/3)*(5*b^3*x^6-9*a*b^2*x^4+27*a^2*b*x^2+81*a^3)/b^4
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{8}{3}}}{16\,b^4}-\frac{9\,(bx^2+a)^{\frac{5}{3}}a}{10\,b^4}+\frac{9\,(bx^2+a)^{\frac{2}{3}}a^2}{4\,b^4}+\frac{3\,a^3}{2\,(bx^2+a)^{\frac{1}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] 3/16*(b*x^2 + a)^(8/3)/b^4 - 9/10*(b*x^2 + a)^(5/3)*a/b^4 + 9/4*(b*x^2 + a)
^(2/3)*a^2/b^4 + 3/2*a^3/((b*x^2 + a)^(1/3)*b^4)
```

Fricas [A]
time = 1.00, size = 58, normalized size = 0.72

$$\frac{3\,(5\,b^3x^6-9\,ab^2x^4+27\,a^2bx^2+81\,a^3)(bx^2+a)^{\frac{2}{3}}}{80\,(b^5x^2+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/80*(5*b^3*x^6 - 9*a*b^2*x^4 + 27*a^2*b*x^2 + 81*a^3)*(b*x^2 + a)^(2/3)/(b
^5*x^2 + a*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1584 vs. $2(75) = 150$.
time = 1.38, size = 1584, normalized size = 19.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(4/3),x)
```

```
[Out] 243*a**(68/3)*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 +
1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a*
*15*b**9*x**10 + 80*a**14*b**10*x**12) - 243*a**(68/3)/(80*a**20*b**4 + 480
*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16
*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 1296*a**(65/3)*
b*x**2*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a*
*18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**
9*x**10 + 80*a**14*b**10*x**12) - 1458*a**(65/3)*b*x**2/(80*a**20*b**4 + 48
0*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**1
6*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 2808*a**(62/3)
*b**2*x**4*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 120
0*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15
*b**9*x**10 + 80*a**14*b**10*x**12) - 3645*a**(62/3)*b**2*x**4/(80*a**20*b*
*4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 12
00*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 3120*a*
*(59/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**
2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 48
0*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 4860*a**(59/3)*b**3*x**6/(80*a
**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x*
*6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) +
1830*a**(56/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b
**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x*
*8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 3645*a**(56/3)*b**4*x**
8/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*
b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x*
*12) + 528*a**(53/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*
a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*
b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 1458*a**(53/3)*b
**5*x**10/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 160
0*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*
```

```
b**10*x**12) + 96*a**(50/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4
 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200
*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 243*a**(5
0/3)*b**6*x**12/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4
 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*
a**14*b**10*x**12) + 48*a**(47/3)*b**7*x**14*(1 + b*x**2/a)**(2/3)/(80*a**2
0*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6
+ 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 15*
a**(44/3)*b**8*x**16*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*
x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 +
 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12)
```

Giac [A]
time = 0.61, size = 70, normalized size = 0.88

$$\frac{3\,a^3}{2\,(bx^2+a)^{\frac{1}{3}}b^4}+\frac{3\left(5\,(bx^2+a)^{\frac{8}{3}}b^{28}-24\,(bx^2+a)^{\frac{5}{3}}ab^{28}+60\,(bx^2+a)^{\frac{2}{3}}a^2b^{28}\right)}{80\,b^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/2*a^3/((b*x^2 + a)^(1/3)*b^4) + 3/80*(5*(b*x^2 + a)^(8/3)*b^28 - 24*(b*x^
2 + a)^(5/3)*a*b^28 + 60*(b*x^2 + a)^(2/3)*a^2*b^28)/b^32
```

Mupad [B]
time = 5.44, size = 55, normalized size = 0.69

$$\frac{180\,a^2\,(b\,x^2+a)-72\,a\,{(b\,x^2+a)}^2+15\,{(b\,x^2+a)}^3+120\,a^3}{80\,b^4\,{(b\,x^2+a)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(4/3),x)
```

```
[Out] (180*a^2*(a + b*x^2) - 72*a*(a + b*x^2)^2 + 15*(a + b*x^2)^3 + 120*a^3)/(80
*b^4*(a + b*x^2)^(1/3))
```

3.727 $\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=59

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a(a+bx^2)^{2/3}}{2b^3}+\frac{3(a+bx^2)^{5/3}}{10b^3}$$

```
[Out] -3/2*a^2/b^3/(b*x^2+a)^(1/3)-3/2*a*(b*x^2+a)^(2/3)/b^3+3/10*(b*x^2+a)^(5/3)
/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a(a+bx^2)^{2/3}}{2b^3}+\frac{3(a+bx^2)^{5/3}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a^2)/(2*b^3*(a + b*x^2)^(1/3)) - (3*a*(a + b*x^2)^(2/3))/(2*b^3) + (3*(
a + b*x^2)^(5/3))/(10*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{4/3}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{4/3}} - \frac{2a}{b^2\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{b^2}\right)dx, x, x^2\right)$$
$$= -\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}} - \frac{3a(a+bx^2)^{2/3}}{2b^3} + \frac{3(a+bx^2)^{5/3}}{10b^3}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.64

$$\frac{3(-9a^2-3abx^2+b^2x^4)}{10b^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(-9*a^2 - 3*a*b*x^2 + b^2*x^4))/(10*b^3*(a + b*x^2)^(1/3))
```

Maple [A]
time = 0.06, size = 36, normalized size = 0.61

method	result	size
gosper	$-\frac{3(-b^2x^4+3abx^2+9a^2)}{10(bx^2+a)^{\frac{1}{3}}b^3}$	36
trager	$-\frac{3(-b^2x^4+3abx^2+9a^2)}{10(bx^2+a)^{\frac{1}{3}}b^3}$	36
risch	$-\frac{3(-bx^2+4a)(bx^2+a)^{\frac{2}{3}}}{10b^3} - \frac{3a^2}{2b^3(bx^2+a)^{\frac{1}{3}}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/10/(b*x^2+a)^(1/3)*(-b^2*x^4+3*a*b*x^2+9*a^2)/b^3
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.80

$$\frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b^3} - \frac{3\,(bx^2+a)^{\frac{2}{3}}a}{2\,b^3} - \frac{3\,a^2}{2\,(bx^2+a)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b^3 - 3/2*(b*x^2 + a)^(2/3)*a/b^3 - 3/2*a^2/((b*x^2
+ a)^(1/3)*b^3)
```

Fricas [A]
time = 0.98, size = 46, normalized size = 0.78

$$\frac{3\,(b^2x^4 - 3\,abx^2 - 9\,a^2)(bx^2 + a)^{\frac{2}{3}}}{10\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/10*(b^2*x^4 - 3*a*b*x^2 - 9*a^2)*(b*x^2 + a)^(2/3)/(b^4*x^2 + a*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(54) = 108$.
time = 0.88, size = 561, normalized size = 9.51

$$-\frac{27a^{\frac{29}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} + \frac{27a^{\frac{29}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} - \frac{63a^{\frac{26}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} + \frac{81a^{\frac{26}{3}}bx^2}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} - \frac{42a^{\frac{23}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} + \frac{81a^{\frac{23}{3}}b^2x^4}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} - \frac{3a^{\frac{20}{3}}b^3x^6\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} + \frac{27a^{\frac{20}{3}}b^3x^6}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} + \frac{3a^{\frac{17}{3}}b^4x^8\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(4/3),x)
```

```
[Out] -27*a**(29/3)*(1 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*
a**6*b**5*x**4 + 10*a**5*b**6*x**6) + 27*a**(29/3)/(10*a**8*b**3 + 30*a**7*
b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 63*a**(26/3)*b*x**2*(1
 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 +
 10*a**5*b**6*x**6) + 81*a**(26/3)*b*x**2/(10*a**8*b**3 + 30*a**7*b**4*x**2
 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 42*a**(23/3)*b**2*x**4*(1 + b*x
**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a*
*5*b**6*x**6) + 81*a**(23/3)*b**2*x**4/(10*a**8*b**3 + 30*a**7*b**4*x**2 +
30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 3*a**(20/3)*b**3*x**6*(1 + b*x**2/
a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b
**6*x**6) + 27*a**(20/3)*b**3*x**6/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a
**6*b**5*x**4 + 10*a**5*b**6*x**6) + 3*a**(17/3)*b**4*x**8*(1 + b*x**2/a)**
(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*
x**6)
```

Giac [A]
time = 0.52, size = 52, normalized size = 0.88

$$-\frac{3\,a^2}{2\,(bx^2 + a)^{\frac{1}{3}}b^3} + \frac{3\,\left((bx^2 + a)^{\frac{5}{3}}b^{12} - 5\,(bx^2 + a)^{\frac{2}{3}}ab^{12}\right)}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] -3/2*a^2/((b*x^2 + a)^(1/3)*b^3) + 3/10*((b*x^2 + a)^(5/3)*b^12 - 5*(b*x^2
+ a)^(2/3)*a*b^12)/b^15
```

Mupad [B]
time = 5.36, size = 41, normalized size = 0.69

$$-\frac{15\,a\,\left(b\,x^2+a\right)-3\,{\left(b\,x^2+a\right)}^2+15\,a^2}{10\,b^3\,{\left(b\,x^2+a\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(4/3),x)
```

```
[Out] -(15*a*(a + b*x^2) - 3*(a + b*x^2)^2 + 15*a^2)/(10*b^3*(a + b*x^2)^(1/3))
```

3.728 $\int \frac{x^3}{(a+bx^2)^{4/3}} dx$

Optimal. Leaf size=38

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3(a+bx^2)^{2/3}}{4b^2}$$

```
[Out] 3/2*a/b^2/(b*x^2+a)^(1/3)+3/4*(b*x^2+a)^(2/3)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {272, 45}

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3(a+bx^2)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a)/(2*b^2*(a + b*x^2)^(1/3)) + (3*(a + b*x^2)^(2/3))/(4*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{4/3}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{4/3}}+\frac{1}{b\sqrt[3]{a+bx}}\right)\,dx, x, x^2\right)\\ &= \frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3(a+bx^2)^{2/3}}{4b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.71

$$\frac{3(3a+bx^2)}{4b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(4/3),x]
[Out] (3*(3*a + b*x^2))/(4*b^2*(a + b*x^2)^(1/3))
```

Maple [A]
time = 0.05, size = 24, normalized size = 0.63

method	result	size
gosper	$\frac{\frac{3b\,x^2}{4}+\frac{9a}{4}}{(b\,x^2+a)^{\frac{1}{3}}\,b^2}$	24
trager	$\frac{\frac{3b\,x^2}{4}+\frac{9a}{4}}{(b\,x^2+a)^{\frac{1}{3}}\,b^2}$	24
risch	$\frac{3a}{2b^2(b\,x^2+a)^{\frac{1}{3}}}+\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b^2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/4/(b*x^2+a)^(1/3)*(b*x^2+3*a)/b^2
```

Maxima [A]
time = 0.29, size = 30, normalized size = 0.79

$$\frac{3\,(bx^2+a)^{\frac{2}{3}}}{4\,b^2}+\frac{3\,a}{2\,(bx^2+a)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] 3/4*(b*x^2 + a)^(2/3)/b^2 + 3/2*a/((b*x^2 + a)^(1/3)*b^2)
```

Fricas [A]
time = 1.88, size = 35, normalized size = 0.92

$$\frac{3\,(bx^2+3\,a)(bx^2+a)^{\frac{2}{3}}}{4\,(b^3x^2+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x^2 + 3*a)*(b*x^2 + a)^(2/3)/(b^3*x^2 + a*b^2)
```

Sympy [A]
time = 0.28, size = 46, normalized size = 1.21

$$\begin{cases} \frac{9a}{4b^2\sqrt[3]{a+bx^2}} + \frac{3x^2}{4b\sqrt[3]{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((9*a/(4*b**2*(a + b*x**2)**(1/3)) + 3*x**2/(4*b*(a + b*x**2)**(1/
3)), Ne(b, 0)), (x**4/(4*a**(4/3)), True))
```

Giac [A]
time = 0.72, size = 34, normalized size = 0.89

$$\frac{3\left(\frac{(bx^2+a)^{\frac{2}{3}}}{b} + \frac{2\,a}{(bx^2+a)^{\frac{1}{3}}\,b}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/4*((b*x^2 + a)^(2/3)/b + 2*a/((b*x^2 + a)^(1/3)*b))/b
```

Mupad [B]
time = 5.59, size = 24, normalized size = 0.63

$$\frac{3\,b\,x^2+9\,a}{4\,b^2\,{\left(b\,x^2+a\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(4/3),x)
```

```
[Out] (9*a + 3*b*x^2)/(4*b^2*(a + b*x^2)^(1/3))
```

3.729 $\int \frac{x}{(a+bx^2)^{4/3}} dx$

Optimal. Leaf size=18

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

```
[Out] -3/2/b/(b*x^2+a)^(1/3)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(4/3),x]
```

```
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{4/3}} dx = -\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(4/3),x]
```

```
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

Maple [A]
time = 0.06, size = 15, normalized size = 0.83

method	result	size
gosper	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15
derivativedivides	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15
default	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15
trager	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/b/(b*x^2+a)^(1/3)
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.78

$$-\frac{3}{2\,(bx^2+a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -3/2/((b*x^2 + a)^(1/3)*b)
```

Fricas [A]
time = 1.13, size = 24, normalized size = 1.33

$$-\frac{3\,(bx^2+a)^{\frac{2}{3}}}{2\,(b^2x^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/2*(b*x^2 + a)^(2/3)/(b^2*x^2 + a*b)
```

Sympy [A]
time = 0.27, size = 26, normalized size = 1.44

$$\begin{cases} -\frac{3}{2b\sqrt[3]{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-3/(2*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(4/3)), Tru
e))
```

Giac [A]
time = 0.71, size = 14, normalized size = 0.78

$$-\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] -3/2/((b*x^2 + a)^(1/3)*b)
```

Mupad [B]
time = 5.39, size = 14, normalized size = 0.78

$$-\frac{3}{2\,b\,{\left(b\,x^2+a\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(4/3),x)
```

```
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

3.730 $\int \frac{1}{x(a+bx^2)^{4/3}} \, dx$

Optimal. Leaf size=104

$$\frac{3}{2a\sqrt[3]{a+bx^2}}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}$$

```
[Out] 3/2/a/(b*x^2+a)^(1/3)-1/2*ln(x)/a^(4/3)+3/4*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(
4/3)+1/2*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^
(4/3)
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {272, 53, 57, 631, 210, 31}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*(a + b*x^2)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))
/(Sqrt[3]*a^(1/3))])/(2*a^(4/3)) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a
 + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}}\,dx, x, x^2\right)\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{2a}\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}} - \frac{\log(x)}{2a^{4/3}} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{4/3}} + \frac{3\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}
\end{aligned}$$

$$\begin{aligned}
&= \frac{3}{2a\sqrt[3]{a+bx^2}} - \frac{\log(x)}{2a^{4/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{}}{}\right.}{2a^{4/3}}\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{4/3}} - \frac{\log(x)}{2a^{4/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 121, normalized size = 1.16

$$\frac{\frac{6\sqrt[3]{a}}{\sqrt[3]{a+bx^2}}+2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{4a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(4/3)),x]
```

```
[Out] ((6*a^(1/3))/(a + b*x^2)^(1/3) + 2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3)
)/a^(1/3))/Sqrt[3]] + 2*Log[-a^(1/3) + (a + b*x^2)^(1/3)] - Log[a^(2/3) + a
^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(4*a^(4/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(b\,x^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x/(b*x^2+a)^(4/3),x)
```

Maxima [A]
time = 0.49, size = 100, normalized size = 0.96

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{4}{3}}}-\frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{4}{3}}}+\frac{\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a^{\frac{4}{3}}}+\frac{3}{2\,(bx^2+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
4/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
4/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(4/3) + 3/2/((b*x^2 + a)^(1/3
)*a)
```

Fricas [A]
time = 0.71, size = 327, normalized size = 3.14

$$\left[\frac{\sqrt{3}\,(abx^2+a^2)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\,\log\left(\frac{2\,bx^2+\sqrt{3}\left(2\,(bx^2+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-(bx^2+a)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,(bx^2+a)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+6\,(bx^2+a)^{\frac{2}{3}}a}{4\,(a^2bx^2+a^3)},\ -\frac{(bx^2+a)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,(bx^2+a)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{2\sqrt{3}\,(abx^2+a^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-6\,(bx^2+a)^{\frac{2}{3}}a}{4\,(a^2bx^2+a^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(a*b*x^2 + a^2)*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(b
*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - (b*x^2 + a)*a^(2/3)*log((b*x^2 +
 a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x^2 + a)*a^(2/3)*lo
g((b*x^2 + a)^(1/3) - a^(1/3)) + 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a^3),
-1/4*((b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3)
 + a^(2/3)) - 2*(b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) - 2*sq
rt(3)*(a*b*x^2 + a^2)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(1/3) - 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 41, normalized size = 0.39

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(4/3)*hyper((4/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4/
3)*x**(8/3)*gamma(7/3))
```

Giac [A]
time = 1.66, size = 101, normalized size = 0.97

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{4}{3}}}-\frac{\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{4}{3}}}+\frac{\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{4}{3}}}+\frac{3}{2\left(bx^{2}+a\right)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
4/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
4/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/2/((b*x^2 + a)
^(1/3)*a)
```

Mupad [B]
time = 5.59, size = 123, normalized size = 1.18

$$\frac{\ln\left(18\,a\,{\left(b\,x^2+a\right)}^{1/3}-18\,a^{4/3}\right)}{2\,a^{4/3}}+\frac{3}{2\,a\,{\left(b\,x^2+a\right)}^{1/3}}+\frac{\ln\left(18\,a\,{\left(b\,x^2+a\right)}^{1/3}-\frac{9\,a^{4/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{4\,a^{4/3}}-\frac{\ln\left(18\,a\,{\left(b\,x^2+a\right)}^{1/3}-\frac{9\,a^{4/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{4\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(4/3)),x)
```

```
[Out] log(18*a*(a + b*x^2)^(1/3) - 18*a^(4/3))/(2*a^(4/3)) + 3/(2*a*(a + b*x^2)^(
1/3)) + (log(18*a*(a + b*x^2)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1)^2)/2)*(3^
(1/2)*1i - 1))/(4*a^(4/3)) - (log(18*a*(a + b*x^2)^(1/3) - (9*a^(4/3)*(3^(1
/2)*1i + 1)^2)/2)*(3^(1/2)*1i + 1))/(4*a^(4/3))
```

3.731 $\int \frac{1}{x^3(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=123

$$-\frac{2b}{a^2\sqrt[3]{a+bx^2}}-\frac{1}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}$$

```
[Out] -2*b/a^2/(b*x^2+a)^(1/3)-1/2/a/x^2/(b*x^2+a)^(1/3)+2/3*b*ln(x)/a^(7/3)-b*ln
(a^(1/3)-(b*x^2+a)^(1/3))/a^(7/3)-2/3*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/
3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {272, 44, 53, 57, 631, 210, 31}

$$-\frac{2b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b}{a^2\sqrt[3]{a+bx^2}}-\frac{1}{2ax^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(4/3)),x]
```

```
[Out] (-2*b)/(a^2*(a + b*x^2)^(1/3)) - 1/(2*a*x^2*(a + b*x^2)^(1/3)) - (2*b*ArcTa
n[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (
2*b*Log[x])/(3*a^(7/3)) - (b*Log[a^(1/3) - (a + b*x^2)^(1/3)])/a^(7/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^2)^{4/3}} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{4/3}} \, dx, x, x^2\right)$$

$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} + \frac{2\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}} \, dx, x, x^2\right)}{a}$$

$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} - \frac{2(a+bx^2)^{2/3}}{a^2x^2} - \frac{(2b)\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}} \, dx, x, x^2\right)}{3a^2}$$

$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} - \frac{2(a+bx^2)^{2/3}}{a^2x^2} + \frac{2b\log(x)}{3a^{7/3}} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x} \, dx, x, \sqrt[3]{a+bx^2}\right)}{a^{7/3}}$$

$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} - \frac{2(a+bx^2)^{2/3}}{a^2x^2} + \frac{2b\log(x)}{3a^{7/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}} + \frac{(2b)\text{Sul}}{}$$

$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} - \frac{2(a+bx^2)^{2/3}}{a^2x^2} - \frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{2b\log(x)}{3a^{7/3}} - \frac{b\log(}{}$$

Mathematica [A]
time = 0.16, size = 135, normalized size = 1.10

$$\frac{-\frac{3\sqrt[3]{a}\,(a+4bx^2)}{x^2\sqrt[3]{a+bx^2}} - 4\sqrt{3}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 4b\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right) + 2b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{6a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(4/3)),x]
[Out] ((-3*a^(1/3)*(a + 4*b*x^2))/(x^2*(a + b*x^2)^(1/3)) - 4*Sqrt[3]*b*ArcTan[(1
 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 4*b*Log[-a^(1/3) + (a + b*x^2)
^(1/3)] + 2*b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])
/(6*a^(7/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(b\,x^2+a\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(4/3),x)
```

Maxima [A]
time = 0.56, size = 136, normalized size = 1.11

$$-\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,(bx^2+a)b-3\,ab}{2\left((bx^2+a)^{\frac{4}{3}}a^2-(bx^2+a)^{\frac{1}{3}}a^3\right)}+\frac{b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{2\,b\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(7/3) - 1/2*(4*(b*x^2 + a)*b - 3*a*b)/((b*x^2 + a)^(4/3)*a^2 - (b*x^2 + a
)^(1/3)*a^3) + 1/3*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(7/3) - 2/3*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(7/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(96) = 192.
time = 0.78, size = 453, normalized size = 3.68

$$\left[\frac{6\sqrt{\frac{1}{3}}\,(ab^2x^4+a^2bx^2)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx^2-3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx^2+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x^2}\right)+2\,(b^2x^4+abx^2)(-a)^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4\,(b^2x^4+abx^2)(-a)^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\,(4\,abx^2+a^2)(bx^2+a)^{\frac{2}{3}}}{6\,(a^3bx^4+a^4x^2)},\ -\frac{12\sqrt{\frac{1}{3}}\,(ab^2x^4+a^2bx^2)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-2\,(b^2x^4+abx^2)(-a)^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+4\,(b^2x^4+abx^2)(-a)^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\,(4\,abx^2+a^2)(bx^2+a)^{\frac{2}{3}}}{6\,(a^3bx^4+a^4x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt((-a)^(1/3)/a)*log((2*b*x^2 -
 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(
1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + 2
*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(
-a)^(1/3) + (-a)^(2/3)) - 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^
(1/3) + (-a)^(1/3)) - 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*b*x^4 + a
^4*x^2), -1/6*(12*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt(-(-a)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(
b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(1
/3) + (-a)^(1/3)) + 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*b*x^4 + a^4
*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.87, size = 41, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\,\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{14}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(7/3)*hyper((4/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4
/3)*x**(14/3)*gamma(10/3))
```

Giac [A]
time = 1.25, size = 134, normalized size = 1.09

$$-\frac{2\,\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\,\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}+\frac{b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{2\,b\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,(bx^2+a)b-3\,ab}{2\,\left((bx^2+a)^{\frac{4}{3}}-(bx^2+a)^{\frac{1}{3}}a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(7/3) + 1/3*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(7/3) - 2/3*b*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) - 1/2*(4*(b
*x^2 + a)*b - 3*a*b)/(((b*x^2 + a)^(4/3) - (b*x^2 + a)^(1/3)*a)*a^2)
```

Mupad [B]
time = 5.65, size = 178, normalized size = 1.45

$$-\frac{\frac{3\,b}{a}-\frac{4\,b\,(b\,x^2+a)}{a^2}}{2\,a\,(b\,x^2+a)^{1/3}-2\,(b\,x^2+a)^{4/3}}-\frac{2\,b\,\ln\left(4\,a^{7/3}\,b^2-4\,a^2\,b^2\,(b\,x^2+a)^{1/3}\right)}{3\,a^{7/3}}+\frac{\ln\left(a^{7/3}\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2-4\,a^2\,b^2\,(b\,x^2+a)^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{7/3}}+\frac{\ln\left(a^{7/3}\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2-4\,a^2\,b^2\,(b\,x^2+a)^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(4/3)),x)
```

```
[Out] (log(a^(7/3)*(b - 3^(1/2)*b*1i)^2 - 4*a^2*b^2*(a + b*x^2)^(1/3))*(b - 3^(1/
2)*b*1i))/(3*a^(7/3)) - (2*b*log(4*a^(7/3)*b^2 - 4*a^2*b^2*(a + b*x^2)^(1/3
)))/(3*a^(7/3)) - ((3*b)/a - (4*b*(a + b*x^2))/a^2)/(2*a*(a + b*x^2)^(1/3)
- 2*(a + b*x^2)^(4/3)) + (log(a^(7/3)*(b + 3^(1/2)*b*1i)^2 - 4*a^2*b^2*(a +
 b*x^2)^(1/3))*(b + 3^(1/2)*b*1i))/(3*a^(7/3))
```

3.732 $\int \frac{1}{x^5\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=159

$$\frac{7b^2}{3a^3\sqrt[3]{a+bx^2}}-\frac{1}{4ax^4\sqrt[3]{a+bx^2}}+\frac{7b}{12a^2x^2\sqrt[3]{a+bx^2}}+\frac{7b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}\right.}{}$$

```
[Out] 7/3*b^2/a^3/(b*x^2+a)^(1/3)-1/4/a/x^4/(b*x^2+a)^(1/3)+7/12*b/a^2/x^2/(b*x^2
+a)^(1/3)-7/9*b^2*ln(x)/a^(10/3)+7/6*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(10/
3)+7/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(10/3)
*3^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {272, 44, 53, 57, 631, 210, 31}

$$\frac{7b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2}{3a^3\sqrt[3]{a+bx^2}}+\frac{7b}{12a^2x^2\sqrt[3]{a+bx^2}}-\frac{1}{4ax^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(4/3)),x]
```

```
[Out] (7*b^2)/(3*a^3*(a + b*x^2)^(1/3)) - 1/(4*a*x^4*(a + b*x^2)^(1/3)) + (7*b)/(
12*a^2*x^2*(a + b*x^2)^(1/3)) + (7*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3
))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) +
 (7*b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^2)^{4/3}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{4/3}}\,dx, x, x^2\right)$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} + \frac{7\text{Subst}\left(\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{2a}$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7(a+bx^2)^{2/3}}{4a^2x^4} - \frac{(7b)\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{3a^2}$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7(a+bx^2)^{2/3}}{4a^2x^4} + \frac{7b(a+bx^2)^{2/3}}{3a^3x^2} + \frac{(7b^2)\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x,\right.}{9a^3}$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7(a+bx^2)^{2/3}}{4a^2x^4} + \frac{7b(a+bx^2)^{2/3}}{3a^3x^2} - \frac{7b^2\log(x)}{9a^{10/3}} - \frac{(7b^2)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}\right.}{6}$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7(a+bx^2)^{2/3}}{4a^2x^4} + \frac{7b(a+bx^2)^{2/3}}{3a^3x^2} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a} - \sqrt[3]{a}\right.}{6a^{10/3}}$$

$$= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7(a+bx^2)^{2/3}}{4a^2x^4} + \frac{7b(a+bx^2)^{2/3}}{3a^3x^2} + \frac{7b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} -$$

Mathematica [A]
time = 0.21, size = 154, normalized size = 0.97

$$\frac{\frac{3\sqrt[3]{a}\,(-3a^2+7abx^2+28b^2x^4)}{x^4\sqrt[3]{a+bx^2}} + 28\sqrt{3}\,b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 28b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right) - 14b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(4/3)),x]
```

```
[Out] ((3*a^(1/3)*(-3*a^2 + 7*a*b*x^2 + 28*b^2*x^4))/(x^4*(a + b*x^2)^(1/3)) + 28
*Sqrt[3]*b^2*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 28*b^2*L
og[-a^(1/3) + (a + b*x^2)^(1/3)] - 14*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)
^(1/3) + (a + b*x^2)^(2/3)])/(36*a^(10/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\,(b\,x^2+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(4/3),x)
```

Maxima [A]
time = 0.50, size = 176, normalized size = 1.11

$$\frac{7\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}}+\frac{28\,(bx^2+a)^2b^2-49\,(bx^2+a)ab^2+18\,a^2b^2}{12\left((bx^2+a)^{\frac{7}{3}}a^3-2\,(bx^2+a)^{\frac{4}{3}}a^4+(bx^2+a)^{\frac{1}{3}}a^5\right)}-\frac{7\,b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{10}{3}}}+\frac{7\,b^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(10/3) + 1/12*(28*(b*x^2 + a)^2*b^2 - 49*(b*x^2 + a)*a*b^2 + 18*a^2*b^2)
/((b*x^2 + a)^(7/3)*a^3 - 2*(b*x^2 + a)^(4/3)*a^4 + (b*x^2 + a)^(1/3)*a^5)
- 7/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
10/3) + 7/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(10/3)
```

Fricas [A]
time = 0.77, size = 437, normalized size = 2.75

$$\left[\frac{42\sqrt{\frac{1}{3}}\,(ab^3x^6+a^2b^2x^4)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-14\,(b^3x^6+ab^2x^4)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+28\,(b^3x^6+ab^2x^4)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(28\,ab^2x^4+7\,a^2bx^2-3\,a^3)(bx^2+a)^{\frac{2}{3}}}{36\,(a^4bx^6+a^5x^4)},\ -\frac{14\,(b^3x^6+ab^2x^4)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-28\,(b^3x^6+ab^2x^4)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{84\sqrt{\frac{1}{3}}\,(ab^3x^6+a^2b^2x^4)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-3\,(28\,ab^2x^4+7\,a^2bx^2-3\,a^3)(bx^2+a)^{\frac{2}{3}}}{36\,(a^4bx^6+a^5x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/36*(42*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4)*sqrt(-1/a^(2/3))*log((2*b*x^2
 + 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3)
)*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 14*(b^3*x^6
+ a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3)) + 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3))
+ 3*(28*a*b^2*x^4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^6 + a^
5*x^4), -1/36*(14*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*
x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b
*x^2 + a)^(1/3) - a^(1/3)) - 84*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4)*arctan(
sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^
4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^6 + a^5*x^4)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.43, size = 41, normalized size = 0.26

$$-\frac{\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{20}{3}}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(10/3)*hyper((4/3, 10/3), (13/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**
(4/3)*x**(20/3)*gamma(13/3))
```

Giac [A]
time = 1.50, size = 154, normalized size = 0.97

$$\frac{7\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}} - \frac{7\,b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{10}{3}}} + \frac{7\,b^2\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{10}{3}}} + \frac{3\,b^2}{2\,(bx^2+a)^{\frac{1}{3}}a^3} + \frac{10\,(bx^2+a)^{\frac{5}{3}}b^2-13\,(bx^2+a)^{\frac{2}{3}}ab^2}{12\,a^3b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(10/3) - 7/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(10/3) + 7/9*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(10/3) +
3/2*b^2/((b*x^2 + a)^(1/3)*a^3) + 1/12*(10*(b*x^2 + a)^(5/3)*b^2 - 13*(b*x^
2 + a)^(2/3)*a*b^2)/(a^3*b^2*x^4)
```

Mupad [B]
time = 5.67, size = 224, normalized size = 1.41

$$\frac{\frac{3b^2}{a}-\frac{49\,b^2\,(b\,x^2+a)}{6\,a^2}+\frac{14\,b^2\,(b\,x^2+a)^2}{3\,a^3}}{2\,(b\,x^2+a)^{7/3}-4\,a\,(b\,x^2+a)^{4/3}+2\,a^2\,(b\,x^2+a)^{1/3}} + \frac{7\,b^2\,\ln\left(147\,a^3\,b^4\,(b\,x^2+a)^{1/3}-147\,a^{10/3}\,b^4\right)}{9\,a^{10/3}} + \frac{7\,b^2\,\ln\left(147\,a^3\,b^4\,(b\,x^2+a)^{1/3}-147\,a^{10/3}\,b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}} - \frac{7\,b^2\,\ln\left(147\,a^3\,b^4\,(b\,x^2+a)^{1/3}-147\,a^{10/3}\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(4/3)),x)
```

```
[Out] ((3*b^2)/a - (49*b^2*(a + b*x^2))/(6*a^2) + (14*b^2*(a + b*x^2)^2)/(3*a^3))
/(2*(a + b*x^2)^(7/3) - 4*a*(a + b*x^2)^(4/3) + 2*a^2*(a + b*x^2)^(1/3)) +
(7*b^2*log(147*a^3*b^4*(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4))/(9*a^(10/3))
+ (7*b^2*log(147*a^3*b^4*(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4*((3^(1/2)*1i)
/2 - 1/2)^2)*((3^(1/2)*1i)/2 - 1/2))/(9*a^(10/3)) - (7*b^2*log(147*a^3*b^4*
(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i
)/2 + 1/2))/(9*a^(10/3))
```

3.733 $\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=577

$$-\frac{3x^3}{2b\sqrt[3]{a+bx^2}}+\frac{27x(a+bx^2)^{2/3}}{14b^2}+\frac{81ax}{14b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a\right.}}{\quad}$$

```
[Out] -3/2*x^3/b/(b*x^2+a)^(1/3)+27/14*x*(b*x^2+a)^(2/3)/b^2+81/14*a*x/b^2/(-(b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+27/14*3^(3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(
1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+
a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^3/x*2^(1/2)/(-a
^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)-81/28*3^(1/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)
^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^
(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^3/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {294, 327, 241, 310, 225, 1893}

$$\frac{27\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{7\sqrt{2}\,b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{28b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{27x(a+bx^2)^{2/3}}{14b^2}+\frac{81ax}{14b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{3x^3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*x^3)/(2*b*(a + b*x^2)^(1/3)) + (27*x*(a + b*x^2)^(2/3))/(14*b^2) + (81*
a*x)/(14*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (81*3^(1/4)*Sqr
t[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3
)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/(
(1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(28*b^3*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))^2)]) + (27*3^(3/4)*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[
```

```
(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(7*Sqrt[2]*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx = -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{9\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx}{2b}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x(a+bx^2)^{2/3}}{14b^2} - \frac{(27a)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{14b^2}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x(a+bx^2)^{2/3}}{14b^2} - \frac{\left(81a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx\right.}{28b^3x}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x(a+bx^2)^{2/3}}{14b^2} + \frac{\left(81a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a\right.}{28b^3x}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x(a+bx^2)^{2/3}}{14b^2} + \frac{81ax}{14b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{81\sqrt[4]{3}\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.54, size = 65, normalized size = 0.11

$$\frac{3x\left(9a+2bx^2-9a\sqrt[3]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{14b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(9*a + 2*b*x^2 - 9*a*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2,
 3/2, -((b*x^2)/a)]))/(14*b^2*(a + b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(4/3),x)
[Out] int(x^4/(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A]
time = 0.42, size = 27, normalized size = 0.05

$$\frac{x^5{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(4/3),x)
[Out] x**5*hyper((4/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(4/3),x)
[Out] int(x^4/(a + b*x^2)^(4/3), x)
```

3.734 $\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=553

$$-\frac{3x}{2b\sqrt[3]{a+bx^2}}-\frac{9x}{2b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\cdots}{\left(\left(1-\cdots\right.\right.}}\cdots}{4b^2x\sqrt{-\cdots}}$$

```
[Out] -3/2*x/b/(b*x^2+a)^(1/3)-9/2*x/b/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-3/2
*3^(3/4)*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1
/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1
-3^(1/2)))^2)^(1/2)/b^2/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+9/4*3^(1/4)*a^(1/3)*(a^(1/3)-(b*
x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)
+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1
/2)+1/2*2^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {294, 241, 310, 225, 1893}

$$-\frac{3\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\,b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{4b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{3x}{2b\sqrt[3]{a+bx^2}}-\frac{9x}{2b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*x)/(2*b*(a + b*x^2)^(1/3)) - (9*x)/(2*b*((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
- Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(4*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((
1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (3*3^(3/4)*a^(1/3)*(a^(1/3
) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x
```

```
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (
a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx = -\frac{3x}{2b\sqrt[3]{a+bx^2}} + \frac{3\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{2b}$$

$$= -\frac{3x}{2b\sqrt[3]{a+bx^2}} + \frac{\left(9\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4b^2x}$$

$$= -\frac{3x}{2b\sqrt[3]{a+bx^2}} - \frac{\left(9\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4b^2x} + \frac{\left(9\sqrt{\frac{1}{2}\left(2\right.}\right.}{}$$

$$= -\frac{3x}{2b\sqrt[3]{a+bx^2}} - \frac{9x}{2b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{\right.}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.99, size = 55, normalized size = 0.10

$$\frac{3x\left(-1+\sqrt[3]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(-1 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)
/a)]))/(2*b*(a + b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(4/3),x)
```

```
[Out] int(x^2/(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A]
time = 0.42, size = 27, normalized size = 0.05

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(4/3),x)
[Out] x**3*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate(x^2/(b*x^2 + a)^(4/3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(4/3),x)
[Out] int(x^2/(a + b*x^2)^(4/3), x)
```

3.735 $\int \frac{1}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=552

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}+\frac{3x}{2a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{4a^{2/3}bx\sqrt{-\cdots}}$$

[Out] 3/2*x/a/(b*x^2+a)^(1/3)+3/2*x/a/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/2*3^(3/4)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-3/4*3^(1/4)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.22, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {205, 241, 310, 225, 1893}

$$\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\,a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{4a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3x}{2a\sqrt[3]{a+bx^2}}+\frac{3x}{2a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-4/3),x]

[Out] (3*x)/(2*a*(a + b*x^2)^(1/3)) + (3*x)/(2*a*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (3^(3/4)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((

```
1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(Sqrt[2]*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x
^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \frac{3x}{2a\sqrt[3]{a+bx^2}} - \frac{\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{2a}$$

$$= \frac{3x}{2a\sqrt[3]{a+bx^2}} - \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4abx}$$

$$= \frac{3x}{2a\sqrt[3]{a+bx^2}} + \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4abx} - \frac{\left(3\sqrt{\frac{1}{2}\left(2-\right.\right.}{}$$

$$= \frac{3x}{2a\sqrt[3]{a+bx^2}} + \frac{3x}{2a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.69, size = 58, normalized size = 0.11

$$\frac{3x - x\sqrt[3]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-4/3),x]
```

```
[Out] (3*x - x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a
)])/(2*a*(a + b*x^2)^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A]
time = 0.40, size = 24, normalized size = 0.04

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{4}{3} \\ \frac{3}{2}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(4/3),x)
[Out] x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(4/3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-4/3), x)
```

Mupad [B]
time = 5.42, size = 37, normalized size = 0.07

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{4/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{4}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(4/3),x)
[Out] (x*((b*x^2)/a + 1)^(4/3)*hypergeom([1/2, 4/3], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(4/3)
```

3.736 $\int \frac{1}{x^2(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=571

$$\frac{3}{2ax\sqrt[3]{a+bx^2}}-\frac{5(a+bx^2)^{2/3}}{2a^2x}-\frac{5bx}{2a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{}$$

[Out] 3/2/a/x/(b*x^2+a)^(1/3)-5/2*(b*x^2+a)^(2/3)/a^2/x-5/2*b*x/a^2/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-5/6*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+5/4*3^(1/4)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/a^(5/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.26, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {296, 331, 241, 310, 225, 1893}

$$-\frac{5\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{4a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{5bx}{2a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{5(a+bx^2)^{2/3}}{2a^2x}+\frac{3}{2ax\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^2)^(4/3)),x]

[Out] 3/(2*a*x*(a + b*x^2)^(1/3)) - (5*(a + b*x^2)^(2/3))/(2*a^2*x) - (5*b*x)/(2*a^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (5*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(

```
1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*El
lipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*3^(1/4)*a^(5/3)*x*S
qrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{4/3}}\,dx = \frac{3}{2ax\sqrt[3]{a+bx^2}} + \frac{5\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx}{2a}$$

$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} + \frac{(5b)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{6a^2}$$

$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} + \frac{\left(5\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^2x}$$

$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} - \frac{\left(5\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right.}{4a^2x}$$

$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} - \frac{5bx}{2a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{5\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.45, size = 52, normalized size = 0.09

$$-\frac{\sqrt[3]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(4/3)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 4/3, 1/2, -((b*x^2)/a)])/(
a*x*(a + b*x^2)^(1/3)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(4/3),x)
[Out] int(1/x^2/(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [A]
time = 0.49, size = 27, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{1}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{a^{\frac{4}{3}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(4/3),x)
[Out] -hyper((-1/2, 4/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(4/3)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^2), x)
```

Mupad [B]
time = 5.57, size = 40, normalized size = 0.07

$$-\frac{3\left(\frac{a}{b\,x^2}+1\right)^{4/3}{}_2\mathrm{F}_1\left(\frac{4}{3},\frac{11}{6};\ \frac{17}{6};\ -\frac{a}{b\,x^2}\right)}{11\,x\,\left(b\,x^2+a\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(4/3)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(4/3)*hypergeom([4/3, 11/6], 17/6, -a/(b*x^2)))/(11*x*(
a + b*x^2)^(4/3))
```

3.737 $\int \frac{1}{x^4 (a+bx^2)^{4/3}} \, dx$

Optimal. Leaf size=599

$$\frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11(a+bx^2)^{2/3}}{6a^2x^3} + \frac{55b(a+bx^2)^{2/3}}{18a^3x} + \frac{55b^2x}{18a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{55\sqrt{2+\sqrt{3}}\,b\Big(}{}$$

```
[Out] 3/2/a/x^3/(b*x^2+a)^(1/3)-11/6*(b*x^2+a)^(2/3)/a^2/x^3+55/18*b*(b*x^2+a)^(2
/3)/a^3/x+55/18*b^2*x/a^3/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+55/54*b*(a
^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-
(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^
2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*3^(3/4)/a^(8/3)/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-55/36*b*(a^(1/3)-(b*x^2+a)^(1/3))*Ellip
ticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(
-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^
(1/4)/a^(8/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 599, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {296, 331, 241, 310, 225, 1893}

$$\frac{55b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt{2}\sqrt[4]{3}\,a^{8/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}} - \frac{55\sqrt{2+\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{12\,3^{3/4}a^{8/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}} + \frac{55b^2x}{18a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{55b(a+bx^2)^{2/3}}{18a^3x} - \frac{11(a+bx^2)^{2/3}}{6a^2x^3} + \frac{3}{2ax^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x^3*(a + b*x^2)^(1/3)) - (11*(a + b*x^2)^(2/3))/(6*a^2*x^3) + (55*b*
(a + b*x^2)^(2/3))/(18*a^3*x) + (55*b^2*x)/(18*a^3*((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))) - (55*Sqrt[2 + Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3)
)*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(12*3^(3/4)*a^(8/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (55*b*(a^(1/3) - (a
```

```
 + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(9*Sqrt[2]*3^(1/4)*a^(8/3)*x*Sqrt[-((a^(1/3)*(a^(
1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^2)^{4/3}}\,dx = \frac{3}{2ax^3\sqrt[3]{a+bx^2}} + \frac{11\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx}{2a}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11(a+bx^2)^{2/3}}{6a^2x^3} - \frac{(55b)\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx}{18a^2}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11(a+bx^2)^{2/3}}{6a^2x^3} + \frac{55b(a+bx^2)^{2/3}}{18a^3x} - \frac{(55b^2)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{54a^3}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11(a+bx^2)^{2/3}}{6a^2x^3} + \frac{55b(a+bx^2)^{2/3}}{18a^3x} - \frac{\left(55b\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{-a}}\right.}{36a^3}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11(a+bx^2)^{2/3}}{6a^2x^3} + \frac{55b(a+bx^2)^{2/3}}{18a^3x} + \frac{\left(55b\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{(1+\sqrt{}}{\sqrt{-}}\right.}{36}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11(a+bx^2)^{2/3}}{6a^2x^3} + \frac{55b(a+bx^2)^{2/3}}{18a^3x} + \frac{55b^2x}{18a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a} - \sqrt[3]{a}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.09

$$-\frac{\sqrt[3]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{3}{2},\frac{4}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(4/3)),x]
[Out] -1/3*((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-3/2, 4/3, -1/2, -((b*x^2)/a)
])/(a*x^3*(a + b*x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(b\,x^{2}+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(4/3),x)
[Out] int(1/x^4/(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [A]
time = 0.54, size = 32, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{4}{3} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{3a^{\frac{4}{3}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(4/3),x)
```

```
[Out] -hyper((-3/2, 4/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(4/3)),x)
[Out] int(1/(x^4*(a + b*x^2)^(4/3)), x)
```

3.738 $\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=195

$$-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}-\frac{5a^3c^{13/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{54\sqrt{3}\,b^{8/3}}-\frac{5}{}$$

[Out] -5/108*a^2*c^3*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^2+1/36*a*c*(c*x)^(10/3)*(b*x^2+a)^(1/3)/b+1/6*(c*x)^(16/3)*(b*x^2+a)^(1/3)/c-5/108*a^3*c^(13/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(8/3)-5/162*a^3*c^(13/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(8/3)*3^(1/2)

Rubi [A]
time = 0.14, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 327, 335, 281, 337}

$$-\frac{5a^3c^{13/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{54\sqrt{3}\,b^{8/3}}-\frac{5a^3c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{108b^{8/3}}-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}$$

Antiderivative was successfully verified.

[In] Int[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]

[Out] (-5*a^2*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(108*b^2) + (a*c*(c*x)^(10/3)*(a + b*x^2)^(1/3))/(36*b) + ((c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*c) - (5*a^3*c^(13/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(54*Sqrt[3]*b^(8/3)) - (5*a^3*c^(13/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(108*b^(8/3))

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{1}{9}a\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx$$

$$= \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{(5a^2c^2)\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx}{54b}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{(5a^3c^4)\int}{8}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{(5a^3c^3)\,\mathrm{S}}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{(5a^3c^3)\,\mathrm{S}}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{(5a^3c^3)\,\mathrm{S}}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^{11/3}\right.}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{5a^3c^{13/3}\,\mathrm{l}}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{5a^3c^{13/3}\,\mathrm{l}}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{5a^3c^{13/3}\,\mathrm{t}}{}$$

Mathematica [A]
time = 1.79, size = 233, normalized size = 1.19

$$\frac{c^4\sqrt[3]{cx}\left(-15a^2b^{2/3}x^{4/3}\sqrt[3]{a+bx^2} + 9ab^{5/3}x^{10/3}\sqrt[3]{a+bx^2} + 54b^{8/3}x^{16/3}\sqrt[3]{a+bx^2} - 10\sqrt{3}\,a^3\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right) - 10a^3\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right) + 5a^3\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{324b^{8/3}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c^4*(c*x)^(1/3)*(-15*a^2*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 9*a*b^(5/3)*x
^(10/3)*(a + b*x^2)^(1/3) + 54*b^(8/3)*x^(16/3)*(a + b*x^2)^(1/3) - 10*Sqrt
[3]*a^3*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(
1/3))] - 10*a^3*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] + 5*a^3*Log[b^(
2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(32
4*b^(8/3)*x^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{13}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4961 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{13/3}\,{\left(b\,x^2+a\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(a + b*x^2)^(1/3),x)
[Out] int((c*x)^(13/3)*(a + b*x^2)^(1/3), x)
```

3.739 $\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=164

$$\frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{5/3}}+\frac{a^2c^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{5/3}}$$

```
[Out] 1/12*a*c*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b+1/4*(c*x)^(10/3)*(b*x^2+a)^(1/3)/c+1
/12*a^2*c^(7/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(5/3)+1/1
8*a^2*c^(7/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*
3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 327, 335, 281, 337}

$$\frac{a^2c^{7/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{5/3}}+\frac{a^2c^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{5/3}}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (a*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b) + ((c*x)^(10/3)*(a + b*x^2)^(1/3
))/(4*c) + (a^2*c^(7/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b
*x^2)^(1/3)))/Sqrt[3]])/(6*Sqrt[3]*b^(5/3)) + (a^2*c^(7/3)*Log[b^(1/3)*(c*x
)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(12*b^(5/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} + \frac{1}{6}a\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{(a^2c^2)\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx}{9b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{(a^2c)\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right.}{3b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{(a^2c)\,\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/}\right.}{6b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{(a^2c)\,\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right.}{6b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{\left(a^2c^{5/3}\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(c}{\sqrt[3]{a}}\right.}{18b^{4/3}}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} + \frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}} + \frac{(a^2}{}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} + \frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}} - \frac{a^2c}{}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} + \frac{a^2c^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{5/3}} + \frac{a}{}$$

Mathematica [A]
time = 1.15, size = 204, normalized size = 1.24

$$\frac{(cx)^{7/3}\left(3ab^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+9b^{5/3}x^{10/3}\sqrt[3]{a+bx^2}+2\sqrt{3}\,a^2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)+2a^2\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)-a^2\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{36b^{5/3}x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] ((c*x)^(7/3)*(3*a*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 9*b^(5/3)*x^(10/3)*(a
 + b*x^2)^(1/3) + 2*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x
^(2/3) + 2*(a + b*x^2)^(1/3))] + 2*a^2*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)
^(1/3)] - a^2*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a
+ b*x^2)^(2/3)]))/(36*b^(5/3)*x^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{7}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 21.16, size = 46, normalized size = 0.28

$$\frac{\sqrt[3]{a}\,c^{\frac{7}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(7/3)*x**(10/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(8/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/3}\,{\left(b\,x^2+a\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(a + b*x^2)^(1/3),x)
```

```
[Out] int((c*x)^(7/3)*(a + b*x^2)^(1/3), x)
```

3.740 $\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=133

$$\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}-\frac{a\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{2/3}}-\frac{a\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}$$

```
[Out] 1/2*(c*x)^(4/3)*(b*x^2+a)^(1/3)/c-1/4*a*c^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2
/3)*(b*x^2+a)^(1/3))/b^(2/3)-1/6*a*c^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2
/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {285, 335, 281, 337}

$$-\frac{a\sqrt[3]{c}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{2/3}}-\frac{a\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] ((c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*c) - (a*c^(1/3)*ArcTan[(1 + (2*b^(1/3)*(
c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(2*Sqrt[3]*b^(2/3)) - (a
*c^(1/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(4*b^(2/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{1}{3}a\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6\sqrt[3]{b}\,\sqrt[3]{c}} - \frac{a\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^2}{c}}\right)}{6\sqrt[3]{b}}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} - \frac{a\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx,\right)}{4\sqrt[3]{b}\,\sqrt[3]{c}}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c}\,\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\right)}{12b^{2/3}}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{2/3}} - \frac{a\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}}$$

Mathematica [A]
time = 0.81, size = 173, normalized size = 1.30

$$\frac{\sqrt[3]{cx}\left(6b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}-2\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)-2a\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)+a\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{12b^{2/3}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] ((c*x)^(1/3)*(6*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) - 2*Sqrt[3]*a*ArcTan[(Sqr
t[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] - 2*a*Log[-(
```

```
b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] + a*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(
2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(12*b^(2/3)*x^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{1}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
[Out] int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 0.77, size = 46, normalized size = 0.35

$$\frac{\sqrt[3]{a}\,\sqrt[3]{c}\,x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)*(b*x**2+a)**(1/3),x)
[Out] a**(1/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(5/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{1/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(a + b*x^2)^(1/3),x)
[Out] int((c*x)^(1/3)*(a + b*x^2)^(1/3), x)
```

3.741 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx$

Optimal. Leaf size=131

$$-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{5/3}}-\frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{5/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/c/(c*x)^(2/3)-3/4*b^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/
3)*(b*x^2+a)^(1/3))/c^(5/3)-1/2*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)
/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5/3)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {283, 335, 281, 337}

$$-\frac{\sqrt{3}\,\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2c^{5/3}}-\frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*c*(c*x)^(2/3)) - (Sqrt[3]*b^(1/3)*ArcTan[(1 + (2*
b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(2*c^(5/3)) - (
3*b^(1/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(4*c^(5/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{b\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx}{c^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{7/3}} - \frac{b^{2/3}\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x}{c^{4/3}}}\right.}{2c^{7/3}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} - \frac{(3b^{2/3})\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x,\right.}{4c^{7/3}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} + \frac{\sqrt[3]{b}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)}{\sqrt[3]{a+bx^2}}\right.}{4c^{5/3}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{5/3}} - \frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} +$$

Mathematica [A]

time = 0.59, size = 182, normalized size = 1.39

$$-\frac{x\left(6\sqrt[3]{a+bx^2}+2\sqrt{3}\sqrt[3]{b}x^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2/3}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a+bx^2}}\right)+2\sqrt[3]{b}x^{2/3}\log\left(-\sqrt[3]{b}x^{2/3}+\sqrt[3]{a+bx^2}\right)-\sqrt[3]{b}x^{2/3}\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{4(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
[Out] -1/4*(x*(6*(a + b*x^2)^(1/3) + 2*Sqrt[3]*b^(1/3)*x^(2/3)*ArcTan[(Sqrt[3]*b^
(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] + 2*b^(1/3)*x^(2/3)
```

```
*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] - b^(1/3)*x^(2/3)*Log[b^(2/3)*
x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(c*x)^(5
/3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 1.60, size = 49, normalized size = 0.37

$$\frac{\sqrt[3]{a}\,\Gamma\left(-\frac{1}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3}, -\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(5/3),x)
```

```
[Out] a**(1/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(5/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(5/3), x)
```

3.742 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{8/3}}$$

```
[Out] -3/8*(b*x^2+a)^(4/3)/a/c/(c*x)^(8/3)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {270}

$$-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(8/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{8/3}}$$

Mathematica [A]
time = 0.74, size = 26, normalized size = 0.93

$$-\frac{3x(a+bx^2)^{4/3}}{8a(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(4/3))/(8*a*(c*x)^(11/3))
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.75

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{8a\left(cx\right)^{\frac{11}{3}}}$	21
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{8c^{3}\left(cx\right)^{\frac{2}{3}}x^{2}a}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(11/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*x*(b*x^2+a)^(4/3)/a/(c*x)^(11/3)
```

Maxima [A]
time = 0.29, size = 35, normalized size = 1.25

$$-\frac{3\left(bc^{\frac{1}{3}}x^{3}+ac^{\frac{1}{3}}x\right)\left(bx^{2}+a\right)^{\frac{1}{3}}}{8\,ac^{4}x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="maxima")
```

```
[Out] -3/8*(b*c^(1/3)*x^3 + a*c^(1/3)*x)*(b*x^2 + a)^(1/3)/(a*c^4*x^(11/3))
```

Fricas [A]
time = 1.56, size = 25, normalized size = 0.89

$$-\frac{3\left(bx^{2}+a\right)^{\frac{4}{3}}\left(cx\right)^{\frac{1}{3}}}{8\,ac^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="fricas")
```

```
[Out] -3/8*(b*x^2 + a)^(4/3)*(c*x)^(1/3)/(a*c^4*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(24) = 48.
time = 39.42, size = 78, normalized size = 2.79

$$\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{4}{3}\right)}{2c^{\frac{11}{3}}x^{2}\Gamma\left(-\frac{1}{3}\right)}+\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{4}{3}\right)}{2ac^{\frac{11}{3}}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(11/3),x)
```

```
[Out] b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*c**(11/3)*x**2*gamma(-1/3))
 + b**(4/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*a*c**(11/3)*gamma(-1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(11/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{11/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(11/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(11/3), x)
```

3.743 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx$

Optimal. Leaf size=57

$$-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{14/3}}+\frac{9(a+bx^2)^{7/3}}{56a^2c(cx)^{14/3}}$$

```
[Out] -3/8*(b*x^2+a)^(4/3)/a/c/(c*x)^(14/3)+9/56*(b*x^2+a)^(7/3)/a^2/c/(c*x)^(14/
3)
```

Rubi [A]

time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 270}

$$\frac{9(a+bx^2)^{7/3}}{56a^2c(cx)^{14/3}}-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(7/3))/(56*a^2
*c*(c*x)^(14/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx &= -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{14/3}}-\frac{3\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx}{4a}\\ &= -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{14/3}}+\frac{9(a+bx^2)^{7/3}}{56a^2c(cx)^{14/3}}\end{aligned}$$

Mathematica [A]
time = 1.73, size = 46, normalized size = 0.81

$$-\frac{3x\sqrt[3]{a+bx^2}\,(4a^2+abx^2-3b^2x^4)}{56a^2(cx)^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
[Out] (-3*x*(a + b*x^2)^(1/3)*(4*a^2 + a*b*x^2 - 3*b^2*x^4))/(56*a^2*(c*x)^(17/3)
)
```

Maple [A]
time = 0.04, size = 31, normalized size = 0.54

method	result	size
gosper	$-\frac{3x(b\,x^2+a)^{\frac{4}{3}}(-3b\,x^2+4a)}{56a^2(cx)^{\frac{17}{3}}}$	31
risch	$-\frac{3(b\,x^2+a)^{\frac{1}{3}}(-3b^2x^4+ab\,x^2+4a^2)}{56c^5(cx)^{\frac{2}{3}}x^4a^2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(17/3),x,method=_RETURNVERBOSE)
[Out] -3/56*x*(b*x^2+a)^(4/3)*(-3*b*x^2+4*a)/a^2/(c*x)^(17/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

Fricas [A]
time = 0.86, size = 46, normalized size = 0.81

$$\frac{3\,(3\,b^2x^4-abx^2-4\,a^2)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{56\,a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="fricas")
```

```
[Out] 3/56*(3*b^2*x^4 - a*b*x^2 - 4*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^6*x
^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(17/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b x^2 + a)^{1/3}}{(c x)^{17/3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(17/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(17/3), x)
```

3.744 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{20/3}}+\frac{9(a+bx^2)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{27(a+bx^2)^{10/3}}{280a^3c(cx)^{20/3}}$$

```
[Out] -3/8*(b*x^2+a)^(4/3)/a/c/(c*x)^(20/3)+9/28*(b*x^2+a)^(7/3)/a^2/c/(c*x)^(20/
3)-27/280*(b*x^2+a)^(10/3)/a^3/c/(c*x)^(20/3)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$-\frac{27(a+bx^2)^{10/3}}{280a^3c(cx)^{20/3}}+\frac{9(a+bx^2)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(7/3))/(28*a^2
*c*(c*x)^(20/3)) - (27*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(20/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{20/3}} - \frac{3\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx}{2a}$$
$$= -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{20/3}} + \frac{9(a+bx^2)^{7/3}}{28a^2c(cx)^{20/3}} + \frac{9\int \frac{(a+bx^2)^{7/3}}{(cx)^{23/3}}\,dx}{14a^2}$$
$$= -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{20/3}} + \frac{9(a+bx^2)^{7/3}}{28a^2c(cx)^{20/3}} - \frac{27(a+bx^2)^{10/3}}{280a^3c(cx)^{20/3}}$$

Mathematica [A]
time = 2.46, size = 47, normalized size = 0.55

$$-\frac{3x(a+bx^2)^{4/3}(14a^2-12abx^2+9b^2x^4)}{280a^3(cx)^{23/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(4/3)*(14*a^2 - 12*a*b*x^2 + 9*b^2*x^4))/(280*a^3*(c*x)^(
23/3))
```

Maple [A]
time = 0.04, size = 42, normalized size = 0.49

method	result	size
gosper	$-\frac{3x(b\,x^2+a)^{\frac{4}{3}}(9b^2x^4-12ab\,x^2+14a^2)}{280a^3(cx)^{\frac{23}{3}}}$	42
risch	$-\frac{3(b\,x^2+a)^{\frac{1}{3}}(9b^3x^6-3a\,b^2x^4+2a^2b\,x^2+14a^3)}{280c^7(cx)^{\frac{2}{3}}x^6a^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(23/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/280*x*(b*x^2+a)^(4/3)*(9*b^2*x^4-12*a*b*x^2+14*a^2)/a^3/(c*x)^(23/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

Fricas [A]
time = 0.98, size = 57, normalized size = 0.67

$$-\frac{3\,(9\,b^3x^6-3\,ab^2x^4+2\,a^2bx^2+14\,a^3)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280\,a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="fricas")
```

```
[Out] -3/280*(9*b^3*x^6 - 3*a*b^2*x^4 + 2*a^2*b*x^2 + 14*a^3)*(b*x^2 + a)^(1/3)*(
c*x)^(1/3)/(a^3*c^8*x^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(23/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{23/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(23/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(23/3), x)
```

3.745 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx$

Optimal. Leaf size=113

$$-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{26/3}}+\frac{27(a+bx^2)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{81(a+bx^2)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{243(a+bx^2)^{13/3}}{3640a^4c(cx)^{26/3}}$$

```
[Out] -3/8*(b*x^2+a)^(4/3)/a/c/(c*x)^(26/3)+27/56*(b*x^2+a)^(7/3)/a^2/c/(c*x)^(26
/3)-81/280*(b*x^2+a)^(10/3)/a^3/c/(c*x)^(26/3)+243/3640*(b*x^2+a)^(13/3)/a^
4/c/(c*x)^(26/3)
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$\frac{243(a+bx^2)^{13/3}}{3640a^4c(cx)^{26/3}}-\frac{81(a+bx^2)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{27(a+bx^2)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(26/3)) + (27*(a + b*x^2)^(7/3))/(56*a^
2*c*(c*x)^(26/3)) - (81*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(26/3)) + (243
*(a + b*x^2)^(13/3))/(3640*a^4*c*(c*x)^(26/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{26/3}} - \frac{9\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx}{4a}$$
$$= -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{26/3}} + \frac{27(a+bx^2)^{7/3}}{56a^2c(cx)^{26/3}} + \frac{27\int \frac{(a+bx^2)^{7/3}}{(cx)^{29/3}}\,dx}{14a^2}$$
$$= -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{26/3}} + \frac{27(a+bx^2)^{7/3}}{56a^2c(cx)^{26/3}} - \frac{81(a+bx^2)^{10/3}}{280a^3c(cx)^{26/3}} - \frac{81\int \frac{(a+bx^2)^{10/3}}{(cx)^{29/3}}\,dx}{140a^3}$$
$$= -\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{26/3}} + \frac{27(a+bx^2)^{7/3}}{56a^2c(cx)^{26/3}} - \frac{81(a+bx^2)^{10/3}}{280a^3c(cx)^{26/3}} + \frac{243(a+bx^2)^{13/3}}{3640a^4c(cx)^{26/3}}$$

Mathematica [A]
time = 4.26, size = 58, normalized size = 0.51

$$-\frac{3x(a+bx^2)^{4/3}\left(140a^3-126a^2bx^2+108ab^2x^4-81b^3x^6\right)}{3640a^4(cx)^{29/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
[Out] (-3*x*(a + b*x^2)^(4/3)*(140*a^3 - 126*a^2*b*x^2 + 108*a*b^2*x^4 - 81*b^3*x
^6))/(3640*a^4*(c*x)^(29/3))
```

Maple [A]
time = 0.05, size = 53, normalized size = 0.47

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-81b^{3}x^{6}+108a\,b^{2}x^{4}-126a^{2}b\,x^{2}+140a^{3}\right)}{3640a^{4}\left(cx\right)^{\frac{29}{3}}}$	53
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-81b^{4}x^{8}+27a\,b^{3}x^{6}-18a^{2}b^{2}x^{4}+14a^{3}b\,x^{2}+140a^{4}\right)}{3640c^{9}\left(cx\right)^{\frac{2}{3}}x^{8}a^{4}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(29/3),x,method=_RETURNVERBOSE)
[Out] -3/3640*x*(b*x^2+a)^(4/3)*(-81*b^3*x^6+108*a*b^2*x^4-126*a^2*b*x^2+140*a^3)
/a^4/(c*x)^(29/3)
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.57

$$\frac{3\left(81\,b^4x^9-27\,ab^3x^7+18\,a^2b^2x^5-14\,a^3bx^3-140\,a^4x\right)(bx^2+a)^{\frac{1}{3}}}{3640\,a^4c^{\frac{29}{3}}x^{\frac{29}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="maxima")

[Out] 3/3640*(81*b^4*x^9 - 27*a*b^3*x^7 + 18*a^2*b^2*x^5 - 14*a^3*b*x^3 - 140*a^4*x)*(b*x^2 + a)^(1/3)/(a^4*c^(29/3)*x^(29/3))

Fricas [**A**]
time = 0.97, size = 68, normalized size = 0.60

$$\frac{3\left(81\,b^{4}x^{8}-27\,ab^{3}x^{6}+18\,a^{2}b^{2}x^{4}-14\,a^{3}bx^{2}-140\,a^{4}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}\left(cx\right)^{\frac{1}{3}}}{3640\,a^{4}c^{10}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="fricas")

[Out] 3/3640*(81*b^4*x^8 - 27*a*b^3*x^6 + 18*a^2*b^2*x^4 - 14*a^3*b*x^2 - 140*a^4)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^4*c^10*x^9)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(1/3)/(c*x)**(29/3),x)

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(29/3), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{29/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^(1/3)/(c*x)^(29/3),x)

[Out] int((a + b*x^2)^(1/3)/(c*x)^(29/3), x)

3.746 $\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=451

$$-\frac{14a^2c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b^2}+\frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b}+\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}+\frac{7a^2c^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)\cdots}{\sqrt[3]{a+b\cdots}}\right.\cdots}{13\cdots}$$

[Out] -14/135*a^2*c^3*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b^2+2/45*a*c*(c*x)^(7/3)*(b*x^2+a)^(1/3)/b+1/5*(c*x)^(13/3)*(b*x^2+a)^(1/3)/c+7/405*a^2*c^(7/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/b^2/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)

Rubi [A]
time = 0.68, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 327, 335, 247, 231}

$$\frac{7a^2c^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{14a^2c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b^2}+\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}+\frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b}$$

Antiderivative was successfully verified.

[In] Int[(c*x)^(10/3)*(a + b*x^2)^(1/3),x]

```
[Out] (-14*a^2*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(135*b^2) + (2*a*c*(c*x)^(7/3)*
(a + b*x^2)^(1/3))/(45*b) + ((c*x)^(13/3)*(a + b*x^2)^(1/3))/(5*c) + (7*a^2
*c^(7/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
+ b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 -
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(135*3^(1/4)*b
^2*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/
(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{1}{15}(2a)\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx$$

$$= \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} - \frac{(14a^2c^2)\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx}{135b}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{(14a^3c^4)}{}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{(14a^3c^3)}{}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{(14a^3c^3)}{}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{7a^2c^{7/3}\sqrt[3]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 103, normalized size = 0.23

$$\frac{c^3 \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left(\sqrt[3]{1+\frac{bx^2}{a}} \left(-7a^2+2abx^2+9b^2x^4\right)+7a^2 \,_2F_1\left(-\frac{1}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{45b^2 \sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(10/3)*(a + b*x^2)^(1/3),x]

[Out] (c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-7*a^2 + 2*a*b*x^2 + 9*b^2*x^4) + 7*a^2*Hypergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)]))/(45*b^2*(1 + (b*x^2)/a)^(1/3))

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{10}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)

[Out] int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)*c^3*x^3, x)

Sympy [C] Result contains complex when optimal does not.
time = 92.49, size = 46, normalized size = 0.10

$$\frac{\sqrt[3]{a}\,c^{\frac{10}{3}}x^{\frac{13}{3}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(10/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(10/3)*x**(13/3)*gamma(13/6)*hyper((-1/3, 13/6), (19/6,), b*x**
2*exp_polar(I*pi)/a)/(2*gamma(19/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,x\right)}^{10/3}\,{\left(b\,x^2+a\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)*(a + b*x^2)^(1/3),x)
```

```
[Out] int((c*x)^(10/3)*(a + b*x^2)^(1/3), x)
```

3.747 $\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=418

$$\frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}-\frac{a\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b\cdots}}{\sqrt[3]{a+b\cdots}}\right.}}\cdots}{9\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(\cdots\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}\cdots\right.}}}$$

```
[Out] 2/9*a*c*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b+1/3*(c*x)^(7/3)*(b*x^2+a)^(1/3)/c-1/2
7*a*c^(1/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2
+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^
(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)
*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-
3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2
+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/
(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/
3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/b/(-b^(1/3)*(c
*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 327, 335, 247, 231}

$$-\frac{a\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}+\frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (2*a*c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*b) + ((c*x)^(7/3)*(a + b*x^2)^(1/3
))/(3*c) - (a*c^(1/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x
^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1
 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^
(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - (
(1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(
9*3^(1/4)*b*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x
)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}+\frac{1}{9}(2a)\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx$$

$$= \frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}-\frac{(2a^2c^2)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{27b}$$

$$= \frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}-\frac{(2a^2c)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right.}{9b}$$

$$= \frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}-\frac{(2a^2c)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{}{\sqrt[6]{a}}\right.}{9b\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= \frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}-\frac{a\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/}}{\sqrt[3]{a+bx}}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 85, normalized size = 0.20

$$\frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left((a+bx^2)\sqrt[3]{1+\frac{bx^2}{a}}-a\,{}_2F_1\left(-\frac{1}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{3b\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(4/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/3) - a*Hyp
ergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)]))/(3*b*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{4}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)*c*x, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.73, size = 46, normalized size = 0.11

$$\frac{\sqrt[3]{a}\,c^{\frac{4}{3}}x^{\frac{7}{3}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(4/3)*x**(7/3)*gamma(7/6)*hyper((-1/3, 7/6), (13/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,x\right)}^{4/3}\,{\left(b\,x^2+a\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)*(a + b*x^2)^(1/3),x)
```

```
[Out] int((c*x)^(4/3)*(a + b*x^2)^(1/3), x)
```

3.748 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx$

Optimal. Leaf size=381

$$\frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c}+\frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\cdots}{c^{2/3}-\cdots}\right.\right.}{\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] (c*x)^(1/3)*(b*x^2+a)^(1/3)/c+1/3*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1
/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/
(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3)
)^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/
3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b
^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2
/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3
)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)
^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*
3^(3/4)/c^(5/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a
)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)
^(1/3))^2)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {285, 335, 247, 231}

$$\frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(2/3),x]
```

```
[Out] ((c*x)^(1/3)*(a + b*x^2)^(1/3))/c + ((c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3)
 - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^
(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))
/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*Ellip
ticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3
))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 +
 Sqrt[3])/4])/(3^(1/4)*c^(5/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1
/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sq
rt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx = \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c} + \frac{1}{3}(2a)\int \frac{1}{(cx)^{2/3}\,(a+bx^2)^{2/3}}\,dx$$

$$= \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c} + \frac{(2a)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c} + \frac{(2a)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{c\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c} + \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/\cdots}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\cdots}}}{\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\cdots\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1\cdots}{\cdots}\right)\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.14

$$\frac{3x\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)}{(cx)^{2/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)])/((c
*x)^(2/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(2/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.62, size = 46, normalized size = 0.12

$$\frac{\sqrt[3]{a}\,\sqrt[3]{x}\,\Gamma\left(\frac{1}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(2/3),x)
```

```
[Out] a**(1/3)*x**(1/3)*gamma(1/6)*hyper((-1/3, 1/6), (7/6,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(2/3)*gamma(7/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(2/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(2/3), x)
```

3.749 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx$

Optimal. Leaf size=391

$$-\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}+\frac{3^{3/4}b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/\cdots}}{c^{2/\cdots}}\right.\right.}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -3/5*(b*x^2+a)^(1/3)/c/(c*x)^(5/3)+1/5*3^(3/4)*b*(c*x)^(1/3)*(b*x^2+a)^(1/3
)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/
3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/
(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a
)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*Elliptic
F((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b
^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^
(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^
(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(
1/3))^2)^(1/2)/a/c^(11/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3
)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))
/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {283, 335, 247, 231}

$$\frac{3^{3/4}b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(8/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(5*c*(c*x)^(5/3)) + (3^(3/4)*b*(c*x)^(1/3)*(a + b*x^
2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3)
+ (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(
a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2
)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x
^2)^(1/3))], (2 + Sqrt[3])/4])/(5*a*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c
^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2
/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}} + \frac{(2b)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{5c^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}} + \frac{(6b)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{5c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}} + \frac{(6b)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{5c^3\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}} + \frac{3^{3/4}b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}}{\sqrt[3]{a+}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2}}{\sqrt[3]{a+bx^2}}\right.}}}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.14

$$-\frac{3x\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{5}{6},-\frac{1}{3};\frac{1}{6};-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(8/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-5/6, -1/3, 1/6, -((b*x^2)/a)])/(
5*(c*x)^(8/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(8/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(8/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c^3*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.61, size = 32, normalized size = 0.08

$$-\frac{\sqrt[3]{b}\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{8}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(8/3),x)
```

```
[Out] -b**(1/3)*hyper((-1/3, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(c**(8/3)*
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(8/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(8/3), x)
```

3.750 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx$

Optimal. Leaf size=422

$$-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}-\frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}}-\frac{3\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^2c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -3/11*(b*x^2+a)^(1/3)/c/(c*x)^(11/3)-6/55*b*(b*x^2+a)^(1/3)/a/c^3/(c*x)^(5/
3)-3/55*3^(3/4)*b^2*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3
)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3
))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*
x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(
2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2)
)/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x
)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/
3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a^2/c^(17/3)/(
-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a
)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {283, 331, 335, 247, 231}

$$-\frac{3\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^2c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(14/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(11*c*(c*x)^(11/3)) - (6*b*(a + b*x^2)^(1/3))/(55*a*
c^3*(c*x)^(5/3)) - (3*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/
3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c
^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*Elliptic
F[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/
(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sq
rt[3])/4])/(55*a^2*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} + \frac{(2b)\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx}{11c^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} - \frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{(6b^2)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{55ac^4}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} - \frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{(18b^2)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{55ac^5}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} - \frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{(18b^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{55ac^5\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}} - \frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{3\;3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^2 \cdots}{(a \cdots}}{\left(c^{2/3}- \cdots\right.}}\cdots}{55a^2c^{17/3}\sqrt{-\frac{\cdots}{\sqrt[3]{a \cdots}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 56, normalized size = 0.13

$$-\frac{3x\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{11}{6},-\frac{1}{3};-\frac{5}{6};-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(14/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-11/6, -1/3, -5/6, -((b*x^2)/a)])
/(11*(c*x)^(14/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c^5*x^5), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(14/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3278 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{14/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(14/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(14/3), x)
```

3.751 $\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}$$

```
[Out] 3/5*(c*x)^(5/3)*(b*x^2+a)^(1/3)*hypergeom([-1/3, 5/6],[11/6],-b*x^2/a)/c/(1
+b*x^2/a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 5/6, 11/6, -((b*x^
2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx = \frac{\sqrt[3]{a+bx^2}\int (cx)^{2/3}\sqrt[3]{1+\frac{bx^2}{a}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}$$
$$= \frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}$$

Mathematica [A]
time = 10.02, size = 56, normalized size = 0.97

$$\frac{3x(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2/3)*(a + b*x^2)^(1/3),x]
[Out] (3*x*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 5/6, 11/6, -((b*
x^2)/a)])/(5*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{2}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)
[Out] int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 46, normalized size = 0.79

$$\frac{\sqrt[3]{a}\,c^{\frac{2}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(2/3)*x**(5/3)*gamma(5/6)*hyper((-1/3, 5/6), (11/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,x\right)}^{2/3}\,{\left(b\,x^2+a\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(a + b*x^2)^(1/3),x)
```

```
[Out] int((c*x)^(2/3)*(a + b*x^2)^(1/3), x)
```

3.752 $\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}$$

```
[Out] 3/2*(c*x)^(2/3)*(b*x^2+a)^(1/3)*hypergeom([-1/3, 1/3],[4/3],-b*x^2/a)/c/(1+
b*x^2/a)^(1/3)
```

Rubi [A]

time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(1/3),x]
```

```
[Out] (3*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^2
)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx = \frac{\sqrt[3]{a+bx^2}\int \frac{\sqrt[3]{1+\frac{bx^2}{a}}}{\sqrt[3]{cx}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}$$
$$= \frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}$$

Mathematica [A]
time = 10.01, size = 56, normalized size = 0.97

$$\frac{3x\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^2)/a)])/(2*
(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(1/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="maxima")
```

[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(c*x), x)

Sympy [C] Result contains complex when optimal does not.
time = 0.58, size = 46, normalized size = 0.79

$$\frac{\sqrt[3]{a}\,x^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[3]{c}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(1/3)/(c*x)**(1/3),x)

[Out] a**(1/3)*x**(2/3)*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**2*exp_polar(I*pi)/a)/(2*c**(1/3)*gamma(4/3))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^(1/3)/(c*x)^(1/3),x)

[Out] int((a + b*x^2)^(1/3)/(c*x)^(1/3), x)

3.753 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\;\sqrt[3]{1+\frac{bx^2}{a}}}$$

```
[Out] -3*(b*x^2+a)^(1/3)*hypergeom([-1/3, -1/6],[5/6],-b*x^2/a)/c/(c*x)^(1/3)/(1+
b*x^2/a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$-\frac{3\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\;\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -((b*x^2)/a)])/(c*
(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx = \frac{\sqrt[3]{a+bx^2}\int \frac{\sqrt[3]{1+\frac{bx^2}{a}}}{(cx)^{4/3}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}$$
$$= -\frac{3\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\ \sqrt[3]{1+\frac{bx^2}{a}}}$$

Mathematica [A]
time = 10.02, size = 54, normalized size = 0.96

$$-\frac{3x\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{(cx)^{4/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -((b*x^2)/a)])/(
(c*x)^(4/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(4/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(c^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.02, size = 49, normalized size = 0.88

$$\frac{\sqrt[3]{a}\,\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}}\sqrt[3]{x}\,\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(4/3),x)
[Out] a**(1/3)*gamma(-1/6)*hyper((-1/3, -1/6), (5/6,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(4/3)*x**(1/3)*gamma(5/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(4/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(4/3), x)
```

3.754 $\int (cx)^{13/3} (a+bx^2)^{4/3} \, dx$

Optimal. Leaf size=223

$$-\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}(a+bx^2)^{4/3}}{8c}-\frac{5a^4c^{13/3}\tan^{-1}}{16}$$

```
[Out] -5/324*a^3*c^3*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^2+1/108*a^2*c*(c*x)^(10/3)*(b*
x^2+a)^(1/3)/b+1/18*a*(c*x)^(16/3)*(b*x^2+a)^(1/3)/c+1/8*(c*x)^(16/3)*(b*x^
2+a)^(4/3)/c-5/324*a^4*c^(13/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1
/3))/b^(8/3)-5/486*a^4*c^(13/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)
/(b*x^2+a)^(1/3))*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 327, 335, 281, 337}

$$-\frac{5a^4c^{13/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{162\sqrt{3}\,b^{8/3}}-\frac{5a^4c^{13/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{324b^{8/3}}-\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{(cx)^{16/3}(a+bx^2)^{4/3}}{8c}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (-5*a^3*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(324*b^2) + (a^2*c*(c*x)^(10/3)*
(a + b*x^2)^(1/3))/(108*b) + (a*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(18*c) + ((
c*x)^(16/3)*(a + b*x^2)^(4/3))/(8*c) - (5*a^4*c^(13/3)*ArcTan[(1 + (2*b^(1/
3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(162*Sqrt[3]*b^(8/3)
) - (5*a^4*c^(13/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(
324*b^(8/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{(cx)^{16/3} \left(a+bx^2\right)^{4/3}}{8c} + \frac{1}{3} a \int (cx)^{13/3} \sqrt[3]{a+bx^2} \, dx$$

$$= \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3} \left(a+bx^2\right)^{4/3}}{8c} + \frac{1}{27} a^2 \int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}} \, dx$$

$$= \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3} \left(a+bx^2\right)^{4/3}}{8c} - \frac{(5a^3c^2) \int}{}$$

$$= -\frac{5a^3c^3(cx)^{4/3} \sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)}{}$$

$$= -\frac{5a^3c^3(cx)^{4/3} \sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)}{}$$

$$= -\frac{5a^3c^3(cx)^{4/3} \sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)}{}$$

$$= -\frac{5a^3c^3(cx)^{4/3} \sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)}{}$$

$$= -\frac{5a^3c^3(cx)^{4/3} \sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)}{}$$

$$= -\frac{5a^3c^3(cx)^{4/3} \sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)}{}$$

$$= -\frac{5a^3c^3(cx)^{4/3} \sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)}{}$$

$$= -\frac{5a^3c^3(cx)^{4/3} \sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)}{}$$

$$= -\frac{5a^3c^3(cx)^{4/3} \sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2 c(cx)^{10/3} \sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)}{}$$

Mathematica [A]
time = 2.40, size = 259, normalized size = 1.16

$$\frac{c^4 \sqrt[3]{cx} \left(-30a^3b^{2/3}x^{4/3}\sqrt[3]{a+bx^2} + 18a^2b^{5/3}x^{10/3}\sqrt[3]{a+bx^2} + 351ab^{8/3}x^{16/3}\sqrt[3]{a+bx^2} + 243b^{11/3}x^{22/3}\sqrt[3]{a+bx^2} - 20\sqrt{3}\,a^4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right) - 20a^4\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right) + 10a^4\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)\right)}{1944b^{8/3}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c^4*(c*x)^(1/3)*(-30*a^3*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 18*a^2*b^(5/3
)*x^(10/3)*(a + b*x^2)^(1/3) + 351*a*b^(8/3)*x^(16/3)*(a + b*x^2)^(1/3) + 2
43*b^(11/3)*x^(22/3)*(a + b*x^2)^(1/3) - 20*Sqrt[3]*a^4*ArcTan[(Sqrt[3]*b^(
1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] - 20*a^4*Log[-(b^(1/
3)*x^(2/3)) + (a + b*x^2)^(1/3)] + 10*a^4*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(
2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(1944*b^(8/3)*x^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{13}{3}} \left(b\,x^2 + a\right)^{\frac{4}{3}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)*(b*x**2+a)**(4/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6546 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{13/3}\,{\left(b\,x^2+a\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(a + b*x^2)^(4/3),x)
[Out] int((c*x)^(13/3)*(a + b*x^2)^(4/3), x)
```

3.755 $\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=192

$$\frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{2a^3c^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{27\sqrt{3}\,b^{5/3}}+\frac{a^3c^7}{}$$

```
[Out] 1/27*a^2*c*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b+1/9*a*(c*x)^(10/3)*(b*x^2+a)^(1/3)
/c+1/6*(c*x)^(10/3)*(b*x^2+a)^(4/3)/c+1/27*a^3*c^(7/3)*ln(b^(1/3)*(c*x)^(2/
3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(5/3)+2/81*a^3*c^(7/3)*arctan(1/3*(1+2*b^(1/3
)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 327, 335, 281, 337}

$$\frac{2a^3c^{7/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{27\sqrt{3}\,b^{5/3}}+\frac{a^3c^{7/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27b^{5/3}}+\frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (a^2*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b) + (a*(c*x)^(10/3)*(a + b*x^2)^
(1/3))/(9*c) + ((c*x)^(10/3)*(a + b*x^2)^(4/3))/(6*c) + (2*a^3*c^(7/3)*ArcT
an[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(27*
Sqrt[3]*b^(5/3)) + (a^3*c^(7/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^
2)^(1/3)])/(27*b^(5/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (cx)^{7/3}\left(a+bx^2\right)^{4/3}\,dx = \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{1}{9}(4a)\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{1}{27}\left(2a^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(4a^3c^2\right)\int \cdots}{\cdots}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(4a^3c\right)\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(2a^3c\right)\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(2a^3c\right)\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(2a^3c^{5/3}\right)\cdots}{\cdots}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{2a^3c^{7/3}\,\mathrm{lo}\cdots}{\cdots}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{2a^3c^{7/3}\,\mathrm{lo}\cdots}{\cdots}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{2a^3c^{7/3}\,\mathrm{ta}\cdots}{\cdots}$$

Mathematica [A]
time = 1.37, size = 230, normalized size = 1.20

$$\frac{(cx)^{7/3}\left(6a^2b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+45ab^{5/3}x^{10/3}\sqrt[3]{a+bx^2}+27b^{8/3}x^{16/3}\sqrt[3]{a+bx^2}+4\sqrt{3}\,a^3\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)+4a^3\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)-2a^3\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)\right)}{162b^{5/3}x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] ((c*x)^(7/3)*(6*a^2*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 45*a*b^(5/3)*x^(10/
3)*(a + b*x^2)^(1/3) + 27*b^(8/3)*x^(16/3)*(a + b*x^2)^(1/3) + 4*Sqrt[3]*a^
3*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))]
 + 4*a^3*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] - 2*a^3*Log[b^(2/3)*x^
(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(162*b^(5/
3)*x^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{7}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 45.09, size = 46, normalized size = 0.24

$$\frac{a^{\frac{4}{3}} c^{\frac{7}{3}} x^{\frac{10}{3}} \Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{2 \Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(7/3)*x**(10/3)*gamma(5/3)*hyper((-4/3, 5/3), (8/3,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/3}\,{\left(b\,x^2+a\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(a + b*x^2)^(4/3),x)
```

```
[Out] int((c*x)^(7/3)*(a + b*x^2)^(4/3), x)
```

3.756 $\int \sqrt[3]{cx}\,(a+bx^2)^{4/3}\,dx$

Optimal. Leaf size=163

$$\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}-\frac{a^2\sqrt[3]{c}\,\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{\right.}}{6b^{2/3}}$$

```
[Out] 1/3*a*(c*x)^(4/3)*(b*x^2+a)^(1/3)/c+1/4*(c*x)^(4/3)*(b*x^2+a)^(4/3)/c-1/6*a
^2*c^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(2/3)-1/9*a^2*
c^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2
))/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {285, 335, 281, 337}

$$-\frac{a^2\sqrt[3]{c}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}-\frac{a^2\sqrt[3]{c}\,\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{6b^{2/3}}+\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (a*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(3*c) + ((c*x)^(4/3)*(a + b*x^2)^(4/3))/(
4*c) - (a^2*c^(1/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2
)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) - (a^2*c^(1/3)*Log[b^(1/3)*(c*x)^(2
/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(6*b^(2/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{cx}\,\left(a+bx^2\right)^{4/3}\,dx &= \frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{1}{3}(2a)\int \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,dx \\
&= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{1}{9}\left(2a^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx \\
&= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right.}{3c} \\
&= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx,x,(cx)^{2/3}\right)}{3c} \\
&= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c} \\
&= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right.}{9\sqrt[3]{b}\sqrt[3]{c}} \\
&= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}-\frac{a^2 S\ldots}{} \\
&= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}+\frac{a^2\ldots}{} \\
&= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}-\ldots
\end{aligned}$$

Mathematica [A]
time = 0.95, size = 204, normalized size = 1.25

$$\frac{\sqrt[3]{cx}\left(21ab^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+9b^{5/3}x^{10/3}\sqrt[3]{a+bx^2}-4\sqrt{3}\,a^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)-4a^2\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)+2a^2\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)\right)}{36b^{2/3}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] ((c*x)^(1/3)*(21*a*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 9*b^(5/3)*x^(10/3)*(
a + b*x^2)^(1/3) - 4*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*
x^(2/3) + 2*(a + b*x^2)^(1/3))] - 4*a^2*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2
)^(1/3)] + 2*a^2*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3)]))/(36*b^(2/3)*x^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{1}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 4.00, size = 46, normalized size = 0.28

$$\frac{a^{\frac{4}{3}}\sqrt[3]{c}\,x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{4}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{1/3}\,\left(b\,x^2+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(a + b*x^2)^(4/3),x)
```

```
[Out] int((c*x)^(1/3)*(a + b*x^2)^(4/3), x)
```

3.757 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx$

Optimal. Leaf size=153

$$\frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3}-\frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}}-\frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}}-\frac{a\sqrt[3]{b}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{c^{5/3}}$$

```
[Out] 2*b*(c*x)^(4/3)*(b*x^2+a)^(1/3)/c^3-3/2*(b*x^2+a)^(4/3)/c/(c*x)^(2/3)-a*b^(
1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/c^(5/3)-2/3*a*b^(1/3)*
arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/c^(5/
3)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {283, 285, 335, 281, 337}

$$-\frac{2a\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}}-\frac{a\sqrt[3]{b}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{c^{5/3}}+\frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3}-\frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

```
[Out] (2*b*(c*x)^(4/3)*(a + b*x^2)^(1/3))/c^3 - (3*(a + b*x^2)^(4/3))/(2*c*(c*x)^
(2/3)) - (2*a*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x
^2)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)) - (a*b^(1/3)*Log[b^(1/3)*(c*x)^(2/3
) - c^(2/3)*(a + b*x^2)^(1/3)])/c^(5/3)
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} + \frac{(4b)\int \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,dx}{c^2}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} + \frac{(4ab)\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx}{3c^2}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} + \frac{(4ab)\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} + \frac{(2ab)\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} + \frac{(2ab)\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} + \frac{(2ab^{2/3})\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{7/3}}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}} - \frac{(ab^{2/3})\,\mathrm{Subs}}{}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}} + \frac{a\sqrt[3]{b}\,\log\left(c\right.}{}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}} - \frac{2a\sqrt[3]{b}\,\log}{}$$

Mathematica [A]

time = 0.74, size = 203, normalized size = 1.33

$$\frac{x\left(-9a\sqrt[3]{a+bx^2}+3bx^2\sqrt[3]{a+bx^2}-4\sqrt{3}\,a\sqrt[3]{b}\,x^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)-4a\sqrt[3]{b}\,x^{2/3}\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)+2a\sqrt[3]{b}\,x^{2/3}\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{6(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

```
[Out] (x*(-9*a*(a + b*x^2)^(1/3) + 3*b*x^2*(a + b*x^2)^(1/3) - 4*Sqrt[3]*a*b^(1/3
)*x^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)
^(1/3))] - 4*a*b^(1/3)*x^(2/3)*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)]
+ 2*a*b^(1/3)*x^(2/3)*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/
3) + (a + b*x^2)^(2/3)]))/(6*(c*x)^(5/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 5.03, size = 49, normalized size = 0.32

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(5/3),x)
```

```
[Out] a**(4/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{\left(c\,x\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(5/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(5/3), x)
```

3.758 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx$

Optimal. Leaf size=157

$$-\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}}-\frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}}-\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{11/3}}-\frac{3b^{4/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{11/3}}$$

```
[Out] -3/2*b*(b*x^2+a)^(1/3)/c^3/(c*x)^(2/3)-3/8*(b*x^2+a)^(4/3)/c/(c*x)^(8/3)-3/
4*b^(4/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/c^(11/3)-1/2*b^(4
/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))*3
^(1/2)/c^(11/3)
```

Rubi [A]

time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {283, 335, 281, 337}

$$-\frac{\sqrt{3}\,b^{4/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2c^{11/3}}-\frac{3b^{4/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{11/3}}-\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}}-\frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*b*(a + b*x^2)^(1/3))/(2*c^3*(c*x)^(2/3)) - (3*(a + b*x^2)^(4/3))/(8*c*(
c*x)^(8/3)) - (Sqrt[3]*b^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)
*(a + b*x^2)^(1/3)))/Sqrt[3]])/(2*c^(11/3)) - (3*b^(4/3)*Log[b^(1/3)*(c*x)^
(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(4*c^(11/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{b\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx}{c^2}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{b^2\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx}{c^4}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{(3b^2)\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{(3b^2)\,\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{(3b^2)\,\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} + \frac{b^{5/3}\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{13/3}} - \frac{b^{5/3}\mathrm{Subs}}{}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} - \frac{(3b^{5/3})\,\mathrm{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} + \frac{b^{4/3}\log\left(c^{4/3}+\frac{b^{2/3}(c}{(a+bx^2}\right.}{4c^1}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} - \frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{11/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\right.}{2c^1}$$

<u>Mathematica</u> [A]
time = 0.78, size = 200, normalized size = 1.27

$$-\frac{x\left(3a\sqrt[3]{a+bx^2}+15bx^2\sqrt[3]{a+bx^2}+4\sqrt{3}\,b^{4/3}x^{8/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)+4b^{4/3}x^{8/3}\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)-2b^{4/3}x^{8/3}\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{8(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

```
[Out] -1/8*(x*(3*a*(a + b*x^2)^(1/3) + 15*b*x^2*(a + b*x^2)^(1/3) + 4*Sqrt[3]*b^(
4/3)*x^(8/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x
^2)^(1/3))] + 4*b^(4/3)*x^(8/3)*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)]
 - 2*b^(4/3)*x^(8/3)*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3
) + (a + b*x^2)^(2/3)]))/(c*x)^(11/3)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{\left(c x\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**C**] Result contains complex when optimal does not.
time = 39.35, size = 53, normalized size = 0.34

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{4}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{11}{3}}x^{\frac{8}{3}}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(11/3),x)
```

```
[Out] a**(4/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(11/3)*x**(8/3)*gamma(-1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{11/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(11/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(11/3), x)
```

3.759 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}} dx$

Optimal. Leaf size=28

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

```
[Out] -3/14*(b*x^2+a)^(7/3)/a/c/(c*x)^(14/3)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {270}

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(14/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}} dx = -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

Mathematica [A]
time = 1.97, size = 26, normalized size = 0.93

$$-\frac{3x(a+bx^2)^{7/3}}{14a(cx)^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(7/3))/(14*a*(c*x)^(17/3))
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.75

method	result	size
gosper	$-\frac{3x(bx^2+a)^{\frac{7}{3}}}{14a(cx)^{\frac{17}{3}}}$	21
risch	$-\frac{3(bx^2+a)^{\frac{1}{3}}(b^2x^4+2abx^2+a^2)}{14c^5(cx)^{\frac{2}{3}}x^4a}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(17/3),x,method=_RETURNVERBOSE)
[Out] -3/14*x*(b*x^2+a)^(7/3)/a/(c*x)^(17/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

Fricas [A]
time = 1.38, size = 43, normalized size = 1.54

$$-\frac{3\,(b^2x^4+2\,abx^2+a^2)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{14\,ac^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="fricas")
[Out] -3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^6*x^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(17/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{17/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(17/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(17/3), x)
```

3.760 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx$

Optimal. Leaf size=57

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}+\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}$$

```
[Out] -3/14*(b*x^2+a)^(7/3)/a/c/(c*x)^(20/3)+9/140*(b*x^2+a)^(10/3)/a^2/c/(c*x)^(
20/3)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(10/3))/(140*
a^2*c*(c*x)^(20/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx &= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}-\frac{3\int \frac{(a+bx^2)^{7/3}}{(cx)^{23/3}}\,dx}{7a}\\ &= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}+\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}\end{aligned}$$

Mathematica [A]
time = 2.63, size = 36, normalized size = 0.63

$$-\frac{3x(7a-3bx^2)\,(a+bx^2)^{7/3}}{140a^2(cx)^{23/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
[Out] (-3*x*(7*a - 3*b*x^2)*(a + b*x^2)^(7/3))/(140*a^2*(c*x)^(23/3))
```

Maple [A]
time = 0.05, size = 31, normalized size = 0.54

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(-3b\,x^{2}+7a\right)}{140a^{2}\left(cx\right)^{\frac{23}{3}}}$	31
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-3b^{3}x^{6}+a\,b^{2}x^{4}+11a^{2}b\,x^{2}+7a^{3}\right)}{140c^{7}\left(cx\right)^{\frac{2}{3}}x^{6}a^{2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(23/3),x,method=_RETURNVERBOSE)
[Out] -3/140*x*(b*x^2+a)^(7/3)*(-3*b*x^2+7*a)/a^2/(c*x)^(23/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

Fricas [A]
time = 0.95, size = 57, normalized size = 1.00

$$\frac{3\,(3\,b^3x^6-ab^2x^4-11\,a^2bx^2-7\,a^3)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{140\,a^2c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="fricas")
[Out] 3/140*(3*b^3*x^6 - a*b^2*x^4 - 11*a^2*b*x^2 - 7*a^3)*(b*x^2 + a)^(1/3)*(c*x
)^(1/3)/(a^2*c^8*x^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(23/3),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{23/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(23/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(23/3), x)
```

3.761 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}}+\frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{27(a+bx^2)^{13/3}}{910a^3c(cx)^{26/3}}$$

```
[Out] -3/14*(b*x^2+a)^(7/3)/a/c/(c*x)^(26/3)+9/70*(b*x^2+a)^(10/3)/a^2/c/(c*x)^(2
6/3)-27/910*(b*x^2+a)^(13/3)/a^3/c/(c*x)^(26/3)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 270}

$$-\frac{27(a+bx^2)^{13/3}}{910a^3c(cx)^{26/3}}+\frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(26/3)) + (9*(a + b*x^2)^(10/3))/(70*a
^2*c*(c*x)^(26/3)) - (27*(a + b*x^2)^(13/3))/(910*a^3*c*(c*x)^(26/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx = -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}} - \frac{6\int \frac{(a+bx^2)^{7/3}}{(cx)^{29/3}}\,dx}{7a}$$
$$= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}} + \frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}} + \frac{9\int \frac{(a+bx^2)^{10/3}}{(cx)^{29/3}}\,dx}{35a^2}$$
$$= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}} + \frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}} - \frac{27(a+bx^2)^{13/3}}{910a^3c(cx)^{26/3}}$$

Mathematica [A]
time = 3.82, size = 47, normalized size = 0.55

$$-\frac{3x(a+bx^2)^{7/3}(35a^2-21abx^2+9b^2x^4)}{910a^3(cx)^{29/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(7/3)*(35*a^2 - 21*a*b*x^2 + 9*b^2*x^4))/(910*a^3*(c*x)^(
29/3))
```

Maple [A]
time = 0.05, size = 42, normalized size = 0.49

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(9b^{2}x^{4}-21ab\,x^{2}+35a^{2}\right)}{910a^{3}\left(cx\right)^{\frac{29}{3}}}$	42
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(9b^{4}x^{8}-3a\,b^{3}x^{6}+2a^{2}b^{2}x^{4}+49a^{3}b\,x^{2}+35a^{4}\right)}{910c^{9}\left(cx\right)^{\frac{2}{3}}x^{8}a^{3}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(29/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/910*x*(b*x^2+a)^(7/3)*(9*b^2*x^4-21*a*b*x^2+35*a^2)/a^3/(c*x)^(29/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

Fricas [A]
time = 1.02, size = 68, normalized size = 0.80

$$-\frac{3\,(9\,b^4x^8-3\,ab^3x^6+2\,a^2b^2x^4+49\,a^3bx^2+35\,a^4)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{910\,a^3c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="fricas")
```

```
[Out] -3/910*(9*b^4*x^8 - 3*a*b^3*x^6 + 2*a^2*b^2*x^4 + 49*a^3*b*x^2 + 35*a^4)*(b
*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c^10*x^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(29/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{29/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(29/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(29/3), x)
```

3.762 $\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=479

$$-\frac{16a^3c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}+\frac{8a^3c^{7/3}\sqrt[3]{cx}}{\cdots}$$

```
[Out] -16/405*a^3*c^3*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b^2+16/945*a^2*c*(c*x)^(7/3)*(b
*x^2+a)^(1/3)/b+8/105*a*(c*x)^(13/3)*(b*x^2+a)^(1/3)/c+1/7*(c*x)^(13/3)*(b*
x^2+a)^(4/3)/c+8/1215*a^3*c^(7/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1
/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/
(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3)
)^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/
3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b
^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2
/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3
)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)
^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*
3^(3/4)/b^2/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1
/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/
3))^2)^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 327, 335, 247, 231}

$$\frac{8a^3c^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{405\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{16a^3c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(10/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (-16*a^3*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(405*b^2) + (16*a^2*c*(c*x)^(7/
3)*(a + b*x^2)^(1/3))/(945*b) + (8*a*(c*x)^(13/3)*(a + b*x^2)^(1/3))/(105*c
) + ((c*x)^(13/3)*(a + b*x^2)^(4/3))/(7*c) + (8*a^3*c^(7/3)*(c*x)^(1/3)*(a
+ b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^
(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^
(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(405*3^(1/4)*b^2*Sqrt[-((b^(1/3)*(c*x)
^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1
/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{10/3}\left(a+bx^2\right)^{4/3}\,dx = \frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}+\frac{1}{21}(8a)\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}+\frac{1}{315}\left(16a^2\right)\int \frac{(cx)^{10/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}-\frac{(16a^3}{}$$

$$= -\frac{16a^3c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(c}{}$$

$$= -\frac{16a^3c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(c}{}$$

$$= -\frac{16a^3c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(c}{}$$

$$= -\frac{16a^3c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(c}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 102, normalized size = 0.21

$$\frac{c^3 \sqrt[3]{cx}\, \sqrt[3]{a+bx^2}\left(-\left((7a-15bx^2)\,(a+bx^2)^2\sqrt[3]{1+\frac{bx^2}{a}}\right)+7a^3\,{}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{105b^2\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(10/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(-((7*a - 15*b*x^2)*(a + b*x^2)^2*(1 + (
b*x^2)/a)^(1/3)) + 7*a^3*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)]))/
(105*b^2*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{10}{3}} \left(b\,x^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*c^3*x^5 + a*c^3*x^3)*(b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(10/3)*(b*x**2+a)**(4/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{10/3}\,\left(b\,x^2+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)*(a + b*x^2)^(4/3),x)
[Out] int((c*x)^(10/3)*(a + b*x^2)^(4/3), x)
```

3.763 $\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=448

$$8a^2\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx}}\right.$$

$$\frac{16a^2c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}-$$

13

```
[Out] 16/135*a^2*c*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b+8/45*a*(c*x)^(7/3)*(b*x^2+a)^(1/
3)/c+1/5*(c*x)^(7/3)*(b*x^2+a)^(4/3)/c-8/405*a^2*c^(1/3)*(c*x)^(1/3)*(b*x^2
+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(
c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3
^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/
(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*
EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c
^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3
)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*
x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/b/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*
x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3
^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {285, 327, 335, 247, 231}

$$-\frac{8a^2\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{16a^2c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (16*a^2*c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(135*b) + (8*a*(c*x)^(7/3)*(a + b*
x^2)^(1/3))/(45*c) + ((c*x)^(7/3)*(a + b*x^2)^(4/3))/(5*c) - (8*a^2*c^(1/3)
*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*
c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3]
)*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(135*3^(1/4)*b*Sqrt[-(
(b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/(
(a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2
)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{4/3}\left(a+bx^2\right)^{4/3}\,dx = \frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}+\frac{1}{15}(8a)\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx$$

$$=\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}+\frac{1}{135}\left(16a^2\right)\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$=\frac{16a^2c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}-\frac{\left(16a^3c^2\right)}{}$$

$$=\frac{16a^2c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}-\frac{\left(16a^3c\right)}{}$$

$$=\frac{16a^2c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}-\frac{\left(16a^3c\right)}{}$$

$$8a^2\sqrt[3]{c}$$

$$=\frac{16a^2c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b}+\frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c}+\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 89, normalized size = 0.20

$$\frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt[3]{1+\frac{bx^2}{a}}-a^2\,{}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{5b\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(4/3)*(a + b*x^2)^(4/3),x]

[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((a + b*x^2)^2*(1 + (b*x^2)/a)^(1/3) - a^2*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)]))/(5*b*(1 + (b*x^2)/a)^(1/3))

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{4}{3}} \left(b\,x^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)

[Out] int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(4/3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")

[Out] integral((b*c*x^3 + a*c*x)*(b*x^2 + a)^(1/3)*(c*x)^(1/3), x)

Sympy [C] Result contains complex when optimal does not.
time = 9.02, size = 46, normalized size = 0.10

$$\frac{a^{\frac{4}{3}}c^{\frac{4}{3}}x^{\frac{7}{3}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(4/3)*x**(7/3)*gamma(7/6)*hyper((-4/3, 7/6), (13/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{4/3}\,{\left(b\,x^2+a\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)*(a + b*x^2)^(4/3),x)
```

```
[Out] int((c*x)^(4/3)*(a + b*x^2)^(4/3), x)
```

3.764 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}} dx$

Optimal. Leaf size=414

$$\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c}+\frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c}+\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(\ldots}{\sqrt[3]{a+\ldots}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/\ldots}}{\sqrt[3]{a+bx^2}}\ldots\right.}}\ldots}{9\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2\ldots}\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\ldots\right.}}}$$

```
[Out] 8/9*a*(c*x)^(1/3)*(b*x^2+a)^(1/3)/c+1/3*(c*x)^(1/3)*(b*x^2+a)^(4/3)/c+8/27*
a*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))
*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1
/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x
)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2
))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(
b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))
^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^
(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(
2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/c^(5/3)/(-b^(1/3)*(c*x)^
(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {285, 335, 247, 231}

$$\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c}+\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(2/3),x]
```

```
[Out] (8*a*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*c) + ((c*x)^(1/3)*(a + b*x^2)^(4/3))
/(3*c) + (8*a*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3
) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) -
((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqr
t[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4
)*c^(5/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^
(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx = \frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c} + \frac{1}{9}(8a)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx$$

$$= \frac{8a\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c} + \frac{1}{27}(16a^2)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx$$

$$= \frac{8a\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c} + \frac{(16a^2)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{9c}$$

$$= \frac{8a\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c} + \frac{(16a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}} \cdots\right.}{9c\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= \frac{8a\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c} + \frac{8a\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/}\cdots}{(\cdots}}\cdots}{9\sqrt[4]{3}\,c^{5/3}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 55, normalized size = 0.13

$$\frac{3ax\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)}{(cx)^{2/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(2/3),x]
```

```
[Out] (3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)])/(
(c*x)^(2/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(2/3),x)
[Out] int((b*x^2+a)^(4/3)/(c*x)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.75, size = 46, normalized size = 0.11

$$\frac{a^{\frac{4}{3}}\sqrt[3]{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{1}{6}\\\frac{7}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(2/3),x)
[Out] a**(4/3)*x**(1/3)*gamma(1/6)*hyper((-4/3, 1/6), (7/6,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(2/3)*gamma(7/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(2/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(2/3), x)
```

3.765 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx$

Optimal. Leaf size=414

$$\frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3}-\frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}+\frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^2 \cdots}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right) \cdots}} \cdots}{5\sqrt[4]{3}\,c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3} \cdots\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1 \cdots}{\cdots}\right.}}}$$

```
[Out] 8/5*b*(c*x)^(1/3)*(b*x^2+a)^(1/3)/c^3-3/5*(b*x^2+a)^(4/3)/c/(c*x)^(5/3)+8/1
5*b*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3
))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^
(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c
*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1
/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))
/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a
)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)
^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/c^(11/3)/(-b^(1/3)*(c*
x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(
2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {283, 285, 335, 247, 231}

$$\frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}\,c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3}-\frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(8/3),x]
```

```
[Out] (8*b*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(5*c^3) - (3*(a + b*x^2)^(4/3))/(5*c*(c
*x)^(5/3)) + (8*b*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(
2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^
(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + S
qrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3
) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 +
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(5*3^
(1/4)*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}} + \frac{(8b)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx}{5c^2}$$

$$= \frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3} - \frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}} + \frac{(16ab)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{15c^2}$$

$$= \frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3} - \frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}} + \frac{(16ab)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{5c^3}$$

$$= \frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3} - \frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}} + \frac{(16ab)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{5c^3\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= \frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3} - \frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}} + \frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\cdots}{\left(c^{2/3}\cdots\right.}}}{5\sqrt[4]{3}\,c^{11/3}\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 57, normalized size = 0.14

$$-\frac{3ax\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},-\frac{5}{6};\frac{1}{6};-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(8/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -5/6, 1/6, -((b*x^2)/a)])
/(5*(c*x)^(8/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{\left(c x\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(8/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^3*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.73, size = 32, normalized size = 0.08

$$\frac{b^{\frac{4}{3}} x\,{}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{a e^{i\pi}}{b x^{2}}\right)}{c^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(8/3),x)
```

```
[Out] b**(4/3)*x*hyper((-4/3, -1/2), (1/2,), a*exp_polar(I*pi)/(b*x**2))/c**(8/3)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(8/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{8/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(8/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(8/3), x)
```

3.766 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx$

Optimal. Leaf size=419

$$-\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}}-\frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}+\frac{8\,3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\cdots}{55ac^{17/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\cdots\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\cdots}{}\right)}}}$$

```
[Out] -24/55*b*(b*x^2+a)^(1/3)/c^3/(c*x)^(5/3)-3/11*(b*x^2+a)^(4/3)/c/(c*x)^(11/3
)+8/55*3^(3/4)*b^2*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)
/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3)
)^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(
2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x
)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2
/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))
/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)
^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a/c^(17/3)/(-b^
(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(
1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {283, 335, 247, 231}

$$\frac{8\,3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55ac^{17/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}}-\frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(14/3),x]
```

```
[Out] (-24*b*(a + b*x^2)^(1/3))/(55*c^3*(c*x)^(5/3)) - (3*(a + b*x^2)^(4/3))/(11*
c*(c*x)^(11/3)) + (8*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3
))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^
(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF
[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(
c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqr
t[3])/4])/(55*a*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c
*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}} + \frac{(8b)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx}{11c^2}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}} + \frac{(16b^2)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{55c^4}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}} + \frac{(48b^2)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{55c^5}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}} + \frac{(48b^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{55c^5\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}} + \frac{8\,3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}\cdots}{\left(c^{\cdots}\right.\cdots}}}{55ac^{17/3}\sqrt{-\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.14

$$-\frac{3ax\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{11}{6},-\frac{4}{3};-\frac{5}{6};-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(14/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-11/6, -4/3, -5/6, -((b*x^2)/a)
])/(11*(c*x)^(14/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(14/3),x)
[Out] int((b*x^2+a)^(4/3)/(c*x)^(14/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(14/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^5*x^5), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(14/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3278 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(14/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{\left(c\,x\right)^{14/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(14/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(14/3), x)
```

3.767 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx$

Optimal. Leaf size=450

$$-\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}}-\frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}}-\frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}}-\frac{24\,3^{3/4}b^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\cdots}{\left(c^{2/3}-\cdots\right)}}\cdots}{935a^2c^{23/3}\sqrt{-\frac{\cdots}{\sqrt[3]{a}\cdots}}}$$

```
[Out] -24/187*b*(b*x^2+a)^(1/3)/c^3/(c*x)^(11/3)-48/935*b^2*(b*x^2+a)^(1/3)/a/c^5
/(c*x)^(5/3)-3/17*(b*x^2+a)^(4/3)/c/(c*x)^(17/3)-24/935*3^(3/4)*b^3*(c*x)^(
1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)
^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(
1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2
+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^
(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
,1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(
1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3
^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a^2/c^(23/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/
3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c
*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {283, 331, 335, 247, 231}

$$-\frac{24\,3^{3/4}b^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{935a^2c^{23/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}}-\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}}-\frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(20/3),x]
```

```
[Out] (-24*b*(a + b*x^2)^(1/3))/(187*c^3*(c*x)^(11/3)) - (48*b^2*(a + b*x^2)^(1/3
))/(935*a*c^5*(c*x)^(5/3)) - (3*(a + b*x^2)^(4/3))/(17*c*(c*x)^(17/3)) - (2
4*3^(3/4)*b^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3
) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) -
((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqr
t[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(935*a^2*
c^(23/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a +
b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(
2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}} + \frac{(8b)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx}{17c^2}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}} + \frac{(16b^2)\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx}{187c^4}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}} - \frac{(48b^3)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{935ac^6}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}} - \frac{(144b^3)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x\right.}{935ac^7}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}} - \frac{(144b^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx\right.}{935ac^7\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a}}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}} - \frac{24\,3^{3/4}b^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 57, normalized size = 0.13

$$-\frac{3ax\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{17}{6},-\frac{4}{3};-\frac{11}{6};-\frac{bx^2}{a}\right)}{17(cx)^{20/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(20/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-17/6, -4/3, -11/6, -((b*x^2)/a
)])/(17*(c*x)^(20/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^7*x^7), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(20/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{\left(c\,x\right)^{20/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(20/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(20/3), x)
```

3.768 $\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=59

$$\frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}$$

```
[Out] 3/5*a*(c*x)^(5/3)*(b*x^2+a)^(1/3)*hypergeom([-4/3, 5/6],[11/6],-b*x^2/a)/c/
(1+b*x^2/a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 5/6, 11/6, -((b*
x^2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (cx)^{2/3}\left(a+bx^2\right)^{4/3}\,dx=\frac{\left(a\sqrt[3]{a+bx^2}\right)\int (cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{4/3}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}$$
$$=\frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}$$

Mathematica [A]
time = 10.01, size = 57, normalized size = 0.97

$$\frac{3ax(cx)^{2/3}\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2/3)*(a + b*x^2)^(4/3),x]
[Out] (3*a*x*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 5/6, 11/6, -((
b*x^2)/a)])/(5*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{2}{3}}\left(b\,x^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)
[Out] int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.09, size = 46, normalized size = 0.78

$$\frac{a^{\frac{4}{3}} c^{\frac{2}{3}} x^{\frac{5}{3}} \Gamma\left(\frac{5}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^{2} e^{i\pi}}{a} \right)}{2\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(2/3)*x**(5/3)*gamma(5/6)*hyper((-4/3, 5/6), (11/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,x\right)}^{2/3}\,{\left(b\,x^2+a\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(a + b*x^2)^(4/3),x)
```

```
[Out] int((c*x)^(2/3)*(a + b*x^2)^(4/3), x)
```

3.769 $\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}} \, dx$

Optimal. Leaf size=59

$$\frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}$$

```
[Out] 3/2*a*(c*x)^(2/3)*(b*x^2+a)^(1/3)*hypergeom([-4/3, 1/3],[4/3],-b*x^2/a)/c/(
1+b*x^2/a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(1/3),x]
```

```
[Out] (3*a*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x
^2)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx = \frac{\left(a\sqrt[3]{a+bx^2}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}}{\sqrt[3]{cx}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}$$
$$= \frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}$$

Mathematica [**A**]
time = 10.01, size = 57, normalized size = 0.97

$$\frac{3ax\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(1/3),x]
```

```
[Out] (3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^2)/a)])/(
2*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(1/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(2/3)/(c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.72, size = 46, normalized size = 0.78

$$\frac{a^{\frac{4}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[3]{c}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(1/3),x)
[Out] a**(4/3)*x**(2/3)*gamma(1/3)*hyper((-4/3, 1/3), (4/3,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(1/3)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(1/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(1/3), x)
```

3.770 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx$

Optimal. Leaf size=57

$$-\frac{3a\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\sqrt[3]{1+\frac{bx^2}{a}}}$$

```
[Out] -3*a*(b*x^2+a)^(1/3)*hypergeom([-4/3, -1/6],[5/6],-b*x^2/a)/c/(c*x)^(1/3)/(
1+b*x^2/a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$-\frac{3a\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6, -((b*x^2)/a)])/(
c*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx = \frac{\left(a\sqrt[3]{a+bx^2}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}}{(cx)^{4/3}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}$$
$$= -\frac{3a\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\ \sqrt[3]{1+\frac{bx^2}{a}}}$$

Mathematica [A]
time = 10.01, size = 55, normalized size = 0.96

$$-\frac{3ax\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{(cx)^{4/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(4/3),x]
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6, -((b*x^2)/a)])
/((c*x)^(4/3)*(1 + (b*x^2)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(4/3),x)
[Out] int((b*x^2+a)^(4/3)/(c*x)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="maxima")
```

[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(2/3)/(c^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.76, size = 49, normalized size = 0.86

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}}\sqrt[3]{x}\,\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(4/3),x)
[Out] a**(4/3)*gamma(-1/6)*hyper((-4/3, -1/6), (5/6,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(4/3)*x**(1/3)*gamma(5/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(4/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(4/3), x)
```

3.771 $\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=198

$$\frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3}-\frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b}+\frac{20a^3c^{19/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{27\sqrt{3}\,b^{11/3}}$$

```
[Out] 10/27*a^2*c^5*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^3-2/9*a*c^3*(c*x)^(10/3)*(b*x^2
+a)^(1/3)/b^2+1/6*c*(c*x)^(16/3)*(b*x^2+a)^(1/3)/b+10/27*a^3*c^(19/3)*ln(b^
(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(11/3)+20/81*a^3*c^(19/3)*arct
an(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(11/3)*
3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {327, 335, 281, 337}

$$\frac{20a^3c^{19/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{27\sqrt{3}\,b^{11/3}}+\frac{10a^3c^{19/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27b^{11/3}}+\frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3}-\frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (10*a^2*c^5*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b^3) - (2*a*c^3*(c*x)^(10/3)
*(a + b*x^2)^(1/3))/(9*b^2) + (c*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*b) + (2
0*a^3*c^(19/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/
3)))/Sqrt[3]])/(27*Sqrt[3]*b^(11/3)) + (10*a^3*c^(19/3)*Log[b^(1/3)*(c*x)^(
2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(27*b^(11/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{(8ac^2)\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx}{9b}$$

$$= -\frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{(20a^2c^4)\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx}{27b^2}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{(40a^3c^6)\int}{8}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{(40a^3c^5)\,\mathrm{S}}{}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{(20a^3c^5)\,\mathrm{S}}{}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{(20a^3c^5)\,\mathrm{S}}{}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{(20a^3c^{17/3}}{}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\,\mathrm{l}}{}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\,\mathrm{l}}{}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\,\mathrm{t}}{}$$

Mathematica [A]
time = 5.52, size = 233, normalized size = 1.18

$$\frac{c^6\sqrt[3]{cx}\left(60a^2b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}-36ab^{5/3}x^{10/3}\sqrt[3]{a+bx^2}+27b^{8/3}x^{16/3}\sqrt[3]{a+bx^2}+40\sqrt{3}\,a^3\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)+40a^3\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)-20a^3\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{162b^{11/3}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^6*(c*x)^(1/3)*(60*a^2*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) - 36*a*b^(5/3)*x
^(10/3)*(a + b*x^2)^(1/3) + 27*b^(8/3)*x^(16/3)*(a + b*x^2)^(1/3) + 40*Sqrt
[3]*a^3*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(
1/3))] + 40*a^3*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] - 20*a^3*Log[b^
(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(1
62*b^(11/3)*x^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{19}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(19/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(c\,x)}^{19/3}}{{(b\,x^2+a)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(19/3)/(a + b*x^2)^(2/3),x)
```

```
[Out] int((c*x)^(19/3)/(a + b*x^2)^(2/3), x)
```

3.772 $\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=167

$$-\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2}+\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5a^2c^{13/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{8/3}}-\frac{5a^2c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}\ldots\right.}{12b^{8/3}}$$

```
[Out] -5/12*a*c^3*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^2+1/4*c*(c*x)^(10/3)*(b*x^2+a)^(1
/3)/b-5/12*a^2*c^(13/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(
8/3)-5/18*a^2*c^(13/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a
)^(1/3))*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {327, 335, 281, 337}

$$-\frac{5a^2c^{13/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{8/3}}-\frac{5a^2c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{8/3}}-\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2}+\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (-5*a*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b^2) + (c*(c*x)^(10/3)*(a + b*
x^2)^(1/3))/(4*b) - (5*a^2*c^(13/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^
(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(6*Sqrt[3]*b^(8/3)) - (5*a^2*c^(13/3)*L
og[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(12*b^(8/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{(5ac^2)\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx}{6b}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{(5a^2c^4)\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx}{9b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{(5a^2c^3)\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right.}{3b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{(5a^2c^3)\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)\right.}{6b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{(5a^2c^3)\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx}}\right.}{6b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^{11/3}\right)\,\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(}{\sqrt[3]{a}}\right.}{18b^{7/3}}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}} - \frac{(5}{}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}} + \frac{5a}{}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{8/3}} - $$

Mathematica [A]
time = 2.23, size = 207, normalized size = 1.24

$$\frac{c^4\sqrt[3]{cx}\left(-15ab^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+9b^{5/3}x^{10/3}\sqrt[3]{a+bx^2}-10\sqrt{3}\,a^2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)-10a^2\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)+5a^2\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{36b^{8/3}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^4*(c*x)^(1/3)*(-15*a*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 9*b^(5/3)*x^(10
/3)*(a + b*x^2)^(1/3) - 10*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^
(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] - 10*a^2*Log[-(b^(1/3)*x^(2/3)) + (a
+ b*x^2)^(1/3)] + 5*a^2*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(
1/3) + (a + b*x^2)^(2/3)]))/(36*b^(8/3)*x^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4497 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{13/3}}{(b\,x^2+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)/(a + b*x^2)^(2/3),x)
[Out] int((c*x)^(13/3)/(a + b*x^2)^(2/3), x)
```

3.773 $\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=131

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}+\frac{ac^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{ac^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{2b^{5/3}}$$

```
[Out] 1/2*c*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b+1/2*a*c^(7/3)*ln(b^(1/3)*(c*x)^(2/3)-c^
(2/3)*(b*x^2+a)^(1/3))/b^(5/3)+1/3*a*c^(7/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^
(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {327, 335, 281, 337}

$$\frac{ac^{7/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{ac^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{2b^{5/3}}+\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) + (a*c^(7/3)*ArcTan[(1 + (2*b^(1/3)
*(c*x)^(2/3))/(c^(2/3)*(a + b*x^2)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(5/3)) + (a
*c^(7/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(2*b^(5/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(2ac^2)\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx}{3b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(2ac)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(ac)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(ac)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{4/3}} + \frac{\left(ac^{5/3}\right)\text{Subst}\Big(\int$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} + \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\right.}{2b^{4/3}}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} - \frac{ac^{7/3}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\right.}{6b^{5/3}}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}}$$

Mathematica [A]

time = 1.24, size = 174, normalized size = 1.33

$$\frac{(cx)^{7/3}\left(3b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+2\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)+2a\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)-a\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{6b^{5/3}x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] ((c*x)^(7/3)*(3*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 2*Sqrt[3]*a*ArcTan[(Sqr
t[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] + 2*a*Log[-(
```

```
b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] - a*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(
2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(6*b^(5/3)*x^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{3}}}{(b\,x^{2}+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
[Out] int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 18.73, size = 44, normalized size = 0.34

$$\frac{c^{\frac{7}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3},\frac{5}{3}\\\frac{8}{3}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(7/3)*x**(10/3)*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/3}}{(b\,x^2+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)/(a + b*x^2)^(2/3),x)
```

```
[Out] int((c*x)^(7/3)/(a + b*x^2)^(2/3), x)
```

3.774 $\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=106

$$-\frac{\sqrt{3}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2b^{2/3}}-\frac{3\sqrt[3]{c}\,\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}$$

```
[Out] -3/4*c^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(2/3)-1/2*c^
(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))
*3^(1/2)/b^(2/3)
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {335, 281, 337}

$$-\frac{\sqrt{3}\,\sqrt[3]{c}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2b^{2/3}}-\frac{3\sqrt[3]{c}\,\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] -1/2*(Sqrt[3]*c^(1/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + b*x
^2)^(1/3)))/Sqrt[3]])/b^(2/3) - (3*c^(1/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3
)*(a + b*x^2)^(1/3)])/(4*b^(2/3))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx = \frac{3\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{3\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c}$$

$$= \frac{3\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2\sqrt[3]{b}\,\sqrt[3]{c}} - \frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2\sqrt[3]{b}\,\sqrt[3]{c}}$$

$$= -\frac{\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} - \frac{3\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4\sqrt[3]{b}\,\sqrt[3]{c}} + \frac{\sqrt[3]{c}}{}$$

$$= -\frac{\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c}\,\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4b^{2/3}} + \frac{(3\sqrt[3]{c}}{}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2b^{2/3}} - \frac{\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c}\,\log\left(c^{4/3}\right.}{}$$

Mathematica [A]
time = 0.81, size = 146, normalized size = 1.38

$$\frac{\sqrt[3]{cx}\left(-2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{b}\,x^{2/3}+2\sqrt[3]{a+bx^2}}\right)-2\log\left(-\sqrt[3]{b}\,x^{2/3}+\sqrt[3]{a+bx^2}\right)+\log\left(b^{2/3}x^{4/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)\right)}{4b^{2/3}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] ((c*x)^(1/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3)
+ 2*(a + b*x^2)^(1/3))] - 2*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] + L
og[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]
))/(4*b^(2/3)*x^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{3}}}{(b\,x^{2}+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 0.68, size = 44, normalized size = 0.42

$$\frac{\sqrt[3]{c}\,x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(1/3)*x**(4/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(2/3)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{1/3}}{(b\,x^2+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)/(a + b*x^2)^(2/3),x)
```

```
[Out] int((c*x)^(1/3)/(a + b*x^2)^(2/3), x)
```

3.775 $\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/a/c/(c*x)^(2/3)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{270\}$

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(2/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Mathematica [A]
time = 0.81, size = 26, normalized size = 0.93

$$-\frac{3x\sqrt[3]{a+bx^2}}{2a(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3))/(2*a*(c*x)^(5/3))
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.75

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{2a\left(cx\right)^{\frac{5}{3}}}$	21
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{2ac\left(cx\right)^{\frac{2}{3}}}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x*(b*x^2+a)^(1/3)/a/(c*x)^(5/3)
```

Maxima [A]
time = 0.30, size = 35, normalized size = 1.25

$$-\frac{3\left(bc^{\frac{1}{3}}x^{3}+ac^{\frac{1}{3}}x\right)}{2\left(bx^{2}+a\right)^{\frac{2}{3}}ac^{2}x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -3/2*(b*c^(1/3)*x^3 + a*c^(1/3)*x)/((b*x^2 + a)^(2/3)*a*c^2*x^(5/3))
```

Fricas [A]
time = 2.22, size = 25, normalized size = 0.89

$$-\frac{3\left(bx^{2}+a\right)^{\frac{1}{3}}\left(cx\right)^{\frac{1}{3}}}{2\,ac^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/2*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^2*x)
```

Sympy [A]
time = 2.13, size = 36, normalized size = 1.29

$$\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{1}{3}\right)}{2ac^{\frac{5}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/3)/(b*x**2+a)**(2/3),x)
[Out] b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-1/3)/(2*a*c**(5/3)*gamma(2/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(5/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(c\,x\right)}^{5/3}\,{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x)
[Out] int(1/((c*x)^(5/3)*(a + b*x^2)^(2/3)), x)
```

3.776 $\int \frac{1}{(cx)^{11/3}(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}+\frac{9(a+bx^2)^{4/3}}{8a^2c(cx)^{8/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/a/c/(c*x)^(8/3)+9/8*(b*x^2+a)^(4/3)/a^2/c/(c*x)^(8/3)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 270}

$$\frac{9(a+bx^2)^{4/3}}{8a^2c(cx)^{8/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(8/3)) + (9*(a + b*x^2)^(4/3))/(8*a^2*c
*(c*x)^(8/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}-\frac{3\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx}{a}\\ &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}+\frac{9(a+bx^2)^{4/3}}{8a^2c(cx)^{8/3}}\end{aligned}$$

Mathematica [A]
time = 1.30, size = 34, normalized size = 0.60

$$-\frac{3x(a-3bx^2)\sqrt[3]{a+bx^2}}{8a^2(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
[Out] (-3*x*(a - 3*b*x^2)*(a + b*x^2)^(1/3))/(8*a^2*(c*x)^(11/3))
```

Maple [A]
time = 0.04, size = 29, normalized size = 0.51

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-3b\,x^{2}+a\right)}{8a^{2}\left(cx\right)^{\frac{11}{3}}}$	29
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-3b\,x^{2}+a\right)}{8c^{3}\left(cx\right)^{\frac{2}{3}}a^{2}x^{2}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
[Out] -3/8*x*(b*x^2+a)^(1/3)*(-3*b*x^2+a)/a^2/(c*x)^(11/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

Fricas [A]
time = 1.12, size = 35, normalized size = 0.61

$$\frac{3\,(3\,bx^2-a)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{8\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/8*(3*b*x^2 - a)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^4*x^3)
```

Sympy [A]
time = 81.74, size = 78, normalized size = 1.37

$$-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{4}{3}\right)}{6ac^{\frac{11}{3}}x^2\Gamma\left(\frac{2}{3}\right)}+\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{4}{3}\right)}{2a^2c^{\frac{11}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] -b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(6*a*c**(11/3)*x**2*gamma(2/3
)) + b**(4/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*a**2*c**(11/3)*gamma(2
/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{11/3}\,\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(11/3)*(a + b*x^2)^(2/3)), x)
```

3.777 $\int \frac{1}{(cx)^{17/3}(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}+\frac{9(a+bx^2)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{27(a+bx^2)^{7/3}}{28a^3c(cx)^{14/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/a/c/(c*x)^(14/3)+9/4*(b*x^2+a)^(4/3)/a^2/c/(c*x)^(14/3
)-27/28*(b*x^2+a)^(7/3)/a^3/c/(c*x)^(14/3)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$-\frac{27(a+bx^2)^{7/3}}{28a^3c(cx)^{14/3}}+\frac{9(a+bx^2)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(4/3))/(4*a^2*
c*(c*x)^(14/3)) - (27*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(14/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}} - \frac{6\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx}{a}$$
$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}} + \frac{9(a+bx^2)^{4/3}}{4a^2c(cx)^{14/3}} + \frac{9\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx}{2a^2}$$
$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}} + \frac{9(a+bx^2)^{4/3}}{4a^2c(cx)^{14/3}} - \frac{27(a+bx^2)^{7/3}}{28a^3c(cx)^{14/3}}$$

Mathematica [A]
time = 3.28, size = 47, normalized size = 0.55

$$-\frac{3x\sqrt[3]{a+bx^2}\,(2a^2-3abx^2+9b^2x^4)}{28a^3(cx)^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*(2*a^2 - 3*a*b*x^2 + 9*b^2*x^4))/(28*a^3*(c*x)^(17/
3))
```

Maple [A]
time = 0.04, size = 42, normalized size = 0.49

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{4}-3ab\,x^{2}+2a^{2}\right)}{28a^{3}\left(cx\right)^{\frac{17}{3}}}$	42
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{4}-3ab\,x^{2}+2a^{2}\right)}{28c^{5}\left(cx\right)^{\frac{2}{3}}a^{3}x^{4}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/28*x*(b*x^2+a)^(1/3)*(9*b^2*x^4-3*a*b*x^2+2*a^2)/a^3/(c*x)^(17/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)
```

Fricas [A]
time = 0.93, size = 46, normalized size = 0.54

$$-\frac{3\,(9\,b^2x^4-3\,abx^2+2\,a^2)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{28\,a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/28*(9*b^2*x^4 - 3*a*b*x^2 + 2*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c^
6*x^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(17/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7143 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{17/3}\,{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(17/3)*(a + b*x^2)^(2/3)), x)
```

3.778 $\int \frac{1}{(cx)^{23/3}(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=113

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}+\frac{27(a+bx^2)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{81(a+bx^2)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{243(a+bx^2)^{10/3}}{280a^4c(cx)^{20/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/a/c/(c*x)^(20/3)+27/8*(b*x^2+a)^(4/3)/a^2/c/(c*x)^(20/
3)-81/28*(b*x^2+a)^(7/3)/a^3/c/(c*x)^(20/3)+243/280*(b*x^2+a)^(10/3)/a^4/c/
(c*x)^(20/3)
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$\frac{243(a+bx^2)^{10/3}}{280a^4c(cx)^{20/3}}-\frac{81(a+bx^2)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{27(a+bx^2)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(20/3)) + (27*(a + b*x^2)^(4/3))/(8*a^2
*c*(c*x)^(20/3)) - (81*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(20/3)) + (243*(a
 + b*x^2)^(10/3))/(280*a^4*c*(c*x)^(20/3))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}} - \frac{9\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx}{a}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}} + \frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}} + \frac{27\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{23/3}}\,dx}{2a^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}} + \frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}} - \frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}} - \frac{81\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{23/3}}\,dx}{14a^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}} + \frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}} - \frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}} + \frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}$$

Mathematica [A]
time = 4.95, size = 58, normalized size = 0.51

$$-\frac{3x\sqrt[3]{a+bx^2}\left(14a^3-18a^2bx^2+27ab^2x^4-81b^3x^6\right)}{280a^4(cx)^{23/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
[Out] (-3*x*(a + b*x^2)^(1/3)*(14*a^3 - 18*a^2*b*x^2 + 27*a*b^2*x^4 - 81*b^3*x^6)
)/(280*a^4*(c*x)^(23/3))
```

Maple [A]
time = 0.04, size = 53, normalized size = 0.47

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-81b^{3}x^{6}+27a\,b^{2}x^{4}-18a^{2}b\,x^{2}+14a^{3}\right)}{280a^{4}\left(cx\right)^{\frac{23}{3}}}$	53
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-81b^{3}x^{6}+27a\,b^{2}x^{4}-18a^{2}b\,x^{2}+14a^{3}\right)}{280c^{7}\left(cx\right)^{\frac{2}{3}}a^{4}x^{6}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
[Out] -3/280*x*(b*x^2+a)^(1/3)*(-81*b^3*x^6+27*a*b^2*x^4-18*a^2*b*x^2+14*a^3)/a^4
/(c*x)^(23/3)
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.57

$$\frac{3\left(81\,b^4x^9+54\,ab^3x^7-9\,a^2b^2x^5+4\,a^3bx^3-14\,a^4x\right)}{280\left(bx^2+a\right)^{\frac{2}{3}}a^4c^{\frac{23}{3}}x^{\frac{23}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/280*(81*b^4*x^9 + 54*a*b^3*x^7 - 9*a^2*b^2*x^5 + 4*a^3*b*x^3 - 14*a^4*x)/
((b*x^2 + a)^(2/3)*a^4*c^(23/3)*x^(23/3))
```

Fricas [**A**]
time = 0.66, size = 57, normalized size = 0.50

$$\frac{3\,(81\,b^3x^6-27\,ab^2x^4+18\,a^2bx^2-14\,a^3)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280\,a^4c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/280*(81*b^3*x^6 - 27*a*b^2*x^4 + 18*a^2*b*x^2 - 14*a^3)*(b*x^2 + a)^(1/3)
*(c*x)^(1/3)/(a^4*c^8*x^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(23/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(23/3)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{23/3}\,{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(23/3)*(a + b*x^2)^(2/3)), x)
```

3.779 $\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=421

$$-\frac{7ac^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}+\frac{7ac^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\cdots}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\cdots}{\sqrt[3]{a\cdots}}\right.\cdots}}\cdots}{18\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2\cdots}}{\sqrt[3]{a+bx^2}\left(\cdots\right.}\cdots}}$$

```
[Out] -7/9*a*c^3*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b^2+1/3*c*(c*x)^(7/3)*(b*x^2+a)^(1/3
)/b+7/54*a*c^(7/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)
/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3)
)^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(
2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x
)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2
/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))
/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)
^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/b^2/(-b
^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^
(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {327, 335, 247, 231}

$$\frac{7ac^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{18\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{7ac^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(10/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (-7*a*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*b^2) + (c*(c*x)^(7/3)*(a + b*x^
2)^(1/3))/(3*b) + (7*a*c^(7/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^
(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))
/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2
/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[A
rcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^
(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[
3])/4])/(18*3^(1/4)*b^2*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} - \frac{(7ac^2)\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx}{9b}$$

$$= -\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} + \frac{(7a^2c^4)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{27b^2}$$

$$= -\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} + \frac{(7a^2c^3)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{9b^2}$$

$$= -\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} + \frac{(7a^2c^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{}}{\sqrt[6]{a-}}\right.}{9b^2\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= -\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} + \frac{7ac^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3} - \frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right.}{18}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 87, normalized size = 0.21

$$\frac{c^3\sqrt[3]{cx}\left(-7a^2-4abx^2+3b^2x^4+7a^2\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{6},\frac{2}{3};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{9b^2\,(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(10/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^3*(c*x)^(1/3)*(-7*a^2 - 4*a*b*x^2 + 3*b^2*x^4 + 7*a^2*(1 + (b*x^2)/a)^(2
/3)*Hypergeometric2F1[1/6, 2/3, 7/6, -((b*x^2)/a)]))/(9*b^2*(a + b*x^2)^(2/
3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{10}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/3)*c^3*x^3/(b*x^2 + a)^(2/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 99.02, size = 44, normalized size = 0.10

$$\frac{c^{\frac{10}{3}}x^{\frac{13}{3}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(10/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(10/3)*x**(13/3)*gamma(13/6)*hyper((2/3, 13/6), (19/6,), b*x**2*exp_pola
r(I*pi)/a)/(2*a**(2/3)*gamma(19/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{10/3}}{(b\,x^2+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)/(a + b*x^2)^(2/3),x)
[Out] int((c*x)^(10/3)/(a + b*x^2)^(2/3), x)
```

3.780 $\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=388

$$\frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b}-\frac{\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\right.\right.}{2\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}}}$$

```
[Out] c*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b-1/6*c^(1/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^
(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1
-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^
2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/
3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-
(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3
)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)
/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))
^2)^(1/2)*3^(3/4)/b/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x
^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^
2+a)^(1/3))^2)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {327, 335, 247, 231}

$$\frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b}-\frac{\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/b - (c^(1/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3
)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2
/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x
^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)
)^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b
*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) -
 (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((
1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b} - \frac{(ac^2)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{3b}$$

$$= \frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b} - \frac{(ac)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{b}$$

$$= \frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b} - \frac{(ac)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{b\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= \frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b} - \frac{\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}}{\sqrt[3]{}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}}{\sqrt[3]{a+bx}}\right.}}}{2\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\,(}{\sqrt[3]{a+bx^2}\left(c^{2/3}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 66, normalized size = 0.17

$$\frac{c\sqrt[3]{cx}\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{6},\frac{2}{3};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{b\,(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(4/3)/(a + b*x^2)^(2/3),x]
[Out] (c*(c*x)^(1/3)*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/6,
2/3, 7/6, -((b*x^2)/a)]))/(b*(a + b*x^2)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{4}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/3)*c*x/(b*x^2 + a)^(2/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.21, size = 44, normalized size = 0.11

$$\frac{c^{\frac{4}{3}}x^{\frac{7}{3}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(4/3)*x**(7/3)*gamma(7/6)*hyper((2/3, 7/6), (13/6,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(13/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{4/3}}{\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)/(a + b*x^2)^(2/3),x)
[Out] int((c*x)^(4/3)/(a + b*x^2)^(2/3), x)
```

3.781 $\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=364

$$\frac{3^{3/4}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right)\right)}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] 1/2*3^(3/4)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2
+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^
(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)
*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-
3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2
+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/
(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/
3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a/c^(5/3)/(-b^(1/3)*(c
*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {335, 247, 231}

$$\frac{3^{3/4}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3^(3/4)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
+ b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 -
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(2*a*c^(5/3)*S
qrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx = \frac{3\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{c\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= \frac{3^{3/4}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\right.}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)}{\sqrt[3]{a+b}}\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}}{\sqrt[3]{a+bx}}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 54, normalized size = 0.15

$$\frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{6},\frac{2}{3};\frac{7}{6};-\frac{bx^2}{a}\right)}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/6, 2/3, 7/6, -((b*x^2)/a)])/
((c*x)^(2/3)*(a + b*x^2)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{2}{3}}\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x)

[Out] int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c*x^3 + a*c*x), x)

Sympy [C] Result contains complex when optimal does not.
time = 0.98, size = 31, normalized size = 0.09

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^{2}}\right)}{b^{\frac{2}{3}}c^{\frac{2}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(2/3)/(b*x**2+a)**(2/3),x)

[Out] -hyper((1/2, 2/3), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(2/3)*c**(2/3)*x
)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="giac")

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{2/3}\,(b\,x^2+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x)
[Out] int(1/((c*x)^(2/3)*(a + b*x^2)^(2/3)), x)
```

3.782 $\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=394

$$-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}-\frac{3\ 3^{3/4}b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\;F\left(\cos^{-1}\left(\cdots\right.\right.}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\cdots}}}$$

```
[Out] -3/5*(b*x^2+a)^(1/3)/a/c/(c*x)^(5/3)-3/10*3^(3/4)*b*(c*x)^(1/3)*(b*x^2+a)^(
1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^
(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2
))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^
2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*Ellip
ticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4
*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*
x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a
)^(1/3))^2)^(1/2)/a^2/c^(11/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)
^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(
1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {331, 335, 247, 231}

$$-\frac{3\ 3^{3/4} b \sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\;F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(5*a*c*(c*x)^(5/3)) - (3*3^(3/4)*b*(c*x)^(1/3)*(a +
b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4
/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b
*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(10*a^2*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(
2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3
)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}} - \frac{(3b)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{5ac^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}} - \frac{(9b)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{5ac^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}} - \frac{(9b)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{5ac^3\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}} - \frac{3\,3^{3/4}b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}\cdots}{\left(c^{2/3}-\frac{\left(1+\sqrt{\cdots}\right.}{\sqrt[3]{\cdots}}\right.\cdots}}}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(c\cdots}{\sqrt[3]{a+bx^2}\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 56, normalized size = 0.14

$$-\frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{5}{6},\frac{2}{3};\frac{1}{6};-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x]
[Out] (-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-5/6, 2/3, 1/6, -((b*x^2)/a)]
)/(5*(c*x)^(8/3)*(a + b*x^2)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{8}{3}}\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(8/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c^3*x^5 + a*c^3*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.03, size = 48, normalized size = 0.12

$$\frac{\Gamma\left(-\frac{5}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{6}, \frac{2}{3} \\ \frac{1}{6} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}c^{\frac{8}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(8/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] gamma(-5/6)*hyper((-5/6, 2/3), (1/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3
)*c**(8/3)*x**(5/3)*gamma(1/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(8/3)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{8/3}\,(b\,x^2+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x)
[Out] int(1/((c*x)^(8/3)*(a + b*x^2)^(2/3)), x)
```

3.783 $\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=425

$$-\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}+\frac{27\,3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(c\cdots}{\sqrt[3]{a+b\cdots}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\cdots\right.}}}{110a^3c^{17/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\cdots\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\cdots}{\cdots}\right.}}}$$

```
[Out] -3/11*(b*x^2+a)^(1/3)/a/c/(c*x)^(11/3)+27/55*b*(b*x^2+a)^(1/3)/a^2/c^3/(c*x
)^(5/3)+27/110*3^(3/4)*b^2*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*
x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+
a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1
/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1
/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*
(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+
3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/
3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))
/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a^3/c^(
17/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(
b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)
^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {331, 335, 247, 231}

$$\frac{27\,3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{110a^3c^{17/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(11*a*c*(c*x)^(11/3)) + (27*b*(a + b*x^2)^(1/3))/(55
*a^2*c^3*(c*x)^(5/3)) + (27*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2
/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*
x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*El
lipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (
2 + Sqrt[3])/4])/(110*a^3*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1
+ Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}} - \frac{(9b)\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx}{11ac^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}} + \frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}} + \frac{(27b^2)\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx}{55a^2c^4}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}} + \frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}} + \frac{(81b^2)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{55a^2c^5}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}} + \frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}} + \frac{(81b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right.}{55a^2c^5\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}} + \frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}} + \frac{27\ 3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3} - \frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right.}{110a^3c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.13

$$-\frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{11}{6},\frac{2}{3};-\frac{5}{6};-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-11/6, 2/3, -5/6, -((b*x^2)/a
)])/(11*(c*x)^(14/3)*(a + b*x^2)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{14}{3}} (b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c^5*x^7 + a*c^5*x^5), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(14/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{14/3}\,\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(14/3)*(a + b*x^2)^(2/3)), x)
```

3.784 $\int \frac{(cx)^{2/3}}{\left(a+bx^2\right)^{2/3}} \, dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{5/3}\left(1+\frac{bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\left(a+bx^2\right)^{2/3}}$$

```
[Out] 3/5*(c*x)^(5/3)*(1+b*x^2/a)^(2/3)*hypergeom([2/3, 5/6],[11/6],-b*x^2/a)/c/(
b*x^2+a)^(2/3)
```

Rubi [A]

time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{3(cx)^{5/3}\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(c*x)^(5/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -((b
*x^2)/a)])/(5*c*(a + b*x^2)^(2/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{(cx)^{2/3}}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{(a+bx^2)^{2/3}}$$
$$= \frac{3(cx)^{5/3}\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\,(a+bx^2)^{2/3}}$$

Mathematica [A]
time = 10.01, size = 56, normalized size = 0.97

$$\frac{3x(cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5\,(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(c*x)^(2/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -(
(b*x^2)/a)])/(5*(a + b*x^2)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{2}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.84, size = 44, normalized size = 0.76

$$\frac{c^{\frac{2}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{5}{6}\\\frac{11}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2/3)/(b*x**2+a)**(2/3),x)
[Out] c**(2/3)*x**(5/3)*gamma(5/6)*hyper((2/3, 5/6), (11/6,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(11/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x\right)}^{2/3}}{{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)/(a + b*x^2)^(2/3),x)
[Out] int((c*x)^(2/3)/(a + b*x^2)^(2/3), x)
```

3.785 $\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\,(a+bx^2)^{2/3}}$$

```
[Out] 3/2*(c*x)^(2/3)*(1+b*x^2/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^2/a)/c/(b
*x^2+a)^(2/3)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{3(cx)^{2/3}\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\,(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*(c*x)^(2/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*
x^2)/a)])/(2*c*(a + b*x^2)^(2/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{\sqrt[3]{cx}\,\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{(a+bx^2)^{2/3}}$$
$$= \frac{3(cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\,(a+bx^2)^{2/3}}$$

Mathematica [A]
time = 10.02, size = 56, normalized size = 0.97

$$\frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\,(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x]
[Out] (3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^2)/a)])/
(2*(c*x)^(1/3)*(a + b*x^2)^(2/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{1}{3}}\,(b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)
[Out] int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.70, size = 46, normalized size = 0.79

$$\frac{\Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{a e^{i \pi}}{b x^{2}}} \right)}{2 b^{\frac{2}{3}} \sqrt[3]{c} x^{\frac{2}{3}} \Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] gamma(-1/3)*hyper((1/3, 2/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*c**(1/3)*x**(2/3)*gamma(2/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{1/3}\,{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(1/3)*(a + b*x^2)^(2/3)), x)
```

3.786 $\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\left(1+\frac{bx^2}{a}\right)^{2/3} {}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\left(a+bx^2\right)^{2/3}}$$

```
[Out] -3*(1+b*x^2/a)^(2/3)*hypergeom([-1/6, 2/3],[5/6],-b*x^2/a)/c/(c*x)^(1/3)/(b
*x^2+a)^(2/3)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$-\frac{3\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -((b*x^2)/a)])/
(c*(c*x)^(1/3)*(a + b*x^2)^(2/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(cx)^{4/3}(a+bx^2)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{(cx)^{4/3}\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{(a+bx^2)^{2/3}}$$
$$= -\frac{3\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,(a+bx^2)^{2/3}}$$

Mathematica [**A**]
time = 10.01, size = 54, normalized size = 0.96

$$-\frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{(cx)^{4/3}(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x]
[Out] (-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -((b*x^2)/a)]
)/((c*x)^(4/3)*(a + b*x^2)^(2/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{4}{3}}(b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x)
[Out] int(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.73, size = 48, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{2}{3} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}c^{\frac{4}{3}}\sqrt[3]{x}\,\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(4/3)/(b*x**2+a)**(2/3),x)
[Out] gamma(-1/6)*hyper((-1/6, 2/3), (5/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3
)*c**(4/3)*x**(1/3)*gamma(5/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{4/3}\,{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x)
[Out] int(1/((c*x)^(4/3)*(a + b*x^2)^(2/3)), x)
```

3.787 $\int x^4 \sqrt[4]{a+bx^2}\, dx$

Optimal. Leaf size=121

$$-\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{8a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] -4/77*a^2*x*(b*x^2+a)^(1/4)/b^2+2/77*a*x^3*(b*x^2+a)^(1/4)/b+2/11*x^5*(b*x^
2+a)^(1/4)+8/77*a^(7/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b
^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {285, 327, 239, 237}

$$\frac{8a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(1/4),x]
```

```
[Out] (-4*a^2*x*(a + b*x^2)^(1/4))/(77*b^2) + (2*a*x^3*(a + b*x^2)^(1/4))/(77*b)
+ (2*x^5*(a + b*x^2)^(1/4))/11 + (8*a^(7/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt[4]{a+bx^2}\,dx &= \frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{1}{11}a\int\frac{x^4}{(a+bx^2)^{3/4}}\,dx\\
&= \frac{2ax^3\sqrt[4]{a+bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}-\frac{(6a^2)\int\frac{x^2}{(a+bx^2)^{3/4}}\,dx}{77b}\\
&= -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{(4a^3)\int\frac{1}{(a+bx^2)^{3/4}}\,dx}{77b^2}\\
&= -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{\left(4a^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right.}}{77b^2\,(a+bx^2)^{3/4}}\\
&= -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{8a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-}\right.}{77b^{5/2}\,(a+bx^2)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.03, size = 93, normalized size = 0.77

$$\frac{2x\sqrt[4]{a+bx^2}\left(\sqrt[4]{1+\frac{bx^2}{a}}\,(-6a^2+abx^2+7b^2x^4)+6a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4)*((1 + (b*x^2)/a)^(1/4)*(-6*a^2 + a*b*x^2 + 7*b^2*x^4
) + 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)]))/(77*b^2*(1 + (b
*x^2)/a)^(1/4))
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^4*(b*x^2+a)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*x^4, x)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.48, size = 29, normalized size = 0.24

$$\frac{\sqrt[4]{a}\,x^5\,{}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**5*hyper((-1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(b\,x^2+a\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(1/4),x)
```

```
[Out] int(x^4*(a + b*x^2)^(1/4), x)
```

3.788 $\int x^2\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=97

$$\frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{4a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] 2/21*a*x*(b*x^2+a)^(1/4)/b+2/7*x^3*(b*x^2+a)^(1/4)-4/21*a^(5/2)*(1+b*x^2/a)
^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(
b*x^2+a)^(3/4)
```

Rubi [A]

time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {285, 327, 239, 237}

$$-\frac{4a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*a*x*(a + b*x^2)^(1/4))/(21*b) + (2*x^3*(a + b*x^2)^(1/4))/7 - (4*a^(5/2)
*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^(
3/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt[4]{a+bx^2}\,dx &= \frac{2}{7}x^3\sqrt[4]{a+bx^2}+\frac{1}{7}a\int\frac{x^2}{(a+bx^2)^{3/4}}\,dx\\ &= \frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{(2a^2)\int\frac{1}{(a+bx^2)^{3/4}}\,dx}{21b}\\ &= \frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{\left(2a^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{21b\,(a+bx^2)^{3/4}}\\ &= \frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{4a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\,(a+bx^2)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.98, size = 62, normalized size = 0.64

$$\frac{2x\sqrt[4]{a+bx^2}\left(a+bx^2-\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(1/4),x]
[Out] (2*x*(a + b*x^2)^(1/4)*(a + b*x^2 - (a*Hypergeometric2F1[-1/4, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(1/4)))/(7*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{2} \left(b \,x^{2}+a \right)^{\frac{1}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/4),x)
[Out] int(x^2*(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 29, normalized size = 0.30

$$\frac{\sqrt[4]{a}\, x^{3} {{}_{2}F_{1}\left(\begin{matrix} - \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/4),x)
[Out] a**(1/4)*x**3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(b\,x^2+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/4),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/4), x)
```

3.789 $\int \sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=75

$$\frac{2}{3}x\sqrt[4]{a+bx^2}+\frac{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\,(a+bx^2)^{3/4}}$$

[Out] 2/3*x*(b*x^2+a)^(1/4)+2/3*a^(3/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(3/4)/b^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {201, 239, 237}

$$\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\,(a+bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(1/4),x]

[Out] (2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^2)^(3/4))

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \sqrt[4]{a+bx^2}\,dx = \frac{2}{3}x\sqrt[4]{a+bx^2} + \frac{1}{3}a\int \frac{1}{(a+bx^2)^{3/4}}\,dx$$
$$= \frac{2}{3}x\sqrt[4]{a+bx^2} + \frac{\left(a\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3(a+bx^2)^{3/4}}$$
$$= \frac{2}{3}x\sqrt[4]{a+bx^2} + \frac{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.96, size = 46, normalized size = 0.61

$$\frac{x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4),x]
[Out] (x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(1/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4),x)
[Out] int((b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 26, normalized size = 0.35

$$\sqrt[4]{a}\, x\, {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4), x)
```

Mupad [B]
time = 4.66, size = 37, normalized size = 0.49

$$\frac{x\,{\left(b\,x^2+a\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4),x)
```

```
[Out] (x*(a + b*x^2)^(1/4)*hypergeom([-1/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(1/4)
```

3.790 $\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt[4]{a+bx^2}}{x}+\frac{\sqrt{a}\,\sqrt{b}\,\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{(a+bx^2)^{3/4}}$$

```
[Out] -(b*x^2+a)^(1/4)/x+(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)
^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2
)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {283, 239, 237}

$$\frac{\sqrt{a}\,\sqrt{b}\,\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/x^2,x]
```

```
[Out] -((a + b*x^2)^(1/4)/x) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(3/4)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx &= -\frac{\sqrt[4]{a+bx^2}}{x}+\frac{1}{2}b\int \frac{1}{(a+bx^2)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{a+bx^2}}{x}+\frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{2\,(a+bx^2)^{3/4}}\\ &= -\frac{\sqrt[4]{a+bx^2}}{x}+\frac{\sqrt{a}\,\sqrt{b}\,\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{(a+bx^2)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.24, size = 49, normalized size = 0.68

$$-\frac{\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/x^2,x]
[Out] -(((a + b*x^2)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(1/4)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/x^2,x)
[Out] int((b*x^2+a)^(1/4)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 29, normalized size = 0.40

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/x**2,x)
[Out] -a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/x^2, x)
```

Mupad [B]
time = 4.81, size = 40, normalized size = 0.56

$$-\frac{2\,{\left(b\,x^2+a\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a}{b\,x^2}\right)}{x\,{\left(\frac{a}{b\,x^2}+1\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/x^2,x)
[Out] -(2*(a + b*x^2)^(1/4)*hypergeom([-1/4, 1/4], 5/4, -a/(b*x^2)))/(x*(a/(b*x^2
) + 1)^(1/4))
```

3.791 $\int \frac{\sqrt[4]{a+bx^2}}{x^4} \, dx$

Optimal. Leaf size=99

$$-\frac{\sqrt[4]{a+bx^2}}{3x^3}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\,\left(a+bx^2\right)^{3/4}}$$

```
[Out] -1/3*(b*x^2+a)^(1/4)/x^3-1/6*b*(b*x^2+a)^(1/4)/a/x-1/6*b^(3/2)*(1+b*x^2/a)^
(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)
/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^
(3/4)/a^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 331, 239, 237}

$$-\frac{b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\,\left(a+bx^2\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{\sqrt[4]{a+bx^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/x^4,x]
```

```
[Out] -1/3*(a + b*x^2)^(1/4)/x^3 - (b*(a + b*x^2)^(1/4))/(6*a*x) - (b^(3/2)*(1 +
(b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a
 + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx &= -\frac{\sqrt[4]{a+bx^2}}{3x^3}+\frac{1}{6}b\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{a+bx^2}}{3x^3}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{b^2\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{12a} \\
&= -\frac{\sqrt[4]{a+bx^2}}{3x^3}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{\left(b^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a\left(a+bx^2\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^2}}{3x^3}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\,\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.52

$$-\frac{\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/x^4,x]
```

```
[Out] -1/3*((a + b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/x^4,x)
[Out] int((b*x^2+a)^(1/4)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 34, normalized size = 0.34

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/x**4,x)
[Out] -a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^{1/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/x^4,x)
```

```
[Out] int((a + b*x^2)^(1/4)/x^4, x)
```

3.792 $\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] -1/5*(b*x^2+a)^(1/4)/x^5-1/30*b*(b*x^2+a)^(1/4)/a/x^3+1/12*b^2*(b*x^2+a)^(1
/4)/a^2/x+1/12*b^(5/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2))
)^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^
(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 331, 239, 237}

$$\frac{b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}-\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/x^6,x]
```

```
[Out] -1/5*(a + b*x^2)^(1/4)/x^5 - (b*(a + b*x^2)^(1/4))/(30*a*x^3) + (b^2*(a + b
*x^2)^(1/4))/(12*a^2*x) + (b^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*a^(3/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx &= -\frac{\sqrt[4]{a+bx^2}}{5x^5}+\frac{1}{10}b\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}-\frac{b^2\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx}{12a}\\
&= -\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{b^3\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{24a^2}\\
&= -\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{\left(b^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{24a^2\left(a+bx^2\right)^{3/4}}\\
&= -\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.41

$$-\frac{\sqrt[4]{a+bx^2}\ {}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/x^6,x]
```

```
[Out] -1/5*((a + b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, -((b*x^2)/a)])/
(x^5*(1 + (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/x^6,x)
[Out] int((b*x^2+a)^(1/4)/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/x^6, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 34, normalized size = 0.28

$$-\frac{\sqrt[4]{a}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},-\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/x**6,x)
[Out] -a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/x^6,x)
```

```
[Out] int((a + b*x^2)^(1/4)/x^6, x)
```

3.793 $\int x^4 \sqrt[4]{a-bx^2}\, dx$

Optimal. Leaf size=126

$$-\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a-bx^2\right)^{3/4}}$$

```
[Out] -4/77*a^2*x*(-b*x^2+a)^(1/4)/b^2-2/77*a*x^3*(-b*x^2+a)^(1/4)/b+2/11*x^5*(-b
*x^2+a)^(1/4)+8/77*a^(7/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1
/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(
x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(-b*x^2+a)^(3/4)
```

Rubi [A]

time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {285, 327, 239, 238}

$$\frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a - b*x^2)^(1/4),x]
```

```
[Out] (-4*a^2*x*(a - b*x^2)^(1/4))/(77*b^2) - (2*a*x^3*(a - b*x^2)^(1/4))/(77*b)
+ (2*x^5*(a - b*x^2)^(1/4))/11 + (8*a^(7/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\sqrt[4]{a-bx^2}\,dx = \frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{1}{11}a\int\frac{x^4}{(a-bx^2)^{3/4}}\,dx$$

$$= -\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{(6a^2)\int\frac{x^2}{(a-bx^2)^{3/4}}\,dx}{77b}$$

$$= -\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{(4a^3)\int\frac{1}{(a-bx^2)^{3/4}}\,dx}{77b^2}$$

$$= -\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{\left(4a^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int\frac{1}{\left(1-\frac{bx^2}{a}\right.}}{77b^2\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}+\frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-}\right.}{77b^{5/2}\left(a-bx^2\right)^{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.02, size = 95, normalized size = 0.75

$$-\frac{2x\sqrt[4]{a-bx^2}\left(\sqrt[4]{1-\frac{bx^2}{a}}\,(6a^2+abx^2-7b^2x^4)-6a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{77b^2\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a - b*x^2)^(1/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*((1 - (b*x^2)/a)^(1/4)*(6*a^2 + a*b*x^2 - 7*b^2*x^4
) - 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^2)/a]))/(77*b^2*(1 - (b*x^
2)/a)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{4} \left(-b\,x^{2}+a \right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-b*x^2+a)^(1/4),x)
[Out] int(x^4*(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)*x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 31, normalized size = 0.25

$$\frac{\sqrt[4]{a}\, x^{5} {{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-b*x**2+a)**(1/4),x)
[Out] a**(1/4)*x**5*hyper((-1/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a-b\,x^2\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a - b*x^2)^(1/4),x)
[Out] int(x^4*(a - b*x^2)^(1/4), x)
```

3.794 $\int x^2\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=101

$$-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2/21*a*x*(-b*x^2+a)^(1/4)/b+2/7*x^3*(-b*x^2+a)^(1/4)+4/21*a^(5/2)*(1-b*x^2
/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(
1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2
)/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {285, 327, 239, 238}

$$\frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a - b*x^2)^(1/4),x]
```

```
[Out] (-2*a*x*(a - b*x^2)^(1/4))/(21*b) + (2*x^3*(a - b*x^2)^(1/4))/7 + (4*a^(5/2
)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^
(3/2)*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt[4]{a-bx^2}\,dx &= \frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{1}{7}a\int\frac{x^2}{(a-bx^2)^{3/4}}\,dx\\&=-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{(2a^2)\int\frac{1}{(a-bx^2)^{3/4}}\,dx}{21b}\\&=-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{\left(2a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int\frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{21b\,(a-bx^2)^{3/4}}\\&=-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\,(a-bx^2)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.94, size = 64, normalized size = 0.63

$$\frac{2x\sqrt[4]{a-bx^2}\left(-a+bx^2+\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(1/4)*(-a + b*x^2 + (a*Hypergeometric2F1[-1/4, 1/2, 3/2, (
b*x^2)/a])/(1 - (b*x^2)/a)^(1/4)))/(7*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{2} \left(-b\,x^{2}+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-b*x^2+a)^(1/4),x)
[Out] int(x^2*(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)*x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 31, normalized size = 0.31

$$\frac{\sqrt[4]{a}\,x^{3}{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-b*x**2+a)**(1/4),x)
[Out] a**(1/4)*x**3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a-b\,x^2\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - b*x^2)^(1/4),x)
[Out] int(x^2*(a - b*x^2)^(1/4), x)
```

3.795 $\int \sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=78

$$\frac{2}{3}x\sqrt[4]{a-bx^2}+\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\,\left(a-bx^2\right)^{3/4}}$$

```
[Out] 2/3*x*(-b*x^2+a)^(1/4)+2/3*a^(3/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1
/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2
*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(3/4)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {201, 239, 238}

$$\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\,\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \sqrt[4]{a-bx^2}\,dx = \frac{2}{3}x\sqrt[4]{a-bx^2}+\frac{1}{3}a\int \frac{1}{(a-bx^2)^{3/4}}\,dx$$
$$= \frac{2}{3}x\sqrt[4]{a-bx^2}+\frac{\left(a\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{3(a-bx^2)^{3/4}}$$
$$= \frac{2}{3}x\sqrt[4]{a-bx^2}+\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\,(a-bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.89, size = 47, normalized size = 0.60

$$\frac{x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4),x]
[Out] (x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b*
x^2)/a)^(1/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4),x)
[Out] int((-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 27, normalized size = 0.35

$$\sqrt[4]{a}\, x\, {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{2i\pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4), x)
```

Mupad [B]
time = 4.80, size = 38, normalized size = 0.49

$$\frac{x\,{\left(a-b\,x^2\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(1-\frac{b\,x^2}{a}\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4),x)
```

```
[Out] (x*(a - b*x^2)^(1/4)*hypergeom([-1/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a
)^(1/4)
```

3.796 $\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt[4]{a-bx^2}}{x}-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\left(a-bx^2\right)^{3/4}}$$

```
[Out] -(-b*x^2+a)^(1/4)/x-(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2
)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/
2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {283, 239, 238}

$$-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/x^2,x]
```

```
[Out] -((a - b*x^2)^(1/4)/x) - (Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(3/4)
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx = -\frac{\sqrt[4]{a-bx^2}}{x} - \frac{1}{2}b\int \frac{1}{(a-bx^2)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{a-bx^2}}{x} - \frac{\left(b\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{2(a-bx^2)^{3/4}}$$
$$= -\frac{\sqrt[4]{a-bx^2}}{x} - \frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{(a-bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.01, size = 50, normalized size = 0.66

$$-\frac{\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{x\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/x^2,x]
[Out] -(((a - b*x^2)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, (b*x^2)/a])/(x*(1 -
 (b*x^2)/a)^(1/4)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/x^2,x)
[Out] int((-b*x^2+a)^(1/4)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 31, normalized size = 0.41

$$-\frac{\sqrt[4]{a}\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/x**2,x)
```

```
[Out] -a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^2, x)
```

Mupad [B]
time = 4.98, size = 41, normalized size = 0.54

$$-\frac{2\,{\left(a-b\,x^2\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a}{b\,x^2}\right)}{x\,{\left(1-\frac{a}{b\,x^2}\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/x^2,x)
```

```
[Out] -(2*(a - b*x^2)^(1/4)*hypergeom([-1/4, 1/4], 5/4, a/(b*x^2)))/(x*(1 - a/(b*
x^2))^(1/4))
```

3.797 $\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt[4]{a-bx^2}}{3x^3}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\,\left(a-bx^2\right)^{3/4}}$$

```
[Out] -1/3*(-b*x^2+a)^(1/4)/x^3+1/6*b*(-b*x^2+a)^(1/4)/a/x-1/6*b^(3/2)*(1-b*x^2/a
)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/
2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+
a)^(3/4)/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {283, 331, 239, 238}

$$-\frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\,\left(a-bx^2\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{\sqrt[4]{a-bx^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/x^4,x]
```

```
[Out] -1/3*(a - b*x^2)^(1/4)/x^3 + (b*(a - b*x^2)^(1/4))/(6*a*x) - (b^(3/2)*(1 -
(b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a
 - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx &= -\frac{\sqrt[4]{a-bx^2}}{3x^3}-\frac{1}{6}b\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{a-bx^2}}{3x^3}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{b^2\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{12a}\\
&= -\frac{\sqrt[4]{a-bx^2}}{3x^3}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{\left(b^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a\left(a-bx^2\right)^{3/4}}\\
&= -\frac{\sqrt[4]{a-bx^2}}{3x^3}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 52, normalized size = 0.50

$$
-\frac{\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{1-\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/x^4,x]
[Out] -1/3*((a - b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (b*x^2)/a])/(x^
3*(1 - (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^{2}+a)^{\frac{1}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/x^4,x)
```

```
[Out] int((-b*x^2+a)^(1/4)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)/x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 36, normalized size = 0.35

$$-\frac{\sqrt[4]{a}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/x**4,x)
```

```
[Out] -a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^2\right)^{1/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/x^4,x)
[Out] int((a - b*x^2)^(1/4)/x^4, x)
```

3.798 $\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx$

Optimal. Leaf size=128

$$-\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a-bx^2\right)^{3/4}}$$

```
[Out] -1/5*(-b*x^2+a)^(1/4)/x^5+1/30*b*(-b*x^2+a)^(1/4)/a/x^3+1/12*b^2*(-b*x^2+a)
^(1/4)/a^2/x-1/12*b^(5/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x
*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {283, 331, 239, 238}

$$-\frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a-bx^2\right)^{3/4}}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/x^6,x]
```

```
[Out] -1/5*(a - b*x^2)^(1/4)/x^5 + (b*(a - b*x^2)^(1/4))/(30*a*x^3) + (b^2*(a - b
*x^2)^(1/4))/(12*a^2*x) - (b^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*a^(3/2)*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx &= -\frac{\sqrt[4]{a-bx^2}}{5x^5}-\frac{1}{10}b\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}-\frac{b^2\int \frac{1}{x^2(a-bx^2)^{3/4}}\,dx}{12a}\\ &= -\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{b^3\int \frac{1}{(a-bx^2)^{3/4}}\,dx}{24a^2}\\ &= -\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{\left(b^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{24a^2\left(a-bx^2\right)^{3/4}}\\ &= -\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a-bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 52, normalized size = 0.41

$$-\frac{\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/x^6,x]
```

```
[Out] -1/5*((a - b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, (b*x^2)/a])/(x^
5*(1 - (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/x^6,x)
[Out] int((-b*x^2+a)^(1/4)/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)/x^6, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.62, size = 36, normalized size = 0.28

$$-\frac{\sqrt[4]{a}\, {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}{5 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/x**6,x)
[Out] -a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x^2\right)}^{1/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/x^6,x)
[Out] int((a - b*x^2)^(1/4)/x^6, x)
```

3.799 $\int x^4(a+bx^2)^{3/4}\,dx$

Optimal. Leaf size=143

$$\frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x(a+bx^2)^{3/4}}{65b^2}+\frac{2ax^3(a+bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{8a^{7/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.\right.}{65b^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 8/65*a^3*x/b^2/(b*x^2+a)^(1/4)-4/65*a^2*x*(b*x^2+a)^(3/4)/b^2+2/39*a*x^3*(b
*x^2+a)^(3/4)/b+2/13*x^5*(b*x^2+a)^(3/4)-8/65*a^(7/2)*(1+b*x^2/a)^(1/4)*(co
s(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2))
)*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^2+a)^(
1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {285, 327, 235, 233, 202}

$$-\frac{8a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx^2}}+\frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x(a+bx^2)^{3/4}}{65b^2}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{2ax^3(a+bx^2)^{3/4}}{39b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/4),x]
```

```
[Out] (8*a^3*x)/(65*b^2*(a + b*x^2)^(1/4)) - (4*a^2*x*(a + b*x^2)^(3/4))/(65*b^2)
 + (2*a*x^3*(a + b*x^2)^(3/4))/(39*b) + (2*x^5*(a + b*x^2)^(3/4))/13 - (8*a
^(7/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
65*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{3/4}\,dx = \frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{1}{13}(3a)\int\frac{x^4}{\sqrt[4]{a+bx^2}}\,dx$$

$$= \frac{2ax^3(a+bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{(2a^2)\int\frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{13b}$$

$$= -\frac{4a^2x(a+bx^2)^{3/4}}{65b^2}+\frac{2ax^3(a+bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{(4a^3)\int\frac{1}{\sqrt[4]{a+bx^2}}\,dx}{65b^2}$$

$$= -\frac{4a^2x(a+bx^2)^{3/4}}{65b^2}+\frac{2ax^3(a+bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{\left(4a^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int}{65b^2\sqrt[4]{a+}}$$

$$= \frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x(a+bx^2)^{3/4}}{65b^2}+\frac{2ax^3(a+bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{\left(4a\right.}{}$$

$$= \frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x(a+bx^2)^{3/4}}{65b^2}+\frac{2ax^3(a+bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{8a^{7/2}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 8.40, size = 93, normalized size = 0.65

$$\frac{2x(a+bx^2)^{3/4}\left(\left(1+\frac{bx^2}{a}\right)^{3/4}(-2a^2+abx^2+3b^2x^4)+2a^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{39b^2\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(3/4)*((1 + (b*x^2)/a)^(3/4)*(-2*a^2 + a*b*x^2 + 3*b^2*x^4
) + 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)]))/(39*b^2*(1 + (b
*x^2)/a)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^4\left(b\,x^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^4*(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 29, normalized size = 0.20

$$\frac{a^{\frac{3}{4}}x^5{}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/4),x)
[Out] a**(3/4)*x**5*hyper((-3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/4)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(b\,x^2+a\right)}^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(3/4),x)
[Out] int(x^4*(a + b*x^2)^(3/4), x)
```

3.800 $\int x^2(a+bx^2)^{3/4}\,dx$

Optimal. Leaf size=119

$$-\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax(a+bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -4/15*a^2*x/b/(b*x^2+a)^(1/4)+2/15*a*x*(b*x^2+a)^(3/4)/b+2/9*x^3*(b*x^2+a)^
(3/4)+4/15*a^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)
^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2
)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {285, 327, 235, 233, 202}

$$\frac{4a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax(a+bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/4),x]
```

```
[Out] (-4*a^2*x)/(15*b*(a + b*x^2)^(1/4)) + (2*a*x*(a + b*x^2)^(3/4))/(15*b) + (2
*x^3*(a + b*x^2)^(3/4))/9 + (4*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^2\right)^{3/4}\,dx &= \frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{1}{3}a\int\frac{x^2}{\sqrt[4]{a+bx^2}}\,dx\\
&= \frac{2ax(a+bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}-\frac{(2a^2)\int\frac{1}{\sqrt[4]{a+bx^2}}\,dx}{15b}\\
&= \frac{2ax(a+bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}-\frac{\left(2a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{15b\sqrt[4]{a+bx^2}}\\
&= -\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax(a+bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{\left(2a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int\frac{1}{\left(1+\frac{bx}{a}\right.}}{15b\sqrt[4]{a+bx^2}}\\
&= -\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax(a+bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan\right.}{15b^{3/2}\sqrt[4]{a+b}}
\end{aligned}$$

Mathematica **[C]** Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 8.09, size = 62, normalized size = 0.52

$$\frac{2x(a+bx^2)^{3/4}\left(a+bx^2-\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/4),x]
[Out] (2*x*(a + b*x^2)^(3/4)*(a + b*x^2 - (a*Hypergeometric2F1[-3/4, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(3/4)))/(9*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2\left(b\,x^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/4),x)
[Out] int(x^2*(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/4)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 29, normalized size = 0.24

$$\frac{a^{\frac{3}{4}}x^3\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/4)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(b\,x^2+a\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(3/4),x)
```

```
[Out] int(x^2*(a + b*x^2)^(3/4), x)
```

3.801 $\int \left(a+bx^2\right)^{3/4}\,dx$

Optimal. Leaf size=92

$$\frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}-\frac{6a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

```
[Out] 6/5*a*x/(b*x^2+a)^(1/4)+2/5*x*(b*x^2+a)^(3/4)-6/5*a^(3/2)*(1+b*x^2/a)^(1/4)
*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1
/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(1/4)
/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {201, 235, 233, 202}

$$-\frac{6a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a+bx^2}}+\frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4),x]
```

```
[Out] (6*a*x)/(5*(a + b*x^2)^(1/4)) + (2*x*(a + b*x^2)^(3/4))/5 - (6*a^(3/2)*(1 +
 (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(
a + b*x^2)^(1/4))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^{3/4}\,dx &= \frac{2}{5}x\left(a+bx^2\right)^{3/4}+\frac{1}{5}(3a)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx\\ &= \frac{2}{5}x\left(a+bx^2\right)^{3/4}+\frac{\left(3a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{5\sqrt[4]{a+bx^2}}\\ &= \frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}-\frac{\left(3a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5\sqrt[4]{a+bx^2}}\\ &= \frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}-\frac{6a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.71, size = 46, normalized size = 0.50

$$\frac{x(a+bx^2)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4),x]
[Out] (x*(a + b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(3/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4),x)
```

```
[Out] int((b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.57, size = 26, normalized size = 0.28

$$a^{\frac{3}{4}}x\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/4), x)
```

Mupad [B]
time = 4.78, size = 37, normalized size = 0.40

$$\frac{x\,{\left(b\,x^2+a\right)}^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4),x)
[Out] (x*(a + b*x^2)^(3/4)*hypergeom([-3/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(3/4)
```

3.802 $\int \frac{(a+bx^2)^{3/4}}{x^2} dx$

Optimal. Leaf size=88

$$\frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}}$$

[Out] 3*b*x/(b*x^2+a)^(1/4)-(b*x^2+a)^(3/4)/x-3*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(b*x^2+a)^(1/4)

Rubi [A]
time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 235, 233, 202}

$$-\frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}} + \frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(3/4)/x^2,x]

[Out] (3*b*x)/(a + b*x^2)^(1/4) - (a + b*x^2)^(3/4)/x - (3*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(1/4)

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2]))*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4)), x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx &= -\frac{(a+bx^2)^{3/4}}{x}+\frac{1}{2}(3b)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx\\
&= -\frac{(a+bx^2)^{3/4}}{x}+\frac{\left(3b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2\sqrt[4]{a+bx^2}}\\
&= \frac{3bx}{\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{x}-\frac{\left(3b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2\sqrt[4]{a+bx^2}}\\
&= \frac{3bx}{\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{x}-\frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.18, size = 49, normalized size = 0.56

$$
-\frac{(a+bx^2)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4)/x^2,x]
[Out] -(((a + b*x^2)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(3/4)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/x^2,x)
```

```
[Out] int((b*x^2+a)^(3/4)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 29, normalized size = 0.33

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/x**2,x)
```

```
[Out] -a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^2, x)
```

Mupad [B]
time = 5.03, size = 40, normalized size = 0.45

$$\frac{2\,{\left(b\,x^2+a\right)}^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{a}{b\,x^2}\right)}{x\,{\left(\frac{a}{b\,x^2}+1\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4)/x^2,x)
[Out] (2*(a + b*x^2)^(3/4)*hypergeom([-3/4, -1/4], 3/4, -a/(b*x^2)))/(x*(a/(b*x^2
) + 1)^(3/4))
```

3.803 $\int \frac{(a+bx^2)^{3/4}}{x^4} dx$

Optimal. Leaf size=121

$$\frac{b^2x}{2a\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{3x^3} - \frac{b(a+bx^2)^{3/4}}{2ax} - \frac{b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a+bx^2}}$$

```
[Out] 1/2*b^2*x/a/(b*x^2+a)^(1/4)-1/3*(b*x^2+a)^(3/4)/x^3-1/2*b*(b*x^2+a)^(3/4)/a
/x-1/2*b^(3/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^
(1/2))),2^(1/2))/(b*x^2+a)^(1/4)/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {283, 331, 235, 233, 202}

$$-\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a+bx^2}} + \frac{b^2x}{2a\sqrt[4]{a+bx^2}} - \frac{b(a+bx^2)^{3/4}}{2ax} - \frac{(a+bx^2)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/x^4,x]
```

```
[Out] (b^2*x)/(2*a*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(3*x^3) - (b*(a + b*x^2
)^(3/4))/(2*a*x) - (b^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx &= -\frac{(a+bx^2)^{3/4}}{3x^3}+\frac{1}{2}b\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx \\
&= -\frac{(a+bx^2)^{3/4}}{3x^3}-\frac{b(a+bx^2)^{3/4}}{2ax}+\frac{b^2\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4a} \\
&= -\frac{(a+bx^2)^{3/4}}{3x^3}-\frac{b(a+bx^2)^{3/4}}{2ax}+\frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4a\sqrt[4]{a+bx^2}} \\
&= \frac{b^2x}{2a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{3x^3}-\frac{b(a+bx^2)^{3/4}}{2ax}-\frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a\sqrt[4]{a+bx^2}} \\
&= \frac{b^2x}{2a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{3x^3}-\frac{b(a+bx^2)^{3/4}}{2ax}-\frac{b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right.}{2\sqrt{a}\,\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 51, normalized size = 0.42

$$-\frac{(a+bx^2)^{3/4}\ {}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4)/x^4,x]
```

```
[Out] -1/3*((a + b*x^2)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/x^4,x)
```

```
[Out] int((b*x^2+a)^(3/4)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 34, normalized size = 0.28

$$-\frac{a^{\frac{3}{4}}{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},-\frac{3}{4}\\-\frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/x**4,x)
```

```
[Out] -a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4)/x^4,x)
```

```
[Out] int((a + b*x^2)^(3/4)/x^4, x)
```

3.804 $\int \frac{(a+bx^2)^{3/4}}{x^6} dx$

Optimal. Leaf size=145

$$-\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{5x^5}-\frac{b(a+bx^2)^{3/4}}{10ax^3}+\frac{3b^2(a+bx^2)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -3/20*b^3*x/a^2/(b*x^2+a)^(1/4)-1/5*(b*x^2+a)^(3/4)/x^5-1/10*b*(b*x^2+a)^(3
/4)/a/x^3+3/20*b^2*(b*x^2+a)^(3/4)/a^2/x+3/20*b^(5/2)*(1+b*x^2/a)^(1/4)*(co
s(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2))
)*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(
1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {283, 331, 235, 233, 202}

$$\frac{3b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a+bx^2}}-\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}+\frac{3b^2(a+bx^2)^{3/4}}{20a^2x}-\frac{(a+bx^2)^{3/4}}{5x^5}-\frac{b(a+bx^2)^{3/4}}{10ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/x^6,x]
```

```
[Out] (-3*b^3*x)/(20*a^2*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*x^5) - (b*(a +
 b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a + b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/
2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a
^(3/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/4}}{x^6}\,dx &= -\frac{(a+bx^2)^{3/4}}{5x^5}+\frac{1}{10}(3b)\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx \\
&= -\frac{(a+bx^2)^{3/4}}{5x^5}-\frac{b(a+bx^2)^{3/4}}{10ax^3}-\frac{(3b^2)\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{20a} \\
&= -\frac{(a+bx^2)^{3/4}}{5x^5}-\frac{b(a+bx^2)^{3/4}}{10ax^3}+\frac{3b^2(a+bx^2)^{3/4}}{20a^2x}-\frac{(3b^3)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{40a^2} \\
&= -\frac{(a+bx^2)^{3/4}}{5x^5}-\frac{b(a+bx^2)^{3/4}}{10ax^3}+\frac{3b^2(a+bx^2)^{3/4}}{20a^2x}-\frac{\left(3b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,d}{40a^2\sqrt[4]{a+bx^2}} \\
&= -\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{5x^5}-\frac{b(a+bx^2)^{3/4}}{10ax^3}+\frac{3b^2(a+bx^2)^{3/4}}{20a^2x}+\frac{\left(3b^3\sqrt[4]{1+\frac{bx^2}{a}}\right.}{40a^2\sqrt{}} \\
&= -\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{5x^5}-\frac{b(a+bx^2)^{3/4}}{10ax^3}+\frac{3b^2(a+bx^2)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}}{20a}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.01, size = 51, normalized size = 0.35

$$-\frac{(a+bx^2)^{3/4}\ {}_2F_1\left(-\frac{5}{2},-\frac{3}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4)/x^6,x]
```

```
[Out] -1/5*((a + b*x^2)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, -((b*x^2)/a)])/
(x^5*(1 + (b*x^2)/a)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/x^6,x)
```

```
[Out] int((b*x^2+a)^(3/4)/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/x^6, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.68, size = 34, normalized size = 0.23

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},-\frac{3}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/x**6,x)
```

```
[Out] -a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4)/x^6,x)
```

```
[Out] int((a + b*x^2)^(3/4)/x^6, x)
```

3.805 $\int x^4 (a-bx^2)^{3/4}\,dx$

Optimal. Leaf size=126

$$-\frac{4a^2x(a-bx^2)^{3/4}}{65b^2}-\frac{2ax^3(a-bx^2)^{3/4}}{39b}+\frac{2}{13}x^5(a-bx^2)^{3/4}+\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a-bx^2}}$$

```
[Out] -4/65*a^2*x*(-b*x^2+a)^(3/4)/b^2-2/39*a*x^3*(-b*x^2+a)^(3/4)/b+2/13*x^5*(-b
*x^2+a)^(3/4)+8/65*a^(7/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1
/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(
x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {285, 327, 235, 234}

$$\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a-bx^2}}-\frac{4a^2x(a-bx^2)^{3/4}}{65b^2}+\frac{2}{13}x^5(a-bx^2)^{3/4}-\frac{2ax^3(a-bx^2)^{3/4}}{39b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a - b*x^2)^(3/4),x]
```

```
[Out] (-4*a^2*x*(a - b*x^2)^(3/4))/(65*b^2) - (2*a*x^3*(a - b*x^2)^(3/4))/(39*b)
+ (2*x^5*(a - b*x^2)^(3/4))/13 + (8*a^(7/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(65*b^(5/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a-bx^2\right)^{3/4}\,dx &= \frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{1}{13}(3a)\int\frac{x^4}{\sqrt[4]{a-bx^2}}\,dx\\ &= -\frac{2ax^3(a-bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{(2a^2)\int\frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{13b}\\ &= -\frac{4a^2x(a-bx^2)^{3/4}}{65b^2}-\frac{2ax^3(a-bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{(4a^3)\int\frac{1}{\sqrt[4]{a-bx^2}}\,dx}{65b^2}\\ &= -\frac{4a^2x(a-bx^2)^{3/4}}{65b^2}-\frac{2ax^3(a-bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{\left(4a^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int}{65b^2\sqrt[4]{a-}}\\ &= -\frac{4a^2x(a-bx^2)^{3/4}}{65b^2}-\frac{2ax^3(a-bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}+\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\Big(}{65b^{5/2}\sqrt[4]{a}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.45, size = 95, normalized size = 0.75

$$-\frac{2x(a-bx^2)^{3/4}\left(\left(1-\frac{bx^2}{a}\right)^{3/4}(2a^2+abx^2-3b^2x^4)-2a^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{39b^2\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a - b*x^2)^(3/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4)*((1 - (b*x^2)/a)^(3/4)*(2*a^2 + a*b*x^2 - 3*b^2*x^4
) - 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, (b*x^2)/a]))/(39*b^2*(1 - (b*x^
2)/a)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4}\left(-b\,x^{2}+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-b*x^2+a)^(3/4),x)
[Out] int(x^4*(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(3/4)*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)*x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.67, size = 31, normalized size = 0.25

$$\frac{a^{\frac{3}{4}}x^{5}{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-b*x**2+a)**(3/4),x)
[Out] a**(3/4)*x**5*hyper((-3/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(a-b\,x^2\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a - b*x^2)^(3/4),x)
```

```
[Out] int(x^4*(a - b*x^2)^(3/4), x)
```

3.806 $\int x^2(a-bx^2)^{3/4}\,dx$

Optimal. Leaf size=101

$$-\frac{2ax(a-bx^2)^{3/4}}{15b}+\frac{2}{9}x^3(a-bx^2)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}$$

```
[Out] -2/15*a*x*(-b*x^2+a)^(3/4)/b+2/9*x^3*(-b*x^2+a)^(3/4)+4/15*a^(5/2)*(1-b*x^2
/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(
1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2
)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {285, 327, 235, 234}

$$\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2ax(a-bx^2)^{3/4}}{15b}+\frac{2}{9}x^3(a-bx^2)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a - b*x^2)^(3/4),x]
```

```
[Out] (-2*a*x*(a - b*x^2)^(3/4))/(15*b) + (2*x^3*(a - b*x^2)^(3/4))/9 + (4*a^(5/2
)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^
(3/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a-bx^2\right)^{3/4}\,dx &= \frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{1}{3}a\int\frac{x^2}{\sqrt[4]{a-bx^2}}\,dx\\ &= -\frac{2ax(a-bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{(2a^2)\int\frac{1}{\sqrt[4]{a-bx^2}}\,dx}{15b}\\ &= -\frac{2ax(a-bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{\left(2a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{15b\sqrt[4]{a-bx^2}}\\ &= -\frac{2ax(a-bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.09, size = 64, normalized size = 0.63

$$\frac{2x(a-bx^2)^{3/4}\left(-a+bx^2+\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a - b*x^2)^(3/4),x]
[Out] (2*x*(a - b*x^2)^(3/4)*(-a + b*x^2 + (a*Hypergeometric2F1[-3/4, 1/2, 3/2, (
b*x^2)/a])/(1 - (b*x^2)/a)^(3/4)))/(9*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{2} \left(-b\,x^{2}+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^2*(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 31, normalized size = 0.31

$$\frac{a^{\frac{3}{4}} x^3 {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{b x^2 e^{2 i \pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a-b\,x^2\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - b*x^2)^(3/4),x)
[Out] int(x^2*(a - b*x^2)^(3/4), x)
```

3.807 $\int (a-bx^2)^{3/4}\,dx$

Optimal. Leaf size=78

$$\frac{2}{5}x\left(a-bx^2\right)^{3/4}+\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

```
[Out] 2/5*x*(-b*x^2+a)^(3/4)+6/5*a^(3/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1
/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2
*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {201, 235, 234}

$$\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a-bx^2}}+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(3/4))/5 + (6*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(a - b*x^2)^(1/4))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rubi steps

$$\int \left(a-bx^2\right)^{3/4}\,dx=\frac{2}{5}x\left(a-bx^2\right)^{3/4}+\frac{1}{5}(3a)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$$
$$=\frac{2}{5}x\left(a-bx^2\right)^{3/4}+\frac{\left(3a\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{5\sqrt[4]{a-bx^2}}$$
$$=\frac{2}{5}x\left(a-bx^2\right)^{3/4}+\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.80, size = 47, normalized size = 0.60

$$\frac{x\left(a-bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4),x]
```

```
[Out] (x*(a - b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b*
x^2)/a)^(3/4)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{2}+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4),x)
```

```
[Out] int((-b*x^2+a)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 27, normalized size = 0.35

$$a^{\frac{3}{4}}x\,{}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/4), x)
```

Mupad [B]
time = 4.85, size = 38, normalized size = 0.49

$$\frac{x\,{\left(a-b\,x^2\right)}^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(1-\frac{b\,x^2}{a}\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(3/4),x)
```

```
[Out] (x*(a - b*x^2)^(3/4)*hypergeom([-3/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a
)^(3/4)
```

3.808 $\int \frac{(a-bx^2)^{3/4}}{x^2} dx$

Optimal. Leaf size=76

$$-\frac{(a-bx^2)^{3/4}}{x}-\frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

```
[Out] -(-b*x^2+a)^(3/4)/x-3*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(
1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {283, 235, 234}

$$-\frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}-\frac{(a-bx^2)^{3/4}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4)/x^2,x]
```

```
[Out] -((a - b*x^2)^(3/4)/x) - (3*Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(1/4)
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx = -\frac{(a-bx^2)^{3/4}}{x} - \frac{1}{2}(3b)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$$
$$= -\frac{(a-bx^2)^{3/4}}{x} - \frac{\left(3b\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{2\sqrt[4]{a-bx^2}}$$
$$= -\frac{(a-bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.26, size = 50, normalized size = 0.66

$$-\frac{(a-bx^2)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4)/x^2,x]
[Out] -(((a - b*x^2)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, (b*x^2)/a])/(x*(1 -
 (b*x^2)/a)^(3/4)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4)/x^2,x)
[Out] int((-b*x^2+a)^(3/4)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(3/4)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.59, size = 31, normalized size = 0.41

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4)/x**2,x)
[Out] -a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(3/4)/x^2, x)
```

Mupad [B]
time = 5.17, size = 41, normalized size = 0.54

$$\frac{2\,{\left(a-b\,x^2\right)}^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{a}{b\,x^2}\right)}{x\,{\left(1-\frac{a}{b\,x^2}\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(3/4)/x^2,x)
[Out] (2*(a - b*x^2)^(3/4)*hypergeom([-3/4, -1/4], 3/4, a/(b*x^2)))/(x*(1 - a/(b*
x^2))^(3/4))
```

3.809 $\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx$

Optimal. Leaf size=103

$$-\frac{(a-bx^2)^{3/4}}{3x^3}+\frac{b(a-bx^2)^{3/4}}{2ax}+\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a-bx^2}}$$

```
[Out] -1/3*(-b*x^2+a)^(3/4)/x^3+1/2*b*(-b*x^2+a)^(3/4)/a/x+1/2*b^(3/2)*(1-b*x^2/a
)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/
2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+
a)^(1/4)/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {283, 331, 235, 234}

$$\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a-bx^2}}+\frac{b(a-bx^2)^{3/4}}{2ax}-\frac{(a-bx^2)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4)/x^4,x]
```

```
[Out] -1/3*(a - b*x^2)^(3/4)/x^3 + (b*(a - b*x^2)^(3/4))/(2*a*x) + (b^(3/2)*(1 -
(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a
 - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx &= -\frac{(a-bx^2)^{3/4}}{3x^3}-\frac{1}{2}b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx\\
&= -\frac{(a-bx^2)^{3/4}}{3x^3}+\frac{b(a-bx^2)^{3/4}}{2ax}+\frac{b^2\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{4a}\\
&= -\frac{(a-bx^2)^{3/4}}{3x^3}+\frac{b(a-bx^2)^{3/4}}{2ax}+\frac{\left(b^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{4a\sqrt[4]{a-bx^2}}\\
&= -\frac{(a-bx^2)^{3/4}}{3x^3}+\frac{b(a-bx^2)^{3/4}}{2ax}+\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 52, normalized size = 0.50

$$-\frac{(a-bx^2)^{3/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4)/x^4,x]
```

```
[Out] -1/3*((a - b*x^2)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, (b*x^2)/a])/(x^
3*(1 - (b*x^2)/a)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4)/x^4,x)
[Out] int((-b*x^2+a)^(3/4)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(3/4)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.62, size = 36, normalized size = 0.35

$$-\frac{a^{\frac{3}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},-\frac{3}{4}\\ -\frac{1}{2}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4)/x**4,x)
[Out] -a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(3/4)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x^2\right)}^{3/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(3/4)/x^4,x)
[Out] int((a - b*x^2)^(3/4)/x^4, x)
```

3.810 $\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx$

Optimal. Leaf size=128

$$-\frac{(a-bx^2)^{3/4}}{5x^5}+\frac{b(a-bx^2)^{3/4}}{10ax^3}+\frac{3b^2(a-bx^2)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a-bx^2}}$$

```
[Out] -1/5*(-b*x^2+a)^(3/4)/x^5+1/10*b*(-b*x^2+a)^(3/4)/a/x^3+3/20*b^2*(-b*x^2+a)
^(3/4)/a^2/x+3/20*b^(5/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x
*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {283, 331, 235, 234}

$$\frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a-bx^2}}+\frac{3b^2(a-bx^2)^{3/4}}{20a^2x}-\frac{(a-bx^2)^{3/4}}{5x^5}+\frac{b(a-bx^2)^{3/4}}{10ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4)/x^6,x]
```

```
[Out] -1/5*(a - b*x^2)^(3/4)/x^5 + (b*(a - b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a -
 b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(3/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx &= -\frac{(a-bx^2)^{3/4}}{5x^5}-\frac{1}{10}(3b)\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx \\
&= -\frac{(a-bx^2)^{3/4}}{5x^5}+\frac{b(a-bx^2)^{3/4}}{10ax^3}-\frac{(3b^2)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{20a} \\
&= -\frac{(a-bx^2)^{3/4}}{5x^5}+\frac{b(a-bx^2)^{3/4}}{10ax^3}+\frac{3b^2(a-bx^2)^{3/4}}{20a^2x}+\frac{(3b^3)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{40a^2} \\
&= -\frac{(a-bx^2)^{3/4}}{5x^5}+\frac{b(a-bx^2)^{3/4}}{10ax^3}+\frac{3b^2(a-bx^2)^{3/4}}{20a^2x}+\frac{\left(3b^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}\cdots}\,d\cdots}{40a^2\sqrt[4]{a-bx^2}} \\
&= -\frac{(a-bx^2)^{3/4}}{5x^5}+\frac{b(a-bx^2)^{3/4}}{10ax^3}+\frac{3b^2(a-bx^2)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{a}}\cdots\right.\right.}{20a^{3/2}\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 52, normalized size = 0.41

$$-\frac{(a-bx^2)^{3/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4)/x^6,x]
```

```
[Out] -1/5*((a - b*x^2)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, (b*x^2)/a])/(x^
5*(1 - (b*x^2)/a)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4)/x^6,x)
[Out] int((-b*x^2+a)^(3/4)/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(3/4)/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/x^6, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.66, size = 36, normalized size = 0.28

$$-\frac{a^{\frac{3}{4}}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, -\frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{2i\pi}}{a}} \right)}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4)/x**6,x)
[Out] -a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x^2\right)}^{3/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(3/4)/x^6,x)
```

```
[Out] int((a - b*x^2)^(3/4)/x^6, x)
```

3.811 $\int (a+bx^2)^{5/4}\,dx$

Optimal. Leaf size=92

$$\frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{10a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\,\left(a+bx^2\right)^{3/4}}$$

```
[Out] 10/21*a*x*(b*x^2+a)^(1/4)+2/7*x*(b*x^2+a)^(5/4)+10/21*a^(5/2)*(1+b*x^2/a)^(
3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/
a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(
3/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {201, 239, 237}

$$\frac{10a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\,\left(a+bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/4),x]
```

```
[Out] (10*a*x*(a + b*x^2)^(1/4))/21 + (2*x*(a + b*x^2)^(5/4))/7 + (10*a^(5/2)*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]
*(a + b*x^2)^(3/4))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^{5/4}\,dx &= \frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{1}{7}(5a)\int \sqrt[4]{a+bx^2}\,dx\\
&= \frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{1}{21}\left(5a^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx\\
&= \frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{\left(5a^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{21\left(a+bx^2\right)^{3/4}}\\
&= \frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{10a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.53, size = 47, normalized size = 0.51

$$
\frac{ax\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/4),x]
[Out] (a*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^(1/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \left(b\,x^2+a\right)^{\frac{5}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/4),x)
[Out] int((b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(5/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.62, size = 26, normalized size = 0.28

$$a^{\frac{5}{4}}x{}_2F_1\left(\begin{matrix} -\frac{5}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/4),x)
[Out] a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/4), x)
```

Mupad [B]
time = 4.86, size = 37, normalized size = 0.40

$$\frac{x\,{\left(b\,x^2+a\right)}^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/4),x)
```

```
[Out] (x*(a + b*x^2)^(5/4)*hypergeom([-5/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(5/4)
```

3.812 $\int (a-bx^2)^{5/4}\,dx$

Optimal. Leaf size=96

$$\frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\,\left(a-bx^2\right)^{3/4}}$$

```
[Out] 10/21*a*x*(-b*x^2+a)^(1/4)+2/7*x*(-b*x^2+a)^(5/4)+10/21*a^(5/2)*(1-b*x^2/a)
^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a
)^(3/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {201, 239, 238}

$$\frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\,\left(a-bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/4),x]
```

```
[Out] (10*a*x*(a - b*x^2)^(1/4))/21 + (2*x*(a - b*x^2)^(5/4))/7 + (10*a^(5/2)*(1
- (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]
*(a - b*x^2)^(3/4))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \left(a-bx^2\right)^{5/4}\,dx &= \frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{1}{7}(5a)\int \sqrt[4]{a-bx^2}\,dx\\ &= \frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{1}{21}\left(5a^2\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx\\ &= \frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{\left(5a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{21\left(a-bx^2\right)^{3/4}}\\ &= \frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a-bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.60, size = 48, normalized size = 0.50

$$\frac{ax\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/4),x]
[Out] (a*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(1/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/4),x)
[Out] int((-b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(5/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.64, size = 27, normalized size = 0.28

$$a^{\frac{5}{4}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/4),x)
```

```
[Out] a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/4), x)
```

Mupad [B]
time = 4.84, size = 38, normalized size = 0.40

$$\frac{x\,{\left(a-b\,x^2\right)}^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(1-\frac{b\,x^2}{a}\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/4),x)
```

```
[Out] (x*(a - b*x^2)^(5/4)*hypergeom([-5/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a
)^(5/4)
```

3.813 $\int (a+bx^2)^{7/4}\, dx$

Optimal. Leaf size=111

$$\frac{14a^2x}{15\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}-\frac{14a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

```
[Out] 14/15*a^2*x/(b*x^2+a)^(1/4)+14/45*a*x*(b*x^2+a)^(3/4)+2/9*x*(b*x^2+a)^(7/4)
-14/15*a^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^
(1/2))),2^(1/2))/(b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {201, 235, 233, 202}

$$-\frac{14a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\,\sqrt[4]{a+bx^2}}+\frac{14a^2x}{15\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/4),x]
```

```
[Out] (14*a^2*x)/(15*(a + b*x^2)^(1/4)) + (14*a*x*(a + b*x^2)^(3/4))/45 + (2*x*(a
 + b*x^2)^(7/4))/9 - (14*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(15*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int (a+bx^2)^{7/4}\,dx = \frac{2}{9}x(a+bx^2)^{7/4} + \frac{1}{9}(7a)\int (a+bx^2)^{3/4}\,dx$$

$$= \frac{14}{45}ax(a+bx^2)^{3/4} + \frac{2}{9}x(a+bx^2)^{7/4} + \frac{1}{15}(7a^2)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx$$

$$= \frac{14}{45}ax(a+bx^2)^{3/4} + \frac{2}{9}x(a+bx^2)^{7/4} + \frac{\left(7a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{15\sqrt[4]{a+bx^2}}$$

$$= \frac{14a^2x}{15\sqrt[4]{a+bx^2}} + \frac{14}{45}ax(a+bx^2)^{3/4} + \frac{2}{9}x(a+bx^2)^{7/4} - \frac{\left(7a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}}{15\sqrt[4]{a+bx^2}}$$

$$= \frac{14a^2x}{15\sqrt[4]{a+bx^2}} + \frac{14}{45}ax(a+bx^2)^{3/4} + \frac{2}{9}x(a+bx^2)^{7/4} - \frac{14a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\right.\right.}{15\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.91, size = 47, normalized size = 0.42

$$\frac{ax(a+bx^2)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(7/4),x]
[Out] (a*x*(a + b*x^2)^(3/4)*Hypergeometric2F1[-7/4, 1/2, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^(3/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/4),x)
```

```
[Out] int((b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(7/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.80, size = 26, normalized size = 0.23

$$a^{\frac{7}{4}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/4),x)
```

```
[Out] a**(7/4)*x*hyper((-7/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(7/4), x)
```

Mupad [B]
time = 4.82, size = 37, normalized size = 0.33

$$\frac{x\,(b\,x^2+a)^{7/4}\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(7/4),x)
[Out] (x*(a + b*x^2)^(7/4)*hypergeom([-7/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(7/4)
```

3.814 $\int \left(a-bx^2\right)^{7/4}\,dx$

Optimal. Leaf size=96

$$\frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}+\frac{14a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

```
[Out] 14/45*a*x*(-b*x^2+a)^(3/4)+2/9*x*(-b*x^2+a)^(7/4)+14/15*a^(5/2)*(1-b*x^2/a)
^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2
)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a
)^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {201, 235, 234}

$$\frac{14a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\,\sqrt[4]{a-bx^2}}+\frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(7/4),x]
```

```
[Out] (14*a*x*(a - b*x^2)^(3/4))/45 + (2*x*(a - b*x^2)^(7/4))/9 + (14*a^(5/2)*(1
- (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*Sqrt[b]
*(a - b*x^2)^(1/4))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

`& PosQ[a]`

Rubi steps

$$
\begin{aligned}
\int \left(a-bx^2\right)^{7/4}\,dx &= \frac{2}{9}x\left(a-bx^2\right)^{7/4}+\frac{1}{9}(7a)\int \left(a-bx^2\right)^{3/4}\,dx\\
&= \frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}+\frac{1}{15}\left(7a^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx\\
&= \frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}+\frac{\left(7a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{15\sqrt[4]{a-bx^2}}\\
&= \frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}+\frac{14a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\,\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.77, size = 48, normalized size = 0.50

$$\frac{ax\left(a-bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(7/4),x]
[Out] (a*x*(a - b*x^2)^(3/4)*Hypergeometric2F1[-7/4, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(3/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(7/4),x)
[Out] int((-b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(7/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.79, size = 27, normalized size = 0.28

$$a^{\frac{7}{4}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(7/4),x)
```

```
[Out] a**(7/4)*x*hyper((-7/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(7/4), x)
```

Mupad [B]
time = 4.82, size = 38, normalized size = 0.40

$$\frac{x\,{\left(a-b\,x^2\right)}^{7/4}\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(1-\frac{b\,x^2}{a}\right)}^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(7/4),x)
```

```
[Out] (x*(a - b*x^2)^(7/4)*hypergeom([-7/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a
)^(7/4)
```

3.815 $\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}}+\frac{8a^2x(a+bx^2)^{3/4}}{39b^3}-\frac{20ax^3(a+bx^2)^{3/4}}{117b^2}+\frac{2x^5(a+bx^2)^{3/4}}{13b}+\frac{16a^{7/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{39b^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -16/39*a^3*x/b^3/(b*x^2+a)^(1/4)+8/39*a^2*x*(b*x^2+a)^(3/4)/b^3-20/117*a*x^
3*(b*x^2+a)^(3/4)/b^2+2/13*x^5*(b*x^2+a)^(3/4)/b+16/39*a^(7/2)*(1+b*x^2/a)^
(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)
/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b
*x^2+a)^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {327, 235, 233, 202}

$$\frac{16a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx^2}}-\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}}+\frac{8a^2x(a+bx^2)^{3/4}}{39b^3}-\frac{20ax^3(a+bx^2)^{3/4}}{117b^2}+\frac{2x^5(a+bx^2)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(1/4),x]
```

```
[Out] (-16*a^3*x)/(39*b^3*(a + b*x^2)^(1/4)) + (8*a^2*x*(a + b*x^2)^(3/4))/(39*b^
3) - (20*a*x^3*(a + b*x^2)^(3/4))/(117*b^2) + (2*x^5*(a + b*x^2)^(3/4))/(13
*b) + (16*a^(7/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(39*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx = \frac{2x^5(a+bx^2)^{3/4}}{13b} - \frac{(10a)\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx}{13b}$$

$$= -\frac{20ax^3(a+bx^2)^{3/4}}{117b^2} + \frac{2x^5(a+bx^2)^{3/4}}{13b} + \frac{(20a^2)\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{39b^2}$$

$$= \frac{8a^2x(a+bx^2)^{3/4}}{39b^3} - \frac{20ax^3(a+bx^2)^{3/4}}{117b^2} + \frac{2x^5(a+bx^2)^{3/4}}{13b} - \frac{(8a^3)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{39b^3}$$

$$= \frac{8a^2x(a+bx^2)^{3/4}}{39b^3} - \frac{20ax^3(a+bx^2)^{3/4}}{117b^2} + \frac{2x^5(a+bx^2)^{3/4}}{13b} - \frac{\left(8a^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+}}}{39b^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}} + \frac{8a^2x(a+bx^2)^{3/4}}{39b^3} - \frac{20ax^3(a+bx^2)^{3/4}}{117b^2} + \frac{2x^5(a+bx^2)^{3/4}}{13b} + \frac{\left(8a^3\sqrt[4]{\right.}}{}$$

$$= -\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}} + \frac{8a^2x(a+bx^2)^{3/4}}{39b^3} - \frac{20ax^3(a+bx^2)^{3/4}}{117b^2} + \frac{2x^5(a+bx^2)^{3/4}}{13b} + \frac{16a^{7/2}\sqrt[4]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.88, size = 90, normalized size = 0.62

$$\frac{2\left(12a^3x+2a^2bx^3-ab^2x^5+9b^3x^7-12a^3x\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{117b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*(12*a^3*x + 2*a^2*b*x^3 - a*b^2*x^5 + 9*b^3*x^7 - 12*a^3*x*(1 + (b*x^2)/
a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(117*b^3*(a + b*x
^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^6/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 27, normalized size = 0.18

$$\frac{x^{7}{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(1/4),x)
```

```
[Out] x**7*hyper((1/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^6/(b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(1/4),x)
[Out] int(x^6/(a + b*x^2)^(1/4), x)
```

3.816 $\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=122

$$\frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}}-\frac{4ax(a+bx^2)^{3/4}}{15b^2}+\frac{2x^3(a+bx^2)^{3/4}}{9b}-\frac{8a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 8/15*a^2*x/b^2/(b*x^2+a)^(1/4)-4/15*a*x*(b*x^2+a)^(3/4)/b^2+2/9*x^3*(b*x^2+
a)^(3/4)/b-8/15*a^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b
^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^2+a)^(1/4)
```

Rubi [A]

time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {327, 235, 233, 202}

$$-\frac{8a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^2}}+\frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}}-\frac{4ax(a+bx^2)^{3/4}}{15b^2}+\frac{2x^3(a+bx^2)^{3/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(1/4),x]
```

```
[Out] (8*a^2*x)/(15*b^2*(a + b*x^2)^(1/4)) - (4*a*x*(a + b*x^2)^(3/4))/(15*b^2) +
 (2*x^3*(a + b*x^2)^(3/4))/(9*b) - (8*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ellipti
cE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx = \frac{2x^3(a+bx^2)^{3/4}}{9b} - \frac{(2a)\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{3b}$$

$$= -\frac{4ax(a+bx^2)^{3/4}}{15b^2} + \frac{2x^3(a+bx^2)^{3/4}}{9b} + \frac{(4a^2)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{15b^2}$$

$$= -\frac{4ax(a+bx^2)^{3/4}}{15b^2} + \frac{2x^3(a+bx^2)^{3/4}}{9b} + \frac{\left(4a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{15b^2\sqrt[4]{a+bx^2}}$$

$$= \frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}} - \frac{4ax(a+bx^2)^{3/4}}{15b^2} + \frac{2x^3(a+bx^2)^{3/4}}{9b} - \frac{\left(4a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}}{15b^2\sqrt[4]{a+bx^2}}$$

$$= \frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}} - \frac{4ax(a+bx^2)^{3/4}}{15b^2} + \frac{2x^3(a+bx^2)^{3/4}}{9b} - \frac{8a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\right.\right.}{15b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.66, size = 79, normalized size = 0.65

$$\frac{2\left(-6a^2x - abx^3 + 5b^2x^5 + 6a^2x\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{45b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*(-6*a^2*x - a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeo
metric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(45*b^2*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/4),x)
[Out] int(x^4/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(x^4/(b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 27, normalized size = 0.22

$$\frac{x^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/4),x)
[Out] x**5*hyper((1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/4),x)
[Out] int(x^4/(a + b*x^2)^(1/4), x)
```

3.817 $\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=98

$$-\frac{4ax}{5b\sqrt[4]{a+bx^2}}+\frac{2x(a+bx^2)^{3/4}}{5b}+\frac{4a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -4/5*a*x/b/(b*x^2+a)^(1/4)+2/5*x*(b*x^2+a)^(3/4)/b+4/5*a^(3/2)*(1+b*x^2/a)^
(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)
/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b
*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {327, 235, 233, 202}

$$\frac{4a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4ax}{5b\sqrt[4]{a+bx^2}}+\frac{2x(a+bx^2)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(1/4),x]
```

```
[Out] (-4*a*x)/(5*b*(a + b*x^2)^(1/4)) + (2*x*(a + b*x^2)^(3/4))/(5*b) + (4*a^(3/
2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^
(3/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \frac{2x(a+bx^2)^{3/4}}{5b} - \frac{(2a)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{5b}$$

$$= \frac{2x(a+bx^2)^{3/4}}{5b} - \frac{\left(2a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{5b\sqrt[4]{a+bx^2}}$$

$$= -\frac{4ax}{5b\sqrt[4]{a+bx^2}} + \frac{2x(a+bx^2)^{3/4}}{5b} + \frac{\left(2a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5b\sqrt[4]{a+bx^2}}$$

$$= -\frac{4ax}{5b\sqrt[4]{a+bx^2}} + \frac{2x(a+bx^2)^{3/4}}{5b} + \frac{4a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.52, size = 62, normalized size = 0.63

$$\frac{2x\left(a+bx^2-a\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
-((b*x^2)/a)]))/(5*b*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/4),x)
[Out] int(x^2/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(x^2/(b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 27, normalized size = 0.28

$$\frac{x^3\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/4),x)
[Out] x**3*hyper((1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/4),x)
```

```
[Out] int(x^2/(a + b*x^2)^(1/4), x)
```

3.818 $\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=71

$$\frac{2x}{\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

```
[Out] 2*x/(b*x^2+a)^(1/4)-2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(
1/2)/a^(1/2))),2^(1/2))*a^(1/2)/(b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {235, 233, 202}

$$\frac{2x}{\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1/4),x]
```

```
[Out] (2*x)/(a + b*x^2)^(1/4) - (2*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{2x}{\sqrt[4]{a+bx^2}} - \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{2x}{\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.46, size = 46, normalized size = 0.65

$$\frac{x\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1/4),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(1/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.43, size = 24, normalized size = 0.34

$$\frac{x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4),x)
```

```
[Out] x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-1/4), x)
```

Mupad [B]
time = 4.85, size = 37, normalized size = 0.52

$$\frac{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/4),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(1/4)*hypergeom([1/4, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(1/4)
```

3.819 $\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=93

$$\frac{bx}{a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}-\frac{\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a+bx^2}}$$

```
[Out] b*x/a/(b*x^2+a)^(1/4)-(b*x^2+a)^(3/4)/a/x-(1+b*x^2/a)^(1/4)*(cos(1/2*arctan
(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(
sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(b*x^2+a)^(1/4)/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {331, 235, 233, 202}

$$-\frac{\sqrt{b}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a+bx^2}}+\frac{bx}{a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(1/4)),x]
```

```
[Out] (b*x)/(a*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(a*x) - (Sqrt[b]*(1 + (b*x^
2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^
2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx &= -\frac{(a+bx^2)^{3/4}}{ax}+\frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{2a}\\ &= -\frac{(a+bx^2)^{3/4}}{ax}+\frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2a\sqrt[4]{a+bx^2}}\\ &= \frac{bx}{a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{ax}-\frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2a\sqrt[4]{a+bx^2}}\\ &= \frac{bx}{a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{ax}-\frac{\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.68, size = 49, normalized size = 0.53

$$-\frac{\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(1/4)),x]
[Out] -(((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(1/4)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b*x^4 + a*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 27, normalized size = 0.29

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[4]{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-1/2, 1/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/4)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^2), x)
```

Mupad [B]
time = 5.09, size = 40, normalized size = 0.43

$$-\frac{2\,{\left(\frac{a}{b\,x^2}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{b\,x^2}\right)}{3\,x\,{\left(b\,x^2+a\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/4)),x)
[Out] -(2*(a/(b*x^2) + 1)^(1/4)*hypergeom([1/4, 3/4], 7/4, -a/(b*x^2)))/(3*x*(a +
 b*x^2)^(1/4))
```

3.820 $\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=124

$$-\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{3ax^3}+\frac{b(a+bx^2)^{3/4}}{2a^2x}+\frac{b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -1/2*b^2*x/a^2/(b*x^2+a)^(1/4)-1/3*(b*x^2+a)^(3/4)/a/x^3+1/2*b*(b*x^2+a)^(3
/4)/a^2/x+1/2*b^(3/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(
1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {331, 235, 233, 202}

$$\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^2}}-\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}}+\frac{b(a+bx^2)^{3/4}}{2a^2x}-\frac{(a+bx^2)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(1/4)),x]
```

```
[Out] -1/2*(b^2*x)/(a^2*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(3*a*x^3) + (b*(a
+ b*x^2)^(3/4))/(2*a^2*x) + (b^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx &= -\frac{(a+bx^2)^{3/4}}{3ax^3}-\frac{b\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{2a}\\
&= -\frac{(a+bx^2)^{3/4}}{3ax^3}+\frac{b(a+bx^2)^{3/4}}{2a^2x}-\frac{b^2\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4a^2}\\
&= -\frac{(a+bx^2)^{3/4}}{3ax^3}+\frac{b(a+bx^2)^{3/4}}{2a^2x}-\frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4a^2\sqrt[4]{a+bx^2}}\\
&= -\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{3ax^3}+\frac{b(a+bx^2)^{3/4}}{2a^2x}+\frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a^2\sqrt[4]{a+bx^2}}\\
&= -\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{3ax^3}+\frac{b(a+bx^2)^{3/4}}{2a^2x}+\frac{b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right.}{2a^{3/2}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.41

$$
-\frac{\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(1/4)),x]
[Out] -1/3*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(b\,x^{2}+a \right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/4),x)
[Out] int(1/x^4/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/(b*x^6 + a*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.56, size = 32, normalized size = 0.26

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{3 \sqrt[4]{a} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/4),x)
[Out] -hyper((-3/2, 1/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/4)),x)
[Out] int(1/(x^4*(a + b*x^2)^(1/4)), x)
```

3.821 $\int \frac{1}{x^6\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=148

$$\frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{5ax^5}+\frac{7b(a+bx^2)^{3/4}}{30a^2x^3}-\frac{7b^2(a+bx^2)^{3/4}}{20a^3x}-\frac{7b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 7/20*b^3*x/a^3/(b*x^2+a)^(1/4)-1/5*(b*x^2+a)^(3/4)/a/x^5+7/30*b*(b*x^2+a)^(
3/4)/a^2/x^3-7/20*b^2*(b*x^2+a)^(3/4)/a^3/x-7/20*b^(5/2)*(1+b*x^2/a)^(1/4)*
(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/
2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^2+a
)^(1/4)
```

Rubi [A]

time = 0.04, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {331, 235, 233, 202}

$$-\frac{7b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a+bx^2}}+\frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}}-\frac{7b^2(a+bx^2)^{3/4}}{20a^3x}+\frac{7b(a+bx^2)^{3/4}}{30a^2x^3}-\frac{(a+bx^2)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(1/4)),x]
```

```
[Out] (7*b^3*x)/(20*a^3*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*a*x^5) + (7*b*(
a + b*x^2)^(3/4))/(30*a^2*x^3) - (7*b^2*(a + b*x^2)^(3/4))/(20*a^3*x) - (7*
b^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(20*a^(5/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[4]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{3/4}}{5ax^5} - \frac{(7b)\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx}{10a}$$

$$= -\frac{(a+bx^2)^{3/4}}{5ax^5} + \frac{7b(a+bx^2)^{3/4}}{30a^2x^3} + \frac{(7b^2)\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{20a^2}$$

$$= -\frac{(a+bx^2)^{3/4}}{5ax^5} + \frac{7b(a+bx^2)^{3/4}}{30a^2x^3} - \frac{7b^2(a+bx^2)^{3/4}}{20a^3x} + \frac{(7b^3)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{40a^3}$$

$$= -\frac{(a+bx^2)^{3/4}}{5ax^5} + \frac{7b(a+bx^2)^{3/4}}{30a^2x^3} - \frac{7b^2(a+bx^2)^{3/4}}{20a^3x} + \frac{\left(7b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,d}{40a^3\sqrt[4]{a+bx^2}}$$

$$= \frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{5ax^5} + \frac{7b(a+bx^2)^{3/4}}{30a^2x^3} - \frac{7b^2(a+bx^2)^{3/4}}{20a^3x} - \frac{\left(7b^3\sqrt[4]{1+\frac{bx^2}{a}}\right.}{40a^3\sqrt[4]{a}}$$

$$= \frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{5ax^5} + \frac{7b(a+bx^2)^{3/4}}{30a^2x^3} - \frac{7b^2(a+bx^2)^{3/4}}{20a^3x} - \frac{7b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\ E}{20a^{5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.34

$$-\frac{\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(1/4)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, -((b*x^2)/a)
])/(x^5*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b*x^8 + a*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.66, size = 32, normalized size = 0.22

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{5 \sqrt[4]{a} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-5/2, 1/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(1/4)),x)
[Out] int(1/(x^6*(a + b*x^2)^(1/4)), x)
```

3.822 $\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=129

$$-\frac{8a^2x(a-bx^2)^{3/4}}{39b^3}-\frac{20ax^3(a-bx^2)^{3/4}}{117b^2}-\frac{2x^5(a-bx^2)^{3/4}}{13b}+\frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^2}}$$

```
[Out] -8/39*a^2*x*(-b*x^2+a)^(3/4)/b^3-20/117*a*x^3*(-b*x^2+a)^(3/4)/b^2-2/13*x^5
*(-b*x^2+a)^(3/4)/b+16/39*a^(7/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*
arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {327, 235, 234}

$$\frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^2}}-\frac{8a^2x(a-bx^2)^{3/4}}{39b^3}-\frac{20ax^3(a-bx^2)^{3/4}}{117b^2}-\frac{2x^5(a-bx^2)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^2)^(1/4),x]
```

```
[Out] (-8*a^2*x*(a - b*x^2)^(3/4))/(39*b^3) - (20*a*x^3*(a - b*x^2)^(3/4))/(117*b
^2) - (2*x^5*(a - b*x^2)^(3/4))/(13*b) + (16*a^(7/2)*(1 - (b*x^2)/a)^(1/4)*
EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(39*b^(7/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx &= -\frac{2x^5(a-bx^2)^{3/4}}{13b} + \frac{(10a)\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx}{13b} \\
&= -\frac{20ax^3(a-bx^2)^{3/4}}{117b^2} - \frac{2x^5(a-bx^2)^{3/4}}{13b} + \frac{(20a^2)\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{39b^2} \\
&= -\frac{8a^2x(a-bx^2)^{3/4}}{39b^3} - \frac{20ax^3(a-bx^2)^{3/4}}{117b^2} - \frac{2x^5(a-bx^2)^{3/4}}{13b} + \frac{(8a^3)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{39b^3} \\
&= -\frac{8a^2x(a-bx^2)^{3/4}}{39b^3} - \frac{20ax^3(a-bx^2)^{3/4}}{117b^2} - \frac{2x^5(a-bx^2)^{3/4}}{13b} + \frac{\left(8a^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{}{\sqrt[4]{1-}}}{39b^3\sqrt[4]{a-bx^2}} \\
&= -\frac{8a^2x(a-bx^2)^{3/4}}{39b^3} - \frac{20ax^3(a-bx^2)^{3/4}}{117b^2} - \frac{2x^5(a-bx^2)^{3/4}}{13b} + \frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin\right.}{39b^{7/2}\sqrt[4]{a-b}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.81, size = 89, normalized size = 0.69

$$\frac{2x\left(-12a^3+2a^2bx^2+ab^2x^4+9b^3x^6+12a^3\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{117b^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^2)^(1/4),x]
[Out] (2*x*(-12*a^3 + 2*a^2*b*x^2 + a*b^2*x^4 + 9*b^3*x^6 + 12*a^3*(1 - (b*x^2)/a
)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(117*b^3*(a - b*x^2)^
(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(x^6/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*x^6/(b*x^2 - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.57, size = 29, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**2+a)**(1/4),x)
```

```
[Out] x**7*hyper((1/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^2)^(1/4),x)
[Out] int(x^6/(a - b*x^2)^(1/4), x)
```

3.823 $\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=104

$$-\frac{4ax(a-bx^2)^{3/4}}{15b^2}-\frac{2x^3(a-bx^2)^{3/4}}{9b}+\frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}$$

```
[Out] -4/15*a*x*(-b*x^2+a)^(3/4)/b^2-2/9*x^3*(-b*x^2+a)^(3/4)/b+8/15*a^(5/2)*(1-b
*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x
*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^
(5/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {327, 235, 234}

$$\frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}-\frac{4ax(a-bx^2)^{3/4}}{15b^2}-\frac{2x^3(a-bx^2)^{3/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2)^(1/4),x]
```

```
[Out] (-4*a*x*(a - b*x^2)^(3/4))/(15*b^2) - (2*x^3*(a - b*x^2)^(3/4))/(9*b) + (8*
a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(15*b^(5/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx &= -\frac{2x^3(a-bx^2)^{3/4}}{9b}+\frac{(2a)\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{3b}\\
&= -\frac{4ax(a-bx^2)^{3/4}}{15b^2}-\frac{2x^3(a-bx^2)^{3/4}}{9b}+\frac{(4a^2)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{15b^2}\\
&= -\frac{4ax(a-bx^2)^{3/4}}{15b^2}-\frac{2x^3(a-bx^2)^{3/4}}{9b}+\frac{\left(4a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{15b^2\sqrt[4]{a-bx^2}}\\
&= -\frac{4ax(a-bx^2)^{3/4}}{15b^2}-\frac{2x^3(a-bx^2)^{3/4}}{9b}+\frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.68, size = 79, normalized size = 0.76

$$\frac{2\left(-6a^2x+abx^3+5b^2x^5+6a^2x\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{45b^2\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2)^(1/4),x]
```

```
[Out] (2*(-6*a^2*x + a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeo
metric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(45*b^2*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^2+a)^(1/4),x)
[Out] int(x^4/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^4/(-b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)*x^4/(b*x^2 - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 29, normalized size = 0.28

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**2+a)**(1/4),x)
[Out] x**5*hyper((1/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^4/(-b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^2)^(1/4),x)
[Out] int(x^4/(a - b*x^2)^(1/4), x)
```

3.824 $\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2x(a-bx^2)^{3/4}}{5b}+\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}$$

```
[Out] -2/5*x*(-b*x^2+a)^(3/4)/b+4/5*a^(3/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {327, 235, 234}

$$\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x(a-bx^2)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^(1/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx &= -\frac{2x(a-bx^2)^{3/4}}{5b}+\frac{(2a)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{5b}\\ &= -\frac{2x(a-bx^2)^{3/4}}{5b}+\frac{\left(2a\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{5b\sqrt[4]{a-bx^2}}\\ &= -\frac{2x(a-bx^2)^{3/4}}{5b}+\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.54, size = 64, normalized size = 0.79

$$\frac{2x\left(-a+bx^2+a\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{5b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^(1/4),x]
[Out] (2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
 (b*x^2)/a]))/(5*b*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(1/4),x)
[Out] int(x^2/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*x^2/(b*x^2 - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 29, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(1/4),x)
```

```
[Out] x**3*hyper((1/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a-b\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^(1/4),x)
```

```
[Out] int(x^2/(a - b*x^2)^(1/4), x)
```

3.825 $\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{a}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

```
[Out] 2*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
csin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/
2))*a^(1/2)/(-b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {235, 234}

$$\frac{2\sqrt{a}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-1/4),x]
```

```
[Out] (2*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx = \frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a-bx^2}}$$
$$= \frac{2\sqrt{a}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.53, size = 47, normalized size = 0.81

$$\frac{x\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-1/4),x]
[Out] (x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(a -
b*x^2)^(1/4)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/4),x)
[Out] int(1/(-b*x^2+a)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^2 - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 26, normalized size = 0.45

$$\frac{x\,{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/4),x)
[Out] x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(-1/4), x)
```

Mupad [B]
time = 4.87, size = 38, normalized size = 0.66

$$\frac{x\,{\left(1-\frac{b\,x^2}{a}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(a-b\,x^2\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(1/4),x)
[Out] (x*(1 - (b*x^2)/a)^(1/4)*hypergeom([1/4, 1/2], 3/2, (b*x^2)/a))/(a - b*x^2)
^(1/4)
```

3.826 $\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{(a-bx^2)^{3/4}}{ax}-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}$$

```
[Out] -(-b*x^2+a)^(3/4)/a/x-(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(
1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(-b*x^2+a)^(1/4)/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {331, 235, 234}

$$-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}-\frac{(a-bx^2)^{3/4}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^(1/4)),x]
```

```
[Out] -((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx &= -\frac{\left(a-bx^2\right)^{3/4}}{ax}-\frac{b\int\frac{1}{\sqrt[4]{a-bx^2}}\,dx}{2a}\\
&= -\frac{\left(a-bx^2\right)^{3/4}}{ax}-\frac{\left(b\sqrt[4]{1-\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{2a\sqrt[4]{a-bx^2}}\\
&= -\frac{\left(a-bx^2\right)^{3/4}}{ax}-\frac{\sqrt{b}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.69, size = 50, normalized size = 0.63

$$
-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{x\sqrt[4]{a-bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^(1/4)),x]
[Out] -(((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (b*x^2)/a])/(x*(
a - b*x^2)^(1/4)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^{2}\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^(1/4),x)
[Out] int(1/x^2/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^4 - a*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 29, normalized size = 0.37

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[4]{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-1/2, 1/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(1/4)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^2), x)
```

Mupad [B]
time = 5.08, size = 41, normalized size = 0.52

$$-\frac{2\,{\left(1-\frac{a}{b\,x^2}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{a}{b\,x^2}\right)}{3\,x\,{\left(a-b\,x^2\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^(1/4)),x)
```

```
[Out] -(2*(1 - a/(b*x^2))^(1/4)*hypergeom([1/4, 3/4], 7/4, a/(b*x^2)))/(3*x*(a -
b*x^2)^(1/4))
```

3.827 $\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=106

$$-\frac{(a-bx^2)^{3/4}}{3ax^3}-\frac{b(a-bx^2)^{3/4}}{2a^2x}-\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a-bx^2}}$$

```
[Out] -1/3*(-b*x^2+a)^(3/4)/a/x^3-1/2*b*(-b*x^2+a)^(3/4)/a^2/x-1/2*b^(3/2)*(1-b*x
^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3
/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {331, 235, 234}

$$-\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a-bx^2}}-\frac{b(a-bx^2)^{3/4}}{2a^2x}-\frac{(a-bx^2)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^2)^(1/4)),x]
```

```
[Out] -1/3*(a - b*x^2)^(3/4)/(a*x^3) - (b*(a - b*x^2)^(3/4))/(2*a^2*x) - (b^(3/2)
*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3
/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx &= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}+\frac{b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{2a}\\
&= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}-\frac{b\left(a-bx^2\right)^{3/4}}{2a^2x}-\frac{b^2\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{4a^2}\\
&= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}-\frac{b\left(a-bx^2\right)^{3/4}}{2a^2x}-\frac{\left(b^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{4a^2\sqrt[4]{a-bx^2}}\\
&= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}-\frac{b\left(a-bx^2\right)^{3/4}}{2a^2x}-\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 52, normalized size = 0.49

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^2)^(1/4)),x]
[Out] -1/3*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (b*x^2)/a])/
(x^3*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^4/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^6 - a*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 34, normalized size = 0.32

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[4]{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**2+a)**(1/4),x)
[Out] -hyper((-3/2, 1/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/4)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(a-b\,x^2\right)^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^2)^(1/4)),x)
[Out] int(1/(x^4*(a - b*x^2)^(1/4)), x)
```

3.828 $\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=131

$$-\frac{(a-bx^2)^{3/4}}{5ax^5}-\frac{7b(a-bx^2)^{3/4}}{30a^2x^3}-\frac{7b^2(a-bx^2)^{3/4}}{20a^3x}-\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a-bx^2}}$$

```
[Out] -1/5*(-b*x^2+a)^(3/4)/a/x^5-7/30*b*(-b*x^2+a)^(3/4)/a^2/x^3-7/20*b^2*(-b*x^
2+a)^(3/4)/a^3/x-7/20*b^(5/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcs
in(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {331, 235, 234}

$$-\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b^2(a-bx^2)^{3/4}}{20a^3x}-\frac{7b(a-bx^2)^{3/4}}{30a^2x^3}-\frac{(a-bx^2)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^2)^(1/4)),x]
```

```
[Out] -1/5*(a - b*x^2)^(3/4)/(a*x^5) - (7*b*(a - b*x^2)^(3/4))/(30*a^2*x^3) - (7*
b^2*(a - b*x^2)^(3/4))/(20*a^3*x) - (7*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*Ellipt
icE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(5/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx &= -\frac{(a-bx^2)^{3/4}}{5ax^5}+\frac{(7b)\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx}{10a}\\
&= -\frac{(a-bx^2)^{3/4}}{5ax^5}-\frac{7b(a-bx^2)^{3/4}}{30a^2x^3}+\frac{(7b^2)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{20a^2}\\
&= -\frac{(a-bx^2)^{3/4}}{5ax^5}-\frac{7b(a-bx^2)^{3/4}}{30a^2x^3}-\frac{7b^2(a-bx^2)^{3/4}}{20a^3x}-\frac{(7b^3)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{40a^3}\\
&= -\frac{(a-bx^2)^{3/4}}{5ax^5}-\frac{7b(a-bx^2)^{3/4}}{30a^2x^3}-\frac{7b^2(a-bx^2)^{3/4}}{20a^3x}-\frac{\left(7b^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}}{40a^3\sqrt[4]{a-bx^2}}\\
&= -\frac{(a-bx^2)^{3/4}}{5ax^5}-\frac{7b(a-bx^2)^{3/4}}{30a^2x^3}-\frac{7b^2(a-bx^2)^{3/4}}{20a^3x}-\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{20a^{5/2}\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 52, normalized size = 0.40

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^2)^(1/4)),x]
```

```
[Out] -1/5*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (b*x^2)/a])/
(x^5*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^6/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^8 - a*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 34, normalized size = 0.26

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[4]{a}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-5/2, 1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/4)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(a-b\,x^2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a - b*x^2)^(1/4)),x)
[Out] int(1/(x^6*(a - b*x^2)^(1/4)), x)
```

3.829 $\int \frac{x^6}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=124

$$\frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2}+\frac{2x^5\sqrt[4]{a+bx^2}}{11b}-\frac{80a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] 40/77*a^2*x*(b*x^2+a)^(1/4)/b^3-20/77*a*x^3*(b*x^2+a)^(1/4)/b^2+2/11*x^5*(b
*x^2+a)^(1/4)/b-80/77*a^(7/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arct
an(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 239, 237}

$$-\frac{80a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a+bx^2\right)^{3/4}}+\frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2}+\frac{2x^5\sqrt[4]{a+bx^2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(3/4),x]
```

```
[Out] (40*a^2*x*(a + b*x^2)^(1/4))/(77*b^3) - (20*a*x^3*(a + b*x^2)^(1/4))/(77*b^
2) + (2*x^5*(a + b*x^2)^(1/4))/(11*b) - (80*a^(7/2)*(1 + (b*x^2)/a)^(3/4)*E
llipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{(a+bx^2)^{3/4}}\,dx &= \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{(10a)\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx}{11b} \\
&= -\frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} + \frac{(60a^2)\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx}{77b^2} \\
&= \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{(40a^3)\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{77b^3} \\
&= \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{\left(40a^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right.}}{77b^3\,(a+bx^2)^{3/4}} \\
&= \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{80a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-}\right.}{77b^{7/2}\,(a+bx^2)^{3}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.13, size = 90, normalized size = 0.73

$$
\frac{2\left(20a^3x+10a^2bx^3-3ab^2x^5+7b^3x^7-20a^3x\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{77b^3\,(a+bx^2)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*(20*a^3*x + 10*a^2*b*x^3 - 3*a*b^2*x^5 + 7*b^3*x^7 - 20*a^3*x*(1 + (b*x^
2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(77*b^3*(a + b
*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{x^6}{(b\,x^2+a)^{\frac{3}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^6/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 27, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{7}{2}\\ \frac{9}{2}\end{matrix}\right| \frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(3/4),x)
```

```
[Out] x**7*hyper((3/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(3/4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(3/4),x)
[Out] int(x^6/(a + b*x^2)^(3/4), x)
```

3.830 $\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{4ax\sqrt[4]{a+bx^2}}{7b^2}+\frac{2x^3\sqrt[4]{a+bx^2}}{7b}+\frac{8a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] -4/7*a*x*(b*x^2+a)^(1/4)/b^2+2/7*x^3*(b*x^2+a)^(1/4)/b+8/7*a^(5/2)*(1+b*x^2
/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2
)/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {327, 239, 237}

$$\frac{8a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a+bx^2}}{7b^2}+\frac{2x^3\sqrt[4]{a+bx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(3/4),x]
```

```
[Out] (-4*a*x*(a + b*x^2)^(1/4))/(7*b^2) + (2*x^3*(a + b*x^2)^(1/4))/(7*b) + (8*a
^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
7*b^(5/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx &= \frac{2x^3\sqrt[4]{a+bx^2}}{7b} - \frac{(6a)\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx}{7b}\\
&= -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{(4a^2)\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{7b^2}\\
&= -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{\left(4a^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{7b^2\,(a+bx^2)^{3/4}}\\
&= -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{8a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\,(a+bx^2)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.04, size = 78, normalized size = 0.78

$$\frac{2\left(-2a^2x-abx^3+b^2x^5+2a^2x\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{7b^2\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*(-2*a^2*x - a*b*x^3 + b^2*x^5 + 2*a^2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeome
tric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(7*b^2*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(3/4),x)
```

[Out] int(x^4/(b*x^2+a)^(3/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(x^4/(b*x^2 + a)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 27, normalized size = 0.27

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(3/4),x)
[Out] x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(3/4),x)
```

```
[Out] int(x^4/(a + b*x^2)^(3/4), x)
```

3.831 $\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] 2/3*x*(b*x^2+a)^(1/4)/b-4/3*a^(3/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/
2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 239, 237}

$$\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx &= \frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{(2a)\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{3b}\\ &= \frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{\left(2a\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3b\,(a+bx^2)^{3/4}}\\ &= \frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\,(a+bx^2)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.72, size = 62, normalized size = 0.79

$$\frac{2x\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{3b\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(3/4),x]
[Out] (2*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(3*b*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/4),x)
[Out] int(x^2/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(x^2/(b*x^2 + a)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.43, size = 27, normalized size = 0.35

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/4),x)
[Out] x**3*hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(x^2/(b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(3/4),x)
[Out] int(x^2/(a + b*x^2)^(3/4), x)
```

3.832 $\int \frac{1}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,(a+bx^2)^{3/4}}$$

```
[Out] 2*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/
2))*a^(1/2)/(b*x^2+a)^(3/4)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {239, 237}

$$\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[a]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{(a+bx^2)^{3/4}}$$

$$= \frac{2\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.52, size = 46, normalized size = 0.82

$$\frac{x\left(1+\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3/4),x]
[Out] (x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(3/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/4),x)
[Out] int(1/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 24, normalized size = 0.43

$$\frac{x\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/4),x)
```

```
[Out] x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-3/4), x)
```

Mupad [B]
time = 4.88, size = 37, normalized size = 0.66

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(3/4),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(3/4)*hypergeom([1/2, 3/4], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(3/4)
```

3.833 $\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt[4]{a+bx^2}}{ax}-\frac{\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,(a+bx^2)^{3/4}}$$

```
[Out] -(b*x^2+a)^(1/4)/a/x-(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^
2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1
/2)/a^(1/2))),2^(1/2))*b^(1/2)/(b*x^2+a)^(3/4)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {331, 239, 237}

$$-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(3/4)),x]
```

```
[Out] -((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTa
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\int \frac{1}{x^2 (a+bx^2)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{b\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{2a}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{2a (a+bx^2)^{3/4}}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.07, size = 49, normalized size = 0.64

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{x (a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(3/4)),x]
[Out] -(((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(3/4)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(3/4),x)
[Out] int(1/x^2/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/(b*x^4 + a*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 27, normalized size = 0.36

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-1/2, 3/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(3/4)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^2), x)
```

Mupad [B]
time = 5.07, size = 40, normalized size = 0.53

$$-\frac{2\left(\frac{a}{b\,x^2}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{a}{b\,x^2}\right)}{5\,x\,\left(b\,x^2+a\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(3/4)),x)
```

```
[Out] -(2*(a/(b*x^2) + 1)^(3/4)*hypergeom([3/4, 5/4], 9/4, -a/(b*x^2)))/(5*x*(a +
 b*x^2)^(3/4))
```

3.834 $\int \frac{1}{x^4(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt[4]{a+bx^2}}{3ax^3}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}+\frac{5b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] -1/3*(b*x^2+a)^(1/4)/a/x^3+5/6*b*(b*x^2+a)^(1/4)/a^2/x+5/6*b^(3/2)*(1+b*x^2
/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2
)/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 239, 237}

$$\frac{5b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(3/4)),x]
```

```
[Out] -1/3*(a + b*x^2)^(1/4)/(a*x^3) + (5*b*(a + b*x^2)^(1/4))/(6*a^2*x) + (5*b^(
3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*
a^(3/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a+bx^2}}{3ax^3}-\frac{(5b)\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx}{6a}\\ &= -\frac{\sqrt[4]{a+bx^2}}{3ax^3}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}+\frac{(5b^2)\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{12a^2}\\ &= -\frac{\sqrt[4]{a+bx^2}}{3ax^3}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}+\frac{\left(5b^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a^2\left(a+bx^2\right)^{3/4}}\\ &= -\frac{\sqrt[4]{a+bx^2}}{3ax^3}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}+\frac{5b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.50

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(3/4)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/(b*x^6 + a*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.59, size = 32, normalized size = 0.31

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(3/4),x)
[Out] -hyper((-3/2, 3/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(3/4)),x)
[Out] int(1/(x^4*(a + b*x^2)^(3/4)), x)
```

3.835 $\int \frac{1}{x^6\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{\sqrt[4]{a+bx^2}}{5ax^5}+\frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x}-\frac{3b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] -1/5*(b*x^2+a)^(1/4)/a/x^5+3/10*b*(b*x^2+a)^(1/4)/a^2/x^3-3/4*b^2*(b*x^2+a)
^(1/4)/a^3/x-3/4*b^(5/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*
b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^2+a)^(3/4)
```

Rubi [A]

time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 239, 237}

$$-\frac{3b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x}+\frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(3/4)),x]
```

```
[Out] -1/5*(a + b*x^2)^(1/4)/(a*x^5) + (3*b*(a + b*x^2)^(1/4))/(10*a^2*x^3) - (3*
b^2*(a + b*x^2)^(1/4))/(4*a^3*x) - (3*b^(5/2)*(1 + (b*x^2)/a)^(3/4)*Ellipti
cF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^(5/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6 (a+bx^2)^{3/4}}\,dx &= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} - \frac{(9b)\int \frac{1}{x^4(a+bx^2)^{3/4}}\,dx}{10a}\\ &= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} + \frac{(3b^2)\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx}{4a^2}\\ &= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{(3b^3)\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{8a^3}\\ &= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{\left(3b^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{8a^3(a+bx^2)^{3/4}}\\ &= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{3b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right.}{4a^{5/2}(a+bx^2)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.40

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(3/4)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, -((b*x^2)/a)
])/(x^5*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/(b*x^8 + a*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.70, size = 32, normalized size = 0.25

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-5/2, 3/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^6), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(b\,x^2+a\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(3/4)),x)
[Out] int(1/(x^6*(a + b*x^2)^(3/4)), x)
```

3.836 $\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=129

$$-\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}+\frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a-bx^2\right)^{3/4}}$$

```
[Out] -40/77*a^2*x*(-b*x^2+a)^(1/4)/b^3-20/77*a*x^3*(-b*x^2+a)^(1/4)/b^2-2/11*x^5
*(-b*x^2+a)^(1/4)/b+80/77*a^(7/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {327, 239, 238}

$$\frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a-bx^2\right)^{3/4}}-\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^2)^(3/4),x]
```

```
[Out] (-40*a^2*x*(a - b*x^2)^(1/4))/(77*b^3) - (20*a*x^3*(a - b*x^2)^(1/4))/(77*b
^2) - (2*x^5*(a - b*x^2)^(1/4))/(11*b) + (80*a^(7/2)*(1 - (b*x^2)/a)^(3/4)*
EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx &= -\frac{2x^5\sqrt[4]{a-bx^2}}{11b}+\frac{(10a)\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx}{11b}\\
&= -\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}+\frac{(60a^2)\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx}{77b^2}\\
&= -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}+\frac{(40a^3)\int \frac{1}{(a-bx^2)^{3/4}}\,dx}{77b^3}\\
&= -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}+\frac{\left(40a^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right.}}{77b^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}+\frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-}\right.}{77b^{7/2}\left(a-bx^2\right)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.13, size = 91, normalized size = 0.71

$$\frac{2\left(-20a^3x+10a^2bx^3+3ab^2x^5+7b^3x^7+20a^3x\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{77b^3\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^2)^(3/4),x]
```

```
[Out] (2*(-20*a^3*x + 10*a^2*b*x^3 + 3*a*b^2*x^5 + 7*b^3*x^7 + 20*a^3*x*(1 - (b*x
^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(77*b^3*(a - b*x
^2)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^6/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*x^6/(b*x^2 - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 29, normalized size = 0.22

$$\frac{x^7{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**2+a)**(3/4),x)
```

```
[Out] x**7*hyper((3/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(3/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{(a-b\,x^2)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^2)^(3/4),x)
[Out] int(x^6/(a - b*x^2)^(3/4), x)
```

3.837 $\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a-bx^2\right)^{3/4}}$$

```
[Out] -4/7*a*x*(-b*x^2+a)^(1/4)/b^2-2/7*x^3*(-b*x^2+a)^(1/4)/b+8/7*a^(5/2)*(1-b*x
^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5
/2)/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {327, 239, 238}

$$\frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2)^(3/4),x]
```

```
[Out] (-4*a*x*(a - b*x^2)^(1/4))/(7*b^2) - (2*x^3*(a - b*x^2)^(1/4))/(7*b) + (8*a
^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
7*b^(5/2)*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx &= -\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{(6a)\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx}{7b}\\ &= -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{(4a^2)\int \frac{1}{(a-bx^2)^{3/4}}\,dx}{7b^2}\\ &= -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{\left(4a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{7b^2(a-bx^2)^{3/4}}\\ &= -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}(a-bx^2)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.01, size = 77, normalized size = 0.74

$$\frac{2x\left(-2a^2+abx^2+b^2x^4+2a^2\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{7b^2(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2)^(3/4),x]
```

```
[Out] (2*x*(-2*a^2 + a*b*x^2 + b^2*x^4 + 2*a^2*(1 - (b*x^2)/a)^(3/4)*Hypergeometr
ic2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(7*b^2*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^2+a)^(3/4),x)
```

[Out] int(x^4/(-b*x^2+a)^(3/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] integrate(x^4/(-b*x^2 + a)^(3/4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="fricas")

[Out] integral(-(-b*x^2 + a)^(1/4)*x^4/(b*x^2 - a), x)

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 29, normalized size = 0.28

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(-b*x**2+a)**(3/4),x)

[Out] x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(3/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="giac")

[Out] integrate(x^4/(-b*x^2 + a)^(3/4), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^2)^(3/4),x)
[Out] int(x^4/(a - b*x^2)^(3/4), x)
```

3.838 $\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=81

$$-\frac{2x\sqrt[4]{a-bx^2}}{3b}+\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2/3*x*(-b*x^2+a)^(1/4)/b+4/3*a^(3/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(
1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {327, 239, 238}

$$\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^(3/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = -\frac{2x\sqrt[4]{a-bx^2}}{3b}+\frac{(2a)\int \frac{1}{(a-bx^2)^{3/4}}\,dx}{3b}$$
$$= -\frac{2x\sqrt[4]{a-bx^2}}{3b}+\frac{\left(2a\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{3b\,(a-bx^2)^{3/4}}$$
$$= -\frac{2x\sqrt[4]{a-bx^2}}{3b}+\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\,(a-bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.54, size = 64, normalized size = 0.79

$$\frac{2x\left(-a+bx^2+a\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{3b\,(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^(3/4),x]
[Out] (2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
 (b*x^2)/a]))/(3*b*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(3/4),x)
[Out] int(x^2/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^2/(-b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)*x^2/(b*x^2 - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 29, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(3/4),x)
[Out] x**3*hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(x^2/(-b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a-b\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^(3/4),x)
[Out] int(x^2/(a - b*x^2)^(3/4), x)
```

3.839 $\int \frac{1}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

```
[Out] 2*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
csin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/
2))*a^(1/2)/(-b*x^2+a)^(3/4)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {239, 238}

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[a]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{(a-bx^2)^{3/4}}$$
$$= \frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.38, size = 47, normalized size = 0.81

$$\frac{x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)}{(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-3/4),x]
```

```
[Out] (x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(a - b*x^2)^(3/4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^2 - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 26, normalized size = 0.45

$$\frac{x{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(3/4),x)
[Out] x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(3/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(-3/4), x)
```

Mupad [B]
time = 4.91, size = 38, normalized size = 0.66

$$\frac{x\left(1-\frac{b\,x^2}{a}\right)^{3/4}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(3/4),x)
[Out] (x*(1 - (b*x^2)/a)^(3/4)*hypergeom([1/2, 3/4], 3/2, (b*x^2)/a))/(a - b*x^2)
^(3/4)
```

3.840 $\int \frac{1}{x^2(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt[4]{a-bx^2}}{ax}+\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\left(a-bx^2\right)^{3/4}}$$

```
[Out] -(-b*x^2+a)^(1/4)/a/x+(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(
1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(-b*x^2+a)^(3/4)/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {331, 239, 238}

$$\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^(3/4)),x]
```

```
[Out] -((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a-bx^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{b\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{2a}$$
$$= -\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{\left(b\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{2a\left(a-bx^2\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.94, size = 50, normalized size = 0.64

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{x\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^(3/4)),x]
```

```
[Out] -(((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (b*x^2)/a])/(x*(
a - b*x^2)^(3/4)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^2/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^4 - a*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 29, normalized size = 0.37

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-1/2, 3/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(3/4)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^2), x)
```

Mupad [B]
time = 5.10, size = 41, normalized size = 0.53

$$-\frac{2\left(1-\frac{a}{b\,x^2}\right)^{3/4}{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ \frac{a}{b\,x^2}\right)}{5\,x\,\left(a-b\,x^2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^(3/4)),x)
```

```
[Out] -(2*(1 - a/(b*x^2))^(3/4)*hypergeom([3/4, 5/4], 9/4, a/(b*x^2)))/(5*x*(a - b*x^2)^(3/4))
```

3.841 $\int \frac{1}{x^4(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=106

$$-\frac{\sqrt[4]{a-bx^2}}{3ax^3}-\frac{5b\sqrt[4]{a-bx^2}}{6a^2x}+\frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a-bx^2\right)^{3/4}}$$

```
[Out] -1/3*(-b*x^2+a)^(1/4)/a/x^3-5/6*b*(-b*x^2+a)^(1/4)/a^2/x+5/6*b^(3/2)*(1-b*x
^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3
/2)/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {331, 239, 238}

$$\frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{5b\sqrt[4]{a-bx^2}}{6a^2x}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^2)^(3/4)),x]
```

```
[Out] -1/3*(a - b*x^2)^(1/4)/(a*x^3) - (5*b*(a - b*x^2)^(1/4))/(6*a^2*x) + (5*b^(
3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*
a^(3/2)*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4 (a-bx^2)^{3/4}} \, dx &= -\frac{\sqrt[4]{a-bx^2}}{3ax^3} + \frac{(5b) \int \frac{1}{x^2 (a-bx^2)^{3/4}} \, dx}{6a} \\ &= -\frac{\sqrt[4]{a-bx^2}}{3ax^3} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} + \frac{(5b^2) \int \frac{1}{(a-bx^2)^{3/4}} \, dx}{12a^2} \\ &= -\frac{\sqrt[4]{a-bx^2}}{3ax^3} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} + \frac{\left(5b^2 \left(1-\frac{bx^2}{a}\right)^{3/4}\right) \int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}} \, dx}{12a^2 (a-bx^2)^{3/4}} \\ &= -\frac{\sqrt[4]{a-bx^2}}{3ax^3} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} + \frac{5b^{3/2} \left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) \middle| 2\right)}{6a^{3/2} (a-bx^2)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 52, normalized size = 0.49

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3 (a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^2)^(3/4)),x]
[Out] -1/3*((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (b*x^2)/a])/
(x^3*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(-b\,x^2+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^2+a)^(3/4),x)
```

[Out] int(1/x^4/(-b*x^2+a)^(3/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="fricas")

[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^6 - a*x^4), x)

Sympy [C] Result contains complex when optimal does not.
time = 0.56, size = 34, normalized size = 0.32

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(-b*x**2+a)**(3/4),x)

[Out] -hyper((-3/2, 3/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(3/4)*x**3)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="giac")

[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^4), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(a-b\,x^2\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^2)^(3/4)),x)
[Out] int(1/(x^4*(a - b*x^2)^(3/4)), x)
```

3.842 $\int \frac{1}{x^6(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=131

$$-\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a-bx^2\right)^{3/4}}$$

```
[Out] -1/5*(-b*x^2+a)^(1/4)/a/x^5-3/10*b*(-b*x^2+a)^(1/4)/a^2/x^3-3/4*b^2*(-b*x^2
+a)^(1/4)/a^3/x+3/4*b^(5/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin
(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {331, 239, 238}

$$\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^2)^(3/4)),x]
```

```
[Out] -1/5*(a - b*x^2)^(1/4)/(a*x^5) - (3*b*(a - b*x^2)^(1/4))/(10*a^2*x^3) - (3*
b^2*(a - b*x^2)^(1/4))/(4*a^3*x) + (3*b^(5/2)*(1 - (b*x^2)/a)^(3/4)*Ellipti
cF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^(5/2)*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6 (a-bx^2)^{3/4}}\,dx &= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}+\frac{(9b)\int \frac{1}{x^4(a-bx^2)^{3/4}}\,dx}{10a}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}+\frac{(3b^2)\int \frac{1}{x^2(a-bx^2)^{3/4}}\,dx}{4a^2}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{(3b^3)\int \frac{1}{(a-bx^2)^{3/4}}\,dx}{8a^3}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{\left(3b^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{8a^3(a-bx^2)^{3/4}}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.\right.}{4a^{5/2}(a-bx^2)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 52, normalized size = 0.40

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^2)^(3/4)),x]
```

```
[Out] -1/5*((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (b*x^2)/a])/
(x^5*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^2+a)^(3/4),x)
[Out] int(1/x^6/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^8 - a*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.68, size = 34, normalized size = 0.26

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**2+a)**(3/4),x)
[Out] -hyper((-5/2, 3/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(3/4)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^6), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,\left(a-b\,x^2\right)^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a - b*x^2)^(3/4)),x)
[Out] int(1/(x^6*(a - b*x^2)^(3/4)), x)
```

3.843 $\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=124

$$\frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}}-\frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}}+\frac{2x^5}{9b\sqrt[4]{a+bx^2}}-\frac{16a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 8/3*a^2*x/b^3/(b*x^2+a)^(1/4)-4/9*a*x^3/b^2/(b*x^2+a)^(1/4)+2/9*x^5/b/(b*x^
2+a)^(1/4)-16/3*a^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b
^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {291, 203, 202}

$$-\frac{16a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}}+\frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}}-\frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}}+\frac{2x^5}{9b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(5/4),x]
```

```
[Out] (8*a^2*x)/(3*b^3*(a + b*x^2)^(1/4)) - (4*a*x^3)/(9*b^2*(a + b*x^2)^(1/4)) +
 (2*x^5)/(9*b*(a + b*x^2)^(1/4)) - (16*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ellipt
icE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
```

```
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx &= \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{(10a)\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx}{9b} \\
&= -\frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} + \frac{(4a^2)\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx}{3b^2} \\
&= \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{(8a^3)\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{3b^3} \\
&= \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{\left(8a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{3b^3\sqrt[4]{a+bx^2}} \\
&= \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{16a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.\right.}{3b^{7/2}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.66, size = 78, normalized size = 0.63

$$
\frac{2\left(-12a^2x-2abx^3+b^2x^5+12a^2x\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{9b^3\sqrt[4]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*(-12*a^2*x - 2*a*b*x^3 + b^2*x^5 + 12*a^2*x*(1 + (b*x^2)/a)^(1/4)*Hyperg
eometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(9*b^3*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{x^6}{(b\,x^2+a)^{\frac{5}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/4),x)
[Out] int(x^6/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(x^6/(b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 27, normalized size = 0.22

$$\frac{x^7{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/4),x)
[Out] x**7*hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(x^6/(b*x^2 + a)^(5/4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(5/4),x)
```

```
[Out] int(x^6/(a + b*x^2)^(5/4), x)
```

3.844 $\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{12ax}{5b^2\sqrt[4]{a+bx^2}}+\frac{2x^3}{5b\sqrt[4]{a+bx^2}}+\frac{24a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -12/5*a*x/b^2/(b*x^2+a)^(1/4)+2/5*x^3/b/(b*x^2+a)^(1/4)+24/5*a^(3/2)*(1+b*x
^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5
/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {291, 203, 202}

$$\frac{24a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}-\frac{12ax}{5b^2\sqrt[4]{a+bx^2}}+\frac{2x^3}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/4),x]
```

```
[Out] (-12*a*x)/(5*b^2*(a + b*x^2)^(1/4)) + (2*x^3)/(5*b*(a + b*x^2)^(1/4)) + (24
*a^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(5*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
```

```
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx &= \frac{2x^3}{5b\sqrt[4]{a+bx^2}} - \frac{(6a)\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx}{5b} \\
&= -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}} + \frac{2x^3}{5b\sqrt[4]{a+bx^2}} + \frac{(12a^2)\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{5b^2} \\
&= -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}} + \frac{2x^3}{5b\sqrt[4]{a+bx^2}} + \frac{\left(12a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5b^2\sqrt[4]{a+bx^2}} \\
&= -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}} + \frac{2x^3}{5b\sqrt[4]{a+bx^2}} + \frac{24a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.53, size = 65, normalized size = 0.65

$$\frac{2\left(6ax+bx^3-6ax\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{5b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*(6*a*x + b*x^3 - 6*a*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2,
 3/2, -((b*x^2)/a)]))/(5*b^2*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/4),x)
```

[Out] int(x^4/(b*x^2+a)^(5/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="maxima")

[Out] integrate(x^4/(b*x^2 + a)^(5/4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(3/4)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)

Sympy [C] Result contains complex when optimal does not.
time = 0.50, size = 27, normalized size = 0.27

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**2+a)**(5/4),x)

[Out] x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="giac")

[Out] integrate(x^4/(b*x^2 + a)^(5/4), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(5/4),x)
[Out] int(x^4/(a + b*x^2)^(5/4), x)
```

3.845 $\int \frac{x^2}{(a+bx^2)^{5/4}} \, dx$

Optimal. Leaf size=74

$$\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 2*x/b/(b*x^2+a)^(1/4)-4*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b
^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/b^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {291, 203, 202}

$$\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x)/(b*(a + b*x^2)^(1/4)) - (4*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[Ar
cTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx = \frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{(2a)\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{b}$$

$$= \frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{\left(2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{b\sqrt[4]{a+bx^2}}$$

$$= \frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{4\sqrt{a}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.17, size = 53, normalized size = 0.72

$$\frac{2x\left(-1+\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x*(-1 + (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)
/a)]))/(b*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/4),x)
```

```
[Out] int(x^2/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 27, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/4),x)
```

```
[Out] x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(5/4),x)
```

```
[Out] int(x^2/(a + b*x^2)^(5/4), x)
```

3.846 $\int \frac{1}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

```
[Out] 2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/
2))/(b*x^2+a)^(1/4)/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {203, 202}

$$\frac{2\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/4),x]
```

```
[Out] (2*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt
[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{a\sqrt[4]{a+bx^2}}$$

$$= \frac{2\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.94, size = 55, normalized size = 0.98

$$\frac{2x - x\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/4),x]
```

```
[Out] (2*x - x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a
)])/(a*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 24, normalized size = 0.43

$$\frac{x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{5}{4}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/4),x)
[Out] x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-5/4), x)
```

Mupad [B]
time = 4.92, size = 37, normalized size = 0.66

$$\frac{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(5/4),x)
[Out] (x*((b*x^2)/a + 1)^(5/4)*hypergeom([1/2, 5/4], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(5/4)
```

3.847 $\int \frac{1}{x^2(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{ax\sqrt[4]{a+bx^2}}-\frac{3\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -1/a/x/(b*x^2+a)^(1/4)-3*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*
b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {292, 203, 202}

$$-\frac{3\sqrt{b}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/4)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
 + 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^2)^{5/4}}\,dx = -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{(3b)\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{2a}$$
$$= -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{\left(3b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2a^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{3\sqrt{b}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.41, size = 52, normalized size = 0.68

$$-\frac{\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/4)),x]
[Out] -(((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, -((b*x^2)/a)])/(
a*x*(a + b*x^2)^(1/4)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/4),x)
[Out] int(1/x^2/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.57, size = 27, normalized size = 0.36

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-1/2, 5/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/4)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^2), x)
```

Mupad [B]
time = 5.16, size = 40, normalized size = 0.53

$$-\frac{2\,{\left(\frac{a}{b\,x^2}+1\right)}^{5/4}\,{}_2\mathrm{F}_1\left(\frac{5}{4},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^2}\right)}{7\,x\,{\left(b\,x^2+a\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(5/4)),x)
```

```
[Out] -(2*(a/(b*x^2) + 1)^(5/4)*hypergeom([5/4, 7/4], 11/4, -a/(b*x^2)))/(7*x*(a
+ b*x^2)^(5/4))
```

3.848 $\int \frac{1}{x^4(a+bx^2)^{5/4}} dx$

Optimal. Leaf size=102

$$-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}}+\frac{7b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -1/3/a/x^3/(b*x^2+a)^(1/4)+7/6*b/a^2/x/(b*x^2+a)^(1/4)+7/2*b^(3/2)*(1+b*x^2
/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2
)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {292, 203, 202}

$$\frac{7b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/4)),x]
```

```
[Out] -1/3*1/(a*x^3*(a + b*x^2)^(1/4)) + (7*b)/(6*a^2*x*(a + b*x^2)^(1/4)) + (7*b
^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
2*a^(5/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
```

```
+ 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
&& PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)^{5/4}}\,dx &= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}}-\frac{(7b)\int \frac{1}{x^2(a+bx^2)^{5/4}}\,dx}{6a}\\ &= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}}+\frac{(7b^2)\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{4a^2}\\ &= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}}+\frac{\left(7b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a^3\sqrt[4]{a+bx^2}}\\ &= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}}+\frac{7b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 54, normalized size = 0.53

$$-\frac{\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/4)),x]
[Out] -1/3*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, -((b*x^2)/a)
])/(a*x^3*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/4),x)
[Out] int(1/x^4/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.66, size = 32, normalized size = 0.31

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/4),x)
[Out] -hyper((-3/2, 5/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(5/4)),x)
[Out] int(1/(x^4*(a + b*x^2)^(5/4)), x)
```

3.849 $\int \frac{1}{x^6 (a+bx^2)^{5/4}} \, dx$

Optimal. Leaf size=126

$$-\frac{1}{5ax^5\sqrt[4]{a+bx^2}}+\frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}}-\frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}}-\frac{77b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -1/5/a/x^5/(b*x^2+a)^(1/4)+11/30*b/a^2/x^3/(b*x^2+a)^(1/4)-77/60*b^2/a^3/x/
(b*x^2+a)^(1/4)-77/20*b^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arct
an(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {292, 203, 202}

$$-\frac{77b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^2}}-\frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}}+\frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}}-\frac{1}{5ax^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(5/4)),x]
```

```
[Out] -1/5*1/(a*x^5*(a + b*x^2)^(1/4)) + (11*b)/(30*a^2*x^3*(a + b*x^2)^(1/4)) -
(77*b^2)/(60*a^3*x*(a + b*x^2)^(1/4)) - (77*b^(5/2)*(1 + (b*x^2)/a)^(1/4)*E
llipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(7/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
 + 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
```

```
&& PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{5/4}} \, dx = -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} - \frac{(11b) \int \frac{1}{x^4 \left(a+bx^2\right)^{5/4}} \, dx}{10a}$$
$$= -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3 \sqrt[4]{a+bx^2}} + \frac{\left(77b^2\right) \int \frac{1}{x^2 \left(a+bx^2\right)^{5/4}} \, dx}{60a^2}$$
$$= -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3 \sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x \sqrt[4]{a+bx^2}} - \frac{\left(77b^3\right) \int \frac{1}{\left(a+bx^2\right)^{5/4}} \, dx}{40a^3}$$
$$= -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3 \sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x \sqrt[4]{a+bx^2}} - \frac{\left(77b^3 \sqrt[4]{1+\frac{bx^2}{a}}\right) \int}{40a^4 \sqrt[4]{a+bx}}$$
$$= -\frac{1}{5ax^5 \sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3 \sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x \sqrt[4]{a+bx^2}} - \frac{77b^{5/2} \sqrt[4]{1+\frac{bx^2}{a}} E\left(\frac{1}{2}\right.}{20a^{7/2} \sqrt[4]{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.43

$$-\frac{\sqrt[4]{1+\frac{bx^2}{a}} \, {}_2F_1\left(-\frac{5}{2},\frac{5}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5ax^5 \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(5/4)),x]
[Out] -1/5*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, -((b*x^2)/a)
])/(a*x^5*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(b\,x^2+a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.78, size = 32, normalized size = 0.25

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(5/4),x)
[Out] -hyper((-5/2, 5/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6 \left(b x^2 + a\right)^{5/4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(5/4)),x)
[Out] int(1/(x^6*(a + b*x^2)^(5/4)), x)
```

3.850 $\int \frac{x^6}{\left(a-bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=124

$$\frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax(a-bx^2)^{3/4}}{3b^3}+\frac{20x^3(a-bx^2)^{3/4}}{9b^2}-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a-bx^2}}$$

```
[Out] 2*x^5/b/(-b*x^2+a)^(1/4)+8/3*a*x*(-b*x^2+a)^(3/4)/b^3+20/9*x^3*(-b*x^2+a)^(
3/4)/b^2-16/3*a^(5/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(
1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {294, 327, 235, 234}

$$-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a-bx^2}}+\frac{8ax(a-bx^2)^{3/4}}{3b^3}+\frac{20x^3(a-bx^2)^{3/4}}{9b^2}+\frac{2x^5}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^2)^(5/4),x]
```

```
[Out] (2*x^5)/(b*(a - b*x^2)^(1/4)) + (8*a*x*(a - b*x^2)^(3/4))/(3*b^3) + (20*x^3
*(a - b*x^2)^(3/4))/(9*b^2) - (16*a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx &= \frac{2x^5}{b\sqrt[4]{a-bx^2}}-\frac{10\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx}{b}\\ &= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{20x^3(a-bx^2)^{3/4}}{9b^2}-\frac{(20a)\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{3b^2}\\ &= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax(a-bx^2)^{3/4}}{3b^3}+\frac{20x^3(a-bx^2)^{3/4}}{9b^2}-\frac{(8a^2)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{3b^3}\\ &= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax(a-bx^2)^{3/4}}{3b^3}+\frac{20x^3(a-bx^2)^{3/4}}{9b^2}-\frac{\left(8a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}}{3b^3\sqrt[4]{a-bx^2}}\\ &= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax(a-bx^2)^{3/4}}{3b^3}+\frac{20x^3(a-bx^2)^{3/4}}{9b^2}-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\right.\right.}{3b^{7/2}\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.65, size = 78, normalized size = 0.63

$$-\frac{2x\left(-12a^2+2abx^2+b^2x^4+12a^2\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{9b^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^2)^(5/4),x]
```

```
[Out] (-2*x*(-12*a^2 + 2*a*b*x^2 + b^2*x^4 + 12*a^2*(1 - (b*x^2)/a)^(1/4)*Hyperge
ometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(9*b^3*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(-b\,x^{2}+a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^2+a)^(5/4),x)
[Out] int(x^6/(-b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(x^6/(-b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)*x^6/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 29, normalized size = 0.23

$$\frac{x^{7} {{}_{2}F_{1}\left(\begin{matrix} \frac{5}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}}{7 a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**2+a)**(5/4),x)
[Out] x**7*hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(x^6/(-b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(a-b\,x^2\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^2)^(5/4),x)
[Out] int(x^6/(a - b*x^2)^(5/4), x)
```

3.851 $\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx$

Optimal. Leaf size=101

$$\frac{2x^3}{b\sqrt[4]{a-bx^2}}+\frac{12x(a-bx^2)^{3/4}}{5b^2}-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}$$

```
[Out] 2*x^3/b/(-b*x^2+a)^(1/4)+12/5*x*(-b*x^2+a)^(3/4)/b^2-24/5*a^(3/2)*(1-b*x^2/
a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1
/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)
/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {294, 327, 235, 234}

$$-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}+\frac{12x(a-bx^2)^{3/4}}{5b^2}+\frac{2x^3}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2)^(5/4),x]
```

```
[Out] (2*x^3)/(b*(a - b*x^2)^(1/4)) + (12*x*(a - b*x^2)^(3/4))/(5*b^2) - (24*a^(3
/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b
^(5/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx &= \frac{2x^3}{b\sqrt[4]{a-bx^2}} - \frac{6\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{b}\\ &= \frac{2x^3}{b\sqrt[4]{a-bx^2}} + \frac{12x(a-bx^2)^{3/4}}{5b^2} - \frac{(12a)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{5b^2}\\ &= \frac{2x^3}{b\sqrt[4]{a-bx^2}} + \frac{12x(a-bx^2)^{3/4}}{5b^2} - \frac{\left(12a\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{5b^2\sqrt[4]{a-bx^2}}\\ &= \frac{2x^3}{b\sqrt[4]{a-bx^2}} + \frac{12x(a-bx^2)^{3/4}}{5b^2} - \frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.65, size = 66, normalized size = 0.65

$$-\frac{2\left(-6ax+bx^3+6ax\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{5b^2\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2)^(5/4),x]
```

```
[Out] (-2*(-6*a*x + b*x^3 + 6*a*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/
2, 3/2, (b*x^2)/a]))/(5*b^2*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^2+a)^(5/4),x)
[Out] int(x^4/(-b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(x^4/(-b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)*x^4/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 29, normalized size = 0.29

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**2+a)**(5/4),x)
[Out] x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(a-b\,x^2\right)^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^2)^(5/4),x)
```

```
[Out] int(x^4/(a - b*x^2)^(5/4), x)
```

3.852 $\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx$

Optimal. Leaf size=77

$$\frac{2x}{b\sqrt[4]{a-bx^2}}-\frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

[Out] 2*x/b/(-b*x^2+a)^(1/4)-4*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/b^(3/2)/(-b*x^2+a)^(1/4)

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {294, 235, 234}

$$\frac{2x}{b\sqrt[4]{a-bx^2}}-\frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a - b*x^2)^(5/4),x]

[Out] (2*x)/(b*(a - b*x^2)^(1/4)) - (4*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a - b*x^2)^(1/4))

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{2\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{b}$$
$$= \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{\left(2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{b\sqrt[4]{a-bx^2}}$$
$$= \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{4\sqrt{a}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.11, size = 56, normalized size = 0.73

$$\frac{2x-2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^(5/4),x]
[Out] (2*x - 2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a
])/(b*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(5/4),x)
[Out] int(x^2/(-b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^2/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 29, normalized size = 0.38

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(5/4),x)
```

```
[Out] x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a-b\,x^2\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^(5/4),x)
```

```
[Out] int(x^2/(a - b*x^2)^(5/4), x)
```

3.853 $\int \frac{1}{(a-bx^2)^{5/4}}\,dx$

Optimal. Leaf size=77

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

```
[Out] 2*x/a/(-b*x^2+a)^(1/4)-2*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*
b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(1/4)/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {205, 235, 234}

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-5/4),x]
```

```
[Out] (2*x)/(a*(a - b*x^2)^(1/4)) - (2*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]*(a - b*x^2)^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-bx^2)^{5/4}}\,dx &= \frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{a}\\ &= \frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{a\sqrt[4]{a-bx^2}}\\ &= \frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.03, size = 56, normalized size = 0.73

$$\frac{2x-x\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{a\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-5/4),x]
[Out] (2*x - x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])
/(a*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(5/4),x)
[Out] int(1/(-b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 26, normalized size = 0.34

$$\frac{x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{2i\pi}}{a}} \right)}{a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(5/4),x)
```

```
[Out] x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(5/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-5/4), x)
```

Mupad [B]
time = 4.91, size = 38, normalized size = 0.49

$$\frac{x\,{\left(1-\frac{b\,x^2}{a}\right)}^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(a-b\,x^2\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(5/4),x)
```

```
[Out] (x*(1 - (b*x^2)/a)^(5/4)*hypergeom([1/2, 5/4], 3/2, (b*x^2)/a))/(a - b*x^2)
^(5/4)
```

3.854 $\int \frac{1}{x^2(a-bx^2)^{5/4}}\,dx$

Optimal. Leaf size=99

$$\frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3(a-bx^2)^{3/4}}{a^2x}-\frac{3\sqrt{b}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}$$

```
[Out] 2/a/x/(-b*x^2+a)^(1/4)-3*(-b*x^2+a)^(3/4)/a^2/x-3*(1-b*x^2/a)^(1/4)*(cos(1/
2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*El
lipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(-b*x^2
+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {296, 331, 235, 234}

$$-\frac{3\sqrt{b}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}-\frac{3(a-bx^2)^{3/4}}{a^2x}+\frac{2}{ax\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*x*(a - b*x^2)^(1/4)) - (3*(a - b*x^2)^(3/4))/(a^2*x) - (3*Sqrt[b]*(1 -
 (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a
- b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a-bx^2\right)^{5/4}}\,dx &= \frac{2}{ax\sqrt[4]{a-bx^2}}+\frac{3\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{a}\\
&= \frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3(a-bx^2)^{3/4}}{a^2x}-\frac{(3b)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{2a^2}\\
&= \frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3(a-bx^2)^{3/4}}{a^2x}-\frac{\left(3b\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{2a^2\sqrt[4]{a-bx^2}}\\
&= \frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3(a-bx^2)^{3/4}}{a^2x}-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.75, size = 53, normalized size = 0.54

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{ax\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^(5/4)),x]
```

```
[Out] -(((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, (b*x^2)/a])/(a*x
*(a - b*x^2)^(1/4)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-b \,x^{2}+a \right)^{\frac{5}{4}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^(5/4),x)
[Out] int(1/x^2/(-b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^6 - 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 29, normalized size = 0.29

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{1}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}{a^{\frac{5}{4}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**(5/4),x)
[Out] -hyper((-1/2, 5/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(5/4)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^2), x)
```

Mupad [B]
time = 5.10, size = 41, normalized size = 0.41

$$-\frac{2\,{\left(1-\frac{a}{b\,x^2}\right)}^{5/4}\,{}_2\mathrm{F}_1\left(\frac{5}{4},\frac{7}{4};\ \frac{11}{4};\ \frac{a}{b\,x^2}\right)}{7\,x\,{\left(a-b\,x^2\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^(5/4)),x)
```

```
[Out] -(2*(1 - a/(b*x^2))^(5/4)*hypergeom([5/4, 7/4], 11/4, a/(b*x^2)))/(7*x*(a -
 b*x^2)^(5/4))
```

3.855 $\int \frac{1}{x^4(a-bx^2)^{5/4}}\,dx$

Optimal. Leaf size=126

$$\frac{2}{ax^3\sqrt[4]{a-bx^2}}-\frac{7(a-bx^2)^{3/4}}{3a^2x^3}-\frac{7b(a-bx^2)^{3/4}}{2a^3x}-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}$$

```
[Out] 2/a/x^3/(-b*x^2+a)^(1/4)-7/3*(-b*x^2+a)^(3/4)/a^2/x^3-7/2*b*(-b*x^2+a)^(3/4
)/a^3/x-7/2*b^(3/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2
)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/
2)/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {296, 331, 235, 234}

$$-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b(a-bx^2)^{3/4}}{2a^3x}-\frac{7(a-bx^2)^{3/4}}{3a^2x^3}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*x^3*(a - b*x^2)^(1/4)) - (7*(a - b*x^2)^(3/4))/(3*a^2*x^3) - (7*b*(a -
 b*x^2)^(3/4))/(2*a^3*x) - (7*b^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(5/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a-bx^2\right)^{5/4}}\,dx = \frac{2}{ax^3\sqrt[4]{a-bx^2}}+\frac{7\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx}{a}$$
$$= \frac{2}{ax^3\sqrt[4]{a-bx^2}}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}+\frac{(7b)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{2a^2}$$
$$= \frac{2}{ax^3\sqrt[4]{a-bx^2}}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{\left(7b^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{4a^3}$$
$$= \frac{2}{ax^3\sqrt[4]{a-bx^2}}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{\left(7b^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}$$
$$= \frac{2}{ax^3\sqrt[4]{a-bx^2}}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 55, normalized size = 0.44

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3ax^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^2)^(5/4)),x]
```

```
[Out] -1/3*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, (b*x^2)/a])/
(a*x^3*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(-b\,x^2+a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^2+a)^(5/4),x)
[Out] int(1/x^4/(-b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^8 - 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.67, size = 34, normalized size = 0.27

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**2+a)**(5/4),x)
[Out] -hyper((-3/2, 5/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(5/4)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(a-b\,x^2\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^2)^(5/4)),x)
[Out] int(1/(x^4*(a - b*x^2)^(5/4)), x)
```

3.856 $\int \frac{1}{x^6 (a-bx^2)^{5/4}} dx$

Optimal. Leaf size=151

$$\frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11(a-bx^2)^{3/4}}{5a^2x^5}-\frac{77b(a-bx^2)^{3/4}}{30a^3x^3}-\frac{77b^2(a-bx^2)^{3/4}}{20a^4x}-\frac{77b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{20a^{7/2}\sqrt[4]{a-bx^2}}$$

```
[Out] 2/a/x^5/(-b*x^2+a)^(1/4)-11/5*(-b*x^2+a)^(3/4)/a^2/x^5-77/30*b*(-b*x^2+a)^(
3/4)/a^3/x^3-77/20*b^2*(-b*x^2+a)^(3/4)/a^4/x-77/20*b^(5/2)*(1-b*x^2/a)^(1/
4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^
(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(-b*x
^2+a)^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {296, 331, 235, 234}

$$-\frac{77b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{20a^{7/2}\sqrt[4]{a-bx^2}}-\frac{77b^2(a-bx^2)^{3/4}}{20a^4x}-\frac{77b(a-bx^2)^{3/4}}{30a^3x^3}-\frac{11(a-bx^2)^{3/4}}{5a^2x^5}+\frac{2}{ax^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*x^5*(a - b*x^2)^(1/4)) - (11*(a - b*x^2)^(3/4))/(5*a^2*x^5) - (77*b*(a
 - b*x^2)^(3/4))/(30*a^3*x^3) - (77*b^2*(a - b*x^2)^(3/4))/(20*a^4*x) - (77
*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(20*a^(7/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a-bx^2\right)^{5/4}}\,dx = \frac{2}{ax^5\sqrt[4]{a-bx^2}}+\frac{11\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx}{a}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11(a-bx^2)^{3/4}}{5a^2x^5}+\frac{(77b)\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx}{10a^2}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11(a-bx^2)^{3/4}}{5a^2x^5}-\frac{77b(a-bx^2)^{3/4}}{30a^3x^3}+\frac{(77b^2)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{20a^3}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11(a-bx^2)^{3/4}}{5a^2x^5}-\frac{77b(a-bx^2)^{3/4}}{30a^3x^3}-\frac{77b^2(a-bx^2)^{3/4}}{20a^4x}-\frac{(77b^3)\int \cdots}{\cdots}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11(a-bx^2)^{3/4}}{5a^2x^5}-\frac{77b(a-bx^2)^{3/4}}{30a^3x^3}-\frac{77b^2(a-bx^2)^{3/4}}{20a^4x}-\frac{\left(77b^3\sqrt[4]{1\cdots}\right.}{\cdots}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11(a-bx^2)^{3/4}}{5a^2x^5}-\frac{77b(a-bx^2)^{3/4}}{30a^3x^3}-\frac{77b^2(a-bx^2)^{3/4}}{20a^4x}-\frac{77b^{5/2}\sqrt[4]{1\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 55, normalized size = 0.36

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{2},\frac{5}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5ax^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^2)^(5/4)),x]
```

```
[Out] -1/5*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, (b*x^2)/a])/
(a*x^5*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6} \left(-b\,x^{2}+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^6/(-b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^10 - 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.84, size = 34, normalized size = 0.23

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}{5 a^{\frac{5}{4}} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-5/2, 5/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(5/4)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(a-b\,x^2\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a - b*x^2)^(5/4)),x)
[Out] int(1/(x^6*(a - b*x^2)^(5/4)), x)
```

3.857 $\int \frac{1}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2x}{3a\left(a+bx^2\right)^{3/4}}+\frac{2\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\sqrt{b}\,\left(a+bx^2\right)^{3/4}}$$

```
[Out] 2/3*x/a/(b*x^2+a)^(3/4)+2/3*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan
(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(3/4)/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {205, 239, 237}

$$\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\sqrt{b}\,\left(a+bx^2\right)^{3/4}}+\frac{2x}{3a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-7/4),x]
```

```
[Out] (2*x)/(3*a*(a + b*x^2)^(3/4)) + (2*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \frac{2x}{3a\,(a+bx^2)^{3/4}} + \frac{\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{3a}$$
$$= \frac{2x}{3a\,(a+bx^2)^{3/4}} + \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3a\,(a+bx^2)^{3/4}}$$
$$= \frac{2x}{3a\,(a+bx^2)^{3/4}} + \frac{2\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\sqrt{b}\,(a+bx^2)^{3/4}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.94, size = 55, normalized size = 0.71

$$\frac{x\left(2+\left(1+\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{3a\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-7/4),x]
[Out] (x*(2 + (1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)
]))/(3*a*(a + b*x^2)^(3/4))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/4),x)
[Out] int(1/(b*x^2+a)^(7/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 24, normalized size = 0.31

$$\frac{x\,{}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{7}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/4),x)
[Out] x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-7/4), x)
```

Mupad [B]
time = 4.88, size = 37, normalized size = 0.47

$$\frac{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{7/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(7/4),x)
[Out] (x*((b*x^2)/a + 1)^(7/4)*hypergeom([1/2, 7/4], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(7/4)
```

3.858 $\int \frac{1}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2x}{5a\,(a+bx^2)^{5/4}}+\frac{6\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

```
[Out] 2/5*x/a/(b*x^2+a)^(5/4)+6/5*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan
(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {205, 203, 202}

$$\frac{6\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a+bx^2}}+\frac{2x}{5a\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-9/4),x]
```

```
[Out] (2*x)/(5*a*(a + b*x^2)^(5/4)) + (6*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{9/4}}\,dx &= \frac{2x}{5a\,(a+bx^2)^{5/4}}+\frac{3\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{5a}\\ &= \frac{2x}{5a\,(a+bx^2)^{5/4}}+\frac{\left(3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5a^2\sqrt[4]{a+bx^2}}\\ &= \frac{2x}{5a\,(a+bx^2)^{5/4}}+\frac{6\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.76, size = 72, normalized size = 0.92

$$\frac{8ax+6bx^3-3x(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{5a^2\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-9/4),x]
[Out] (8*a*x + 6*b*x^3 - 3*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, -((b*x^2)/a)])/(5*a^2*(a + b*x^2)^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/4),x)
[Out] int(1/(b*x^2+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.77, size = 24, normalized size = 0.31

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/4),x)
[Out] x*hyper((1/2, 9/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(9/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-9/4), x)
```

Mupad [B]
time = 4.88, size = 37, normalized size = 0.47

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{9/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(9/4),x)
[Out] (x*((b*x^2)/a + 1)^(9/4)*hypergeom([1/2, 9/4], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(9/4)
```

3.859 $\int \frac{1}{(a+bx^2)^{11/4}}\,dx$

Optimal. Leaf size=97

$$\frac{2x}{7a\,(a+bx^2)^{7/4}}+\frac{10x}{21a^2\,(a+bx^2)^{3/4}}+\frac{10\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\,\left(a+bx^2\right)^{3/4}}$$

```
[Out] 2/7*x/a/(b*x^2+a)^(7/4)+10/21*x/a^2/(b*x^2+a)^(3/4)+10/21*(1+b*x^2/a)^(3/4)
*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1
/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+
a)^(3/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {205, 239, 237}

$$\frac{10\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\,\left(a+bx^2\right)^{3/4}}+\frac{10x}{21a^2\,(a+bx^2)^{3/4}}+\frac{2x}{7a\,(a+bx^2)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-11/4),x]
```

```
[Out] (2*x)/(7*a*(a + b*x^2)^(7/4)) + (10*x)/(21*a^2*(a + b*x^2)^(3/4)) + (10*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)
*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{11/4}}\,dx &= \frac{2x}{7a\,(a+bx^2)^{7/4}}+\frac{5\int \frac{1}{(a+bx^2)^{7/4}}\,dx}{7a}\\ &= \frac{2x}{7a\,(a+bx^2)^{7/4}}+\frac{10x}{21a^2\,(a+bx^2)^{3/4}}+\frac{5\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{21a^2}\\ &= \frac{2x}{7a\,(a+bx^2)^{7/4}}+\frac{10x}{21a^2\,(a+bx^2)^{3/4}}+\frac{\left(5\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{21a^2\,(a+bx^2)^{3/4}}\\ &= \frac{2x}{7a\,(a+bx^2)^{7/4}}+\frac{10x}{21a^2\,(a+bx^2)^{3/4}}+\frac{10\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\,(a+bx^2)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.84, size = 75, normalized size = 0.77

$$\frac{2x(8a+5bx^2)+5x(a+bx^2)\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)}{21a^2\,(a+bx^2)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-11/4),x]
```

```
[Out] (2*x*(8*a + 5*b*x^2) + 5*x*(a + b*x^2)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric
2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(21*a^2*(a + b*x^2)^(7/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(11/4),x)
```

```
[Out] int(1/(b*x^2+a)^(11/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-11/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.00, size = 24, normalized size = 0.25

$$\frac{x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{11}{4} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{a^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(11/4),x)
[Out] x*hyper((1/2, 11/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(11/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-11/4), x)
```

Mupad [B]
time = 4.89, size = 37, normalized size = 0.38

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{11/4}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(11/4),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(11/4)*hypergeom([1/2, 11/4], 3/2, -(b*x^2)/a))/(a + b*x
^2)^(11/4)
```

3.860 $\int \frac{1}{(a-bx^2)^{7/4}}\,dx$

Optimal. Leaf size=81

$$\frac{2x}{3a\left(a-bx^2\right)^{3/4}}+\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\sqrt{b}\,\left(a-bx^2\right)^{3/4}}$$

```
[Out] 2/3*x/a/(-b*x^2+a)^(3/4)+2/3*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^
(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsi
n(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(3/4)/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {205, 239, 238}

$$\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\sqrt{b}\,\left(a-bx^2\right)^{3/4}}+\frac{2x}{3a\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-7/4),x]
```

```
[Out] (2*x)/(3*a*(a - b*x^2)^(3/4)) + (2*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \frac{2x}{3a(a-bx^2)^{3/4}} + \frac{\int \frac{1}{(a-bx^2)^{3/4}}\,dx}{3a}$$
$$= \frac{2x}{3a(a-bx^2)^{3/4}} + \frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{3a(a-bx^2)^{3/4}}$$
$$= \frac{2x}{3a(a-bx^2)^{3/4}} + \frac{2\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\sqrt{b}\,(a-bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.91, size = 56, normalized size = 0.69

$$\frac{x\left(2+\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{3a(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-7/4),x]
[Out] (x*(2 + (1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))
/(3*a*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(-b\,x^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(7/4),x)
[Out] int(1/(-b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(-7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 26, normalized size = 0.32

$$\frac{x{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(7/4),x)
[Out] x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(7/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(-7/4), x)
```

Mupad [B]
time = 4.87, size = 38, normalized size = 0.47

$$\frac{x\left(1-\frac{b\,x^2}{a}\right)^{7/4}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(7/4),x)
[Out] (x*(1 - (b*x^2)/a)^(7/4)*hypergeom([1/2, 7/4], 3/2, (b*x^2)/a))/(a - b*x^2)
^(7/4)
```

3.861 $\int \frac{1}{(a-bx^2)^{9/4}}\,dx$

Optimal. Leaf size=101

$$\frac{2x}{5a\left(a-bx^2\right)^{5/4}}+\frac{6x}{5a^2\sqrt[4]{a-bx^2}}-\frac{6\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

```
[Out] 2/5*x/a/(-b*x^2+a)^(5/4)+6/5*x/a^2/(-b*x^2+a)^(1/4)-6/5*(1-b*x^2/a)^(1/4)*(
cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2
)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^2+a
)^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {205, 235, 234}

$$-\frac{6\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a-bx^2}}+\frac{6x}{5a^2\sqrt[4]{a-bx^2}}+\frac{2x}{5a\left(a-bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-9/4),x]
```

```
[Out] (2*x)/(5*a*(a - b*x^2)^(5/4)) + (6*x)/(5*a^2*(a - b*x^2)^(1/4)) - (6*(1 - (
b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*Sqr
t[b]*(a - b*x^2)^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-bx^2)^{9/4}}\,dx &= \frac{2x}{5a\,(a-bx^2)^{5/4}}+\frac{3\int \frac{1}{(a-bx^2)^{5/4}}\,dx}{5a}\\
&= \frac{2x}{5a\,(a-bx^2)^{5/4}}+\frac{6x}{5a^2\sqrt[4]{a-bx^2}}-\frac{3\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{5a^2}\\
&= \frac{2x}{5a\,(a-bx^2)^{5/4}}+\frac{6x}{5a^2\sqrt[4]{a-bx^2}}-\frac{\left(3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{5a^2\sqrt[4]{a-bx^2}}\\
&= \frac{2x}{5a\,(a-bx^2)^{5/4}}+\frac{6x}{5a^2\sqrt[4]{a-bx^2}}-\frac{6\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.70, size = 74, normalized size = 0.73

$$\frac{8ax-6bx^3-3x(a-bx^2)\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{5a^2\,(a-bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-9/4),x]
```

```
[Out] (8*a*x - 6*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, (b*x^2)/a])/(5*a^2*(a - b*x^2)^(5/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(9/4),x)
[Out] int(1/(-b*x^2+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(9/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(-9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(9/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x
)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.75, size = 26, normalized size = 0.26

$$\frac{x\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{3}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{2i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(9/4),x)
[Out] x*hyper((1/2, 9/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(9/4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(9/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(-9/4), x)
```

Mupad [B]
time = 4.86, size = 38, normalized size = 0.38

$$\frac{x\left(1-\frac{b\,x^2}{a}\right)^{9/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(9/4),x)
```

```
[Out] (x*(1 - (b*x^2)/a)^(9/4)*hypergeom([1/2, 9/4], 3/2, (b*x^2)/a))/(a - b*x^2)
^(9/4)
```

3.862 $\int \frac{1}{(a-bx^2)^{11/4}}\,dx$

Optimal. Leaf size=101

$$\frac{2x}{7a\left(a-bx^2\right)^{7/4}}+\frac{10x}{21a^2\left(a-bx^2\right)^{3/4}}+\frac{10\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

```
[Out] 2/7*x/a/(-b*x^2+a)^(7/4)+10/21*x/a^2/(-b*x^2+a)^(3/4)+10/21*(1-b*x^2/a)^(3/
4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^
(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x
^2+a)^(3/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {205, 239, 238}

$$\frac{10\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{10x}{21a^2\left(a-bx^2\right)^{3/4}}+\frac{2x}{7a\left(a-bx^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-11/4),x]
```

```
[Out] (2*x)/(7*a*(a - b*x^2)^(7/4)) + (10*x)/(21*a^2*(a - b*x^2)^(3/4)) + (10*(1
- (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)
*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-bx^2)^{11/4}}\,dx &= \frac{2x}{7a\,(a-bx^2)^{7/4}}+\frac{5\int \frac{1}{(a-bx^2)^{7/4}}\,dx}{7a}\\
&= \frac{2x}{7a\,(a-bx^2)^{7/4}}+\frac{10x}{21a^2\,(a-bx^2)^{3/4}}+\frac{5\int \frac{1}{(a-bx^2)^{3/4}}\,dx}{21a^2}\\
&= \frac{2x}{7a\,(a-bx^2)^{7/4}}+\frac{10x}{21a^2\,(a-bx^2)^{3/4}}+\frac{\left(5\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{21a^2\,(a-bx^2)^{3/4}}\\
&= \frac{2x}{7a\,(a-bx^2)^{7/4}}+\frac{10x}{21a^2\,(a-bx^2)^{3/4}}+\frac{10\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{21a^{3/2}\sqrt{b}\,(a-bx^2)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.88, size = 77, normalized size = 0.76

$$\frac{2x(8a-5bx^2)+5x(a-bx^2)\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)}{21a^2\,(a-bx^2)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-11/4),x]
```

```
[Out] (2*x*(8*a - 5*b*x^2) + 5*x*(a - b*x^2)*(1 - (b*x^2)/a)^(3/4)*Hypergeometric
2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(21*a^2*(a - b*x^2)^(7/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(11/4),x)
```

[Out] int(1/(-b*x^2+a)^(11/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^(11/4),x, algorithm="maxima")

[Out] integrate((-b*x^2 + a)^(-11/4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^(11/4),x, algorithm="fricas")

[Out] integral(-(-b*x^2 + a)^(1/4)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x
)

Sympy [C] Result contains complex when optimal does not.
time = 1.01, size = 26, normalized size = 0.26

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{11}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x**2+a)**(11/4),x)

[Out] x*hyper((1/2, 11/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(11/4)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^(11/4),x, algorithm="giac")

[Out] integrate((-b*x^2 + a)^(-11/4), x)

Mupad [B]
time = 4.81, size = 38, normalized size = 0.38

$$\frac{x\left(1-\frac{b\,x^2}{a}\right)^{11/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(11/4),x)
```

```
[Out] (x*(1 - (b*x^2)/a)^(11/4)*hypergeom([1/2, 11/4], 3/2, (b*x^2)/a))/(a - b*x^
2)^(11/4)
```

3.863 $\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=99

$$-\frac{128x}{1053\sqrt[4]{2+3x^2}}+\frac{32x(2+3x^2)^{3/4}}{1053}-\frac{40x^3(2+3x^2)^{3/4}}{1053}+\frac{2}{39}x^5\left(2+3x^2\right)^{3/4}+\frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\right.}{1053\sqrt{3}}$$

```
[Out] -128/1053*x/(3*x^2+2)^(1/4)+32/1053*x*(3*x^2+2)^(3/4)-40/1053*x^3*(3*x^2+2)
^(3/4)+2/39*x^5*(3*x^2+2)^(3/4)+128/3159*2^(1/4)*(cos(1/2*arctan(1/2*x*6^(1
/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arctan(1/2*
x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 233, 202}

$$\frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{1053\sqrt{3}}+\frac{32(3x^2+2)^{3/4}x}{1053}-\frac{128x}{1053\sqrt[4]{3x^2+2}}+\frac{2}{39}(3x^2+2)^{3/4}x^5-\frac{40(3x^2+2)^{3/4}x^3}{1053}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (-128*x)/(1053*(2 + 3*x^2)^(1/4)) + (32*x*(2 + 3*x^2)^(3/4))/1053 - (40*x^3
*(2 + 3*x^2)^(3/4))/1053 + (2*x^5*(2 + 3*x^2)^(3/4))/39 + (128*2^(1/4)*Elli
pticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(1053*Sqrt[3])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx &= \frac{2}{39}x^5\left(2+3x^2\right)^{3/4}-\frac{20}{39}\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx\\ &= -\frac{40x^3(2+3x^2)^{3/4}}{1053}+\frac{2}{39}x^5\left(2+3x^2\right)^{3/4}+\frac{80}{351}\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx\\ &= \frac{32x(2+3x^2)^{3/4}}{1053}-\frac{40x^3(2+3x^2)^{3/4}}{1053}+\frac{2}{39}x^5\left(2+3x^2\right)^{3/4}-\frac{64\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx}{1053}\\ &= -\frac{128x}{1053\sqrt[4]{2+3x^2}}+\frac{32x(2+3x^2)^{3/4}}{1053}-\frac{40x^3(2+3x^2)^{3/4}}{1053}+\frac{2}{39}x^5\left(2+3x^2\right)^{3/4}+\frac{128\int \frac{}{(2+}}{10}\\ &= -\frac{128x}{1053\sqrt[4]{2+3x^2}}+\frac{32x(2+3x^2)^{3/4}}{1053}-\frac{40x^3(2+3x^2)^{3/4}}{1053}+\frac{2}{39}x^5\left(2+3x^2\right)^{3/4}+\frac{128\sqrt[4]{2}\,E}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.15, size = 54, normalized size = 0.55

$$\frac{2x\left(\left(2+3x^2\right)^{3/4}\left(16-20x^2+27x^4\right)-16\ 2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{1053}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + 3*x^2)^(1/4),x]
[Out] (2*x*((2 + 3*x^2)^(3/4)*(16 - 20*x^2 + 27*x^4) - 16*2^(3/4)*Hypergeometric2
F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/1053
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 20, normalized size = 0.20

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right],\left[\frac{9}{2}\right],-\frac{3 x^{2}}{2}\right)}{14}$	20
risch	$\frac{2 x\left(27 x^{4}-20 x^{2}+16\right)\left(3 x^{2}+2\right)^{\frac{3}{4}}}{1053}-\frac{32\, 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{1053}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*2^(3/4)*x^7*hypergeom([1/4,7/2],[9/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(3*x^2 + 2)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 27, normalized size = 0.27

$$\frac{2^{\frac{3}{4}} x^{7} {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{3 x^{2} e^{i \pi}}{2}} \right)}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)/2)/14
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(x^6/(3*x^2 + 2)^(1/4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{(3\,x^2+2)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2 + 2)^(1/4),x)
[Out] int(x^6/(3*x^2 + 2)^(1/4), x)
```

3.864 $\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=81

$$\frac{32x}{135\sqrt[4]{2+3x^2}}-\frac{8}{135}x\left(2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(2+3x^2\right)^{3/4}-\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{135\sqrt{3}}$$

```
[Out] 32/135*x/(3*x^2+2)^(1/4)-8/135*x*(3*x^2+2)^(3/4)+2/27*x^3*(3*x^2+2)^(3/4)-3
2/405*2^(1/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x
*6^(1/2)))*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {327, 233, 202}

$$-\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{135\sqrt{3}}-\frac{8}{135}\left(3x^2+2\right)^{3/4}x+\frac{32x}{135\sqrt[4]{3x^2+2}}+\frac{2}{27}\left(3x^2+2\right)^{3/4}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (32*x)/(135*(2 + 3*x^2)^(1/4)) - (8*x*(2 + 3*x^2)^(3/4))/135 + (2*x^3*(2 +
3*x^2)^(3/4))/27 - (32*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(135*Sq
rt[3])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx &= \frac{2}{27}x^3\left(2+3x^2\right)^{3/4}-\frac{4}{9}\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx\\ &= -\frac{8}{135}x\left(2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(2+3x^2\right)^{3/4}+\frac{16}{135}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx\\ &= \frac{32x}{135\sqrt[4]{2+3x^2}}-\frac{8}{135}x\left(2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(2+3x^2\right)^{3/4}-\frac{32}{135}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx\\ &= \frac{32x}{135\sqrt[4]{2+3x^2}}-\frac{8}{135}x\left(2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(2+3x^2\right)^{3/4}-\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\,\right.}{135\sqrt{3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.86, size = 49, normalized size = 0.60

$$\frac{2}{135}x\left(\left(2+3x^2\right)^{3/4}\left(-4+5x^2\right)+4\,2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + 3*x^2)^(1/4),x]
[Out] (2*x*((2 + 3*x^2)^(3/4)*(-4 + 5*x^2) + 4*2^(3/4)*Hypergeometric2F1[1/4, 1/2
, 3/2, (-3*x^2)/2]))/135
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 20, normalized size = 0.25

method	result	size
meijerg	$\frac{2^{\frac{3}{4}}x^5\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{7}{2}\right],-\frac{3x^2}{2}\right)}{10}$	20
risch	$\frac{2x\left(5x^2-4\right)\left(3x^2+2\right)^{\frac{3}{4}}}{135}+\frac{8\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{135}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
[Out] 1/10*2^(3/4)*x^5*hypergeom([1/4,5/2],[7/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="maxima")
[Out] integrate(x^4/(3*x^2 + 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral(x^4/(3*x^2 + 2)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 27, normalized size = 0.33

$$\frac{2^{\frac{3}{4}}x^5{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2+2)**(1/4),x)
[Out] 2**(3/4)*x**5*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)/2)/10
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(x^4/(3*x^2 + 2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(3\,x^2+2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2 + 2)^(1/4),x)
[Out] int(x^4/(3*x^2 + 2)^(1/4), x)
```

3.865 $\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{8x}{15\sqrt[4]{2+3x^2}}+\frac{2}{15}x\left(2+3x^2\right)^{3/4}+\frac{8\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{15\sqrt{3}}$$

```
[Out] -8/15*x/(3*x^2+2)^(1/4)+2/15*x*(3*x^2+2)^(3/4)+8/45*2^(1/4)*(cos(1/2*arctan
(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*
arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {327, 233, 202}

$$\frac{8\sqrt[4]{2}\,E\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{15\sqrt{3}}+\frac{2}{15}\left(3x^2+2\right)^{3/4}x-\frac{8x}{15\sqrt[4]{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (-8*x)/(15*(2 + 3*x^2)^(1/4)) + (2*x*(2 + 3*x^2)^(3/4))/15 + (8*2^(1/4)*Ell
ipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx &= \frac{2}{15}x\left(2+3x^2\right)^{3/4}-\frac{4}{15}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx\\ &= -\frac{8x}{15\sqrt[4]{2+3x^2}}+\frac{2}{15}x\left(2+3x^2\right)^{3/4}+\frac{8}{15}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx\\ &= -\frac{8x}{15\sqrt[4]{2+3x^2}}+\frac{2}{15}x\left(2+3x^2\right)^{3/4}+\frac{8\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{15\sqrt{3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.77, size = 41, normalized size = 0.65

$$\frac{2}{15}x\left(\left(2+3x^2\right)^{3/4}-2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 3*x^2)^(1/4),x]
[Out] (2*x*((2 + 3*x^2)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)
/2]))/15
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.05, size = 20, normalized size = 0.32

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^3 \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],-\frac{3x^2}{2}\right)}{6}$	20
risch	$\frac{2x\left(3x^2+2\right)^{\frac{3}{4}}}{15}-\frac{2\,2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{15}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(3/4)*x^3*hypergeom([1/4,3/2],[5/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="maxima")
[Out] integrate(x^2/(3*x^2 + 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral(x^2/(3*x^2 + 2)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.38, size = 27, normalized size = 0.43

$$\frac{2^{\frac{3}{4}}x^3 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(1/4),x)
[Out] 2**(3/4)*x**3*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)/2)/6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(x^2/(3*x^2 + 2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(3\,x^2+2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2 + 2)^(1/4),x)
[Out] int(x^2/(3*x^2 + 2)^(1/4), x)
```

3.866 $\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{2x}{\sqrt[4]{2+3x^2}}-\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

[Out] 2*x/(3*x^2+2)^(1/4)-2/3*2^(1/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {233, 202}

$$\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)^(-1/4),x]

[Out] (2*x)/(2 + 3*x^2)^(1/4) - (2*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2]))*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4)), x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx = \frac{2x}{\sqrt[4]{2+3x^2}} - 2\int \frac{1}{(2+3x^2)^{5/4}}\,dx$$
$$= \frac{2x}{\sqrt[4]{2+3x^2}} - \frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.74, size = 24, normalized size = 0.56

$$\frac{x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)^(-1/4),x]
```

```
[Out] (x*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2])/2^(1/4)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.09, size = 18, normalized size = 0.42

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)^(-1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(-1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.37, size = 26, normalized size = 0.60

$$\frac{2^{\frac{3}{4}}x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{3x^{2}e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(1/4),x)
[Out] 2**(3/4)*x*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(I*pi)/2)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)^(-1/4), x)
```

Mupad [B]
time = 0.09, size = 16, normalized size = 0.37

$$\frac{8^{1/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 2)^(1/4),x)
[Out] (8^(1/4)*x*hypergeom([1/4, 1/2], 3/2, -(3*x^2)/2))/2
```

3.867 $\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{3x}{2\sqrt[4]{2+3x^2}}-\frac{(2+3x^2)^{3/4}}{2x}-\frac{\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}$$

```
[Out] 3/2*x/(3*x^2+2)^(1/4)-1/2*(3*x^2+2)^(3/4)/x-1/2*2^(1/4)*(cos(1/2*arctan(1/2
*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arct
an(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 233, 202}

$$-\frac{\sqrt{3}\,E\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}+\frac{3x}{2\sqrt[4]{3x^2+2}}-\frac{(3x^2+2)^{3/4}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] (3*x)/(2*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*EllipticE[
ArcTan[Sqrt[3/2]*x]/2, 2])/2^(3/4)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx &= -\frac{(2+3x^2)^{3/4}}{2x}+\frac{3}{4}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx\\ &= \frac{3x}{2\sqrt[4]{2+3x^2}}-\frac{(2+3x^2)^{3/4}}{2x}-\frac{3}{2}\int \frac{1}{(2+3x^2)^{5/4}}\,dx\\ &= \frac{3x}{2\sqrt[4]{2+3x^2}}-\frac{(2+3x^2)^{3/4}}{2x}-\frac{\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.86, size = 27, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/4, 1/2, (-3*x^2)/2]/(2^(1/4)*x))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.07, size = 20, normalized size = 0.32

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{2x}$	20
risch	$-\frac{(3x^2+2)^{\frac{3}{4}}}{2x}+\frac{3\,2^{\frac{3}{4}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{8}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(3/4)/x*hypergeom([-1/2,1/4],[1/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(3/4)/(3*x^4 + 2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.39, size = 29, normalized size = 0.46

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2+2)**(1/4),x)
[Out] -2**(3/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(2*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)
```

Mupad [B]
time = 5.02, size = 36, normalized size = 0.57

$$-\frac{2\,3^{3/4}\,{\left(\frac{2}{x^2}+3\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{2}{3\,x^2}\right)}{9\,x\,{\left(3\,x^2+2\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 + 2)^(1/4)),x)
[Out] -(2*3^(3/4)*(2/x^2 + 3)^(1/4)*hypergeom([1/4, 3/4], 7/4, -2/(3*x^2)))/(9*x*
(3*x^2 + 2)^(1/4))
```

3.868 $\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{9x}{8\sqrt[4]{2+3x^2}}-\frac{\left(2+3x^2\right)^{3/4}}{6x^3}+\frac{3\left(2+3x^2\right)^{3/4}}{8x}+\frac{3\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{4\,2^{3/4}}$$

```
[Out] -9/8*x/(3*x^2+2)^(1/4)-1/6*(3*x^2+2)^(3/4)/x^3+3/8*(3*x^2+2)^(3/4)/x+3/8*2^
(1/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)
))*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {331, 233, 202}

$$\frac{3\sqrt{3}\,E\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{4\,2^{3/4}}-\frac{9x}{8\sqrt[4]{3x^2+2}}+\frac{3\left(3x^2+2\right)^{3/4}}{8x}-\frac{\left(3x^2+2\right)^{3/4}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-9*x)/(8*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(6*x^3) + (3*(2 + 3*x^2)^(
3/4))/(8*x) + (3*Sqrt[3]*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx = -\frac{(2+3x^2)^{3/4}}{6x^3} - \frac{3}{4}\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{(2+3x^2)^{3/4}}{6x^3} + \frac{3(2+3x^2)^{3/4}}{8x} - \frac{9}{16}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{9x}{8\sqrt[4]{2+3x^2}} - \frac{(2+3x^2)^{3/4}}{6x^3} + \frac{3(2+3x^2)^{3/4}}{8x} + \frac{9}{8}\int \frac{1}{(2+3x^2)^{5/4}}\,dx$$
$$= -\frac{9x}{8\sqrt[4]{2+3x^2}} - \frac{(2+3x^2)^{3/4}}{6x^3} + \frac{3(2+3x^2)^{3/4}}{8x} + \frac{3\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{4\ 2^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 29, normalized size = 0.35

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3\sqrt[4]{2}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1/4, -1/2, (-3*x^2)/2]/(2^(1/4)*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.11, size = 20, normalized size = 0.24

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[-\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{6x^3}$	20
risch	$\frac{27x^4+6x^2-8}{24x^3\left(3x^2+2\right)^{\frac{1}{4}}}-\frac{9\,2^{\frac{3}{4}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{32}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(3/4)/x^3*hypergeom([-3/2,1/4],[-1/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(3/4)/(3*x^6 + 2*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.54, size = 32, normalized size = 0.39

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2+2)**(1/4),x)
[Out] -2**(3/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(3\,x^2+2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 + 2)^(1/4)),x)
[Out] int(1/(x^4*(3*x^2 + 2)^(1/4)), x)
```

3.869 $\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=101

$$\frac{189x}{160\sqrt[4]{2+3x^2}}-\frac{(2+3x^2)^{3/4}}{10x^5}+\frac{7(2+3x^2)^{3/4}}{40x^3}-\frac{63(2+3x^2)^{3/4}}{160x}-\frac{63\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{80\ 2^{3/4}}$$

```
[Out] 189/160*x/(3*x^2+2)^(1/4)-1/10*(3*x^2+2)^(3/4)/x^5+7/40*(3*x^2+2)^(3/4)/x^3
-63/160*(3*x^2+2)^(3/4)/x-63/160*2^(1/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)
^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2
))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 233, 202}

$$-\frac{63\sqrt{3}\,E\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{80\ 2^{3/4}}+\frac{189x}{160\sqrt[4]{3x^2+2}}-\frac{63(3x^2+2)^{3/4}}{160x}-\frac{(3x^2+2)^{3/4}}{10x^5}+\frac{7(3x^2+2)^{3/4}}{40x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] (189*x)/(160*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(10*x^5) + (7*(2 + 3*x^
2)^(3/4))/(40*x^3) - (63*(2 + 3*x^2)^(3/4))/(160*x) - (63*Sqrt[3]*EllipticE
[ArcTan[Sqrt[3/2]*x]/2, 2])/(80*2^(3/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx &= -\frac{(2+3x^2)^{3/4}}{10x^5}-\frac{21}{20}\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx\\ &= -\frac{(2+3x^2)^{3/4}}{10x^5}+\frac{7(2+3x^2)^{3/4}}{40x^3}+\frac{63}{80}\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx\\ &= -\frac{(2+3x^2)^{3/4}}{10x^5}+\frac{7(2+3x^2)^{3/4}}{40x^3}-\frac{63(2+3x^2)^{3/4}}{160x}+\frac{189}{320}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx\\ &= \frac{189x}{160\sqrt[4]{2+3x^2}}-\frac{(2+3x^2)^{3/4}}{10x^5}+\frac{7(2+3x^2)^{3/4}}{40x^3}-\frac{63(2+3x^2)^{3/4}}{160x}-\frac{189}{160}\int \frac{1}{(2+3x^2)^5}\\ &= \frac{189x}{160\sqrt[4]{2+3x^2}}-\frac{(2+3x^2)^{3/4}}{10x^5}+\frac{7(2+3x^2)^{3/4}}{40x^3}-\frac{63(2+3x^2)^{3/4}}{160x}-\frac{63\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\right.}{80\,2}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 29, normalized size = 0.29

$$-\frac{{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5\sqrt[4]{2}\,x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/2, 1/4, -3/2, (-3*x^2)/2]/(2^(1/4)*x^5)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 20, normalized size = 0.20

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{1}{4}\right],\left[-\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{10x^5}$	20
risch	$-\frac{189x^6+42x^4-8x^2+32}{160x^5(3x^2+2)^{\frac{1}{4}}}+\frac{189\,2^{\frac{3}{4}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{640}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*2^(3/4)/x^5*hypergeom([-5/2,1/4],[-3/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(3/4)/(3*x^8 + 2*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 32, normalized size = 0.32

$$-\frac{2^{\frac{3}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(3\,x^2+2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(3*x^2 + 2)^(1/4)),x)
[Out] int(1/(x^6*(3*x^2 + 2)^(1/4)), x)
```

3.870 $\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{32x(2-3x^2)^{3/4}}{1053}-\frac{40x^3(2-3x^2)^{3/4}}{1053}-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}+\frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{1053\sqrt{3}}$$

```
[Out] -32/1053*x*(-3*x^2+2)^(3/4)-40/1053*x^3*(-3*x^2+2)^(3/4)-2/39*x^5*(-3*x^2+2
)^(3/4)+128/3159*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*a
rcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {327, 234}

$$\frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{1053\sqrt{3}}-\frac{32\left(2-3x^2\right)^{3/4}x}{1053}-\frac{2}{39}\left(2-3x^2\right)^{3/4}x^5-\frac{40\left(2-3x^2\right)^{3/4}x^3}{1053}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-32*x*(2 - 3*x^2)^(3/4))/1053 - (40*x^3*(2 - 3*x^2)^(3/4))/1053 - (2*x^5*(
2 - 3*x^2)^(3/4))/39 + (128*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(1
053*Sqrt[3])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx = -\frac{2}{39}x^5\left(2-3x^2\right)^{3/4} + \frac{20}{39}\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx$$
$$= -\frac{40x^3(2-3x^2)^{3/4}}{1053} - \frac{2}{39}x^5\left(2-3x^2\right)^{3/4} + \frac{80}{351}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx$$
$$= -\frac{32x(2-3x^2)^{3/4}}{1053} - \frac{40x^3(2-3x^2)^{3/4}}{1053} - \frac{2}{39}x^5\left(2-3x^2\right)^{3/4} + \frac{64\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx}{1053}$$
$$= -\frac{32x(2-3x^2)^{3/4}}{1053} - \frac{40x^3(2-3x^2)^{3/4}}{1053} - \frac{2}{39}x^5\left(2-3x^2\right)^{3/4} + \frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\right.\right.}{1053\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.10, size = 54, normalized size = 0.65

$$-\frac{2x\left(\left(2-3x^2\right)^{3/4}\left(16+20x^2+27x^4\right)-16\ 2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)}{1053}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*((2 - 3*x^2)^(3/4)*(16 + 20*x^2 + 27*x^4) - 16*2^(3/4)*Hypergeometric
2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/1053
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.10, size = 20, normalized size = 0.24

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^7 \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{7}{2}\right],\left[\frac{9}{2}\right],\frac{3x^2}{2}\right)}{14}$	20
risch	$\frac{2x\left(27x^4+20x^2+16\right)\left(3x^2-2\right)}{1053\left(-3x^2+2\right)^{\frac{1}{4}}}+\frac{32\,2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{1053}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*2^(3/4)*x^7*hypergeom([1/4,7/2],[9/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)*x^6/(3*x^2 - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 29, normalized size = 0.35

$$\frac{2^{\frac{3}{4}}x^7{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(2*I*pi)/2)/14
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(2-3\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2 - 3*x^2)^(1/4),x)
```

```
[Out] int(x^6/(2 - 3*x^2)^(1/4), x)
```

3.871 $\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{8}{135}x\left(2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{135\sqrt{3}}$$

```
[Out] -8/135*x*(-3*x^2+2)^(3/4)-2/27*x^3*(-3*x^2+2)^(3/4)+32/405*2^(1/4)*(cos(1/2
*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(s
in(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {327, 234}

$$\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{135\sqrt{3}}-\frac{8}{135}\left(2-3x^2\right)^{3/4}x-\frac{2}{27}\left(2-3x^2\right)^{3/4}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-8*x*(2 - 3*x^2)^(3/4))/135 - (2*x^3*(2 - 3*x^2)^(3/4))/27 + (32*2^(1/4)*E
llipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(135*Sqrt[3])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx = -\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{4}{9}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx$$
$$= -\frac{8}{135}x\left(2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{16}{135}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx$$
$$= -\frac{8}{135}x\left(2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.86, size = 49, normalized size = 0.75

$$-\frac{2}{135}x\left(\left(2-3x^2\right)^{3/4}\left(4+5x^2\right)-4\ 2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*((2 - 3*x^2)^(3/4)*(4 + 5*x^2) - 4*2^(3/4)*Hypergeometric2F1[1/4, 1/2
, 3/2, (3*x^2)/2]))/135
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 20, normalized size = 0.31

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^5 \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{7}{2}\right],\frac{3x^2}{2}\right)}{10}$	20
risch	$\frac{2x\left(5x^2+4\right)\left(3x^2-2\right)}{135\left(-3x^2+2\right)^{\frac{1}{4}}}+\frac{8\,2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{135}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*2^(3/4)*x^5*hypergeom([1/4,5/2],[7/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="maxima")

[Out] integrate(x^4/(-3*x^2 + 2)^(1/4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="fricas")

[Out] integral(-(-3*x^2 + 2)^(3/4)*x^4/(3*x^2 - 2), x)

Sympy [C] Result contains complex when optimal does not.
time = 0.43, size = 29, normalized size = 0.45

$$\frac{2^{\frac{3}{4}}x^5{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(-3*x**2+2)**(1/4),x)

[Out] 2**(3/4)*x**5*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(2*I*pi)/2)/10

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="giac")

[Out] integrate(x^4/(-3*x^2 + 2)^(1/4), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(2-3\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(2 - 3*x^2)^(1/4),x)

[Out] int(x^4/(2 - 3*x^2)^(1/4), x)

3.872 $\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2}{15}x\left(2-3x^2\right)^{3/4}+\frac{8\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{15\sqrt{3}}$$

```
[Out] -2/15*x*(-3*x^2+2)^(3/4)+8/45*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1
/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2)))
,2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {327, 234}

$$\frac{8\sqrt[4]{2}\,E\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{15\sqrt{3}}-\frac{2}{15}x\left(2-3x^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*(2 - 3*x^2)^(3/4))/15 + (8*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2
])/(15*Sqrt[3])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx = -\frac{2}{15}x(2-3x^2)^{3/4} + \frac{4}{15}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx$$
$$= -\frac{2}{15}x(2-3x^2)^{3/4} + \frac{8\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{15\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.75, size = 41, normalized size = 0.87

$$-\frac{2}{15}x\left((2-3x^2)^{3/4} - 2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*((2 - 3*x^2)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)
/2]))/15
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.07, size = 20, normalized size = 0.43

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^3 \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],\frac{3x^2}{2}\right)}{6}$	20
risch	$\frac{2x(3x^2-2)}{15(-3x^2+2)^{\frac{1}{4}}} + \frac{2\,2^{\frac{3}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{15}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(3/4)*x^3*hypergeom([1/4,3/2],[5/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(3/4)*x^2/(3*x^2 - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 29, normalized size = 0.62

$$\frac{2^{\frac{3}{4}} x^{3} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{3x^{2} e^{2i\pi}}{2}} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/4),x)
[Out] 2**(3/4)*x**3*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(2*I*pi)/2)/6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(x^2/(-3*x^2 + 2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(2-3\,x^2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2 - 3*x^2)^(1/4),x)
[Out] int(x^2/(2 - 3*x^2)^(1/4), x)
```

3.873 $\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] 2/3*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6
^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{234\}$

$$\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)^(-1/4),x]
```

```
[Out] (2*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx = \frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 5.77, size = 24, normalized size = 0.86

$$\frac{x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)^(-1/4),x]
```

```
[Out] (x*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2])/2^(1/4)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.05, size = 18, normalized size = 0.64

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 + 2)^(-1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^2 - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.36, size = 27, normalized size = 0.96

$$\frac{2^{\frac{3}{4}} x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{3x^{2}e^{2i\pi}}{2}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)/2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 + 2)^(-1/4), x)
```

Mupad [**B**]
time = 0.09, size = 16, normalized size = 0.57

$$\frac{2^{3/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^2)^(1/4),x)
```

```
[Out] (2^(3/4)*x*hypergeom([1/4, 1/2], 3/2, (3*x^2)/2))/2
```

3.874 $\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{(2-3x^2)^{3/4}}{2x}-\frac{\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}$$

```
[Out] -1/2*(-3*x^2+2)^(3/4)/x-1/2*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2
)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2
^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {331, 234}

$$-\frac{\sqrt{3}\,E\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}-\frac{(2-3x^2)^{3/4}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*(2 - 3*x^2)^(3/4)/x - (Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/2^
(3/4)
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx = -\frac{(2-3x^2)^{3/4}}{2x} - \frac{3}{4}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx$$
$$= -\frac{(2-3x^2)^{3/4}}{2x} - \frac{\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.87, size = 27, normalized size = 0.57

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/4, 1/2, (3*x^2)/2]/(2^(1/4)*x))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 20, normalized size = 0.43

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{1}{2}\right],\frac{3x^2}{2}\right)}{2x}$	20
risch	$\frac{3x^2-2}{2x(-3x^2+2)^{\frac{1}{4}}} - \frac{3\,2^{\frac{3}{4}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{8}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(3/4)/x*hypergeom([-1/2,1/4],[1/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 31, normalized size = 0.66

$$-\frac{2^{\frac{3}{4}}{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| {\frac{3x^{2}e^{2i\pi}}{2}} \right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(2*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)
```

Mupad [B]
time = 5.05, size = 36, normalized size = 0.77

$$-\frac{2\,3^{3/4}\,{\left(3-\frac{2}{x^2}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{2}{3\,x^2}\right)}{9\,x\,{\left(2-3\,x^2\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2 - 3*x^2)^(1/4)),x)
```

```
[Out] -(2*3^(3/4)*(3 - 2/x^2)^(1/4)*hypergeom([1/4, 3/4], 7/4, 2/(3*x^2)))/(9*x*(
2 - 3*x^2)^(1/4))
```

3.875 $\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=67

$$-\frac{(2-3x^2)^{3/4}}{6x^3}-\frac{3(2-3x^2)^{3/4}}{8x}-\frac{3\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{4\,2^{3/4}}$$

```
[Out] -1/6*(-3*x^2+2)^(3/4)/x^3-3/8*(-3*x^2+2)^(3/4)/x-3/8*2^(1/4)*(cos(1/2*arcsi
n(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2
*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {331, 234}

$$-\frac{3\sqrt{3}\,E\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{4\,2^{3/4}}-\frac{3(2-3x^2)^{3/4}}{8x}-\frac{(2-3x^2)^{3/4}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/6*(2 - 3*x^2)^(3/4)/x^3 - (3*(2 - 3*x^2)^(3/4))/(8*x) - (3*Sqrt[3]*Ellip
ticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx = -\frac{(2-3x^2)^{3/4}}{6x^3} + \frac{3}{4}\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx$$
$$= -\frac{(2-3x^2)^{3/4}}{6x^3} - \frac{3(2-3x^2)^{3/4}}{8x} - \frac{9}{16}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx$$
$$= -\frac{(2-3x^2)^{3/4}}{6x^3} - \frac{3(2-3x^2)^{3/4}}{8x} - \frac{3\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{4\ 2^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 29, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3\sqrt[4]{2}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1/4, -1/2, (3*x^2)/2]/(2^(1/4)*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 20, normalized size = 0.30

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[-\frac{1}{2}\right],\frac{3x^2}{2}\right)}{6x^3}$	20
risch	$\frac{27x^4-6x^2-8}{24x^3\left(-3x^2+2\right)^{\frac{1}{4}}}-\frac{9\,2^{\frac{3}{4}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{32}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(3/4)/x^3*hypergeom([-3/2,1/4],[-1/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 34, normalized size = 0.51

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(1/4),x)
[Out] -2**(3/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(2-3\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(2 - 3*x^2)^(1/4)),x)
[Out] int(1/(x^4*(2 - 3*x^2)^(1/4)), x)
```

3.876 $\int \frac{1}{x^6\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{(2-3x^2)^{3/4}}{10x^5}-\frac{7(2-3x^2)^{3/4}}{40x^3}-\frac{63(2-3x^2)^{3/4}}{160x}-\frac{63\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{80\ 2^{3/4}}$$

```
[Out] -1/10*(-3*x^2+2)^(3/4)/x^5-7/40*(-3*x^2+2)^(3/4)/x^3-63/160*(-3*x^2+2)^(3/4
)/x-63/160*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(
1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {331, 234}

$$-\frac{63\sqrt{3}\,E\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{80\ 2^{3/4}}-\frac{63(2-3x^2)^{3/4}}{160x}-\frac{(2-3x^2)^{3/4}}{10x^5}-\frac{7(2-3x^2)^{3/4}}{40x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/10*(2 - 3*x^2)^(3/4)/x^5 - (7*(2 - 3*x^2)^(3/4))/(40*x^3) - (63*(2 - 3*x
^2)^(3/4))/(160*x) - (63*Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(80*2
^(3/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\sqrt[4]{2-3x^2}}\,dx &= -\frac{(2-3x^2)^{3/4}}{10x^5} + \frac{21}{20}\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx \\
&= -\frac{(2-3x^2)^{3/4}}{10x^5} - \frac{7(2-3x^2)^{3/4}}{40x^3} + \frac{63}{80}\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx \\
&= -\frac{(2-3x^2)^{3/4}}{10x^5} - \frac{7(2-3x^2)^{3/4}}{40x^3} - \frac{63(2-3x^2)^{3/4}}{160x} - \frac{189}{320}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx \\
&= -\frac{(2-3x^2)^{3/4}}{10x^5} - \frac{7(2-3x^2)^{3/4}}{40x^3} - \frac{63(2-3x^2)^{3/4}}{160x} - \frac{63\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{80\ 2^{3/4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 29, normalized size = 0.34

$$
-\frac{{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5\sqrt[4]{2}\,x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/2, 1/4, -3/2, (3*x^2)/2]/(2^(1/4)*x^5)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 20, normalized size = 0.24

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{1}{4}\right],\left[-\frac{3}{2}\right],\frac{3x^2}{2}\right)}{10x^5}$	20
risch	$\frac{189x^6-42x^4-8x^2-32}{160x^5\left(-3x^2+2\right)^{\frac{1}{4}}}-\frac{189\,2^{\frac{3}{4}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{640}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*2^(3/4)/x^5*hypergeom([-5/2,1/4],[-3/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.58, size = 34, normalized size = 0.40

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2+2)**(1/4),x)
[Out] -2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(2-3\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(2 - 3*x^2)^(1/4)),x)
[Out] int(1/(x^6*(2 - 3*x^2)^(1/4)), x)
```

3.877 $\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=83

$$\frac{160x\sqrt[4]{2+3x^2}}{2079}-\frac{40}{693}x^3\sqrt[4]{2+3x^2}+\frac{2}{33}x^5\sqrt[4]{2+3x^2}-\frac{320\,2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2079\sqrt{3}}$$

```
[Out] 160/2079*x*(3*x^2+2)^(1/4)-40/693*x^3*(3*x^2+2)^(1/4)+2/33*x^5*(3*x^2+2)^(1
/4)-320/6237*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcta
n(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {327, 237}

$$-\frac{320\,2^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2079\sqrt{3}}+\frac{160\sqrt[4]{3x^2+2}\,x}{2079}+\frac{2}{33}\sqrt[4]{3x^2+2}\,x^5-\frac{40}{693}\sqrt[4]{3x^2+2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (160*x*(2 + 3*x^2)^(1/4))/2079 - (40*x^3*(2 + 3*x^2)^(1/4))/693 + (2*x^5*(2
 + 3*x^2)^(1/4))/33 - (320*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(20
79*Sqrt[3])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx &= \frac{2}{33}x^5\sqrt[4]{2+3x^2} - \frac{20}{33}\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx \\ &= -\frac{40}{693}x^3\sqrt[4]{2+3x^2} + \frac{2}{33}x^5\sqrt[4]{2+3x^2} + \frac{80}{231}\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx \\ &= \frac{160x\sqrt[4]{2+3x^2}}{2079} - \frac{40}{693}x^3\sqrt[4]{2+3x^2} + \frac{2}{33}x^5\sqrt[4]{2+3x^2} - \frac{320\int \frac{1}{(2+3x^2)^{3/4}}\,dx}{2079} \\ &= \frac{160x\sqrt[4]{2+3x^2}}{2079} - \frac{40}{693}x^3\sqrt[4]{2+3x^2} + \frac{2}{33}x^5\sqrt[4]{2+3x^2} - \frac{320\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\right.}{2079\sqrt{3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.15, size = 54, normalized size = 0.65

$$\frac{2x\left(\sqrt[4]{2+3x^2}\,(80-60x^2+63x^4)-80\sqrt[4]{2}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{2079}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(1/4)*(80 - 60*x^2 + 63*x^4) - 80*2^(1/4)*Hypergeometric2
F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/2079
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.09, size = 20, normalized size = 0.24

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}x^7\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{7}{2}\right],\left[\frac{9}{2}\right],-\frac{3x^2}{2}\right)}{14}$	20
risch	$\frac{2x\,(63x^4-60x^2+80)\,(3x^2+2)^{\frac{1}{4}}}{2079}-\frac{160\,2^{\frac{1}{4}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{2079}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*2^(1/4)*x^7*hypergeom([3/4,7/2],[9/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(3*x^2 + 2)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.43, size = 27, normalized size = 0.33

$$\frac{\sqrt[4]{2}\,x^7\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**7*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)/2)/14
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(3\,x^2+2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2 + 2)^(3/4),x)
```

```
[Out] int(x^6/(3*x^2 + 2)^(3/4), x)
```

3.878 $\int \frac{x^4}{(2+3x^2)^{3/4}} dx$

Optimal. Leaf size=65

$$-\frac{8}{63}x\sqrt[4]{2+3x^2}+\frac{2}{21}x^3\sqrt[4]{2+3x^2}+\frac{16\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{63\sqrt{3}}$$

```
[Out] -8/63*x*(3*x^2+2)^(1/4)+2/21*x^3*(3*x^2+2)^(1/4)+16/189*2^(3/4)*(cos(1/2*ar
ctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticF(sin(
1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {327, 237}

$$\frac{16\ 2^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{63\sqrt{3}}-\frac{8}{63}\sqrt[4]{3x^2+2}\,x+\frac{2}{21}\sqrt[4]{3x^2+2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (-8*x*(2 + 3*x^2)^(1/4))/63 + (2*x^3*(2 + 3*x^2)^(1/4))/21 + (16*2^(3/4)*El
lipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx = \frac{2}{21}x^3\sqrt[4]{2+3x^2} - \frac{4}{7}\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx$$
$$= -\frac{8}{63}x\sqrt[4]{2+3x^2} + \frac{2}{21}x^3\sqrt[4]{2+3x^2} + \frac{16}{63}\int \frac{1}{(2+3x^2)^{3/4}}\,dx$$
$$= -\frac{8}{63}x\sqrt[4]{2+3x^2} + \frac{2}{21}x^3\sqrt[4]{2+3x^2} + \frac{16\,2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{63\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.92, size = 49, normalized size = 0.75

$$\frac{2}{63}x\left(\left(-4+3x^2\right)\sqrt[4]{2+3x^2}+4\sqrt[4]{2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*((-4 + 3*x^2)*(2 + 3*x^2)^(1/4) + 4*2^(1/4)*Hypergeometric2F1[1/2, 3/4
, 3/2, (-3*x^2)/2]))/63
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 20, normalized size = 0.31

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}x^5\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{7}{2}\right],-\frac{3x^2}{2}\right)}{10}$	20
risch	$\frac{2x\left(3x^2-4\right)\left(3x^2+2\right)^{\frac{1}{4}}}{63}+\frac{8\,2^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{63}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*2^(1/4)*x^5*hypergeom([3/4,5/2],[7/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(x^4/(3*x^2 + 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(x^4/(3*x^2 + 2)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 27, normalized size = 0.42

$$\frac{\sqrt[4]{2}\, x^5 {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{3x^2 e^{i\pi}}{2}} \right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2+2)**(3/4),x)
[Out] 2**(1/4)*x**5*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)/2)/10
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(x^4/(3*x^2 + 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{(3\,x^2+2)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2 + 2)^(3/4),x)
[Out] int(x^4/(3*x^2 + 2)^(3/4), x)
```

3.879 $\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=47

$$\frac{2}{9}x\sqrt[4]{2+3x^2}-\frac{4\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}$$

```
[Out] 2/9*x*(3*x^2+2)^(1/4)-4/27*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)
/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^
(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {327, 237}

$$\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4\ 2^{3/4}F\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(2 + 3*x^2)^(1/4))/9 - (4*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])
/(9*Sqrt[3])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx = \frac{2}{9}x\sqrt[4]{2+3x^2} - \frac{4}{9}\int \frac{1}{(2+3x^2)^{3/4}}\,dx$$
$$= \frac{2}{9}x\sqrt[4]{2+3x^2} - \frac{4\,2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.56, size = 41, normalized size = 0.87

$$\frac{2}{9}x\left(\sqrt[4]{2+3x^2} - \sqrt[4]{2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(1/4) - 2^(1/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)
/2]))/9
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 20, normalized size = 0.43

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],-\frac{3x^2}{2}\right)}{6}$	20
risch	$\frac{2x(3x^2+2)^{\frac{1}{4}}}{9} - \frac{2\,2^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{9}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/4)*x^3*hypergeom([3/4,3/2],[5/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 + 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(x^2/(3*x^2 + 2)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.39, size = 27, normalized size = 0.57

$$\frac{\sqrt[4]{2}\,x^3{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(3/4),x)
[Out] 2**(1/4)*x**3*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)/2)/6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(x^2/(3*x^2 + 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(3\,x^2+2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2 + 2)^(3/4),x)
[Out] int(x^2/(3*x^2 + 2)^(3/4), x)
```

3.880 $\int \frac{1}{(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=27

$$\frac{2^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] 1/3*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6
^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{237\}$

$$\frac{2^{3/4} F\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)^(-3/4),x]
```

```
[Out] (2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(2+3x^2)^{3/4}}\,dx = \frac{2^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.39, size = 24, normalized size = 0.89

$$\frac{x\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)^(-3/4),x]
```

```
[Out] (x*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2])/2^(3/4)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.04, size = 18, normalized size = 0.67

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)^(-3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(-3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.39, size = 26, normalized size = 0.96

$$\frac{\sqrt[4]{2}\, x\, {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| {\frac{3 x^{2} e^{i \pi}}{2}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(I*pi)/2)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)^(-3/4), x)
```

Mupad [B]
time = 0.08, size = 16, normalized size = 0.59

$$\frac{2^{1/4}\,x\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 2)^(3/4),x)
```

```
[Out] (2^(1/4)*x*hypergeom([1/2, 3/4], 3/2, -(3*x^2)/2))/2
```

3.881 $\int \frac{1}{x^2(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt[4]{2+3x^2}}{2x}-\frac{\sqrt{3}\,F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2\sqrt[4]{2}}$$

```
[Out] -1/2*(3*x^2+2)^(1/4)/x-1/4*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)
/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^
(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {331, 237}

$$-\frac{\sqrt{3}\,F\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{3x^2+2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/2*(2 + 3*x^2)^(1/4)/x - (Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(2
*2^(1/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2 (2+3x^2)^{3/4}} \, dx = -\frac{\sqrt[4]{2+3x^2}}{2x} - \frac{3}{4} \int \frac{1}{(2+3x^2)^{3/4}} \, dx$$
$$= -\frac{\sqrt[4]{2+3x^2}}{2x} - \frac{\sqrt{3}\, F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\, x\right)\middle| 2\right)}{2\sqrt[4]{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.87, size = 27, normalized size = 0.55

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 3/4, 1/2, (-3*x^2)/2]/(2^(3/4)*x))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 20, normalized size = 0.41

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}}\, \mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{2x}$	20
risch	$-\frac{(3x^2+2)^{\frac{1}{4}}}{2x} - \frac{3\, 2^{\frac{1}{4}}\, x\, \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{8}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/4)/x*hypergeom([-1/2,3/4],[1/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(1/4)/(3*x^4 + 2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 29, normalized size = 0.59

$$-\frac{\sqrt[4]{2}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| {\frac{3x^{2}e^{i\pi}}{2}} \right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(2*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)
```

Mupad [B]
time = 4.99, size = 36, normalized size = 0.73

$$-\frac{2\,3^{1/4}\,{\left(\frac{2}{x^2}+3\right)}^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{2}{3\,x^2}\right)}{15\,x\,{\left(3\,x^2+2\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 + 2)^(3/4)),x)
```

```
[Out] -(2*3^(1/4)*(2/x^2 + 3)^(3/4)*hypergeom([3/4, 5/4], 9/4, -2/(3*x^2)))/(15*x
*(3*x^2 + 2)^(3/4))
```

3.882 $\int \frac{1}{x^4(2+3x^2)^{3/4}} dx$

Optimal. Leaf size=67

$$-\frac{\sqrt[4]{2+3x^2}}{6x^3}+\frac{5\sqrt[4]{2+3x^2}}{8x}+\frac{5\sqrt{3}\,F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

```
[Out] -1/6*(3*x^2+2)^(1/4)/x^3+5/8*(3*x^2+2)^(1/4)/x+5/16*2^(3/4)*(cos(1/2*arctan
(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticF(sin(1/2*
arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {331, 237}

$$\frac{5\sqrt{3}\,F\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{8\sqrt[4]{2}}+\frac{5\sqrt[4]{3x^2+2}}{8x}-\frac{\sqrt[4]{3x^2+2}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/6*(2 + 3*x^2)^(1/4)/x^3 + (5*(2 + 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Ellip
ticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(8*2^(1/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4 (2+3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2+3x^2}}{6x^3} - \frac{5}{4}\int \frac{1}{x^2 (2+3x^2)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2+3x^2}}{6x^3} + \frac{5\sqrt[4]{2+3x^2}}{8x} + \frac{15}{16}\int \frac{1}{(2+3x^2)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2+3x^2}}{6x^3} + \frac{5\sqrt[4]{2+3x^2}}{8x} + \frac{5\sqrt{3}\,F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 29, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3\ 2^{3/4}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 3/4, -1/2, (-3*x^2)/2]/(2^(3/4)*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 20, normalized size = 0.30

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[-\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{6x^3}$	20
risch	$\frac{45x^4+18x^2-8}{24x^3(3x^2+2)^{\frac{3}{4}}}+\frac{15\,2^{\frac{1}{4}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{32}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/4)/x^3*hypergeom([-3/2,3/4],[-1/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(1/4)/(3*x^6 + 2*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.50, size = 32, normalized size = 0.48

$$-\frac{\sqrt[4]{2}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^{2}e^{i\pi}}{2} \right)}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(3\,x^2+2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 + 2)^(3/4)),x)
```

```
[Out] int(1/(x^4*(3*x^2 + 2)^(3/4)), x)
```

3.883 $\int \frac{1}{x^6(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt[4]{2+3x^2}}{10x^5}+\frac{9\sqrt[4]{2+3x^2}}{40x^3}-\frac{27\sqrt[4]{2+3x^2}}{32x}-\frac{27\sqrt{3}\,F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

```
[Out] -1/10*(3*x^2+2)^(1/4)/x^5+9/40*(3*x^2+2)^(1/4)/x^3-27/32*(3*x^2+2)^(1/4)/x-
27/64*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x
*6^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {331, 237}

$$-\frac{27\sqrt{3}\,F\left(\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{32\sqrt[4]{2}}-\frac{27\sqrt[4]{3x^2+2}}{32x}-\frac{\sqrt[4]{3x^2+2}}{10x^5}+\frac{9\sqrt[4]{3x^2+2}}{40x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/10*(2 + 3*x^2)^(1/4)/x^5 + (9*(2 + 3*x^2)^(1/4))/(40*x^3) - (27*(2 + 3*x
^2)^(1/4))/(32*x) - (27*Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(32*2^
(1/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6 \left(2+3 x^2\right)^{3/4}} \, dx &= -\frac{\sqrt[4]{2+3 x^2}}{10 x^5}-\frac{27}{20} \int \frac{1}{x^4 \left(2+3 x^2\right)^{3/4}} \, dx \\
&= -\frac{\sqrt[4]{2+3 x^2}}{10 x^5}+\frac{9 \sqrt[4]{2+3 x^2}}{40 x^3}+\frac{27}{16} \int \frac{1}{x^2 \left(2+3 x^2\right)^{3/4}} \, dx \\
&= -\frac{\sqrt[4]{2+3 x^2}}{10 x^5}+\frac{9 \sqrt[4]{2+3 x^2}}{40 x^3}-\frac{27 \sqrt[4]{2+3 x^2}}{32 x}-\frac{81}{64} \int \frac{1}{\left(2+3 x^2\right)^{3/4}} \, dx \\
&= -\frac{\sqrt[4]{2+3 x^2}}{10 x^5}+\frac{9 \sqrt[4]{2+3 x^2}}{40 x^3}-\frac{27 \sqrt[4]{2+3 x^2}}{32 x}-\frac{27 \sqrt{3} F\left(\frac{1}{2} \tan ^{-1}\left(\sqrt{\frac{3}{2}} x\right)\middle|2\right)}{32 \sqrt[4]{2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 29, normalized size = 0.34

$$
-\frac{{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{3 x^2}{2}\right)}{5\ 2^{3/4} x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/2, 3/4, -3/2, (-3*x^2)/2]/(2^(3/4)*x^5)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.07, size = 20, normalized size = 0.24

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}} \mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{3}{4}\right],\left[-\frac{3}{2}\right],-\frac{3 x^2}{2}\right)}{10 x^5}$	20
risch	$-\frac{405 x^6+162 x^4-24 x^2+32}{160 x^5 \left(3 x^2+2\right)^{\frac{3}{4}}}-\frac{81\, 2^{\frac{1}{4}} x\, \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3 x^2}{2}\right)}{128}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*2^(1/4)/x^5*hypergeom([-5/2,3/4],[-3/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)^(1/4)/(3*x^8 + 2*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 32, normalized size = 0.38

$$-\frac{\sqrt[4]{2}\ {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(3\,x^2+2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(3*x^2 + 2)^(3/4)),x)
```

```
[Out] int(1/(x^6*(3*x^2 + 2)^(3/4)), x)
```

3.884 $\int \frac{x^6}{\left(2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=83

$$-\frac{160x\sqrt[4]{2-3x^2}}{2079}-\frac{40}{693}x^3\sqrt[4]{2-3x^2}-\frac{2}{33}x^5\sqrt[4]{2-3x^2}+\frac{320\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2079\sqrt{3}}$$

```
[Out] -160/2079*x*(-3*x^2+2)^(1/4)-40/693*x^3*(-3*x^2+2)^(1/4)-2/33*x^5*(-3*x^2+2
)^(1/4)+320/6237*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*a
rcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {327, 238}

$$\frac{320\ 2^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2079\sqrt{3}}-\frac{160\sqrt[4]{2-3x^2}\,x}{2079}-\frac{2}{33}\sqrt[4]{2-3x^2}\,x^5-\frac{40}{693}\sqrt[4]{2-3x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-160*x*(2 - 3*x^2)^(1/4))/2079 - (40*x^3*(2 - 3*x^2)^(1/4))/693 - (2*x^5*(
2 - 3*x^2)^(1/4))/33 + (320*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2
079*Sqrt[3])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = -\frac{2}{33}x^5\sqrt[4]{2-3x^2} + \frac{20}{33}\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{40}{693}x^3\sqrt[4]{2-3x^2} - \frac{2}{33}x^5\sqrt[4]{2-3x^2} + \frac{80}{231}\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{160x\sqrt[4]{2-3x^2}}{2079} - \frac{40}{693}x^3\sqrt[4]{2-3x^2} - \frac{2}{33}x^5\sqrt[4]{2-3x^2} + \frac{320\int \frac{1}{(2-3x^2)^{3/4}}\,dx}{2079}$$
$$= -\frac{160x\sqrt[4]{2-3x^2}}{2079} - \frac{40}{693}x^3\sqrt[4]{2-3x^2} - \frac{2}{33}x^5\sqrt[4]{2-3x^2} + \frac{320\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\right.\right.}{2079\sqrt{3}}$$

Mathematica [A]
time = 6.26, size = 59, normalized size = 0.71

$$\frac{-6x\sqrt[4]{2-3x^2}\,(80+60x^2+63x^4)+320\ 2^{3/4}\sqrt{3}\,F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\,2\right)}{6237}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-6*x*(2 - 3*x^2)^(1/4)*(80 + 60*x^2 + 63*x^4) + 320*2^(3/4)*Sqrt[3]*Ellipt
icF[ArcSin[Sqrt[3/2]*x]/2, 2])/6237
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.10, size = 20, normalized size = 0.24

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}x^7\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{7}{2}\right],\left[\frac{9}{2}\right],\frac{3x^2}{2}\right)}{14}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*2^(1/4)*x^7*hypergeom([3/4,7/2],[9/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(x^6/(-3*x^2 + 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)*x^6/(3*x^2 - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 29, normalized size = 0.35

$$\frac{\sqrt[4]{2}\, x^7 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2+2)**(3/4),x)
[Out] 2**(1/4)*x**7*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(2*I*pi)/2)/14
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(x^6/(-3*x^2 + 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(2-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2 - 3*x^2)^(3/4),x)
[Out] int(x^6/(2 - 3*x^2)^(3/4), x)
```

3.885 $\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=65

$$-\frac{8}{63}x\sqrt[4]{2-3x^2}-\frac{2}{21}x^3\sqrt[4]{2-3x^2}+\frac{16\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{63\sqrt{3}}$$

```
[Out] -8/63*x*(-3*x^2+2)^(1/4)-2/21*x^3*(-3*x^2+2)^(1/4)+16/189*2^(3/4)*(cos(1/2*
arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(si
n(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {327, 238}

$$\frac{16\ 2^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{63\sqrt{3}}-\frac{8}{63}\sqrt[4]{2-3x^2}\,x-\frac{2}{21}\sqrt[4]{2-3x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-8*x*(2 - 3*x^2)^(1/4))/63 - (2*x^3*(2 - 3*x^2)^(1/4))/21 + (16*2^(3/4)*El
lipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx = -\frac{2}{21}x^3\sqrt[4]{2-3x^2} + \frac{4}{7}\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{8}{63}x\sqrt[4]{2-3x^2} - \frac{2}{21}x^3\sqrt[4]{2-3x^2} + \frac{16}{63}\int \frac{1}{(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{8}{63}x\sqrt[4]{2-3x^2} - \frac{2}{21}x^3\sqrt[4]{2-3x^2} + \frac{16\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{63\sqrt{3}}$$

Mathematica [A]
time = 6.17, size = 54, normalized size = 0.83

$$-\frac{2}{189}\left(3x\sqrt[4]{2-3x^2}\,(4+3x^2) - 8\,2^{3/4}\sqrt{3}\,F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 - 3*x^2)^(3/4),x]
[Out] (-2*(3*x*(2 - 3*x^2)^(1/4)*(4 + 3*x^2) - 8*2^(3/4)*Sqrt[3]*EllipticF[ArcSin
[Sqrt[3/2]*x]/2, 2]))/189
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 20, normalized size = 0.31

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x^5 \,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{7}{2}\right],\frac{3x^2}{2}\right)}{10}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
[Out] 1/10*2^(1/4)*x^5*hypergeom([3/4,5/2],[7/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 + 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)*x^4/(3*x^2 - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 29, normalized size = 0.45

$$\frac{\sqrt[4]{2}\, x^5\, {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2+2)**(3/4),x)
[Out] 2**(1/4)*x**5*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(2*I*pi)/2)/10
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(x^4/(-3*x^2 + 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(2-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2 - 3*x^2)^(3/4),x)
[Out] int(x^4/(2 - 3*x^2)^(3/4), x)
```

3.886 $\int \frac{x^2}{(2-3x^2)^{3/4}} \, dx$

Optimal. Leaf size=47

$$-\frac{2}{9}x\sqrt[4]{2-3x^2}+\frac{4\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}$$

```
[Out] -2/9*x*(-3*x^2+2)^(1/4)+4/27*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/
2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),
2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {327, 238}

$$\frac{4\ 2^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2]
)/(9*Sqrt[3])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx = -\frac{2}{9}x\sqrt[4]{2-3x^2} + \frac{4}{9}\int \frac{1}{(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{2}{9}x\sqrt[4]{2-3x^2} + \frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}$$

Mathematica [A]
time = 5.76, size = 47, normalized size = 1.00

$$-\frac{2}{9}x\sqrt[4]{2-3x^2} + \frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 3*x^2)^(3/4),x]
[Out] (-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2]
)/(9*Sqrt[3])
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 20, normalized size = 0.43

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],\frac{3x^2}{2}\right)}{6}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/4)*x^3*hypergeom([3/4,3/2],[5/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(x^2/(-3*x^2 + 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)*x^2/(3*x^2 - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.39, size = 29, normalized size = 0.62

$$\frac{\sqrt[4]{2}\,x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**3*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(2*I*pi)/2)/6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(2-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2 - 3*x^2)^(3/4),x)
```

```
[Out] int(x^2/(2 - 3*x^2)^(3/4), x)
```

3.887 $\int \frac{1}{(2-3x^2)^{3/4}} \, dx$

Optimal. Leaf size=27

$$\frac{2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}} x\right) \middle| 2\right)}{\sqrt{3}}$$

[Out] 1/3*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{238\}$

$$\frac{2^{3/4} F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}} x\right) \middle| 2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3*x^2)^(-3/4),x]

[Out] (2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{(2-3x^2)^{3/4}} \, dx = \frac{2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}} x\right) \middle| 2\right)}{\sqrt{3}}$$

Mathematica [A]
time = 5.55, size = 27, normalized size = 1.00

$$\frac{2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}} x\right) \middle| 2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)^(-3/4),x]
```

```
[Out] (2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.05, size = 18, normalized size = 0.67

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 + 2)^(-3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^2 - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.35, size = 27, normalized size = 1.00

$$\frac{\sqrt[4]{2}\, x\, {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| {\frac{3x^{2} e^{2i\pi}}{2}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(3/4),x)
[Out] 2**(1/4)*x*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate((-3*x^2 + 2)^(-3/4), x)
```

Mupad [B]
time = 4.76, size = 16, normalized size = 0.59

$$\frac{2^{1/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^2)^(3/4),x)
[Out] (2^(1/4)*x*hypergeom([1/2, 3/4], 3/2, (3*x^2)/2))/2
```

3.888 $\int \frac{1}{x^2(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt[4]{2-3x^2}}{2x}+\frac{\sqrt{3}\,F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2\sqrt[4]{2}}$$

```
[Out] -1/2*(-3*x^2+2)^(1/4)/x+1/4*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2
)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2
^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {331, 238}

$$\frac{\sqrt{3}\,F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/2*(2 - 3*x^2)^(1/4)/x + (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2
*2^(1/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2 (2-3x^2)^{3/4}} \, dx = -\frac{\sqrt[4]{2-3x^2}}{2x} + \frac{3}{4} \int \frac{1}{(2-3x^2)^{3/4}} \, dx$$
$$= -\frac{\sqrt[4]{2-3x^2}}{2x} + \frac{\sqrt{3} F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.99, size = 27, normalized size = 0.55

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(3/4)),x]
[Out] -(Hypergeometric2F1[-1/2, 3/4, 1/2, (3*x^2)/2]/(2^(3/4)*x))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 20, normalized size = 0.41

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{1}{2}\right],\frac{3x^2}{2}\right)}{2x}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
[Out] -1/2*2^(1/4)/x*hypergeom([-1/2,3/4],[1/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 31, normalized size = 0.63

$$-\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(3/4),x)
[Out] -2**(1/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(2*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)
```

Mupad [B]
time = 5.03, size = 36, normalized size = 0.73

$$-\frac{2\,3^{1/4}\,\left(3-\frac{2}{x^2}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ \frac{2}{3\,x^2}\right)}{15\,x\,\left(2-3\,x^2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2 - 3*x^2)^(3/4)),x)
[Out] -(2*3^(1/4)*(3 - 2/x^2)^(3/4)*hypergeom([3/4, 5/4], 9/4, 2/(3*x^2)))/(15*x*
(2 - 3*x^2)^(3/4))
```

3.889 $\int \frac{1}{x^4(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt[4]{2-3x^2}}{6x^3}-\frac{5\sqrt[4]{2-3x^2}}{8x}+\frac{5\sqrt{3}\,F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

```
[Out] -1/6*(-3*x^2+2)^(1/4)/x^3-5/8*(-3*x^2+2)^(1/4)/x+5/16*2^(3/4)*(cos(1/2*arcs
in(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/
2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {331, 238}

$$\frac{5\sqrt{3}\,F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{8\sqrt[4]{2}}-\frac{5\sqrt[4]{2-3x^2}}{8x}-\frac{\sqrt[4]{2-3x^2}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/6*(2 - 3*x^2)^(1/4)/x^3 - (5*(2 - 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Ellip
ticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(8*2^(1/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4(2-3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2-3x^2}}{6x^3}+\frac{5}{4}\int \frac{1}{x^2(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2-3x^2}}{6x^3}-\frac{5\sqrt[4]{2-3x^2}}{8x}+\frac{15}{16}\int \frac{1}{(2-3x^2)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{2-3x^2}}{6x^3}-\frac{5\sqrt[4]{2-3x^2}}{8x}+\frac{5\sqrt{3}\,F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 29, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3\;2^{3/4}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 3/4, -1/2, (3*x^2)/2]/(2^(3/4)*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 20, normalized size = 0.30

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[-\frac{1}{2}\right],\frac{3x^2}{2}\right)}{6x^3}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/4)/x^3*hypergeom([-3/2,3/4],[-1/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.50, size = 34, normalized size = 0.51

$$-\frac{\sqrt[4]{2}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^{2}e^{2i\pi}}{2}\right)}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(3/4),x)
[Out] -2**(1/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(2-3\,x^2\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(2 - 3*x^2)^(3/4)),x)
[Out] int(1/(x^4*(2 - 3*x^2)^(3/4)), x)
```

3.890 $\int \frac{1}{x^6(2-3x^2)^{3/4}} dx$

Optimal. Leaf size=85

$$-\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}-\frac{27\sqrt[4]{2-3x^2}}{32x}+\frac{27\sqrt{3}\,F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

```
[Out] -1/10*(-3*x^2+2)^(1/4)/x^5-9/40*(-3*x^2+2)^(1/4)/x^3-27/32*(-3*x^2+2)^(1/4)
/x+27/64*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/
2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {331, 238}

$$\frac{27\sqrt{3}\,F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{32\sqrt[4]{2}}-\frac{27\sqrt[4]{2-3x^2}}{32x}-\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/10*(2 - 3*x^2)^(1/4)/x^5 - (9*(2 - 3*x^2)^(1/4))/(40*x^3) - (27*(2 - 3*x
^2)^(1/4))/(32*x) + (27*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(32*2^
(1/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6 (2-3x^2)^{3/4}}\,dx &= -\frac{\sqrt[4]{2-3x^2}}{10x^5}+\frac{27}{20}\int \frac{1}{x^4 (2-3x^2)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}+\frac{27}{16}\int \frac{1}{x^2 (2-3x^2)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}-\frac{27\sqrt[4]{2-3x^2}}{32x}+\frac{81}{64}\int \frac{1}{(2-3x^2)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}-\frac{27\sqrt[4]{2-3x^2}}{32x}+\frac{27\sqrt{3}\,F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{32\sqrt[4]{2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 29, normalized size = 0.34

$$
-\frac{{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5\ 2^{3/4}x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/2, 3/4, -3/2, (3*x^2)/2]/(2^(3/4)*x^5)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.09, size = 20, normalized size = 0.24

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{3}{4}\right],\left[-\frac{3}{2}\right],\frac{3x^2}{2}\right)}{10x^5}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*2^(1/4)/x^5*hypergeom([-5/2,3/4],[-3/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.58, size = 34, normalized size = 0.40

$$-\frac{\sqrt[4]{2}\ {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2+2)**(3/4),x)
[Out] -2**(1/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(2-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(2 - 3*x^2)^(3/4)),x)
[Out] int(1/(x^6*(2 - 3*x^2)^(3/4)), x)
```

3.891 $\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=258

$$\frac{32x(-2+3x^2)^{3/4}}{1053}+\frac{40x^3(-2+3x^2)^{3/4}}{1053}+\frac{2}{39}x^5(-2+3x^2)^{3/4}+\frac{128x\sqrt[4]{-2+3x^2}}{1053\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}-\frac{128\sqrt[4]{2}\sqrt{\frac{}{\left(\sqrt{2}\right.}}}{}$$

```
[Out] 32/1053*x*(3*x^2-2)^(3/4)+40/1053*x^3*(3*x^2-2)^(3/4)+2/39*x^5*(3*x^2-2)^(3
/4)+128/1053*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))-128/3159*2^(1/4)*(
cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2
-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/
2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2
)/x*3^(1/2)+64/3159*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^
(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2
*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(
1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {327, 236, 311, 226, 1210}

$$\frac{64\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{1053\sqrt{3}x}-\frac{128\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{1053\sqrt{3}x}+\frac{32(3x^2-2)^{3/4}x}{1053}+\frac{128\sqrt[4]{3x^2-2}x}{1053\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{2}{39}(3x^2-2)^{3/4}x^5+\frac{40(3x^2-2)^{3/4}x^3}{1053}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (32*x*(-2 + 3*x^2)^(3/4))/1053 + (40*x^3*(-2 + 3*x^2)^(3/4))/1053 + (2*x^5*
(-2 + 3*x^2)^(3/4))/39 + (128*x*(-2 + 3*x^2)^(1/4))/(1053*(Sqrt[2] + Sqrt[-
2 + 3*x^2])) - (128*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[
2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2]
)/(1053*Sqrt[3]*x) + (64*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(
Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)],
 1/2])/(1053*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{20}{39}\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{80}{351}\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{64\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx}{1053}$$

$$= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{\left(64\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Su}\cdots}{\cdots}$$

$$= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{\left(128\sqrt{x^2}\right)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4} + \frac{128x\sqrt[4]{-2+3x\cdots}}{1053\left(\sqrt{2}+\sqrt{-2\cdots}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.88, size = 68, normalized size = 0.26

$$\frac{2x\left(-32+8x^2+6x^4+81x^6+16\ 2^{3/4}\sqrt[4]{2-3x^2}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)}{1053\sqrt[4]{-2+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-32 + 8*x^2 + 6*x^4 + 81*x^6 + 16*2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeom
etric2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/(1053*(-2 + 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 42, normalized size = 0.16

method	result	size

meijerg	$\frac{2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}x^7\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{7}{2}\right],\left[\frac{9}{2}\right],\frac{3x^2}{2}\right)}{14\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	42
risch	$\frac{2x(27x^4+20x^2+16)\,(3x^2-2)^{\frac{3}{4}}}{1053}+\frac{32\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{1053\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x^7*hyper
geom([1/4,7/2],[9/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 - 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(3*x^2 - 2)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 29, normalized size = 0.11

$$\frac{2^{\frac{3}{4}}x^7e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*exp(-I*pi/4)*hyper((1/4, 7/2), (9/2,), 3*x**2/2)/14
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate(x^6/(3*x^2 - 2)^(1/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(3\,x^2-2)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2 - 2)^(1/4),x)
[Out] int(x^6/(3*x^2 - 2)^(1/4), x)
```

3.892 $\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=240

$$\frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{32x\sqrt[4]{-2+3x^2}}{135\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}-\frac{32\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{\right.}}{}$$

```
[Out] 8/135*x*(3*x^2-2)^(3/4)+2/27*x^3*(3*x^2-2)^(3/4)+32/135*x*(3*x^2-2)^(1/4)/(
2^(1/2)+(3*x^2-2)^(1/2))-32/405*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2
^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(
2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2
))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)+16/405*2^(1/4)*(cos(2*
arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1
/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1
/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^
(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {327, 236, 311, 226, 1210}

$$\frac{16\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{135\sqrt{3}x}-\frac{32\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{135\sqrt{3}x}+\frac{8}{135}\left(3x^2-2\right)^{3/4}x+\frac{32\sqrt[4]{3x^2-2}\,x}{135\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (8*x*(-2 + 3*x^2)^(3/4))/135 + (2*x^3*(-2 + 3*x^2)^(3/4))/27 + (32*x*(-2 +
3*x^2)^(1/4))/(135*(Sqrt[2] + Sqrt[-2 + 3*x^2])) - (32*2^(1/4)*Sqrt[x^2/(Sq
rt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTa
n[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x) + (16*2^(1/4)*Sqrt[x^2
/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*A
rcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2}{27}x^3\left(-2+3x^2\right)^{3/4} + \frac{4}{9}\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx$$
$$= \frac{8}{135}x\left(-2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(-2+3x^2\right)^{3/4} + \frac{16}{135}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$$
$$= \frac{8}{135}x\left(-2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(-2+3x^2\right)^{3/4} + \frac{\left(16\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,\right.}{135x}$$
$$= \frac{8}{135}x\left(-2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(-2+3x^2\right)^{3/4} + \frac{\left(32\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-}\right.}{135\sqrt{3}\,x}$$
$$= \frac{8}{135}x\left(-2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(-2+3x^2\right)^{3/4} + \frac{32x\sqrt[4]{-2+3x^2}}{135\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} - \frac{32\sqrt[4]{2}\sqrt{\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.68, size = 63, normalized size = 0.26

$$\frac{2x\left(-8+2x^2+15x^4+4\,2^{3/4}\sqrt[4]{2-3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)}{135\sqrt[4]{-2+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-8 + 2*x^2 + 15*x^4 + 4*2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeometric2F1[1
/4, 1/2, 3/2, (3*x^2)/2]))/(135*(-2 + 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 42, normalized size = 0.18

method	result	size
meijerg	$\frac{2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}x^5\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{7}{2}\right],\frac{3x^2}{2}\right)}{10\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	42

risch	$\frac{2x(5x^2+4)(3x^2-2)^{\frac{3}{4}}}{135}+\frac{8\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{135\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x^5*hyper
geom([1/4,5/2],[7/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 - 2)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.42, size = 29, normalized size = 0.12

$$\frac{2^{\frac{3}{4}}x^5e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**5*exp(-I*pi/4)*hyper((1/4, 5/2), (7/2,), 3*x**2/2)/10
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(3\,x^2-2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2 - 2)^(1/4),x)
```

```
[Out] int(x^4/(3*x^2 - 2)^(1/4), x)
```

3.893 $\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=222

$$\frac{2}{15}x\left(-2+3x^2\right)^{3/4}+\frac{8x\sqrt[4]{-2+3x^2}}{15\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}-\frac{8\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\Big(}{15\sqrt{3}\,x}$$

```
[Out] 2/15*x*(3*x^2-2)^(3/4)+8/15*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))-8/4
5*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan
(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2
^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/
2))^2)^(1/2)/x*3^(1/2)+4/45*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/
4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*ar
ctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(
x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {327, 236, 311, 226, 1210}

$$\frac{4\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{15\sqrt{3}\,x}-\frac{8\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{15\sqrt{3}\,x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-2}\,x}{15\left(\sqrt{3x^2-2}+\sqrt{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2)^(3/4))/15 + (8*x*(-2 + 3*x^2)^(1/4))/(15*(Sqrt[2] + Sqrt[
-2 + 3*x^2])) - (8*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2
] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(15*Sqrt[3]*x) + (4*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt
[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2
])/(15*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
```

```
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2}{15}x\left(-2+3x^2\right)^{3/4} + \frac{4}{15}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$$

$$= \frac{2}{15}x\left(-2+3x^2\right)^{3/4} + \frac{\left(4\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{15x}$$

$$= \frac{2}{15}x\left(-2+3x^2\right)^{3/4} + \frac{\left(8\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{15\sqrt{3}\,x} - \frac{\left(8\sqrt{x^2}\right)\text{S}}{}$$

$$= \frac{2}{15}x\left(-2+3x^2\right)^{3/4} + \frac{8x\sqrt[4]{-2+3x^2}}{15\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} - \frac{8\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.59, size = 57, normalized size = 0.26

$$\frac{2x\left(-2+3x^2+2^{3/4}\sqrt[4]{2-3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)}{15\sqrt[4]{-2+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2 + 2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/
2, (3*x^2)/2]))/(15*(-2 + 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 42, normalized size = 0.19

method	result	size
meijerg	$\frac{2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],\frac{3x^2}{2}\right)}{6\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	42
risch	$\frac{2x\left(3x^2-2\right)^{\frac{3}{4}}}{15}+\frac{2\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{15\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x^3*hyperg
eom([1/4,3/2],[5/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="fricas")
[Out] integral(x^2/(3*x^2 - 2)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 29, normalized size = 0.13

$$\frac{2^{\frac{3}{4}}x^3e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)**(1/4),x)
[Out] 2**(3/4)*x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 3*x**2/2)/6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate(x^2/(3*x^2 - 2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(3\,x^2-2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2 - 2)^(1/4),x)
[Out] int(x^2/(3*x^2 - 2)^(1/4), x)
```

3.894 $\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=199

$$\frac{2x\sqrt[4]{-2+3x^2}}{\sqrt{2}+\sqrt{-2+3x^2}}-\frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right.}{\sqrt{3}\,x}$$

```
[Out] 2*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))-2/3*2^(1/4)*(cos(2*arctan(1/2
*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4
)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1
/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)+1/3*
2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1
/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(
3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2)
)^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {236, 311, 226, 1210}

$$\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}\,x}-\frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}\,x}+\frac{2\sqrt[4]{3x^2-2}\,x}{\sqrt{3x^2-2}+\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^2)^(-1/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2]) - (2*2^(1/4)*Sqrt[x^2
/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*A
rcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x) + (2^(1/4)*Sqrt[x^2/(S
qrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcT
an[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
```

```
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx = \frac{\left(\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{x}$$

$$= \frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{\sqrt{3}\,x} - \frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\,d\right.}{\sqrt{3}\,x}$$

$$= \frac{2x\sqrt[4]{-2+3x^2}}{\sqrt{2}+\sqrt{-2+3x^2}} - \frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\tan\right.}{\sqrt{3}\,x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 5.57, size = 43, normalized size = 0.22

$$\frac{x\sqrt[4]{1-\frac{3x^2}{2}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)}{\sqrt[4]{-2+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^2)^(-1/4),x]
```

```
[Out] (x*(1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2])/(-2 +
 3*x^2)^(1/4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 40, normalized size = 0.20

method	result	size
meijerg	$\frac{2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x*hypergeo
m([1/4,1/2],[3/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - 2)^(-1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(-1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.36, size = 27, normalized size = 0.14

$$\frac{2^{\frac{3}{4}} x e^{-\frac{i\pi}{4}} {}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2}\end{matrix}\right| \frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)**(1/4),x)
[Out] 2**(3/4)*x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 3*x**2/2)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate((3*x^2 - 2)^(-1/4), x)
```

Mupad [B]
time = 4.74, size = 34, normalized size = 0.17

$$\frac{2^{3/4}\,x\,{\left(2-3\,x^2\right)}^{1/4}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2\,{\left(3\,x^2-2\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 2)^(1/4),x)
[Out] (2^(3/4)*x*(2 - 3*x^2)^(1/4)*hypergeom([1/4, 1/2], 3/2, (3*x^2)/2))/(2*(3*x
^2 - 2)^(1/4))
```

3.895 $\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=221

$$\frac{(-2+3x^2)^{3/4}}{2x}-\frac{3x\sqrt[4]{-2+3x^2}}{2\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\tan^{-}\right.}{2^{3/4}x}$$

```
[Out] 1/2*(3*x^2-2)^(3/4)/x-3/2*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))+1/2*2
^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/
2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3
/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))
^2)^(1/2)/x*3^(1/2)-1/4*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))
^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan
(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/
(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {331, 236, 311, 226, 1210}

$$-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2\ 2^{3/4}x}+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2^{3/4}x}-\frac{3\sqrt[4]{3x^2-2}\,x}{2\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{(3x^2-2)^{3/4}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-2 + 3*x^2)^(3/4)/(2*x) - (3*x*(-2 + 3*x^2)^(1/4))/(2*(Sqrt[2] + Sqrt[-2 +
 3*x^2])) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sq
rt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3
/4)*x) - (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[
-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2*2^(3/
4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
```

```
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx &= \frac{\left(-2+3x^2\right)^{3/4}}{2x} - \frac{3}{4}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{2x} - \frac{\left(\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{2x} \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{2x} - \frac{\left(\sqrt{3}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{2x} + \frac{\left(\sqrt{3}\sqrt{x^2}\right.\cdots}{\cdots} \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{2x} - \frac{3x\sqrt[4]{-2+3x^2}}{2\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} + \frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\cdots\right.}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.66, size = 46, normalized size = 0.21

$$-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{x\sqrt[4]{-2+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)^(1/4)),x]
[Out] -(((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (3*x^2)/2])/(x*(
-2 + 3*x^2)^(1/4)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 42, normalized size = 0.19

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}\mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{1}{2}\right],\frac{3x^2}{2}\right)}{2\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}x}$	42
risch	$\frac{\left(3x^2-2\right)^{\frac{3}{4}}}{2x}-\frac{3\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{8\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
[Out] -1/2*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)/x*hyperge
om([-1/2,1/4],[1/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(3/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 31, normalized size = 0.14

$$\frac{2^{\frac{3}{4}} e^{\frac{3i\pi}{4}} {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{3x^2}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2/2)/(2*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^2), x)
```

Mupad [B]
time = 4.89, size = 36, normalized size = 0.16

$$-\frac{2\,3^{3/4}\,\left(3-\frac{2}{x^2}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{2}{3\,x^2}\right)}{9\,x\,\left(3\,x^2-2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 - 2)^(1/4)),x)
```

```
[Out] -(2*3^(3/4)*(3 - 2/x^2)^(1/4)*hypergeom([1/4, 3/4], 7/4, 2/(3*x^2)))/(9*x*(
3*x^2 - 2)^(1/4))
```

3.896 $\int \frac{1}{x^4 \sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=242

$$\frac{(-2+3x^2)^{3/4}}{6x^3}+\frac{3(-2+3x^2)^{3/4}}{8x}-\frac{9x\sqrt[4]{-2+3x^2}}{8\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}+\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2\right.}}{4\,2^{3/4}x}$$

```
[Out] 1/6*(3*x^2-2)^(3/4)/x^3+3/8*(3*x^2-2)^(3/4)/x-9/8*x*(3*x^2-2)^(1/4)/(2^(1/2
)+(3*x^2-2)^(1/2))+3/8*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^
2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(
1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(
2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)-3/16*2^(1/4)*(cos(2*arctan(1/2*
(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)
))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/
2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {331, 236, 311, 226, 1210}

$$-\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{8\,2^{3/4}x}+\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\,2^{3/4}x}-\frac{9\sqrt[4]{3x^2-2}\,x}{8\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{3(3x^2-2)^{3/4}}{8x}+\frac{(3x^2-2)^{3/4}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-2 + 3*x^2)^(3/4)/(6*x^3) + (3*(-2 + 3*x^2)^(3/4))/(8*x) - (9*x*(-2 + 3*x^
2)^(1/4))/(8*(Sqrt[2] + Sqrt[-2 + 3*x^2])) + (3*Sqrt[3]*Sqrt[x^2/(Sqrt[2] +
 Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 +
 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(3/4)*x) - (3*Sqrt[3]*Sqrt[x^2/(Sqrt[2]
+ Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2
+ 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8*2^(3/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
```

```
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = \frac{(-2+3x^2)^{3/4}}{6x^3} + \frac{3}{4}\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx$$
$$= \frac{(-2+3x^2)^{3/4}}{6x^3} + \frac{3(-2+3x^2)^{3/4}}{8x} - \frac{9}{16}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$$
$$= \frac{(-2+3x^2)^{3/4}}{6x^3} + \frac{3(-2+3x^2)^{3/4}}{8x} - \frac{\left(3\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2\right.}{8x}$$
$$= \frac{(-2+3x^2)^{3/4}}{6x^3} + \frac{3(-2+3x^2)^{3/4}}{8x} - \frac{\left(3\sqrt{3}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2\right.}{8x}$$
$$= \frac{(-2+3x^2)^{3/4}}{6x^3} + \frac{3(-2+3x^2)^{3/4}}{8x} - \frac{9x\sqrt[4]{-2+3x^2}}{8\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} + \frac{3\sqrt{3}\sqrt{\frac{x}{\left(\sqrt{2}+\sqrt{-\right.}}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 48, normalized size = 0.20

$$-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3x^3\sqrt[4]{-2+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/3*((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (3*x^2)/2])/
(x^3*(-2 + 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 42, normalized size = 0.17

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[-\frac{1}{2}\right],\frac{3x^2}{2}\right)}{6\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}x^3}$	42

risch	$\frac{27x^4-6x^2-8}{24x^3(3x^2-2)^{\frac{1}{4}}}-\frac{9\,2^{\frac{3}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{32\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)/x^3*hyper
geom([-3/2,1/4],[-1/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(3/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 34, normalized size = 0.14

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2/2)/(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(3\,x^2-2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 - 2)^(1/4)),x)
```

```
[Out] int(1/(x^4*(3*x^2 - 2)^(1/4)), x)
```

3.897 $\int \frac{1}{x^6\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=260

$$\frac{(-2+3x^2)^{3/4}}{10x^5}+\frac{7(-2+3x^2)^{3/4}}{40x^3}+\frac{63(-2+3x^2)^{3/4}}{160x}-\frac{189x\sqrt[4]{-2+3x^2}}{160\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}+\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\cdots}{\cdots}$$

```
[Out] 1/10*(3*x^2-2)^(3/4)/x^5+7/40*(3*x^2-2)^(3/4)/x^3+63/160*(3*x^2-2)^(3/4)/x-
189/160*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))+63/160*2^(1/4)*(cos(2*a
rctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/
4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/
2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(
1/2)-63/320*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/co
s(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-
2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*
x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {331, 236, 311, 226, 1210}

$$-\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{160\ 2^{3/4}x}+\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{80\ 2^{3/4}x}-\frac{189\sqrt[4]{3x^2-2}\,x}{160\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{63(3x^2-2)^{3/4}}{160x}+\frac{(3x^2-2)^{3/4}}{10x^5}+\frac{7(3x^2-2)^{3/4}}{40x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-2 + 3*x^2)^(3/4)/(10*x^5) + (7*(-2 + 3*x^2)^(3/4))/(40*x^3) + (63*(-2 + 3
*x^2)^(3/4))/(160*x) - (189*x*(-2 + 3*x^2)^(1/4))/(160*(Sqrt[2] + Sqrt[-2 +
 3*x^2])) + (63*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] +
 Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8
0*2^(3/4)*x) - (63*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2
] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(160*2^(3/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[4]{-2+3x^2}}\,dx = \frac{(-2+3x^2)^{3/4}}{10x^5} + \frac{21}{20}\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx$$
$$= \frac{(-2+3x^2)^{3/4}}{10x^5} + \frac{7(-2+3x^2)^{3/4}}{40x^3} + \frac{63}{80}\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx$$
$$= \frac{(-2+3x^2)^{3/4}}{10x^5} + \frac{7(-2+3x^2)^{3/4}}{40x^3} + \frac{63(-2+3x^2)^{3/4}}{160x} - \frac{189}{320}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$$
$$= \frac{(-2+3x^2)^{3/4}}{10x^5} + \frac{7(-2+3x^2)^{3/4}}{40x^3} + \frac{63(-2+3x^2)^{3/4}}{160x} - \frac{\left(63\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int -\right.}{16}$$
$$= \frac{(-2+3x^2)^{3/4}}{10x^5} + \frac{7(-2+3x^2)^{3/4}}{40x^3} + \frac{63(-2+3x^2)^{3/4}}{160x} - \frac{\left(63\sqrt{3}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{160}$$
$$= \frac{(-2+3x^2)^{3/4}}{10x^5} + \frac{7(-2+3x^2)^{3/4}}{40x^3} + \frac{63(-2+3x^2)^{3/4}}{160x} - \frac{189x\sqrt[4]{-2+3x^2}}{160\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 48, normalized size = 0.18

$$-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\ {}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5x^5\sqrt[4]{-2+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/5*((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (3*x^2)/2])/
(x^5*(-2 + 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 42, normalized size = 0.16

method	result	size

meijerg	$-\frac{2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}\mathrm{hypergeom}\left(\left[-\frac{5}{2},\frac{1}{4}\right],\left[-\frac{3}{2}\right],\frac{3x^2}{2}\right)}{10\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}x^5}$	42
risch	$\frac{189x^6-42x^4-8x^2-32}{160x^5(3x^2-2)^{\frac{1}{4}}}-\frac{189\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{640\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)/x^5*hype
rgeom([-5/2,1/4],[-3/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(3/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 34, normalized size = 0.13

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2/2)/(10*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,{\left(3\,x^2-2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(3*x^2 - 2)^(1/4)),x)
[Out] int(1/(x^6*(3*x^2 - 2)^(1/4)), x)
```

3.898 $\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=260

$$-\frac{32x(-2-3x^2)^{3/4}}{1053}+\frac{40x^3(-2-3x^2)^{3/4}}{1053}-\frac{2}{39}x^5(-2-3x^2)^{3/4}-\frac{128x\sqrt[4]{-2-3x^2}}{1053\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}-\frac{128\sqrt[4]{2}\sqrt{-\cdots}}{\cdots}$$

```
[Out] -32/1053*x*(-3*x^2-2)^(3/4)+40/1053*x^3*(-3*x^2-2)^(3/4)-2/39*x^5*(-3*x^2-2
)^(3/4)-128/1053*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))-128/3159*2^(
1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2
*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(
3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1
/2))^2)^(1/2)/x*3^(1/2)+64/3159*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*
2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(si
n(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^
(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {327, 236, 311, 226, 1210}

$$\frac{64\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{1053\sqrt{3}x}-\frac{128\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{1053\sqrt{3}x}-\frac{32(-3x^2-2)^{3/4}x}{1053}-\frac{128\sqrt[4]{-3x^2-2}\,x}{1053\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}-\frac{2}{39}(-3x^2-2)^{3/4}x^5+\frac{40(-3x^2-2)^{3/4}x^3}{1053}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (-32*x*(-2 - 3*x^2)^(3/4))/1053 + (40*x^3*(-2 - 3*x^2)^(3/4))/1053 - (2*x^5
*(-2 - 3*x^2)^(3/4))/39 - (128*x*(-2 - 3*x^2)^(1/4))/(1053*(Sqrt[2] + Sqrt[
-2 - 3*x^2])) - (128*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(S
qrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)],
1/2])/(1053*Sqrt[3]*x) + (64*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2]
)^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^
(1/4)], 1/2])/(1053*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} - \frac{20}{39}\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{40x^3(-2-3x^2)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} + \frac{80}{351}\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= -\frac{32x(-2-3x^2)^{3/4}}{1053} + \frac{40x^3(-2-3x^2)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} - \frac{64\int \frac{1}{\sqrt[4]{-2-3x^2}}}{1053}$$

$$= -\frac{32x(-2-3x^2)^{3/4}}{1053} + \frac{40x^3(-2-3x^2)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} + \frac{\left(64\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)}{}$$

$$= -\frac{32x(-2-3x^2)^{3/4}}{1053} + \frac{40x^3(-2-3x^2)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} + \frac{\left(128\sqrt{-x^2}\right)\mathrm{Sub}}{}$$

$$= -\frac{32x(-2-3x^2)^{3/4}}{1053} + \frac{40x^3(-2-3x^2)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} - \frac{128x\sqrt[4]{-2-}}{1053\left(\sqrt{2}+\sqrt{-}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.90, size = 68, normalized size = 0.26

$$\frac{2x\left(32+8x^2-6x^4+81x^6-16\,2^{3/4}\sqrt[4]{2+3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{1053\sqrt[4]{-2-3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(32 + 8*x^2 - 6*x^4 + 81*x^6 - 16*2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeome
tric2F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/(1053*(-2 - 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.07, size = 23, normalized size = 0.09

method	result	size

meijerg	$-\frac{(-1)^{\frac{3}{4}}2^{\frac{3}{4}}x^7 \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{7}{2}\right],\left[\frac{9}{2}\right],-\frac{3x^2}{2}\right)}{14}$	23
risch	$\frac{2x(27x^4-20x^2+16)(3x^2+2)}{1053(-3x^2-2)^{\frac{1}{4}}}+\frac{32(-1)^{\frac{3}{4}}2^{\frac{3}{4}}x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{1053}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
[Out] -1/14*(-1)^(3/4)*2^(3/4)*x^7*hypergeom([1/4,7/2],[9/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="maxima")
[Out] integrate(x^6/(-3*x^2 - 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="fricas")
[Out] 1/3159*(3159*x*integral(256/3159*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2
*(81*x^6 - 60*x^4 + 48*x^2 - 64)*(-3*x^2 - 2)^(3/4))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 34, normalized size = 0.13

$$\frac{2^{\frac{3}{4}}x^7e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2-2)**(1/4),x)
[Out] 2**(3/4)*x**7*exp(-I*pi/4)*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)
/2)/14
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate(x^6/(-3*x^2 - 2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(-3\,x^2-2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(- 3*x^2 - 2)^(1/4),x)
[Out] int(x^6/(- 3*x^2 - 2)^(1/4), x)
```

3.899 $\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=242

$$\frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}+\frac{32x\sqrt[4]{-2-3x^2}}{135\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}+\frac{32\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\,\left(\right.}{}$$

```
[Out] 8/135*x*(-3*x^2-2)^(3/4)-2/27*x^3*(-3*x^2-2)^(3/4)+32/135*x*(-3*x^2-2)^(1/4
)/(2^(1/2)+(-3*x^2-2)^(1/2))+32/405*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1
/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*Elliptic
E(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2
-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)-16/405*2^(1/
4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(
-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/
4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2
))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {327, 236, 311, 226, 1210}

$$-\frac{16\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{135\sqrt{3}x}+\frac{32\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{135\sqrt{3}x}+\frac{8}{135}\left(-3x^2-2\right)^{3/4}x+\frac{32\sqrt[4]{-3x^2-2}\,x}{135\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (8*x*(-2 - 3*x^2)^(3/4))/135 - (2*x^3*(-2 - 3*x^2)^(3/4))/27 + (32*x*(-2 -
3*x^2)^(1/4))/(135*(Sqrt[2] + Sqrt[-2 - 3*x^2])) + (32*2^(1/4)*Sqrt[-(x^2/(
Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*Ar
cTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x) - (16*2^(1/4)*Sqrt[
-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Ellipti
cF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}-\frac{4}{9}\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$$
$$=\frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}+\frac{16}{135}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$
$$=\frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}-\frac{\left(16\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx\right.}{135x}$$
$$=\frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}-\frac{\left(32\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{\right.}{135\sqrt{3}\,x}$$
$$=\frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}+\frac{32x\sqrt[4]{-2-3x^2}}{135\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}+\frac{32\sqrt[4]{2}\sqrt{-\cdots}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.68, size = 63, normalized size = 0.26

$$\frac{2x\left(-8-2x^2+15x^4+4\,2^{3/4}\sqrt[4]{2+3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{135\sqrt[4]{-2-3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-8 - 2*x^2 + 15*x^4 + 4*2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeometric2F1[1
/4, 1/2, 3/2, (-3*x^2)/2]))/(135*(-2 - 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.11, size = 23, normalized size = 0.10

method	result	size
meijerg	$-\frac{(-1)^{\frac{3}{4}}2^{\frac{3}{4}}x^5\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{5}{2}\right],\left[\frac{7}{2}\right],-\frac{3x^2}{2}\right)}{10}$	23

risch	$\frac{2x(5x^2-4)(3x^2+2)}{135(-3x^2-2)^{\frac{1}{4}}}-\frac{8(-1)^{\frac{3}{4}}2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{135}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(-1)^(3/4)*2^(3/4)*x^5*hypergeom([1/4,5/2],[7/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/405*(405*x*integral(-64/405*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(1
5*x^4 - 12*x^2 + 16)*(-3*x^2 - 2)^(3/4))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 34, normalized size = 0.14

$$\frac{2^{\frac{3}{4}}x^5e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**5*exp(-I*pi/4)*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)
/2)/10
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(-3x^2-2)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(- 3*x^2 - 2)^(1/4),x)
```

```
[Out] int(x^4/(- 3*x^2 - 2)^(1/4), x)
```

3.900 $\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=224

$$-\frac{2}{15}x\left(-2-3x^2\right)^{3/4}-\frac{8x\sqrt[4]{-2-3x^2}}{15\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}-\frac{8\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}{15\sqrt{3}\,x}$$

```
[Out] -2/15*x*(-3*x^2-2)^(3/4)-8/15*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))
-8/45*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*a
rctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^
(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*
x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)+4/45*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^
(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*Ellipt
icF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x
^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {327, 236, 311, 226, 1210}

$$\frac{4\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{15\sqrt{3}\,x}-\frac{8\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{15\sqrt{3}\,x}-\frac{2}{15}\left(-3x^2-2\right)^{3/4}x-\frac{8\sqrt[4]{-3x^2-2}\,x}{15\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*(-2 - 3*x^2)^(3/4))/15 - (8*x*(-2 - 3*x^2)^(1/4))/(15*(Sqrt[2] + Sqrt
[-2 - 3*x^2])) - (8*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sq
rt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1
/2])/(15*Sqrt[3]*x) + (4*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)
]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4
)], 1/2])/(15*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
```

```
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx &= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}-\frac{4}{15}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx \\
&= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}+\frac{\left(4\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{15x} \\
&= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}+\frac{\left(8\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{15\sqrt{3}\,x}-\frac{\left(8\sqrt{-x^2}\right.\cdots}{\cdots} \\
&= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}-\frac{8x\sqrt[4]{-2-3x^2}}{15\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}-\frac{8\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\cdots}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.60, size = 58, normalized size = 0.26

$$\frac{2x\left(2+3x^2-2^{3/4}\sqrt[4]{2+3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{15\sqrt[4]{-2-3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 - 3*x^2)^(1/4),x]
[Out] (2*x*(2 + 3*x^2 - 2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2
, (-3*x^2)/2]))/(15*(-2 - 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 23, normalized size = 0.10

method	result	size
meijerg	$-\frac{(-1)^{\frac{3}{4}}2^{\frac{3}{4}}x^3\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],-\frac{3x^2}{2}\right)}{6}$	23
risch	$\frac{2x(3x^2+2)}{15(-3x^2-2)^{\frac{1}{4}}}+\frac{2(-1)^{\frac{3}{4}}2^{\frac{3}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{15}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
[Out] -1/6*(-1)^(3/4)*2^(3/4)*x^3*hypergeom([1/4,3/2],[5/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="maxima")
[Out] integrate(x^2/(-3*x^2 - 2)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/45*(45*x*integral(16/45*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(3*x^2
 - 4)*(-3*x^2 - 2)^(3/4))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 34, normalized size = 0.15

$$\frac{2^{\frac{3}{4}}x^3e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)
/2)/6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-3*x^2 - 2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(-3\,x^2-2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(- 3*x^2 - 2)^(1/4),x)
```

```
[Out] int(x^2/(- 3*x^2 - 2)^(1/4), x)
```

3.901 $\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=202

$$\frac{2x\sqrt[4]{-2-3x^2}}{\sqrt{2}+\sqrt{-2-3x^2}}+\frac{2\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)\right.}{\sqrt{3}\,x}$$

```
[Out] 2*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))+2/3*2^(1/4)*(cos(2*arctan(1
/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^
(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*
(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1
/2)-1/3*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2
*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2
)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-
3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {236, 311, 226, 1210}

$$-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}\,x}+\frac{2\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}\,x}+\frac{2\sqrt[4]{-3x^2-2}\,x}{\sqrt{-3x^2-2}+\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x^2)^(-1/4),x]
```

```
[Out] (2*x*(-2 - 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^2]) + (2*2^(1/4)*Sqrt[-(x
^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[
2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x) - (2^(1/4)*Sqrt[-(x
^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[
2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
```

```
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{\left(\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{x}$$

$$= -\frac{\left(2\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{\sqrt{3}\,x} + \frac{\left(2\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\cdots}} \cdots\right)}{\sqrt{3}\,x}$$

$$= \frac{2x\sqrt[4]{-2-3x^2}}{\sqrt{2}+\sqrt{-2-3x^2}} + \frac{2\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\,\text{t}\cdots\right.}{\sqrt{3}\,x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.56, size = 43, normalized size = 0.21

$$\frac{x\sqrt[4]{1+\frac{3x^2}{2}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)}{\sqrt[4]{-2-3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x^2)^(-1/4),x]
```

```
[Out] (x*(1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2])/(-2
- 3*x^2)^(1/4)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.05, size = 21, normalized size = 0.10

method	result	size
meijerg	$-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-1)^(3/4)*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 - 2)^(-1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*integral(-4/3*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(-3*x^2 -
 2)^(3/4))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 32, normalized size = 0.16

$$\frac{2^{\frac{3}{4}} x e^{-\frac{i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{3x^{2} e^{i\pi}}{2}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(I*pi)/2)
/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 - 2)^(-1/4), x)
```

Mupad [B]
time = 4.88, size = 34, normalized size = 0.17

$$\frac{2^{3/4}\,x\,{\left(3\,x^2+2\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2\,{\left(-3\,x^2-2\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 3*x^2 - 2)^(1/4),x)
```

```
[Out] (2^(3/4)*x*(3*x^2 + 2)^(1/4)*hypergeom([1/4, 1/2], 3/2, -(3*x^2)/2))/(2*(-
3*x^2 - 2)^(1/4))
```

3.902 $\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=223

$$\frac{(-2-3x^2)^{3/4}}{2x}+\frac{3x\sqrt[4]{-2-3x^2}}{2\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}+\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\Big(2\tan}{2^{3/4}x}$$

```
[Out] 1/2*(-3*x^2-2)^(3/4)/x+3/2*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))+1/
2*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arcta
n(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4
)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-
2)^(1/2))^2)^(1/2)/x*3^(1/2)-1/4*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)
*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(s
in(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)
^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {331, 236, 311, 226, 1210}

$$-\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2\ 2^{3/4}x}+\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2^{3/4}x}+\frac{3\sqrt[4]{-3x^2-2}\,x}{2\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{(-3x^2-2)^{3/4}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] (-2 - 3*x^2)^(3/4)/(2*x) + (3*x*(-2 - 3*x^2)^(1/4))/(2*(Sqrt[2] + Sqrt[-2 -
 3*x^2])) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] +
 Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2
^(3/4)*x) - (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] +
 Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2
*2^(3/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
```

```
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx = \frac{\left(-2-3x^2\right)^{3/4}}{2x}+\frac{3}{4}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{2x}-\frac{\left(\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{2x}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{2x}-\frac{\left(\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{2x}+\frac{\left(\sqrt{3}\sqrt{\right.}{}}{}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{2x}+\frac{3x\sqrt[4]{-2-3x^2}}{2\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}+\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.70, size = 46, normalized size = 0.21

$$-\frac{\sqrt[4]{1+\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{x\sqrt[4]{-2-3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (-3*x^2)/2])/(x*
(-2 - 3*x^2)^(1/4)))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 23, normalized size = 0.10

method	result	size
meijerg	$\frac{(-1)^{\frac{3}{4}}2^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{2x}$	23
risch	$-\frac{3x^2+2}{2x\left(-3x^2-2\right)^{\frac{1}{4}}}-\frac{3(-1)^{\frac{3}{4}}2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{8}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1)^(3/4)*2^(3/4)/x*hypergeom([-1/2,1/4],[1/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*integral(-3/4*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) + (-3*x^2 - 2)^(3
/4))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 36, normalized size = 0.16

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)
/(2*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^2), x)
```

Mupad [B]
time = 5.04, size = 36, normalized size = 0.16

$$-\frac{2\,3^{3/4}\,{\left(\frac{2}{x^2}+3\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{2}{3\,x^2}\right)}{9\,x\,{\left(-3\,x^2-2\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- 3*x^2 - 2)^(1/4)),x)
```

```
[Out] -(2*3^(3/4)*(2/x^2 + 3)^(1/4)*hypergeom([1/4, 3/4], 7/4, -2/(3*x^2)))/(9*x*
(- 3*x^2 - 2)^(1/4))
```

3.903 $\int \frac{1}{x^4 \sqrt[4]{-2-3x^2}} \, dx$

Optimal. Leaf size=244

$$\frac{(-2-3x^2)^{3/4}}{6x^3} - \frac{3(-2-3x^2)^{3/4}}{8x} - \frac{9x\sqrt[4]{-2-3x^2}}{8\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} - \frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-\right.}}{4\,2^{3/4}}$$

```
[Out] 1/6*(-3*x^2-2)^(3/4)/x^3-3/8*(-3*x^2-2)^(3/4)/x-9/8*x*(-3*x^2-2)^(1/4)/(2^(
1/2)+(-3*x^2-2)^(1/2))-3/8*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/
4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*a
rctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2)
)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)+3/16*2^(1/4)*(cos(2*a
rctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(
1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^
(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)
/x*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {331, 236, 311, 226, 1210}

$$\frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{8\,2^{3/4}x} - \frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\,2^{3/4}x} - \frac{9\sqrt[4]{-3x^2-2}\,x}{8\left(\sqrt{-3x^2-2}+\sqrt{2}\right)} - \frac{3(-3x^2-2)^{3/4}}{8x} + \frac{(-3x^2-2)^{3/4}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] (-2 - 3*x^2)^(3/4)/(6*x^3) - (3*(-2 - 3*x^2)^(3/4))/(8*x) - (9*x*(-2 - 3*x^
2)^(1/4))/(8*(Sqrt[2] + Sqrt[-2 - 3*x^2])) - (3*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2]
 + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-
2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(3/4)*x) + (3*Sqrt[3]*Sqrt[-(x^2/(Sqr
t[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTa
n[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8*2^(3/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx = \frac{(-2-3x^2)^{3/4}}{6x^3} - \frac{3}{4}\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{(-2-3x^2)^{3/4}}{6x^3} - \frac{3(-2-3x^2)^{3/4}}{8x} - \frac{9}{16}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{(-2-3x^2)^{3/4}}{6x^3} - \frac{3(-2-3x^2)^{3/4}}{8x} + \frac{\left(3\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{\cdot}\right.}{8x}$$

$$= \frac{(-2-3x^2)^{3/4}}{6x^3} - \frac{3(-2-3x^2)^{3/4}}{8x} + \frac{\left(3\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2}\right.}{8x}$$

$$= \frac{(-2-3x^2)^{3/4}}{6x^3} - \frac{3(-2-3x^2)^{3/4}}{8x} - \frac{9x\sqrt[4]{-2-3x^2}}{8\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} - \frac{3\sqrt{3}\sqrt{-\frac{\cdots}{\left(\sqrt{2}+\cdots\right.}}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 48, normalized size = 0.20

$$-\frac{\sqrt[4]{1+\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3x^3\sqrt[4]{-2-3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/3*((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (-3*x^2)/2])
/(x^3*(-2 - 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 23, normalized size = 0.09

method	result	size
meijerg	$\frac{(-1)^{\frac{3}{4}}2^{\frac{3}{4}}\text{hypergeom}\left([-\frac{3}{2},\frac{1}{4}],[-\frac{1}{2}],-\frac{3x^2}{2}\right)}{6x^3}$	23

risch	$\frac{27x^4+6x^2-8}{24x^3(-3x^2-2)^{\frac{1}{4}}}+\frac{9(-1)^{\frac{3}{4}}2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left([\frac{1}{4},\frac{1}{2}],[\frac{3}{2}],-\frac{3x^2}{2}\right)}{32}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-1)^(3/4)*2^(3/4)/x^3*hypergeom([-3/2,1/4],[-1/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^3*integral(9/16*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) - (9*x^2 - 4)
*(-3*x^2 - 2)^(3/4))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 39, normalized size = 0.16

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2
)/(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(-3\,x^2-2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(- 3*x^2 - 2)^(1/4)),x)
```

```
[Out] int(1/(x^4*(- 3*x^2 - 2)^(1/4)), x)
```

3.904 $\int \frac{1}{x^6 \sqrt[4]{-2-3x^2}} \, dx$

Optimal. Leaf size=262

$$\frac{(-2-3x^2)^{3/4}}{10x^5} - \frac{7(-2-3x^2)^{3/4}}{40x^3} + \frac{63(-2-3x^2)^{3/4}}{160x} + \frac{189x\sqrt[4]{-2-3x^2}}{160\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} + \frac{63\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-\right.}}}}{}$$

[Out] 1/10*(-3*x^2-2)^(3/4)/x^5-7/40*(-3*x^2-2)^(3/4)/x^3+63/160*(-3*x^2-2)^(3/4)/x+189/160*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))+63/160*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)-63/320*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {331, 236, 311, 226, 1210}

$$-\frac{63\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{160\ 2^{3/4}x} + \frac{63\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{80\ 2^{3/4}x} + \frac{189\sqrt[4]{-3x^2-2}\,x}{160\left(\sqrt{-3x^2-2}+\sqrt{2}\right)} + \frac{63(-3x^2-2)^{3/4}}{160x} + \frac{(-3x^2-2)^{3/4}}{10x^5} - \frac{7(-3x^2-2)^{3/4}}{40x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(-2 - 3*x^2)^(1/4)),x]

[Out] (-2 - 3*x^2)^(3/4)/(10*x^5) - (7*(-2 - 3*x^2)^(3/4))/(40*x^3) + (63*(-2 - 3*x^2)^(3/4))/(160*x) + (189*x*(-2 - 3*x^2)^(1/4))/(160*(Sqrt[2] + Sqrt[-2 - 3*x^2])) + (63*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(80*2^(3/4)*x) - (63*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(160*2^(3/4)*x)

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[4]{-2-3x^2}}\,dx = \frac{(-2-3x^2)^{3/4}}{10x^5} - \frac{21}{20}\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx$$
$$= \frac{(-2-3x^2)^{3/4}}{10x^5} - \frac{7(-2-3x^2)^{3/4}}{40x^3} + \frac{63}{80}\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx$$
$$= \frac{(-2-3x^2)^{3/4}}{10x^5} - \frac{7(-2-3x^2)^{3/4}}{40x^3} + \frac{63(-2-3x^2)^{3/4}}{160x} + \frac{189}{320}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$
$$= \frac{(-2-3x^2)^{3/4}}{10x^5} - \frac{7(-2-3x^2)^{3/4}}{40x^3} + \frac{63(-2-3x^2)^{3/4}}{160x} - \frac{\left(63\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \right.}{1}$$
$$= \frac{(-2-3x^2)^{3/4}}{10x^5} - \frac{7(-2-3x^2)^{3/4}}{40x^3} + \frac{63(-2-3x^2)^{3/4}}{160x} - \frac{\left(63\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int - \right.}{16}$$
$$= \frac{(-2-3x^2)^{3/4}}{10x^5} - \frac{7(-2-3x^2)^{3/4}}{40x^3} + \frac{63(-2-3x^2)^{3/4}}{160x} + \frac{189x\sqrt[4]{-2-3x^2}}{160\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 48, normalized size = 0.18

$$-\frac{\sqrt[4]{1+\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5x^5\sqrt[4]{-2-3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/5*((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (-3*x^2)/2])
/(x^5*(-2 - 3*x^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 23, normalized size = 0.09

method	result	size

meijerg	$\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{1}{4}\right],\left[-\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{10 x^{5}}$	23
risch	$-\frac{189 x^{6}+42 x^{4}-8 x^{2}+32}{160 x^{5}\left(-3 x^{2}-2\right)^{\frac{1}{4}}}-\frac{189(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{640}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(-1)^(3/4)*2^(3/4)/x^5*hypergeom([-5/2,1/4],[-3/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/160*(160*x^5*integral(-189/320*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) + (63*x
^4 - 28*x^2 + 16)*(-3*x^2 - 2)^(3/4))/x^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 39, normalized size = 0.15

$$\frac{2^{\frac{3}{4}} e^{\frac{3 i \pi}{4}} {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{3 x^{2} e^{i \pi}}{2}} \right)}{10 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2
)/(10*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,{\left(-3\,x^2-2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(- 3*x^2 - 2)^(1/4)),x)
[Out] int(1/(x^6*(- 3*x^2 - 2)^(1/4)), x)
```

3.905 $\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=138

$$\frac{160x\sqrt[4]{-2+3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2+3x^2}+\frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{160\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-\right.}}{2079\sqrt{3}}$$

```
[Out] 160/2079*x*(3*x^2-2)^(1/4)+40/693*x^3*(3*x^2-2)^(1/4)+2/33*x^5*(3*x^2-2)^(1
/4)+160/6237*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/c
os(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2
-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3
*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 240, 226}

$$\frac{160\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2079\sqrt{3}\,x}+\frac{160\sqrt[4]{3x^2-2}\,x}{2079}+\frac{2}{33}\sqrt[4]{3x^2-2}\,x^5+\frac{40}{693}\sqrt[4]{3x^2-2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (160*x*(-2 + 3*x^2)^(1/4))/2079 + (40*x^3*(-2 + 3*x^2)^(1/4))/693 + (2*x^5*
(-2 + 3*x^2)^(1/4))/33 + (160*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])
^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1
/4)], 1/2])/(2079*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx &= \frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{20}{33}\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx\\ &= \frac{40}{693}x^3\sqrt[4]{-2+3x^2}+\frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{80}{231}\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx\\ &= \frac{160x\sqrt[4]{-2+3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2+3x^2}+\frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{320\int \frac{1}{(-2+3x^2)^{3/4}}\,dx}{2079}\\ &= \frac{160x\sqrt[4]{-2+3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2+3x^2}+\frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{\left(320\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subs}}{}\\ &= \frac{160x\sqrt[4]{-2+3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2+3x^2}+\frac{2}{33}x^5\sqrt[4]{-2+3x^2}+\frac{160\,2^{3/4}\sqrt{\frac{x}{\left(\sqrt{2}+\sqrt{-}\right.}}}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.08, size = 68, normalized size = 0.49

$$\frac{2x\left(-160+120x^2+54x^4+189x^6+80\sqrt[4]{2}\,(2-3x^2)^{3/4}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)\right)}{2079\,(-2+3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-160 + 120*x^2 + 54*x^4 + 189*x^6 + 80*2^(1/4)*(2 - 3*x^2)^(3/4)*Hype
rgeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2]))/(2079*(-2 + 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 42, normalized size = 0.30

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^{2}}{2}\right)\right)^{\frac{3}{4}}x^{7}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{7}{2}\right],\left[\frac{9}{2}\right],\frac{3x^{2}}{2}\right)}{14\mathrm{signum}\left(-1+\frac{3x^{2}}{2}\right)^{\frac{3}{4}}}$	42
risch	$\frac{2x\left(63x^{4}+60x^{2}+80\right)\left(3x^{2}-2\right)^{\frac{1}{4}}}{2079}+\frac{160\,2^{\frac{1}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^{2}}{2}\right)\right)^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^{2}}{2}\right)}{2079\mathrm{signum}\left(-1+\frac{3x^{2}}{2}\right)^{\frac{3}{4}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x^7*hyper
geom([3/4,7/2],[9/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 - 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(3*x^2 - 2)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 31, normalized size = 0.22

$$\frac{\sqrt[4]{2}\,x^{7}e^{-\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^{2}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**7*exp(-3*I*pi/4)*hyper((3/4, 7/2), (9/2,), 3*x**2/2)/14
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(x^6/(3*x^2 - 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(3\,x^2-2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2 - 2)^(3/4),x)
[Out] int(x^6/(3*x^2 - 2)^(3/4), x)
```

3.906 $\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=120

$$\frac{8}{63}x\sqrt[4]{-2+3x^2}+\frac{2}{21}x^3\sqrt[4]{-2+3x^2}+\frac{8\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\right.\right.}{63\sqrt{3}\,x}$$

[Out] 8/63*x*(3*x^2-2)^(1/4)+2/21*x^3*(3*x^2-2)^(1/4)+8/189*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 240, 226}

$$\frac{8\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{63\sqrt{3}\,x}+\frac{8}{63}\sqrt[4]{3x^2-2}\,x+\frac{2}{21}\sqrt[4]{3x^2-2}\,x^3$$

Antiderivative was successfully verified.

[In] Int[x^4/(-2 + 3*x^2)^(3/4),x]

[Out] (8*x*(-2 + 3*x^2)^(1/4))/63 + (2*x^3*(-2 + 3*x^2)^(1/4))/21 + (8*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(63*Sqrt[3]*x)

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx &= \frac{2}{21}x^3\sqrt[4]{-2+3x^2}+\frac{4}{7}\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx\\ &= \frac{8}{63}x\sqrt[4]{-2+3x^2}+\frac{2}{21}x^3\sqrt[4]{-2+3x^2}+\frac{16}{63}\int \frac{1}{(-2+3x^2)^{3/4}}\,dx\\ &= \frac{8}{63}x\sqrt[4]{-2+3x^2}+\frac{2}{21}x^3\sqrt[4]{-2+3x^2}+\frac{\left(16\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{\ldots}\right.}{63x}\\ &= \frac{8}{63}x\sqrt[4]{-2+3x^2}+\frac{2}{21}x^3\sqrt[4]{-2+3x^2}+\frac{8\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-\ldots}\right.}{63\sqrt{3}\ldots}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.93, size = 63, normalized size = 0.52

$$\frac{2x\left(-8+6x^2+9x^4+4\sqrt[4]{2}\,(2-3x^2)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)\right)}{63\,(-2+3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-8 + 6*x^2 + 9*x^4 + 4*2^(1/4)*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[1/
2, 3/4, 3/2, (3*x^2)/2]))/(63*(-2 + 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 42, normalized size = 0.35

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}x^5\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{7}{2}\right],\frac{3x^2}{2}\right)}{10\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	42

risch	$\frac{2x(3x^2+4)(3x^2-2)^{\frac{1}{4}}}{63}+\frac{8\,2^{\frac{1}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{63\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x^5*hyper
geom([3/4,5/2],[7/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 - 2)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 31, normalized size = 0.26

$$\frac{\sqrt[4]{2}\,x^5e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**5*exp(-3*I*pi/4)*hyper((3/4, 5/2), (7/2,), 3*x**2/2)/10
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{(3\,x^2-2)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2 - 2)^(3/4),x)
```

```
[Out] int(x^4/(3*x^2 - 2)^(3/4), x)
```

3.907 $\int \frac{x^2}{(-2+3x^2)^{3/4}} dx$

Optimal. Leaf size=102

$$\frac{2}{9}x\sqrt[4]{-2+3x^2}+\frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}\,x}$$

```
[Out] 2/9*x*(3*x^2-2)^(1/4)+2/27*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4
)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arc
tan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x
^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {327, 240, 226}

$$\frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}\,x}+\frac{2}{9}\sqrt[4]{3x^2-2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2
])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^
(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx = \frac{2}{9}x\sqrt[4]{-2+3x^2} + \frac{4}{9}\int \frac{1}{(-2+3x^2)^{3/4}}\,dx$$

$$= \frac{2}{9}x\sqrt[4]{-2+3x^2} + \frac{\left(4\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{9x}$$

$$= \frac{2}{9}x\sqrt[4]{-2+3x^2} + \frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\right.}{9\sqrt{3}\,x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.58, size = 57, normalized size = 0.56

$$\frac{2x\left(-2+3x^2+\sqrt[4]{2}\,(2-3x^2)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)\right)}{9\,(-2+3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2 + 2^(1/4)*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/
2, (3*x^2)/2]))/(9*(-2 + 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 42, normalized size = 0.41

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}\left(-\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}x^3\,\text{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],\frac{3x^2}{2}\right)}{6\,\text{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	42

risch	$\frac{2x(3x^2-2)^{\frac{1}{4}}}{9}+\frac{2\,2^{\frac{1}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{9\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x^3*hyperg
eom([3/4,3/2],[5/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(3*x^2 - 2)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.38, size = 31, normalized size = 0.30

$$\frac{\sqrt[4]{2}\,x^3e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**3*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2,), 3*x**2/2)/6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(3\,x^2-2)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2 - 2)^(3/4),x)
```

```
[Out] int(x^2/(3*x^2 - 2)^(3/4), x)
```

3.908 $\int \frac{1}{(-2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}$$

```
[Out] 1/6*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arct
an(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)
*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(
1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {240, 226}

$$\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^2)^(-3/4),x]
```

```
[Out] (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Elli
pticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{(-2+3x^2)^{3/4}}\,dx = \frac{\left(\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{x}$$

$$= \frac{\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.57, size = 43, normalized size = 0.52

$$\frac{x\left(1-\frac{3x^2}{2}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)}{(-2+3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^2)^(-3/4),x]
```

```
[Out] (x*(1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2])/(-2 +
 3*x^2)^(3/4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 40, normalized size = 0.49

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}\left(-\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}} x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2\text{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hypergeo
m([1/2,3/4],[3/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - 2)^(-3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(-3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.38, size = 29, normalized size = 0.35

$$\frac{\sqrt[4]{2}\, x e^{-\frac{3i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{3x^{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x*exp(-3*I*pi/4)*hyper((1/2, 3/4), (3/2,), 3*x**2/2)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 - 2)^(-3/4), x)
```

Mupad [B]
time = 4.87, size = 34, normalized size = 0.41

$$\frac{2^{1/4}\, x\, {\left(2-3\,x^2\right)}^{3/4}\, {}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2\,{\left(3\,x^2-2\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 2)^(3/4),x)
```

```
[Out] (2^(1/4)*x*(2 - 3*x^2)^(3/4)*hypergeom([1/2, 3/4], 3/2, (3*x^2)/2))/(2*(3*x
^2 - 2)^(3/4))
```

3.909 $\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt[4]{-2+3x^2}}{2x}+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}$$

```
[Out] 1/2*(3*x^2-2)^(1/4)/x+1/8*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)
))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arct
an(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^
2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 240, 226}

$$\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}+\frac{\sqrt[4]{3x^2-2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2
]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4
)], 1/2])/(4*2^(1/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2 (-2+3x^2)^{3/4}} dx = \frac{\sqrt[4]{-2+3x^2}}{2x} + \frac{3}{4}\int \frac{1}{(-2+3x^2)^{3/4}} dx$$
$$= \frac{\sqrt[4]{-2+3x^2}}{2x} + \frac{\left(\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}} dx, x, \sqrt[4]{-2+3x^2}\right)}{2x}$$
$$= \frac{\sqrt[4]{-2+3x^2}}{2x} + \frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\right.}{4\sqrt[4]{2}x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.86, size = 46, normalized size = 0.44

$$-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4} {}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{x(-2+3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (3*x^2)/2])/(x*(
-2 + 3*x^2)^(3/4)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 42, normalized size = 0.40

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}}\left(-\text{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}\text{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{1}{2}\right],\frac{3x^2}{2}\right)}{2\text{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}x}$	42

risch	$\frac{(3x^2-2)^{\frac{1}{4}}}{2x}+\frac{3\,2^{\frac{1}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{8\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)/x*hyperge
om([-1/2,3/4],[1/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(1/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 29, normalized size = 0.28

$$\frac{\sqrt[4]{2}\,e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2/2)/(2*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^2), x)
```

Mupad [B]
time = 5.07, size = 23, normalized size = 0.22

$$-\frac{2\,3^{1/4}\,\left(\frac{1}{x^2}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ \frac{2}{3\,x^2}\right)}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 - 2)^(3/4)),x)
```

```
[Out] -(2*3^(1/4)*(1/x^2)^(3/4)*hypergeom([3/4, 5/4], 9/4, 2/(3*x^2)))/(15*x)
```

3.910 $\int \frac{1}{x^4(-2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=122

$$\frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5\sqrt[4]{-2+3x^2}}{8x}+\frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3}}{\sqrt[4]{2}}\right.\right.}{16\sqrt[4]{2}\,x}$$

```
[Out] 1/6*(3*x^2-2)^(1/4)/x^3+5/8*(3*x^2-2)^(1/4)/x+5/32*2^(3/4)*(cos(2*arctan(1/
2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/
4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(
1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 240, 226}

$$\frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{16\sqrt[4]{2}\,x}+\frac{5\sqrt[4]{3x^2-2}}{8x}+\frac{\sqrt[4]{3x^2-2}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2 + 3*x^2)^(1/4)/(6*x^3) + (5*(-2 + 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Sqrt
[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF
[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(16*2^(1/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4(-2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{-2+3x^2}}{6x^3} + \frac{5}{4}\int \frac{1}{x^2(-2+3x^2)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2+3x^2}}{6x^3} + \frac{5\sqrt[4]{-2+3x^2}}{8x} + \frac{15}{16}\int \frac{1}{(-2+3x^2)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2+3x^2}}{6x^3} + \frac{5\sqrt[4]{-2+3x^2}}{8x} + \frac{\left(5\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2\right.}{8x}$$
$$= \frac{\sqrt[4]{-2+3x^2}}{6x^3} + \frac{5\sqrt[4]{-2+3x^2}}{8x} + \frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+\right.}{16\sqrt[4]{2}\,x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 48, normalized size = 0.39

$$-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4} {}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3x^3(-2+3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/3*((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (3*x^2)/2])/
(x^3*(-2 + 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 42, normalized size = 0.34

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[-\frac{1}{2}\right],\frac{3x^2}{2}\right)}{6\,\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}x^3}$	42

risch	$\frac{45x^4-18x^2-8}{24x^3(3x^2-2)^{\frac{3}{4}}}+\frac{15\,2^{\frac{1}{4}}\left(-\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{32\mathrm{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)/x^3*hyper
geom([-3/2,3/4],[-1/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(1/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 32, normalized size = 0.26

$$\frac{\sqrt[4]{2}\,e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\middle|\,\frac{3x^2}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2/2)/(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(3\,x^2-2\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 - 2)^(3/4)),x)
[Out] int(1/(x^4*(3*x^2 - 2)^(3/4)), x)
```

3.911 $\int \frac{1}{x^6\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27\sqrt[4]{-2+3x^2}}{32x}+\frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\right.}{64\sqrt[4]{2}\,x}$$

```
[Out] 1/10*(3*x^2-2)^(1/4)/x^5+9/40*(3*x^2-2)^(1/4)/x^3+27/32*(3*x^2-2)^(1/4)/x+2
7/128*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*ar
ctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/
4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)
^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 240, 226}

$$\frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{64\sqrt[4]{2}\,x}+\frac{27\sqrt[4]{3x^2-2}}{32x}+\frac{\sqrt[4]{3x^2-2}}{10x^5}+\frac{9\sqrt[4]{3x^2-2}}{40x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2 + 3*x^2)^(1/4)/(10*x^5) + (9*(-2 + 3*x^2)^(1/4))/(40*x^3) + (27*(-2 + 3
*x^2)^(1/4))/(32*x) + (27*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*
(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)]
, 1/2])/(64*2^(1/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(-2+3x^2\right)^{3/4}}\,dx &= \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{27}{20}\int \frac{1}{x^4\left(-2+3x^2\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27}{16}\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27\sqrt[4]{-2+3x^2}}{32x}+\frac{81}{64}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27\sqrt[4]{-2+3x^2}}{32x}+\frac{\left(27\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{32x}\\ &= \frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27\sqrt[4]{-2+3x^2}}{32x}+\frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right.}}}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 48, normalized size = 0.34

$$-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5x^5\left(-2+3x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/5*((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (3*x^2)/2])/
(x^5*(-2 + 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 42, normalized size = 0.30

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}\operatorname{hypergeom}\left(\left[-\frac{5}{2},\frac{3}{4}\right],\left[-\frac{3}{2}\right],\frac{3x^2}{2}\right)}{10\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}x^5}$	42
risch	$\frac{405x^6-162x^4-24x^2-32}{160x^5\left(3x^2-2\right)^{\frac{3}{4}}}+\frac{81\,2^{\frac{1}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}x\operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{128\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)/x^5*hype
rgeom([-5/2,3/4],[-3/2],3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(1/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.58, size = 32, normalized size = 0.23

$$\frac{\sqrt[4]{2}\,e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{3}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2-2)**(3/4),x)
```

[Out] 2**(1/4)*exp(I*pi/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2/2)/(10*x**5)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(3\,x^2-2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(3*x^2 - 2)^(3/4)),x)
[Out] int(1/(x^6*(3*x^2 - 2)^(3/4)), x)
```

3.912 $\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=139

$$-\frac{160x\sqrt[4]{-2-3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2-3x^2}-\frac{2}{33}x^5\sqrt[4]{-2-3x^2}+\frac{160\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{\ldots}\right)}{2079\sqrt{\ldots}}$$

```
[Out] -160/2079*x*(-3*x^2-2)^(1/4)+40/693*x^3*(-3*x^2-2)^(1/4)-2/33*x^5*(-3*x^2-2
)^(1/4)+160/6237*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1
/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*
(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2
^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 240, 226}

$$\frac{160\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2079\sqrt{3}\,x}-\frac{160\sqrt[4]{-3x^2-2}\,x}{2079}-\frac{2}{33}\sqrt[4]{-3x^2-2}\,x^5+\frac{40}{693}\sqrt[4]{-3x^2-2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (-160*x*(-2 - 3*x^2)^(1/4))/2079 + (40*x^3*(-2 - 3*x^2)^(1/4))/693 - (2*x^5
*(-2 - 3*x^2)^(1/4))/33 + (160*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^
2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/
2^(1/4)], 1/2])/(2079*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx &= -\frac{2}{33}x^5\sqrt[4]{-2-3x^2}-\frac{20}{33}\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx\\
&= \frac{40}{693}x^3\sqrt[4]{-2-3x^2}-\frac{2}{33}x^5\sqrt[4]{-2-3x^2}+\frac{80}{231}\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx\\
&= -\frac{160x\sqrt[4]{-2-3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2-3x^2}-\frac{2}{33}x^5\sqrt[4]{-2-3x^2}-\frac{320\int \frac{1}{(-2-3x^2)^{3/4}}\,dx}{2079}\\
&= -\frac{160x\sqrt[4]{-2-3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2-3x^2}-\frac{2}{33}x^5\sqrt[4]{-2-3x^2}+\frac{\left(320\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)}{}\\
&= -\frac{160x\sqrt[4]{-2-3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2-3x^2}-\frac{2}{33}x^5\sqrt[4]{-2-3x^2}+\frac{160\ 2^{3/4}\sqrt{-\frac{}{\left(\sqrt{2}\right.}}}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.17, size = 68, normalized size = 0.49

$$\frac{2x\left(160+120x^2-54x^4+189x^6-80\sqrt[4]{2}\left(2+3x^2\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{2079\left(-2-3x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(160 + 120*x^2 - 54*x^4 + 189*x^6 - 80*2^(1/4)*(2 + 3*x^2)^(3/4)*Hyper
geometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/(2079*(-2 - 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.09, size = 23, normalized size = 0.17

method	result	size
meijerg	$-\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{7}{2}\right], \left[\frac{9}{2}\right], -\frac{3x^{2}}{2}\right)}{14}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*(-1)^(1/4)*2^(1/4)*x^7*hypergeom([3/4,7/2],[9/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-3*x^2 - 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/2079*(63*x^5 - 60*x^3 + 80*x)*(-3*x^2 - 2)^(1/4) + integral(320/2079*(-3
*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.42, size = 36, normalized size = 0.26

$$\frac{\sqrt[4]{2}\, x^{7} e^{-\frac{3i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{3x^{2} e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**7*exp(-3*I*pi/4)*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(I*p
i)/2)/14
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(x^6/(-3*x^2 - 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(-3\,x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(- 3*x^2 - 2)^(3/4),x)
[Out] int(x^6/(- 3*x^2 - 2)^(3/4), x)
```

3.913 $\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=121

$$\frac{8}{63}x\sqrt[4]{-2-3x^2}-\frac{2}{21}x^3\sqrt[4]{-2-3x^2}-\frac{8\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\right.\right.}{63\sqrt{3}\,x}$$

[Out] 8/63*x*(-3*x^2-2)^(1/4)-2/21*x^3*(-3*x^2-2)^(1/4)-8/189*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 240, 226}

$$-\frac{8\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{63\sqrt{3}\,x}+\frac{8}{63}\sqrt[4]{-3x^2-2}\,x-\frac{2}{21}\sqrt[4]{-3x^2-2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (8*x*(-2 - 3*x^2)^(1/4))/63 - (2*x^3*(-2 - 3*x^2)^(1/4))/21 - (8*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(63*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx = -\frac{2}{21}x^3\sqrt[4]{-2-3x^2} - \frac{4}{7}\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx$$
$$= \frac{8}{63}x\sqrt[4]{-2-3x^2} - \frac{2}{21}x^3\sqrt[4]{-2-3x^2} + \frac{16}{63}\int \frac{1}{(-2-3x^2)^{3/4}}\,dx$$
$$= \frac{8}{63}x\sqrt[4]{-2-3x^2} - \frac{2}{21}x^3\sqrt[4]{-2-3x^2} - \frac{\left(16\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, \right.}{63x}$$
$$= \frac{8}{63}x\sqrt[4]{-2-3x^2} - \frac{2}{21}x^3\sqrt[4]{-2-3x^2} - \frac{8\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\right.}{63}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.03, size = 63, normalized size = 0.52

$$\frac{2x\left(-8-6x^2+9x^4+4\sqrt[4]{2}\,(2+3x^2)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{63\,(-2-3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-8 - 6*x^2 + 9*x^4 + 4*2^(1/4)*(2 + 3*x^2)^(3/4)*Hypergeometric2F1[1/
2, 3/4, 3/2, (-3*x^2)/2]))/(63*(-2 - 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.06, size = 23, normalized size = 0.19

method	result	size
meijerg	$-\frac{(-1)^{\frac{1}{4}}2^{\frac{1}{4}}x^5\,\text{hypergeom}\left([\frac{3}{4},\frac{5}{2}],[\frac{7}{2}],-\frac{3x^2}{2}\right)}{10}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(-1)^(1/4)*2^(1/4)*x^5*hypergeom([3/4,5/2],[7/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/63*(3*x^3 - 4*x)*(-3*x^2 - 2)^(1/4) + integral(-16/63*(-3*x^2 - 2)^(1/4)
/(3*x^2 + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 36, normalized size = 0.30

$$\frac{\sqrt[4]{2}\,x^5 e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**5*exp(-3*I*pi/4)*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(I*p
i)/2)/10
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(-3\,x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(- 3*x^2 - 2)^(3/4),x)
```

```
[Out] int(x^4/(- 3*x^2 - 2)^(3/4), x)
```

3.914 $\int \frac{x^2}{(-2-3x^2)^{3/4}} \, dx$

Optimal. Leaf size=103

$$-\frac{2}{9}x\sqrt[4]{-2-3x^2}+\frac{2\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}\,x}$$

```
[Out] -2/9*x*(-3*x^2-2)^(1/4)+2/27*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(
3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2
*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/
2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {327, 240, 226}

$$\frac{2\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}\,x}-\frac{2}{9}x\sqrt[4]{-3x^2-2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (-2*x*(-2 - 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*
x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4
)/2^(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx = -\frac{2}{9}x\sqrt[4]{-2-3x^2} - \frac{4}{9}\int \frac{1}{(-2-3x^2)^{3/4}}\,dx$$

$$= -\frac{2}{9}x\sqrt[4]{-2-3x^2} + \frac{\left(4\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{9x}$$

$$= -\frac{2}{9}x\sqrt[4]{-2-3x^2} + \frac{2\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\Big(2\ldots}{9\sqrt{3}\,x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.60, size = 58, normalized size = 0.56

$$\frac{2x\left(2+3x^2-\sqrt[4]{2}\,(2+3x^2)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{9\,(-2-3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(2 + 3*x^2 - 2^(1/4)*(2 + 3*x^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2
, (-3*x^2)/2]))/(9*(-2 - 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.07, size = 23, normalized size = 0.22

method	result	size
meijerg	$-\frac{(-1)^{\frac{1}{4}}2^{\frac{1}{4}}x^3\,\text{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],-\frac{3x^2}{2}\right)}{6}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-1)^(1/4)*2^(1/4)*x^3*hypergeom([3/4,3/2],[5/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="maxima")
[Out] integrate(x^2/(-3*x^2 - 2)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="fricas")
[Out] -2/9*(-3*x^2 - 2)^(1/4)*x + integral(4/9*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.40, size = 36, normalized size = 0.35

$$\frac{\sqrt[4]{2}\,x^{3}e^{-\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\,\frac{3x^{2}e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2)**(3/4),x)
[Out] 2**(1/4)*x**3*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(I*p
i)/2)/6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(x^2/(-3*x^2 - 2)^(3/4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(-3\,x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(- 3*x^2 - 2)^(3/4),x)
```

```
[Out] int(x^2/(- 3*x^2 - 2)^(3/4), x)
```

3.915 $\int \frac{1}{(-2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}$$

```
[Out] -1/6*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*ar
ctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(
1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x
^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {240, 226}

$$-\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x^2)^(-3/4),x]
```

```
[Out] -((Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])
*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x))
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{(-2-3x^2)^{3/4}}\,dx = -\frac{\left(\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{x}$$

$$= -\frac{\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right.}{\sqrt[4]{2}\sqrt{3}\,x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.43, size = 43, normalized size = 0.51

$$\frac{x\left(1+\frac{3x^2}{2}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)}{(-2-3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x^2)^(-3/4),x]
```

```
[Out] (x*(1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2])/(-2
- 3*x^2)^(3/4)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.04, size = 21, normalized size = 0.25

method	result	size
meijerg	$-\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} x \,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-1)^(1/4)*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 - 2)^(-3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.36, size = 34, normalized size = 0.40

$$\frac{\sqrt[4]{2}\, x e^{-\frac{3i\pi}{4}} {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x*exp(-3*I*pi/4)*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(I*pi)/
2)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 - 2)^(-3/4), x)
```

Mupad [B]
time = 4.61, size = 34, normalized size = 0.40

$$\frac{2^{1/4}\, x\, (3\, x^2 + 2)^{3/4}\, {}_2\mathrm{F}_1\left(\frac{1}{2}, \frac{3}{4};\ \frac{3}{2};\ -\frac{3\, x^2}{2}\right)}{2\, (-3\, x^2 - 2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 3*x^2 - 2)^(3/4),x)
```

```
[Out] (2^(1/4)*x*(3*x^2 + 2)^(3/4)*hypergeom([1/2, 3/4], 3/2, -(3*x^2)/2))/(2*(-
3*x^2 - 2)^(3/4))
```

3.916 $\int \frac{1}{x^2(-2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt[4]{-2-3x^2}}{2x}+\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}$$

```
[Out] 1/2*(-3*x^2-2)^(1/4)/x+1/8*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/
4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*a
rctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2)
)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 240, 226}

$$\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}+\frac{\sqrt[4]{-3x^2-2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] (-2 - 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])
^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(
1/4)], 1/2])/(4*2^(1/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{-2-3x^2}}{2x} - \frac{3}{4}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2-3x^2}}{2x} + \frac{\left(\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{2x}$$

$$= \frac{\sqrt[4]{-2-3x^2}}{2x} + \frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\right.}{4\sqrt[4]{2}\,x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.89, size = 46, normalized size = 0.44

$$-\frac{\left(1+\frac{3x^2}{2}\right)^{3/4}{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{x\left(-2-3x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 - 3*x^2)^(3/4)),x]
[Out] -(((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (-3*x^2)/2])/(x*
(-2 - 3*x^2)^(3/4)))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 23, normalized size = 0.22

method	result	size
meijerg	$\frac{(-1)^{\frac{1}{4}}2^{\frac{1}{4}}\,\text{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{2x}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1)^(1/4)*2^(1/4)/x*hypergeom([-1/2,3/4],[1/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="fricas")
[Out] 1/2*(2*x*integral(3/4*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) + (-3*x^2 - 2)^(1/
4))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 34, normalized size = 0.32

$$\frac{\sqrt[4]{2}\,e^{\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\middle|\,\frac{3x^{2}e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2-2)**(3/4),x)
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(
2*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^2), x)
```

Mupad [B]
time = 4.73, size = 36, normalized size = 0.34

$$-\frac{2\,3^{1/4}\,{\left(\frac{2}{x^2}+3\right)}^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{2}{3\,x^2}\right)}{15\,x\,{\left(-3\,x^2-2\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- 3*x^2 - 2)^(3/4)),x)
```

```
[Out] -(2*3^(1/4)*(2/x^2 + 3)^(3/4)*hypergeom([3/4, 5/4], 9/4, -2/(3*x^2)))/(15*x
*(- 3*x^2 - 2)^(3/4))
```

3.917 $\int \frac{1}{x^4(-2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5\sqrt[4]{-2-3x^2}}{8x}-\frac{5\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2}}{\sqrt[4]{}}\right.\right.}{16\sqrt[4]{2}\,x}$$

```
[Out] 1/6*(-3*x^2-2)^(1/4)/x^3-5/8*(-3*x^2-2)^(1/4)/x-5/32*2^(3/4)*(cos(2*arctan(
1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2
^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))
*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(
1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 240, 226}

$$-\frac{5\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{16\sqrt[4]{2}\,x}-\frac{5\sqrt[4]{-3x^2-2}}{8x}+\frac{\sqrt[4]{-3x^2-2}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] (-2 - 3*x^2)^(1/4)/(6*x^3) - (5*(-2 - 3*x^2)^(1/4))/(8*x) - (5*Sqrt[3]*Sqrt
[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Ellipt
icF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(16*2^(1/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \left(-2-3 x^2\right)^{3/4}} \, dx &= \frac{\sqrt[4]{-2-3 x^2}}{6 x^3}-\frac{5}{4} \int \frac{1}{x^2 \left(-2-3 x^2\right)^{3/4}} \, dx \\
&= \frac{\sqrt[4]{-2-3 x^2}}{6 x^3}-\frac{5 \sqrt[4]{-2-3 x^2}}{8 x}+\frac{15}{16} \int \frac{1}{\left(-2-3 x^2\right)^{3/4}} \, dx \\
&= \frac{\sqrt[4]{-2-3 x^2}}{6 x^3}-\frac{5 \sqrt[4]{-2-3 x^2}}{8 x}-\frac{\left(5 \sqrt{\frac{3}{2}} \sqrt{-x^2}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}} \, dx,x,\sqrt[4]{-2\right.}{8 x} \\
&= \frac{\sqrt[4]{-2-3 x^2}}{6 x^3}-\frac{5 \sqrt[4]{-2-3 x^2}}{8 x}-\frac{5 \sqrt{3} \sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3 x^2}\right)^2}} \left(\sqrt{2}+\sqrt{-2-}\right.}{16 \sqrt[4]{2} x}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 48, normalized size = 0.39

$$-\frac{\left(1+\frac{3 x^2}{2}\right)^{3/4} {}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{3 x^2}{2}\right)}{3 x^3 \left(-2-3 x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/3*((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (-3*x^2)/2])
/(x^3*(-2 - 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 23, normalized size = 0.19

method	result	size
meijerg	$\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} \text{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[-\frac{1}{2}\right],-\frac{3 x^2}{2}\right)}{6 x^3}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-1)^(1/4)*2^(1/4)/x^3*hypergeom([-3/2,3/4],[-1/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^3*integral(-15/16*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) - (15*x^2 -
 4)*(-3*x^2 - 2)^(1/4))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.53, size = 37, normalized size = 0.30

$$\frac{\sqrt[4]{2}\, e^{\frac{i\pi}{4}}\, {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{3x^{2}e^{i\pi}}{2}} \right)}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/
(6*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(-3\,x^2-2\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(- 3*x^2 - 2)^(3/4)),x)
```

```
[Out] int(1/(x^4*(- 3*x^2 - 2)^(3/4)), x)
```

3.918 $\int \frac{1}{x^6(-2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt[4]{-2-3x^2}}{10x^5}-\frac{9\sqrt[4]{-2-3x^2}}{40x^3}+\frac{27\sqrt[4]{-2-3x^2}}{32x}+\frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F}{64\sqrt[4]{2}\,x}$$

```
[Out] 1/10*(-3*x^2-2)^(1/4)/x^5-9/40*(-3*x^2-2)^(1/4)/x^3+27/32*(-3*x^2-2)^(1/4)/
x+27/128*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(
2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-
2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(
-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {331, 240, 226}

$$\frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{64\sqrt[4]{2}\,x}+\frac{27\sqrt[4]{-3x^2-2}}{32x}+\frac{\sqrt[4]{-3x^2-2}}{10x^5}-\frac{9\sqrt[4]{-3x^2-2}}{40x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] (-2 - 3*x^2)^(1/4)/(10*x^5) - (9*(-2 - 3*x^2)^(1/4))/(40*x^3) + (27*(-2 - 3
*x^2)^(1/4))/(32*x) + (27*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2
)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/
4)], 1/2])/(64*2^(1/4)*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6 \left(-2-3x^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{27}{20}\int \frac{1}{x^4\left(-2-3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27}{16}\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27\sqrt[4]{-2-3x^2}}{32x} - \frac{81}{64}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27\sqrt[4]{-2-3x^2}}{32x} + \frac{\left(27\sqrt{\frac{3}{2}}\,\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{32x}$$

$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27\sqrt[4]{-2-3x^2}}{32x} + \frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right.}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 48, normalized size = 0.34

$$-\frac{\left(1+\frac{3x^2}{2}\right)^{3/4} {}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5x^5\left(-2-3x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 - 3*x^2)^(3/4)),x]
[Out] -1/5*((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (-3*x^2)/2])
/(x^5*(-2 - 3*x^2)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 23, normalized size = 0.16

method	result	size
meijerg	$\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} \text{hypergeom}\left(\left[-\frac{5}{2},\frac{3}{4}\right],\left[-\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{10x^5}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(-1)^(1/4)*2^(1/4)/x^5*hypergeom([-5/2,3/4],[-3/2],-3/2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/160*(160*x^5*integral(81/64*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) + (135*x^4
 - 36*x^2 + 16)*(-3*x^2 - 2)^(1/4))/x^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.59, size = 37, normalized size = 0.26

$$\frac{\sqrt[4]{2}\, e^{\frac{i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/
(10*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(-3\,x^2-2\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(- 3*x^2 - 2)^(3/4)),x)
[Out] int(1/(x^6*(- 3*x^2 - 2)^(3/4)), x)
```

3.919 $\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=152

$$-\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}-\frac{a^{5/2}c^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] 1/30*a*c*(c*x)^(5/2)*(b*x^2+a)^(1/4)/b+1/5*(c*x)^(9/2)*(b*x^2+a)^(1/4)/c-1/
12*a^(5/2)*c^2*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1
/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(
x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^(3/4)-1/12*a^2*c^3*(b*x^2+a)
^(1/4)*(c*x)^(1/2)/b^2
```

Rubi [A]

time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {285, 327, 335, 243, 342, 281, 237}

$$-\frac{a^{5/2}c^2(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] -1/12*(a^2*c^3*Sqrt[c*x]*(a + b*x^2)^(1/4))/b^2 + (a*c*(c*x)^(5/2)*(a + b*x
^2)^(1/4))/(30*b) + ((c*x)^(9/2)*(a + b*x^2)^(1/4))/(5*c) - (a^(5/2)*c^2*(1
 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/(12*b^(3/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx = \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} + \frac{1}{10}a\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/4}}\,dx$$

$$= \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} - \frac{(a^2c^2)\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx}{12b}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} + \frac{(a^3c^4)\int \frac{}{\sqrt{cx}}}{24}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} + \frac{(a^3c^3)\,\mathrm{Subst}}{}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} + \frac{\left(a^3c^3\left(1+\frac{a}{bx}\right.\right.}{}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} - \frac{\left(a^3c^3\left(1+\frac{a}{bx}\right.\right.}{}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} - \frac{\left(a^3c^3\left(1+\frac{a}{bx}\right.\right.}{}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2} + \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b} + \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c} - \frac{a^{5/2}c^2\left(1+\frac{a}{bx}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 102, normalized size = 0.67

$$\frac{c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}\left(\sqrt[4]{1+\frac{bx^2}{a}}\,(-5a^2+abx^2+6b^2x^4)+5a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{30b^2\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (c^3*Sqrt[c*x]*(a + b*x^2)^(1/4)*((1 + (b*x^2)/a)^(1/4)*(-5*a^2 + a*b*x^2 +
 6*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)]))/(30*b
^2*(1 + (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{7}{2}} \left(b\,x^{2}+a\right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(7/2)*(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)*c^3*x^3, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 16.98, size = 46, normalized size = 0.30

$$\frac{\sqrt[4]{a}\, c^{\frac{7}{2}} x^{\frac{9}{2}} \Gamma\left(\frac{9}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{2 \Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((-1/4, 9/4), (13/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/2}\,{\left(b\,x^2+a\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(a + b*x^2)^(1/4),x)
[Out] int((c*x)^(7/2)*(a + b*x^2)^(1/4), x)
```

3.920 $\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=118

$$\frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{a^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\,\left(a+bx^2\right)^{3/4}}$$

```
[Out] 1/3*(c*x)^(5/2)*(b*x^2+a)^(1/4)/c+1/6*a^(3/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)
*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1
/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(3/4)
/b^(1/2)+1/6*a*c*(b*x^2+a)^(1/4)*(c*x)^(1/2)/b
```

Rubi [A]

time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {285, 327, 335, 243, 342, 281, 237}

$$\frac{a^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\,\left(a+bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (a*c*Sqrt[c*x]*(a + b*x^2)^(1/4))/(6*b) + ((c*x)^(5/2)*(a + b*x^2)^(1/4))/(
3*c) + (a^(3/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(6*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx = \frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{1}{6}a\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx$$

$$= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}-\frac{(a^2c^2)\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx}{12b}$$

$$= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}-\frac{(a^2c)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{6b}$$

$$= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}-\frac{\left(a^2c\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{\left(1+\frac{ac}{bx}\right.}\right.}{6b\,(a+bx^2)^{3/4}}$$

$$= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{\left(a^2c\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{\left(1+\frac{ac}{}\right.}\right.}{6b\,(a+bx^2)^{3/4}}$$

$$= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{\left(a^2c\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{\left(1+\frac{ac}{}\right.}\right.}{12b\,(a+bx^2)^{3/4}}$$

$$= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{a^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.\right.}{6\sqrt{b}\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 85, normalized size = 0.72

$$\frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}\left((a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}-a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(1/4)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/4) - a*Hyper
geometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)]))/(3*b*(1 + (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{3}{2}}\left(b\,x^{2}+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)*c*x, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.56, size = 46, normalized size = 0.39

$$\frac{\sqrt[4]{a}\, c^{\frac{3}{2}} x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^{2} e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{3/2}\,{\left(b\,x^2+a\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(a + b*x^2)^(1/4),x)
[Out] int((c*x)^(3/2)*(a + b*x^2)^(1/4), x)
```

3.921 $\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\,\sqrt{b}\,\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] -(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)
/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1
/2))),2^(1/2))*a^(1/2)*b^(1/2)/c^2/(b*x^2+a)^(3/4)+(b*x^2+a)^(1/4)*(c*x)^(1
/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {285, 335, 243, 342, 281, 237}

$$\frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\,\sqrt{b}\,(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (Sqrt[c*x]*(a + b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(c
*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(c^2*(a + b*x^2)^(3/
4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx &= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}+\frac{1}{2}a\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx\\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}+\frac{a\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c}\\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}+\frac{\left(a\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{c\,(a+bx^2)^{3/4}}\\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\left(a\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{c\,(a+bx^2)^{3/4}}\\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\left(a\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{2c\,(a+bx^2)^{3/4}}\\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\,\sqrt{b}\,\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{c^2\,(a+bx^2)^{3/4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.61

$$
\frac{2x\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\,\sqrt[4]{1+\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)])/(Sq
rt[c*x]*(1 + (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{\sqrt{cx}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.70, size = 46, normalized size = 0.52

$$\frac{\sqrt[4]{a}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(1/2),x)
```

```
[Out] a**(1/4)*sqrt(x)*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(c)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(1/2),x)
[Out] int((a + b*x^2)^(1/4)/(c*x)^(1/2), x)
```

3.922 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}-\frac{2b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,c^4\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2/3*(b*x^2+a)^(1/4)/c/(c*x)^(3/2)-2/3*b^(3/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2
)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(
1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/c^4/(b*x^2+a)^
(3/4)/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {283, 335, 243, 342, 281, 237}

$$-\frac{2b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) - (2*b^(3/2)*(1 + a/(b*x^2))^(3/4)
*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*c^4*(a
 + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}} + \frac{b\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx}{3c^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{3c^3}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}} + \frac{\left(2b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{3c^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}} - \frac{\left(2b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{3c^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}} - \frac{\left(b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}} - \frac{2b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,c^4\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 56, normalized size = 0.60

$$-\frac{2x\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, -((b*x^2)/a)])/(
3*(c*x)^(5/2)*(1 + (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^3*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.17, size = 32, normalized size = 0.34

$$-\frac{\sqrt[4]{b}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(5/2),x)
```

```
[Out] -b**(1/4)*hyper((-1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(c**(5/2)*
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{{\left(c\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(5/2),x)
[Out] int((a + b*x^2)^(1/4)/(c*x)^(5/2), x)
```

3.923 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}} \, dx$

Optimal. Leaf size=123

$$-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}+\frac{4b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2/7*(b*x^2+a)^(1/4)/c/(c*x)^(7/2)-2/21*b*(b*x^2+a)^(1/4)/a/c^3/(c*x)^(3/2)
+4/21*b^(5/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x
*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/c^6/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {283, 331, 335, 243, 342, 281, 237}

$$\frac{4b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(9/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) - (2*b*(a + b*x^2)^(1/4))/(21*a*c^
3*(c*x)^(3/2)) + (4*b^(5/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[Arc
Cot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*c^6*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}} + \frac{b\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/4}}\,dx}{7c^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}} - \frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} - \frac{(2b^2)\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx}{21ac^4}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}} - \frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} - \frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{21ac^5}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}} - \frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} - \frac{\left(4b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x,\right.}{21ac^5\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}} - \frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} + \frac{\left(4b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x,\right.}{21ac^5\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}} - \frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} + \frac{\left(2b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x,\right.}{21ac^5\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}} - \frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} + \frac{4b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.46

$$-\frac{2x\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(9/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, -((b*x^2)/a)])/
(7*(c*x)^(9/2)*(1 + (b*x^2)/a)^(1/4))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^5*x^5), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 19.41, size = 36, normalized size = 0.29

$$-\frac{\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{3c^{\frac{9}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(9/2),x)
```

```
[Out] -b**(1/4)*hyper((-1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**2))/(3*c**(9/2
)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{{\left(c\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(9/2),x)
```

```
[Out] int((a + b*x^2)^(1/4)/(c*x)^(9/2), x)
```

3.924 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{8b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2/11*(b*x^2+a)^(1/4)/c/(c*x)^(11/2)-2/77*b*(b*x^2+a)^(1/4)/a/c^3/(c*x)^(7/
2)+4/77*b^2*(b*x^2+a)^(1/4)/a^2/c^5/(c*x)^(3/2)-8/77*b^(7/2)*(1+a/b/x^2)^(3
/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot
(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/
a^(5/2)/c^8/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {283, 331, 335, 243, 342, 281, 237}

$$-\frac{8b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a+bx^2\right)^{3/4}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) - (2*b*(a + b*x^2)^(1/4))/(77*a*
c^3*(c*x)^(7/2)) + (4*b^2*(a + b*x^2)^(1/4))/(77*a^2*c^5*(c*x)^(3/2)) - (8*
b^(7/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(77*a^(5/2)*c^8*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} + \frac{b\int \frac{1}{(cx)^{9/2}(a+bx^2)^{3/4}}\,dx}{11c^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} - \frac{(6b^2)\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/4}}\,dx}{77ac^4}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{(4b^3)\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx}{77a^2c^6}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{(8b^3)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{77a^2c^7}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{\left(8b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{(1+\cdots}\right.}{77a^2c^7\,(a+bx^2)^{3/\cdots}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{\left(8b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{(1+\cdots}\right.}{77a^2c^7\,(a+bx^2)^{3/\cdots}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{\left(4b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{(1+\cdots}\right.}{77a^2c^7\,(a+bx^2)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{8b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.\right.}{77a^{5/2}c^8\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 56, normalized size = 0.36

$$-\frac{2x\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt[4]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, -((b*x^2)/a)])
/(11*(c*x)^(13/2)*(1 + (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(13/2),x)
[Out] int((b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^7*x^7), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 177.70, size = 36, normalized size = 0.23

$$-\frac{\sqrt[4]{b}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\,\frac{ae^{i\pi}}{bx^{2}}\right)}{5c^{\frac{13}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(13/2),x)
[Out] -b**(1/4)*hyper((-1/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*c**(13/
2)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^{1/4}}{(c\,x)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(13/2),x)
[Out] int((a + b*x^2)^(1/4)/(c*x)^(13/2), x)
```

3.925 $\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=147

$$\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}$$

```
[Out] 1/16*a*c*(c*x)^(3/2)*(b*x^2+a)^(1/4)/b+1/4*(c*x)^(7/2)*(b*x^2+a)^(1/4)/c+3/
32*a^2*c^(5/2)*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)-
3/32*a^2*c^(5/2)*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(7/
4)
```

Rubi [A]
time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.368$, Rules used = {285, 327, 335, 338, 304, 211, 214}

$$\frac{3a^2c^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b) + ((c*x)^(7/2)*(a + b*x^2)^(1/4)
)/(4*c) + (3*a^2*c^(5/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1
/4))])/(32*b^(7/4)) - (3*a^2*c^(5/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(32*b^(7/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx = \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{1}{8}a\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx$$

$$=\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{(3a^2c^2)\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx}{32b}$$

$$=\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{(3a^2c)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right.}{16b}$$

$$=\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{(3a^2c)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right.}{16b}$$

$$=\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{(3a^2c^3)\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{c}}{\sqrt[4]{a+}}\right.}{32b^{3/2}}$$

$$=\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a}{}$$

Mathematica [A]
time = 0.32, size = 109, normalized size = 0.74

$$\frac{(cx)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}\,(a+4bx^2)+3a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-3a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{32b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4)*(a + 4*b*x^2) + 3*a^2*Arc
Tan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] - 3*a^2*ArcTanh[(b^(1/4)*Sqrt[x])/
(a + b*x^2)^(1/4)]))/(32*b^(7/4)*x^(5/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{5}{2}}\left(b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 6.18, size = 46, normalized size = 0.31

$$\frac{\sqrt[4]{a}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(1/4),x)
[Out] a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{5/2}\,\left(b\,x^2+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(a + b*x^2)^(1/4), x)
```

3.926 $\int \sqrt{cx} \sqrt[4]{a+bx^2}\, dx$

Optimal. Leaf size=116

$$\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}-\frac{a\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}$$

```
[Out] 1/2*(c*x)^(3/2)*(b*x^2+a)^(1/4)/c-1/4*a*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a
)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)+1/4*a*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a
)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {285, 335, 338, 304, 211, 214}

$$-\frac{a\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(3/4)) + (a*Sqrt[c]*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx}\,\sqrt[4]{a+bx^2}\,dx &= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}+\frac{1}{4}a\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx\\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}+\frac{a\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{2c}\\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}+\frac{a\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2c}\\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}+\frac{(ac)\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4\sqrt{b}}-\frac{(ac)\text{Subst}\left(\int \frac{}{c+\sqrt{}}\right.}{}\\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}-\frac{a\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right.}{4b^{3/4}}\end{aligned}$$

Mathematica [A]

time = 0.19, size = 96, normalized size = 0.83

$$\frac{\sqrt{cx}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}-a\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+a\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

```
[Out] (Sqrt[c*x]*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4) - a*ArcTan[(b^(1/4)*Sqrt[x]
)/(a + b*x^2)^(1/4)] + a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(4*
b^(3/4)*Sqrt[x])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\,\left(b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 46, normalized size = 0.40

$$\frac{\sqrt[4]{a}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(1/4),x)
[Out] a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x}\,\left(b\,x^2+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a + b*x^2)^(1/4),x)
[Out] int((c*x)^(1/2)*(a + b*x^2)^(1/4), x)
```

3.927 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}} \, dx$

Optimal. Leaf size=107

$$-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}$$

```
[Out] -b^(1/4)*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/c^(3/2)+b^(1/4
)*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/c^(3/2)-2*(b*x^2+a)^
(1/4)/c/(c*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {283, 335, 338, 304, 211, 214}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(c*Sqrt[c*x]) - (b^(1/4)*ArcTan[(b^(1/4)*Sqrt[c*x])/
(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2) + (b^(1/4)*ArcTanh[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{b\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx}{c^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{(2b)\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{(2b)\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}-\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}-\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.18, size = 92, normalized size = 0.86

$$\frac{x\left(-2\sqrt[4]{a+bx^2}-\sqrt[4]{b}\sqrt{x}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+\sqrt[4]{b}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (x*(-2*(a + b*x^2)^(1/4) - b^(1/4)*Sqrt[x]*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*
x^2)^(1/4)] + b^(1/4)*Sqrt[x]*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)])
)/(c*x)^(3/2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 1.30, size = 49, normalized size = 0.46

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(3/2),x)
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{1/4}}{{\left(c\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(3/2),x)
[Out] int((a + b*x^2)^(1/4)/(c*x)^(3/2), x)
```

3.928 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

```
[Out] -2/5*(b*x^2+a)^(5/4)/a/c/(c*x)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{270\}$

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx = -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

Mathematica [A]
time = 0.12, size = 26, normalized size = 0.93

$$-\frac{2x(a+bx^2)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
[Out] (-2*x*(a + b*x^2)^(5/4))/(5*a*(c*x)^(7/2))
```

Maple [A]
time = 0.05, size = 21, normalized size = 0.75

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{5}{4}}}{5a\left(cx\right)^{\frac{7}{2}}}$	21
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{5}{4}}}{5c^{3}\sqrt{cx}\;x^{2}a}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*x*(b*x^2+a)^(5/4)/a/(c*x)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Fricas [A]
time = 1.07, size = 25, normalized size = 0.89

$$-\frac{2\,{\left(b x^{2} + a\right)}^{\frac{5}{4}} \sqrt{c x}}{5\,a c^{4} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")
[Out] -2/5*(b*x^2 + a)^(5/4)*sqrt(c*x)/(a*c^4*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(24) = 48.
time = 5.53, size = 78, normalized size = 2.79

$$\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^{2}\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(7/2),x)
```

```
[Out] b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4))
+ b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Mupad [B]
time = 4.92, size = 37, normalized size = 1.32

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2}{5\,c^3}+\frac{2\,b\,x^2}{5\,a\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(7/2),x)
[Out] -((a + b*x^2)^(1/4)*(2/(5*c^3) + (2*b*x^2)/(5*a*c^3)))/(x^2*(c*x)^(1/2))
```

3.929 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{9/2}}+\frac{8(a+bx^2)^{9/4}}{45a^2c(cx)^{9/2}}$$

```
[Out] -2/5*(b*x^2+a)^(5/4)/a/c/(c*x)^(9/2)+8/45*(b*x^2+a)^(9/4)/a^2/c/(c*x)^(9/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$\frac{8(a+bx^2)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx &= -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{9/2}}-\frac{4\int \frac{(a+bx^2)^{5/4}}{(cx)^{11/2}}\,dx}{5a}\\ &= -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{9/2}}+\frac{8(a+bx^2)^{9/4}}{45a^2c(cx)^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 46, normalized size = 0.81

$$-\frac{2x\sqrt[4]{a+bx^2}\,(5a^2+abx^2-4b^2x^4)}{45a^2(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*(5*a^2 + a*b*x^2 - 4*b^2*x^4))/(45*a^2*(c*x)^(11/2)
)
```

Maple [A]
time = 0.05, size = 31, normalized size = 0.54

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(-4b\,x^{2}+5a\right)}{45a^{2}\left(cx\right)^{\frac{11}{2}}}$	31
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-4b^{2}x^{4}+ab\,x^{2}+5a^{2}\right)}{45c^{5}\sqrt{cx}\;x^{4}a^{2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*x*(b*x^2+a)^(5/4)*(-4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Fricas [A]
time = 1.04, size = 46, normalized size = 0.81

$$\frac{2\,(4\,b^2x^4-abx^2-5\,a^2)(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{45\,a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*b^2*x^4 - a*b*x^2 - 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x^5
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(48) = 96$.
time = 55.15, size = 124, normalized size = 2.18

$$-\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{8c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{8ac^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{2a^2c^{\frac{11}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

```
[Out] -5*b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(8*c**(11/2)*x**4*gamma(-1/
4)) - b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(8*a*c**(11/2)*x**2*gamm
a(-1/4)) + b**(9/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(2*a**2*c**(11/2)*g
amma(-1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Mupad [B]
time = 4.98, size = 51, normalized size = 0.89

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2}{9\,c^5}+\frac{2\,b\,x^2}{45\,a\,c^5}-\frac{8\,b^2\,x^4}{45\,a^2\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(9*c^5) + (2*b*x^2)/(45*a*c^5) - (8*b^2*x^4)/(45*a^2
*c^5)))/(x^4*(c*x)^(1/2))
```

3.930 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}} \, dx$

Optimal. Leaf size=85

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{13/2}}+\frac{16(a+bx^2)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{64(a+bx^2)^{13/4}}{585a^3c(cx)^{13/2}}$$

```
[Out] -2/5*(b*x^2+a)^(5/4)/a/c/(c*x)^(13/2)+16/45*(b*x^2+a)^(9/4)/a^2/c/(c*x)^(13
/2)-64/585*(b*x^2+a)^(13/4)/a^3/c/(c*x)^(13/2)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$-\frac{64(a+bx^2)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16(a+bx^2)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a + b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a + b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{13/2}} - \frac{8\int \frac{(a+bx^2)^{5/4}}{(cx)^{15/2}}\,dx}{5a}$$
$$= -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{13/2}} + \frac{16(a+bx^2)^{9/4}}{45a^2c(cx)^{13/2}} + \frac{32\int \frac{(a+bx^2)^{9/4}}{(cx)^{15/2}}\,dx}{45a^2}$$
$$= -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{13/2}} + \frac{16(a+bx^2)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{64(a+bx^2)^{13/4}}{585a^3c(cx)^{13/2}}$$

Mathematica [A]
time = 0.15, size = 47, normalized size = 0.55

$$-\frac{2x(a+bx^2)^{5/4}(45a^2-40abx^2+32b^2x^4)}{585a^3(cx)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(5/4)*(45*a^2 - 40*a*b*x^2 + 32*b^2*x^4))/(585*a^3*(c*x)^
(15/2))
```

Maple [A]
time = 0.06, size = 42, normalized size = 0.49

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(32b^{2}x^{4}-40ab\,x^{2}+45a^{2}\right)}{585a^{3}\left(cx\right)^{\frac{15}{2}}}$	42
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(32b^{3}x^{6}-8a\,b^{2}x^{4}+5a^{2}b\,x^{2}+45a^{3}\right)}{585c^{7}\sqrt{cx}\;x^{6}a^{3}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/585*x*(b*x^2+a)^(5/4)*(32*b^2*x^4-40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Fricas [A]
time = 1.36, size = 57, normalized size = 0.67

$$-\frac{2\left(32\,b^{3}x^{6}-8\,ab^{2}x^{4}+5\,a^{2}bx^{2}+45\,a^{3}\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{585\,a^{3}c^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
[Out] -2/585*(32*b^3*x^6 - 8*a*b^2*x^4 + 5*a^2*b*x^2 + 45*a^3)*(b*x^2 + a)^(1/4)*
sqrt(c*x)/(a^3*c^8*x^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(15/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4497 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Mupad [B]
time = 5.00, size = 65, normalized size = 0.76

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2}{13\,c^7}+\frac{2\,b\,x^2}{117\,a\,c^7}-\frac{16\,b^2\,x^4}{585\,a^2\,c^7}+\frac{64\,b^3\,x^6}{585\,a^3\,c^7}\right)}{x^6\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(15/2),x)
[Out] -((a + b*x^2)^(1/4)*(2/(13*c^7) + (2*b*x^2)/(117*a*c^7) - (16*b^2*x^4)/(585
*a^2*c^7) + (64*b^3*x^6)/(585*a^3*c^7)))/(x^6*(c*x)^(1/2))
```

3.931 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}}+\frac{8(a+bx^2)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64(a+bx^2)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{256(a+bx^2)^{17/4}}{3315a^4c(cx)^{17/2}}$$

```
[Out] -2/5*(b*x^2+a)^(5/4)/a/c/(c*x)^(17/2)+8/15*(b*x^2+a)^(9/4)/a^2/c/(c*x)^(17/
2)-64/195*(b*x^2+a)^(13/4)/a^3/c/(c*x)^(17/2)+256/3315*(b*x^2+a)^(17/4)/a^4
/c/(c*x)^(17/2)
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$\frac{256(a+bx^2)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64(a+bx^2)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8(a+bx^2)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a + b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a + b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a + b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}} - \frac{12\int \frac{(a+bx^2)^{5/4}}{(cx)^{19/2}}\,dx}{5a}$$
$$= -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}} + \frac{8(a+bx^2)^{9/4}}{15a^2c(cx)^{17/2}} + \frac{32\int \frac{(a+bx^2)^{9/4}}{(cx)^{19/2}}\,dx}{15a^2}$$
$$= -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}} + \frac{8(a+bx^2)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{64(a+bx^2)^{13/4}}{195a^3c(cx)^{17/2}} - \frac{128\int \frac{(a+bx^2)^{13/4}}{(cx)^{19/2}}\,dx}{195a^3}$$
$$= -\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}} + \frac{8(a+bx^2)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{64(a+bx^2)^{13/4}}{195a^3c(cx)^{17/2}} + \frac{256(a+bx^2)^{17/4}}{3315a^4c(cx)^{17/2}}$$

Mathematica [A]
time = 1.14, size = 58, normalized size = 0.51

$$-\frac{2x(a+bx^2)^{5/4}\left(195a^3-180a^2bx^2+160ab^2x^4-128b^3x^6\right)}{3315a^4(cx)^{19/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(5/4)*(195*a^3 - 180*a^2*b*x^2 + 160*a*b^2*x^4 - 128*b^3*
x^6))/(3315*a^4*(c*x)^(19/2))
```

Maple [A]
time = 0.06, size = 53, normalized size = 0.47

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(-128b^{3}x^{6}+160a\,b^{2}x^{4}-180a^{2}b\,x^{2}+195a^{3}\right)}{3315a^{4}\left(cx\right)^{\frac{19}{2}}}$	53
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-128b^{4}x^{8}+32a\,b^{3}x^{6}-20a^{2}b^{2}x^{4}+15a^{3}b\,x^{2}+195a^{4}\right)}{3315c^{9}\sqrt{cx}\;x^{8}a^{4}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(19/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3315*x*(b*x^2+a)^(5/4)*(-128*b^3*x^6+160*a*b^2*x^4-180*a^2*b*x^2+195*a^3
)/a^4/(c*x)^(19/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Fricas [A]
time = 1.10, size = 68, normalized size = 0.60

$$\frac{2\left(128\,b^{4}x^{8}-32\,ab^{3}x^{6}+20\,a^{2}b^{2}x^{4}-15\,a^{3}bx^{2}-195\,a^{4}\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^{4}c^{10}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(128*b^4*x^8 - 32*a*b^3*x^6 + 20*a^2*b^2*x^4 - 15*a^3*b*x^2 - 195*a^
4)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(19/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Mupad [B]
time = 5.02, size = 79, normalized size = 0.70

$$-\frac{{\left(b\,x^2+a\right)}^{1/4}\,\left(\frac{2}{17\,c^9}+\frac{2\,b\,x^2}{221\,a\,c^9}-\frac{8\,b^2\,x^4}{663\,a^2\,c^9}+\frac{64\,b^3\,x^6}{3315\,a^3\,c^9}-\frac{256\,b^4\,x^8}{3315\,a^4\,c^9}\right)}{x^8\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(17*c^9) + (2*b*x^2)/(221*a*c^9) - (8*b^2*x^4)/(663*
a^2*c^9) + (64*b^3*x^6)/(3315*a^3*c^9) - (256*b^4*x^8)/(3315*a^4*c^9)))/(x^
8*(c*x)^(1/2))
```

3.932 $\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=122

$$-\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{a^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\,\left(a-bx^2\right)^{3/4}}$$

```
[Out] 1/3*(c*x)^(5/2)*(-b*x^2+a)^(1/4)/c-1/6*a^(3/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2
)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(
1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(3/
4)/b^(1/2)-1/6*a*c*(-b*x^2+a)^(1/4)*(c*x)^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {285, 327, 335, 243, 342, 281, 238}

$$-\frac{a^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\,\left(a-bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] -1/6*(a*c*Sqrt[c*x]*(a - b*x^2)^(1/4))/b + ((c*x)^(5/2)*(a - b*x^2)^(1/4))/
(3*c) - (a^(3/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b
]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c} + \frac{1}{6}a\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx$$

$$= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b} + \frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c} + \frac{(a^2c^2)\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx}{12b}$$

$$= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b} + \frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c} + \frac{(a^2c)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{6b}$$

$$= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b} + \frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c} + \frac{\left(a^2c\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{6b\,(a-bx^2)^{3/4}}$$

$$= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b} + \frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c} - \frac{\left(a^2c\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{6b\,(a-bx^2)^{3/4}}$$

$$= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b} + \frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c} - \frac{\left(a^2c\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{12b\,(a-bx^2)^{3/4}}$$

$$= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b} + \frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c} - \frac{a^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{6\sqrt{b}\,(a-bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 88, normalized size = 0.72

$$\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}\left((-a+bx^2)\sqrt[4]{1-\frac{bx^2}{a}} + a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*(a - b*x^2)^(1/4),x]
[Out] (c*Sqrt[c*x]*(a - b*x^2)^(1/4)*((-a + b*x^2)*(1 - (b*x^2)/a)^(1/4) + a*Hype
rgeometric2F1[-1/4, 1/4, 5/4, (b*x^2)/a]))/(3*b*(1 - (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{3}{2}}\left(-b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(-b*x^2+a)^(1/4),x)
[Out] int((c*x)^(3/2)*(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)*c*x, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.49, size = 48, normalized size = 0.39

$$\frac{\sqrt[4]{a}\,c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(-b*x**2+a)**(1/4),x)
[Out] a**(1/4)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**2*exp
_polar(2*I*pi)/a)/(2*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{3/2}\,{\left(a-b\,x^2\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(a - b*x^2)^(1/4),x)
[Out] int((c*x)^(3/2)*(a - b*x^2)^(1/4), x)
```

3.933 $\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c}-\frac{\sqrt{a}\,\sqrt{b}\,\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a-bx^2\right)^{3/4}}$$

```
[Out] -(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)
/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1
/2))),2^(1/2))*a^(1/2)*b^(1/2)/c^2/(-b*x^2+a)^(3/4)+(-b*x^2+a)^(1/4)*(c*x)^
(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {285, 335, 243, 342, 281, 238}

$$\frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c}-\frac{\sqrt{a}\,\sqrt{b}\,(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (Sqrt[c*x]*(a - b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(c
*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(c^2*(a - b*x^2)^(3/
4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx = \frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c} + \frac{1}{2}a\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx$$

$$= \frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c} + \frac{a\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c} + \frac{\left(a\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{c\,(a-bx^2)^{3/4}}$$

$$= \frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c} - \frac{\left(a\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{c\,(a-bx^2)^{3/4}}$$

$$= \frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c} - \frac{\left(a\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{2c\,(a-bx^2)^{3/4}}$$

$$= \frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c} - \frac{\sqrt{a}\,\sqrt{b}\,\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{c^2\,(a-bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 55, normalized size = 0.60

$$\frac{2x\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^2}{a}\right)}{\sqrt{cx}\,\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^2)/a])/(Sqrt[
c*x]*(1 - (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(1/2),x)
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.72, size = 39, normalized size = 0.42

$$-\frac{i\sqrt[4]{b}\,xe^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{1}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(1/2),x)
[Out] -I*b**(1/4)*x*exp(3*I*pi/4)*hyper((-1/2, -1/4), (1/2,), a/(b*x**2))/sqrt(c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^2\right)^{1/4}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(1/2),x)
[Out] int((a - b*x^2)^(1/4)/(c*x)^(1/2), x)
```

3.934 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}} \, dx$

Optimal. Leaf size=97

$$-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}+\frac{2b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,c^4\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2/3*(-b*x^2+a)^(1/4)/c/(c*x)^(3/2)+2/3*b^(3/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/
2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^
(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))/c^4/(-b*x^2+a
)^(3/4)/a^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {283, 335, 243, 342, 281, 238}

$$\frac{2b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) + (2*b^(3/2)*(1 - a/(b*x^2))^(3/4)
*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*c^4*(a
 - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} - \frac{b\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx}{3c^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{3c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} - \frac{\left(2b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{3c^3\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} + \frac{\left(2b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{3c^3\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} + \frac{\left(b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} + \frac{2b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{3\sqrt{a}\,c^4\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.59

$$-\frac{2x\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, (b*x^2)/a])/(3*(
c*x)^(5/2)*(1 - (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^3*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.18, size = 36, normalized size = 0.37

$$-\frac{i\sqrt[4]{b}\,e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{a}{bx^2}\right)}{c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(5/2),x)
```

```
[Out] -I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 1/2), (3/2,), a/(b*x**2))/(c**(5/2)*x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x^2)^{1/4}}{(c\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(5/2),x)
[Out] int((a - b*x^2)^(1/4)/(c*x)^(5/2), x)
```

3.935 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}+\frac{4b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2/7*(-b*x^2+a)^(1/4)/c/(c*x)^(7/2)+2/21*b*(-b*x^2+a)^(1/4)/a/c^3/(c*x)^(3/
2)+4/21*b^(5/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc
(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/c^6/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {283, 331, 335, 243, 342, 281, 238}

$$\frac{4b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(9/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) + (2*b*(a - b*x^2)^(1/4))/(21*a*c^
3*(c*x)^(3/2)) + (4*b^(5/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[Arc
Csc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*c^6*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}} - \frac{b\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx}{7c^2} \\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}} + \frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}} - \frac{(2b^2)\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx}{21ac^4} \\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}} + \frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}} - \frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{21ac^5} \\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}} + \frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}} - \frac{\left(4b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \ldots\right)}{21ac^5\left(a-bx^2\right)^{3/4}} \\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}} + \frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}} + \frac{\left(4b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \ldots\right)}{21ac^5\left(a-bx^2\right)^{3/4}} \\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}} + \frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}} + \frac{\left(2b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \ldots\right)}{21ac^5\left(a-bx^2\right)^{3/4}} \\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}} + \frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}} + \frac{4b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.45

$$
-\frac{2x\sqrt[4]{a-bx^2}\ {}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt[4]{1-\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(9/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, (b*x^2)/a])/(7*
(c*x)^(9/2)*(1 - (b*x^2)/a)^(1/4))
```

Maple [**F**]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^5*x^5), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 19.12, size = 39, normalized size = 0.31

$$\frac{i\sqrt[4]{b}\,e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{a}{bx^2}\right)}{3c^{\frac{9}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(9/2),x)
```

```
[Out] I*b**(1/4)*exp(3*I*pi/4)*hyper((-1/4, 3/2), (5/2,), a/(b*x**2))/(3*c**(9/2)
*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^2\right)^{1/4}}{{\left(c\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(9/2),x)
[Out] int((a - b*x^2)^(1/4)/(c*x)^(9/2), x)
```

3.936 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{8b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2/11*(-b*x^2+a)^(1/4)/c/(c*x)^(11/2)+2/77*b*(-b*x^2+a)^(1/4)/a/c^3/(c*x)^(
7/2)+4/77*b^2*(-b*x^2+a)^(1/4)/a^2/c^5/(c*x)^(3/2)+8/77*b^(7/2)*(1-a/b/x^2)
^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arc
csc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2
))/a^(5/2)/c^8/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {283, 331, 335, 243, 342, 281, 238}

$$\frac{8b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a-bx^2\right)^{3/4}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) + (2*b*(a - b*x^2)^(1/4))/(77*a*
c^3*(c*x)^(7/2)) + (4*b^2*(a - b*x^2)^(1/4))/(77*a^2*c^5*(c*x)^(3/2)) + (8*
b^(7/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(77*a^(5/2)*c^8*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} - \frac{b\int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}}\,dx}{11c^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} - \frac{(6b^2)\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx}{77ac^4}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{(4b^3)\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx}{77a^2c^6}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{(8b^3)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{77a^2c^7}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{\left(8b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{77a^2c^7\,(a-bx^2)^{3/}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{\left(8b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{77a^2c^7\,(a-bx^2)^{3/}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{\left(4b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{77a^2c^7\,(a-bx^2)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{8b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{77a^{5/2}c^8\,(a-bx^2)^{3/4}}$$

<u>**Mathematica**</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.36

$$-\frac{2x\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, (b*x^2)/a])/(1
1*(c*x)^(13/2)*(1 - (b*x^2)/a)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(13/2),x)
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^7*x^7), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 177.50, size = 39, normalized size = 0.25

$$-\frac{i\sqrt[4]{b}\,e^{-\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\ \frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{a}{bx^{2}}\right)}{5c^{\frac{13}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(13/2),x)
[Out] -I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 5/2), (7/2,), a/(b*x**2))/(5*c**(13/2
)*x**5)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x^2\right)}^{1/4}}{{\left(c\,x\right)}^{13/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(13/2),x)
```

```
[Out] int((a - b*x^2)^(1/4)/(c*x)^(13/2), x)
```

3.937 $\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=343

$$-\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}-\frac{3a^2c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}\,b^{7/4}}+\frac{3a^2c^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\cdots}{\sqrt{c}\,\cdots}\right)}{32\sqrt{2}\,b^{7/4}}$$

```
[Out] -1/16*a*c*(c*x)^(3/2)*(-b*x^2+a)^(1/4)/b+1/4*(c*x)^(7/2)*(-b*x^2+a)^(1/4)/c
+3/64*a^2*c^(5/2)*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^
(1/2))/b^(7/4)*2^(1/2)+3/64*a^2*c^(5/2)*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2
)/(-b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)*2^(1/2)+3/128*a^2*c^(5/2)*ln(c^(1/2)-b^
(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/
2))/b^(7/4)*2^(1/2)-3/128*a^2*c^(5/2)*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2
)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {285, 327, 335, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3a^2c^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}\,b^{7/4}}+\frac{3a^2c^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}+1\right)}{32\sqrt{2}\,b^{7/4}}+\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}-\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] -1/16*(a*c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/b + ((c*x)^(7/2)*(a - b*x^2)^(1/4
))/(4*c) - (3*a^2*c^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) + (3*a^2*c^(5/2)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) +
(3*a^2*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4)) - (3*a^2*c^(5/
2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqr
t[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{1}{8}a\int\frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{(3a^2c^2)\int\frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx}{32b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{(3a^2c)\,\mathrm{Subst}\left(\int\frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{c}\cdots\right.}{16b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{(3a^2c)\,\mathrm{Subst}\left(\int\frac{x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-b\cdots}}\right.}{16b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}-\frac{(3a^2c)\,\mathrm{Subst}\left(\int\frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{\cdots}}{\sqrt[4]{a-\cdots}}\right.}{32b^{3/2}}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{(3a^2c^{5/2})\,\mathrm{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x\cdots}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}\cdots}\right.}{64\sqrt{2}\,b^{7/4}}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{3a^2c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\cdots\right.}{64\sqrt{2}\,b^{7/4}}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}-\frac{3a^2c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}\,b^{7/4}}\cdots$$

Mathematica [A]
time = 0.59, size = 177, normalized size = 0.52

$$\frac{(cx)^{5/2}\left(4b^{3/4}x^{3/2}\sqrt[4]{a-bx^2}\,(-a+4bx^2)+3\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\sqrt[4]{a-bx^2}}{-\sqrt{b}\,x+\sqrt{a-bx^2}}\right)-3\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x+\sqrt{a-bx^2}}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\sqrt[4]{a-bx^2}}\right)\right)}{64b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(5/2)*(4*b^(3/4)*x^(3/2)*(a - b*x^2)^(1/4)*(-a + 4*b*x^2) + 3*Sqrt[2
]*a^2*ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*x) + Sq
rt[a - b*x^2])] - 3*Sqrt[2]*a^2*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt
[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))]))/(64*b^(7/4)*x^(5/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{5}{2}} \left(-b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 6.20, size = 48, normalized size = 0.14

$$\frac{\sqrt[4]{a}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*ex
p_polar(2*I*pi)/a)/(2*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{5/2}\,\left(a-b\,x^2\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(a - b*x^2)^(1/4), x)
```

3.938 $\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=307

$$\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}-\frac{a\sqrt{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}}+\frac{a\sqrt{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}}+\frac{a\sqrt{c}\,\log\left(\sqrt{c}\right.}{}$$

```
[Out] 1/2*(c*x)^(3/2)*(-b*x^2+a)^(1/4)/c+1/8*a*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1
/2)/(-b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)+1/8*a*arctan(1+b^(1/4
)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)+1/1
6*a*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/
2)/(-b*x^2+a)^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)-1/16*a*ln(c^(1/2)+b^(1/4)*2^(1
/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))*c^(1/2
)/b^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {285, 335, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{a\sqrt{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}}+\frac{a\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{4\sqrt{2}\,b^{3/4}}+\frac{a\sqrt{c}\,\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\,\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(3/4)) + (a*Sqrt[
c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*
Sqrt[2]*b^(3/4)) + (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*
x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
- (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b
^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
 p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \sqrt{cx}\,\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{1}{4}a\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} - \frac{a\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4\sqrt{b}\,c} + \frac{a\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx\right.}{4\sqrt{b}\,c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{\left(a\sqrt{c}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{3/4}} + \frac{(a}{}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{3/4}} - \frac{a\sqrt{c}\,\log\left(\sqrt{}\right.}{}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} - \frac{a\sqrt{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}} + \frac{a\sqrt{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt{c}\sqrt[4]{a}}\right.}{4\sqrt{2}\,b^{3/4}}$$

Mathematica [A]
time = 0.36, size = 162, normalized size = 0.53

$$\frac{\sqrt{cx}\left(4b^{3/4}x^{3/2}\sqrt[4]{a-bx^2}+\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{-\sqrt{b}\,x+\sqrt{a-bx^2}}\right)-\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{b}\,x+\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}\right)\right)}{8b^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

```
[Out] (Sqrt[c*x]*(4*b^(3/4)*x^(3/2)*(a - b*x^2)^(1/4) + Sqrt[2]*a*ArcTan[(Sqrt[2]
*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*x) + Sqrt[a - b*x^2])] - Sqr
t[2]*a*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1/4)*Sqrt[x]*(a -
b*x^2)^(1/4))]))/(8*b^(3/4)*Sqrt[x])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\,\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
[Out] int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.97, size = 48, normalized size = 0.16

$$\frac{\sqrt[4]{a}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp_
polar(2*I*pi)/a)/(2*gamma(7/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x}\,\left(a-b\,x^2\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a - b*x^2)^(1/4),x)
[Out] int((c*x)^(1/2)*(a - b*x^2)^(1/4), x)
```

3.939 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}}-\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}}-\frac{\sqrt[4]{b}\log\left(\sqrt{c}+\frac{\sqrt{b}\cdots}{\sqrt{a\cdots}}\right.}{2\cdots}$$

```
[Out] -1/2*b^(1/4)*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2)
)/c^(3/2)*2^(1/2)-1/2*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+
a)^(1/4)/c^(1/2))/c^(3/2)*2^(1/2)-1/4*b^(1/4)*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c
*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/c^(3/2)*2^(1
/2)+1/4*b^(1/4)*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b
^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/c^(3/2)*2^(1/2)-2*(-b*x^2+a)^(1/4)/c/(c*x)
^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {283, 335, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\,c^{3/2}}-\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(c*Sqrt[c*x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c^(3/2)) - (b^(1/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c
^(3/2)) - (b^(1/4)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqr
t[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2)) + (b^(1/4)*
Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c
*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{b\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx}{c^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{(2b)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{(2b)\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, \right.}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,c^{3/2}} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,c^{3/2}} + \frac{\sqrt[4]{b}\,\log\left(\sqrt{c}+\frac{\sqrt{b}}{\sqrt{a-}}\right.}{2\sqrt{}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} + \frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}}$$

Mathematica [A]
time = 0.32, size = 161, normalized size = 0.54

$$\frac{x\left(-4\sqrt[4]{a-bx^2}+\sqrt{2}\sqrt[4]{b}\sqrt{x}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{\sqrt{b}\,x-\sqrt{a-bx^2}}\right)+\sqrt{2}\sqrt[4]{b}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{b}\,x+\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}\right)\right)}{2(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (x*(-4*(a - b*x^2)^(1/4) + Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[(Sqrt[2]*b^(1/4)*
Sqrt[x]*(a - b*x^2)^(1/4))/(Sqrt[b]*x - Sqrt[a - b*x^2])] + Sqrt[2]*b^(1/4)
*Sqrt[x]*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1/4)*Sqrt[x]*(a
- b*x^2)^(1/4))]))/(2*(c*x)^(3/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 1.26, size = 51, normalized size = 0.17

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(3/2),x)
```

```
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(2*I*pi)/a
)/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-b\,x^2\right)}^{1/4}}{{\left(c\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(3/2),x)
```

```
[Out] int((a - b*x^2)^(1/4)/(c*x)^(3/2), x)
```

3.940 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

```
[Out] -2/5*(-b*x^2+a)^(5/4)/a/c/(c*x)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {270}

$$-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx=-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

Mathematica [A]
time = 0.13, size = 27, normalized size = 0.93

$$-\frac{2x(a-bx^2)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]
[Out] (-2*x*(a - b*x^2)^(5/4))/(5*a*(c*x)^(7/2))
```

Maple [A]
time = 0.05, size = 22, normalized size = 0.76

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}}{5a\left(cx\right)^{\frac{7}{2}}}$	22
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}}{5\sqrt{cx}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}}c^{3}x^{2}a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*x*(-b*x^2+a)^(5/4)/a/(c*x)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Fricas [A]
time = 1.71, size = 35, normalized size = 1.21

$$\frac{2\left(bx^2-a\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{5\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")
[Out] 2/5*(b*x^2 - a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.82, size = 178, normalized size = 6.14

$$\begin{cases} \frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{for}\ \left|\frac{a}{bx^2}\right|>1 \\ \frac{\sqrt[4]{b}\,\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(7/2),x)
```

```
[Out] Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*ga
mma(-1/4)) - b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*gam
ma(-1/4)), Abs(a/(b*x**2)) > 1), (b**(1/4)*(-a/(b*x**2) + 1)**(1/4)*exp(I*p
i/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4)) - b**(5/4)*(-a/(b*x**2) + 1)
**(1/4)*exp(I*pi/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Mupad [B]
time = 4.85, size = 38, normalized size = 1.31

$$-\frac{{\left(a-b\,x^2\right)}^{1/4}\,\left(\frac{2}{5\,c^3}-\frac{2\,b\,x^2}{5\,a\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(7/2),x)
```

```
[Out] -((a - b*x^2)^(1/4)*(2/(5*c^3) - (2*b*x^2)/(5*a*c^3)))/(x^2*(c*x)^(1/2))
```

3.941 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{9/2}}+\frac{8(a-bx^2)^{9/4}}{45a^2c(cx)^{9/2}}$$

```
[Out] -2/5*(-b*x^2+a)^(5/4)/a/c/(c*x)^(9/2)+8/45*(-b*x^2+a)^(9/4)/a^2/c/(c*x)^(9/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {279, 270}

$$\frac{8(a-bx^2)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a - b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx &= -\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{9/2}}-\frac{4\int \frac{(a-bx^2)^{5/4}}{(cx)^{11/2}}\,dx}{5a}\\ &= -\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{9/2}}+\frac{8(a-bx^2)^{9/4}}{45a^2c(cx)^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 48, normalized size = 0.81

$$-\frac{2x\sqrt[4]{a-bx^2}\,(5a^2-abx^2-4b^2x^4)}{45a^2(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
[Out] (-2*x*(a - b*x^2)^(1/4)*(5*a^2 - a*b*x^2 - 4*b^2*x^4))/(45*a^2*(c*x)^(11/2)
)
```

Maple [A]
time = 0.06, size = 32, normalized size = 0.54

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(4b\,x^{2}+5a\right)}{45a^{2}\left(cx\right)^{\frac{11}{2}}}$	32
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}\left(-4b^{2}x^{4}-ab\,x^{2}+5a^{2}\right)}{45\sqrt{cx}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}}c^{5}x^{4}a^{2}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
[Out] -2/45*x*(-b*x^2+a)^(5/4)*(4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Fricas [A]
time = 1.32, size = 46, normalized size = 0.78

$$\frac{2\,(4\,b^2x^4+abx^2-5\,a^2)(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{45\,a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*b^2*x^4 + a*b*x^2 - 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x^
5)
```

Sympy [C] Result contains complex when optimal does not.
time = 55.50, size = 462, normalized size = 7.83

$$\begin{cases} -\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{9}{4}\right)}{8c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{9}{4}\right)}{8ac^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{9}{4}\right)}{2a^2c^{\frac{11}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^2}\right|>1 \\ \frac{5a^3b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{x^2\left(-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+8a^2b^2c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)\right)}-\frac{6a^2b^{\frac{9}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+8a^2b^2c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)}-\frac{3ab^{\frac{13}{4}}x^2\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+8a^2b^2c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)}+\frac{4b^{\frac{17}{4}}x^4\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+8a^2b^2c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

```
[Out] Piecewise((-5*b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(8*c**(11/2)*x**
4*gamma(-1/4)) + b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(8*a*c**(11/2
)*x**2*gamma(-1/4)) + b**(9/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(2*a**2*
c**(11/2)*gamma(-1/4)), Abs(a/(b*x**2)) > 1), (5*a**3*b**(5/4)*(-a/(b*x**2)
 + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(x**2*(-8*a**3*b*c**(11/2)*x**2*gamma(
-1/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4))) - 6*a**2*b**(9/4)*(-a/(b*x
**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*gamma(-1
/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)) - 3*a*b**(13/4)*x**2*(-a/(b*x
**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*gamma(-1
/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)) + 4*b**(17/4)*x**4*(-a/(b*x**
2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*gamma(-1/4
) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Mupad [B]
time = 4.89, size = 51, normalized size = 0.86

$$\frac{{\left(a-b\,x^2\right)}^{1/4}\,\left(\frac{2\,b\,x^2}{45\,a\,c^5}-\frac{2}{9\,c^5}+\frac{8\,b^2\,x^4}{45\,a^2\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] ((a - b*x^2)^(1/4)*((2*b*x^2)/(45*a*c^5) - 2/(9*c^5) + (8*b^2*x^4)/(45*a^2*
c^5)))/(x^4*(c*x)^(1/2))
```

3.942 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{13/2}}+\frac{16(a-bx^2)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{64(a-bx^2)^{13/4}}{585a^3c(cx)^{13/2}}$$

```
[Out] -2/5*(-b*x^2+a)^(5/4)/a/c/(c*x)^(13/2)+16/45*(-b*x^2+a)^(9/4)/a^2/c/(c*x)^(
13/2)-64/585*(-b*x^2+a)^(13/4)/a^3/c/(c*x)^(13/2)
```

Rubi [A]
time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {279, 270}

$$-\frac{64(a-bx^2)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16(a-bx^2)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a - b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a - b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx = -\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{13/2}} - \frac{8\int \frac{(a-bx^2)^{5/4}}{(cx)^{15/2}}\,dx}{5a}$$
$$= -\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{13/2}} + \frac{16(a-bx^2)^{9/4}}{45a^2c(cx)^{13/2}} + \frac{32\int \frac{(a-bx^2)^{9/4}}{(cx)^{15/2}}\,dx}{45a^2}$$
$$= -\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{13/2}} + \frac{16(a-bx^2)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{64(a-bx^2)^{13/4}}{585a^3c(cx)^{13/2}}$$

Mathematica [A]
time = 0.15, size = 48, normalized size = 0.55

$$-\frac{2x(a-bx^2)^{5/4}(45a^2+40abx^2+32b^2x^4)}{585a^3(cx)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
[Out] (-2*x*(a - b*x^2)^(5/4)*(45*a^2 + 40*a*b*x^2 + 32*b^2*x^4))/(585*a^3*(c*x)^
(15/2))
```

Maple [A]
time = 0.06, size = 43, normalized size = 0.49

method	result	size
gosper	$-\frac{2x\left(-b\,x^2+a\right)^{\frac{5}{4}}\left(32b^2x^4+40ab\,x^2+45a^2\right)}{585a^3(cx)^{\frac{15}{2}}}$	43
risch	$-\frac{2\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(\left(-b\,x^2+a\right)^3\right)^{\frac{1}{4}}\left(-32b^3x^6-8a\,b^2x^4-5a^2b\,x^2+45a^3\right)}{585\sqrt{cx}\,\left(-\left(b\,x^2-a\right)^3\right)^{\frac{1}{4}}c^7x^6a^3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(15/2),x,method=_RETURNVERBOSE)
[Out] -2/585*x*(-b*x^2+a)^(5/4)*(32*b^2*x^4+40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Fricas [A]
time = 1.63, size = 58, normalized size = 0.66

$$\frac{2\left(32\,b^{3}x^{6}+8\,ab^{2}x^{4}+5\,a^{2}bx^{2}-45\,a^{3}\right)\left(-bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{585\,a^{3}c^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
```

```
[Out] 2/585*(32*b^3*x^6 + 8*a*b^2*x^4 + 5*a^2*b*x^2 - 45*a^3)*(-b*x^2 + a)^(1/4)*
sqrt(c*x)/(a^3*c^8*x^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(15/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4497 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Mupad [B]
time = 4.91, size = 65, normalized size = 0.74

$$\frac{\left(a-b\,x^{2}\right)^{1/4}\left(\frac{2\,b\,x^{2}}{117\,a\,c^{7}}-\frac{2}{13\,c^{7}}+\frac{16\,b^{2}\,x^{4}}{585\,a^{2}\,c^{7}}+\frac{64\,b^{3}\,x^{6}}{585\,a^{3}\,c^{7}}\right)}{x^{6}\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] ((a - b*x^2)^(1/4)*((2*b*x^2)/(117*a*c^7) - 2/(13*c^7) + (16*b^2*x^4)/(585*
a^2*c^7) + (64*b^3*x^6)/(585*a^3*c^7)))/(x^6*(c*x)^(1/2))
```

3.943 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{17/2}}+\frac{8(a-bx^2)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64(a-bx^2)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{256(a-bx^2)^{17/4}}{3315a^4c(cx)^{17/2}}$$

```
[Out] -2/5*(-b*x^2+a)^(5/4)/a/c/(c*x)^(17/2)+8/15*(-b*x^2+a)^(9/4)/a^2/c/(c*x)^(1
7/2)-64/195*(-b*x^2+a)^(13/4)/a^3/c/(c*x)^(17/2)+256/3315*(-b*x^2+a)^(17/4)
/a^4/c/(c*x)^(17/2)
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {279, 270}

$$\frac{256(a-bx^2)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64(a-bx^2)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8(a-bx^2)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a - b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a - b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a - b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = -\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{17/2}} - \frac{12\int \frac{(a-bx^2)^{5/4}}{(cx)^{19/2}}\,dx}{5a}$$
$$= -\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{17/2}} + \frac{8(a-bx^2)^{9/4}}{15a^2c(cx)^{17/2}} + \frac{32\int \frac{(a-bx^2)^{9/4}}{(cx)^{19/2}}\,dx}{15a^2}$$
$$= -\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{17/2}} + \frac{8(a-bx^2)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{64(a-bx^2)^{13/4}}{195a^3c(cx)^{17/2}} - \frac{128\int \frac{(a-bx^2)^{13/4}}{(cx)^{19/2}}\,dx}{195a^3}$$
$$= -\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{17/2}} + \frac{8(a-bx^2)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{64(a-bx^2)^{13/4}}{195a^3c(cx)^{17/2}} + \frac{256(a-bx^2)^{17/4}}{3315a^4c(cx)^{17/2}}$$

Mathematica [A]
time = 1.15, size = 59, normalized size = 0.50

$$-\frac{2x(a-bx^2)^{5/4}(195a^3+180a^2bx^2+160ab^2x^4+128b^3x^6)}{3315a^4(cx)^{19/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(5/4)*(195*a^3 + 180*a^2*b*x^2 + 160*a*b^2*x^4 + 128*b^3*
x^6))/(3315*a^4*(c*x)^(19/2))
```

Maple [A]
time = 0.06, size = 54, normalized size = 0.46

method	result	size
gosper	$-\frac{2x(-bx^2+a)^{\frac{5}{4}}(128b^3x^6+160ab^2x^4+180a^2bx^2+195a^3)}{3315a^4(cx)^{\frac{19}{2}}}$	54
risch	$-\frac{2(-bx^2+a)^{\frac{1}{4}}\left((-bx^2+a)^3\right)^{\frac{1}{4}}(-128b^4x^8-32ab^3x^6-20a^2b^2x^4-15a^3bx^2+195a^4)}{3315\sqrt{cx}\,\left(-(bx^2-a)^3\right)^{\frac{1}{4}}c^9x^8a^4}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(19/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3315*x*(-b*x^2+a)^(5/4)*(128*b^3*x^6+160*a*b^2*x^4+180*a^2*b*x^2+195*a^3
)/a^4/(c*x)^(19/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Fricas [A]
time = 1.75, size = 69, normalized size = 0.59

$$\frac{2\,(128\,b^4x^8+32\,ab^3x^6+20\,a^2b^2x^4+15\,a^3bx^2-195\,a^4)(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
[Out] 2/3315*(128*b^4*x^8 + 32*a*b^3*x^6 + 20*a^2*b^2*x^4 + 15*a^3*b*x^2 - 195*a^
4)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(19/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Mupad [B]
time = 4.93, size = 79, normalized size = 0.68

$$\frac{{\left(a-b\,x^2\right)}^{1/4}\,\left(\frac{2\,b\,x^2}{221\,a\,c^9}-\frac{2}{17\,c^9}+\frac{8\,b^2\,x^4}{663\,a^2\,c^9}+\frac{64\,b^3\,x^6}{3315\,a^3\,c^9}+\frac{256\,b^4\,x^8}{3315\,a^4\,c^9}\right)}{x^8\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(19/2),x)
[Out] ((a - b*x^2)^(1/4)*((2*b*x^2)/(221*a*c^9) - 2/(17*c^9) + (8*b^2*x^4)/(663*a
^2*c^9) + (64*b^3*x^6)/(3315*a^3*c^9) + (256*b^4*x^8)/(3315*a^4*c^9)))/(x^8
*(c*x)^(1/2))
```

3.944 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\, dx$

Optimal. Leaf size=117

$$\frac{c\sqrt{cx}\,(a+bx^2)^{3/4}}{2b}-\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}-\frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}$$

```
[Out] -1/4*a*c^(3/2)*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(5/4)-
1/4*a*c^(3/2)*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(5/4)+
1/2*c*(b*x^2+a)^(3/4)*(c*x)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {327, 335, 246, 218, 214, 211}

$$-\frac{ac^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}-\frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}+\frac{c\sqrt{cx}\,(a+bx^2)^{3/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4)) - (a*c^(3/2)*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c\sqrt{cx}\,(a+bx^2)^{3/4}}{2b} - \frac{(ac^2)\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx}{4b}$$

$$= \frac{c\sqrt{cx}\,(a+bx^2)^{3/4}}{2b} - \frac{(ac)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b}$$

$$= \frac{c\sqrt{cx}\,(a+bx^2)^{3/4}}{2b} - \frac{(ac)\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b}$$

$$= \frac{c\sqrt{cx}\,(a+bx^2)^{3/4}}{2b} - \frac{(ac^2)\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b} - \frac{(ac^2)\,\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\ldots}\right.}{}$$

$$= \frac{c\sqrt{cx}\,(a+bx^2)^{3/4}}{2b} - \frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}} - \frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}$$

Mathematica [A]

time = 0.27, size = 97, normalized size = 0.83

$$\frac{(cx)^{3/2}\left(2\sqrt[4]{b}\,\sqrt{x}\,(a+bx^2)^{3/4}-a\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(2*b^(1/4)*Sqrt[x]*(a + b*x^2)^(3/4) - a*ArcTan[(b^(1/4)*Sqrt[
x])/(a + b*x^2)^(1/4)] - a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(
4*b^(5/4)*x^(3/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(83) = 166.
time = 1.61, size = 314, normalized size = 2.68

$$\frac{4\,(bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,c+4\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}(bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,ab^4c-(b^5x^2+ab^4)\left(\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}\,a^2c^3x+\sqrt{\frac{a^4c^6}{b^5}}\,(b^3x^2+ab^2)}{bx^2+a}}}{a^4bc^6x^2+a^5c^6}\right)-\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,ac+\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}(b^2x^2+ab)}{bx^2+a}\right)+\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,ac-\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}(b^2x^2+ab)}{bx^2+a}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(a^4*c^6/b^5)^(1/4)*b*arctan(-((a^
4*c^6/b^5)^(3/4)*(b*x^2 + a)^(3/4)*sqrt(c*x)*a*b^4*c - (b^5*x^2 + a*b^4)*(a
^4*c^6/b^5)^(3/4)*sqrt((sqrt(b*x^2 + a)*a^2*c^3*x + sqrt(a^4*c^6/b^5)*(b^3*
x^2 + a*b^2))/(b*x^2 + a)))/(a^4*b*c^6*x^2 + a^5*c^6)) - (a^4*c^6/b^5)^(1/4
)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c + (a^4*c^6/b^5)^(1/4)*(b^2*x^2 + a
*b))/(b*x^2 + a)) + (a^4*c^6/b^5)^(1/4)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*
a*c - (a^4*c^6/b^5)^(1/4)*(b^2*x^2 + a*b))/(b*x^2 + a)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 1.29, size = 44, normalized size = 0.38

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(1/4)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^{3/2}}{{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(a + b*x^2)^(1/4), x)
```

3.945 $\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

```
[Out] arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(1/4)/c^(1/2)+arctanh
(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(1/4)/c^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {335, 246, 218, 214, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c]) +
 ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}$$
$$= \text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right) + \text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

Mathematica [A]
time = 0.17, size = 65, normalized size = 0.78

$$\frac{\sqrt{x}\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{\sqrt[4]{b}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4)*Sq
rt[x])/(a + b*x^2)^(1/4)]))/(b^(1/4)*Sqrt[c*x])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(59) = 118$.
time = 1.24, size = 241, normalized size = 2.90

$$-2\left(\frac{1}{bc^{2}}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx^{2}+a)^{\frac{3}{4}}\sqrt{cx}\,bc\left(\frac{1}{bc^{2}}\right)^{\frac{3}{4}}-(b^{2}cx^{2}+abc)\sqrt{\frac{\sqrt{bx^{2}+a}\,cx+(bc^{2}x^{2}+ac^{2})\sqrt{\frac{1}{bc^{2}}}}{bx^{2}+a}}\left(\frac{1}{bc^{2}}\right)^{\frac{3}{4}}}{bx^{2}+a}\right)+\frac{1}{2}\left(\frac{1}{bc^{2}}\right)^{\frac{1}{4}}\log\left(\frac{(bx^{2}+a)^{\frac{3}{4}}\sqrt{cx}+(bcx^{2}+ac)\left(\frac{1}{bc^{2}}\right)^{\frac{1}{4}}}{bx^{2}+a}\right)-\frac{1}{2}\left(\frac{1}{bc^{2}}\right)^{\frac{1}{4}}\log\left(\frac{(bx^{2}+a)^{\frac{3}{4}}\sqrt{cx}-(bcx^{2}+ac)\left(\frac{1}{bc^{2}}\right)^{\frac{1}{4}}}{bx^{2}+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2*(1/(b*c^2))^(1/4)*arctan(-((b*x^2 + a)^(3/4)*sqrt(c*x)*b*c*(1/(b*c^2))^(
3/4) - (b^2*c*x^2 + a*b*c)*sqrt((sqrt(b*x^2 + a)*c*x + (b*c^2*x^2 + a*c^2)*
sqrt(1/(b*c^2)))/(b*x^2 + a))*(1/(b*c^2))^(3/4))/(b*x^2 + a)) + 1/2*(1/(b*c
^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 + a*c)*(1/(b*c^2))^(
1/4))/(b*x^2 + a)) - 1/2*(1/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)
 - (b*c*x^2 + a*c)*(1/(b*c^2))^(1/4))/(b*x^2 + a))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.75, size = 44, normalized size = 0.53

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(1/4)*sqrt(c)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/4)), x)
```

3.946 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

```
[Out] -2/3*(b*x^2+a)^(3/4)/a/c/(c*x)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {270}

$$-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

Mathematica [A]
time = 0.13, size = 26, normalized size = 0.93

$$-\frac{2x(a+bx^2)^{3/4}}{3a(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]
[Out] (-2*x*(a + b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

Maple [A]
time = 0.05, size = 21, normalized size = 0.75

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{3a\left(cx\right)^{\frac{5}{2}}}$	21
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{3c^{2}\sqrt{cx}\;ax}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
[Out] -2/3*x*(b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Fricas [A]
time = 1.20, size = 25, normalized size = 0.89

$$-\frac{2\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\,ac^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/3*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

Sympy [A]
time = 2.12, size = 36, normalized size = 1.29

$$\frac{b^{\frac{3}{4}}\left(\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/4),x)
[Out] b**(3/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Mupad [B]
time = 4.95, size = 25, normalized size = 0.89

$$-\frac{2\,{\left(b\,x^2+a\right)}^{3/4}}{3\,a\,c^2\,x\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x)
[Out] -(2*(a + b*x^2)^(3/4))/(3*a*c^2*x*(c*x)^(1/2))
```

3.947 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{7/2}}+\frac{8(a+bx^2)^{7/4}}{21a^2c(cx)^{7/2}}$$

```
[Out] -2/3*(b*x^2+a)^(3/4)/a/c/(c*x)^(7/2)+8/21*(b*x^2+a)^(7/4)/a^2/c/(c*x)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$\frac{8(a+bx^2)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx &= -\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{7/2}}-\frac{4\int \frac{(a+bx^2)^{3/4}}{(cx)^{9/2}}\,dx}{3a}\\
&= -\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{7/2}}+\frac{8(a+bx^2)^{7/4}}{21a^2c(cx)^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 36, normalized size = 0.63

$$-\frac{2x(3a-4bx^2)\left(a+bx^2\right)^{3/4}}{21a^2(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
[Out] (-2*x*(3*a - 4*b*x^2)*(a + b*x^2)^(3/4))/(21*a^2*(c*x)^(9/2))
```

Maple [A]
time = 0.05, size = 31, normalized size = 0.54

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(-4b\,x^{2}+3a\right)}{21a^{2}\left(cx\right)^{\frac{9}{2}}}$	31
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(-4b\,x^{2}+3a\right)}{21c^{4}\sqrt{cx}\;a^{2}x^{3}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
[Out] -2/21*x*(b*x^2+a)^(3/4)*(-4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Fricas [A]
time = 1.10, size = 35, normalized size = 0.61

$$\frac{2\,(4\,bx^2-3\,a)(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{21\,a^2c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] 2/21*(4*b*x^2 - 3*a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

Sympy [A]
time = 25.48, size = 80, normalized size = 1.40

$$-\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{8ac^{\frac{9}{2}}x^2\Gamma\left(\frac{1}{4}\right)}+\frac{b^{\frac{7}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2a^2c^{\frac{9}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -3*b**(3/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(8*a*c**(9/2)*x**2*gamma(1/
4)) + b**(7/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(2*a**2*c**(9/2)*gamma(1
/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Mupad [B]
time = 5.00, size = 40, normalized size = 0.70

$$-\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{2}{7\,a\,c^4}-\frac{8\,b\,x^2}{21\,a^2\,c^4}\right)}{x^3\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*(2/(7*a*c^4) - (8*b*x^2)/(21*a^2*c^4)))/(x^3*(c*x)^(1/2
))
```

3.948 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}}+\frac{16(a+bx^2)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{64(a+bx^2)^{11/4}}{231a^3c(cx)^{11/2}}$$

```
[Out] -2/3*(b*x^2+a)^(3/4)/a/c/(c*x)^(11/2)+16/21*(b*x^2+a)^(7/4)/a^2/c/(c*x)^(11
/2)-64/231*(b*x^2+a)^(11/4)/a^3/c/(c*x)^(11/2)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$-\frac{64(a+bx^2)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16(a+bx^2)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a + b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a + b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}} - \frac{8\int \frac{(a+bx^2)^{3/4}}{(cx)^{13/2}}\,dx}{3a}$$
$$= -\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}} + \frac{16(a+bx^2)^{7/4}}{21a^2c(cx)^{11/2}} + \frac{32\int \frac{(a+bx^2)^{7/4}}{(cx)^{13/2}}\,dx}{21a^2}$$
$$= -\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}} + \frac{16(a+bx^2)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{64(a+bx^2)^{11/4}}{231a^3c(cx)^{11/2}}$$

Mathematica [A]
time = 0.19, size = 47, normalized size = 0.55

$$-\frac{2x(a+bx^2)^{3/4}(21a^2-24abx^2+32b^2x^4)}{231a^3(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
[Out] (-2*x*(a + b*x^2)^(3/4)*(21*a^2 - 24*a*b*x^2 + 32*b^2*x^4))/(231*a^3*(c*x)^
(13/2))
```

Maple [A]
time = 0.05, size = 42, normalized size = 0.49

method	result	size
gosper	$-\frac{2x(b\,x^2+a)^{\frac{3}{4}}(32b^2x^4-24ab\,x^2+21a^2)}{231a^3(cx)^{\frac{13}{2}}}$	42
risch	$-\frac{2(b\,x^2+a)^{\frac{3}{4}}(32b^2x^4-24ab\,x^2+21a^2)}{231c^6\sqrt{cx}\;a^3x^5}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
[Out] -2/231*x*(b*x^2+a)^(3/4)*(32*b^2*x^4-24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Fricas [A]
time = 1.30, size = 46, normalized size = 0.54

$$-\frac{2\left(32\,b^2x^4-24\,abx^2+21\,a^2\right)(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{231\,a^3c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/231*(32*b^2*x^4 - 24*a*b*x^2 + 21*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3*
c^7*x^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(1/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3279 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Mupad [B]
time = 5.00, size = 54, normalized size = 0.64

$$-\frac{{\left(b\,x^2+a\right)}^{3/4}\,\left(\frac{2}{11\,a\,c^6}-\frac{16\,b\,x^2}{77\,a^2\,c^6}+\frac{64\,b^2\,x^4}{231\,a^3\,c^6}\right)}{x^5\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x)
[Out] -((a + b*x^2)^(3/4)*(2/(11*a*c^6) - (16*b*x^2)/(77*a^2*c^6) + (64*b^2*x^4)/
(231*a^3*c^6)))/(x^5*(c*x)^(1/2))
```

3.949 $\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=156

$$\frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2}+\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}+\frac{7a^{5/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right.}{20b^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -7/30*a*c^3*(c*x)^(3/2)*(b*x^2+a)^(3/4)/b^2+1/5*c*(c*x)^(7/2)*(b*x^2+a)^(3/
4)/b+7/20*a^2*c^4*x*(c*x)^(1/2)/b^2/(b*x^2+a)^(1/4)+7/20*a^(5/2)*c^4*(1+a/b
/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*x
)^(1/2)/b^(5/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.05, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {327, 320, 290, 342, 202}

$$\frac{7a^{5/2}c^4\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20b^{5/2}\sqrt[4]{a+bx^2}}+\frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2}+\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(9/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (7*a^2*c^4*x*Sqrt[c*x])/(20*b^2*(a + b*x^2)^(1/4)) - (7*a*c^3*(c*x)^(3/2)*(
a + b*x^2)^(3/4))/(30*b^2) + (c*(c*x)^(7/2)*(a + b*x^2)^(3/4))/(5*b) + (7*a
^(5/2)*c^4*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(20*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[x*(Sqrt[
c*x]/(a + b*x^2)^(1/4)), x] - Dist[a/2, Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x]
```

```
, x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(7ac^2\right)\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx}{10b}$$

$$= -\frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{\left(7a^2c^4\right)\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx}{20b^2}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(7a^3c^4\right)\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,d}{40b^2}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(7a^3c^4\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{}\right.}{40b^3\sqrt[4]{}}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{\left(7a^3c^4\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{}\right.}{}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{7a^{5/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{}}{20b^5}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 87, normalized size = 0.56

$$\frac{c^3(cx)^{3/2}\left(-7a^2-abx^2+6b^2x^4+7a^2\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{30b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(9/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c^3*(c*x)^(3/2)*(-7*a^2 - a*b*x^2 + 6*b^2*x^4 + 7*a^2*(1 + (b*x^2)/a)^(1/4
)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(30*b^2*(a + b*x^2)^(1/4
))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{9}{2}}}{(b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^4*x^4/(b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 37.36, size = 44, normalized size = 0.28

$$\frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(9/2)/(b*x**2+a)**(1/4),x)
[Out] c**(9/2)*x**(11/2)*gamma(11/4)*hyper((1/4, 11/4), (15/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(1/4)*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{9/2}}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(a + b*x^2)^(1/4),x)
[Out] int((c*x)^(9/2)/(a + b*x^2)^(1/4), x)
```

3.950 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}-\frac{a^{3/2}c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 1/3*c*(c*x)^(3/2)*(b*x^2+a)^(3/4)/b-1/2*a*c^2*x*(c*x)^(1/2)/b/(b*x^2+a)^(1/
4)-1/2*a^(3/2)*c^2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)
^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2
)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/b^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {327, 320, 290, 342, 202}

$$-\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}-\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] -1/2*(a*c^2*x*Sqrt[c*x])/(b*(a + b*x^2)^(1/4)) + (c*(c*x)^(3/2)*(a + b*x^2)
^(3/4))/(3*b) - (a^(3/2)*c^2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcC
ot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[x*(Sqrt[
c*x]/(a + b*x^2)^(1/4)), x] - Dist[a/2, Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x]
```

```
, x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{\left(ac^2\right)\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx}{2b} \\
&= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} + \frac{\left(a^2c^2\right)\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx}{4b} \\
&= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} + \frac{\left(a^2c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{4b^2\sqrt[4]{a+bx^2}} \\
&= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{\left(a^2c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx\right.}{4b^2\sqrt[4]{a+bx^2}} \\
&= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{a^{3/2}c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 69, normalized size = 0.55

$$\frac{c(cx)^{3/2}\left(a+bx^2-a\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4,
3/4, 7/4, -((b*x^2)/a)]))/(3*b*(a + b*x^2)^(1/4))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{(b\,x^{2}+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^2*x^2/(b*x^2 + a)^(1/4), x)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 3.99, size = 44, normalized size = 0.35

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(11/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^{5/2}}{{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(1/4), x)
```

3.951 $\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}+\frac{\sqrt{a}\,\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

```
[Out] x*(c*x)^(1/2)/(b*x^2+a)^(1/4)+(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcc
ot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {320, 290, 342, 202}

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}+\frac{\sqrt{a}\,\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(1/4),x]
```

```
[Out] (x*Sqrt[c*x])/(a + b*x^2)^(1/4) + (Sqrt[a]*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*
EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[x*(Sqrt[
c*x]/(a + b*x^2)^(1/4)), x] - Dist[a/2, Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x]
, x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} - \frac{1}{2}a\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx \\
&= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} - \frac{\left(a\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{2b\sqrt[4]{a+bx^2}} \\
&= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} + \frac{\left(a\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2b\sqrt[4]{a+bx^2}} \\
&= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} + \frac{\sqrt{a}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 56, normalized size = 0.67

$$\frac{2x\sqrt{cx}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(1/4),x]
[Out] (2*x*Sqrt[c*x]*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*
x^2)/a)])/(3*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 44, normalized size = 0.53

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(1/4)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(1/4),x)
[Out] int((c*x)^(1/2)/(a + b*x^2)^(1/4), x)
```

3.952 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=90

$$-\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}+\frac{2\sqrt{b}\,\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a+bx^2}}$$

```
[Out] -2/c/(b*x^2+a)^(1/4)/(c*x)^(1/2)+2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1
/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2
*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(c*x)^(1/2)/c^2/(b*x^2+a)^(1/4
)/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {322, 290, 342, 202}

$$\frac{2\sqrt{b}\,\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] -2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (2*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt[
c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^
(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 322

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Dist[b/c^2, Int[Sqrt[c*x]/(a + b*x^
```

```
2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx &= -\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}-\frac{b\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx}{c^2}\\
&= -\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}-\frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{c^2\sqrt[4]{a+bx^2}}\\
&= -\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}+\frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{c^2\sqrt[4]{a+bx^2}}\\
&= -\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}+\frac{2\sqrt{b}\,\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 54, normalized size = 0.60

$$-\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x^2)/a)]
)/((c*x)^(3/2)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\,(b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x)
[Out] int(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.08, size = 31, normalized size = 0.34

$$-\frac{{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{\sqrt[4]{b}\,c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(1/4),x)
[Out] -hyper((1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(1/4)*c**(3/2)*x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x)

[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(1/4)), x)

3.953 $\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=126

$$\frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}-\frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}}-\frac{4b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}$$

```
[Out] -2/5*(b*x^2+a)^(3/4)/a/c/(c*x)^(5/2)+4/5*b/a/c^3/(b*x^2+a)^(1/4)/(c*x)^(1/2
)-4/5*b^(3/2)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2
)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(
1/2))),2^(1/2))*(c*x)^(1/2)/a^(3/2)/c^4/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {331, 322, 290, 342, 202}

$$-\frac{4b^{3/2}\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}+\frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}-\frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*b)/(5*a*c^3*Sqrt[c*x]*(a + b*x^2)^(1/4)) - (2*(a + b*x^2)^(3/4))/(5*a*c*
(c*x)^(5/2)) - (4*b^(3/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*c^4*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 322

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Dist[b/c^2, Int[Sqrt[c*x]/(a + b*x^
```

```
2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}} - \frac{(2b)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx}{5ac^2}$$

$$= \frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}} + \frac{(2b^2)\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx}{5ac^4}$$

$$= \frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}} + \frac{\left(2b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{5ac^4\sqrt[4]{a+bx^2}}$$

$$= \frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}} - \frac{\left(2b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\right.}{5ac^4\sqrt[4]{a+bx^2}}$$

$$= \frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}} - \frac{4b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right.}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.44

$$-\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, -((b*x^2)/a)
])/(5*(c*x)^(7/2)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{7}{2}} \left(b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^4*x^6 + a*c^4*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.11, size = 34, normalized size = 0.27

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{a e^{i\pi}}{b x^{2}}} \right)}{3 \sqrt[4]{b}\, c^{\frac{7}{2}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**2))/(3*b**(1/4)*c**(7/2)
*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(1/4)), x)
```

3.954 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=157

$$-\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}}+\frac{4b(a+bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}}+\frac{8b^{5/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a+bx^2}}$$

```
[Out] -2/9*(b*x^2+a)^(3/4)/a/c/(c*x)^(9/2)+4/15*b*(b*x^2+a)^(3/4)/a^2/c^3/(c*x)^(
5/2)-8/15*b^2/a^2/c^5/(b*x^2+a)^(1/4)/(c*x)^(1/2)+8/15*b^(5/2)*(1+a/b/x^2)^
(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)
/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*x)^(1/2
)/a^(5/2)/c^6/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.05, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {331, 322, 290, 342, 202}

$$\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a+bx^2}}-\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{4b(a+bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-8*b^2)/(15*a^2*c^5*Sqrt[c*x]*(a + b*x^2)^(1/4)) - (2*(a + b*x^2)^(3/4))/(
9*a*c*(c*x)^(9/2)) + (4*b*(a + b*x^2)^(3/4))/(15*a^2*c^3*(c*x)^(5/2)) + (8*
b^(5/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(15*a^(5/2)*c^6*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 322

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Dist[b/c^2, Int[Sqrt[c*x]/(a + b*x^
```

```
2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx &= -\frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}} - \frac{(2b)\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx}{3ac^2} \\
&= -\frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b(a+bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{(4b^2)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx}{15a^2c^4} \\
&= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b(a+bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{(4b^3)\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx}{15a^2c^6} \\
&= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b(a+bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{\left(4b^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{c \cdots\right.}{15a^2c^6\sqrt[4]{\cdots}} \\
&= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b(a+bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{\left(4b^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{c \cdots\right.}{1\cdots} \\
&= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b(a+bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{8b^{5/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{c \cdots}}{15a^{5/2}\cdots}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 56, normalized size = 0.36

$$-\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x]
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, -((b*x^2)/a)
])/(9*(c*x)^(11/2)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{11}{2}}\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x)
[Out] int(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^6*x^8 + a*c^6*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 73.09, size = 34, normalized size = 0.22

$$-\frac{{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{ae^{i\pi}}{bx^2}\right)}{5\sqrt[4]{b}\,c^{\frac{11}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((1/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(1/4)*c**(11/2
)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{11/2}\,{(b\,x^2+a)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(11/2)*(a + b*x^2)^(1/4)), x)
```

3.955 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=308

$$-\frac{c\sqrt{cx}\,(a-bx^2)^{3/4}}{2b}-\frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}-\frac{ac^{3/2}\log\left(\sqrt{}\right.}{}$$

```
[Out] 1/8*a*c^(3/2)*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2
))/b^(5/4)*2^(1/2)+1/8*a*c^(3/2)*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x
^2+a)^(1/4)/c^(1/2))/b^(5/4)*2^(1/2)-1/16*a*c^(3/2)*ln(c^(1/2)-b^(1/4)*2^(1
/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(5/4
)*2^(1/2)+1/16*a*c^(3/2)*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^
(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(5/4)*2^(1/2)-1/2*c*(-b*x^2+a)^
(3/4)*(c*x)^(1/2)/b
```

Rubi [A]
time = 0.20, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {327, 335, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{ac^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}+1\right)}{4\sqrt{2}\,b^{5/4}}-\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}-\frac{c\sqrt{cx}\,(a-bx^2)^{3/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] -1/2*(c*Sqrt[c*x]*(a - b*x^2)^(3/4))/b - (a*c^(3/2)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(5/4)) + (a*c^(3
/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4
*Sqrt[2]*b^(5/4)) - (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b
*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(5/4))
 + (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*
b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = -\frac{c\sqrt{cx}\,(a-bx^2)^{3/4}}{2b}+\frac{(ac^2)\int\frac{1}{\sqrt{cx}\,\sqrt[4]{a-bx^2}}\,dx}{4b}$$

$$= -\frac{c\sqrt{cx}\,(a-bx^2)^{3/4}}{2b}+\frac{(ac)\text{Subst}\left(\int\frac{1}{\sqrt[4]{a-\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{2b}$$

$$= -\frac{c\sqrt{cx}\,(a-bx^2)^{3/4}}{2b}+\frac{(ac)\text{Subst}\left(\int\frac{1}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2b}$$

$$= -\frac{c\sqrt{cx}\,(a-bx^2)^{3/4}}{2b}+\frac{a\text{Subst}\left(\int\frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b}+\frac{a\text{Subst}\left(\int\frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx\right.}{4b}$$

$$= -\frac{c\sqrt{cx}\,(a-bx^2)^{3/4}}{2b}-\frac{(ac^{3/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\,\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{5/4}}-\frac{(ac}{}$$

$$= -\frac{c\sqrt{cx}\,(a-bx^2)^{3/4}}{2b}-\frac{ac^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\log\left(\sqrt{c}\right.}{}$$

$$= -\frac{c\sqrt{cx}\,(a-bx^2)^{3/4}}{2b}-\frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt{c}\,\sqrt[4]{a-}}\right.}{4\sqrt{2}\,b^{5/4}}$$

Mathematica [A]
time = 0.50, size = 161, normalized size = 0.52

$$\frac{(cx)^{3/2}\left(-4\sqrt[4]{b}\,\sqrt{x}\,(a-bx^2)^{3/4}+\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\sqrt[4]{a-bx^2}}{-\sqrt{b}\,x+\sqrt{a-bx^2}}\right)+\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{b}\,x+\sqrt{a-bx^2}}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\sqrt[4]{a-bx^2}}\right)\right)}{8b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(-4*b^(1/4)*Sqrt[x]*(a - b*x^2)^(3/4) + Sqrt[2]*a*ArcTan[(Sqrt
[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*x) + Sqrt[a - b*x^2])] +
Sqrt[2]*a*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1/4)*Sqrt[x]*(a
 - b*x^2)^(1/4))]))/(8*b^(5/4)*x^(3/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{(-b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

Fricas [A]
time = 1.41, size = 340, normalized size = 1.10

$$-\frac{4\,(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,c+4\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,ab^4c-(b^5x^2-ab^4)\left(-\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{-\frac{\sqrt{-bx^2+a}\,a^2c^3x-\sqrt{-\frac{a^4c^6}{b^5}}\,(b^3x^2-ab^2)}{bx^2-a}}}{a^4bc^6x^2-a^5c^6}\right)+\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\frac{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,ac+\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}(b^2x^2-ab)}{bx^2-a}\right)-\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\frac{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,ac-\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}(b^2x^2-ab)}{bx^2-a}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(-b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(-a^4*c^6/b^5)^(1/4)*b*arctan(-(
(-a^4*c^6/b^5)^(3/4)*(-b*x^2 + a)^(3/4)*sqrt(c*x)*a*b^4*c - (b^5*x^2 - a*b^
4)*(-a^4*c^6/b^5)^(3/4)*sqrt(-(sqrt(-b*x^2 + a)*a^2*c^3*x - sqrt(-a^4*c^6/b
^5)*(b^3*x^2 - a*b^2))/(b*x^2 - a)))/(a^4*b*c^6*x^2 - a^5*c^6)) + (-a^4*c^6
/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3/4)*sqrt(c*x)*a*c + (-a^4*c^6/b^5)^(1/4)*
(b^2*x^2 - a*b))/(b*x^2 - a)) - (-a^4*c^6/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3
/4)*sqrt(c*x)*a*c - (-a^4*c^6/b^5)^(1/4)*(b^2*x^2 - a*b))/(b*x^2 - a)))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 1.28, size = 46, normalized size = 0.15

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(2*I
*pi)/a)/(2*a**(1/4)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x\right)}^{3/2}}{{\left(a-b\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(a - b*x^2)^(1/4), x)
```

3.956 $\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=272

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+$$

```
[Out] 1/2*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))/b^(1/4)
*2^(1/2)/c^(1/2)+1/2*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/
c^(1/2))/b^(1/4)*2^(1/2)/c^(1/2)-1/4*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c*x)^(1/2)
/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(1/4)*2^(1/2)/c^(1/
2)+1/4*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^
(1/2)/(-b*x^2+a)^(1/2))/b^(1/4)*2^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt
[2]*b^(1/4)*Sqrt[c])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a
- b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c]) - Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]
*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqr
t[2]*b^(1/4)*Sqrt[c]) + Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqrt[2]*b^(1/4)*Sqrt[c])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a-bx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^2} + \frac{\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}+x^2}\,dx,\right.}{2\sqrt{b}}$$

$$= -\frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}}{\sqrt[4]{a-bx^2}}\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} - \frac{\log\left(\sqrt{c}+\frac{\sqrt{b}}{\sqrt{a-}}\right.}{2\sqrt{2}}$$

Mathematica [A]
time = 0.31, size = 125, normalized size = 0.46

$$\frac{\sqrt{x}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{-\sqrt{b}\,x+\sqrt{a-bx^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{b}\,x+\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}\right)\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
[Out] (Sqrt[x]*(ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*x)
+ Sqrt[a - b*x^2])] + ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1/4
)*Sqrt[x]*(a - b*x^2)^(1/4))]))/(Sqrt[2]*b^(1/4)*Sqrt[c*x])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Fricas [A]
time = 1.22, size = 267, normalized size = 0.98

$$-2\left(-\frac{1}{bc^2}\right)^{\frac14}\arctan\left(-\frac{(-bx^2+a)^{\frac34}\sqrt{cx}\,bc\left(-\frac{1}{bc^2}\right)^{\frac34}-(b^2cx^2-abc)\sqrt{-\frac{\sqrt{-bx^2+a}\,cx-(bc^2x^2-ac^2)\sqrt{-\frac{1}{bc^2}}}{bx^2-a}}\left(-\frac{1}{bc^2}\right)^{\frac34}}{bx^2-a}\right)-\frac12\left(-\frac{1}{bc^2}\right)^{\frac14}\log\left(\frac{(-bx^2+a)^{\frac34}\sqrt{cx}+(bcx^2-ac)\left(-\frac{1}{bc^2}\right)^{\frac14}}{bx^2-a}\right)+\frac12\left(-\frac{1}{bc^2}\right)^{\frac14}\log\left(\frac{(-bx^2+a)^{\frac34}\sqrt{cx}-(bcx^2-ac)\left(-\frac{1}{bc^2}\right)^{\frac14}}{bx^2-a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2*(-1/(b*c^2))^(1/4)*arctan(-((-b*x^2 + a)^(3/4)*sqrt(c*x)*b*c*(-1/(b*c^2)
)^(3/4) - (b^2*c*x^2 - a*b*c)*sqrt(-(sqrt(-b*x^2 + a)*c*x - (b*c^2*x^2 - a*
c^2)*sqrt(-1/(b*c^2)))/(b*x^2 - a))*(-1/(b*c^2))^(3/4))/(b*x^2 - a)) - 1/2*
(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 - a*c)*(-1/
(b*c^2))^(1/4))/(b*x^2 - a)) + 1/2*(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/
4)*sqrt(c*x) - (b*c*x^2 - a*c)*(-1/(b*c^2))^(1/4))/(b*x^2 - a))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.81, size = 46, normalized size = 0.17

$$\frac{\sqrt{x}\,\Gamma\left(\frac14\right){}_2F_1\left(\begin{matrix}\frac14,\frac14\\\frac54\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\sqrt{c}\,\Gamma\left(\frac54\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(2*I*pi)/a)/(2
*a**(1/4)*sqrt(c)*gamma(5/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x}\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a - b*x^2)^(1/4)),x)
[Out] int(1/((c*x)^(1/2)*(a - b*x^2)^(1/4)), x)
```

3.957 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

```
[Out] -2/3*(-b*x^2+a)^(3/4)/a/c/(c*x)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{270\}$

$$-\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

Mathematica [A]
time = 0.14, size = 27, normalized size = 0.93

$$-\frac{2x(a-bx^2)^{3/4}}{3a(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

Maple [A]
time = 0.05, size = 22, normalized size = 0.76

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}{3a\left(cx\right)^{\frac{5}{2}}}$	22
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}{3c^{2}\sqrt{cx}\;ax}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
[Out] -2/3*x*(-b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Fricas [A]
time = 1.29, size = 26, normalized size = 0.90

$$-\frac{2\left(-bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\,ac^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/3*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.19, size = 88, normalized size = 3.03

$$\begin{cases} \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^{2}}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{for}\: \left|\frac{a}{bx^{2}}\right| > 1 \\ -\frac{b^{\frac{3}{4}}\left(-\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Piecewise((b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma
(1/4)), Abs(a/(b*x**2)) > 1), (-b**(3/4)*(-a/(b*x**2) + 1)**(3/4)*exp(-I*pi
/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Mupad [B]
time = 5.12, size = 26, normalized size = 0.90

$$-\frac{2\,{\left(a-b\,x^2\right)}^{3/4}}{3\,a\,c^2\,x\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] -(2*(a - b*x^2)^(3/4))/(3*a*c^2*x*(c*x)^(1/2))
```

3.958 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{7/2}}+\frac{8(a-bx^2)^{7/4}}{21a^2c(cx)^{7/2}}$$

```
[Out] -2/3*(-b*x^2+a)^(3/4)/a/c/(c*x)^(7/2)+8/21*(-b*x^2+a)^(7/4)/a^2/c/(c*x)^(7/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {279, 270}

$$\frac{8(a-bx^2)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a - b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx &= -\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{7/2}}-\frac{4\int \frac{(a-bx^2)^{3/4}}{(cx)^{9/2}}\,dx}{3a}\\ &= -\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{7/2}}+\frac{8(a-bx^2)^{7/4}}{21a^2c(cx)^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 37, normalized size = 0.63

$$-\frac{2x(a-bx^2)^{3/4}(3a+4bx^2)}{21a^2(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
[Out] (-2*x*(a - b*x^2)^(3/4)*(3*a + 4*b*x^2))/(21*a^2*(c*x)^(9/2))
```

Maple [A]
time = 0.05, size = 32, normalized size = 0.54

method	result	size
gosper	$-\frac{2x\left(-b\,x^2+a\right)^{\frac{3}{4}}\left(4b\,x^2+3a\right)}{21a^2\left(cx\right)^{\frac{9}{2}}}$	32
risch	$-\frac{2\left(-b\,x^2+a\right)^{\frac{3}{4}}\left(4b\,x^2+3a\right)}{21c^4\sqrt{cx}\,a^2x^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
[Out] -2/21*x*(-b*x^2+a)^(3/4)*(4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Fricas [A]
time = 1.44, size = 36, normalized size = 0.61

$$-\frac{2\,(4\,bx^2+3\,a)(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{21\,a^2c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/21*(4*b*x^2 + 3*a)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 25.97, size = 343, normalized size = 5.81

$$\begin{cases} -\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{8ac^{\frac{9}{2}}x^2\Gamma\left(\frac{1}{4}\right)} - \frac{b^{\frac{7}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2a^2c^{\frac{9}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^2}\right| > 1 \\ -\frac{3a^2b^{\frac{7}{4}}\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^3bc^{\frac{9}{2}}x^2e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^2b^2c^{\frac{9}{2}}x^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} - \frac{ab^{\frac{11}{4}}x^2\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^3bc^{\frac{9}{2}}x^2e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^2b^2c^{\frac{9}{2}}x^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} + \frac{4b^{\frac{15}{4}}x^4\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^3bc^{\frac{9}{2}}x^2e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^2b^2c^{\frac{9}{2}}x^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Piecewise((-3*b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-7/4)/(8*a*c**(9/2)*x*
*2*gamma(1/4)) - b**(7/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-7/4)/(2*a**2*c**(9
/2)*gamma(1/4)), Abs(a/(b*x**2)) > 1), (-3*a**2*b**(7/4)*(-a/(b*x**2) + 1)*
*(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1/4) + 8*a**2
*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)) - a*b**(11/4)*x**2*(-a/(b*x**2)
 + 1)**(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1/4) +
8*a**2*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)) + 4*b**(15/4)*x**4*(-a/(b
*x**2) + 1)**(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1
/4) + 8*a**2*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Mupad [B]
time = 5.14, size = 41, normalized size = 0.69

$$-\frac{(a-b\,x^2)^{3/4}\left(\frac{2}{7\,a\,c^4}+\frac{8\,b\,x^2}{21\,a^2\,c^4}\right)}{x^3\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] -((a - b*x^2)^(3/4)*(2/(7*a*c^4) + (8*b*x^2)/(21*a^2*c^4)))/(x^3*(c*x)^(1/2
))
```

3.959 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=88

$$-\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{11/2}}+\frac{16(a-bx^2)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{64(a-bx^2)^{11/4}}{231a^3c(cx)^{11/2}}$$

```
[Out] -2/3*(-b*x^2+a)^(3/4)/a/c/(c*x)^(11/2)+16/21*(-b*x^2+a)^(7/4)/a^2/c/(c*x)^(
11/2)-64/231*(-b*x^2+a)^(11/4)/a^3/c/(c*x)^(11/2)
```

Rubi [A]
time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {279, 270}

$$-\frac{64(a-bx^2)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16(a-bx^2)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a - b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a - b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{11/2}} - \frac{8\int \frac{(a-bx^2)^{3/4}}{(cx)^{13/2}}\,dx}{3a}$$
$$= -\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{11/2}} + \frac{16(a-bx^2)^{7/4}}{21a^2c(cx)^{11/2}} + \frac{32\int \frac{(a-bx^2)^{7/4}}{(cx)^{13/2}}\,dx}{21a^2}$$
$$= -\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{11/2}} + \frac{16(a-bx^2)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{64(a-bx^2)^{11/4}}{231a^3c(cx)^{11/2}}$$

Mathematica [A]
time = 0.19, size = 48, normalized size = 0.55

$$-\frac{2x(a-bx^2)^{3/4}\left(21a^2+24abx^2+32b^2x^4\right)}{231a^3(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4)*(21*a^2 + 24*a*b*x^2 + 32*b^2*x^4))/(231*a^3*(c*x)^
(13/2))
```

Maple [A]
time = 0.12, size = 43, normalized size = 0.49

method	result	size
gosper	$-\frac{2x(-bx^2+a)^{\frac{3}{4}}(32b^2x^4+24abx^2+21a^2)}{231a^3(cx)^{\frac{13}{2}}}$	43
risch	$-\frac{2(-bx^2+a)^{\frac{3}{4}}(32b^2x^4+24abx^2+21a^2)}{231c^6\sqrt{cx}\,a^3x^5}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231*x*(-b*x^2+a)^(3/4)*(32*b^2*x^4+24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Fricas [A]
time = 1.32, size = 47, normalized size = 0.53

$$-\frac{2\,(32\,b^2x^4+24\,abx^2+21\,a^2)(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{231\,a^3c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/231*(32*b^2*x^4 + 24*a*b*x^2 + 21*a^2)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3
*c^7*x^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3279 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Mupad [B]
time = 5.25, size = 55, normalized size = 0.62

$$-\frac{{\left(a-b\,x^2\right)}^{3/4}\,\left(\frac{2}{11\,a\,c^6}+\frac{16\,b\,x^2}{77\,a^2\,c^6}+\frac{64\,b^2\,x^4}{231\,a^3\,c^6}\right)}{x^5\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] -((a - b*x^2)^(3/4)*(2/(11*a*c^6) + (16*b*x^2)/(77*a^2*c^6) + (64*b^2*x^4)/
(231*a^3*c^6)))/(x^5*(c*x)^(1/2))
```

3.960 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=128

$$-\frac{ac^3(a-bx^2)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\,(a-bx^2)^{3/4}}{3b}+\frac{a^{3/2}c^2\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}$$

```
[Out] -1/3*c*(c*x)^(3/2)*(-b*x^2+a)^(3/4)/b-1/2*a*c^3*(-b*x^2+a)^(3/4)/b^2/(c*x)^
(1/2)+1/2*a^(3/2)*c^2*(1-a/b/x^2)^(1/4)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccsc(x*b^(
1/2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/b^(3/2)/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {327, 321, 323, 342, 234}

$$\frac{a^{3/2}c^2\sqrt{cx}\,\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}-\frac{ac^3(a-bx^2)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\,(a-bx^2)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] -1/2*(a*c^3*(a - b*x^2)^(3/4))/(b^2*Sqrt[c*x]) - (c*(c*x)^(3/2)*(a - b*x^2)
^(3/4))/(3*b) + (a^(3/2)*c^2*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcC
sc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(3/2)*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[c*((a +
b*x^2)^(3/4)/(b*Sqrt[c*x])), x] + Dist[a*(c^2/(2*b)), Int[1/((c*x)^(3/2)*(a
 + b*x^2)^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 323

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a
```

```
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx &= -\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{\left(ac^2\right)\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx}{2b}\\
&= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}-\frac{\left(a^2c^4\right)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{4b^2}\\
&= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}-\frac{\left(a^2c^2\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{4b^2\sqrt[4]{a-bx^2}}\\
&= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{\left(a^2c^2\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{1-\ldots}}\ldots\right)}{4b^2\sqrt[4]{a-bx^2}}\\
&= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{a^{3/2}c^2\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\ldots\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 71, normalized size = 0.55

$$\frac{c(cx)^{3/2}\left(-a+bx^2+a\sqrt[4]{1-\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, (b*x^2)/a]))/(3*b*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{(-b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2/(b*x^2 - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.98, size = 46, normalized size = 0.36

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**2*exp_polar(2*
I*pi)/a)/(2*a**(1/4)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{(a-b\,x^2)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)/(a - b*x^2)^(1/4), x)
```

3.961 $\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=90

$$-\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}+\frac{\sqrt{a}\,\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

```
[Out] -c*(-b*x^2+a)^(3/4)/b/(c*x)^(1/2)+(1-a/b/x^2)^(1/4)*(cos(1/2*arccsc(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*
arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/b^
(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {321, 323, 342, 234}

$$\frac{\sqrt{a}\,\sqrt{cx}\,\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^2}}-\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a - b*x^2)^(1/4),x]
```

```
[Out] -((c*(a - b*x^2)^(3/4))/(b*Sqrt[c*x])) + (Sqrt[a]*(1 - a/(b*x^2))^(1/4)*Sqr
t[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a - b*x^2)^(1
/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[c*((a +
b*x^2)^(3/4)/(b*Sqrt[c*x])), x] + Dist[a*(c^2/(2*b)), Int[1/((c*x)^(3/2)*(a
 + b*x^2)^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 323

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a
```

```
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx &= -\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}} - \frac{(ac^2)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{2b} \\
&= -\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}} - \frac{\left(a\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{2b\sqrt[4]{a-bx^2}} \\
&= -\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}} + \frac{\left(a\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x}\right)}{2b\sqrt[4]{a-bx^2}} \\
&= -\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}} + \frac{\sqrt{a}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.63

$$\frac{2x\sqrt{cx}\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)}{3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a - b*x^2)^(1/4),x]
[Out] (2*x*Sqrt[c*x]*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (b*x^
2)/a])/(3*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(-b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/2)/(-b*x^2+a)^(1/4),x)

[Out] int((c*x)^(1/2)/(-b*x^2+a)^(1/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")

[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")

[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*x^2 - a), x)

Sympy [C] Result contains complex when optimal does not.
time = 0.56, size = 46, normalized size = 0.51

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(1/2)/(-b*x**2+a)**(1/4),x)

[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**2*exp_polar(2*I*pi)/a)/(2*a**(1/4)*gamma(7/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{{\left(a-b\,x^2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a - b*x^2)^(1/4),x)
[Out] int((c*x)^(1/2)/(a - b*x^2)^(1/4), x)
```

3.962 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{b}\,\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a-bx^2}}$$

```
[Out] -2*(1-a/b/x^2)^(1/4)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*a
rccsc(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1
/2))*b^(1/2)*(c*x)^(1/2)/c^2/(-b*x^2+a)^(1/4)/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {323, 342, 234}

$$-\frac{2\sqrt{b}\,\sqrt{cx}\,\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[b]*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sq
rt[a]]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 323

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = \frac{\left(\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{c^2\sqrt[4]{a-bx^2}}$$

$$= -\frac{\left(\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x}\right)}{c^2\sqrt[4]{a-bx^2}}$$

$$= -\frac{2\sqrt{b}\,\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{\sqrt{a}\,c^2\sqrt[4]{a-bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 55, normalized size = 0.81

$$-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x]
[Out] (-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (b*x^2)/a])/(
(c*x)^(3/2)*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
[Out] int(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^2*x^4 - a*c^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.15, size = 32, normalized size = 0.47

$$\frac{ie^{\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{\sqrt[4]{b}\,c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(1/4),x)
[Out] I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), a/(b*x**2))/(b**(1/4)*c**(3/2)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{3/2}\,{\left(a-b\,x^2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x)
[Out] int(1/((c*x)^(3/2)*(a - b*x^2)^(1/4)), x)
```

3.963 $\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}-\frac{4b^{3/2}\sqrt[4]{1-\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}$$

```
[Out] -2/5*(-b*x^2+a)^(3/4)/a/c/(c*x)^(5/2)-4/5*b^(3/2)*(1-a/b/x^2)^(1/4)*(cos(1/
2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*El
lipticE(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/a^(3/2)/c^4
/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 323, 342, 234}

$$-\frac{4b^{3/2}\sqrt{cx}\,\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}-\frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(5*a*c*(c*x)^(5/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(1/
4)*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*c^4*(a
 - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 323

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx &= -\frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}+\frac{(2b)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{5ac^2}\\
&= -\frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}+\frac{\left(2b\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{5ac^4\sqrt[4]{a-bx^2}}\\
&= -\frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}-\frac{\left(2b\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx,x,\frac{1}{x}\right)}{5ac^4\sqrt[4]{a-bx^2}}\\
&= -\frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}-\frac{4b^{3/2}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.57

$$-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x]
[Out] (-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, (b*x^2)/a])/
(5*(c*x)^(7/2)*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{7}{2}} (-b\,x^{2}+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^4*x^6 - a*c^4*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.09, size = 39, normalized size = 0.39

$$-\frac{ie^{-\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{a}{bx^{2}}\right)}{3\sqrt[4]{b}\,c^{\frac{7}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] -I*exp(-3*I*pi/4)*hyper((1/4, 3/2), (5/2,), a/(b*x**2))/(3*b**(1/4)*c**(7/2
)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{7/2}\,(a-b\,x^2)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(7/2)*(a - b*x^2)^(1/4)), x)
```

3.964 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=130

$$-\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b(a-bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{8b^{5/2}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}$$

```
[Out] -2/9*(-b*x^2+a)^(3/4)/a/c/(c*x)^(9/2)-4/15*b*(-b*x^2+a)^(3/4)/a^2/c^3/(c*x)
^(5/2)-8/15*b^(5/2)*(1-a/b/x^2)^(1/4)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2
)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccsc(x*b^(1/
2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/a^(5/2)/c^6/(-b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {331, 323, 342, 234}

$$-\frac{8b^{5/2}\sqrt{cx}\,\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}-\frac{4b(a-bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(9*a*c*(c*x)^(9/2)) - (4*b*(a - b*x^2)^(3/4))/(15*a^
2*c^3*(c*x)^(5/2)) - (8*b^(5/2)*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[A
rcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(5/2)*c^6*(a - b*x^2)^(1/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 323

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx &= -\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}+\frac{(2b)\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx}{3ac^2}\\ &= -\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b(a-bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}}+\frac{(4b^2)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{15a^2c^4}\\ &= -\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b(a-bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}}+\frac{\left(4b^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{15a^2c^6\sqrt[4]{a-bx^2}}\\ &= -\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b(a-bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{\left(4b^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\right.}{15a^2c^6\sqrt[4]{a-bx^2}}\\ &= -\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b(a-bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{8b^{5/2}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.44

$$-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, (b*x^2)/a])/
(9*(c*x)^(11/2)*(a - b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{11}{2}} \left(-b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^6*x^8 - a*c^6*x^6), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 72.84, size = 36, normalized size = 0.28

$$\frac{ie^{\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle|\,\frac{a}{bx^{2}}\right)}{5\sqrt[4]{b}\,c^{\frac{11}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] I*exp(I*pi/4)*hyper((1/4, 5/2), (7/2,), a/(b*x**2))/(5*b**(1/4)*c**(11/2)*x
**5)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{11/2}\,(a-b\,x^2)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x)
[Out] int(1/((c*x)^(11/2)*(a - b*x^2)^(1/4)), x)
```

3.965 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=86

$$\frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}+\frac{\sqrt{a}\,\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\left(a+bx^2\right)^{3/4}}$$

```
[Out] (1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/
cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/
2))),2^(1/2))*a^(1/2)/(b*x^2+a)^(3/4)/b^(1/2)+c*(b*x^2+a)^(1/4)*(c*x)^(1/2)
/b
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {327, 335, 243, 342, 281, 237}

$$\frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}+\frac{\sqrt{a}\,(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(1/4))/b + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3
/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(3/4)
)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx = \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b} - \frac{(ac^2)\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx}{2b}$$

$$= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b} - \frac{(ac)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{b}$$

$$= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b} - \frac{\left(ac\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{b\,(a+bx^2)^{3/4}}$$

$$= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b} + \frac{\left(ac\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{b\,(a+bx^2)^{3/4}}$$

$$= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b} + \frac{\left(ac\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{2b\,(a+bx^2)^{3/4}}$$

$$= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b} + \frac{\sqrt{a}\,\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{\sqrt{b}\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 66, normalized size = 0.77

$$\frac{c\sqrt{cx}\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{b\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2 - a*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/
4, 5/4, -((b*x^2)/a)]))/(b*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c*x/(b*x^2 + a)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.09, size = 44, normalized size = 0.51

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(3/4)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(3/4),x)
[Out] int((c*x)^(3/2)/(a + b*x^2)^(3/4), x)
```

3.966 $\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt{b}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^
(1/2))),2^(1/2))*b^(1/2)/c^2/(b*x^2+a)^(3/4)/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {335, 243, 342, 281, 237}

$$-\frac{2\sqrt{b}\,(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\left(2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{c\,(a+bx^2)^{3/4}}$$

$$= -\frac{\left(2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{c\,(a+bx^2)^{3/4}}$$

$$= -\frac{\left(\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{c\,(a+bx^2)^{3/4}}$$

$$= -\frac{2\sqrt{b}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.82

$$\frac{2x\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)])/
(Sqrt[c*x]*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.99, size = 31, normalized size = 0.47

$$-\frac{{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{b^{\frac{3}{4}}\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/4),x)
[Out] -hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*sqrt(c)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x}\,{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(3/4)), x)
```

3.967 $\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=97

$$-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}+\frac{4b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2/3*(b*x^2+a)^(1/4)/a/c/(c*x)^(3/2)+4/3*b^(3/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3
/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a
^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/c^4/
(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {331, 335, 243, 342, 281, 237}

$$\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3/
4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*c^4*
(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} - \frac{(2b)\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx}{3ac^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} - \frac{(4b)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{3ac^3}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} - \frac{\left(4b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{3ac^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} + \frac{\left(4b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{3ac^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} + \frac{\left(2b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{3ac^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} + \frac{4b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 56, normalized size = 0.58

$$-\frac{2x\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x^2)/a)]
)/(3*(c*x)^(5/2)*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^3*x^5 + a*c^3*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.83, size = 48, normalized size = 0.49

$$\frac{\Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{2a^{\frac{3}{4}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{5/2}\,{\left(b\,x^2+a\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(5/2)*(a + b*x^2)^(3/4)), x)
```

3.968 $\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}+\frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{8b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2/7*(b*x^2+a)^(1/4)/a/c/(c*x)^(7/2)+4/7*b*(b*x^2+a)^(1/4)/a^2/c^3/(c*x)^(3
/2)-8/7*b^(5/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot
(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/c^6/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {331, 335, 243, 342, 281, 237}

$$-\frac{8b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a+bx^2\right)^{3/4}}+\frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) + (4*b*(a + b*x^2)^(1/4))/(7*a^2
*c^3*(c*x)^(3/2)) - (8*b^(5/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[
ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^(5/2)*c^6*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{9/2}\,(a+bx^2)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} - \frac{(6b)\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/4}}\,dx}{7ac^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{(4b^2)\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx}{7a^2c^4}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{(8b^2)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{7a^2c^5}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{\left(8b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x}\right.}{7a^2c^5\,(a+bx^2)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{\left(8b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/}}\right.}{7a^2c^5\,(a+bx^2)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{\left(4b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/}}\right.}{7a^2c^5\,(a+bx^2)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{8b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right|}{7a^{5/2}c^6\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.44

$$-\frac{2x\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x^2)/a)
])/(7*(c*x)^(9/2)*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{9}{2}}\,(b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x)

[Out] int(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^5*x^7 + a*c^5*x^5), x)

Sympy [C] Result contains complex when optimal does not.
time = 52.37, size = 34, normalized size = 0.27

$$-\frac{{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{5b^{\frac{3}{4}}c^{\frac{9}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(3/4),x)

[Out] -hyper((3/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(3/4)*c**(9/2)*x**5)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="giac")

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{9/2}\,{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(9/2)*(a + b*x^2)^(3/4)), x)
```

3.969 $\int \frac{1}{(cx)^{13/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=157

$$-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{80b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}c^8\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2/11*(b*x^2+a)^(1/4)/a/c/(c*x)^(11/2)+20/77*b*(b*x^2+a)^(1/4)/a^2/c^3/(c*x
)^(7/2)-40/77*b^2*(b*x^2+a)^(1/4)/a^3/c^5/(c*x)^(3/2)+80/77*b^(7/2)*(1+a/b/
x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2
*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^
(1/2))/a^(7/2)/c^8/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {331, 335, 243, 342, 281, 237}

$$\frac{80b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}c^8\left(a+bx^2\right)^{3/4}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) + (20*b*(a + b*x^2)^(1/4))/(77
*a^2*c^3*(c*x)^(7/2)) - (40*b^2*(a + b*x^2)^(1/4))/(77*a^3*c^5*(c*x)^(3/2))
 + (80*b^(7/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*c^8*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} - \frac{(10b)\int \frac{1}{(cx)^{9/2}(a+bx^2)^{3/4}}\,dx}{11ac^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} + \frac{(60b^2)\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/4}}\,dx}{77a^2c^4}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{(40b^3)\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx}{77a^3c^6}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{(80b^3)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\right.}{77a^3c^7}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{\left(80b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)}{77a^3}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{\left(80b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)}{77a^3}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{\left(40b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)}{77a^3c^7}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{80b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F}{77a^{7/2}c^8\,(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.36

$$-\frac{2x\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{11}{4},\frac{3}{4};-\frac{7}{4};-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, -((b*x^2)/a
)])/(11*(c*x)^(13/2)*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{13}{2}} (b\,x^2+a)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x)
[Out] int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^7*x^9 + a*c^7*x^7), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(3/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{13/2}\,(b\,x^2+a)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] int(1/((c*x)^(13/2)*(a + b*x^2)^(3/4)), x)
```

3.970 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=117

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}+\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}$$

```
[Out] 1/2*c*(c*x)^(3/2)*(b*x^2+a)^(1/4)/b+3/4*a*c^(5/2)*arctan(b^(1/4)*(c*x)^(1/2
)/(b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)-3/4*a*c^(5/2)*arctanh(b^(1/4)*(c*x)^(1/2
)/(b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)
```

Rubi [A]

time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {327, 335, 338, 304, 211, 214}

$$\frac{3ac^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b) + (3*a*c^(5/2)*ArcTan[(b^(1/4)*Sqrt
[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4)) - (3*a*c^(5/2)*ArcTanh[(b^
(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx &= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac^2)\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx}{4b} \\
&= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac)\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2b} \\
&= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac)\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b} \\
&= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac^3)\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b^{3/2}} + \frac{(3ac^3)\,\text{Subst}\left(\int \frac{}{c+}\right.}{} \\
&= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} + \frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} - \frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right.}{4b^{7/4}}
\end{aligned}$$

Mathematica [A]

time = 0.32, size = 97, normalized size = 0.83

$$\frac{(cx)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}+3a\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] ((c*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4) + 3*a*ArcTan[(b^(1/4)*Sqr
t[x])/(a + b*x^2)^(1/4)] - 3*a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]
))/(4*b^(7/4)*x^(5/2))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{(b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 4.56, size = 44, normalized size = 0.38

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/4)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^{5/2}}{{\left(b\,x^2+a\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(3/4), x)
```

3.971 $\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

```
[Out] -arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)+arctan
h(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {335, 338, 304, 211, 214}

$$\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3/4)
) + (Sqrt[c]*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3
/4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx &= \frac{2\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}\\ &= \frac{2\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}\\ &= \frac{c\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}} - \frac{c\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}}\\ &= -\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 67, normalized size = 0.80

$$\frac{\sqrt{cx}\left(-\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{b^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
```

```
[Out] (Sqrt[c*x]*(-ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4)
*Sqrt[x])/(a + b*x^2)^(1/4)]))/(b^(3/4)*Sqrt[x])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 0.75, size = 44, normalized size = 0.52

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(3/4)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(3/4),x)
[Out] int((c*x)^(1/2)/(a + b*x^2)^(3/4), x)
```

3.972 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

```
[Out] -2*(b*x^2+a)^(1/4)/a/c/(c*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{270\}$

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\,(a+bx^2)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Mathematica [A]
time = 0.17, size = 24, normalized size = 0.92

$$-\frac{2x\sqrt[4]{a+bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]
[Out] (-2*x*(a + b*x^2)^(1/4))/(a*(c*x)^(3/2))
```

Maple [A]
time = 0.05, size = 21, normalized size = 0.81

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{a\left(cx\right)^{\frac{3}{2}}}$	21
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{ac\sqrt{cx}}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x*(b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Fricas [A]
time = 1.69, size = 25, normalized size = 0.96

$$-\frac{2\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{ac^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)
```

Sympy [A]
time = 1.69, size = 36, normalized size = 1.38

$$\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Mupad [B]
time = 4.90, size = 22, normalized size = 0.85

$$-\frac{2\,{\left(b\,x^2+a\right)}^{1/4}}{a\,c\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x)
[Out] -(2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(1/2))
```

3.973 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=55

$$-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}+\frac{8(a+bx^2)^{5/4}}{5a^2c(cx)^{5/2}}$$

```
[Out] -2*(b*x^2+a)^(1/4)/a/c/(c*x)^(5/2)+8/5*(b*x^2+a)^(5/4)/a^2/c/(c*x)^(5/2)
```

Rubi [A]

time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$\frac{8(a+bx^2)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}-\frac{4\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx}{a}\\ &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}+\frac{8(a+bx^2)^{5/4}}{5a^2c(cx)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 34, normalized size = 0.62

$$-\frac{2x(a-4bx^2)\sqrt[4]{a+bx^2}}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
[Out] (-2*x*(a - 4*b*x^2)*(a + b*x^2)^(1/4))/(5*a^2*(c*x)^(7/2))
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.53

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-4b\,x^{2}+a\right)}{5a^{2}\left(cx\right)^{\frac{7}{2}}}$	29
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-4b\,x^{2}+a\right)}{5c^{3}\sqrt{cx}\;a^{2}x^{2}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
[Out] -2/5*x*(b*x^2+a)^(1/4)*(-4*b*x^2+a)/a^2/(c*x)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Fricas [A]
time = 1.30, size = 35, normalized size = 0.64

$$\frac{2\,(4\,bx^2-a)(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{5\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] 2/5*(4*b*x^2 - a)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

Sympy [**A**]
time = 17.07, size = 78, normalized size = 1.42

$$-\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{8ac^{\frac{7}{2}}x^2\Gamma\left(\frac{3}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2a^2c^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(8*a*c**(7/2)*x**2*gamma(3/4)
) + b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a**2*c**(7/2)*gamma(3/4
))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Mupad [**B**]
time = 4.98, size = 40, normalized size = 0.73

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2}{5\,a\,c^3}-\frac{8\,b\,x^2}{5\,a^2\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(5*a*c^3) - (8*b*x^2)/(5*a^2*c^3)))/(x^2*(c*x)^(1/2)
)
```

3.974 $\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=83

$$-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}$$

```
[Out] -2*(b*x^2+a)^(1/4)/a/c/(c*x)^(9/2)+16/5*(b*x^2+a)^(5/4)/a^2/c/(c*x)^(9/2)-6
4/45*(b*x^2+a)^(9/4)/a^3/c/(c*x)^(9/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 270}

$$-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a + b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a + b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}(a+bx^2)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}} - \frac{8\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx}{a}$$
$$= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}} + \frac{16(a+bx^2)^{5/4}}{5a^2c(cx)^{9/2}} + \frac{32\int \frac{(a+bx^2)^{5/4}}{(cx)^{11/2}}\,dx}{5a^2}$$
$$= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}} + \frac{16(a+bx^2)^{5/4}}{5a^2c(cx)^{9/2}} - \frac{64(a+bx^2)^{9/4}}{45a^3c(cx)^{9/2}}$$

Mathematica [A]
time = 0.27, size = 47, normalized size = 0.57

$$-\frac{2x\sqrt[4]{a+bx^2}\,(5a^2-8abx^2+32b^2x^4)}{45a^3(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*(5*a^2 - 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*(c*x)^(11
/2))
```

Maple [A]
time = 0.05, size = 42, normalized size = 0.51

method	result	size
gosper	$-\frac{2x(bx^2+a)^{\frac{1}{4}}(32b^2x^4-8abx^2+5a^2)}{45a^3(cx)^{\frac{11}{2}}}$	42
risch	$-\frac{2(bx^2+a)^{\frac{1}{4}}(32b^2x^4-8abx^2+5a^2)}{45c^5\sqrt{cx}\,a^3x^4}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*x*(b*x^2+a)^(1/4)*(32*b^2*x^4-8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Fricas [A]
time = 1.38, size = 46, normalized size = 0.55

$$-\frac{2\left(32\,b^{2}x^{4}-8\,abx^{2}+5\,a^{2}\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^{3}c^{6}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*b^2*x^4 - 8*a*b*x^2 + 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c^6
*x^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(71) = 142.
time = 141.65, size = 483, normalized size = 5.82

$$\frac{5a^{4}b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^{5}b^{4}c^{\frac{11}{2}}x^{4}\Gamma\left(\frac{3}{4}\right)+64a^{4}b^{5}c^{\frac{11}{2}}x^{6}\Gamma\left(\frac{3}{4}\right)+32a^{3}b^{6}c^{\frac{11}{2}}x^{8}\Gamma\left(\frac{3}{4}\right)}+\frac{2a^{3}b^{\frac{21}{4}}x^{2}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^{5}b^{4}c^{\frac{11}{2}}x^{4}\Gamma\left(\frac{3}{4}\right)+64a^{4}b^{5}c^{\frac{11}{2}}x^{6}\Gamma\left(\frac{3}{4}\right)+32a^{3}b^{6}c^{\frac{11}{2}}x^{8}\Gamma\left(\frac{3}{4}\right)}+\frac{21a^{2}b^{\frac{25}{4}}x^{4}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^{5}b^{4}c^{\frac{11}{2}}x^{4}\Gamma\left(\frac{3}{4}\right)+64a^{4}b^{5}c^{\frac{11}{2}}x^{6}\Gamma\left(\frac{3}{4}\right)+32a^{3}b^{6}c^{\frac{11}{2}}x^{8}\Gamma\left(\frac{3}{4}\right)}+\frac{56ab^{\frac{29}{4}}x^{6}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^{5}b^{4}c^{\frac{11}{2}}x^{4}\Gamma\left(\frac{3}{4}\right)+64a^{4}b^{5}c^{\frac{11}{2}}x^{6}\Gamma\left(\frac{3}{4}\right)+32a^{3}b^{6}c^{\frac{11}{2}}x^{8}\Gamma\left(\frac{3}{4}\right)}+\frac{32b^{\frac{33}{4}}x^{8}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^{5}b^{4}c^{\frac{11}{2}}x^{4}\Gamma\left(\frac{3}{4}\right)+64a^{4}b^{5}c^{\frac{11}{2}}x^{6}\Gamma\left(\frac{3}{4}\right)+32a^{3}b^{6}c^{\frac{11}{2}}x^{8}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] 5*a**4*b**(17/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2
)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c
**(11/2)*x**8*gamma(3/4)) + 2*a**3*b**(21/4)*x**2*(a/(b*x**2) + 1)**(1/4)*g
amma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)
*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4)) + 21*a**2*b**(25
/4)*x**4*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*g
amma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)
*x**8*gamma(3/4)) + 56*a*b**(29/4)*x**6*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)
/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamm
a(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4)) + 32*b**(33/4)*x**8*(a/(b*
x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*
a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Mupad [B]
time = 5.17, size = 54, normalized size = 0.65

$$-\frac{(b\,x^2+a)^{1/4}\left(\frac{2}{9\,a\,c^5}-\frac{16\,b\,x^2}{45\,a^2\,c^5}+\frac{64\,b^2\,x^4}{45\,a^3\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x)
[Out] -((a + b*x^2)^(1/4)*(2/(9*a*c^5) - (16*b*x^2)/(45*a^2*c^5) + (64*b^2*x^4)/(
45*a^3*c^5)))/(x^4*(c*x)^(1/2))
```

3.975 $\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=91

$$-\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}-\frac{\sqrt{a}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

```
[Out] -(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)
/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1
/2))),2^(1/2))*a^(1/2)/(-b*x^2+a)^(3/4)/b^(1/2)-c*(-b*x^2+a)^(1/4)*(c*x)^(1
/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {327, 335, 243, 342, 281, 238}

$$-\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}-\frac{\sqrt{a}\,(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -((c*Sqrt[c*x]*(a - b*x^2)^(1/4))/b) - (Sqrt[a]*(1 - a/(b*x^2))^(3/4)*(c*x)
^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a - b*x^2)^(3
/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx = -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b} + \frac{(ac^2)\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx}{2b}$$

$$= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b} + \frac{(ac)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{b}$$

$$= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b} + \frac{\left(ac\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{b\,(a-bx^2)^{3/4}}$$

$$= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b} - \frac{\left(ac\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{b\,(a-bx^2)^{3/4}}$$

$$= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b} - \frac{\left(ac\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{2b\,(a-bx^2)^{3/4}}$$

$$= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b} - \frac{\sqrt{a}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 68, normalized size = 0.75

$$\frac{c\sqrt{cx}\left(-a+bx^2+a\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^2}{a}\right)\right)}{b\,(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] (c*Sqrt[c*x]*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3
/4, 5/4, (b*x^2)/a]))/(b*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{(-b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(3/2)/(-b*x^2+a)^(3/4),x)

[Out] int((c*x)^(3/2)/(-b*x^2+a)^(3/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")

[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)*c*x/(b*x^2 - a), x)

Sympy [C] Result contains complex when optimal does not.
time = 1.19, size = 46, normalized size = 0.51

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^{2} e^{2i\pi}}{a} \right)}{2a^{\frac{3}{4}} \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(3/2)/(-b*x**2+a)**(3/4),x)

[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(2*I*pi)/a)/(2*a**(3/4)*gamma(9/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{{\left(a-b\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a - b*x^2)^(3/4),x)
[Out] int((c*x)^(3/2)/(a - b*x^2)^(3/4), x)
```

3.976 $\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{b}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^
(1/2))),2^(1/2))*b^(1/2)/c^2/(-b*x^2+a)^(3/4)/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {335, 243, 342, 281, 238}

$$-\frac{2\sqrt{b}\,(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}\\
&= \frac{\left(2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{c\,(a-bx^2)^{3/4}}\\
&= -\frac{\left(2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{c\,(a-bx^2)^{3/4}}\\
&= -\frac{\left(\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{c\,(a-bx^2)^{3/4}}\\
&= -\frac{2\sqrt{b}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{\sqrt{a}\,c^2\,(a-bx^2)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 55, normalized size = 0.81

$$\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^2}{a}\right)}{\sqrt{cx}\,(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a - b*x^2)^(3/4)),x]
```

```
[Out] (2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^2)/a])/(Sq
rt[c*x]*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c*x^3 - a*c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 32, normalized size = 0.47

$$\frac{ie^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{a}{bx^2}\right)}{b^{\frac{3}{4}}\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(3/4),x)
[Out] I*exp(-I*pi/4)*hyper((1/2, 3/4), (3/2,), a/(b*x**2))/(b**(3/4)*sqrt(c)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a - b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(1/2)*(a - b*x^2)^(3/4)), x)
```

3.977 $\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{4b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2/3*(-b*x^2+a)^(1/4)/a/c/(c*x)^(3/2)-4/3*b^(3/2)*(1-a/b/x^2)^(3/4)*(c*x)^(
3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/
a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/c^4
/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {331, 335, 243, 342, 281, 238}

$$-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(3/
4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*c^4*
(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}+\frac{(2b)\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx}{3ac^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}+\frac{(4b)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{3ac^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}+\frac{\left(4b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{3ac^3\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{\left(4b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{3ac^3\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{\left(2b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{3ac^3\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{4b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 57, normalized size = 0.57

$$-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{bx^2}{a}\right)}{3(cx)^{5/2}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x]
[Out] (-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, (b*x^2)/a])/(
3*(c*x)^(5/2)*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
[Out] int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^3*x^5 - a*c^3*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.71, size = 39, normalized size = 0.39

$$-\frac{ie^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{3b^{\frac{3}{4}}c^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(3/4),x)
[Out] -I*exp(3*I*pi/4)*hyper((3/4, 3/2), (5/2,), a/(b*x**2))/(3*b**(3/4)*c**(5/2)
*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{5/2}\,{\left(a-b\,x^2\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(5/2)*(a - b*x^2)^(3/4)), x)
```

3.978 $\int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=130

$$-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{8b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2/7*(-b*x^2+a)^(1/4)/a/c/(c*x)^(7/2)-4/7*b*(-b*x^2+a)^(1/4)/a^2/c^3/(c*x)^
(3/2)-8/7*b^(5/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcc
sc(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/c^6/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {331, 335, 243, 342, 281, 238}

$$-\frac{8b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a-bx^2\right)^{3/4}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) - (4*b*(a - b*x^2)^(1/4))/(7*a^2
*c^3*(c*x)^(3/2)) - (8*b^(5/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[
ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^(5/2)*c^6*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}} + \frac{(6b)\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx}{7ac^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{(4b^2)\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx}{7a^2c^4}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{(8b^2)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{7a^2c^5}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{\left(8b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4} x}\right.}{7a^2c^5\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{\left(8b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/}}\right.}{7a^2c^5\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{\left(4b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/}}\right.}{7a^2c^5\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{8b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right|}{7a^{5/2}c^6\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.44

$$-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{bx^2}{a}\right)}{7(cx)^{9/2}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x]
[Out] (-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, (b*x^2)/a])/
(7*(c*x)^(9/2)*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{9}{2}}\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x)

[Out] int(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")

[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^5*x^7 - a*c^5*x^5), x)

Sympy [C] Result contains complex when optimal does not.
time = 52.06, size = 36, normalized size = 0.28

$$\frac{ie^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{5b^{\frac{3}{4}}c^{\frac{9}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(3/4),x)

[Out] I*exp(-I*pi/4)*hyper((3/4, 5/2), (7/2,), a/(b*x**2))/(5*b**(3/4)*c**(9/2)*x
**5)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{9/2}\,{\left(a-b\,x^2\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(9/2)*(a - b*x^2)^(3/4)), x)
```

3.979 $\int \frac{1}{(cx)^{13/2}(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=162

$$-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{80b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}c^8\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2/11*(-b*x^2+a)^(1/4)/a/c/(c*x)^(11/2)-20/77*b*(-b*x^2+a)^(1/4)/a^2/c^3/(c
*x)^(7/2)-40/77*b^2*(-b*x^2+a)^(1/4)/a^3/c^5/(c*x)^(3/2)-80/77*b^(7/2)*(1-a
/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(
1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2)))
,2^(1/2))/a^(7/2)/c^8/(-b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {331, 335, 243, 342, 281, 238}

$$-\frac{80b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}c^8\left(a-bx^2\right)^{3/4}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) - (20*b*(a - b*x^2)^(1/4))/(77
*a^2*c^3*(c*x)^(7/2)) - (40*b^2*(a - b*x^2)^(1/4))/(77*a^3*c^5*(c*x)^(3/2))
 - (80*b^(7/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*c^8*(a - b*x^2)^(3/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}(a-bx^2)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} + \frac{(10b)\int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}}\,dx}{11ac^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} + \frac{(60b^2)\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx}{77a^2c^4}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{(40b^3)\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx}{77a^3c^6}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{(80b^3)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\right.}{77a^3c^7}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{\left(80b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)}{77a^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{\left(80b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)}{77a^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{\left(40b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)}{77a^3c^7}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{80b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}E}{77a^{7/2}c^8(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.35

$$-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{11}{4},\frac{3}{4};-\frac{7}{4};\frac{bx^2}{a}\right)}{11(cx)^{13/2}(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, (b*x^2)/a])
/(11*(c*x)^(13/2)*(a - b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{13}{2}} (-b\,x^2+a)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x)
[Out] int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^7*x^9 - a*c^7*x^7), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(3/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{13/2}\,{\left(a-b\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(13/2)*(a - b*x^2)^(3/4)), x)
```

3.980 $\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=308

$$-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}-\frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3ac^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3ac^{5/2}\log}{}$$

```
[Out] -1/2*c*(c*x)^(3/2)*(-b*x^2+a)^(1/4)/b+3/8*a*c^(5/2)*arctan(-1+b^(1/4)*2^(1/
2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)*2^(1/2)+3/8*a*c^(5/2)*arct
an(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)*2^(1/2)+
3/16*a*c^(5/2)*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^
(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(7/4)*2^(1/2)-3/16*a*c^(5/2)*ln(c^(1/2)+b
^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1
/2))/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {327, 335, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3ac^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3ac^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -1/2*(c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/b - (3*a*c^(5/2)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(7/4)) + (3*
a*c^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4)
)])/(4*Sqrt[2]*b^(7/4)) + (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*
b^(7/4)) - (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx = -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{(3ac^2)\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx}{4b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{(3ac)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{(3ac)\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} - \frac{(3ac)\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b^{3/2}} + \frac{(3ac)\text{Subst}\left(\int \frac{c+}{}\right.}{}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{\left(3ac^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{7/4}} + -$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{3ac^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{7/4}} - \frac{3ac^{5/2}\log\left(\right.}{}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} - \frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}} + \frac{3ac^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}}{\sqrt{c}}\right.}{4\sqrt{2}\,b^{7/4}}$$

Mathematica [A]
time = 0.51, size = 164, normalized size = 0.53

$$-\frac{(cx)^{5/2}\left(4b^{3/4}x^{3/2}\sqrt[4]{a-bx^2}+3\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{\sqrt{b}\,x-\sqrt{a-bx^2}}\right)+3\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{b}\,x+\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}\right)\right)}{8b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -1/8*((c*x)^(5/2)*(4*b^(3/4)*x^(3/2)*(a - b*x^2)^(1/4) + 3*Sqrt[2]*a*ArcTan
[(Sqrt[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(Sqrt[b]*x - Sqrt[a - b*x^2])]
 + 3*Sqrt[2]*a*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1/4)*Sqrt[
x]*(a - b*x^2)^(1/4))]))/(b^(7/4)*x^(5/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{(-b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 4.31, size = 46, normalized size = 0.15

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(2*
I*pi)/a)/(2*a**(3/4)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{5/2}}{{\left(a-b\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a - b*x^2)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(a - b*x^2)^(3/4), x)
```

3.981 $\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=272

$$-\frac{\sqrt{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{\cdots}}{\sqrt[4]{a\cdots}}\right.}{2\sqrt{2}\,b^{3/4}}\cdots$$

```
[Out] 1/2*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)
/b^(3/4)*2^(1/2)+1/2*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/
c^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)+1/4*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c*x)^(1/2)
/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))*c^(1/2)/b^(3/4)*2^(1/
2)-1/4*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^
(1/2)/(-b*x^2+a)^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\,\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,b^{3/4}}-\frac{\sqrt{c}\,\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4
))]])/(Sqrt[2]*b^(3/4))) + (Sqrt[c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(
Sqrt[c]*(a - b*x^2)^(1/4))]])/(Sqrt[2]*b^(3/4)) + (Sqrt[c]*Log[Sqrt[c] + (Sq
rt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^
(1/4)]])/(2*Sqrt[2]*b^(3/4)) - (Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]])/(2*Sqrt[2]*
b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}$$
$$= \frac{2\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt{b}\,c} + \frac{\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt{b}\,c}$$
$$= \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}-2x}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}\cdots}\right.}{2\sqrt{2}\,b^{3/4}\cdots}$$
$$= \frac{\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}} - \frac{\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a-\cdots}}\right.}{2\sqrt{2}\,b^{3/4}}$$
$$= -\frac{\sqrt{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}} + \frac{\sqrt{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}} + \frac{\sqrt{c}\,\log\left(\sqrt{c}\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.33, size = 127, normalized size = 0.47

$$\frac{\sqrt{cx}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{-\sqrt{b}\,x+\sqrt{a-bx^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{b}\,x+\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}\right)\right)}{\sqrt{2}\,b^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

```
[Out] (Sqrt[c*x]*(ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*x
) + Sqrt[a - b*x^2])] - ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1
/4)*Sqrt[x]*(a - b*x^2)^(1/4))]))/(Sqrt[2]*b^(3/4)*Sqrt[x])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(-b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 0.70, size = 46, normalized size = 0.17

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(2*I*
pi)/a)/(2*a**(3/4)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x}}{{\left(a-b\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a - b*x^2)^(3/4),x)
[Out] int((c*x)^(1/2)/(a - b*x^2)^(3/4), x)
```

3.982 $\int \frac{1}{(cx)^{3/2}(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

```
[Out] -2*(-b*x^2+a)^(1/4)/a/c/(c*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {270}

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Mathematica [A]
time = 0.17, size = 25, normalized size = 0.93

$$-\frac{2x\sqrt[4]{a-bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]
[Out] (-2*x*(a - b*x^2)^(1/4))/(a*(c*x)^(3/2))
```

Maple [A]
time = 0.05, size = 22, normalized size = 0.81

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{a\left(cx\right)^{\frac{3}{2}}}$	22
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}}{\sqrt{cx}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}}ca}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
[Out] -2*x*(-b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Fricas [A]
time = 0.98, size = 26, normalized size = 0.96

$$-\frac{2\left(-bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{ac^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] -2*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.54, size = 90, normalized size = 3.33

$$\begin{cases} \frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^{2}}-1}\,\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{for}\: \left|\frac{a}{bx^{2}}\right| > 1 \\ -\frac{\sqrt[4]{b}\,\sqrt[4]{-\frac{a}{bx^{2}}+1}\,e^{-\frac{3i\pi}{4}}\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma
(3/4)), Abs(a/(b*x**2)) > 1), (-b**(1/4)*(-a/(b*x**2) + 1)**(1/4)*exp(-3*I*
pi/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4)), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Mupad [**B**]
time = 5.08, size = 23, normalized size = 0.85

$$-\frac{2\left(a-b\,x^2\right)^{1/4}}{a\,c\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x)
```

```
[Out] -(2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(1/2))
```

3.983 $\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=57

$$-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}+\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}$$

```
[Out] -2*(-b*x^2+a)^(1/4)/a/c/(c*x)^(5/2)+8/5*(-b*x^2+a)^(5/4)/a^2/c/(c*x)^(5/2)
```

Rubi [A]

time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {279, 270}

$$\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a - b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}-\frac{4\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx}{a}\\ &= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}+\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 35, normalized size = 0.61

$$-\frac{2x\sqrt[4]{a-bx^2}\,(a+4bx^2)}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
[Out] (-2*x*(a - b*x^2)^(1/4)*(a + 4*b*x^2))/(5*a^2*(c*x)^(7/2))
```

Maple [A]
time = 0.05, size = 30, normalized size = 0.53

method	result	size
gosper	$-\frac{2x\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(4b\,x^2+a\right)}{5a^2\left(cx\right)^{\frac{7}{2}}}$	30
risch	$-\frac{2\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(\left(-b\,x^2+a\right)^3\right)^{\frac{1}{4}}\left(4b\,x^2+a\right)}{5\sqrt{cx}\,\left(-\left(b\,x^2-a\right)^3\right)^{\frac{1}{4}}c^3a^2x^2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
[Out] -2/5*x*(-b*x^2+a)^(1/4)*(4*b*x^2+a)/a^2/(c*x)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Fricas [A]
time = 0.75, size = 34, normalized size = 0.60

$$-\frac{2\,(4\,bx^2+a)(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{5\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/5*(4*b*x^2 + a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.65, size = 352, normalized size = 6.18

$$\begin{cases} -\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{8ac^{\frac{7}{2}}x^2\Gamma\left(\frac{3}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{2a^2c^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{for } \left|\frac{a}{bx^2}\right| > 1 \\ -\frac{a^2b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-8a^3bc^{\frac{7}{2}}x^2e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+8a^2b^2c^{\frac{7}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)} - \frac{3ab^{\frac{9}{4}}x^2\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-8a^3bc^{\frac{7}{2}}x^2e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+8a^2b^2c^{\frac{7}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)} + \frac{4b^{\frac{13}{4}}x^4\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-8a^3bc^{\frac{7}{2}}x^2e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+8a^2b^2c^{\frac{7}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Piecewise((-b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(8*a*c**(7/2)*x**2
*gamma(3/4)) - b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*a**2*c**(7/2
)*gamma(3/4)), Abs(a/(b*x**2)) > 1), (-a**2*b**(5/4)*(-a/(b*x**2) + 1)**(1/
4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*gamma(3/4) + 8*a**2*b
**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)) - 3*a*b**(9/4)*x**2*(-a/(b*x**2
) + 1)**(1/4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*gamma(3/4)
 + 8*a**2*b**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)) + 4*b**(13/4)*x**4*(
-a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*
gamma(3/4) + 8*a**2*b**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Mupad [B]
time = 5.10, size = 41, normalized size = 0.72

$$-\frac{{\left(a-b\,x^2\right)}^{1/4}\,\left(\frac{2}{5\,a\,c^3}+\frac{8\,b\,x^2}{5\,a^2\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x)
```

```
[Out] -((a - b*x^2)^(1/4)*(2/(5*a*c^3) + (8*b*x^2)/(5*a^2*c^3)))/(x^2*(c*x)^(1/2)
)
```

3.984 $\int \frac{1}{(cx)^{11/2}(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}+\frac{16(a-bx^2)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{64(a-bx^2)^{9/4}}{45a^3c(cx)^{9/2}}$$

```
[Out] -2*(-b*x^2+a)^(1/4)/a/c/(c*x)^(9/2)+16/5*(-b*x^2+a)^(5/4)/a^2/c/(c*x)^(9/2)
-64/45*(-b*x^2+a)^(9/4)/a^3/c/(c*x)^(9/2)
```

Rubi [A]

time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {279, 270}

$$-\frac{64(a-bx^2)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16(a-bx^2)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a - b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a - b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}(a-bx^2)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}} - \frac{8\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx}{a}$$
$$= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}} + \frac{16(a-bx^2)^{5/4}}{5a^2c(cx)^{9/2}} + \frac{32\int \frac{(a-bx^2)^{5/4}}{(cx)^{11/2}}\,dx}{5a^2}$$
$$= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}} + \frac{16(a-bx^2)^{5/4}}{5a^2c(cx)^{9/2}} - \frac{64(a-bx^2)^{9/4}}{45a^3c(cx)^{9/2}}$$

Mathematica [A]
time = 0.30, size = 48, normalized size = 0.56

$$-\frac{2x\sqrt[4]{a-bx^2}\,(5a^2+8abx^2+32b^2x^4)}{45a^3(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*(5*a^2 + 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*(c*x)^(11
/2))
```

Maple [A]
time = 0.12, size = 43, normalized size = 0.50

method	result	size
gosper	$-\frac{2x\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(32b^2x^4+8ab\,x^2+5a^2\right)}{45a^3\left(cx\right)^{\frac{11}{2}}}$	43
risch	$-\frac{2\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(\left(-b\,x^2+a\right)^3\right)^{\frac{1}{4}}\left(32b^2x^4+8ab\,x^2+5a^2\right)}{45\sqrt{cx}\,\left(-\left(b\,x^2-a\right)^3\right)^{\frac{1}{4}}c^5a^3x^4}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*x*(-b*x^2+a)^(1/4)*(32*b^2*x^4+8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Fricas [A]
time = 0.62, size = 47, normalized size = 0.55

$$-\frac{2\,(32\,b^2x^4+8\,abx^2+5\,a^2)(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{45\,a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*b^2*x^4 + 8*a*b*x^2 + 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c^
6*x^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 133.64, size = 1263, normalized size = 14.69

$$\begin{cases} -\frac{5a^4b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,e^{-\frac{i\pi}{4}}\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})}+\frac{2a^3b^{\frac{21}{4}}x^2\sqrt[4]{\frac{a}{bx^2}-1}\,e^{-\frac{i\pi}{4}}\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})}-\frac{21a^2b^{\frac{25}{4}}x^4\sqrt[4]{\frac{a}{bx^2}-1}\,e^{-\frac{i\pi}{4}}\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})}+\frac{56ab^{\frac{29}{4}}x^6\sqrt[4]{\frac{a}{bx^2}-1}\,e^{-\frac{i\pi}{4}}\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})}-\frac{32b^{\frac{33}{4}}x^8\sqrt[4]{\frac{a}{bx^2}-1}\,e^{-\frac{i\pi}{4}}\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})} & \text{for}\ \left|\frac{a}{bx^2}\right|>1 \\ -\frac{5a^4b^{\frac{17}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})}+\frac{2a^3b^{\frac{21}{4}}x^2\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})}-\frac{21a^2b^{\frac{25}{4}}x^4\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})}+\frac{56ab^{\frac{29}{4}}x^6\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})}-\frac{32b^{\frac{33}{4}}x^8\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma(-\frac{9}{4})}{32a^5b^4c^{\frac{11}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})-64a^4b^5c^{\frac{11}{2}}x^6e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})+32a^3b^6c^{\frac{11}{2}}x^8e^{\frac{3i\pi}{4}}\Gamma(\frac{3}{4})} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Piecewise((-5*a**4*b**(17/4)*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/
4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**
(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*
pi/4)*gamma(3/4)) + 2*a**3*b**(21/4)*x**2*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi
/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*
a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*
x**8*exp(3*I*pi/4)*gamma(3/4)) - 21*a**2*b**(25/4)*x**4*(a/(b*x**2) - 1)**(
1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*ga
mma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b
**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) + 56*a*b**(29/4)*x**6*(a/(b*x*
*2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3
*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4)
+ 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) - 32*b**(33/4)*x**8
*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x
**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*ga
mma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)), Abs(a/(b*
x**2)) > 1), (-5*a**4*b**(17/4)*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a*
*5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x*
*6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gam
ma(3/4)) + 2*a**3*b**(21/4)*x**2*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a
**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x
**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*ga
```

```
mma(3/4)) - 21*a**2*b**(25/4)*x***4*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32
*a**5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)
*x***6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*
gamma(3/4)) + 56*a*b**(29/4)*x***6*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*
a**5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*
x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*g
amma(3/4)) - 32*b**(33/4)*x***8*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**
5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x***
6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*gamm
a(3/4)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Mupad [B]
time = 5.15, size = 55, normalized size = 0.64

$$-\frac{{\left(a-b\,x^2\right)}^{1/4}\,\left(\frac{2}{9\,a\,c^5}+\frac{16\,b\,x^2}{45\,a^2\,c^5}+\frac{64\,b^2\,x^4}{45\,a^3\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x)
[Out] -((a - b*x^2)^(1/4)*(2/(9*a*c^5) + (16*b*x^2)/(45*a^2*c^5) + (64*b^2*x^4)/(
45*a^3*c^5)))/(x^4*(c*x)^(1/2))
```

3.985 $\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=146

$$\frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}$$

```
[Out] 1/2*c*(c*x)^(5/2)/b/(b*x^2+a)^(1/4)-5/4*a*c^(7/2)*arctan(b^(1/4)*(c*x)^(1/2
)/(b*x^2+a)^(1/4)/c^(1/2))/b^(9/4)-5/4*a*c^(7/2)*arctanh(b^(1/4)*(c*x)^(1/2
)/(b*x^2+a)^(1/4)/c^(1/2))/b^(9/4)+5/2*a*c^3*(c*x)^(1/2)/b^2/(b*x^2+a)^(1/4
)
```

Rubi [A]

time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {291, 294, 335, 246, 218, 214, 211}

$$-\frac{5ac^{7/2}\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (5*a*c^3*Sqrt[c*x])/(2*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(5/2))/(2*b*(a + b
*x^2)^(1/4)) - (5*a*c^(7/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)
^(1/4))])/(4*b^(9/4)) - (5*a*c^(7/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{(5ac^2)\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx}{4b}$$
$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{(5ac^4)\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx}{4b^2}$$
$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{(5ac^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b^2}$$
$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{(5ac^3)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b^2}$$
$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{(5ac^4)\,\mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b^2} - \frac{(5ac^4)}{}$$
$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} - \frac{5ac^{7/2}\tanh^{-1}\left(\frac{}{\sqrt{c}}\right.}{4b^{9/4}}$$

Mathematica [A]
time = 0.40, size = 131, normalized size = 0.90

$$\frac{c^3\sqrt{cx}\left(2\sqrt[4]{b}\sqrt{x}\,(5a+bx^2) - 5a\sqrt[4]{a+bx^2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) - 5a\sqrt[4]{a+bx^2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{9/4}\sqrt{x}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c^3*Sqrt[c*x]*(2*b^(1/4)*Sqrt[x]*(5*a + b*x^2) - 5*a*(a + b*x^2)^(1/4)*Arc
Tan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] - 5*a*(a + b*x^2)^(1/4)*ArcTanh[(b
^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(4*b^(9/4)*Sqrt[x]*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(106) = 212.
time = 0.78, size = 389, normalized size = 2.66

$$\frac{4\left(bc^3x^2+5ac^3\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}+20\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\arctan\left(-\frac{\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{3}{4}}\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ab^7c^3-\left(b^8x^2+ab^7\right)\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}\,a^2c^7x+\sqrt{\frac{a^4c^{14}}{b^9}}\left(b^5x^2+ab^4\right)}{bx^2+a}}}{a^4bc^{14}x^2+a^5c^{14}}\right)-5\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\log\left(\frac{5\left(\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ac^3+\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\right)}{bx^2+a}\right)+5\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\log\left(\frac{5\left(\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ac^3-\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\right)}{bx^2+a}\right)}{8\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^3*x^2 + 5*a*c^3)*(b*x^2 + a)^(3/4)*sqrt(c*x) + 20*(a^4*c^14/b^9
)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((a^4*c^14/b^9)^(3/4)*(b*x^2 + a)^(3/4)*s
qrt(c*x)*a*b^7*c^3 - (b^8*x^2 + a*b^7)*(a^4*c^14/b^9)^(3/4)*sqrt((sqrt(b*x^
2 + a)*a^2*c^7*x + sqrt(a^4*c^14/b^9)*(b^5*x^2 + a*b^4))/(b*x^2 + a)))/(a^4
*b*c^14*x^2 + a^5*c^14)) - 5*(a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*(
(b*x^2 + a)^(3/4)*sqrt(c*x)*a*c^3 + (a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2))
/(b*x^2 + a)) + 5*(a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*((b*x^2 + a)
^(3/4)*sqrt(c*x)*a*c^3 - (a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2))/(b*x^2 + a
)))/(b^3*x^2 + a*b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.42, size = 44, normalized size = 0.30

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((5/4, 9/4), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/4)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/2}}{\left(b\,x^2+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a + b*x^2)^(5/4),x)
[Out] int((c*x)^(7/2)/(a + b*x^2)^(5/4), x)
```

3.986 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}} dx$

Optimal. Leaf size=107

$$-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}$$

```
[Out] c^(3/2)*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(5/4)+c^(3/2)
*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(5/4)-2*c*(c*x)^(1/
2)/b/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {294, 335, 246, 218, 214, 211}

$$\frac{c^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (-2*c*Sqrt[c*x])/(b*(a + b*x^2)^(1/4)) + (c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4) + (c^(3/2)*ArcTanh[(b^(1/4)*Sqrt[c*
x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx = -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}} + \frac{c^2\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx}{b}$$

$$= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}} + \frac{(2c)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{b}$$

$$= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}} + \frac{(2c)\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}$$

$$= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}} + \frac{c^2\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b} + \frac{c^2\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \ldots\right.}{b}$$

$$= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}} + \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}} + \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}$$

Mathematica [**A**]
time = 0.30, size = 91, normalized size = 0.85

$$\frac{c\sqrt{cx}\left(-\frac{2\sqrt[4]{b}}{\sqrt[4]{a+bx^2}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{x}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{x}}\right)}{b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c*Sqrt[c*x]*((-2*b^(1/4))/(a + b*x^2)^(1/4) + ArcTan[(b^(1/4)*Sqrt[x])/(a
+ b*x^2)^(1/4)]/Sqrt[x] + ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]/Sqrt
[x]))/b^(5/4)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(79) = 158.
time = 1.20, size = 319, normalized size = 2.98

$$-\frac{4\,(bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,c+4\,(b^2x^2+ab)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,b^4c\left(\frac{c^6}{b^5}\right)^{\frac{3}{4}}-(b^5x^2+ab^4)\left(\frac{c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}\,c^3x+(b^3x^2+ab^2)\sqrt{\frac{c^6}{b^5}}}{bx^2+a}}}{bc^6x^2+ac^6}\right)-(b^2x^2+ab)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,c+(b^2x^2+ab)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}}{bx^2+a}\right)+(b^2x^2+ab)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}\,c-(b^2x^2+ab)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}}{bx^2+a}\right)}{2\,(b^2x^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*a
rctan(-((b*x^2 + a)^(3/4)*sqrt(c*x)*b^4*c*(c^6/b^5)^(3/4) - (b^5*x^2 + a*b^
4)*(c^6/b^5)^(3/4)*sqrt((sqrt(b*x^2 + a)*c^3*x + (b^3*x^2 + a*b^2)*sqrt(c^6
/b^5))/(b*x^2 + a)))/(b*c^6*x^2 + a*c^6)) - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)
*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*c + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x
^2 + a)) + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)
*c - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x^2 + a)))/(b^2*x^2 + a*b)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.31, size = 44, normalized size = 0.41

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{5}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{b x^{2} e^{i \pi}}{a}\right)}{2 a^{\frac{5}{4}} \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(5/4)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(5/4),x)
```

```
[Out] int((c*x)^(3/2)/(a + b*x^2)^(5/4), x)
```

3.987 $\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

```
[Out] 2*(c*x)^(1/2)/a/c/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{270\}$

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*Sqrt[c*x])/(a*c*(a + b*x^2)^(1/4))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx = \frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Mathematica [A]
time = 0.19, size = 24, normalized size = 0.92

$$\frac{2x}{a\sqrt{cx}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*x)/(a*Sqrt[c*x]*(a + b*x^2)^(1/4))
```

Maple [A]
time = 0.05, size = 21, normalized size = 0.81

method	result	size
gosper	$\frac{2x}{(b\,x^2+a)^{\frac{1}{4}}a\sqrt{cx}}$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)

[Out] 2*x/(b*x^2+a)^(1/4)/a/(c*x)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)

Fricas [A]
time = 0.86, size = 31, normalized size = 1.19

$$\frac{2\,(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{abcx^2+a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")

[Out] 2*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*b*c*x^2 + a^2*c)

Sympy [A]
time = 1.52, size = 34, normalized size = 1.31

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\,\sqrt{c}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/4),x)

[Out] gamma(1/4)/(2*a*b**(1/4)*sqrt(c)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)
```

Mupad [B]
time = 4.97, size = 29, normalized size = 1.12

$$\frac{2\,x\,{\left(b\,x^2+a\right)}^{3/4}}{\left(a^2+b\,a\,x^2\right)\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(5/4)),x)
[Out] (2*x*(a + b*x^2)^(3/4))/((a^2 + a*b*x^2)*(c*x)^(1/2))
```

3.988 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=55

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}$$

```
[Out] 2/a/c/(c*x)^(3/2)/(b*x^2+a)^(1/4)-8/3*(b*x^2+a)^(3/4)/a^2/c/(c*x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 270}

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(3*a^2*c*(c*x
)^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx &= \frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}+\frac{4\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx}{a}\\ &= \frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 34, normalized size = 0.62

$$-\frac{2x(a+4bx^2)}{3a^2(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
[Out] (-2*x*(a + 4*b*x^2))/(3*a^2*(c*x)^(5/2)*(a + b*x^2)^(1/4))
```

Maple [A]
time = 0.06, size = 29, normalized size = 0.53

method	result	size
gosper	$-\frac{2x\left(4b\,x^{2}+a\right)}{3\left(b\,x^{2}+a\right)^{\frac{1}{4}}a^{2}\left(cx\right)^{\frac{5}{2}}}$	29
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{3a^{2}x\,c^{2}\sqrt{cx}}-\frac{2bx}{a^{2}c^{2}\sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
[Out] -2/3*x*(4*b*x^2+a)/(b*x^2+a)^(1/4)/a^2/(c*x)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

Fricas [A]
time = 1.14, size = 48, normalized size = 0.87

$$-\frac{2\,(4\,bx^2+a)(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{3\,(a^2bc^3x^4+a^3c^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] -2/3*(4*b*x^2 + a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*b*c^3*x^4 + a^3*c^3*x^2
)
```

Sympy [A]
time = 8.53, size = 78, normalized size = 1.42

$$\frac{\Gamma\left(-\frac{3}{4}\right)}{8a\sqrt[4]{b}\,c^{\frac{5}{2}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2a^2c^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] gamma(-3/4)/(8*a*b**(1/4)*c**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(5/4))
 + b**(3/4)*gamma(-3/4)/(2*a**2*c**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

Mupad [B]
time = 5.10, size = 57, normalized size = 1.04

$$-\frac{(b\,x^2+a)^{3/4}\left(\frac{2}{3\,a\,b\,c^2}+\frac{8\,x^2}{3\,a^2\,c^2}\right)}{x^3\,\sqrt{c\,x}+\frac{a\,x\,\sqrt{c\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*(2/(3*a*b*c^2) + (8*x^2)/(3*a^2*c^2)))/(x^3*(c*x)^(1/2)
 + (a*x*(c*x)^(1/2))/b)
```

3.989 $\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=83

$$\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}$$

```
[Out] 2/a/c/(c*x)^(7/2)/(b*x^2+a)^(1/4)-16/3*(b*x^2+a)^(3/4)/a^2/c/(c*x)^(7/2)+64
/21*(b*x^2+a)^(7/4)/a^3/c/(c*x)^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(7/2)*(a + b*x^2)^(1/4)) - (16*(a + b*x^2)^(3/4))/(3*a^2*c*(c*
x)^(7/2)) + (64*(a + b*x^2)^(7/4))/(21*a^3*c*(c*x)^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx = \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}} + \frac{8\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx}{a}$$
$$= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}} - \frac{16(a+bx^2)^{3/4}}{3a^2c(cx)^{7/2}} - \frac{32\int \frac{(a+bx^2)^{3/4}}{(cx)^{9/2}}\,dx}{3a^2}$$
$$= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}} - \frac{16(a+bx^2)^{3/4}}{3a^2c(cx)^{7/2}} + \frac{64(a+bx^2)^{7/4}}{21a^3c(cx)^{7/2}}$$

Mathematica [A]
time = 0.39, size = 47, normalized size = 0.57

$$-\frac{2x(3a^2-8abx^2-32b^2x^4)}{21a^3(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(3*a^2 - 8*a*b*x^2 - 32*b^2*x^4))/(21*a^3*(c*x)^(9/2)*(a + b*x^2)^(1/
4))
```

Maple [A]
time = 0.06, size = 42, normalized size = 0.51

method	result	size
gosper	$-\frac{2x\left(-32b^2x^4-8ab\,x^2+3a^2\right)}{21\left(b\,x^2+a\right)^{\frac{1}{4}}a^3\left(cx\right)^{\frac{9}{2}}}$	42
risch	$-\frac{2\left(b\,x^2+a\right)^{\frac{3}{4}}\left(-11b\,x^2+3a\right)}{21a^3x^3c^4\sqrt{cx}}+\frac{2b^2x}{a^3c^4\sqrt{cx}\,\left(b\,x^2+a\right)^{\frac{1}{4}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*x*(-32*b^2*x^4-8*a*b*x^2+3*a^2)/(b*x^2+a)^(1/4)/a^3/(c*x)^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

Fricas [A]
time = 1.04, size = 61, normalized size = 0.73

$$\frac{2\,(32\,b^2x^4+8\,abx^2-3\,a^2)(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{21\,(a^3bc^5x^6+a^4c^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/21*(32*b^2*x^4 + 8*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3*b*c^
5*x^6 + a^4*c^5*x^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(71) = 142.
time = 80.89, size = 384, normalized size = 4.63

$$-\frac{3a^3b^{\frac{19}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}+\frac{5a^2b^{\frac{23}{4}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}+\frac{40ab^{\frac{27}{4}}x^4\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}+\frac{32b^{\frac{31}{4}}x^6\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -3*a**3*b**(19/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**4*c**(9/2
)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**3*b**6*c*
*(9/2)*x**6*gamma(5/4)) + 5*a**2*b**(23/4)*x**2*(a/(b*x**2) + 1)**(3/4)*gam
ma(-7/4)/(32*a**5*b**4*c**(9/2)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**
4*gamma(5/4) + 32*a**3*b**6*c**(9/2)*x**6*gamma(5/4)) + 40*a*b**(27/4)*x**4
*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**4*c**(9/2)*x**2*gamma(5/4)
 + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**3*b**6*c**(9/2)*x**6*gamma
(5/4)) + 32*b**(31/4)*x**6*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**
4*c**(9/2)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**
3*b**6*c**(9/2)*x**6*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

Mupad [B]
time = 5.15, size = 70, normalized size = 0.84

$$\frac{(b\,x^2+a)^{3/4}\left(\frac{16\,x^2}{21\,a^2\,c^4}-\frac{2}{7\,a\,b\,c^4}+\frac{64\,b\,x^4}{21\,a^3\,c^4}\right)}{x^5\,\sqrt{c\,x}+\frac{a\,x^3\,\sqrt{c\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] ((a + b*x^2)^(3/4)*((16*x^2)/(21*a^2*c^4) - 2/(7*a*b*c^4) + (64*b*x^4)/(21*
a^3*c^4)))/(x^5*(c*x)^(1/2) + (a*x^3*(c*x)^(1/2))/b)
```

3.990 $\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=109

$$\frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}$$

```
[Out] 2/a/c/(c*x)^(11/2)/(b*x^2+a)^(1/4)-8*(b*x^2+a)^(3/4)/a^2/c/(c*x)^(11/2)+64/
7*(b*x^2+a)^(7/4)/a^3/c/(c*x)^(11/2)-256/77*(b*x^2+a)^(11/4)/a^4/c/(c*x)^(1
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 270}

$$-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(11/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(a^2*c*(c*x)
^(11/2)) + (64*(a + b*x^2)^(7/4))/(7*a^3*c*(c*x)^(11/2)) - (256*(a + b*x^2)
^(11/4))/(77*a^4*c*(c*x)^(11/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}(a+bx^2)^{5/4}}\,dx = \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}} + \frac{12\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx}{a}$$
$$= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}} - \frac{8(a+bx^2)^{3/4}}{a^2c(cx)^{11/2}} - \frac{32\int \frac{(a+bx^2)^{3/4}}{(cx)^{13/2}}\,dx}{a^2}$$
$$= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}} - \frac{8(a+bx^2)^{3/4}}{a^2c(cx)^{11/2}} + \frac{64(a+bx^2)^{7/4}}{7a^3c(cx)^{11/2}} + \frac{128\int \frac{(a+bx^2)^{7/4}}{(cx)^{13/2}}\,dx}{7a^3}$$
$$= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}} - \frac{8(a+bx^2)^{3/4}}{a^2c(cx)^{11/2}} + \frac{64(a+bx^2)^{7/4}}{7a^3c(cx)^{11/2}} - \frac{256(a+bx^2)^{11/4}}{77a^4c(cx)^{11/2}}$$

Mathematica [A]
time = 0.47, size = 58, normalized size = 0.53

$$-\frac{2x(7a^3-12a^2bx^2+32ab^2x^4+128b^3x^6)}{77a^4(cx)^{13/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(7*a^3 - 12*a^2*b*x^2 + 32*a*b^2*x^4 + 128*b^3*x^6))/(77*a^4*(c*x)^(1
3/2)*(a + b*x^2)^(1/4))
```

Maple [A]
time = 0.07, size = 53, normalized size = 0.49

method	result	size
gosper	$-\frac{2x(128b^3x^6+32a\,b^2x^4-12a^2b\,x^2+7a^3)}{77(b\,x^2+a)^{\frac{1}{4}}a^4(cx)^{\frac{13}{2}}}$	53
risch	$-\frac{2(b\,x^2+a)^{\frac{3}{4}}(51b^2x^4-19ab\,x^2+7a^2)}{77a^4x^5c^6\sqrt{cx}} - \frac{2b^3x}{a^4c^6\sqrt{cx}\,(b\,x^2+a)^{\frac{1}{4}}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*x*(128*b^3*x^6+32*a*b^2*x^4-12*a^2*b*x^2+7*a^3)/(b*x^2+a)^(1/4)/a^4/(
c*x)^(13/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

Fricas [A]
time = 1.46, size = 72, normalized size = 0.66

$$-\frac{2\,(128\,b^3x^6+32\,ab^2x^4-12\,a^2bx^2+7\,a^3)(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{77\,(a^4bc^7x^8+a^5c^7x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/77*(128*b^3*x^6 + 32*a*b^2*x^4 - 12*a^2*b*x^2 + 7*a^3)*(b*x^2 + a)^(3/4)
*sqrt(c*x)/(a^4*b*c^7*x^8 + a^5*c^7*x^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4963 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

Mupad [B]
time = 5.20, size = 85, normalized size = 0.78

$$-\frac{{\left(b\,x^2+a\right)}^{3/4}\,\left(\frac{2}{11\,a\,b\,c^6}-\frac{24\,x^2}{77\,a^2\,c^6}+\frac{64\,b\,x^4}{77\,a^3\,c^6}+\frac{256\,b^2\,x^6}{77\,a^4\,c^6}\right)}{x^7\,\sqrt{c\,x}+\frac{a\,x^5\,\sqrt{c\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*(2/(11*a*b*c^6) - (24*x^2)/(77*a^2*c^6) + (64*b*x^4)/(7
7*a^3*c^6) + (256*b^2*x^6)/(77*a^4*c^6)))/(x^7*(c*x)^(1/2) + (a*x^5*(c*x)^(
1/2))/b)
```

3.991 $\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=155

$$\frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}}-\frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}+\frac{77a^{5/2}c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 77/60*a^2*c^5*(c*x)^(3/2)/b^3/(b*x^2+a)^(1/4)-11/30*a*c^3*(c*x)^(7/2)/b^2/(
b*x^2+a)^(1/4)+1/5*c*(c*x)^(11/2)/b/(b*x^2+a)^(1/4)+77/20*a^(5/2)*c^6*(1+a/
b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*
b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*
x)^(1/2)/b^(7/2)/(b*x^2+a)^(1/4)
```

Rubi [A]

time = 0.05, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {291, 290, 342, 202}

$$\frac{77a^{5/2}c^6\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}+\frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}}-\frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (77*a^2*c^5*(c*x)^(3/2))/(60*b^3*(a + b*x^2)^(1/4)) - (11*a*c^3*(c*x)^(7/2)
)/(30*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(11/2))/(5*b*(a + b*x^2)^(1/4)) + (
77*a^(5/2)*c^6*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)
/Sqrt[a]]/2, 2])/(20*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} - \frac{(11ac^2)\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx}{10b}$$

$$= -\frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{(77a^2c^4)\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx}{60b^2}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} - \frac{(77a^3c^6)\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx}{40b^3}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} - \frac{\left(77a^3c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right.\cdots}}{40b^4\sqrt[4]{a+bx^2}}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{\left(77a^3c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\Big(\cdots}{40b^4\sqrt[4]{a+bx^2}}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{77a^{5/2}c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\Big(\frac{1}{2}\cot^{-}\cdots}{20b^{7/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 87, normalized size = 0.56

$$\frac{c^5(cx)^{3/2}\left(77a^2-22abx^2+12b^2x^4-77a^2\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{60b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c^5*(c*x)^(3/2)*(77*a^2 - 22*a*b*x^2 + 12*b^2*x^4 - 77*a^2*(1 + (b*x^2)/a)
^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^2)
^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{2}}}{(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^6*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{13/2}}{(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/2)/(a + b*x^2)^(5/4),x)
[Out] int((c*x)^(13/2)/(a + b*x^2)^(5/4), x)
```

3.992 $\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=124

$$-\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7a^{3/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -7/6*a*c^3*(c*x)^(3/2)/b^2/(b*x^2+a)^(1/4)+1/3*c*(c*x)^(7/2)/b/(b*x^2+a)^(1
/4)-7/2*a^(3/2)*c^4*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2
)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/
2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/b^(5/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {291, 290, 342, 202}

$$-\frac{7a^{3/2}c^4\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(9/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (-7*a*c^3*(c*x)^(3/2))/(6*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(7/2))/(3*b*(a
+ b*x^2)^(1/4)) - (7*a^(3/2)*c^4*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[
ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
```

```
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{(7ac^2)\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx}{6b}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} + \frac{(7a^2c^4)\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx}{4b^2}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} + \frac{\left(7a^2c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{4b^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{\left(7a^2c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x,\right.}{4b^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{7a^{3/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 74, normalized size = 0.60

$$\frac{c^3(cx)^{3/2}\left(-7a+2bx^2+7a\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{6b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(9/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c^3*(c*x)^(3/2)*(-7*a + 2*b*x^2 + 7*a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric
2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{9}{2}}}{(b\,x^{2}+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(b*x^2+a)^(5/4),x)
[Out] int((c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^4*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x
)
```

Sympy [C] Result contains complex when optimal does not.
time = 42.38, size = 44, normalized size = 0.35

$$\frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(9/2)/(b*x**2+a)**(5/4),x)
[Out] c**(9/2)*x**(11/2)*gamma(11/4)*hyper((5/4, 11/4), (15/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(5/4)*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{9/2}}{(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(a + b*x^2)^(5/4),x)
[Out] int((c*x)^(9/2)/(a + b*x^2)^(5/4), x)
```

3.993 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=90

$$\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}+\frac{3\sqrt{a}\,c^2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] c*(c*x)^(3/2)/b/(b*x^2+a)^(1/4)+3*c^2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(c*x)^(1/2)/b^(3/2)/(b*x^2+
a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {291, 290, 342, 202}

$$\frac{3\sqrt{a}\,c^2\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c*(c*x)^(3/2))/(b*(a + b*x^2)^(1/4)) + (3*Sqrt[a]*c^2*(1 + a/(b*x^2))^(1/4
)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^
2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
```

```
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx &= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} - \frac{(3ac^2)\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx}{2b} \\
&= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} - \frac{\left(3ac^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{2b^2\sqrt[4]{a+bx^2}} \\
&= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} + \frac{\left(3ac^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2b^2\sqrt[4]{a+bx^2}} \\
&= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} + \frac{3\sqrt{a}\,c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 60, normalized size = 0.67

$$\frac{c(cx)^{3/2}\left(1-\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(5/4),x]
[Out] (c*(c*x)^(3/2)*(1 - (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4,
-((b*x^2)/a)]))/(b*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x
)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.21, size = 44, normalized size = 0.49

$$\frac{c^{\frac{5}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix} \frac{5}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\right| \frac{b x^{2} e^{i \pi}}{a}\right)}{2 a^{\frac{5}{4}} \Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/4)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(5/4),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(5/4), x)
```

3.994 $\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

```
[Out] -2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*a
rccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1
/2))*(c*x)^(1/2)/(b*x^2+a)^(1/4)/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {290, 342, 202}

$$-\frac{2\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(5/4),x]
```

```
[Out] (-2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2
, 2])/(Sqrt[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx = \frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{b\sqrt[4]{a+bx^2}}$$
$$= -\frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{b\sqrt[4]{a+bx^2}}$$
$$= -\frac{2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 59, normalized size = 0.94

$$\frac{2x\sqrt{cx}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(5/4),x]
[Out] (2*x*Sqrt[c*x]*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*
x^2)/a)])/(3*a*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(5/4),x)
[Out] int((c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.04, size = 44, normalized size = 0.70

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(5/4),x)
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(5/4)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(5/4),x)
[Out] int((c*x)^(1/2)/(a + b*x^2)^(5/4), x)
```

3.995 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=93

$$-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{4\sqrt{b}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}$$

```
[Out] -2/a/c/(b*x^2+a)^(1/4)/(c*x)^(1/2)+4*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^
(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1
/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(c*x)^(1/2)/a^(3/2)/c^2/(b*x
^2+a)^(1/4)
```

Rubi [A]

time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {292, 290, 342, 202}

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] -2/(a*c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (4*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqr
t[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*c^2*(a + b*x^2
)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
```

```
+ 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
&& PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx &= -\frac{2}{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}-\frac{(2b)\int\frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{ac^2}\\
&= -\frac{2}{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}-\frac{\left(2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\right)\int\frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{ac^2\sqrt[4]{a+bx^2}}\\
&= -\frac{2}{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}+\frac{\left(2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{ac^2\sqrt[4]{a+bx^2}}\\
&= -\frac{2}{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}+\frac{4\sqrt{b}\,\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.61

$$-\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{a(cx)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x]
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((b*x^2)/a)]
)/(a*(c*x)^(3/2)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c^2
*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.20, size = 48, normalized size = 0.52

$$\frac{\Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{3}{4} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{2a^{\frac{5}{4}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{3/2}\,(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(5/4)), x)
```

3.996 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}+\frac{12b}{5a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}-\frac{24b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}c^4\sqrt[4]{a+bx^2}}$$

```
[Out] -2/5/a/c/(c*x)^(5/2)/(b*x^2+a)^(1/4)+12/5*b/a^2/c^3/(b*x^2+a)^(1/4)/(c*x)^(
1/2)-24/5*b^(3/2)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^
(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)
/a^(1/2))),2^(1/2))*(c*x)^(1/2)/a^(5/2)/c^4/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {292, 290, 342, 202}

$$-\frac{24b^{3/2}\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}c^4\sqrt[4]{a+bx^2}}+\frac{12b}{5a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] -2/(5*a*c*(c*x)^(5/2)*(a + b*x^2)^(1/4)) + (12*b)/(5*a^2*c^3*Sqrt[c*x]*(a +
 b*x^2)^(1/4)) - (24*b^(3/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcC
ot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*c^4*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
```

```
+ 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
&& PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\,(a+bx^2)^{5/4}}\,dx = -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{(6b)\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}}\,dx}{5ac^2}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} + \frac{(12b^2)\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx}{5a^2c^4}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} + \frac{\left(12b\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\right)\int \frac{}{(1}}{5a^2c^4\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{\left(12b\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\right)\text{Subs}}{5a^2c^4\sqrt[4]{a}}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{24b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\right.}{5a^{5/2}c^4\sqrt[4]{a+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 59, normalized size = 0.47

$$-\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, -((b*x^2)/a)
])/(5*a*(c*x)^(7/2)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{7}{2}} (b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^4*x^8 + 2*a*b*c^4*x^6 + a^2*c^4
*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 27.78, size = 34, normalized size = 0.27

$$-\frac{{}_2F_1\left(\begin{matrix}\frac{5}{4}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{5b^{\frac{5}{4}}c^{\frac{7}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((5/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(5/4)*c**(7/2)
*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x)
[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(5/4)), x)
```

3.997 $\int \frac{1}{(cx)^{11/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=157

$$-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{8b^2}{3a^3c^5\sqrt{cx}\,\sqrt[4]{a+bx^2}}+\frac{16b^{5/2}\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\right.\right.}{3a^{7/2}c^6\sqrt[4]{a+bx^2}}$$

```
[Out] -2/9/a/c/(c*x)^(9/2)/(b*x^2+a)^(1/4)+4/9*b/a^2/c^3/(c*x)^(5/2)/(b*x^2+a)^(1
/4)-8/3*b^2/a^3/c^5/(b*x^2+a)^(1/4)/(c*x)^(1/2)+16/3*b^(5/2)*(1+a/b/x^2)^(1
/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a
^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/
a^(7/2)/c^6/(b*x^2+a)^(1/4)
```

Rubi [A]

time = 0.05, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {292, 290, 342, 202}

$$\frac{16b^{5/2}\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}c^6\sqrt[4]{a+bx^2}}-\frac{8b^2}{3a^3c^5\sqrt{cx}\,\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] -2/(9*a*c*(c*x)^(9/2)*(a + b*x^2)^(1/4)) + (4*b)/(9*a^2*c^3*(c*x)^(5/2)*(a
 + b*x^2)^(1/4)) - (8*b^2)/(3*a^3*c^5*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (16*b^(
5/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(3*a^(7/2)*c^6*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
```

```
+ 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
&& PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} - \frac{(10b)\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/4}}\,dx}{9ac^2}$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{(4b^2)\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}}\,dx}{3a^2c^4}$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \cdots$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \cdots$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\,\sqrt[4]{a+bx^2}} + \cdots$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\,\sqrt[4]{a+bx^2}} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 59, normalized size = 0.38

$$-\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9a(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, -((b*x^2)/a)
])/(9*a*(c*x)^(11/2)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{11}{2}} (b\,x^2 + a)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x)
[Out] int(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^6*x^10 + 2*a*b*c^6*x^8 + a^2*c^
6*x^6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(5/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3279 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{11/2}\,(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] int(1/((c*x)^(11/2)*(a + b*x^2)^(5/4)), x)
```

3.998 $\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{9/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

```
[Out] 4/9*(c*x)^(9/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 9/8],[17/8],-b*x^2/a)/c/(
b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{4(cx)^{9/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 9/8, 17/8, -((b
*x^2)/a)])/(9*c*(a + b*x^2)^(1/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{(cx)^{5/4}}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{4(cx)^{9/4}\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

Mathematica [A]
time = 10.01, size = 56, normalized size = 0.97

$$\frac{4x(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/4)/(a + b*x^2)^(1/4),x]
[Out] (4*x*(c*x)^(5/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 9/8, 17/8, -(
(b*x^2)/a)])/(9*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{4}}}{(b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(b*x^2+a)^(1/4),x)
[Out] int((c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((c*x)^(1/4)*c*x/(b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 6.60, size = 44, normalized size = 0.76

$$\frac{c^{\frac{5}{4}} x^{\frac{9}{4}} \Gamma\left(\frac{9}{8}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{9}{8} \\ \frac{17}{8} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{2 \sqrt[4]{a} \Gamma\left(\frac{17}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/4)/(b*x**2+a)**(1/4),x)
[Out] c**(5/4)*x**(9/4)*gamma(9/8)*hyper((1/4, 9/8), (17/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(17/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x\right)}^{5/4}}{{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(a + b*x^2)^(1/4),x)
[Out] int((c*x)^(5/4)/(a + b*x^2)^(1/4), x)
```

3.999 $\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{7/4}\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

```
[Out] 4/7*(c*x)^(7/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 7/8],[15/8],-b*x^2/a)/c/(
b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{4(cx)^{7/4}\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 7/8, 15/8, -((b
*x^2)/a)])/(7*c*(a + b*x^2)^(1/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{(cx)^{3/4}}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{4(cx)^{7/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

Mathematica [**A**]
time = 10.02, size = 56, normalized size = 0.97

$$\frac{4x(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/4)/(a + b*x^2)^(1/4),x]
[Out] (4*x*(c*x)^(3/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 7/8, 15/8, -(
(b*x^2)/a)])/(7*(a + b*x^2)^(1/4))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/4)/(b*x^2+a)^(1/4),x)
[Out] int((c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.85, size = 44, normalized size = 0.76

$$\frac{c^{\frac{3}{4}} x^{\frac{7}{4}} \Gamma\left(\frac{7}{8}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{7}{8} \\ \frac{15}{8} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{2 \sqrt[4]{a} \Gamma\left(\frac{15}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/4)/(b*x**2+a)**(1/4),x)
[Out] c**(3/4)*x**(7/4)*gamma(7/8)*hyper((1/4, 7/8), (15/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(15/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x\right)}^{3/4}}{{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/4)/(a + b*x^2)^(1/4),x)
[Out] int((c*x)^(3/4)/(a + b*x^2)^(1/4), x)
```

3.1000 $\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}} \, dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

```
[Out] 4/5*(c*x)^(5/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 5/8],[13/8],-b*x^2/a)/c/(
b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{4(cx)^{5/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 5/8, 13/8, -((b
*x^2)/a)])/(5*c*(a + b*x^2)^(1/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{\sqrt[4]{cx}}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{4(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

Mathematica [A]
time = 10.01, size = 56, normalized size = 0.97

$$\frac{4x\sqrt[4]{cx}\,\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/4)/(a + b*x^2)^(1/4),x]
[Out] (4*x*(c*x)^(1/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 5/8, 13/8, -(
(b*x^2)/a)])/(5*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{4}}}{(b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(b*x^2+a)^(1/4),x)
[Out] int((c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.58, size = 44, normalized size = 0.76

$$\frac{\sqrt[4]{c}\,x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{8}\\\frac{13}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{13}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(1/4)*x**(5/4)*gamma(5/8)*hyper((1/4, 5/8), (13/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(13/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{(c\,x)}^{1/4}}{{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(1/4)/(a + b*x^2)^(1/4), x)
```

3.1001 $\int \frac{1}{\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}$$

```
[Out] 4/3*(c*x)^(3/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 3/8],[11/8],-b*x^2/a)/c/(
b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{4(cx)^{3/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/8, 11/8, -((b
*x^2)/a)])/(3*c*(a + b*x^2)^(1/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{cx}\,\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{4(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}$$

Mathematica [**A**]
time = 10.02, size = 56, normalized size = 0.97

$$\frac{4x\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x]
[Out] (4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/8, 11/8, -((b*x^2)/a)])
/(3*(c*x)^(1/4)*(a + b*x^2)^(1/4))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{1}{4}}\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)
[Out] int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(c*x)^(3/4)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.63, size = 44, normalized size = 0.76

$$\frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{8}\\\frac{11}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\Gamma\left(\frac{11}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] x**(3/4)*gamma(3/8)*hyper((1/4, 3/8), (11/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(1/4)*c**(1/4)*gamma(11/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{1/4}\,{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(1/4)*(a + b*x^2)^(1/4)), x)
```

3.1002 $\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{4\sqrt[4]{cx}\,\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}$$

```
[Out] 4*(c*x)^(1/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/8, 1/4],[9/8],-b*x^2/a)/c/(b*x
^2+a)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{4\sqrt[4]{cx}\,\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((b*
x^2)/a)])/(c*(a + b*x^2)^(1/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}} = \frac{4\sqrt[4]{cx}\,\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}$$

Mathematica [A]
time = 10.01, size = 54, normalized size = 0.96

$$\frac{4x\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{(cx)^{3/4}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x]
[Out] (4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((b*x^2)/a)])/
((c*x)^(3/4)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{4}}\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x)
[Out] int(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(c*x)^(1/4)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.01, size = 44, normalized size = 0.79

$$\frac{\sqrt[4]{x}\,\Gamma\left(\frac{1}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{8},\frac{1}{4}\\\frac{9}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] x**(1/4)*gamma(1/8)*hyper((1/8, 1/4), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(1/4)*c**(3/4)*gamma(9/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{3/4}\,{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(3/4)*(a + b*x^2)^(1/4)), x)
```

3.1003 $\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{4\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\ \sqrt[4]{a+bx^2}}$$

```
[Out] -4*(1+b*x^2/a)^(1/4)*hypergeom([-1/8, 1/4],[7/8],-b*x^2/a)/c/(c*x)^(1/4)/(b
*x^2+a)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$-\frac{4\sqrt[4]{\frac{bx^2}{a}+1}\ {}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\ \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-4*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/8, 1/4, 7/8, -((b*x^2)/a)])/
(c*(c*x)^(1/4)*(a + b*x^2)^(1/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}$$
$$= -\frac{4\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\ \sqrt[4]{a+bx^2}}$$

Mathematica [**A**]
time = 10.02, size = 54, normalized size = 0.96

$$-\frac{4x\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{(cx)^{5/4}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x]
[Out] (-4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/8, 1/4, 7/8, -((b*x^2)/a)]
)/((c*x)^(5/4)*(a + b*x^2)^(1/4))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{4}}\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x)
[Out] int(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(c*x)^(3/4)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.00, size = 48, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{1}{8}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{8}, \frac{1}{4} \\ \frac{7}{8} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,c^{\frac{5}{4}}\sqrt[4]{x}\,\Gamma\left(\frac{7}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] gamma(-1/8)*hyper((-1/8, 1/4), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4
)*c**(5/4)*x**(1/4)*gamma(7/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{5/4}\,{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(5/4)*(a + b*x^2)^(1/4)), x)
```

3.1004 $\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{9/4}\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] 4/9*(c*x)^(9/4)*(1+b*x^2/a)^(3/4)*hypergeom([9/8, 7/4],[17/8],-b*x^2/a)/a/c
/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{4(cx)^{9/4}\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[9/8, 7/4, 17/8, -((b
*x^2)/a)])/(9*a*c*(a + b*x^2)^(3/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{(cx)^{5/4}}{\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\,(a+bx^2)^{3/4}}$$
$$= \frac{4(cx)^{9/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9ac\,(a+bx^2)^{3/4}}$$

Mathematica [A]
time = 10.03, size = 59, normalized size = 0.97

$$\frac{4x(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9a\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*x*(c*x)^(5/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[9/8, 7/4, 17/8, -(
(b*x^2)/a)])/(9*a*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{4}}}{(b\,x^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)*c*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.30, size = 44, normalized size = 0.72

$$\frac{c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{9}{8},\frac{7}{4}\\\frac{17}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{17}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] c**(5/4)*x**(9/4)*gamma(9/8)*hyper((9/8, 7/4), (17/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(7/4)*gamma(17/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x\right)}^{5/4}}{{\left(b\,x^2+a\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(a + b*x^2)^(7/4),x)
```

```
[Out] int((c*x)^(5/4)/(a + b*x^2)^(7/4), x)
```

3.1005 $\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{7/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7ac\,(a+bx^2)^{3/4}}$$

```
[Out] 4/7*(c*x)^(7/4)*(1+b*x^2/a)^(3/4)*hypergeom([7/8, 7/4],[15/8],-b*x^2/a)/a/c
/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{4(cx)^{7/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7ac\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/8, 7/4, 15/8, -((b
*x^2)/a)])/(7*a*c*(a + b*x^2)^(3/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{(cx)^{3/4}}{\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\,(a+bx^2)^{3/4}}$$
$$= \frac{4(cx)^{7/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7ac\,(a+bx^2)^{3/4}}$$

Mathematica [A]
time = 10.02, size = 59, normalized size = 0.97

$$\frac{4x(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7a\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/4)/(a + b*x^2)^(7/4),x]
[Out] (4*x*(c*x)^(3/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/8, 7/4, 15/8, -(
(b*x^2)/a)])/(7*a*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)
[Out] int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.76, size = 44, normalized size = 0.72

$$\frac{c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{8},\frac{7}{4}\\ \frac{15}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{15}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] c**(3/4)*x**(7/4)*gamma(7/8)*hyper((7/8, 7/4), (15/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(7/4)*gamma(15/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x\right)}^{3/4}}{{\left(b\,x^2+a\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/4)/(a + b*x^2)^(7/4),x)
```

```
[Out] int((c*x)^(3/4)/(a + b*x^2)^(7/4), x)
```

3.1006 $\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5ac\,(a+bx^2)^{3/4}}$$

```
[Out] 4/5*(c*x)^(5/4)*(1+b*x^2/a)^(3/4)*hypergeom([5/8, 7/4],[13/8],-b*x^2/a)/a/c
/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$,
Rules used = {372, 371}

$$\frac{4(cx)^{5/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5ac\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[5/8, 7/4, 13/8, -((b
*x^2)/a)])/(5*a*c*(a + b*x^2)^(3/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{\sqrt[4]{cx}}{\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\,(a+bx^2)^{3/4}}$$
$$= \frac{4(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5ac\,(a+bx^2)^{3/4}}$$

Mathematica [A]
time = 10.02, size = 59, normalized size = 0.97

$$\frac{4x\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5a\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/4)/(a + b*x^2)^(7/4),x]
[Out] (4*x*(c*x)^(1/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[5/8, 7/4, 13/8, -(
(b*x^2)/a)])/(5*a*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{4}}}{(b\,x^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(b*x^2+a)^(7/4),x)
[Out] int((c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.14, size = 44, normalized size = 0.72

$$\frac{\sqrt[4]{c}\,x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{7}{4}\\\frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{13}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] c**(1/4)*x**(5/4)*gamma(5/8)*hyper((5/8, 7/4), (13/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(7/4)*gamma(13/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(7/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{1/4}}{(b\,x^2+a)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(a + b*x^2)^(7/4),x)
```

```
[Out] int((c*x)^(1/4)/(a + b*x^2)^(7/4), x)
```

3.1007 $\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3ac\,(a+bx^2)^{3/4}}$$

```
[Out] 4/3*(c*x)^(3/4)*(1+b*x^2/a)^(3/4)*hypergeom([3/8, 7/4],[11/8],-b*x^2/a)/a/c
/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{4(cx)^{3/4}\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3ac\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/8, 7/4, 11/8, -((b
*x^2)/a)])/(3*a*c*(a + b*x^2)^(3/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\,(a+bx^2)^{3/4}}$$
$$= \frac{4(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3ac\,(a+bx^2)^{3/4}}$$

Mathematica [A]
time = 10.02, size = 59, normalized size = 0.97

$$\frac{4x\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3a\sqrt[4]{cx}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x]
[Out] (4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/8, 7/4, 11/8, -((b*x^2)/a)])
/(3*a*(c*x)^(1/4)*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x)
[Out] int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x),
 x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.48, size = 44, normalized size = 0.72

$$\frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{8},\frac{7}{4}\\\frac{11}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\sqrt[4]{c}\,\Gamma\left(\frac{11}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] x**(3/4)*gamma(3/8)*hyper((3/8, 7/4), (11/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(7/4)*c**(1/4)*gamma(11/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{1/4}\,{\left(b\,x^2+a\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x)
```

```
[Out] int(1/((c*x)^(1/4)*(a + b*x^2)^(7/4)), x)
```

3.1008 $\int \frac{1}{(cx)^{3/4}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=59

$$\frac{4\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{ac\,(a+bx^2)^{3/4}}$$

```
[Out] 4*(c*x)^(1/4)*(1+b*x^2/a)^(3/4)*hypergeom([1/8, 7/4],[9/8],-b*x^2/a)/a/c/(b
*x^2+a)^(3/4)
```

Rubi [A]

time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$\frac{4\sqrt[4]{cx}\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{ac\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/8, 7/4, 9/8, -((b*
x^2)/a)])/(a*c*(a + b*x^2)^(3/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{7/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}$$
$$= \frac{4\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{ac\left(a+bx^2\right)^{3/4}}$$

Mathematica [A]
time = 10.02, size = 57, normalized size = 0.97

$$\frac{4x\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{a(cx)^{3/4}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/8, 7/4, 9/8, -((b*x^2)/a)])/
(a*(c*x)^(3/4)*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{4}}\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x),
 x)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.53, size = 44, normalized size = 0.75

$$\frac{\sqrt[4]{x}\,\Gamma\left(\frac{1}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{8},\frac{7}{4}\\\frac{9}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] x**(1/4)*gamma(1/8)*hyper((1/8, 7/4), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(7/4)*c**(3/4)*gamma(9/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{3/4}\,{\left(b\,x^2+a\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x)
```

```
[Out] int(1/((c*x)^(3/4)*(a + b*x^2)^(7/4)), x)
```

3.1009 $\int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=59

$$-\frac{4\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\,\left(a+bx^2\right)^{3/4}}$$

```
[Out] -4*(1+b*x^2/a)^(3/4)*hypergeom([-1/8, 7/4],[7/8],-b*x^2/a)/a/c/(c*x)^(1/4)/
(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 371}

$$-\frac{4\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\,\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-4*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/8, 7/4, 7/8, -((b*x^2)/a)])/
(a*c*(c*x)^(1/4)*(a + b*x^2)^(3/4))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\,(a+bx^2)^{3/4}}$$
$$= -\frac{4\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\,(a+bx^2)^{3/4}}$$

Mathematica [A]
time = 10.02, size = 57, normalized size = 0.97

$$-\frac{4x\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{a(cx)^{5/4}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x]
[Out] (-4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/8, 7/4, 7/8, -((b*x^2)/a)]
)/(a*(c*x)^(5/4)*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{4}}\,(b\,x^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x)
[Out] int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c
^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.56, size = 48, normalized size = 0.81

$$\frac{\Gamma\left(-\frac{1}{8}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{8}, \frac{7}{4} \\ \frac{7}{8} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{2a^{\frac{7}{4}}c^{\frac{5}{4}}\sqrt[4]{x}\,\Gamma\left(\frac{7}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/4)/(b*x**2+a)**(7/4),x)
[Out] gamma(-1/8)*hyper((-1/8, 7/4), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4
)*c**(5/4)*x**(1/4)*gamma(7/8))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{5/4}\,{\left(b\,x^2+a\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x)
[Out] int(1/((c*x)^(5/4)*(a + b*x^2)^(7/4)), x)
```

3.1010 $\int x^6 \sqrt[6]{a+bx^2}\, dx$

Optimal. Leaf size=345

$$\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\sqrt[6]{a+bx^2}\left(1-\right.}{}$$

[Out] 81/2816*a^3*x*(b*x^2+a)^(1/6)/b^3-9/704*a^2*x^3*(b*x^2+a)^(1/6)/b^2+3/352*a *x^5*(b*x^2+a)^(1/6)/b+3/22*x^7*(b*x^2+a)^(1/6)-81/2816*3^(3/4)*a^4*(b*x^2+ a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/ (1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^ 2)^(1/2)/b^4/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.29, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {285, 327, 247, 242, 225}

$$-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2816b^4x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}$$

Antiderivative was successfully verified.

[In] Int[x^6*(a + b*x^2)^(1/6),x]

[Out] (81*a^3*x*(a + b*x^2)^(1/6))/(2816*b^3) - (9*a^2*x^3*(a + b*x^2)^(1/6))/(70 4*b^2) + (3*a*x^5*(a + b*x^2)^(1/6))/(352*b) + (3*x^7*(a + b*x^2)^(1/6))/22 - (81*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2)) ^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^ 2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2816*b ^4*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^6 \sqrt[6]{a+bx^2}\,dx = \frac{3}{22}x^7\sqrt[6]{a+bx^2} + \frac{1}{22}a\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx$$

$$= \frac{3ax^5\sqrt[6]{a+bx^2}}{352b} + \frac{3}{22}x^7\sqrt[6]{a+bx^2} - \frac{(15a^2)\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx}{352b}$$

$$= -\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2} + \frac{3ax^5\sqrt[6]{a+bx^2}}{352b} + \frac{3}{22}x^7\sqrt[6]{a+bx^2} + \frac{(27a^3)\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx}{704b^2}$$

$$= \frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3} - \frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2} + \frac{3ax^5\sqrt[6]{a+bx^2}}{352b} + \frac{3}{22}x^7\sqrt[6]{a+bx^2} - \frac{(81a^4)\int \frac{}{(a}}{281}$$

$$= \frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3} - \frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2} + \frac{3ax^5\sqrt[6]{a+bx^2}}{352b} + \frac{3}{22}x^7\sqrt[6]{a+bx^2} - \frac{(81a^4)\,\mathrm{Sub}}{281}$$

$$= \frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3} - \frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2} + \frac{3ax^5\sqrt[6]{a+bx^2}}{352b} + \frac{3}{22}x^7\sqrt[6]{a+bx^2} + \frac{\left(243a^4\sqrt{-}\right.}{}$$

$$= \frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3} - \frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2} + \frac{3ax^5\sqrt[6]{a+bx^2}}{352b} + \frac{3}{22}x^7\sqrt[6]{a+bx^2} - \frac{81\ 3^{3/4}\sqrt{2}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 8.79, size = 105, normalized size = 0.30

$$\frac{3x\sqrt[6]{a+bx^2}\left(\sqrt[6]{1+\frac{bx^2}{a}}\,(27a^3-3a^2bx^2+2ab^2x^4+32b^3x^6)-27a^3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{704b^3\sqrt[6]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6)*((1 + (b*x^2)/a)^(1/6)*(27*a^3 - 3*a^2*b*x^2 + 2*a*b
^2*x^4 + 32*b^3*x^6) - 27*a^3*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a
)]))/(704*b^3*(1 + (b*x^2)/a)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{6} \left(b\,x^{2}+a\right)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^(1/6),x)
[Out] int(x^6*(b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/6)*x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/6)*x^6, x)
```

Sympy [A]
time = 0.55, size = 29, normalized size = 0.08

$$\frac{\sqrt[6]{a}\,x^{7}{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**(1/6),x)
[Out] a**(1/6)*x**7*hyper((-1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,\left(b\,x^2+a\right)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^(1/6),x)
```

```
[Out] int(x^6*(a + b*x^2)^(1/6), x)
```

3.1011 $\int x^4\sqrt[6]{a+bx^2}\,dx$

Optimal. Leaf size=321

$$-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\cdots}{640b^3x\sqrt[3]{\frac{a}{a+\cdots}}\cdots}$$

```
[Out] -27/640*a^2*x*(b*x^2+a)^(1/6)/b^2+3/160*a*x^3*(b*x^2+a)^(1/6)/b+3/16*x^5*(b
*x^2+a)^(1/6)+27/640*3^(3/4)*a^3*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*El
lipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I
-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))
^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^3/x/(a/(b*x^2+a))^(1/3)/
((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {285, 327, 247, 242, 225}

$$\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{640b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(1/6),x]
```

```
[Out] (-27*a^2*x*(a + b*x^2)^(1/6))/(640*b^2) + (3*a*x^3*(a + b*x^2)^(1/6))/(160*
b) + (3*x^5*(a + b*x^2)^(1/6))/16 + (27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a +
b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) +
(a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[A
rcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^
(1/3))], -7 + 4*Sqrt[3]])/(640*b^3*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(
a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\sqrt[6]{a+bx^2}\,dx=\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{1}{16}a\int\frac{x^4}{(a+bx^2)^{5/6}}\,dx$$

$$=\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}-\frac{(9a^2)\int\frac{x^2}{(a+bx^2)^{5/6}}\,dx}{160b}$$

$$=-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{(27a^3)\int\frac{1}{(a+bx^2)^{5/6}}\,dx}{640b^2}$$

$$=-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{(27a^3)\,\text{Subst}\left(\int\frac{1}{(1-bx^2)^{2/3}}\,d\right.}{640b^2\sqrt[3]{\frac{a}{a+bx^2}}\,\sqrt[3]{}}$$

$$=-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}-\frac{\left(81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{a+b}\right.}{}$$

$$=-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{27\;3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\sqrt{-\frac{}{a}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.82, size = 93, normalized size = 0.29

$$\frac{3x\sqrt[6]{a+bx^2}\left(\sqrt[6]{1+\frac{bx^2}{a}}\,(-9a^2+abx^2+10b^2x^4)+9a^2\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{160b^2\sqrt[6]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6)*((1 + (b*x^2)/a)^(1/6)*(-9*a^2 + a*b*x^2 + 10*b^2*x^
4) + 9*a^2*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a)]))/(160*b^2*(1 +
(b*x^2)/a)^(1/6))
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(b\,x^2+a\right)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^4*(b*x^2+a)^(1/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)*x^4, x)
```

Sympy [**A**]
time = 0.51, size = 29, normalized size = 0.09

$$\frac{\sqrt[6]{a}\,x^5\,{}_2F_1\left(\begin{matrix} -\frac{1}{6},\frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/6),x)
```

```
[Out] a**(1/6)*x**5*hyper((-1/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(b x^2+a\right)^{1/6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(1/6),x)
```

```
[Out] int(x^4*(a + b*x^2)^(1/6), x)
```

3.1012 $\int x^2 \sqrt[6]{a+bx^2}\, dx$

Optimal. Leaf size=297

$$\frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\cdots}{40b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\cdots\right)^2}}}$$

```
[Out] 3/40*a*x*(b*x^2+a)^(1/6)/b+3/10*x^3*(b*x^2+a)^(1/6)-3/40*3^(3/4)*a^2*(b*x^2
+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))
/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*(
(1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))
^2)^(1/2)/b^2/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+
a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {285, 327, 247, 242, 225}

$$-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{40b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*a*x*(a + b*x^2)^(1/6))/(40*b) + (3*x^3*(a + b*x^2)^(1/6))/10 - (3*3^(3/4
)*Sqrt[2 - Sqrt[3]]*a^2*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[
(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(40*b^2*x*(a/(a + b*
x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2
))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\sqrt[6]{a+bx^2}\,dx = \frac{3}{10}x^3\sqrt[6]{a+bx^2}+\frac{1}{10}a\int\frac{x^2}{(a+bx^2)^{5/6}}\,dx$$

$$=\frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}-\frac{(3a^2)\int\frac{1}{(a+bx^2)^{5/6}}\,dx}{40b}$$

$$=\frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}-\frac{(3a^2)\,\mathrm{Subst}\left(\int\frac{1}{(1-bx^2)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{40b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$=\frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}+\frac{\left(9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{-1+x^3}}\right.}{80b^2x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$=\frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\right.}}{40b^2x\sqrt[3]{\frac{a}{a+b}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.56, size = 62, normalized size = 0.21

$$\frac{3x\sqrt[6]{a+bx^2}\left(a+bx^2-\frac{a\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6)*(a + b*x^2 - (a*Hypergeometric2F1[-1/6, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(1/6)))/(10*b)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(b\,x^{2}+a \right)^{\frac{1}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/6),x)
[Out] int(x^2*(b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/6)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/6)*x^2, x)
```

Sympy [A]
time = 0.45, size = 29, normalized size = 0.10

$$\frac{\sqrt[6]{a}\, x^{3} {{}_{2}F_{1}\left(\begin{matrix} - \frac{1}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/6),x)
[Out] a**(1/6)*x**3*hyper((-1/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(b x^2 + a\right)^{1/6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/6),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/6), x)
```

3.1013 $\int \sqrt[6]{a+bx^2}\,dx$

Optimal. Leaf size=273

$$\frac{3}{4}x\sqrt[6]{a+bx^2}+\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1\cdots}{1\cdots}\right.\right.}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}}$$

```
[Out] 3/4*x*(b*x^2+a)^(1/6)+1/4*3^(3/4)*a*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))
*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),
2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+
a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b/x/(a/(b*x^2+a))^(1/3)
/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {201, 247, 242, 225}

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3}{4}x\sqrt[6]{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6))/4 + (3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a + b*x^2)^(1/6)*
(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2
))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 +
 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3)
)/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rubi steps

$$\int \sqrt[6]{a+bx^2}\,dx = \frac{3}{4}x\sqrt[6]{a+bx^2} + \frac{1}{4}a\int \frac{1}{(a+bx^2)^{5/6}}\,dx$$

$$= \frac{3}{4}x\sqrt[6]{a+bx^2} + \frac{a\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{4\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= \frac{3}{4}x\sqrt[6]{a+bx^2} - \frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{8bx\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= \frac{3}{4}x\sqrt[6]{a+bx^2} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+\cdots}}}{\left(1-\sqrt{\cdots}\right.}}}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a\cdots}}}{\left(1-\sqrt{3}-\sqrt[3]{\cdots}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.52, size = 46, normalized size = 0.17

$$\frac{x\sqrt[6]{a+bx^2}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6),x]
```

```
[Out] (x*(a + b*x^2)^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(1/6)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6),x)
[Out] int((b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/6), x)
```

Sympy [A]
time = 0.43, size = 26, normalized size = 0.10

$$\sqrt[6]{a}\, x\, {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6),x)
[Out] a**(1/6)*x*hyper((-1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/6), x)
```

Mupad [B]
time = 4.88, size = 37, normalized size = 0.14

$$\frac{x\,{\left(b\,x^2+a\right)}^{1/6}\,{}_2\mathrm{F}_1\left(-\frac{1}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/6),x)
[Out] (x*(a + b*x^2)^(1/6)*hypergeom([-1/6, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(1/6)
```

3.1014 $\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=266

$$-\frac{\sqrt[6]{a+bx^2}}{x}+\frac{\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\cdots}{1-\sqrt{3}-\cdots}\right.\right.}{\sqrt[4]{3}\,x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}}$$

```
[Out] -(b*x^2+a)^(1/6)/x+1/3*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1
-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2)
)*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1
-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/x/(a/(b*x^2+a))^(1/3)/((-1+(
a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {283, 247, 242, 225}

$$\frac{\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{\sqrt[6]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^2,x]
```

```
[Out] -((a + b*x^2)^(1/6)/x) + (Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a/(a +
b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1
- Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a
 + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/
(3^(1/4)*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[6]{a+bx^2}}{x} + \frac{1}{3}b\int \frac{1}{(a+bx^2)^{5/6}}\,dx$$

$$= -\frac{\sqrt[6]{a+bx^2}}{x} + \frac{b\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{x} - \frac{\left(\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{x} + \frac{\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\cdots}\right.}}}{\sqrt[4]{3}\,x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+b\cdots}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{}{a\cdots}}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.63, size = 49, normalized size = 0.18

$$-\frac{\sqrt[6]{a+bx^2}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[6]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(1/6)*Hypergeometric2F1[-1/2, -1/6, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(1/6)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{6}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^2,x)
```

```
[Out] int((b*x^2+a)^(1/6)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/x^2, x)
```

Sympy [A]
time = 0.48, size = 29, normalized size = 0.11

$$-\frac{\sqrt[6]{a}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{6} \\ \frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**2,x)
```

```
[Out] -a**(1/6)*hyper((-1/2, -1/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="giac")
```

[Out] integrate((b*x^2 + a)^(1/6)/x^2, x)

Mupad [B]
time = 5.08, size = 40, normalized size = 0.15

$$-\frac{3\left(b\,x^2+a\right)^{1/6}\,{}_2\mathrm{F}_1\left(-\frac{1}{6},\frac{1}{3};\ \frac{4}{3};\ -\frac{a}{b\,x^2}\right)}{2\,x\,{\left(\frac{a}{b\,x^2}+1\right)}^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^(1/6)/x^2,x)

[Out] -(3*(a + b*x^2)^(1/6)*hypergeom([-1/6, 1/3], 4/3, -a/(b*x^2)))/(2*x*(a/(b*x^2) + 1)^(1/6))

3.1015 $\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx$

Optimal. Leaf size=297

$$-\frac{\sqrt[6]{a+bx^2}}{3x^3}-\frac{b\sqrt[6]{a+bx^2}}{9ax}-\frac{2\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\ldots\right.}{9\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}}$$

```
[Out] -1/3*(b*x^2+a)^(1/6)/x^3-1/9*b*(b*x^2+a)^(1/6)/a/x-2/27*b*(b*x^2+a)^(1/6)*(
1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x
^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^
2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3
^(3/4)/a/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(
1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {283, 331, 247, 242, 225}

$$-\frac{2\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{b\sqrt[6]{a+bx^2}}{9ax}-\frac{\sqrt[6]{a+bx^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^4,x]
```

```
[Out] -1/3*(a + b*x^2)^(1/6)/x^3 - (b*(a + b*x^2)^(1/6))/(9*a*x) - (2*Sqrt[2 - Sq
rt[3]]*b*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*
x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a
/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*(a/(a + b*x^2))^(1/3
)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^
2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[6]{a+bx^2}}{3x^3} + \frac{1}{9}b\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx$$
$$= -\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} - \frac{(2b^2)\int \frac{1}{(a+bx^2)^{5/6}}\,dx}{27a}$$
$$= -\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{27a\sqrt[3]{\frac{a}{a+bx^2}}\,\sqrt[3]{a+bx^2}}$$
$$= -\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} + \frac{\left(b\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x,\right.}{9ax\sqrt[3]{\frac{a}{a+bx^2}}}$$
$$= -\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} - \frac{2\sqrt{2-\sqrt{3}}\,b\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)}{9\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.17

$$-\frac{\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[6]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^4,x]
```

```
[Out] -1/3*((a + b*x^2)^(1/6)*Hypergeometric2F1[-3/2, -1/6, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{6}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^4,x)
```

```
[Out] int((b*x^2+a)^(1/6)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/x^4, x)
```

Sympy [A]
time = 0.54, size = 34, normalized size = 0.11

$$-\frac{\sqrt[6]{a}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},-\frac{1}{6}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**4,x)
```

```
[Out] -a**(1/6)*hyper((-3/2, -1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{1/6}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/6)/x^4,x)
```

```
[Out] int((a + b*x^2)^(1/6)/x^4, x)
```

3.1016 $\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx$

Optimal. Leaf size=323

$$-\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}+\frac{16\sqrt{2-\sqrt{3}}\,b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\right.}}}}{135\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{}{\left(\right.}}}$$

```
[Out] -1/5*(b*x^2+a)^(1/6)/x^5-1/45*b*(b*x^2+a)^(1/6)/a/x^3+8/135*b^2*(b*x^2+a)^(
1/6)/a^2/x+16/405*b^2*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-
(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))
*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-
(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a^2/x/(a/(b*x^2+a))^(1/3)/((-
1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {283, 331, 247, 242, 225}

$$\frac{16\sqrt{2-\sqrt{3}}\,b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{135\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}-\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^6,x]
```

```
[Out] -1/5*(a + b*x^2)^(1/6)/x^5 - (b*(a + b*x^2)^(1/6))/(45*a*x^3) + (8*b^2*(a +
 b*x^2)^(1/6))/(135*a^2*x) + (16*Sqrt[2 - Sqrt[3]]*b^2*(a + b*x^2)^(1/6)*(1
 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))
^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt
[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4
*Sqrt[3]])/(135*3^(1/4)*a^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[6]{a+bx^2}}{5x^5} + \frac{1}{15}b\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5x^5} - \frac{b\sqrt[6]{a+bx^2}}{45ax^3} - \frac{\left(8b^2\right)\int \frac{1}{x^2(a+bx^2)^{5/6}}\,dx}{135a}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5x^5} - \frac{b\sqrt[6]{a+bx^2}}{45ax^3} + \frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x} + \frac{\left(16b^3\right)\int \frac{1}{(a+bx^2)^{5/6}}\,dx}{405a^2}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5x^5} - \frac{b\sqrt[6]{a+bx^2}}{45ax^3} + \frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x} + \frac{\left(16b^3\right)\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right.}{405a^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5x^5} - \frac{b\sqrt[6]{a+bx^2}}{45ax^3} + \frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x} - \frac{\left(8b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int\right.}{135a^2x\sqrt[3]{\frac{a}{a+b}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5x^5} - \frac{b\sqrt[6]{a+bx^2}}{45ax^3} + \frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x} + \frac{16\sqrt{2-\sqrt{3}}\,b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(\right.}{135\sqrt[4]{3}\,a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.16

$$-\frac{\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[6]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^6,x]
```

```
[Out] -1/5*((a + b*x^2)^(1/6)*Hypergeometric2F1[-5/2, -1/6, -3/2, -((b*x^2)/a)])/
(x^5*(1 + (b*x^2)/a)^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{6}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^6,x)
[Out] int((b*x^2+a)^(1/6)/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/6)/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/6)/x^6, x)
```

Sympy [A]
time = 0.61, size = 34, normalized size = 0.11

$$-\frac{\sqrt[6]{a}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},-\frac{1}{6}\\-\frac{3}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**6,x)
[Out] -a**(1/6)*hyper((-5/2, -1/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{1/6}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/6)/x^6,x)
```

```
[Out] int((a + b*x^2)^(1/6)/x^6, x)
```

3.1017 $\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx$

Optimal. Leaf size=347

$$-\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x}-\frac{32\sqrt{2-\sqrt{3}}\,b^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)}{405\sqrt[4]{3}\,a^3x}$$

```
[Out] -1/7*(b*x^2+a)^(1/6)/x^7-1/105*b*(b*x^2+a)^(1/6)/a/x^5+2/135*b^2*(b*x^2+a)^
(1/6)/a^2/x^3-16/405*b^3*(b*x^2+a)^(1/6)/a^3/x-32/1215*b^3*(b*x^2+a)^(1/6)*
(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*
x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x
^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*
3^(3/4)/a^3/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a)
)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {283, 331, 247, 242, 225}

$$-\frac{32\sqrt{2-\sqrt{3}}\,b^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{405\sqrt[4]{3}\,a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x}+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^8,x]
```

```
[Out] -1/7*(a + b*x^2)^(1/6)/x^7 - (b*(a + b*x^2)^(1/6))/(105*a*x^5) + (2*b^2*(a
+ b*x^2)^(1/6))/(135*a^2*x^3) - (16*b^3*(a + b*x^2)^(1/6))/(405*a^3*x) - (3
2*Sqrt[2 - Sqrt[3]]*b^3*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[
(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(405*3^(1/4)*a^3*x*(
a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx = -\frac{\sqrt[6]{a+bx^2}}{7x^7} + \frac{1}{21}b\int \frac{1}{x^6\left(a+bx^2\right)^{5/6}}\,dx$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} - \frac{(2b^2)\int \frac{1}{x^4(a+bx^2)^{5/6}}\,dx}{45a}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} + \frac{(16b^3)\int \frac{1}{x^2(a+bx^2)^{5/6}}\,dx}{405a^2}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} - \frac{(32b^4)\int \frac{1}{(a+bx^2)^{5/6}}\,dx}{1215a^3}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} - \frac{(32b^4)\,\text{Subst}\left(\int \frac{1}{(1-bx^2}\right.}{1215a^3\sqrt[3]{\frac{a}{a+}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} + \frac{\left(16b^3\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{}\right.}{}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} - \frac{32\sqrt{2-\sqrt{3}}\,b^3\sqrt{-\frac{}{a}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.15

$$-\frac{\sqrt[6]{a+bx^2}\;{}_2F_1\left(-\frac{7}{2},-\frac{1}{6};-\frac{5}{2};-\frac{bx^2}{a}\right)}{7x^7\sqrt[6]{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^8,x]
```

[Out] -1/7*((a + b*x^2)^(1/6)*Hypergeometric2F1[-7/2, -1/6, -5/2, -((b*x^2)/a)])/(x^7*(1 + (b*x^2)/a)^(1/6))

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{6}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(1/6)/x^8,x)

[Out] int((b*x^2+a)^(1/6)/x^8,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(1/6)/x^8, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(1/6)/x^8, x)

Sympy [A]
time = 0.74, size = 34, normalized size = 0.10

$$-\frac{\sqrt[6]{a}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{7}{2}, -\frac{1}{6} \\ -\frac{5}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(1/6)/x**8,x)

[Out] -a**(1/6)*hyper((-7/2, -1/6), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/6)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{1/6}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/6)/x^8,x)
[Out] int((a + b*x^2)^(1/6)/x^8, x)
```

3.1018 $\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=659

$$-\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}}+\frac{81a^2x(a+bx^2)^{5/6}}{448b^3}-\frac{9ax^3(a+bx^2)^{5/6}}{56b^2}+\frac{3x^5(a+bx^2)^{5/6}}{20b}-\frac{243a^4x}{896b^3\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1\right.}$$

```
[Out] -243/896*a^3*x/b^3/(b*x^2+a)^(1/6)+81/448*a^2*x*(b*x^2+a)^(5/6)/b^3-9/56*a*
x^3*(b*x^2+a)^(5/6)/b^2+3/20*x^5*(b*x^2+a)^(5/6)/b-243/896*a^4*x/b^3/(a/(b*
x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))+81/896*3^(3/4
)*a^4*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-
(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x
^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^4/x/(a/(b*x^2+a))^
(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1
/3)-3^(1/2))^2)^(1/2)-243/1792*3^(1/4)*a^4*(1-(a/(b*x^2+a))^(1/3))*Elliptic
E((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(
1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3
^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^4/x/(a/(b*x^2+a))^(2/3)/(b*x^2
+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2
)
```

Rubi [A]

time = 0.51, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {327, 244, 204, 241, 310, 225, 1893}

$$\frac{81\,3^{3/4}a^4\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{448\sqrt{2}\,b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{243\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^4\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{1792b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{243a^4x}{896b^3\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}}+\frac{81a^2x(a+bx^2)^{5/6}}{448b^3}-\frac{9ax^3(a+bx^2)^{5/6}}{56b^2}+\frac{3x^5(a+bx^2)^{5/6}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(1/6),x]
```

```
[Out] (-243*a^3*x)/(896*b^3*(a + b*x^2)^(1/6)) + (81*a^2*x*(a + b*x^2)^(5/6))/(44
8*b^3) - (9*a*x^3*(a + b*x^2)^(5/6))/(56*b^2) + (3*x^5*(a + b*x^2)^(5/6))/(
20*b) - (243*a^4*x)/(896*b^3*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - S
```

```
qrt[3] - (a/(a + b*x^2))^(1/3))) - (243*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 -
(a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2
/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3]
 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sq
rt[3]])/(1792*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/
(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (81*3^(3/4
)*a^4*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a +
 b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(
1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(448*Sqrt[2]*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x^5(a+bx^2)^{5/6}}{20b} - \frac{(3a)\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx}{4b}$$

$$= -\frac{9ax^3(a+bx^2)^{5/6}}{56b^2} + \frac{3x^5(a+bx^2)^{5/6}}{20b} + \frac{(27a^2)\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{56b^2}$$

$$= \frac{81a^2x(a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3(a+bx^2)^{5/6}}{56b^2} + \frac{3x^5(a+bx^2)^{5/6}}{20b} - \frac{(81a^3)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{448b^3}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x(a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3(a+bx^2)^{5/6}}{56b^2} + \frac{3x^5(a+bx^2)^{5/6}}{20b} + \frac{(81a^4)}{}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x(a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3(a+bx^2)^{5/6}}{56b^2} + \frac{3x^5(a+bx^2)^{5/6}}{20b} + \frac{(81a^4)}{}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x(a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3(a+bx^2)^{5/6}}{56b^2} + \frac{3x^5(a+bx^2)^{5/6}}{20b} - \frac{\left(243a\right.}{}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x(a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3(a+bx^2)^{5/6}}{56b^2} + \frac{3x^5(a+bx^2)^{5/6}}{20b} + \frac{\left(243a\right.}{}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x(a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3(a+bx^2)^{5/6}}{56b^2} + \frac{3x^5(a+bx^2)^{5/6}}{20b} + \frac{}{896b^4x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.75, size = 90, normalized size = 0.14

$$\frac{3\left(135a^3x + 15a^2bx^3 - 8ab^2x^5 + 112b^3x^7 - 135a^3x\sqrt[6]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{2240b^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(1/6),x]
```

```
[Out] (3*(135*a^3*x + 15*a^2*b*x^3 - 8*a*b^2*x^5 + 112*b^3*x^7 - 135*a^3*x*(1 + (
b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)]))/(2240*b^3*
(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^6/(b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(1/6), x)
```

Sympy [A]
time = 0.53, size = 27, normalized size = 0.04

$$\frac{x^{7}{}_{2}F_{1}\left(\begin{matrix}\frac{1}{6},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{7\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(1/6),x)
```

```
[Out] x**7*hyper((1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(b\,x^2+a\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(1/6),x)
```

```
[Out] int(x^6/(a + b*x^2)^(1/6), x)
```

3.1019 $\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=635

$$\frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}}-\frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2}+\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}+\frac{81a^3x}{224b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right.}$$

```
[Out] 81/224*a^2*x/b^2/(b*x^2+a)^(1/6)-27/112*a*x*(b*x^2+a)^(5/6)/b^2+3/14*x^3*(b
*x^2+a)^(5/6)/b+81/224*a^3*x/b^2/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/
(b*x^2+a))^(1/3)-3^(1/2))-27/224*3^(3/4)*a^3*(1-(a/(b*x^2+a))^(1/3))*Ellipt
icF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3
^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)
-3^(1/2))^2)^(1/2)/b^3/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(
a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+81/448*3^(1/4)
*a^3*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(
a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^
2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1
/2))/b^3/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1
-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {327, 244, 204, 241, 310, 225, 1893}

$$-\frac{27\,3^{3/4}a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{112\sqrt{2}\,b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{448b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{81a^3x}{224b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}}-\frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2}+\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(1/6),x]
```

```
[Out] (81*a^2*x)/(224*b^2*(a + b*x^2)^(1/6)) - (27*a*x*(a + b*x^2)^(5/6))/(112*b^
2) + (3*x^3*(a + b*x^2)^(5/6))/(14*b) + (81*a^3*x)/(224*b^2*(a/(a + b*x^2))
^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (81*3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^
```

```
2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]
*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(448*b^3*x*(a/(a + b*x^2))^(2/3)*(a +
 b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))^2)]) - (27*3^(3/4)*a^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a
/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt
[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(112*Sqrt[2]*b^3*x*(a/(a +
b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x^3(a+bx^2)^{5/6}}{14b} - \frac{(9a)\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{14b}$$

$$= -\frac{27ax(a+bx^2)^{5/6}}{112b^2} + \frac{3x^3(a+bx^2)^{5/6}}{14b} + \frac{(27a^2)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{112b^2}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax(a+bx^2)^{5/6}}{112b^2} + \frac{3x^3(a+bx^2)^{5/6}}{14b} - \frac{(27a^3)\int \frac{1}{(a+bx^2)^{7/6}}\,dx}{224b^2}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax(a+bx^2)^{5/6}}{112b^2} + \frac{3x^3(a+bx^2)^{5/6}}{14b} - \frac{(27a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,d\ldots\right.}{224b^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+\ldots}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax(a+bx^2)^{5/6}}{112b^2} + \frac{3x^3(a+bx^2)^{5/6}}{14b} + \frac{\left(81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\ldots\right.}{448b^3x\left(\frac{a}{a+b\ldots}\right.}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax(a+bx^2)^{5/6}}{112b^2} + \frac{3x^3(a+bx^2)^{5/6}}{14b} - \frac{\left(81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\ldots\right.}{448b^3x\left(\frac{a}{a+b\ldots}\right.}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax(a+bx^2)^{5/6}}{112b^2} + \frac{3x^3(a+bx^2)^{5/6}}{14b} - \frac{81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\ \ldots}{224b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\ \ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 8.63, size = 79, normalized size = 0.12

$$\frac{3\left(-9a^2x - abx^3 + 8b^2x^5 + 9a^2x\sqrt[6]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{112b^2\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(1/6),x]
```

```
[Out] (3*(-9*a^2*x - a*b*x^3 + 8*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)]))/(112*b^2*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/6),x)
[Out] int(x^4/(b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="fricas")
[Out] integral(x^4/(b*x^2 + a)^(1/6), x)
```

Sympy [A]
time = 0.46, size = 27, normalized size = 0.04

$$\frac{x^5\,{}_2F_1\left(\begin{matrix} \frac{1}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/6),x)
[Out] x**5*hyper((1/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/6),x)
[Out] int(x^4/(a + b*x^2)^(1/6), x)
```

3.1020 $\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=611

$$-\frac{9ax}{16b\sqrt[6]{a+bx^2}}+\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{9a^2x}{16b\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^2}{}$$

```
[Out] -9/16*a*x/b/(b*x^2+a)^(1/6)+3/8*x*(b*x^2+a)^(5/6)/b-9/16*a^2*x/b/(a/(b*x^2+
a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))+3/16*3^(3/4)*a^2*
(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*
x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))
^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^2/x/(a/(b*x^2+a))^(2/3)/
(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^
(1/2))^2)^(1/2)-9/32*3^(1/4)*a^2*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b
*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+
(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)
^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/
((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {327, 244, 204, 241, 310, 225, 1893}

$$\frac{3\,3^{3/4}a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{8\sqrt{2}\,b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{32b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{9a^2x}{16b\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{9ax}{16b\sqrt[6]{a+bx^2}}+\frac{3x(a+bx^2)^{5/6}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(1/6),x]
```

```
[Out] (-9*a*x)/(16*b*(a + b*x^2)^(1/6)) + (3*x*(a + b*x^2)^(5/6))/(8*b) - (9*a^2*
x)/(16*b*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))) - (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^2*(1 - (a/(a + b*x^2))^(1/3))
*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a
/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/
```

```
3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(32*b^2*x*(a/(
a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (3*3^(3/4)*a^2*(1 - (a/(a + b*x^2)
)^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt
[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(8*Sqr
t[2]*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x(a+bx^2)^{5/6}}{8b} - \frac{(3a)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{8b}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x(a+bx^2)^{5/6}}{8b} + \frac{(3a^2)\int \frac{1}{(a+bx^2)^{7/6}}\,dx}{16b}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x(a+bx^2)^{5/6}}{8b} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x(a+bx^2)^{5/6}}{8b} - \frac{\left(9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\right.}{32b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x(a+bx^2)^{5/6}}{8b} + \frac{\left(9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\right.}{32b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x(a+bx^2)^{5/6}}{8b} + \frac{9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{16b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.41, size = 62, normalized size = 0.10

$$\frac{3x\left(a+bx^2-a\sqrt[6]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{8b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2,
-((b*x^2)/a)]))/(8*b*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(b\,x^{2}+a\right)^{\frac{1}{6}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/6),x)
[Out] int(x^2/(b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="fricas")
[Out] integral(x^2/(b*x^2 + a)^(1/6), x)
```

Sympy [A]
time = 0.46, size = 27, normalized size = 0.04

$$\frac{x^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/6),x)
[Out] x**3*hyper((1/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/6),x)
```

```
[Out] int(x^2/(a + b*x^2)^(1/6), x)
```

3.1021 $\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=577

$$\frac{3x}{2\sqrt[6]{a+bx^2}}+\frac{3ax}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\cdots}}{4bx\left(\frac{a}{a+bx^2}\right)^{2\cdots}}$$

```
[Out] 3/2*x/(b*x^2+a)^(1/6)+3/2*a*x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*
x^2+a))^(1/3)-3^(1/2))-1/2*3^(3/4)*a*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(
a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*
((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)
)^2)^(1/2)/b/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a
))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+3/4*3^(1/4)*a*(1-(a/(b*x
^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1
/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1
-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(a/(b*
x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1
/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$,
Rules used = {244, 204, 241, 310, 225, 1893}

$$-\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\,bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3x}{2\sqrt[6]{a+bx^2}}+\frac{3ax}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1/6),x]
```

```
[Out] (3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*x)/(2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^
(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]
*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*
x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 +
 Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -
```

```
7 + 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (3^(3/4
)*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b
*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1
+ Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))],
-7 + 4*Sqrt[3]])/(Sqrt[2]*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[
-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
```

```
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{1}{2}a\int \frac{1}{(a+bx^2)^{7/6}}\,dx$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{a\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} + \frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} + \frac{\left(3\sqrt{\frac{1}{2}\left(2\right.}\right.}{}$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.39, size = 46, normalized size = 0.08

$$\frac{x\sqrt[6]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1/6),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(1/6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/6),x)
```

```
[Out] int(1/(b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-1/6), x)
```

Sympy [A]
time = 0.41, size = 24, normalized size = 0.04

$$\frac{x\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/6),x)
```

```
[Out] x*hyper((1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/6)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-1/6), x)
```

Mupad [B]
time = 4.88, size = 37, normalized size = 0.06

$$\frac{x\,\left(\frac{b\,x^2}{a}+1\right)^{1/6}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/6),x)
[Out] (x*((b*x^2)/a + 1)^(1/6)*hypergeom([1/6, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(1/6)
```

3.1022 $\int \frac{1}{x^2 \sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=586

$$\frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{(a+bx^2)^{5/6}}{ax}+\frac{bx}{\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{a}{a+}}\right.}{}$$

[Out] b*x/a/(b*x^2+a)^(1/6)-(b*x^2+a)^(5/6)/a/x+b*x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))-1/3*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+1/2*3^(1/4)*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.37, antiderivative size = 586, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {331, 244, 204, 241, 310, 225, 1893}

$$-\frac{\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{bx}{a\sqrt[6]{a+bx^2}}+\frac{bx}{\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{(a+bx^2)^{5/6}}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^2)^(1/6)),x]

[Out] (b*x)/(a*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(a*x) + (b*x)/((a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a +

```
b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1
/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)
)^2)]) - (Sqrt[2]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/
3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)
^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1
/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{5/6}}{ax}+\frac{(2b)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{3a}$$

$$= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{(a+bx^2)^{5/6}}{ax}-\frac{1}{3}b\int \frac{1}{(a+bx^2)^{7/6}}\,dx$$

$$= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{(a+bx^2)^{5/6}}{ax}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{3\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{(a+bx^2)^{5/6}}{ax}+\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{(a+bx^2)^{5/6}}{ax}-\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\,\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{bx}{a\sqrt[6]{a+bx^2}}-\frac{(a+bx^2)^{5/6}}{ax}-\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}+ -$$

$\sqrt[4]{}$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.42, size = 49, normalized size = 0.08

$$-\frac{\sqrt[6]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(1/6)),x]
[Out] -(((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-1/2, 1/6, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(1/6)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(b\,x^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/6),x)
[Out] int(1/x^2/(b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(5/6)/(b*x^4 + a*x^2), x)
```

Sympy [A]
time = 0.46, size = 27, normalized size = 0.05

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[6]{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/6),x)
```

```
[Out] -hyper((-1/2, 1/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/6)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^2), x)
```

Mupad [B]
time = 5.09, size = 40, normalized size = 0.07

$$-\frac{3\left(\frac{a}{b\,x^2}+1\right)^{1/6}{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{2}{3};\ \frac{5}{3};\ -\frac{a}{b\,x^2}\right)}{4\,x\,\left(b\,x^2+a\right)^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/6)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(1/6)*hypergeom([1/6, 2/3], 5/3, -a/(b*x^2)))/(4*x*(a +
 b*x^2)^(1/6))
```

3.1023 $\int \frac{1}{x^4 \sqrt[6]{a+bx^2}}\, dx$

Optimal. Leaf size=633

$$2\sqrt{2\cdot}$$

$$-\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}-\frac{4b^2x}{9a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}-$$

```
[Out] -4/9*b^2*x/a^2/(b*x^2+a)^(1/6)-1/3*(b*x^2+a)^(5/6)/a/x^3+4/9*b*(b*x^2+a)^(5
/6)/a^2/x-4/9*b^2*x/a/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^
(1/3)-3^(1/2))+4/27*b*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1
/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/
(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1
/2)*3^(3/4)/a/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3
))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)-2/9*b*(1-(a/(b*x^2+a))^(1/3))*E
llipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*
I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^
(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a/x/(a/(b*x^2+a))
^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(
1/2))^2)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 633, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {331, 244, 204, 241, 310, 225, 1893}

$$-\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}+\frac{4\sqrt{2}\,b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{2\sqrt{2+\sqrt{3}}\,b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\,3^{3/4}ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{4b^2x}{9a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(1/6)),x]
```

```
[Out] (-4*b^2*x)/(9*a^2*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(3*a*x^3) + (4*b*(
a + b*x^2)^(5/6))/(9*a^2*x) - (4*b^2*x)/(9*a*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (2*Sqrt[2 + Sqrt[3]]*b*(
1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2)
```

```
)^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqr
t[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 +
4*Sqrt[3]])/(3*3^(3/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((
1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (4*
Sqrt[2]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/
(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/
3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{5/6}}{3ax^3} - \frac{(4b)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{9a}$$

$$= -\frac{(a+bx^2)^{5/6}}{3ax^3} + \frac{4b(a+bx^2)^{5/6}}{9a^2x} - \frac{(8b^2)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{27a^2}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{3ax^3} + \frac{4b(a+bx^2)^{5/6}}{9a^2x} + \frac{(4b^2)\int \frac{1}{(a+bx^2)^{7/6}}\,dx}{27a}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{3ax^3} + \frac{4b(a+bx^2)^{5/6}}{9a^2x} + \frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \cdots\right.}{27a\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)\cdots}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{3ax^3} + \frac{4b(a+bx^2)^{5/6}}{9a^2x} - \frac{\left(2b\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \cdots\right.}{9ax\left(\frac{a}{a+bx^2}\right)^{2/3}\cdots}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{3ax^3} + \frac{4b(a+bx^2)^{5/6}}{9a^2x} + \frac{\left(2b\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\cdots}{\sqrt{-\cdots}}\right.}{9ax\left(\frac{a}{a+bx^2}\right)^{2/3}\cdots}$$

$$= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{3ax^3} + \frac{4b(a+bx^2)^{5/6}}{9a^2x} + \frac{4b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\cdots}}{9ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{\cdots}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.08

$$-\frac{\sqrt[6]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(1/6)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-3/2, 1/6, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(b\,x^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/6),x)
[Out] int(1/x^4/(b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(5/6)/(b*x^6 + a*x^4), x)
```

Sympy [A]
time = 0.52, size = 32, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{1}{6} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{3 \sqrt[6]{a}\, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/6),x)
[Out] -hyper((-3/2, 1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/6)),x)
[Out] int(1/(x^4*(a + b*x^2)^(1/6)), x)
```

3.1024 $\int \frac{1}{x^6 \sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=661

$$\frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}}-\frac{(a+bx^2)^{5/6}}{5ax^5}+\frac{2b(a+bx^2)^{5/6}}{9a^2x^3}-\frac{8b^2(a+bx^2)^{5/6}}{27a^3x}+\frac{8b^3x}{27a^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\cdots}\right.}$$

[Out] 8/27*b^3*x/a^3/(b*x^2+a)^(1/6)-1/5*(b*x^2+a)^(5/6)/a/x^5+2/9*b*(b*x^2+a)^(5/6)/a^2/x^3-8/27*b^2*(b*x^2+a)^(5/6)/a^3/x+8/27*b^3*x/a^2/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))-8/81*b^2*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+4/27*b^2*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.47, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {331, 244, 204, 241, 310, 225, 1893}

$$\frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}}-\frac{8b^2(a+bx^2)^{5/6}}{27a^3x}-\frac{8\sqrt{2}\,b^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{27\sqrt[4]{3}\,a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{4\sqrt{2+\sqrt{3}}\,b^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\,3^{3/4}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{8b^3x}{27a^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{2b(a+bx^2)^{5/6}}{9a^2x^3}-\frac{(a+bx^2)^{5/6}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(a + b*x^2)^(1/6)),x]

[Out] (8*b^3*x)/(27*a^3*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(5*a*x^5) + (2*b*(a + b*x^2)^(5/6))/(9*a^2*x^3) - (8*b^2*(a + b*x^2)^(5/6))/(27*a^3*x) + (8*b^3*x)/(27*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (4*Sqrt[2 + Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a

```
 + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^2*x*
(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/
(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (8*Sqrt[2]*b^2*(1 - (a/(a + b*
x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 -
Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2
7*3^(1/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[6]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{5/6}}{5ax^5} - \frac{(2b)\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx}{3a}$$

$$= -\frac{(a+bx^2)^{5/6}}{5ax^5} + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} + \frac{(8b^2)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{27a^2}$$

$$= -\frac{(a+bx^2)^{5/6}}{5ax^5} + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x} + \frac{(16b^3)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{81a^3}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{5ax^5} + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x} - \frac{(8b^3)\int \frac{1}{(a+bx^2)^{7/}}}{81a^2}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{5ax^5} + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x} - \frac{(8b^3)\operatorname{Subst}\left(\int\right.}{81a^2\,(}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{5ax^5} + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x} + \frac{\left(4b^2\sqrt{-\frac{bx^2}{a+b}}\right.}{}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{5ax^5} + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x} - \frac{\left(4b^2\sqrt{-\frac{bx^2}{a+b}}\right.}{}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{(a+bx^2)^{5/6}}{5ax^5} + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x} - \frac{8b^2\sqrt{}}{27a^2x\left(\frac{a}{a+bx^2}\right)^{2/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.08

$$-\frac{\sqrt[6]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(1/6)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-5/2, 1/6, -3/2, -((b*x^2)/a)
])/(x^5*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(1/6),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b*x^8 + a*x^6), x)
```

Sympy [A]
time = 0.60, size = 32, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{1}{6} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{5 \sqrt[6]{a} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(1/6),x)
```

```
[Out] -hyper((-5/2, 1/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6)*x**5)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,{\left(b\,x^2+a\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(1/6)),x)
[Out] int(1/(x^6*(a + b*x^2)^(1/6)), x)
```

3.1025 $\int \frac{x^6}{\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=324

$$\frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3}-\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2}+\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\cdots}{\left(\cdots\right.}}}{128b^4x\sqrt[3]{\frac{a}{a+bx^2\cdots}}}$$

```
[Out] 81/128*a^2*x*(b*x^2+a)^(1/6)/b^3-9/32*a*x^3*(b*x^2+a)^(1/6)/b^2+3/16*x^5*(b
*x^2+a)^(1/6)/b-81/128*3^(3/4)*a^3*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*
EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2
*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a
))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^4/x/(a/(b*x^2+a))^(1/3
)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {327, 247, 242, 225}

$$-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{128b^4x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3}-\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2}+\frac{3x^5\sqrt[6]{a+bx^2}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(5/6),x]
```

```
[Out] (81*a^2*x*(a + b*x^2)^(1/6))/(128*b^3) - (9*a*x^3*(a + b*x^2)^(1/6))/(32*b^
2) + (3*x^5*(a + b*x^2)^(1/6))/(16*b) - (81*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(
a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3
) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ellipti
cF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^
2))^(1/3))], -7 + 4*Sqrt[3]])/(128*b^4*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx = \frac{3x^5\sqrt[6]{a+bx^2}}{16b} - \frac{(15a)\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx}{16b}$$

$$= -\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} + \frac{(27a^2)\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx}{32b^2}$$

$$= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} - \frac{(81a^3)\int \frac{1}{(a+bx^2)^{5/6}}\,dx}{128b^3}$$

$$= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} - \frac{(81a^3)\,\mathrm{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x,\right.}{128b^3\sqrt[3]{\frac{a}{a+bx^2}}\ \sqrt[3]{a+}}$$

$$= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} + \frac{\left(243a^3\sqrt{-\frac{bx^2}{a+bx^2}}\ \sqrt[6]{a+bx^2}\right.}{256}$$

$$= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b} - \frac{81\ 3^{3/4}\sqrt{2-\sqrt{3}}\ a^3\sqrt{-\frac{bx^2}{a+bx}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.50, size = 89, normalized size = 0.27

$$\frac{3x\left(27a^3+15a^2bx^2-4ab^2x^4+8b^3x^6-27a^3\left(1+\frac{bx^2}{a}\right)^{5/6}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{128b^3\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*x*(27*a^3 + 15*a^2*b*x^2 - 4*a*b^2*x^4 + 8*b^3*x^6 - 27*a^3*(1 + (b*x^2)
/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)]))/(128*b^3*(a + b*
x^2)^(5/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{\frac{5}{6}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/6),x)
[Out] int(x^6/(b*x^2+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="maxima")
[Out] integrate(x^6/(b*x^2 + a)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="fricas")
[Out] integral(x^6/(b*x^2 + a)^(5/6), x)
```

Sympy [A]
time = 0.46, size = 27, normalized size = 0.08

$$\frac{x^{7}{}_{2}F_{1}\left(\begin{matrix} \frac{5}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{7a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/6),x)
[Out] x**7*hyper((5/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(5/6), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(5/6),x)
```

```
[Out] int(x^6/(a + b*x^2)^(5/6), x)
```

3.1026 $\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx$

Optimal. Leaf size=300

$$-\frac{27ax\sqrt[6]{a+bx^2}}{40b^2}+\frac{3x^3\sqrt[6]{a+bx^2}}{10b}+\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\cdots}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{\cdots}}\cdots\right.}}\cdots}{40b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\cdots}{\left(1-\sqrt{3}\cdots\right.}}}$$

```
[Out] -27/40*a*x*(b*x^2+a)^(1/6)/b^2+3/10*x^3*(b*x^2+a)^(1/6)/b+27/40*3^(3/4)*a^2
*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3
^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(
1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3
^(1/2))^2)^(1/2)/b^3/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/
(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {327, 247, 242, 225}

$$\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{40b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{27ax\sqrt[6]{a+bx^2}}{40b^2}+\frac{3x^3\sqrt[6]{a+bx^2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/6),x]
```

```
[Out] (-27*a*x*(a + b*x^2)^(1/6))/(40*b^2) + (3*x^3*(a + b*x^2)^(1/6))/(10*b) + (
27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/
3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] -
 (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^
(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(40*b^3*x*(
a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx = \frac{3x^3\sqrt[6]{a+bx^2}}{10b} - \frac{(9a)\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx}{10b}$$

$$= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b} + \frac{(27a^2)\int \frac{1}{(a+bx^2)^{5/6}}\,dx}{40b^2}$$

$$= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b} + \frac{(27a^2)\,\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{40b^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b} - \frac{\left(81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1}\cdots}\right.}{80b^3x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b} + \frac{27\ 3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1\cdots\right.}{40b^3x\sqrt[3]{\frac{\cdots}{a\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 9.19, size = 79, normalized size = 0.26

$$\frac{3\left(-9a^2x-5abx^3+4b^2x^5+9a^2x\left(1+\frac{bx^2}{a}\right)^{5/6}\ {}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{40b^2\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*(-9*a^2*x - 5*a*b*x^3 + 4*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(5/6)*Hyperg
eometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)]))/(40*b^2*(a + b*x^2)^(5/6))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/6),x)
```

```
[Out] int(x^4/(b*x^2+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(5/6), x)
```

Sympy [A]
time = 0.49, size = 27, normalized size = 0.09

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{5}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(5/6),x)
```

```
[Out] x**5*hyper((5/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/6), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(5/6),x)
```

```
[Out] int(x^4/(a + b*x^2)^(5/6), x)
```

3.1027 $\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx$

Optimal. Leaf size=276

$$\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin^{-1}\left(\ldots\right.\right.}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}}$$

```
[Out] 3/4*x*(b*x^2+a)^(1/6)/b-3/4*3^(3/4)*a*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3
))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)
),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^
2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^2/x/(a/(b*x^2+a))^(
1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {327, 247, 242, 225}

$$\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a + b*x^2)^
(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a +
 b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(
1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx = \frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{(3a)\int \frac{1}{(a+bx^2)^{5/6}}\,dx}{4b}$$

$$= \frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{(3a)\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= \frac{3x\sqrt[6]{a+bx^2}}{4b} + \frac{\left(9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= \frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{}}{\left(1-\right.}}}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{}}{\left(1-\sqrt{3}-\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.47, size = 62, normalized size = 0.22

$$\frac{3x\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{5/6}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{4b\,(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2,
-((b*x^2)/a)]))/(4*b*(a + b*x^2)^(5/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/6),x)
```

```
[Out] int(x^2/(b*x^2+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(5/6), x)
```

Sympy [A]
time = 0.44, size = 27, normalized size = 0.10

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{5}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/6),x)
```

```
[Out] x**3*hyper((5/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(b\,x^2+a)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(5/6),x)
```

```
[Out] int(x^2/(a + b*x^2)^(5/6), x)
```

3.1028 $\int \frac{1}{(a+bx^2)^{5/6}}\,dx$

Optimal. Leaf size=252

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}}{1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}}\right.\right.}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}}$$

```
[Out] 3^(3/4)*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^
(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-
1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^
(1/3)-3^(1/2))^2)^(1/2)/b/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(
1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 242, 225}

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/6),x]
```

```
[Out] (3^(3/4)*Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sq
rt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a + b*x^
2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt[3]{\frac{a}{a+bx^2}}\,\sqrt[3]{a+bx^2}} \\
&= -\frac{\left(3\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2bx\sqrt[3]{\frac{a}{a+bx^2}}} \\
&= \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)\cdots}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)\cdots}}\cdots}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-\cdots}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.20, size = 46, normalized size = 0.18

$$\frac{x\left(1+\frac{bx^2}{a}\right)^{5/6}{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/6),x]
[Out] (x*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(5/6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/6),x)
[Out] int(1/(b*x^2+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(-5/6), x)
```

Sympy [A]
time = 0.42, size = 24, normalized size = 0.10

$$\frac{x{}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{5}{6} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/6),x)
```

```
[Out] x*hyper((1/2, 5/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/6)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-5/6), x)
```

Mupad [B]
time = 4.91, size = 37, normalized size = 0.15

$$\frac{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{5/6}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(5/6),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(5/6)*hypergeom([1/2, 5/6], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(5/6)
```

3.1029 $\int \frac{1}{x^2(a+bx^2)^{5/6}}\,dx$

Optimal. Leaf size=273

$$-\frac{\sqrt[6]{a+bx^2}}{ax}-\frac{2\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right.\right.}{\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}}$$

```
[Out] -(b*x^2+a)^(1/6)/a/x-2/3*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF(
(1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/
2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/
(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a/x/(a/(b*x^2+a))^(1/3)/((
-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {331, 247, 242, 225}

$$-\frac{2\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/6)),x]
```

```
[Out] -((a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a
/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3
))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt
[3]])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx = -\frac{\sqrt[6]{a+bx^2}}{ax} - \frac{(2b)\int \frac{1}{(a+bx^2)^{5/6}}\,dx}{3a}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{ax} - \frac{(2b)\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{ax} + \frac{\left(\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right.}{ax\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{ax} - \frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{}{a}}}{\left(1-\sqrt{}\right.}}}{\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-}{\left(1-\sqrt{3}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.08, size = 49, normalized size = 0.18

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{5/6}{}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/6)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-1/2, 5/6, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(5/6)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(b\,x^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/6),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/(b*x^4 + a*x^2), x)
```

Sympy [A]
time = 0.52, size = 27, normalized size = 0.10

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/6),x)
```

```
[Out] -hyper((-1/2, 5/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/6)*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^2), x)
```

Mupad [B]
time = 5.10, size = 40, normalized size = 0.15

$$-\frac{3\left(\frac{a}{b\,x^2}+1\right)^{5/6}\,{}_2\mathrm{F}_1\left(\frac{5}{6},\frac{4}{3};\ \frac{7}{3};\ -\frac{a}{b\,x^2}\right)}{8\,x\,\left(b\,x^2+a\right)^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(5/6)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(5/6)*hypergeom([5/6, 4/3], 7/3, -a/(b*x^2)))/(8*x*(a +
 b*x^2)^(5/6))
```

3.1030 $\int \frac{1}{x^4(a+bx^2)^{5/6}}\,dx$

Optimal. Leaf size=300

$$-\frac{\sqrt[6]{a+bx^2}}{3ax^3}+\frac{8b\sqrt[6]{a+bx^2}}{9a^2x}+\frac{16\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F}{9\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a-}}\right.}}}$$

```
[Out] -1/3*(b*x^2+a)^(1/6)/a/x^3+8/9*b*(b*x^2+a)^(1/6)/a^2/x+16/27*b*(b*x^2+a)^(1
/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a
/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/
(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1
/2)*3^(3/4)/a^2/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^
2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {331, 247, 242, 225}

$$\frac{16\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{8b\sqrt[6]{a+bx^2}}{9a^2x}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/6)),x]
```

```
[Out] -1/3*(a + b*x^2)^(1/6)/(a*x^3) + (8*b*(a + b*x^2)^(1/6))/(9*a^2*x) + (16*Sq
rt[2 - Sqrt[3]]*b*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (
a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^2*x*(a/(a + b
*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^
2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^2)^{5/6}}\,dx = -\frac{\sqrt[6]{a+bx^2}}{3ax^3} - \frac{(8b)\int \frac{1}{x^2(a+bx^2)^{5/6}}\,dx}{9a}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3ax^3} + \frac{8b\sqrt[6]{a+bx^2}}{9a^2x} + \frac{(16b^2)\int \frac{1}{(a+bx^2)^{5/6}}\,dx}{27a^2}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3ax^3} + \frac{8b\sqrt[6]{a+bx^2}}{9a^2x} + \frac{(16b^2)\,\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{27a^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3ax^3} + \frac{8b\sqrt[6]{a+bx^2}}{9a^2x} - \frac{\left(8b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,d\right.}{9a^2x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{3ax^3} + \frac{8b\sqrt[6]{a+bx^2}}{9a^2x} + \frac{16\sqrt{2-\sqrt{3}}\,b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+b}}\right.}{9\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.17

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{5/6} {}_2F_1\left(-\frac{3}{2},\frac{5}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/6)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-3/2, 5/6, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(5/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (b\,x^2+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/6),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/(b*x^6 + a*x^4), x)
```

Sympy [A]
time = 0.60, size = 32, normalized size = 0.11

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/6),x)
```

```
[Out] -hyper((-3/2, 5/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(5/6)),x)
[Out] int(1/(x^4*(a + b*x^2)^(5/6)), x)
```

3.1031 $\int \frac{1}{x^6 (a+bx^2)^{5/6}} \, dx$

Optimal. Leaf size=326

$$-\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}-\frac{224\sqrt{2-\sqrt{3}}\, b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}\cdots}{\left(1-\sqrt{\cdots}\right.\cdots}}\cdots}{135\sqrt[4]{3}\, a^3x\sqrt[3]{\frac{a}{a+bx^2}}\cdots}$$

```
[Out] -1/5*(b*x^2+a)^(1/6)/a/x^5+14/45*b*(b*x^2+a)^(1/6)/a^2/x^3-112/135*b^2*(b*x
^2+a)^(1/6)/a^3/x-224/405*b^2*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*Ellip
ticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*
3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2
/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a^3/x/(a/(b*x^2+a))^(
1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {331, 247, 242, 225}

$$-\frac{224\sqrt{2-\sqrt{3}}\, b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\, F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{135\sqrt[4]{3}\, a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(5/6)),x]
```

```
[Out] -1/5*(a + b*x^2)^(1/6)/(a*x^5) + (14*b*(a + b*x^2)^(1/6))/(45*a^2*x^3) - (1
12*b^2*(a + b*x^2)^(1/6))/(135*a^3*x) - (224*Sqrt[2 - Sqrt[3]]*b^2*(a + b*x
^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/
(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/
3))], -7 + 4*Sqrt[3]])/(135*3^(1/4)*a^3*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 -
 (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 247

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6 (a+bx^2)^{5/6}} dx = -\frac{\sqrt[6]{a+bx^2}}{5ax^5} - \frac{(14b)\int \frac{1}{x^4(a+bx^2)^{5/6}} dx}{15a}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5} + \frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3} + \frac{(112b^2)\int \frac{1}{x^2(a+bx^2)^{5/6}} dx}{135a^2}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5} + \frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3} - \frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x} - \frac{(224b^3)\int \frac{1}{(a+bx^2)^{5/6}} dx}{405a^3}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5} + \frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3} - \frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x} - \frac{(224b^3)\,\text{Subst}\left(\int \frac{1}{(1-bx^2)^{2/3}} dx, x\right.}{405a^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+}}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5} + \frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3} - \frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x} + \frac{\left(112b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right.}{135}$$

$$= -\frac{\sqrt[6]{a+bx^2}}{5ax^5} + \frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3} - \frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x} - \frac{224\sqrt{2-\sqrt{3}}\,b^2\sqrt{-\frac{bx^2}{a+bx^2}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.16

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{5/6} {}_2F_1\left(-\frac{5}{2},\frac{5}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(5/6)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-5/2, 5/6, -3/2, -((b*x^2)/a)
])/(x^5*(a + b*x^2)^(5/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6} \left(b \,x^{2}+a \right)^{\frac{5}{6}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(5/6),x)
[Out] int(1/x^6/(b*x^2+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/6)/(b*x^8 + a*x^6), x)
```

Sympy [A]
time = 0.71, size = 32, normalized size = 0.10

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{5}{6} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{5 a^{\frac{5}{6}} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(5/6),x)
[Out] -hyper((-5/2, 5/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,{\left(b\,x^2+a\right)}^{5/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(5/6)),x)
```

```
[Out] int(1/(x^6*(a + b*x^2)^(5/6)), x)
```

3.1032 $\int \frac{x^6}{(a+bx^2)^{7/6}} \, dx$

Optimal. Leaf size=654

$$\frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}-\frac{405ax(a+bx^2)^{5/6}}{112b^3}+\frac{45x^3(a+bx^2)^{5/6}}{14b^2}+\frac{1215a^3x}{224b^3\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}\right.}$$

```
[Out] 1215/224*a^2*x/b^3/(b*x^2+a)^(1/6)-3*x^5/b/(b*x^2+a)^(1/6)-405/112*a*x*(b*x
^2+a)^(5/6)/b^3+45/14*x^3*(b*x^2+a)^(5/6)/b^2+1215/224*a^3*x/b^3/(a/(b*x^2+
a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))-405/224*3^(3/4)*a
^3*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/
(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+
a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^4/x/(a/(b*x^2+a))^(2/
3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)
-3^(1/2))^2)^(1/2)+1215/448*3^(1/4)*a^3*(1-(a/(b*x^2+a))^(1/3))*EllipticE((
1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2
))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1
/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^4/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)
^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.51, antiderivative size = 654, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {294, 327, 244, 204, 241, 310, 225, 1893}

$$-\frac{405\, 3^{3/4}a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{112\sqrt{2}b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{1215\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{448b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{1215a^3x}{224b^3\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}-\frac{405ax(a+bx^2)^{5/6}}{112b^3}+\frac{45x^3(a+bx^2)^{5/6}}{14b^2}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(7/6),x]
```

```
[Out] (1215*a^2*x)/(224*b^3*(a + b*x^2)^(1/6)) - (3*x^5)/(b*(a + b*x^2)^(1/6)) -
(405*a*x*(a + b*x^2)^(5/6))/(112*b^3) + (45*x^3*(a + b*x^2)^(5/6))/(14*b^2)
 + (1215*a^3*x)/(224*b^3*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))) + (1215*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^3*(1 - (a/
```

```
(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3)
)/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[
3]])/(448*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (405*3^(3/4)*a
^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*
x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 +
 Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -
7 + 4*Sqrt[3]])/(112*Sqrt[2]*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*
Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)
])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx = -\frac{3x^5}{b\sqrt[6]{a+bx^2}} + \frac{15\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx}{b}$$

$$= -\frac{3x^5}{b\sqrt[6]{a+bx^2}} + \frac{45x^3(a+bx^2)^{5/6}}{14b^2} - \frac{(135a)\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{14b^2}$$

$$= -\frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax(a+bx^2)^{5/6}}{112b^3} + \frac{45x^3(a+bx^2)^{5/6}}{14b^2} + \frac{(405a^2)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{112b^3}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax(a+bx^2)^{5/6}}{112b^3} + \frac{45x^3(a+bx^2)^{5/6}}{14b^2} - \frac{(405a^3)\int}{22}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax(a+bx^2)^{5/6}}{112b^3} + \frac{45x^3(a+bx^2)^{5/6}}{14b^2} - \frac{(405a^3)\,\mathrm{Su}}{2}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax(a+bx^2)^{5/6}}{112b^3} + \frac{45x^3(a+bx^2)^{5/6}}{14b^2} + \frac{\left(1215a^3\right.}{}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax(a+bx^2)^{5/6}}{112b^3} + \frac{45x^3(a+bx^2)^{5/6}}{14b^2} - \frac{\left(1215a^3\right.}{}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax(a+bx^2)^{5/6}}{112b^3} + \frac{45x^3(a+bx^2)^{5/6}}{14b^2} - \frac{12}{224b^4x\left(\frac{}{a+}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 9.37, size = 79, normalized size = 0.12

$$\frac{405a^2x - 90abx^3 + 48b^2x^5 - 405a^2x\sqrt[6]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{224b^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(7/6),x]
```

```
[Out] (405*a^2*x - 90*a*b*x^3 + 48*b^2*x^5 - 405*a^2*x*(1 + (b*x^2)/a)^(1/6)*Hype
rgeometric2F1[1/2, 7/6, 3/2, -((b*x^2)/a)])/(224*b^3*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(7/6),x)
```

```
[Out] int(x^6/(b*x^2+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A]
time = 0.49, size = 27, normalized size = 0.04

$$\frac{x^{7}{}_{2}F_{1}\left(\begin{matrix} \frac{7}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{7a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(7/6),x)
[Out] x**7*hyper((7/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(7/6))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="giac")
[Out] integrate(x^6/(b*x^2 + a)^(7/6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(7/6),x)
[Out] int(x^6/(a + b*x^2)^(7/6), x)
```

3.1033 $\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx$

Optimal. Leaf size=630

81

$$-\frac{81ax}{16b^2\sqrt[6]{a+bx^2}}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}+\frac{27x(a+bx^2)^{5/6}}{8b^2}-\frac{81a^2x}{16b^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}-\,-$$

```
[Out] -81/16*a*x/b^2/(b*x^2+a)^(1/6)-3*x^3/b/(b*x^2+a)^(1/6)+27/8*x*(b*x^2+a)^(5/
6)/b^2-81/16*a^2*x/b^2/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))
^(1/3)-3^(1/2))+27/16*3^(3/4)*a^2*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(
b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1
+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2
)^(1/2)/b^3/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a)
)^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)-81/32*3^(1/4)*a^2*(1-(a/(
b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))
^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))
/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^3/x/(
a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a
))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {294, 327, 244, 204, 241, 310, 225, 1893}

$$\frac{27\, 3^{3/4} a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{8\sqrt{2}\,b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{32b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{81a^2x}{16b^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{27x(a+bx^2)^{5/6}}{8b^2}-\frac{81ax}{16b^2\sqrt[6]{a+bx^2}}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(7/6),x]
```

```
[Out] (-81*a*x)/(16*b^2*(a + b*x^2)^(1/6)) - (3*x^3)/(b*(a + b*x^2)^(1/6)) + (27*
x*(a + b*x^2)^(5/6))/(8*b^2) - (81*a^2*x)/(16*b^2*(a/(a + b*x^2))^(2/3)*(a
+ b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (81*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
```

```
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE
[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))], -7 + 4*Sqrt[3]])/(32*b^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)]) + (27*3^(3/4)*a^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2
))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(8*Sqrt[2]*b^3*x*(a/(a + b*x^2))^(2/3)
*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a
+ b*x^2))^(1/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx = -\frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{9\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{b}$$

$$= -\frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x(a+bx^2)^{5/6}}{8b^2} - \frac{(27a)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{8b^2}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x(a+bx^2)^{5/6}}{8b^2} + \frac{(27a^2)\int \frac{1}{(a+bx^2)^{7/6}}\,dx}{16b^2}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x(a+bx^2)^{5/6}}{8b^2} + \frac{(27a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x \ldots\right.}{16b^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx \ldots}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x(a+bx^2)^{5/6}}{8b^2} - \frac{\left(81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \ldots\right.}{32b^3x\left(\frac{a}{a+bx^2}\right)^{2 \ldots}}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x(a+bx^2)^{5/6}}{8b^2} + \frac{\left(81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \ldots\right.}{32b^3x\left(\frac{a}{a+bx^2}\right)^{2 \ldots}}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x(a+bx^2)^{5/6}}{8b^2} + \frac{81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1 \ldots}}{16b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1 - \ldots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.23, size = 65, normalized size = 0.10

$$\frac{3x\left(-9a+2bx^2+9a\sqrt[6]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{16b^2\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(7/6),x]
```

```
[Out] (3*x*(-9*a + 2*b*x^2 + 9*a*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6
, 3/2, -((b*x^2)/a)]))/(16*b^2*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(b\,x^{2}+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(7/6),x)
```

```
[Out] int(x^4/(b*x^2+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A]
time = 0.49, size = 27, normalized size = 0.04

$$\frac{x^{5}{}_{2}F_{1}\left(\begin{matrix} \frac{7}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{5 a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(7/6),x)
```

```
[Out] x**5*hyper((7/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(7/6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(7/6),x)
[Out] int(x^4/(a + b*x^2)^(7/6), x)
```

3.1034 $\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx$

Optimal. Leaf size=583

$$\frac{3x}{2b\sqrt[6]{a+bx^2}}+\frac{9ax}{2b\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\cdots}}{4b^2x\left(\frac{a}{a+bx^2}\right.\cdots}$$

```
[Out] 3/2*x/b/(b*x^2+a)^(1/6)+9/2*a*x/b/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a
/(b*x^2+a))^(1/3)-3^(1/2))-3/2*3^(3/4)*a*(1-(a/(b*x^2+a))^(1/3))*EllipticF(
(1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/
2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(
1/2))^2)^(1/2)/b^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b
*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+9/4*3^(1/4)*a*(1-(
a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+
a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/
3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^2/
x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^
2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 583, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {294, 244, 204, 241, 310, 225, 1893}

$$-\frac{3\,3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\,b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3x}{2b\sqrt[6]{a+bx^2}}+\frac{9ax}{2b\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(7/6),x]
```

```
[Out] (3*x)/(2*b*(a + b*x^2)^(1/6)) + (9*a*x)/(2*b*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a
+ b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[
(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
```

```
], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-
((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (
3*3^(3/4)*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (
a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[Ar
cSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(
1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3
))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx = -\frac{3x}{b\sqrt[6]{a+bx^2}} + \frac{3\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{b}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{(3a)\int \frac{1}{(a+bx^2)^{7/6}}\,dx}{2b}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{(3a)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} + \frac{\left(9a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{\left(9a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} + \frac{\left(9\sqrt{\vphantom{x}}\right.}{}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{2b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{\vphantom{x}}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.68, size = 58, normalized size = 0.10

$$\frac{3x - 3x\sqrt[6]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{2b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(7/6),x]
```

```
[Out] (3*x - 3*x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, -((b*x^2)
/a)])/(2*b*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(7/6),x)
[Out] int(x^2/(b*x^2+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(5/6)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A]
time = 0.49, size = 27, normalized size = 0.05

$$\frac{x^3{}_2F_1\left(\begin{matrix} \frac{7}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(7/6),x)
[Out] x**3*hyper((7/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(7/6),x)
```

```
[Out] int(x^2/(a + b*x^2)^(7/6), x)
```

3.1035 $\int \frac{1}{(a+bx^2)^{7/6}}\,dx$

Optimal. Leaf size=555

$$-\frac{3x}{\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}-\frac{3\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\right.}}}}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

[Out] -3*x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))+3^(3/4)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)-3/2*3^(1/4)*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.28, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {204, 241, 310, 225, 1893}

$$\frac{\sqrt{2}\,3^{3/4}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{3x}{\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-7/6),x]

[Out] (-3*x)/((a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]

```
 - (a/(a + b*x^2))^(1/3))^2)]) + (Sqrt[2]*3^(3/4)*(1 - (a/(a + b*x^2))^(1/3
))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] -
(a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(
1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a +
 b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$= -\frac{\left(3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{\left(3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} - \frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt{-\right.}{}$$

$$= \frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\left(1\right.}{2bx\left(\frac{}{a}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.53, size = 49, normalized size = 0.09

$$\frac{x\sqrt[6]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{a\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-7/6),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, -((b*x^2)/a)])/(a
*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^2+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/6),x)
```

```
[Out] int(1/(b*x^2+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A]
time = 0.47, size = 24, normalized size = 0.04

$$\frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{6} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/6),x)
```

```
[Out] x*hyper((1/2, 7/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/6)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-7/6), x)
```

Mupad [B]
time = 4.94, size = 37, normalized size = 0.07

$$\frac{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{7/6}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{7/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(7/6),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(7/6)*hypergeom([1/2, 7/6], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(7/6)
```

3.1036 $\int \frac{1}{x^2\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=614

$$2\sqrt[4]{3}\ \sqrt{2}$$

$$\frac{3}{ax\sqrt[6]{a+bx^2}}+\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4(a+bx^2)^{5/6}}{a^2x}+\frac{4bx}{a\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)}+$$

```
[Out] 3/a/x/(b*x^2+a)^(1/6)+4*b*x/a^2/(b*x^2+a)^(1/6)-4*(b*x^2+a)^(5/6)/a^2/x+4*b
*x/a/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))-4/
3*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(
b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)+(a/
(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a/x/(a/(
b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^
(1/3)-3^(1/2))^2)^(1/2)+2*3^(1/4)*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(
b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1
+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2
)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/a/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/(
(-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {296, 331, 244, 204, 241, 310, 225, 1893}

$$\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4(a+bx^2)^{5/6}}{a^2x}-\frac{4\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{4bx}{a\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{3}{ax\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(7/6)),x]
```

```
[Out] 3/(a*x*(a + b*x^2)^(1/6)) + (4*b*x)/(a^2*(a + b*x^2)^(1/6)) - (4*(a + b*x^2
)^(5/6))/(a^2*x) + (4*b*x)/(a*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 -
Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (2*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a
 + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/
(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a
```

```
/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]
])/(a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))
^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (4*Sqrt[2]*(1 - (a/(a
+ b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(
1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/
(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]]
)/(3^(1/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^2)^{7/6}}\,dx = \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{a}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x} + \frac{(8b)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{3a^2}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x} - \frac{(4b)\int \frac{1}{(a+bx^2)^{7/6}}\,dx}{3a}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x} - \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{}{\sqrt{a-}}\right.}{3a\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x} + \frac{\left(2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+}}\right.}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a-}}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x} - \frac{\left(2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}}{\sqrt{-1+}}\right.}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a-}}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x} - \frac{4\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{}{a+}}}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.18, size = 52, normalized size = 0.08

$$-\frac{\sqrt[6]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{2},\frac{7}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(7/6)),x]
```

[Out] -(((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-1/2, 7/6, 1/2, -((b*x^2)/a)])/(a*x*(a + b*x^2)^(1/6)))

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a\right)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^2+a)^(7/6),x)

[Out] int(1/x^2/(b*x^2+a)^(7/6),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)^(7/6)*x^2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="fricas")

[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)

Sympy [A]
time = 0.59, size = 27, normalized size = 0.04

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{7}{6} \\ \frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{a^{\frac{7}{6}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**2+a)**(7/6),x)

[Out] -hyper((-1/2, 7/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(7/6)*x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^2), x)
```

Mupad [B]
time = 5.12, size = 40, normalized size = 0.07

$$-\frac{3\,{\left(\frac{a}{b\,x^2}+1\right)}^{7/6}\,{}_2\mathrm{F}_1\left(\frac{7}{6},\frac{5}{3};\ \frac{8}{3};\ -\frac{a}{b\,x^2}\right)}{10\,x\,{\left(b\,x^2+a\right)}^{7/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(7/6)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(7/6)*hypergeom([7/6, 5/3], 8/3, -a/(b*x^2)))/(10*x*(a
+ b*x^2)^(7/6))
```

3.1037 $\int \frac{1}{x^4(a+bx^2)^{7/6}}\,dx$

Optimal. Leaf size=652

$$\frac{3}{ax^3\sqrt[6]{a+bx^2}}-\frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}}-\frac{10(a+bx^2)^{5/6}}{3a^2x^3}+\frac{40b(a+bx^2)^{5/6}}{9a^3x}-\frac{40b^2x}{9a^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(1-\sqrt{3}-\right.}$$

```
[Out] 3/a/x^3/(b*x^2+a)^(1/6)-40/9*b^2*x/a^3/(b*x^2+a)^(1/6)-10/3*(b*x^2+a)^(5/6)
/a^2/x^3+40/9*b*(b*x^2+a)^(5/6)/a^3/x-40/9*b^2*x/a^2/(a/(b*x^2+a))^(2/3)/(b
*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))+40/27*b*(1-(a/(b*x^2+a))^(1/3
))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)
),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a
/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a^2/x/(a/(b*x^2+a))^(2/3)/(b*x^
2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/
2)-20/9*b*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))
/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/
(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2
*2^(1/2))*3^(1/4)/a^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+
a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {296, 331, 244, 204, 241, 310, 225, 1893}

$$-\frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}}+\frac{40b(a+bx^2)^{5/6}}{9a^3x}+\frac{40\sqrt{2}\,b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{20\sqrt{2+\sqrt{3}}\,b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\,3^{3/4}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{40b^2x}{9a^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{10(a+bx^2)^{5/6}}{3a^2x^3}+\frac{3}{ax^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(7/6)),x]
```

```
[Out] 3/(a*x^3*(a + b*x^2)^(1/6)) - (40*b^2*x)/(9*a^3*(a + b*x^2)^(1/6)) - (10*(a
 + b*x^2)^(5/6))/(3*a^2*x^3) + (40*b*(a + b*x^2)^(5/6))/(9*a^3*x) - (40*b^2
*x)/(9*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b
*x^2))^(1/3))) - (20*Sqrt[2 + Sqrt[3]]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(
1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b
```

```
*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
- Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*a^2*x*(a/(
a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (40*Sqrt[2]*b*(1 - (a/(a + b*x^2))
^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1
/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^2)^{7/6}} dx = \frac{3}{ax^3\sqrt[6]{a+bx^2}} + \frac{10\int \frac{1}{x^4\sqrt[6]{a+bx^2}} dx}{a}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} - \frac{(40b)\int \frac{1}{x^2\sqrt[6]{a+bx^2}} dx}{9a^2}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} + \frac{40b(a+bx^2)^{5/6}}{9a^3x} - \frac{(80b^2)\int \frac{1}{\sqrt[6]{a+bx^2}} dx}{27a^3}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} + \frac{40b(a+bx^2)^{5/6}}{9a^3x} + \frac{(40b^2)\int \frac{1}{(a+bx^2}}{27a^2}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} + \frac{40b(a+bx^2)^{5/6}}{9a^3x} + \frac{(40b^2)\,\text{Subst}\Big(}{27a^2}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} + \frac{40b(a+bx^2)^{5/6}}{9a^3x} - \frac{\Big(20b\sqrt{-\frac{bx}{a+}}}{}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} + \frac{40b(a+bx^2)^{5/6}}{9a^3x} + \frac{\Big(20b\sqrt{-\frac{bx}{a+}}}{}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} + \frac{40b(a+bx^2)^{5/6}}{9a^3x} + \frac{40b\sqrt{}}{9a^2x\left(\frac{a}{a+bx^2}\right)^{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.01, size = 54, normalized size = 0.08

$$-\frac{\sqrt[6]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{3}{2},\frac{7}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(7/6)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-3/2, 7/6, -1/2, -((b*x^2)/a)
])/(a*x^3*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(b\,x^{2}+a \right)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(7/6),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [A]
time = 0.71, size = 32, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{7}{6} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{3 a^{\frac{7}{6}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(7/6),x)
[Out] -hyper((-3/2, 7/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/6)*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{7/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(7/6)),x)
[Out] int(1/(x^4*(a + b*x^2)^(7/6)), x)
```

3.1038 $\int \frac{1}{x^6 (a+bx^2)^{7/6}} \, dx$

Optimal. Leaf size=680

$$\frac{3}{ax^5 \sqrt[6]{a+bx^2}} + \frac{128b^3 x}{27a^4 \sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{128b^2(a+bx^2)^{5/6}}{27a^4 x} + \frac{}{27a^3 \left(\frac{a}{a+bx^2}\right)^{2/3} (a+}$$

```
[Out] 3/a/x^5/(b*x^2+a)^(1/6)+128/27*b^3*x/a^4/(b*x^2+a)^(1/6)-16/5*(b*x^2+a)^(5/
6)/a^2/x^5+32/9*b*(b*x^2+a)^(5/6)/a^3/x^3-128/27*b^2*(b*x^2+a)^(5/6)/a^4/x+
128/27*b^3*x/a^3/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)
-3^(1/2))-128/81*b^2*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/
3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(
b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/
2)*3^(3/4)/a^3/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/
3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+64/27*b^2*(1-(a/(b*x^2+a))^(1/
3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2
)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2
+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^3/x/(a/(b*
x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1
/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {296, 331, 244, 204, 241, 310, 225, 1893}

$$\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{128b^2(a+bx^2)^{5/6}}{27a^4x} - \frac{128\sqrt{2}\, b^2 \left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\, F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{27\sqrt[4]{3}\, a^3 x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}} + \frac{64\sqrt{2+\sqrt{3}}\, b^2 \left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\, E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\, 3^{3/4} a^3 x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}} + \frac{128b^3x}{27a^3\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{3}{ax^5\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(7/6)),x]
```

```
[Out] 3/(a*x^5*(a + b*x^2)^(1/6)) + (128*b^3*x)/(27*a^4*(a + b*x^2)^(1/6)) - (16*
(a + b*x^2)^(5/6))/(5*a^2*x^5) + (32*b*(a + b*x^2)^(5/6))/(9*a^3*x^3) - (12
8*b^2*(a + b*x^2)^(5/6))/(27*a^4*x) + (128*b^3*x)/(27*a^3*(a/(a + b*x^2))^(
2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (64*Sqrt[2
```

```
+ Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE
[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2)]) - (128*Sqrt[2]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a
 + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1
/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*a^3*x*(a/(a + b*x^
2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^6 (a+bx^2)^{7/6}}\,dx = \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{16\int \frac{1}{x^6\sqrt[6]{a+bx^2}}\,dx}{a}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} - \frac{(32b)\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx}{3a^2}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} + \frac{(128b^2)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{27a^3}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{128b^2(a+bx^2)^{5/6}}{27a^4x} + \frac{(256b^3)\int}{8}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{128b^2(a+bx}{27a^4x}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{128b^2(a+bx}{27a^4x}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{128b^2(a+bx}{27a^4x}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{128b^2(a+bx}{27a^4x}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{128b^2(a+bx}{27a^4x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.01, size = 54, normalized size = 0.08

$$-\frac{\sqrt[6]{1+\frac{bx^2}{a}}\ {}_2F_1\left(-\frac{5}{2},\frac{7}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5ax^5\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(7/6)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-5/2, 7/6, -3/2, -((b*x^2)/a)
])/(a*x^5*(a + b*x^2)^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(b\,x^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(7/6),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [A]
time = 0.84, size = 32, normalized size = 0.05

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{7}{6}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{7}{6}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(7/6),x)
```

```
[Out] -hyper((-5/2, 7/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6)*x**5)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,{\left(b\,x^2+a\right)}^{7/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(7/6)),x)
```

```
[Out] int(1/(x^6*(a + b*x^2)^(7/6)), x)
```

3.1039 $\int x^7(a+bx^2)^p\,dx$

Optimal. Leaf size=100

$$-\frac{a^3(a+bx^2)^{1+p}}{2b^4(1+p)}+\frac{3a^2(a+bx^2)^{2+p}}{2b^4(2+p)}-\frac{3a(a+bx^2)^{3+p}}{2b^4(3+p)}+\frac{(a+bx^2)^{4+p}}{2b^4(4+p)}$$

```
[Out] -1/2*a^3*(b*x^2+a)^(1+p)/b^4/(1+p)+3/2*a^2*(b*x^2+a)^(2+p)/b^4/(2+p)-3/2*a*
(b*x^2+a)^(3+p)/b^4/(3+p)+1/2*(b*x^2+a)^(4+p)/b^4/(4+p)
```

Rubi [A]

time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$-\frac{a^3(a+bx^2)^{p+1}}{2b^4(p+1)}+\frac{3a^2(a+bx^2)^{p+2}}{2b^4(p+2)}-\frac{3a(a+bx^2)^{p+3}}{2b^4(p+3)}+\frac{(a+bx^2)^{p+4}}{2b^4(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a^3*(a + b*x^2)^(1 + p))/(b^4*(1 + p)) + (3*a^2*(a + b*x^2)^(2 + p))/
(2*b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(2*b^4*(3 + p)) + (a + b*x^2)^(
4 + p)/(2*b^4*(4 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx^2)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^p\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right)dx,x\right.\\ &= -\frac{a^3(a+bx^2)^{1+p}}{2b^4(1+p)}+\frac{3a^2(a+bx^2)^{2+p}}{2b^4(2+p)}-\frac{3a(a+bx^2)^{3+p}}{2b^4(3+p)}+\frac{(a+bx^2)^{4+p}}{2b^4(4+p)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 95, normalized size = 0.95

$$\frac{1}{2}\left(-\frac{a^3(a+bx^2)^{1+p}}{b^4(1+p)}+\frac{3a^2(a+bx^2)^{2+p}}{b^4(2+p)}-\frac{3a(a+bx^2)^{3+p}}{b^4(3+p)}+\frac{(a+bx^2)^{4+p}}{b^4(4+p)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^p,x]
```

```
[Out] (-((a^3*(a + b*x^2)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x^2)^(2 + p))/(
b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(b^4*(3 + p)) + (a + b*x^2)^(4 + p
)/(b^4*(4 + p)))/2
```

Maple [A]
time = 0.06, size = 132, normalized size = 1.32

method	result
gosper	$-\frac{(b\,x^2+a)^{1+p}(-b^3p^3x^6-6b^3p^2x^6-11b^3p\,x^6+3a\,b^2p^2x^4-6b^3x^6+9a\,b^2p\,x^4+6a\,b^2x^4-6a^2bp\,x^2-6a^2b\,x^2+6a^3)}{2b^4(p^4+10p^3+35p^2+50p+24)}$
risch	$-\frac{(-b^4p^3x^8-6b^4p^2x^8-a\,b^3p^3x^6-11b^4p\,x^8-3a\,b^3p^2x^6-6b^4x^8-2ap\,x^6b^3+3a^2b^2p^2x^4+3a^2p\,x^4b^2-6a^3p\,x^2b+6a^4)(b\,x^2+a)^p}{2(3+p)(4+p)(2+p)(1+p)b^4}$
norman	$\frac{x^8e^{p\ln(b\,x^2+a)}}{2p+8}-\frac{3a^4e^{p\ln(b\,x^2+a)}}{b^4(p^4+10p^3+35p^2+50p+24)}+\frac{ap\,x^6e^{p\ln(b\,x^2+a)}}{2b(p^2+7p+12)}-\frac{3a^2p\,x^4e^{p\ln(b\,x^2+a)}}{2b^2(p^3+9p^2+26p+24)}+\frac{3p\,a^3x^2e^{p\ln(b\,x^2+a)}}{b^3(p^4+10p^3+35p^2+50p+24)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)^(1+p)*(-b^3*p^3*x^6-6*b^3*p^2*x^6-11*b^3*p*x^6+3*a*b^2*p^2*x
^4-6*b^3*x^6+9*a*b^2*p*x^4+6*a*b^2*x^4-6*a^2*b*p*x^2-6*a^2*b*x^2+6*a^3)/b^4
/(p^4+10*p^3+35*p^2+50*p+24)
```

Maxima [A]
time = 0.29, size = 106, normalized size = 1.06

$$\frac{((p^3+6\,p^2+11\,p+6)b^4x^8+(p^3+3\,p^2+2\,p)ab^3x^6-3\,(p^2+p)a^2b^2x^4+6\,a^3bpx^2-6\,a^4)(bx^2+a)^p}{2\,(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^8 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^6 - 3*(
p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 6*a^4)*(b*x^2 + a)^p/((p^4 + 10*p^3
+ 35*p^2 + 50*p + 24)*b^4)
```

Fricas [A]
time = 1.30, size = 148, normalized size = 1.48

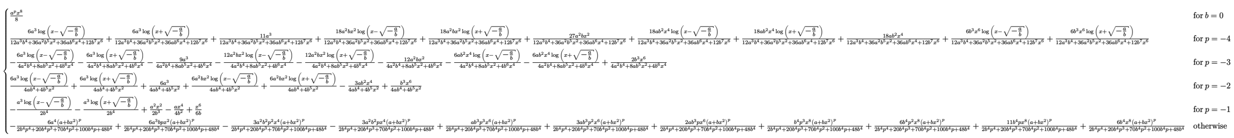
$$\frac{((b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4)x^8+6\,a^3bpx^2+(ab^3p^3+3\,ab^3p^2+2\,ab^3p)x^6-3\,(a^2b^2p^2+a^2b^2p)x^4-6\,a^4)(bx^2+a)^p}{2\,(b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^8 + 6*a^3*b*p*x^2 + (a*b^3*
p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^6 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^4 - 6*a^
4)*(b*x^2 + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1923 vs. $2(85) = 170$.
time = 3.00, size = 1923, normalized size = 19.23



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a**3*b**
4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(x + sqr
t(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6)
 + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**
6) + 18*a**2*b*x**2*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 +
 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a/b))/(12*a*
*3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x*
*2/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*
a*b**2*x**4*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b*
*6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))/(12*a**3*b**4
+ 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a
**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6
*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 1
2*b**7*x**6) + 6*b**3*x**6*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5
*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(x - sqrt(-
a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(x + sqrt(-a/
b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a
*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**4
 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(x + sqrt(-a/b))/(4*a**
2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b
**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**4 +
8*a*b**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b*
*4 + 8*a*b**5*x**2 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**
2 + 4*b**6*x**4), Eq(p, -3)), (6*a**3*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**
5*x**2) + 6*a**3*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4*a
*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5
*x**2) + 6*a**2*b*x**2*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) - 3*a*b
```

```
**2*x**4/(4*a*b**4 + 4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(
p, -2)), (-a**3*log(x - sqrt(-a/b))/(2*b**4) - a**3*log(x + sqrt(-a/b))/(2*
b**4) + a**2*x**2/(2*b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*
a**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*
p + 48*b**4) + 6*a**3*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3
+ 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)
**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*
a**2*b**2*p*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2
 + 100*b**4*p + 48*b**4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 +
20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a
 + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*
b**4) + 2*a*b**3*p*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b*
*4*p**2 + 100*b**4*p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p*
*4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8
*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p +
48*b**4) + 11*b**4*p*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*
b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**
4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(92) = 184.

time = 1.16, size = 260, normalized size = 2.60

$$\frac{(bx^2+a)^4(bx^2+a)^p p^2 - 3\,(bx^2+a)^3(bx^2+a)^p ap^2 + 3\,(bx^2+a)^2(bx^2+a)^p a^2p^2 + 5\,(bx^2+a)^4(bx^2+a)^p p - 18\,(bx^2+a)^3(bx^2+a)^p ap + 21\,(bx^2+a)^2(bx^2+a)^p a^2p + 6\,(bx^2+a)^4(bx^2+a)^p - 24\,(bx^2+a)^3(bx^2+a)^p a + 36\,(bx^2+a)^2(bx^2+a)^p a^2}{2\,(b^4p^3 + 9\,b^4p^2 + 26\,b^4p + 24\,b^4)} - \frac{(bx^2+a)^{p+1}a^3}{2\,b^4(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^4*(b*x^2 + a)^p*p^2 - 3*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p^2
+ 3*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p^2 + 5*(b*x^2 + a)^4*(b*x^2 + a)^p*p -
 18*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p + 21*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p
+ 6*(b*x^2 + a)^4*(b*x^2 + a)^p - 24*(b*x^2 + a)^3*(b*x^2 + a)^p*a + 36*(b*
x^2 + a)^2*(b*x^2 + a)^p*a^2)/(b^4*p^3 + 9*b^4*p^2 + 26*b^4*p + 24*b^4) - 1
/2*(b*x^2 + a)^(p + 1)*a^3/(b^4*(p + 1))
```

Mupad [B]

time = 4.97, size = 183, normalized size = 1.83

$$(b\,x^2+a)^p\,\left(\frac{x^8\,(p^3+6\,p^2+11\,p+6)}{2\,(p^4+10\,p^3+35\,p^2+50\,p+24)}-\frac{3\,a^4}{b^4\,(p^4+10\,p^3+35\,p^2+50\,p+24)}+\frac{3\,a^3\,p\,x^2}{b^3\,(p^4+10\,p^3+35\,p^2+50\,p+24)}+\frac{a\,p\,x^6\,(p^2+3\,p+2)}{2\,b\,(p^4+10\,p^3+35\,p^2+50\,p+24)}-\frac{3\,a^2\,p\,x^4\,(p+1)}{2\,b^2\,(p^4+10\,p^3+35\,p^2+50\,p+24)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^p,x)
```

```
[Out] (a + b*x^2)^p*((x^8*(11*p + 6*p^2 + p^3 + 6))/(2*(50*p + 35*p^2 + 10*p^3 +
p^4 + 24)) - (3*a^4)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (3*a^3*p*x
^2)/(b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^6*(3*p + p^2 + 2))/(
2*b*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*a^2*p*x^4*(p + 1))/(2*b^2*(50
*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

3.1040 $\int x^5(a+bx^2)^p\,dx$

Optimal. Leaf size=72

$$\frac{a^2(a+bx^2)^{1+p}}{2b^3(1+p)}-\frac{a(a+bx^2)^{2+p}}{b^3(2+p)}+\frac{(a+bx^2)^{3+p}}{2b^3(3+p)}$$

```
[Out] 1/2*a^2*(b*x^2+a)^(1+p)/b^3/(1+p)-a*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*(b*x^2+a)
^(3+p)/b^3/(3+p)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 45}

$$\frac{a^2(a+bx^2)^{p+1}}{2b^3(p+1)}-\frac{a(a+bx^2)^{p+2}}{b^3(p+2)}+\frac{(a+bx^2)^{p+3}}{2b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^p,x]
```

```
[Out] (a^2*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*(a + b*x^2)^(2 + p))/(b^3*(2
 + p)) + (a + b*x^2)^(3 + p)/(2*b^3*(3 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5(a+bx^2)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{a^2(a+bx^2)^{1+p}}{2b^3(1+p)}-\frac{a(a+bx^2)^{2+p}}{b^3(2+p)}+\frac{(a+bx^2)^{3+p}}{2b^3(3+p)}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 64, normalized size = 0.89

$$\frac{(a+bx^2)^{1+p}(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)x^4)}{2b^3(1+p)(2+p)(3+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x^4))
/(2*b^3*(1 + p)*(2 + p)*(3 + p))
```

Maple [A]
time = 0.05, size = 80, normalized size = 1.11

method	result	size
gosper	$\frac{(b\,x^2+a)^{1+p}(b^2p^2x^4+3b^2p\,x^4+2b^2x^4-2abp\,x^2-2ab\,x^2+2a^2)}{2b^3(p^3+6p^2+11p+6)}$	80
risch	$\frac{(b^3p^2x^6+3b^3p\,x^6+a\,b^2p^2x^4+2b^3x^6+a\,b^2p\,x^4-2a^2bp\,x^2+2a^3)(b\,x^2+a)^p}{2(2+p)(3+p)(1+p)b^3}$	93
norman	$\frac{a^3e^{p\ln(b\,x^2+a)}}{b^3(p^3+6p^2+11p+6)}+\frac{x^6e^{p\ln(b\,x^2+a)}}{6+2p}+\frac{ap\,x^4e^{p\ln(b\,x^2+a)}}{2b(p^2+5p+6)}-\frac{p\,a^2x^2e^{p\ln(b\,x^2+a)}}{b^2(p^3+6p^2+11p+6)}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x^2+a)^(1+p)*(b^2*p^2*x^4+3*b^2*p*x^4+2*b^2*x^4-2*a*b*p*x^2-2*a*b*x^
2+2*a^2)/b^3/(p^3+6*p^2+11*p+6)
```

Maxima [A]
time = 0.29, size = 73, normalized size = 1.01

$$\frac{((p^2+3\,p+2)b^3x^6+(p^2+p)ab^2x^4-2\,a^2bpx^2+2\,a^3)(bx^2+a)^p}{2\,(p^3+6\,p^2+11\,p+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Fricas [A]
time = 1.47, size = 98, normalized size = 1.36

$$\frac{((b^3p^2+3\,b^3p+2\,b^3)x^6-2\,a^2bpx^2+(ab^2p^2+ab^2p)x^4+2\,a^3)(bx^2+a)^p}{2\,(b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((b^3*p^2 + 3*b^3*p + 2*b^3)*x^6 - 2*a^2*b*p*x^2 + (a*b^2*p^2 + a*b^2*p
)*x^4 + 2*a^3)*(b*x^2 + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(58) = 116$.
time = 1.37, size = 920, normalized size = 12.78

$$\begin{cases} \frac{a^p x^6}{6} & \text{for } b = 0 \\ \frac{2a^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2a^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2b^2x^4 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2b^2x^4 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} & \text{for } p = -3 \\ -\frac{2a^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2}{2ab^3+2b^4x^2} - \frac{2abx^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2abx^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} + \frac{b^2x^4}{2ab^3+2b^4x^2} & \text{for } p = -2 \\ \frac{a^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2b^3} + \frac{a^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b} & \text{for } p = -1 \\ \frac{2a^3(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} - \frac{2a^2bpx^2(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2p^2x^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2px^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{b^3p^2x^6(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{3b^3px^6(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{2b^3x^6(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 +
 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**
5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b
**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4
*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b*
*2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2
*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4),
 Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2
*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x
**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2
*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**
4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt
(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b
*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x
**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*
b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12
*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**
3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2
 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b
**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3
+ 12*b**3*p**2 + 22*b**3*p + 12*b**3), True))
```

Giac [A]
time = 1.22, size = 132, normalized size = 1.83

$$\frac{(bx^2+a)^3(bx^2+a)^p p - 2\,(bx^2+a)^2(bx^2+a)^p a p + 2\,(bx^2+a)^3(bx^2+a)^p - 6\,(bx^2+a)^2(bx^2+a)^p a}{2\,(b^3p^2 + 5\,b^3p + 6\,b^3)} + \frac{(bx^2+a)^{p+1}a^2}{2\,b^3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^3*(b*x^2 + a)^p*p - 2*(b*x^2 + a)^2*(b*x^2 + a)^p*a*p + 2*
(b*x^2 + a)^3*(b*x^2 + a)^p - 6*(b*x^2 + a)^2*(b*x^2 + a)^p*a)/(b^3*p^2 + 5
*b^3*p + 6*b^3) + 1/2*(b*x^2 + a)^(p + 1)*a^2/(b^3*(p + 1))
```

Mupad [B]
time = 4.90, size = 117, normalized size = 1.62

$$(b\,x^2+a)^p\,\left(\frac{a^3}{b^3\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{x^6\,\left(p^2+3\,p+2\right)}{2\,\left(p^3+6\,p^2+11\,p+6\right)}-\frac{a^2\,p\,x^2}{b^2\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{a\,p\,x^4\,\left(p+1\right)}{2\,b\,\left(p^3+6\,p^2+11\,p+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p,x)
```

```
[Out] (a + b*x^2)^p*(a^3/(b^3*(11*p + 6*p^2 + p^3 + 6)) + (x^6*(3*p + p^2 + 2))/(
2*(11*p + 6*p^2 + p^3 + 6)) - (a^2*p*x^2)/(b^2*(11*p + 6*p^2 + p^3 + 6)) +
(a*p*x^4*(p + 1))/(2*b*(11*p + 6*p^2 + p^3 + 6)))
```

3.1041 $\int x^3(a+bx^2)^p\,dx$

Optimal. Leaf size=48

$$-\frac{a(a+bx^2)^{1+p}}{2b^2(1+p)}+\frac{(a+bx^2)^{2+p}}{2b^2(2+p)}$$

```
[Out] -1/2*a*(b*x^2+a)^(1+p)/b^2/(1+p)+1/2*(b*x^2+a)^(2+p)/b^2/(2+p)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {272, 45}

$$\frac{(a+bx^2)^{p+2}}{2b^2(p+2)}-\frac{a(a+bx^2)^{p+1}}{2b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*(a + b*x^2)^(1 + p))/(b^2*(1 + p)) + (a + b*x^2)^(2 + p)/(2*b^2*(2
+ p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx^2)^p\,dx&=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx,x,x^2\right)\\&=-\frac{a(a+bx^2)^{1+p}}{2b^2(1+p)}+\frac{(a+bx^2)^{2+p}}{2b^2(2+p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.83

$$\frac{\left(a+bx^2\right)^{1+p}\left(-a+b(1+p)x^2\right)}{2b^2(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^(1 + p)*(-a + b*(1 + p)*x^2))/(2*b^2*(1 + p)*(2 + p))
```

Maple [A]
time = 0.05, size = 42, normalized size = 0.88

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{1+p}\left(-x^{2}pb-b\,x^{2}+a\right)}{2b^{2}\left(p^{2}+3p+2\right)}$	42
risch	$-\frac{\left(-b^{2}p\,x^{4}-b^{2}x^{4}-abp\,x^{2}+a^{2}\right)\left(b\,x^{2}+a\right)^{p}}{2b^{2}\left(2+p\right)\left(1+p\right)}$	54
norman	$\frac{x^{4}\mathrm{e}^{p\ln\left(b\,x^{2}+a\right)}}{4+2p}-\frac{a^{2}\mathrm{e}^{p\ln\left(b\,x^{2}+a\right)}}{2b^{2}\left(p^{2}+3p+2\right)}+\frac{pa\,x^{2}\mathrm{e}^{p\ln\left(b\,x^{2}+a\right)}}{2b\left(p^{2}+3p+2\right)}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
[Out] -1/2*(b*x^2+a)^(1+p)*(-b*p*x^2-b*x^2+a)/b^2/(p^2+3*p+2)
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.98

$$\frac{\left(b^2(p+1)x^4+abpx^2-a^2\right)\left(bx^2+a\right)^p}{2\,(p^2+3\,p+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p/((p^2 + 3*p + 2)*b^2)
```

Fricas [A]
time = 0.93, size = 58, normalized size = 1.21

$$\frac{\left(abpx^2+(b^2p+b^2)x^4-a^2\right)\left(bx^2+a\right)^p}{2\,(b^2p^2+3\,b^2p+2\,b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*p*x^2 + (b^2*p + b^2)*x^4 - a^2)*(b*x^2 + a)^p/(b^2*p^2 + 3*b^2*p
+ 2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(37) = 74$.
time = 0.50, size = 333, normalized size = 6.94

$$\begin{cases} \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{a \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} & \text{for } p = -2 \\ -\frac{a \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**
3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*
b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*l
og(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-
a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (
-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b
*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(
2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 +
 6*b**2*p + 4*b**2), True))
```

Giac [A]
time = 1.05, size = 51, normalized size = 1.06

$$\frac{(bx^2+a)^2(bx^2+a)^p}{2\,b^2(p+2)} - \frac{(bx^2+a)^{p+1}a}{2\,b^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + a)^2*(b*x^2 + a)^p/(b^2*(p + 2)) - 1/2*(b*x^2 + a)^(p + 1)*a/(
b^2*(p + 1))
```

Mupad [B]
time = 4.89, size = 68, normalized size = 1.42

$$(b\,x^2+a)^p\,\left(\frac{x^4\,(p+1)}{2\,(p^2+3\,p+2)} - \frac{a^2}{2\,b^2\,(p^2+3\,p+2)} + \frac{a\,p\,x^2}{2\,b\,(p^2+3\,p+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p,x)
```

```
[Out] (a + b*x^2)^p*((x^4*(p + 1))/(2*(3*p + p^2 + 2)) - a^2/(2*b^2*(3*p + p^2 +
2)) + (a*p*x^2)/(2*b*(3*p + p^2 + 2)))
```

3.1042 $\int x(a+bx^2)^p \, dx$

Optimal. Leaf size=23

$$\frac{(a+bx^2)^{1+p}}{2b(1+p)}$$

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/(1+p)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\frac{(a+bx^2)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^p,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*(1 + p))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^p \, dx = \frac{\left(a+bx^2\right)^{1+p}}{2b(1+p)}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.96

$$\frac{(a+bx^2)^{1+p}}{2b+2bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^p,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b + 2*b*p)
```

Maple [A]
time = 0.04, size = 22, normalized size = 0.96

method	result	size
gosper	$\frac{(b\,x^2+a)^{1+p}}{2b(1+p)}$	22
derivativedivides	$\frac{(b\,x^2+a)^{1+p}}{2b(1+p)}$	22
default	$\frac{(b\,x^2+a)^{1+p}}{2b(1+p)}$	22
risch	$\frac{(b\,x^2+a)\,(b\,x^2+a)^p}{2b(1+p)}$	27
norman	$\frac{x^2\mathrm{e}^{p\ln\left(b\,x^2+a\right)}}{2+2p}+\frac{a\,\mathrm{e}^{p\ln\left(b\,x^2+a\right)}}{2b(1+p)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/(1+p)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.91

$$\frac{(bx^2+a)^{p+1}}{2\,b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)/(b*(p + 1))
```

Fricas [A]
time = 0.90, size = 25, normalized size = 1.09

$$\frac{(bx^2+a)(bx^2+a)^p}{2\,(bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^p/(b*p + b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

time = 0.20, size = 87, normalized size = 3.78

$$\begin{cases} \frac{x^2}{2a} & \text{for } b = 0 \wedge p = -1 \\ \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{\log\left(x - \sqrt{-\frac{a}{b}}\right)}{2b} + \frac{\log\left(x + \sqrt{-\frac{a}{b}}\right)}{2b} & \text{for } p = -1 \\ \frac{a(a+bx^2)^p}{2bp+2b} + \frac{bx^2(a+bx^2)^p}{2bp+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**2/2, Eq(b, 0)), (log
(x - sqrt(-a/b))/(2*b) + log(x + sqrt(-a/b))/(2*b), Eq(p, -1)), (a*(a + b*x
**2)**p/(2*b*p + 2*b) + b*x**2*(a + b*x**2)**p/(2*b*p + 2*b), True))
```

Giac [A]
time = 1.03, size = 21, normalized size = 0.91

$$\frac{(bx^2 + a)^{p+1}}{2\,b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)/(b*(p + 1))
```

Mupad [B]
time = 4.89, size = 21, normalized size = 0.91

$$\frac{(b\,x^2 + a)^{p+1}}{2\,b\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p,x)
```

```
[Out] (a + b*x^2)^(p + 1)/(2*b*(p + 1))
```

3.1043 $\int \frac{(a+bx^2)^p}{x}\,dx$

Optimal. Leaf size=41

$$-\frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[Out] -1/2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 67}

$$-\frac{(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x,x]
```

```
[Out] -1/2*((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a]
)/(a*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)\\ &= -\frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 41, normalized size = 1.00

$$-\frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x,x]
[Out] -1/2*((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a]
)/(a*(1 + p))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x,x)
[Out] int((b*x^2+a)^p/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/x, x)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.91, size = 39, normalized size = 0.95

$$-\frac{b^p x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x,x)
```

```
[Out] -b**p*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**
2))/(2*gamma(1 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x,x)
```

```
[Out] int((a + b*x^2)^p/x, x)
```

3.1044 $\int \frac{(a+bx^2)^p}{x^3}\,dx$

Optimal. Leaf size=42

$$\frac{b(a+bx^2)^{1+p}\ {}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

```
[Out] 1/2*b*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],1+b*x^2/a)/a^2/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 67}

$$\frac{b(a+bx^2)^{p+1}\ {}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a])/(
2*a^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p}{x^2}\,dx,x,x^2\right)\\ &= \frac{b(a+bx^2)^{1+p}\ {}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$\frac{b(a+bx^2)^{1+p}\ {}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^3,x]
[Out] (b*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a])/(
2*a^2*(1 + p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^3,x)
[Out] int((b*x^2+a)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/x^3, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.03, size = 42, normalized size = 1.00

$$-\frac{b^p x^{2p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^2\Gamma(2-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**3,x)
```

```
[Out] -b**p*x**(2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/
(b*x**2))/(2*x**2*gamma(2 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^3,x)
```

```
[Out] int((a + b*x^2)^p/x^3, x)
```

3.1045 $\int x^6(a+bx^2)^p\,dx$

Optimal. Leaf size=40

$$\frac{x^7(a+bx^2)^{1+p}\;{}_2F_1\left(1,\frac{9}{2}+p;\frac{9}{2};-\frac{bx^2}{a}\right)}{7a}$$

```
[Out] 1/7*x^7*(b*x^2+a)^(1+p)*hypergeom([1, 9/2+p],[9/2],-b*x^2/a)/a
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {372, 371}

$$\frac{1}{7}x^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\;{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^p,x]
```

```
[Out] (x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (
b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^6\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^6\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{1}{7}x^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\;{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 1.22

$$\frac{1}{7}x^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^p,x]
[Out] (x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (
b*x^2)/a)^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^6\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^p,x)
[Out] int(x^6*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^6, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.66, size = 26, normalized size = 0.65

$$\frac{a^px^7{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^6\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^p,x)
```

```
[Out] int(x^6*(a + b*x^2)^p, x)
```

3.1046 $\int x^4(a+bx^2)^p\,dx$

Optimal. Leaf size=40

$$\frac{x^5(a+bx^2)^{1+p}\,{}_2F_1\left(1,\frac{7}{2}+p;\frac{7}{2};-\frac{bx^2}{a}\right)}{5a}$$

```
[Out] 1/5*x^5*(b*x^2+a)^(1+p)*hypergeom([1, 7/2+p],[7/2],-b*x^2/a)/a
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {372, 371}

$$\frac{1}{5}x^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^p,x]
```

```
[Out] (x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (
b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{1}{5}x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 1.22

$$\frac{1}{5}x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^p,x]
[Out] (x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (
b*x^2)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p,x)
[Out] int(x^4*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.20, size = 26, normalized size = 0.65

$$\frac{a^p x^5 {}_2F_1\left(\left.\begin{matrix}\frac{5}{2},-p\\\frac{7}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^4\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^p,x)
```

```
[Out] int(x^4*(a + b*x^2)^p, x)
```

3.1047 $\int x^2(a+bx^2)^p\,dx$

Optimal. Leaf size=40

$$\frac{x^3(a+bx^2)^{1+p}\,{}_2F_1\left(1,\frac{5}{2}+p;\frac{5}{2};-\frac{bx^2}{a}\right)}{3a}$$

[Out] 1/3*x^3*(b*x^2+a)^(1+p)*hypergeom([1, 5/2+p],[5/2],-b*x^2/a)/a

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {372, 371}

$$\frac{1}{3}x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^2)^p,x]

[Out] (x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{1}{3}x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 1.22

$$\frac{1}{3}x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^p,x]
[Out] (x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (
b*x^2)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p,x)
[Out] int(x^2*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.81, size = 26, normalized size = 0.65

$$\frac{a^p x^3 {}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^p,x)
```

```
[Out] int(x^2*(a + b*x^2)^p, x)
```

3.1048 $\int (a+bx^2)^p \, dx$

Optimal. Leaf size=35

$$\frac{x(a+bx^2)^{1+p} \,{}_2F_1\left(1,\frac{3}{2}+p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}$$

```
[Out] x*(b*x^2+a)^(1+p)*hypergeom([1, 3/2+p],[3/2],-b*x^2/a)/a
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {252, 251}

$$x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p,x]
```

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (a+bx^2)^p \, dx &= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \left(1+\frac{bx^2}{a}\right)^p \, dx \\ &= x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right) \end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.26

$$x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p,x]
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p,x)
[Out] int((b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 22, normalized size = 0.63

$$a^p x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p,x)
```

```
[Out] a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p, x)
```

Mupad [B]
time = 5.41, size = 41, normalized size = 1.17

$$\frac{x\,\left(b\,x^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p,x)
```

```
[Out] (x*(a + b*x^2)^p*hypergeom([1/2, -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

3.1049 $\int \frac{(a+bx^2)^p}{x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{(a+bx^2)^{1+p}\ {}_2F_1\left(1,\frac{1}{2}+p;\frac{1}{2};-\frac{bx^2}{a}\right)}{ax}$$

```
[Out] -(b*x^2+a)^(1+p)*hypergeom([1, 1/2+p],[1/2],-b*x^2/a)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {372, 371}

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\ {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b
*x^2)/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^2}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx\\ &= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\ {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 1.24

$$-\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^2,x]
[Out] -(((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b
*x^2)/a)^p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int\frac{(b\,x^2+a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2,x)
[Out] int((b*x^2+a)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.70, size = 26, normalized size = 0.68

$$-\frac{a^p{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^2, x)
```

Mupad [B]
time = 5.03, size = 58, normalized size = 1.53

$$\frac{(b\,x^2+a)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2}-p,-p;\ \frac{3}{2}-p;\ -\frac{a}{b\,x^2}\right)}{x\,\left(2\,p-1\right)\,{\left(\frac{a}{b\,x^2}+1\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^2,x)
```

```
[Out] ((a + b*x^2)^p*hypergeom([1/2 - p, -p], 3/2 - p, -a/(b*x^2)))/(x*(2*p - 1)*
(a/(b*x^2) + 1)^p)
```

3.1050 $\int x^{7/2}(a+bx^2)^p\,dx$

Optimal. Leaf size=42

$$\frac{2x^{9/2}(a+bx^2)^{1+p}\,{}_2F_1\left(1,\frac{13}{4}+p;\frac{13}{4};-\frac{bx^2}{a}\right)}{9a}$$

[Out] 2/9*x^(9/2)*(b*x^2+a)^(1+p)*hypergeom([1, 13/4+p],[13/4],-b*x^2/a)/a

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{2}{9}x^{9/2}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{9}{4},-p;\frac{13}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)*(a + b*x^2)^p,x]

[Out] (2*x^(9/2)*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -((b*x^2)/a)])/(9*(1 + (b*x^2)/a)^p)

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{7/2}(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{7/2}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{2}{9}x^{9/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{9}{4},-p;\frac{13}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 1.21

$$\frac{2}{9}x^{9/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{9}{4},-p;\frac{13}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(9/2)*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -((b*x^2)/a)])/(9
*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{\frac{7}{2}}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(7/2)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(7/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{7/2}\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^p,x)
```

```
[Out] int(x^(7/2)*(a + b*x^2)^p, x)
```

3.1051 $\int x^{5/2}(a+bx^2)^p\,dx$

Optimal. Leaf size=42

$$\frac{2x^{7/2}(a+bx^2)^{1+p}\ {}_2F_1\left(1,\frac{11}{4}+p;\frac{11}{4};-\frac{bx^2}{a}\right)}{7a}$$

[Out] 2/7*x^(7/2)*(b*x^2+a)^(1+p)*hypergeom([1, 11/4+p],[11/4],-b*x^2/a)/a

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {372, 371}

$$\frac{2}{7}x^{7/2}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\ {}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)*(a + b*x^2)^p,x]

[Out] (2*x^(7/2)*(a + b*x^2)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^2)/a)])/(7*(1 + (b*x^2)/a)^p)

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{5/2}\left(1+\frac{bx^2}{a}\right)^p dx\\ &= \frac{2}{7}x^{7/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\ {}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.13, size = 51, normalized size = 1.21

$$\frac{2}{7}x^{7/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(7/2)*(a + b*x^2)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(5/2)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(5/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{5/2}\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^p,x)
```

```
[Out] int(x^(5/2)*(a + b*x^2)^p, x)
```

3.1052 $\int x^{3/2}(a+bx^2)^p\,dx$

Optimal. Leaf size=42

$$\frac{2x^{5/2}(a+bx^2)^{1+p}\,{}_2F_1\left(1,\frac{9}{4}+p;\frac{9}{4};-\frac{bx^2}{a}\right)}{5a}$$

```
[Out] 2/5*x^(5/2)*(b*x^2+a)^(1+p)*hypergeom([1, 9/4+p],[9/4],-b*x^2/a)/a
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{2}{5}x^{5/2}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(5/2)*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^2)/a)])/(5*
(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{3/2}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{2}{5}x^{5/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.11, size = 51, normalized size = 1.21

$$\frac{2}{5}x^{5/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^p,x]
[Out] (2*x^(5/2)*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^2)/a)])/(5*
(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^p,x)
[Out] int(x^(3/2)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(3/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 171.89, size = 37, normalized size = 0.88

$$\frac{a^p x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**(5/2)*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**2*exp_polar(I*pi)/a)
/(2*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{3/2}\,{\left(b\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^p,x)
```

```
[Out] int(x^(3/2)*(a + b*x^2)^p, x)
```

3.1053 $\int \sqrt{x}\,(a+bx^2)^p\,dx$

Optimal. Leaf size=42

$$\frac{2x^{3/2}(a+bx^2)^{1+p}\,{}_2F_1\left(1,\frac{7}{4}+p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a}$$

```
[Out] 2/3*x^(3/2)*(b*x^2+a)^(1+p)*hypergeom([1, 7/4+p],[7/4],-b*x^2/a)/a
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{2}{3}x^{3/2}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(3/2)*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^2)/a)])/(3*
(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \sqrt{x}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{2}{3}x^{3/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 51, normalized size = 1.21

$$\frac{2}{3}x^{3/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^p,x]
[Out] (2*x^(3/2)*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^2)/a)])/(3*
(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2+a)^p,x)
[Out] int(x^(1/2)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*sqrt(x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 25.42, size = 37, normalized size = 0.88

$$\frac{a^p x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**(3/2)*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**2*exp_polar(I*pi)/a)
/(2*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x}\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^p,x)
```

```
[Out] int(x^(1/2)*(a + b*x^2)^p, x)
```

3.1054 $\int \frac{(a+bx^2)^p}{\sqrt{x}} dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}\,(a+bx^2)^{1+p}\,{}_2F_1\left(1,\frac{5}{4}+p;\frac{5}{4};-\frac{bx^2}{a}\right)}{a}$$

[Out] 2*(b*x^2+a)^(1+p)*hypergeom([1, 5/4+p],[5/4],-b*x^2/a)*x^(1/2)/a

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$2\sqrt{x}\,(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^p/Sqrt[x],x]

[Out] (2*Sqrt[x]*(a + b*x^2)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{\sqrt{x}}\,dx\\ &= 2\sqrt{x}\,(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.12, size = 49, normalized size = 1.22

$$2\sqrt{x}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/Sqrt[x],x]
[Out] (2*Sqrt[x]*(a + b*x^2)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^p
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(1/2),x)
[Out] int((b*x^2+a)^p/x^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/sqrt(x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 16.24, size = 37, normalized size = 0.92

$$\frac{a^p\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(1/2),x)
```

```
[Out] a**p*sqrt(x)*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**2*exp_polar(I*pi)/a)/
(2*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^2+a\right)}^p}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(1/2),x)
```

```
[Out] int((a + b*x^2)^p/x^(1/2), x)
```

3.1055 $\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{2(a+bx^2)^{1+p}\,{}_2F_1\left(1,\frac{3}{4}+p;\frac{3}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x}}$$

```
[Out] -2*(b*x^2+a)^(1+p)*hypergeom([1, 3/4+p],[3/4],-b*x^2/a)/a/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$-\frac{2(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{4},-p;\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^2)/a)])/(Sqrt[x]*
(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^{3/2}}\,dx\\ &= -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{4},-p;\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 49, normalized size = 1.22

$$-\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{4},-p;\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(3/2),x]
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^2)/a)])/(Sqrt[x]*
(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(3/2),x)
[Out] int((b*x^2+a)^p/x^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/x^(3/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 86.07, size = 41, normalized size = 1.02

$$\frac{a^p\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-p\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(3/2),x)
```

```
[Out] a**p*gamma(-1/4)*hyper((-1/4, -p), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqr
t(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^2+a\right)}^p}{x^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(3/2),x)
```

```
[Out] int((a + b*x^2)^p/x^(3/2), x)
```

3.1056 $\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2(a+bx^2)^{1+p}\,{}_2F_1\left(1,\frac{1}{4}+p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3ax^{3/2}}$$

```
[Out] -2/3*(b*x^2+a)^(1+p)*hypergeom([1, 1/4+p],[1/4],-b*x^2/a)/a/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$-\frac{2(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{3}{4},-p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/2
)*(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^{5/2}}\,dx\\ &= -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{3}{4},-p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 51, normalized size = 1.21

$$-\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{3}{4},-p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/2
)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(5/2),x)
```

```
[Out] int((b*x^2+a)^p/x^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^(5/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(5/2),x)
```

```
[Out] int((a + b*x^2)^p/x^(5/2), x)
```

3.1057 $\int \frac{(a+bx^2)^p}{x^{7/2}} \, dx$

Optimal. Leaf size=42

$$-\frac{2(a+bx^2)^{1+p} \, {}_2F_1\left(1,-\frac{1}{4}+p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5ax^{5/2}}$$

```
[Out] -2/5*(b*x^2+a)^(1+p)*hypergeom([1, -1/4+p],[-1/4],-b*x^2/a)/a/x^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$-\frac{2(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{5}{4},-p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(7/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-5/4, -p, -1/4, -((b*x^2)/a)])/(5*x^(5/
2)*(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^{7/2}} \, dx &= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^{7/2}} \, dx \\ &= -\frac{2(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{5}{4},-p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 51, normalized size = 1.21

$$-\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{5}{4},-p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(7/2),x]
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-5/4, -p, -1/4, -((b*x^2)/a)])/(5*x^(5/
2)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(7/2),x)
[Out] int((b*x^2+a)^p/x^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(7/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/x^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(7/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/x^(7/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(7/2),x)
```

```
[Out] int((a + b*x^2)^p/x^(7/2), x)
```

3.1058 $\int x^m(a+bx^2)^p\,dx$

Optimal. Leaf size=53

$$\frac{x^{1+m}\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,\frac{1}{2}(3+m+2p);\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

```
[Out] x^(1+m)*(b*x^2+a)^(1+p)*hypergeom([1, 3/2+1/2*m+p],[3/2+1/2*m],-b*x^2/a)/a/
(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {372, 371}

$$\frac{x^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((b*x
^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^m\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 1.19

$$\frac{x^{1+m}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^p,x]
[Out] (x^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^m\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^p,x)
[Out] int(x^m*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.97, size = 51, normalized size = 0.96

$$\frac{a^p x x^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**p,x)
```

```
[Out] a**p*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(b\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^p,x)
```

```
[Out] int(x^m*(a + b*x^2)^p, x)
```

3.1059 $\int (cx)^m \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=66

$$\frac{(cx)^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{c(1+m)}$$

```
[Out] (c*x)^(1+m)*(b*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/c/(
1+m)/((1+b*x^2/a)^p)
```

Rubi [A]

time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{(cx)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^2)^p,x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(b*x^2)/a)])/(c*(1 + m)*(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^m\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int (cx)^m\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{(cx)^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{c(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.97

$$\frac{x(cx)^m(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^2)^p,x]
[Out] (x*(c*x)^m*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^m \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2+a)^p,x)
[Out] int((c*x)^m*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*(c*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(c*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.00, size = 54, normalized size = 0.82

$$\frac{a^pc^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**2+a)**p,x)
```

```
[Out] a**p*c**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(c*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^2)^p,x)
```

```
[Out] int((c*x)^m*(a + b*x^2)^p, x)
```

3.1060 $\int x^{-8-2p}(a+bx^2)^p \, dx$

Optimal. Leaf size=53

$$-\frac{x^{-7-2p}(a+bx^2)^{1+p} \, {}_2F_1\left(-\frac{5}{2},1;\frac{1}{2}(-5-2p);-\frac{bx^2}{a}\right)}{a(7+2p)}$$

```
[Out] -x^(-7-2*p)*(b*x^2+a)^(1+p)*hypergeom([-5/2, 1],[-5/2-p],-b*x^2/a)/a/(7+2*p
)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$-\frac{x^{-2p-7}(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{2}(-2p-7),-p;\frac{1}{2}(-2p-5);-\frac{bx^2}{a}\right)}{2p+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-8 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-7 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-7 - 2*p)/2, -p, (-5 - 2*p
)/2, -((b*x^2)/a)])/((7 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-8-2p}(a+bx^2)^p \, dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-8-2p}\left(1+\frac{bx^2}{a}\right)^p dx\\ &= -\frac{x^{-7-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2}(-7-2p),-p;\frac{1}{2}(-5-2p);-\frac{bx^2}{a}\right)}{7+2p}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 1.25

$$-\frac{x^{-7-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{7}{2}-p,-p;-\frac{5}{2}-p;-\frac{bx^2}{a}\right)}{7+2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-8 - 2*p)*(a + b*x^2)^p,x]
[Out] -((x^(-7 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-7/2 - p, -p, -5/2 - p, -((
b*x^2)/a)])/((7 + 2*p)*(1 + (b*x^2)/a)^p))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-8-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-8-2*p)*(b*x^2+a)^p,x)
[Out] int(x^(-8-2*p)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 8), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p - 8), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-8-2*p)*(b*x**2+a)**p,x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-8-2*p)*(b*x^2+a)^p,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p*x^(-2*p - 8), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{x^{2\,p+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^p/x^(2*p + 8),x)

[Out] int((a + b*x^2)^p/x^(2*p + 8), x)

3.1061 $\int x^{-7-2p}(a+bx^2)^p\,dx$

Optimal. Leaf size=105

$$-\frac{b^2x^{-2(1+p)}(a+bx^2)^{1+p}}{a^3(1+p)(2+p)(3+p)}+\frac{bx^{-2(2+p)}(a+bx^2)^{1+p}}{a^2(2+p)(3+p)}-\frac{x^{-2(3+p)}(a+bx^2)^{1+p}}{2a(3+p)}$$

```
[Out] -b^2*(b*x^2+a)^(1+p)/a^3/(2+p)/(p^2+4*p+3)/(x^(2+2*p))+b*(b*x^2+a)^(1+p)/a^
2/(2+p)/(3+p)/(x^(4+2*p))-1/2*(b*x^2+a)^(1+p)/a/(3+p)/(x^(6+2*p))
```

Rubi [A]
time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {277, 270}

$$-\frac{b^2x^{-2(p+1)}(a+bx^2)^{p+1}}{a^3(p+1)(p+2)(p+3)}+\frac{bx^{-2(p+2)}(a+bx^2)^{p+1}}{a^2(p+2)(p+3)}-\frac{x^{-2(p+3)}(a+bx^2)^{p+1}}{2a(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((b^2*(a + b*x^2)^(1 + p))/(a^3*(1 + p)*(2 + p)*(3 + p)*x^(2*(1 + p)))) +
(b*(a + b*x^2)^(1 + p))/(a^2*(2 + p)*(3 + p)*x^(2*(2 + p))) - (a + b*x^2)^(
1 + p)/(2*a*(3 + p)*x^(2*(3 + p)))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^{-7-2p}(a+bx^2)^p\,dx &= -\frac{x^{-2(3+p)}(a+bx^2)^{1+p}}{2a(3+p)}-\frac{(2b)\int x^{-5-2p}(a+bx^2)^p\,dx}{a(3+p)}\\
&= \frac{bx^{-2(2+p)}(a+bx^2)^{1+p}}{a^2(2+p)(3+p)}-\frac{x^{-2(3+p)}(a+bx^2)^{1+p}}{2a(3+p)}+\frac{(2b^2)\int x^{-3-2p}(a+bx^2)^p\,dx}{a^2(2+p)(3+p)}\\
&= -\frac{b^2x^{-2(1+p)}(a+bx^2)^{1+p}}{a^3(1+p)(2+p)(3+p)}+\frac{bx^{-2(2+p)}(a+bx^2)^{1+p}}{a^2(2+p)(3+p)}-\frac{x^{-2(3+p)}(a+bx^2)^{1+p}}{2a(3+p)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 62, normalized size = 0.59

$$-\frac{x^{-2(3+p)}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-3-p,-p;-2-p;-\frac{bx^2}{a}\right)}{2(3+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*((a + b*x^2)^p*Hypergeometric2F1[-3 - p, -p, -2 - p, -((b*x^2)/a)])/((
3 + p)*x^(2*(3 + p))*(1 + (b*x^2)/a)^p)
```

Maple [A]
time = 0.07, size = 81, normalized size = 0.77

method	result	size
gosper	$-\frac{(b\,x^2+a)^{1+p}x^{-6-2p}(2b^2x^4-2abp\,x^2+a^2p^2-2ab\,x^2+3a^2p+2a^2)}{2(3+p)(2+p)(1+p)a^3}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-7-2*p)*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)^(1+p)*x^(-6-2*p)*(2*b^2*x^4-2*a*b*p*x^2+a^2*p^2-2*a*b*x^2+3*
a^2*p+2*a^2)/(3+p)/(2+p)/(1+p)/a^3
```

Maxima [A]
time = 0.30, size = 84, normalized size = 0.80

$$-\frac{(2\,b^3x^6-2\,ab^2px^4+(p^2+p)a^2bx^2+(p^2+3\,p+2)a^3)e^{(p\log(bx^2+a)-2\,p\log(x))}}{2\,(p^3+6\,p^2+11\,p+6)a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*x^6 - 2*a*b^2*p*x^4 + (p^2 + p)*a^2*b*x^2 + (p^2 + 3*p + 2)*a^3
)*e^(p*log(b*x^2 + a) - 2*p*log(x))/((p^3 + 6*p^2 + 11*p + 6)*a^3*x^6)
```

Fricas [A]
time = 0.89, size = 106, normalized size = 1.01

$$-\frac{(2\,b^3x^7-2\,ab^2px^5+(a^2bp^2+a^2bp)x^3+(a^3p^2+3\,a^3p+2\,a^3)x)(bx^2+a)^px^{-2\,p-7}}{2\,(a^3p^3+6\,a^3p^2+11\,a^3p+6\,a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="fricas")

[Out] -1/2*(2*b^3*x^7 - 2*a*b^2*p*x^5 + (a^2*b*p^2 + a^2*b*p)*x^3 + (a^3*p^2 + 3*a^3*p + 2*a^3)*x)*(b*x^2 + a)^p*x^(-2*p - 7)/(a^3*p^3 + 6*a^3*p^2 + 11*a^3*p + 6*a^3)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-7-2*p)*(b*x**2+a)**p,x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p*x^(-2*p - 7), x)

Mupad [B]
time = 5.09, size = 154, normalized size = 1.47

$$-(b\,x^2+a)^p\,\left(\frac{x\,(p^2+3\,p+2)}{2\,x^{2\,p+7}\,(p^3+6\,p^2+11\,p+6)}+\frac{b^3\,x^7}{a^3\,x^{2\,p+7}\,(p^3+6\,p^2+11\,p+6)}-\frac{b^2\,p\,x^5}{a^2\,x^{2\,p+7}\,(p^3+6\,p^2+11\,p+6)}+\frac{b\,p\,x^3\,(p+1)}{2\,a\,x^{2\,p+7}\,(p^3+6\,p^2+11\,p+6)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^p/x^(2*p + 7),x)

[Out] -(a + b*x^2)^p*((x*(3*p + p^2 + 2))/(2*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)) + (b^3*x^7)/(a^3*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)) - (b^2*p*x^5)/(a^2*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)) + (b*p*x^3*(p + 1))/(2*a*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)))

3.1062 $\int x^{-6-2p}(a+bx^2)^p\,dx$

Optimal. Leaf size=53

$$-\frac{x^{-5-2p}(a+bx^2)^{1+p}\,{}_2F_1\left(-\frac{3}{2},1;\frac{1}{2}(-3-2p);-\frac{bx^2}{a}\right)}{a(5+2p)}$$

```
[Out] -x^(-5-2*p)*(b*x^2+a)^(1+p)*hypergeom([-3/2, 1],[-3/2-p],-b*x^2/a)/a/(5+2*p
)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$-\frac{x^{-2p-5}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-2p-5),-p;\frac{1}{2}(-2p-3);-\frac{bx^2}{a}\right)}{2p+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-6 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-5 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-5 - 2*p)/2, -p, (-3 - 2*p
)/2, -((b*x^2)/a)])/((5 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-6-2p}(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-6-2p}\left(1+\frac{bx^2}{a}\right)^p dx\\ &= -\frac{x^{-5-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-5-2p),-p;\frac{1}{2}(-3-2p);-\frac{bx^2}{a}\right)}{5+2p}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 1.25

$$-\frac{x^{-5-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{5}{2}-p,-p;-\frac{3}{2}-p;-\frac{bx^2}{a}\right)}{5+2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-6 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-5 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-5/2 - p, -p, -3/2 - p, -((
b*x^2)/a)])/((5 + 2*p)*(1 + (b*x^2)/a)^p))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-6-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-6-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-6-2*p)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-6-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{x^{2\,p+6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 6),x)
```

```
[Out] int((a + b*x^2)^p/x^(2*p + 6), x)
```

3.1063 $\int x^{-5-2p}(a+bx^2)^p \, dx$

Optimal. Leaf size=67

$$\frac{bx^{-2(1+p)}(a+bx^2)^{1+p}}{2a^2(1+p)(2+p)}-\frac{x^{-2(2+p)}(a+bx^2)^{1+p}}{2a(2+p)}$$

```
[Out] 1/2*b*(b*x^2+a)^(1+p)/a^2/(1+p)/(2+p)/(x^(2+2*p))-1/2*(b*x^2+a)^(1+p)/a/(2+
p)/(x^(4+2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {277, 270}

$$\frac{bx^{-2(p+1)}(a+bx^2)^{p+1}}{2a^2(p+1)(p+2)}-\frac{x^{-2(p+2)}(a+bx^2)^{p+1}}{2a(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - (a + b*x^2)
^(1 + p)/(2*a*(2 + p)*x^(2*(2 + p)))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{-5-2p}(a+bx^2)^p \, dx &= -\frac{x^{-2(2+p)}(a+bx^2)^{1+p}}{2a(2+p)}-\frac{b\int x^{-3-2p}(a+bx^2)^p \, dx}{a(2+p)}\\ &= \frac{bx^{-2(1+p)}(a+bx^2)^{1+p}}{2a^2(1+p)(2+p)}-\frac{x^{-2(2+p)}(a+bx^2)^{1+p}}{2a(2+p)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 62, normalized size = 0.93

$$-\frac{x^{-2(2+p)}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-2-p,-p;-1-p;-\frac{bx^2}{a}\right)}{2(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*((a + b*x^2)^p*Hypergeometric2F1[-2 - p, -p, -1 - p, -((b*x^2)/a)])/((
2 + p)*x^(2*(2 + p))*(1 + (b*x^2)/a)^p)
```

Maple [A]
time = 0.07, size = 45, normalized size = 0.67

method	result	size
gosper	$-\frac{(b\,x^2+a)^{1+p}x^{-4-2p}(-b\,x^2+ap+a)}{2(2+p)(1+p)a^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-5-2*p)*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)^(1+p)*x^(-4-2*p)*(-b*x^2+a*p+a)/(2+p)/(1+p)/a^2
```

Maxima [A]
time = 0.29, size = 59, normalized size = 0.88

$$\frac{(b^2x^4-abpx^2-a^2(p+1))e^{(p\log(bx^2+a)-2\,p\log(x))}}{2\,(p^2+3\,p+2)a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*x^4 - a*b*p*x^2 - a^2*(p + 1))*e^(p*log(b*x^2 + a) - 2*p*log(x))/(
(p^2 + 3*p + 2)*a^2*x^4)
```

Fricas [A]
time = 0.88, size = 67, normalized size = 1.00

$$\frac{(b^2x^5-abpx^3-(a^2p+a^2)x)(bx^2+a)^px^{-2\,p-5}}{2\,(a^2p^2+3\,a^2p+2\,a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

[Out] 1/2*(b^2*x^5 - a*b*p*x^3 - (a^2*p + a^2)*x)*(b*x^2 + a)^p*x^(-2*p - 5)/(a^2*p^2 + 3*a^2*p + 2*a^2)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-5-2*p)*(b*x**2+a)**p,x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p*x^(-2*p - 5), x)

Mupad [B]
time = 5.02, size = 96, normalized size = 1.43

$$-\left(b\,x^2+a\right)^p\,\left(\frac{x\,\left(p+1\right)}{2\,x^{2\,p+5}\,\left(p^2+3\,p+2\right)}-\frac{b^2\,x^5}{2\,a^2\,x^{2\,p+5}\,\left(p^2+3\,p+2\right)}+\frac{b\,p\,x^3}{2\,a\,x^{2\,p+5}\,\left(p^2+3\,p+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^p/x^(2*p + 5),x)

[Out] -(a + b*x^2)^p*((x*(p + 1))/(2*x^(2*p + 5)*(3*p + p^2 + 2)) - (b^2*x^5)/(2*a^2*x^(2*p + 5)*(3*p + p^2 + 2)) + (b*p*x^3)/(2*a*x^(2*p + 5)*(3*p + p^2 + 2)))

3.1064 $\int x^{-4-2p}(a+bx^2)^p\,dx$

Optimal. Leaf size=53

$$-\frac{x^{-3-2p}(a+bx^2)^{1+p}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2}(-1-2p);-\frac{bx^2}{a}\right)}{a(3+2p)}$$

```
[Out] -x^(-3-2*p)*(b*x^2+a)^(1+p)*hypergeom([-1/2, 1],[-1/2-p],-b*x^2/a)/a/(3+2*p
)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$-\frac{x^{-2p-3}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-2p-3),-p;\frac{1}{2}(-2p-1);-\frac{bx^2}{a}\right)}{2p+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-3 - 2*p)/2, -p, (-1 - 2*p
)/2, -((b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-4-2p}(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-4-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= -\frac{x^{-3-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-3-2p),-p;\frac{1}{2}(-1-2p);-\frac{bx^2}{a}\right)}{3+2p}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 1.25

$$-\frac{x^{-3-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{3}{2}-p,-p;-\frac{1}{2}-p;-\frac{bx^2}{a}\right)}{3+2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-3/2 - p, -p, -1/2 - p, -((
b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a)^p))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-4-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-4-2*p)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{x^{2\,p+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 4),x)
```

```
[Out] int((a + b*x^2)^p/x^(2*p + 4), x)
```

3.1065 $\int x^{-3-2p}(a+bx^2)^p\,dx$

Optimal. Leaf size=30

$$-\frac{x^{-2(1+p)}(a+bx^2)^{1+p}}{2a(1+p)}$$

```
[Out] -1/2*(b*x^2+a)^(1+p)/a/(1+p)/(x^(2+2*p))
```

Rubi [A]

time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {270}

$$-\frac{x^{-2(p+1)}(a+bx^2)^{p+1}}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 2*p)*(a + b*x^2)^p,x]
[Out] -1/2*(a + b*x^2)^(1 + p)/(a*(1 + p)*x^(2*(1 + p)))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-3-2p}(a+bx^2)^p\,dx = -\frac{x^{-2(1+p)}(a+bx^2)^{1+p}}{2a(1+p)}$$

Mathematica [A]

time = 0.03, size = 29, normalized size = 0.97

$$\frac{x^{-2-2p}(a+bx^2)^{1+p}}{a(-2-2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 2*p)*(a + b*x^2)^p,x]
[Out] (x^(-2 - 2*p)*(a + b*x^2)^(1 + p))/(a*(-2 - 2*p))
```

Maple [A]
time = 0.07, size = 29, normalized size = 0.97

method	result	size
gosper	$-\frac{x^{-2-2p}\left(b\,x^{2}+a\right)^{1+p}}{2a(1+p)}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-2*p)*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
[Out] -1/2*x^(-2-2*p)*(b*x^2+a)^(1+p)/a/(1+p)
```

Maxima [A]
time = 0.32, size = 37, normalized size = 1.23

$$-\frac{(bx^2+a)e^{\left(p\log\left(bx^2+a\right)-2\,p\log\left(x\right)\right)}}{2\,a(p+1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] -1/2*(b*x^2 + a)*e^(p*log(b*x^2 + a) - 2*p*log(x))/(a*(p + 1)*x^2)
```

Fricas [A]
time = 1.06, size = 34, normalized size = 1.13

$$-\frac{(bx^3+ax)(bx^2+a)^p x^{-2\,p-3}}{2\,(ap+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] -1/2*(b*x^3 + a*x)*(b*x^2 + a)^p*x^(-2*p - 3)/(a*p + a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-2*p)*(b*x**2+a)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 3), x)
```

Mupad [B]
time = 5.05, size = 52, normalized size = 1.73

$$-\left(b\,x^2+a\right)^p\,\left(\frac{x}{2\,x^{2\,p+3}\,\left(p+1\right)}+\frac{b\,x^3}{2\,a\,x^{2\,p+3}\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 3),x)
```

```
[Out] -(a + b*x^2)^p*(x/(2*x^(2*p + 3)*(p + 1)) + (b*x^3)/(2*a*x^(2*p + 3)*(p + 1
)))
```

3.1066 $\int x^{-2-2p}(a+bx^2)^p\,dx$

Optimal. Leaf size=53

$$-\frac{x^{-1-2p}(a+bx^2)^{1+p}\,{}_2F_1\left(\frac{1}{2},1;\frac{1}{2}(1-2p);-\frac{bx^2}{a}\right)}{a(1+2p)}$$

```
[Out] -x^(-1-2*p)*(b*x^2+a)^(1+p)*hypergeom([1/2, 1],[1/2-p],-b*x^2/a)/a/(1+2*p)
```

Rubi [A]

time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {372, 371}

$$-\frac{x^{-2p-1}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-2p-1),-p;\frac{1}{2}(1-2p);-\frac{bx^2}{a}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-1 - 2*p)/2, -p, (1 - 2*p)
/2, -((b*x^2)/a)])/((1 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-2-2p}(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-2-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= -\frac{x^{-1-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-1-2p),-p;\frac{1}{2}(1-2p);-\frac{bx^2}{a}\right)}{1+2p}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 1.25

$$-\frac{x^{-1-2p}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2}-p,-p;\frac{1}{2}-p;-\frac{bx^2}{a}\right)}{1+2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-1/2 - p, -p, 1/2 - p, -((b
*x^2)/a)])/((1 + 2*p)*(1 + (b*x^2)/a)^p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-2-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-2-2*p)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{2\,p+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 2),x)
```

```
[Out] int((a + b*x^2)^p/x^(2*p + 2), x)
```

3.1067 $\int x^{-1-2p}(a+bx^2)^p \, dx$

Optimal. Leaf size=43

$$-\frac{x^{-2p}(a+bx^2)^{1+p} \, {}_2F_1\left(1,1;1-p;-\frac{bx^2}{a}\right)}{2ap}$$

```
[Out] -1/2*(b*x^2+a)^(1+p)*hypergeom([1, 1],[1-p],-b*x^2/a)/a/p/(x^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$-\frac{x^{-2p}(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-p,-p;1-p;-\frac{bx^2}{a}\right)}{2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*((a + b*x^2)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^2)/a)])/(p*x^(2
*p)*(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-2p}(a+bx^2)^p \, dx &= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^{-1-2p}\left(1+\frac{bx^2}{a}\right)^p dx \\ &= -\frac{x^{-2p}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-p,-p;1-p;-\frac{bx^2}{a}\right)}{2p}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 1.30

$$-\frac{x^{-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{bx^2}{a}\right)}{2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*((a + b*x^2)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^2)/a)])/(p*x^(2
*p)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{-1-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*p)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-1-2*p)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 133.85, size = 37, normalized size = 0.86

$$\frac{a^p x^{-2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*p)*(b*x**2+a)**p,x)
```

```
[Out] a**p*gamma(-p)*hyper((-p, -p), (1 - p,), b*x**2*exp_polar(I*pi)/a)/(2*x**(2
*p)*gamma(1 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{x^{2\,p+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 1),x)
```

```
[Out] int((a + b*x^2)^p/x^(2*p + 1), x)
```

3.1068 $\int x^{-2p}(a+bx^2)^p\,dx$

Optimal. Leaf size=52

$$\frac{x^{1-2p}(a+bx^2)^{1+p}\ {}_2F_1\left(1,\frac{3}{2};\frac{1}{2}(3-2p);-\frac{bx^2}{a}\right)}{a(1-2p)}$$

```
[Out] x^(1-2*p)*(b*x^2+a)^(1+p)*hypergeom([1, 3/2],[3/2-p],-b*x^2/a)/a/(1-2*p)
```

Rubi [A]

time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {372, 371}

$$\frac{x^{1-2p}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(1-2p),-p;\frac{1}{2}(3-2p);-\frac{bx^2}{a}\right)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(2*p),x]
```

```
[Out] (x^(1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(1 - 2*p)/2, -p, (3 - 2*p)/2,
-((b*x^2)/a)])/((1 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-2p}(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{1-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(1-2p),-p;\frac{1}{2}(3-2p);-\frac{bx^2}{a}\right)}{1-2p}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 1.25

$$\frac{x^{1-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}-p,-p;\frac{3}{2}-p;-\frac{bx^2}{a}\right)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(2*p),x]
[Out] (x^(1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1/2 - p, -p, 3/2 - p, -((b*x^2
)/a)])/((1 - 2*p)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^p x^{-2p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(x^(2*p)),x)
[Out] int((b*x^2+a)^p/(x^(2*p)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/x^(2*p), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/x^(2*p), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 6.95, size = 24, normalized size = 0.46

$$b^p x\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(x**(2*p)),x)
```

```
[Out] b**p*x*hyper((-1/2, -p), (1/2,), a*exp_polar(I*pi)/(b*x**2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(2*p), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{x^{2\,p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p),x)
```

```
[Out] int((a + b*x^2)^p/x^(2*p), x)
```

3.1069 $\int x^{1-2p}(a+bx^2)^p \, dx$

Optimal. Leaf size=49

$$\frac{x^{2-2p}(a+bx^2)^{1+p} \, {}_2F_1\left(1,2;2-p;-\frac{bx^2}{a}\right)}{2a(1-p)}$$

```
[Out] 1/2*x^(2-2*p)*(b*x^2+a)^(1+p)*hypergeom([1, 2],[2-p],-b*x^2/a)/a/(1-p)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$\frac{x^{2-2p}(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(1-p,-p;2-p;-\frac{bx^2}{a}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(2 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^2)/a)
])/(2*(1 - p)*(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{1-2p}(a+bx^2)^p \, dx &= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^{1-2p}\left(1+\frac{bx^2}{a}\right)^p \, dx \\ &= \frac{x^{2-2p}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(1-p,-p;2-p;-\frac{bx^2}{a}\right)}{2(1-p)}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 61, normalized size = 1.24

$$\frac{x^{2-2p}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(1-p,-p;2-p;-\frac{bx^2}{a}\right)}{2-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 - 2*p)*(a + b*x^2)^p,x]
[Out] (x^(2 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^2)/a)
])/((2 - 2*p)*(1 + (b*x^2)/a)^p)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{1-2p} \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1-2*p)*(b*x^2+a)^p,x)
[Out] int(x^(1-2*p)*(b*x^2+a)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p + 1), x)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 111.87, size = 41, normalized size = 0.84

$$\frac{a^p x^2 x^{-2p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma(2-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1-2*p)*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**2*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), b*x**2*exp_polar(I*pi)/
a)/(2*x**(2*p)*gamma(2 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{1-2\,p}\,{\left(b\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1 - 2*p)*(a + b*x^2)^p,x)
```

```
[Out] int(x^(1 - 2*p)*(a + b*x^2)^p, x)
```

3.1070 $\int x^{2-2p}(a+bx^2)^p\,dx$

Optimal. Leaf size=52

$$\frac{x^{3-2p}(a+bx^2)^{1+p}\,{}_2F_1\left(1,\frac{5}{2};\frac{1}{2}(5-2p);-\frac{bx^2}{a}\right)}{a(3-2p)}$$

```
[Out] x^(3-2*p)*(b*x^2+a)^(1+p)*hypergeom([1, 5/2],[5/2-p],-b*x^2/a)/a/(3-2*p)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {372, 371}

$$\frac{x^{3-2p}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(3-2p),-p;\frac{1}{2}(5-2p);-\frac{bx^2}{a}\right)}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(3 - 2*p)/2, -p, (5 - 2*p)/2,
-((b*x^2)/a)])/((3 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2-2p}(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{2-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{3-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(3-2p),-p;\frac{1}{2}(5-2p);-\frac{bx^2}{a}\right)}{3-2p}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 1.25

$$\frac{x^{3-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2}-p,-p;\frac{5}{2}-p;-\frac{bx^2}{a}\right)}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 - 2*p)*(a + b*x^2)^p,x]
[Out] (x^(3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[3/2 - p, -p, 5/2 - p, -((b*x^2
)/a)])/((3 - 2*p)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{2-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2-2*p)*(b*x^2+a)^p,x)
[Out] int(x^(2-2*p)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 125.93, size = 48, normalized size = 0.92

$$\frac{a^p x^3 x^{-2p}\Gamma\left(\frac{3}{2}-p\right){}_2F_1\left(\left.\begin{matrix}-p,\frac{3}{2}-p\\\frac{5}{2}-p\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{2}-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2-2*p)*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**3*gamma(3/2 - p)*hyper((-p, 3/2 - p), (5/2 - p,), b*x**2*exp_polar(
I*pi)/a)/(2*x**(2*p)*gamma(5/2 - p))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{2-2\,p}\,{\left(b\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2 - 2*p)*(a + b*x^2)^p,x)
```

```
[Out] int(x^(2 - 2*p)*(a + b*x^2)^p, x)
```

3.1071 $\int x^{3-2p}(a+bx^2)^p\,dx$

Optimal. Leaf size=49

$$\frac{x^{4-2p}(a+bx^2)^{1+p}\,{}_2F_1\left(1,3;3-p;-\frac{bx^2}{a}\right)}{2a(2-p)}$$

```
[Out] 1/2*x^(4-2*p)*(b*x^2+a)^(1+p)*hypergeom([1, 3],[3-p],-b*x^2/a)/a/(2-p)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$\frac{x^{4-2p}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(2-p,-p;3-p;-\frac{bx^2}{a}\right)}{2(2-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(4 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^2)/a)
])/(2*(2 - p)*(1 + (b*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3-2p}(a+bx^2)^p\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{3-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{4-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(2-p,-p;3-p;-\frac{bx^2}{a}\right)}{2(2-p)}\end{aligned}$$

<u>**Mathematica**</u> [**A**]
time = 0.03, size = 61, normalized size = 1.24

$$\frac{x^{4-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;-\frac{bx^2}{a}\right)}{4-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 - 2*p)*(a + b*x^2)^p,x]
[Out] (x^(4 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^2)/a)
])/((4 - 2*p)*(1 + (b*x^2)/a)^p)
```

<u>**Maple**</u> [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{3-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3-2*p)*(b*x^2+a)^p,x)
[Out] int(x^(3-2*p)*(b*x^2+a)^p,x)
```

<u>**Maxima**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 3), x)
```

<u>**Fricas**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p + 3), x)
```

<u>**Sympy**</u> [**C**] Result contains complex when optimal does not.
time = 168.19, size = 41, normalized size = 0.84

$$\frac{a^p x^4 x^{-2p}\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}-p,2-p\\3-p\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(3-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3-2*p)*(b*x**2+a)**p,x)
```

```
[Out] a**p*x**4*gamma(2 - p)*hyper((-p, 2 - p), (3 - p,), b*x**2*exp_polar(I*pi)/
a)/(2*x**(2*p)*gamma(3 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{3-2\,p}\,{\left(b\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3 - 2*p)*(a + b*x^2)^p,x)
```

```
[Out] int(x^(3 - 2*p)*(a + b*x^2)^p, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section 4386
4.2 Listing of Grading functions 4386

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```